CSE 311 - HW 1
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1

1(a)

$$
\begin{aligned}
f &: \text{The stadium is full.}\\
c &: \text{The crowd is subdued.}\\
w &: \text{We will win the game.}\\
&: (f \wedge \neg c) \to w
\end{aligned}
$$

1(b)

$$
\begin{aligned}
w &: \text{The watch is more than \$25.}\\
s &: \text{The watch is on sale.}\\
b &: \text{I will buy the watch.}\\
i) &: b \to \neg w\\
ii) &: \neg b \to \neg s\\
iii) &: b \to (s \wedge \neg w)
\end{aligned}
$$

1(c)

$$
\begin{aligned}
e &: \text{The stack is empty.}\\
f &: \text{The stack is full.}\\
u &: \text{You can push onto the stack.}\\
o &: \text{You can pop from the stack.}\\
i) &: e \to (u \wedge \neg o)\\
ii) &: (\neg u \wedge o) \to f\\
iii) &: (\neg u \vee \neg o) \to (e \vee f)
\end{aligned}
$$

2

We begin by writing a truth table for the possible combinations of p, q, r, and F(p,q,r). From that we see that there are four rows that produce the outcome we're looking for: 1, 2, 3, and 5. Grouping the states of p, q, and r in each of those 4 rows with ”and” and then joining each of the groups with ”or” yields a function that is true when a majority of the inputs are true.

p	q	r	F(p,q,r)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \rightarrow F(p, q, r)$$

3

From the following truth table we see that the desired state of the circuit B(p,q,r) can be made to match a compound proposition, namely, it's last column. We then translate that into a circuit diagram by replacing logical symbols with circuit components.

p	q	r	$\neg p$	$\neg q$	$\neg r$	$B(p, q, r)$	$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	T	F	F

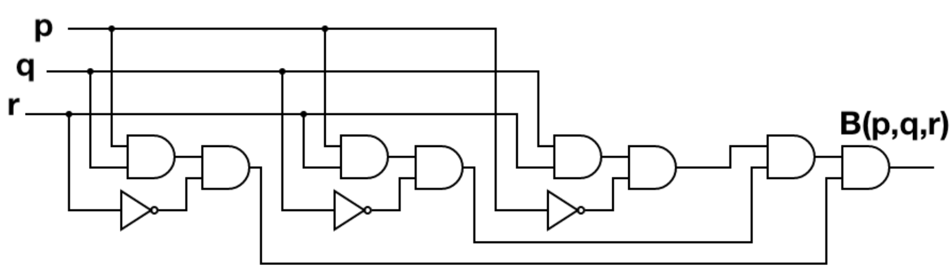

Figure 1: Circuit Diagram of B(p,q,r)

4

4(a)

p	$\neg p$	$A(F,p) = F \vee \neg p$
T	F	F
T	F	F
F	T	T
F	T	T

4(b)

p	q	$(p \vee q)$	$A_1(F,q) = F \vee \neg q$	$A_2(p, A_1) = p \vee \neg A_1$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	F

4(c)

p	q	$\neg(p \wedge q)$	$A_1(F,p) = F \vee \neg p$	$A_2(A_1, q) = A_1 \vee \neg q$
T	T	F	F	F
T	F	T	F	T
F	T	T	T	T
F	F	T	T	T

4(d)

p	q	$(p \wedge q)$	$A_1(F,p) = F \vee \neg p$	$A_2(A_1, q) = A_1 \vee \neg q$	$A_3(F, A_2) = F \vee \neg A_2$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	T	F
F	F	F	T	T	F

5

5(a)

By the last two columns of the truth table we see that the last three rows fail to agree.

p	q	$(p \wedge q)$	$p \vee (p \wedge q)$	$q \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	F	T
F	F	F	F	F

5(b)

By the last two columns of the truth table we see that none of the rows agree.

p	q	$\neg p$	$\neg q$	$(p \oplus q)$	$\neg(p \oplus q)$	$\neg p \oplus \neg q$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	F

5(c)

By the last two rows of the truth table we see that rows six and eight fail to agree.

p	q	r	$(q \rightarrow r)$	$(p \rightarrow q)$	$p \rightarrow (q \rightarrow r)$	$(p \rightarrow q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

5(d)

By the last two rows of the truth table we see that the second and third rows fail to agree.

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$	$(q \rightarrow p) \rightarrow (p \rightarrow q)$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

6

6(a)

We begin by defining the identities of the two people and whether the statement is true. The speaker can either be a student or a TA and the other person can also be either a student or a TA and the truth of the statement is dependent on the speaker's identity. (s will always have the same truth value as p but is retained for clarity.)

$$p : \text{The speaker is a student.}$$
$$q : \text{The other person is a student.}$$
$$s : \text{At least one of them is a TA.}$$

There are four possible cases of the identites of the people: neither person is a student, the speaker is a student and the other person isn't, the speaker isn't a student but the other person is, and both speaker and other person are students. Or, more formally:

$$C1 : \neg p \wedge \neg q$$
$$C2 : p \wedge \neg q$$
$$C3 : \neg p \wedge q$$
$$C4 : p \wedge q$$

Let us now consider which, if any, of these cases are possible.

p	q	s	$s \wedge [(\neg p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)]$	$\neg s \wedge (p \wedge q)$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	F
F	F	F	F	F

Let us consider the two possible cases of the speaker's identity.

If the speaker is a student they are telling the truth and at least one of them is a TA. We already know that the speaker is a student so the only possible scenario is that the speaker is a student and the other person is a TA.

If the speaker is a TA they are lying and neither is a TA, but that would be a contradiction. Therefore the only possibility is that the speaker is a student and the other person is a TA.

6(b)

As above, we begin by defining propositions.

p : The speaker is a student.

q : The second person is a student.

r : The third person is a student.

s : Every TA in the circle has a TA to their immediate right.

For the spoken statement to be true, there are four possible states: the speaker is a student and the other people in the circle are also students (The statement is not false as it doesn't state that there *are* TAs in the circle.) or the speaker is a TA, is lying, and there are no other TAs, or one of the other people is a TA. Notably, the state where all the people are TAs can't be true because then the statement would be true. (Truth table columns truncated for space.)

p	q	r	s	$(p \wedge q \wedge r \wedge s) \vee (\neg p \wedge q \wedge r \wedge \neg s) \vee (\neg p \wedge q \wedge \neg r \wedge \neg s) \vee (\neg p \wedge \wedge q \wedge r \wedge \neg s)$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	F
T	F	F	T	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	F

7

We begin by XNORing R_1 and R_2 and store the result in R_2. Then, we XNOR R_1 and R_2 again and store the result in R_1. Finally, we XNOR R_1 and R_2 again and store the result in R_2. This has the effect of swapping the values as show by match between the last two rows of the step table and truth table.

Steps	R_1	R_2
0	p	q
1	p	$p\bar{\oplus}q$
2	$(p\bar{\oplus}q)\bar{\oplus}p$	$p\bar{\oplus}q$
3	$(p\bar{\oplus}q)\bar{\oplus}p$	$((p\bar{\oplus}q)\bar{\oplus}p)\bar{\oplus}(p \oplus q)$

p	q	$p\bar{\oplus}q$	$(p\bar{\oplus}q)\bar{\oplus}p$	$((p\bar{\oplus}q)\bar{\oplus}p)\bar{\oplus}(p \oplus q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	F	T	F