Homework 4.

1) Which of the following maps $\phi : \mathbb{C} \to \mathbb{C}$ are ring homomorphisms

- $\phi(x + yi) = x$
- $\phi(x + yi) = y$
- $\phi(x + yi) = x - yi$
- $\phi(x + yi) = |x + yi|$
- $\phi(x + yi) = y + xi$

2) Prove that the intersection of two ideals of a ring R is an ideal.

3) Prove that

$$\mathbb{Z}[i] = \Big\{ m + ni, \mid m, \, n \in \mathbb{Z} \Big\}$$

is a commutative ring with identity.

4) Prove that the subset

$$I = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, \quad a, \, b \in \mathbb{Z} \right\} \subset M_2(\mathbb{Z})$$

is a left ideal of the ring $M_2(\mathbb{Z})$ but not a right-ideal.

5) Prove that in any ring R the units form a multiplicative group.

6) If $I = 4\mathbb{Z}$ is an ideal of the ring $R = \mathbb{Z}$ find the quotient ring R/I.

7) Prove that the map $\phi : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_4$ defined by

$$n \mapsto \phi(n) = n \pmod 4$$

is a ring homomorphism and find its kernel.

Is the map $\psi : \mathbb{Z}_{14} \longrightarrow \mathbb{Z}_4$ given by $\psi(n) = n \pmod 4$ a ring homomorphism?

8) If R is a non commutative ring with identity and for a and b in R, the element $1 - ab$ is a unit, so is $1 - ba$.

9) For an ideal I of a non commutative ring R, the quotient ring R/I is commutative if and only if $ab - ba \in I$ for a, b in R.