P4

Claim:

Given n real numbers, where n is even, the optimal partitioning of these numbers into $\frac{n}{2}$ pairs that minimizes the sum of the maximim partition will be one wherein the two sets of pairs will have sums that are as close to equal as possible.

Algorithm:

```
// Where R is a set of n real numbers and n is even
Partition(R):
    Set V to Sum(R) / 2 such that V is the optimal split value
    While R not empty
        // Add to A the largest viable term from R
        // And if no such term exists add the smallest term in R to A
        For each term tu in R
            If tu + Sum(A) <= V and current tu > m
                Set m to tu
            EndIf
            If tu < n
                Set n to tu
            EndIf
        EndFor
        If m is set
            Add m to A
            Remove m from R
        Else
            Add n to A
            Remove n from R
        EndIf

        // Add to B the largest viable term from R
        // And if no such term exists add the smallest term in R to B
        For each term tv in R
            If tv + Sum(B) <= V and current tv > k
                Set k to tv
            EndIf
            If tv < j
                Set j to tv
            EndIf
        EndFor
        If k is set
            Add k to B
        Else
            Add j to B
        EndIf
    EndWhile

    Set tb to the first term added to B
    Set D to |Sum(A) - Sum(B)|
```

```
For term ta in A
    If Swap(ta, tb) would reduce D
        Set P to store pair (ta, tb)
    EndIf
EndFor
If P has stored pair
    Swap(P)
EndIf
```

Claim:

The algorithm terminates in polynomial time.

Proof:

Consider the While loop and note that on every iteration, two elements are removed from R so that the number of iterations of while are at most $\frac{n}{2}$. Within the while loop, each for loop iterates at most n times so that the total iterations with the while loop are 2n. Thus, the run time of the while loop as a whole is O($\frac{n}{2} * n$) or, more simply, O(n^2). Note that because elements are removed from R on every iteration of the while loop, the loop is guaranteed to terminate. The last for loop iterates over the terms of A and since —A— = $\frac{n}{2}$ the total run time of this for loop is O($\frac{n}{2}$) or just O(n). And, because it encounters each term once and only once, it is guaranteed to terminate. Thus the total runtime is the sum of the two outer loop runtimes, O(n^2) + O(n) or just, O(n^2). And because the loops terminate, the algorithm as a whole terminates.

Claim:

The algorithm returns the optimal partitioning of n numbers where n is even into $\frac{n}{2}$ pairs that minimizes the sum of the maximim partition.

Proof: