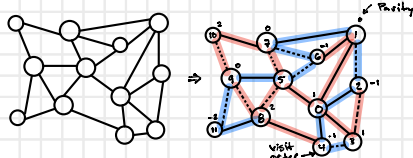


5.



Let parity on a vertex refer to the difference between Red and Blue edges connected to that vertex.
i.e. Parity: $R - B$
So this parity on each node should be $\leq |2|$.

Claim: The edges of any graph can be colored red or blue such that, for every vertex v in the graph, the difference between red and blue edges touching that v is at most 2.

That is $\forall v \in V, R - B \leq |2|$ where $(R \setminus B) \in \text{edges on } v$.

Proof: Consider the following for each connected component of arbitrary graph G . We proceed by choosing an arbitrary vertex v in G and performing BFS on G from v . For each edge u incident to v we color u in alternating red or blue. Note that between changes in v , i.e. visiting all edges incident to v and moving to the next v in the queue, we continue alternating. So if the last edge for v was colored blue, the first edge incident to v will be colored red, and so on. Having explored every vertex and every edge incident to every vertex thus, we will have colored every edge red or blue. Now to show that every vertex in G has a parity, where parity is $\text{RedEdges} - \text{BlueEdges}$ for vertex v , consider an arbitrary vertex v in G . If v happens to be the start vertex from the BFS then we know v has a parity of at most $|1|$, since it had no incoming edges and every outgoing edge was colored in alternating red and blue. Otherwise, .. (ran out of time showing it and why this approach will always work)

