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Content Check 1 Diff. Eq.

1. Verify that the family of functions $y=c_1x^{-1}+c_2x+c_3x\ln x+4x^2$ is a solution of the differential equation $x^3y'''+2x^2y''-xy'+y=12x^2$.

$$y = c_1x^{-1} + c_2x + c_3x\ln(x) + 4x^2 \quad ; \text{ up to } 3$$

$$\begin{aligned} y' &= -c_1x^{-2} + c_2 + \left[c_3x\cdot\frac{1}{x} + c_3\cdot\ln(x)\right] + 8x \\ &= -c_1x^{-2} + c_2 + \left[c_3 + c_3\ln(x)\right] + 8x \\ &= -c_1x^{-2} + c_2 + c_3 + c_3\ln(x) + 8x \end{aligned}$$

$$\begin{aligned} y'' &= 2c_1x^{-3} + 0 + 0 + \left[c_3\cdot\frac{1}{x}\right] + 8 \\ &= 2c_1x^{-3} + c_3x^{-1} + 8 \end{aligned}$$

$$\begin{aligned} y''' &= -6c_1x^{-4} + (-c_3x^{-2}) + 0 \\ &= -6c_1x^{-4} - c_3x^{-2} \end{aligned}$$

$$x^3y''' + 2x^2y'' - xy' + y = 12x^2$$

$$x^3\left(\frac{-6c_1}{x^4} - \frac{c_3}{x^2}\right) + 2x^2\left(\frac{2c_1}{x^3} + \frac{c_3}{x} + 8\right) - x\left(-\frac{c_1}{x} + c_2 + c_3 + c_3\ln(x) + 8x\right) + \left(\frac{c_1}{x} + c_2x + c_3x\ln(x) + 4x^2\right)$$

$$= -\frac{6c_1}{x} - c_3x + 2x^2\left(\frac{2c_1}{x^3} + \frac{c_3}{x} + 8\right) - x\left(-\frac{c_1}{x^2} + c_2 + c_3 + c_3\ln x + 8x\right) + \frac{c_1}{x} + c_2x + c_3x\ln(x) + 4x^2$$

$$= \frac{-6c_1}{x} - c_3x + \frac{4c_1}{x} + 2c_3x + 16x^2 + \frac{c_1}{x} - c_2x - c_3x - c_3x\ln(x) - 8x^2 + \frac{c_1}{x} + c_2x + c_3x\ln(x) + 4x^2$$

$$= \boxed{12x^2 = 12x^2 \checkmark}$$

$\therefore$ since $y$ satisfies the DE, it is a solution

2. Find values of $m$ so that the function $y = x^m$ is a solution of the differential equation $x^2y'' - 7xy' + 15y = 0$.

$y = x^m$ ; up to 2

$y' = m x^{(m-1)}$

$y'' = m \cdot (m-1) \cdot x^{((m-1)-1)}$
$= (m^2 - m) \cdot x^{m-2}$
$= m^2 x^{m-2} - m x^{m-2}$

$x^2 y'' - 7xy' + 15y = 0$

$x^2(m^2 x^{m-2} - m x^{m-2}) - 7x(m x^{m-1}) + 15(x^m) = 0$
$x^2 m^2 x^{m-2} - x^2 m x^{m-2} - 7xmx^{m-1} + 15x^m = 0$
$m^2 x^m - m x^m - 7m x^m + 15 x^m = 0$
$x^m(m^2 - m - 7m + 15) = 0$
$x^m(m^2 - 8m + 15) = 0$
$x^m((m-5)(m-3)) = 0$

$x^m \neq 0$ ✗ $\quad m - 5 = 0 \quad m - 3 = 0$
no sol. $\quad m = 5 \quad m = 3$

$\boxed{\text{Values of } m: \quad m = 3;\ m = 5}$