Parameterized algorithms for thinness via the cluster module number

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In 2007, Mannino et al. defined k-thin graphs as a generalization of interval graphs, and defined the thinness of a graph to be the minimum k such that the graph is k-thin. When given a k-thin representation of a graph, several NP-complete problems can be solved in XP time parameterized by k. Thus, the problem of computing the thinness of a graph, as well as the corresponding representation, has various algorithmic applications. In this work we define a new graph parameter that we call the *cluster module number* of a graph, which generalizes twin-cover and neighborhood diversity, and can be computed in linear time. We then present a linear kernel for the problem of calculating the thinness on graphs with bounded cluster module number. As a corollary, this results in a linear kernel for Thinness when the input graph has bounded neighborhood diversity, and exponential kernels when the input graph has bounded twin-cover or vertex cover. On the negative side, we observe that Thinness parameterized by treewidth, pathwidth, bandwidth, (linear) mim-width, clique-width, modular-width, or the thinness itself, has no polynomial kernel assuming NP $\not\subseteq$ coNP/poly.