P1

Claim:

For any pair of $x, y \in \mathbb{R}$ such that $0 \leq x < y$, $x^2 < y^2$.

Proof:

Assume for a contradiction that there exists a pair of $x, y \in \mathbb{R}$ such that $0 \leq x < y$ but $x^2 \geq y^2$. Let x be less than y by some positive, nonzero amount k such that $x + k = y$. Then, by substitution $x^2 \geq (x+k)^2$. Algebra shows this to be equivalent to $-\frac{k}{2} \geq x$. Since k is positive, the term $-\frac{k}{2}$ is always negative, but then x is negative which is a contradiction. $\square$

Claim:

The minimum spanning tree produced by Kruskal's algorithm on graph G where the cost of edge e is c_e, and all c_e are positive and distinct, will be the same mininimum spanning tree produced by Kruskal's algorithm on G after updating the cost of every e to c_e^2.

Proof:

Let Kruskal's algorithm produce a minimum spanning tree T from arbitrary connected graph $G = (V, E)$. Let E' refer to the set of edges under consideration for inclusion in T during an arbitrary iteration of the algorithm. Let edge e be the minimum cost edge in E' such that the cost $c_e < c_f$ for any other edge $f \in E'$ and, by the cut property, e is in T. If the cost of every edge in E is squared, such that in E' c_e becomes c_e^2 and c_f becomes c_f^2, then by the above proof $c_e^2 < c_f^2$ and e is still the minimum cost edge in E'. Thus, the minimum spanning tree produced by Kruskal's algorithm is the same. $\square$