P3

Claim:

4-Color $\leq_p$ 5-Color.

Proof:

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Suppose G = (V, E) is a 4-colorable graph
If G is 4-Colorable, then G' is 5-Colorable
    since G is 4-colorable, there exists a color mapping c such that
        c: V = {1, 2, 3, 4} where
        for all vertices v in V
            for all vertices u in V
                if v != u then
                    c(v) != c(u)
    Let graph G' = (V', E') such that
        let vertex x be a vertex not in V
        V' = V ∪ x
        E' = E ∪ {(e, x) for all e in E}
    Let there be a mapping c' such that
        c': V' = {1, 2, 3, 4, 5} where
        for all vertices v in V'
            for all vertices u in V'
                if v != u then
                    c(v) != c(u)
        and such that c(v) = c'(v) for all v in V and
        c'(x) = 5 for the vertex x in V'
    By construction c' is a 5-coloring of G'
    Thus if G is 4-Colorable, then G' is 5-colorable

Suppose graph G' = (V', E') is a 5-colorable graph
If G' is 5-colorable, then G is 4-colorable
    since G' is 5' colorable, there exists a color mapping c' such that
        c': V = {1, 2, 3, 4, 5} where
        for all vertices v in V
            for all vertices u in V
                if v != u then
                    c(v) != c(u)
        and such that without loss of generality c'(x) = 5
    x shares an edge with every other vertex in V'
    so no other vertex v in V' has c'(v) = 5
    Let there be a mapping c such that
        c: V = {1, 2, 3, 4} where
        for all vertices v in V
            for all vertices u in V
                if v != u then
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                    c(v) != c(u)
     and such that for all vertices v in V, c(v) = c'(v)
c is a 4 coloring for G
Thus, if G' is 5 colorable, then G is 4-colorable
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