1. Claim: Any tree G can have it's vertices colored in alternating black and white such that the endpoints of every edge are opposite colors.

Proof:

Let n designate the number of edges between a given edge and the root vertex of a given tree. Then, $P(n)$ claims that in an arbitrary tree, the endpoints of any edge, n edges from T's root, can be colored black or white such that they are opposite in color. Note, that under this coloring convention, vertices one edge away from the root will be opposite in color from the root, vertices two edges from the root will be the same color as the root, and so on.

Base Case: $P(1)$ holds because in arbitrary tree T the only vertices will be one edge away from the root and can therefore all be colored opposite to the root's color. And, because T is a tree, no vertices one edge away from the root will be connected to any other vertices also one edge from the root or else there would be a cycle.

Inductive Hypothesis: Assume $P(n-1)$ holds.

Inductive Step: Goal show $P(n)$ holds. Let T be an arbitrary tree. By the inductive hypothesis we know that $P(n-1)$ holds for subtree T' of T. Because T' is a valid tree at $n-1$, we know our defined coloring convention still holds at $n-1$ such that all the vertices at $n-1$ will be the same color. Therefore, all the vertices at n can validly be colored opposite in color to the vertices at $n-1$. $\square$