CSE 312: Foundations of Computing II
Assignment #3
January 21, 2020
due: Wednesday, January 29, 2020, 10:00 p.m.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator, and only then use your calculator, paying attention to the exercise's instructions for the form of your final answer. If it asks for a simplified fraction, this means an integer or a fraction a/b, where $\gcd(a, b) = 1$. If it asks for an answer to (say) 3 significant digits, this means that, when written in scientific notation, your answer would look like $a.bc \times 10^d$, where a, b, c are digits and d is an integer. Highlight or box your final answer to each exercise part.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable. Popular choices for typesetting mathematics are Microsoft Word (be sure to use its Equation Editor and save as PDF) and LaTeX.

1. You are playing a game of Schnapsen with the Maestro and find yourself in the following position:

Your cards	**Concealed**	**Trump:** ♢J
♠ K	♠ TQJ	**Stock:** 3 face-down cards
♡ AT	♡ J	**Trick points:** You 7, Maestro 29
♣ —	♣ AQJ	**On lead:** You
♢ AT	♢ Q	

The 8 cards listed under "Concealed" are all the cards you have not seen yet: the 5 held by the Maestro plus the 3 still face-down in the stock. Assume that you have no further information about which of these 8 cards are in the stock. The face-up trump showing on the table is ♢J, you have already accumulated 7 trick points in your tricks, and you are on lead. With this powerful hand, you decide to close the stock before leading. You then play your cards in the order ♢A, ♢T, ♡A, ♡T. At this point, if you do not yet have enough trick points to claim 66, you play your last card, ♠K. Your chief concern is whether ♠T is still in the stock: if it is, your ♠K is a winner. The steps below will lead you ultimately to the probability that your play succeeds, whether or not ♠T is in the stock. Let W be the event that you accumulate at least 66 points in your tricks, and let S be the event that ♠T is still in the stock at the moment when you close the stock. Give all probabilities exactly as simplified fractions.

(a) Compute $\mathrm{P}(S)$.

(b) Suppose ♠T is still in the stock. In this case, explain why $t = 13$ is the minimum number of trick points the Maestro must have in his hand in order for you to win the deal. Remember that you are starting with 7 trick points in the diagrammed position.

(c) Compute $\mathrm{P}(W \mid S)$.

(d) Suppose the Maestro holds ♠T in his hand. We will assume that he plays well enough to hold on to it until you lead ♠K. In this case, explain why $t' = 17$ is the minimum number of trick points the Maestro must be holding in his other 4 cards combined in order for you to win the deal.

(e) Compute $\mathrm{P}(W \mid \overline{S})$.

(f) Finally, use your answers from parts (a), (c), and (e) to compute the probability $\mathrm{P}(W)$ that your play succeeds.

2. A girl with a taste for intricate schemes has 3 lavender and 2 red marbles in her left pocket, and 1 lavender and 4 red marbles in her right pocket. She chooses one of her pockets (called the "chosen pocket") by drawing a randomly chosen marble D from her left pocket: if D is Lavender the Left pocket is the chosen pocket, and if D is Red the Right pocket is the chosen pocket. She returns D to her left pocket. She then draws a randomly chosen marble M_1 from the chosen pocket, notes its color, returns it to the same pocket, and then again draws a randomly chosen marble M_2 from the same pocket. Let L_1 be the event that M_1 is lavender and L_2 be the event that M_2 is lavender. Give exact answers as simplified fractions.

   (a) Calculate $\mathrm{P}(L_1)$ and $\mathrm{P}(L_2)$.

   (b) Calculate $\mathrm{P}(L_2 \mid L_1)$. (Hint: you cannot assume without proof that L_1 and L_2 are independent.)

   (c) Are L_1 and L_2 are independent? Justify your answer.

   (d) If both M_1 and M_2 are lavender, what is the probability that D was lavender?

3. A pharmaceutical company proudly publishes results from a trial of its new test for a certain genetic disorder. The false negative rate is small: the test returns a negative result for only 4% of patients with the disorder. The false positive rate is also small: the test returns a positive result for only 12% of participants that do not have the disorder. Assume that 0.5% (that is, the fraction 0.005) of the population has the

disorder. Let's see how good a test this will be and what a test result would mean to you as a patient. Calculate your answers to 2 significant digits.

(a) What is the probability of having the disorder if you have a negative test result? (Seeing your answer, how reassured should you be if you were the one that had a negative test result?)

(b) What is the probability of having the disorder if you have a positive test result? (Seeing your answer, how anxious should you be if you were the one that had a positive test result?)

(c) Repeat part (a) assuming that 15% of the population has the disorder.

(d) Repeat part (b) assuming that 15% of the population has the disorder.

4. In some games, such as tennis and ping pong, you can reach an "overtime" state, where the score is tied and the first player who then gets two points ahead of the other player wins the game. Suppose you are at such an overtime tied state. Suppose the probability that you win a single point is p and this is true independently for all points. As a function of p, what is the probability that you win the game? You are required to solve this using the following method: from a given score, condition on the outcome of the next point or points, similar to the Gambler's Ruin solution from lecture. Simplify your final answer as much as possible. Then, evaluate it for $p = 3/4$, giving an exact answer as a simplified fraction.

5. As you may remember from basic biology, the human A/B/O blood type system is controlled by one gene for which 3 variants ("alleles") are common in the human population – unsurprisingly called A, B, and O. As with most genes, everyone has 2 copies of this gene, one inherited from the mother and the other from the father, and everyone passes a randomly selected copy to each of their children (probability 1/2 for each copy, independently for each child). Focusing only on A and O, people with AA or AO gene pairs have type A blood; those with OO have type O blood. (A is "dominant", O is "recessive".) Suppose Apple and both of her parents have type A blood, but her sister Olive has type O. Give exact answers as simplified fractions.

(a) What is the probability that Apple carries an O gene, given that she has blood type A?

(b) Apple marries Oscar, who has type O blood. What is the probability that their first child will have type O blood?

(c) If Apple and Oscar's first child had type A blood, what is the probability that Apple carries an O gene?

(d) If Apple and Oscar's first child had type A blood, what is the probability that their second child will as well?

(e) Are the blood types of Apple and Oscar's first two children independent? Justify your answer.

6. Recall from lecture that the probability of being dealt no trumps in your initial Schnapsen hand is 0.258. Suppose you play a game of Schnapsen that consists of 8 deals, with the cards well shuffled before each deal. Calculate your answers to 3 significant digits.

   (a) What is the probability that you were dealt no trumps in exactly 2 of the 8 initial Schnapsen hands?

   (b) What is the probability that you were dealt no trumps in 2 or more of the 8 initial Schnapsen hands?

7. In the communications networks diagrammed below, suppose that the communication link labeled $1 \le i \le 5$ is working with probability p_i, and fails with probability $1 - p_i$, independently of the others. What is the probability that A can communicate with B in each case, that is, the probability that there is at least one path from A to B all of whose links are working? Give your answer as a function of p_1, p_2, p_3, p_4, p_5, and simplify your formula as much as possible. Then, for each part, evaluate your formula at the following values: $p_1 = 0.75, p_2 = 0.9, p_3 = 0.7, p_4 = 0.95, p_5 = 0.85$. Give your answers to 8 significant digits. (Hint: for network (b), condition on the status of link 3. When link 3 is working, think carefully about subnetworks that are in series and in parallel.)