1. Answer: $\boxed{E[X] \approx 1.41,\ Var(x) = 2.92}$

Explanation:

Because we're given the probabilities and outcomes of x_i, we can plug these into the formula for the expected value of X and solve.

$$E[X] = -3(0.026) - 2(0.138) + 1(0.155) + 2(0.432) + 3(0.249)$$
$$= -\frac{39}{500} - \frac{69}{250} + \frac{31}{200} + \frac{108}{125} + \frac{747}{1000}$$
$$= \frac{353}{250}$$
$$\approx 1.412$$

To find the variance, we'll want to know $E[X^2]$ which we find first by squaring the values X_i can have and summing their products as normal. Then, we plug in that result to the variance formula and solve.

$$E[X^2] = -3^2(0.026) - 2^2(0.138) + 1^2(0.155) + 2^2(0.432) + 3^2(0.249)$$
$$= 9(0.026) + 4(0.138) + 1(0.155) + 4(0.432) + 9(0.249)$$
$$= \frac{491}{100}$$

$$Var(x) = E[(X-\mu)^2] = E[X^2] - (E[X])^2$$
$$= \frac{491}{100} - \left(\frac{353}{250}\right)^2$$
$$\approx 2.916256$$

2. Answer: $\boxed{E[Y] \approx 7.06,\ Var(Y) \approx 7.29 \times 10^{1}}$

Explanation:

Because Y is the result of 5 independent deals, we can use linearity to express their sum as a multiple of 5.

$$E[Y] = E[X] + E[X] + E[X] + E[X] + E[X]$$
$$= (5)E[X]$$
$$= 5\left(\frac{353}{250}\right)$$
$$= 7.06$$

We substitute in for $Var(Y)$. Because X_i is independent we can pull out n and $\sum_{i=1}^{n}$ and solve.

$$Var(Y) = E\left[n \sum_{i=1}^{n} Var(X_i)\right]$$
$$= n \sum_{i=1}^{n} Var(X_i)$$
$$= 5 \cdot (2.916256)$$
$$\approx 14.58128$$

3. Answer: $\boxed{E[z] \approx 1.41,\ Var(z) \approx 5.88 \times 10^{-1}}$

Explanation:

We know that z is the average of X_i over n runs. We can pull out $\frac{1}{n}$ and $\sum_{i=1}^{n}$. We simplify and solve.

$$
\begin{aligned}
E[z] &= E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] \\
&= \frac{1}{n}\sum_{i=1}^{n} E[X_i] \\
&= \left(\frac{n}{n}\right)E[X] \\
&= \left(\frac{353}{250}\right) \\
&\approx 1.412
\end{aligned}
$$

Similarly, we substitute for z. We pull out $\frac{1}{n}$ and $\sum_{i=1}^{n}$ because X_i is independent and solve.

$$
\begin{aligned}
Var(z) &= Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) \\
&= \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) \\
&= \frac{1}{n} Var(X) \\
&= \left(\frac{1}{5}\right)(2.916256) \\
&\approx 0.5832512
\end{aligned}
$$

4.

a. Answer: $\boxed{E[x] = \frac{1}{6}}$

Explanation:

We know the probabilities of each of the outcomes and their points, so we plug these into the formula for expected value and solve.

$$E[x] = -2\left(\tfrac{2}{6}\right) + 1\left(\tfrac{3}{6}\right) + 2\left(\tfrac{1}{6}\right)$$
$$= -\tfrac{4}{6} + \tfrac{3}{6} + \tfrac{2}{6}$$
$$= \tfrac{1}{6}$$

b. Answer: $\boxed{Var(x) = \frac{1}{18}}$

Explanation:

We first find the expected value of x^2 which is simply the sum of the squares of the points times their probabilities. i.e.

$$E(x^2] = -2\left(\tfrac{2}{6}\right)^2 + 1\left(\tfrac{3}{6}\right)^2 + 2\left(\tfrac{1}{6}\right)^2$$
$$= -2\left(\tfrac{4}{36}\right) + \left(\tfrac{9}{36}\right) + 2\left(\tfrac{1}{36}\right)$$
$$= \tfrac{-8}{36} + \tfrac{9}{36} + \tfrac{2}{36}$$
$$= \tfrac{3}{36}$$
$$= \tfrac{1}{12}$$

Than we can plug that result into the variance formula and solve.

$$Var(x) = E[x^2] - (E(x])^2$$
$$= \tfrac{1}{12} - \left(\tfrac{1}{6}\right)^2$$
$$= \tfrac{1}{18}$$

c. Answer: $\boxed{Std(x) \approx 2.36 \times 10^{-1}}$

Explanation:

We know that the standard deviation of x is simply the square root of the variance that we found in part b.

$$\sigma = \sqrt{Var(x)}$$
$$= \sqrt{\tfrac{1}{18}}$$
$$\approx 0.235702$$

5. Answer: $\boxed{E[X] = 0,\ Var(X) = 1}$

Explanation:

We are given Y so we plug that in to the expected value formula. We then separate the parts of the division and use linearity to pull $\frac{1}{\sigma}$ and $\frac{\mu}{\sigma}$ out of the formula. We substitute μ for $E[X]$ and solve.

$$
\begin{aligned}
E[Y] &= E\left[\left(\frac{X-\mu}{\sigma}\right)\right] \\
&= E\left[\frac{X}{\sigma} - \frac{\mu}{\sigma}\right] \\
&= \frac{1}{\sigma}E[X] - \frac{\mu}{\sigma} \\
&= \frac{1}{\sigma}(\mu) - \frac{\mu}{\sigma} \\
&= 0
\end{aligned}
$$

Finding the variance of X is similar. Substitute for Y, pull out $\frac{1}{\sigma^2}$ using the variance theorem, and solve.

$$
\begin{aligned}
Var(Y) &= Var\left(\frac{X-\mu}{\sigma}\right) \\
&= Var\left(\frac{1}{\sigma}X - \frac{\mu}{\sigma}\right) \\
&= \frac{1}{\sigma^2} Var(X) \\
&= \frac{1}{\sigma^2} Var(X) \\
&= \frac{1}{\sigma^2}\sigma^2 \\
&= 1
\end{aligned}
$$

6. Answer: $\boxed{E[M] = \mu}$

Explanation:

We substitute for M, by linearity extract $\frac{1}{n}$ from $E[X]$, and we extract $\sum_{i=1}^{n}$ from $E[X_i]$. Then we substitute for $E[X_i]$ and solve.

$$
\begin{aligned}
E[M] &= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)\right] \\
&= \frac{1}{n}\sum_{i=1}^{n} E[X_i] \\
&= \frac{1}{n}\sum_{i=1}^{n}(\mu) \\
&= \frac{1}{n}(n)\mu \\
&= \mu
\end{aligned}
$$

7. Answer: $\boxed{Var(M) = \frac{\sigma^2}{n}}$

Explanation:

We substitute for M, and extract $\frac{1}{n}$ which becomes $\frac{1}{n^2}$ and $\sum_{i=1}^{n}$ from Var( ). Then we substitute for $Var(X_i)$ and solve.

$$\begin{aligned} Var(M) &= Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) \\ &= \frac{1}{n^2}\sum_{i=1}^{n} Var(x_i) \\ &= \frac{1}{n^2}\sum_{i=1}^{n}\left(\sigma^2\right) \\ &= \frac{1}{n^2}(n)\,\sigma^2 \\ &= \frac{\sigma^2}{n} \end{aligned}$$