Robolocomotion HW 5

Evan Sean Grealish

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1 Section 11

Define
Inputs
a = acceleration
B = steering input

Outputs
y= y-position array of 3 points
x= x-position array of 3 points
v = speed
Φ = relative steering angle
Ω = Global Angle
θ = Angle between Truck and Trailer

Points:
Point 1 = front drive, driving tires and steering
Point 2 = Back tires and connection point for hitch
Point 3 = Back tires of trailer.

The trailer consists of 2 back tires and attached to hitch.

Units are meters, seconds etc
Max Angle of steering is [-45, 45] Degrees.
Max Steering derivative, B_Max is [-45, 45] Degrees/second
Max speed is [-10, 10]
Max acceleration, a_Max [-1, 1]

L1=Length of Car
L2=Length of Trailer

Timestep = ΔT = 0.1 seconds

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_0 = \begin{bmatrix} 0 \\ L1 \\ L1 + L2 \end{bmatrix}$$

$$v_0 = 0$$

$$\Phi_0 = 0$$

$$x_1(k+1) = x(k) + \Delta T \cdot v(k) \cdot \sin(\Phi(k) + \Omega(k))$$
$$y_1(k+1) = x(k) + \Delta T \cdot v(k) \cdot \cos(\Phi(k) + \Omega(k))$$

Calculate line of truck in time step k using:

$$y - y_1 = m(x - x_1)$$

$$y - m(x) + (m \cdot x_1 - y_1) = 0$$

$$m_{truck}(k) = \frac{y_2(k) - y_1(k)}{x_2(k) - x_1(k)}$$

$$c_{truck}(k) = y_1(k) - m_{truck}(k) \cdot x_1(k)$$

$$Ax + By + C = 0$$

$$A(k) = -m$$

$$B(k) = 1$$

$$C(k) = m \cdot x_1(k) - y_1(k)$$

Calculate Point 2 in time step k+1 using the Point 1 in k+1, and calculate the distance between point 1 and 2 in time step k+1 using the distance formula and combine with that point 2 in k+1 is on the line of truck in step k

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$L_1 = \sqrt{(x_2(k+1) - x_1(k+1))^2 + (y_2(k+1) - y_1(k+1))^2}$$
$$(L_1)^2 = (x_2(k+1) - x_1(k+1))^2 + (y_2(k+1) - y_1(k+1))^2$$

Where $x_2(k+1)$ and $y_2(k+1)$ are unknowns. Combining this with the equation below:

$$A(k)x_2(k+1) + B(k)y_2(k+1) + C(k) = 0$$

$$x_2(k+1) = \frac{-B(k)y_2(k+1) - C(k)}{A(k)}$$

$$(L_1)^2 = (\frac{-B(k)y_2(k+1) - C(k)}{A(k)} - x_1(k+1))^2 + (y_2(k+1) - y_1(k+1))^2$$

I will remove the "k" and "k+1". The x_2 and y_2 are unknowns. The x_1, y_1, A, B and C are knowns.

$$L^2 = (\frac{-By_2 - C}{A} - x_1)^2 + (y_2 - y_1)^2$$

$$L^2 = \frac{B^2(y_2)^2}{A^2} + \frac{(C + Ax_1)^2}{A^2} + 2\frac{B(C + Ax_1)}{A} + (y_2)^2 + (y_1)^2 + 2y_2y_1$$

$$0 = (y_2)^2(\frac{B^2}{A^2} + 1) + (y_2)(\frac{2B(C + Ax_1)}{A} + 2y_1) + (\frac{(C + Ax_1)^2}{A^2} + (y_1)^2 - (L_1)^2)$$

$$a_q = (\frac{B^2}{A^2} + 1)$$

$$b_q = (\frac{2B(C + Ax_1)}{A} + 2y_1)$$

$$c_q = (\frac{(C + Ax_1)^2}{A^2} + (y_1)^2)$$

Solve using quadratic Formula

$$y_2(\pm) = \frac{-b_q \pm \sqrt{b_q^2 - 4a_c c_q}}{2a_q}$$

$$x_2(\pm) = \frac{-By_2 - C}{A}$$

$$d(+) = \sqrt{(y_2(k) - y_2(k+1)_{(+)})^2 + (x_2(k) - x_2(k+1)_{(+)}))^2}$$

$$d(-) = \sqrt{(y_2(k) - y_2(k+1)_{(-)})^2 + (x_2(k) - x_2(k+1)_{(-)}))^2}$$

$$Min(d(+), d(-))$$

Select $y_2(k+1)$ and $x_2(k+1)$ =y(+)&x(+) or y(-)&x(-)

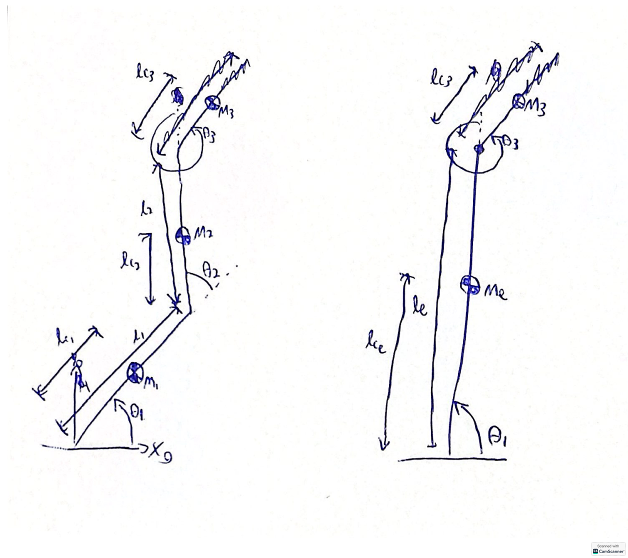

Figure 1: Original Diagram to Equivalent

$$\dot{Q} = \begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_3} \end{bmatrix}$$