

# THÈSE DE DOCTORAT

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Vision Methods for Navigation of Unmanned Aerial Vehicles (UAVs)

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*Dedication*





# Acknowledgements

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# Symbols and Abbreviations

$\alpha$  Sharpness (Gabor function major axis)

$\beta$  Sharpness (Gabor function minor axis)

$\eta$  Sharpness of Gabor filter (minor axis)

$\gamma$  Sharpness of Gabor filter (major axis)

$\mathcal{F}^{-\infty}\{\cdot\}$  Inverse Fourier transform

$\mathcal{F}\{\cdot\}$  Fourier transform

$\phi$  Phase shift Gabor filter

$\sigma$  Spread of 1-d Gaussian function

$\theta$  Orientation angle of Gabor filter

$h(t)$  1-d signal

$h(x, y)$  2-d signal





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# Chapter 1

## Introduction

This document presents the advances and results obtained in the first year of the Ph.D. thesis *Vision Methods for Unmanned Aerial Vehicles Navigation*<sup>1</sup> at the Center for Mathematical Morphology (CMM). In our context, a UAV (Unmanned Aerial vehicle or drone) is a flying robot formed by complex subsystems which allows it to perform certain tasks autonomously. Today, the intensive automatization in the production, transport, construction, and security sectors, demands more autonomous UAVs [1].

Computer vision and the need to perform tasks in more complex environments with non-controlled conditions plays an important role in the evolution of these systems. We can delimit the use of vision systems in UAVs into two purposes: **i)** applications (e.g., freight delivery, monitoring [7], inspection [23] and surveillance [25]) and **ii)** aid to control and operation [8].

In the work of thesis, we will focus on the second purpose to develop a new vision framework able to provide aid in the automated decision chain of UAVs in complex scenarios.

### 1.1 Background and motivation

A UAV mission involves three principal moments: take-off, navigation, and landing. Commonly, the UAV control in these phases is achieved with the use of conventional sensors, such as inertial sensors (IMUs) for orientation, and GPS for position. The drawback with the IMU is that suffers from bias error propagation due to the integral drift, while with the GPS signal is not always guaranteed. In urban or indoor environments, the satellite signal is low or unexisting. A recurrent technique to enhance the position ac-

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curacy implies the data fusion of pressure, ultrasonic, radars and laser range-finders sensors [30]. The fusion of data can provide the advantages of each sensor; however, the use of multiple sensors on board becomes expensive and impractical, taking into account the drone has a maximum payload capacity and the flight time depends on that. Contrariwise, visual sensors are passive, lightweight and can acquire valuable information about the surrounding structures, including color and textures, and UAV self-motion.

Today one can use different visual sensors; such as monocular cameras [22], stereo cameras [27], RGB-D cameras [12], fish-eye cameras [11], thermal [7], among others. This wide range of sensors offers more options and flexibility to deal with the problems mentioned above. Based on the three moments in a UAV mission, we present a brief review of developed works to improve the controllability of a UAV.

### 1.1.1 UAV Navigation

Vision-based techniques for UAV navigation are classified into two groups according to the intention and its consequence result in i) localization and mapping, ii) obstacle avoidance.

#### Localization and Mapping

The so-called Simultaneous Localization and Mapping (SLAM) is a technique that estimates the local pose of a robot and builds a 3D model of its surroundings employing visual sensors. The Visual Odometry (VO) [26] is responsible of the robot motion estimation while the maps are built with occupancy grid algorithms [29]. Taking into account the image information used to perform a SLAM, we can classify the methods in two classes.

**Feature-based Methods** extract a set of image features (e.g., lines, points) in a sequence of images. To do so, the invariant feature detectors most commonly used are Harris [10], SIFT [17], FAST [24], SURF [3]. After that, the algorithm performs a feature matching through an invariant feature descriptor. The last stage is to perform the motion estimation using the data of other sensors; a local optimization is optional.

The Parallel Tracking and Mapping (PTAM) [14] is one of the first and most used methods for SLAM. It is a feature-based algorithm that achieves robustness through tracking and mapping hundreds of features. It is the base for other approaches such as the Multi-Camera Parallel Tracking Mapping (MCPTAM) [9] that uses multiple cameras to build a 3D map and calculate the robot position.

**Direct-Based Methods** make use of the image intensity information to estimate the structure and the

motion of the robot. Unlike the feature-based methods, the direct methods compare the entire image between them to make a scene reconstruction. The use of image intensity information permits using different theoretical frameworks for SLAM. The DTAM method [21] uses the minimization of a global, spatially-regularized energy functional while [20] uses a contrario framework to carry out a structure from motion. Those methods provide more visual information about the environment giving a more meaningful representation to the human eye.

Some of the first direct approaches [13], [18] treated salient feature patches as observations of locally planar regions on 3D world surfaces. This approach allows being robust to images where exits areas with textures and small gradients [16] or to blur images caused for camera-defocus [21].

The use of SLAM techniques for UAV navigation presents remarkable advantages. Feature-based methods can use a wide variety of feature detectors which counts typically with an optimization stage that allows having fast algorithms. Direct-based methods have the advantage to be robust to images degradations; they can lead better with images with texture and blurred zones; besides, the map produced is of an acceptable resolution. An interesting fact is that the strengths of the first group of methods are the weak points of the second and vice versa. A method that tries to gather the benefits of both approaches is the Semi-direct Visual Odometry [4]; however, in general, the SLAM methods works in indoor environments, where the illumination conditions are static or controlled.

## **Obstacle Avoidance**

An indispensable feature to increase the autonomy of the UAV navigation is the detection and avoidance of obstacles. This capability is of great importance for achieving free collisions missions in both, indoor and outdoor environments. A recurrent solution, as we early mentioned, is the multi-sensor data fusion. In [6] present a platform using low-cost ultrasound and IR sensors; however, despite the obtained results, it utilizes several sensors to recovers environment information and yet, it does not get a perceptual representation of the scene due to the low resolution and perceptive capacity of the sensors. On the other hand, vision-based techniques for obstacle avoidance could identify obstacles and in some cases classify the found object [15].

Visual methods for avoidance of obstacles can be classified into two groups. The first, SLAM-based

techniques, make use of the principles described in the last subsection. The 3D reconstruction provides accurate and sophisticated maps and allows the air vehicle to travel with more information about the environment. In [19], takes this advantage to develop an obstacle avoidance approach for static and dynamic obstacles.

The second group is the flow-based methods which historically, were inspired by the navigation of insects such as bees [28] or flies [5]. Many insects in the wild identify obstacles through the intensity of light. During the flight, their eyes produce an optical flow that provides accurate spatial information. Currently, there are also works inspired by the behavior of the human eye [2]. The technique measures the object size from the idea that objects in the robot's field of vision are larger as the obstacle is closer.

The techniques for obstacle detection and avoidance present interesting characteristics and ideas; however, its implementation is strongly linked to an application under certain conditions. Their use would involve a recalibration or readjustment of parameters and, given the conditions in which a drone can operate, it is necessary to have more general and non-supervised methods.

## 1.2 Objectives of the thesis

In this Ph.D. thesis, we aim at developing vision methods for navigation of UAVs. In that context, the primary objective is to propose a new methodological framework capable of providing aid for control and decision taking in UAV navigation. The framework must be robust in environments with non-controlled conditions.

During the thesis, several specific tasks are considered, such as i) environment awareness, ii) obstacle detection and avoidance, iii) target identification and following. Given these tasks, the work also focuses on the scene understanding problem.

Finally, this work includes:

- **Framework implementation.** Referring in a first stage to the implementation in simulated and off-board environments. In a second moment, to the implementation in a real platform. The latter requires the search for a portable platform capable of real-time operation.
- **Framework functional evaluation.** Comparison of the obtained results w.r.t. the approaches of state of the art.

- **Framework validation.** Real test deployment taking into account the constraints of portability and real-time on the industrial scale.

## 1.3 Organization of the thesis

In this report, we present the conducted work and scientific procedure to tackle the UAV landing phase. The proposed methodology seeks the detection<sup>2</sup> and recognition<sup>3</sup> of a particular a landing target.

A general organization of the document is as follows. Chapter ?? contains the landing target detection algorithm developed. We compare some thresholding methods for contour detection, and we show the perception framework. Chapter ?? enlist the perspectives for the thesis and chapter ?? shows the curricular content taken from the beginning of the thesis. Finally, the appendix ?? describes the landing target design as well as the encoding and decoding techniques needed for its detection.

---

<sup>2</sup>**Detection** refers to find a landing target among other objects.

<sup>3</sup>**Recognition** refers to differentiate a landing target among others landing targets.



## Chapter 2

# Perceptual information on images for object detection and segmentation

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### 2.1 Image contours

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### 2.1.1 State of the art

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### 2.1.2 Human perception principles and *a contrario* methods

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**Application: Landing target detection and recognition**

## Chapter 3

# Image spectral decomposition

### 3.1 The Heisenberg uncertainty principle in signal and image processing

The uncertainty principle is one of the most famous ideas in quantum mechanics. An early incarnation of the uncertainty principle appeared in a 1927 paper by the German physicist Heisenberg. The uncertainty principle says that we cannot measure the position ( $x$ ) and the momentum ( $p$ ) of a particle with absolute precision. The more accurately we know one of these values, the less accurately we know the other.

However, the uncertainty principle in the field of quantum mechanics is just a particular case of a more general compromise that appears in many cases of everyday life involving waves. The central idea is connected with the interrelation between frequency and duration. For example, in the case of sound waves, if we want to identify the frequency of a musical note, the shorter the sound lasts in time, the less certain we can be about the exact frequency of the sound; to find a more defined frequency, it would be necessary to listen to the sound for a longer time. In the language of signal processing, we can say that a short signal correlates highly with a wide range of frequencies and only wide signals correlate with a short range of frequencies. Formally this is expressed as

$$(\Delta t)^2(\Delta \omega)^2 \geq \frac{1}{4} \quad (3.1)$$

where  $\Delta t$  is the duration of the signal in the time domain and  $\Delta \omega$  is the bandwidth of the signal in the frequency domain (CITE). The uncertainty principle then says: the product of the spectral bandwidth multiplied with the time duration of the signal cannot be less than a certain minimum value.

The Heisenberg uncertainty principle in the field of signal processing and image processing can be



mathematically proved by **Parseval's theorem**

$$\int_{-\infty}^{\infty} f(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \quad (3.2)$$

where  $f(t)$  is a function and  $F(\omega)$  its the Fourier transform.

The **energy content** of the signal described by  $f(t)$  is defined as:

$$E_{\infty} \equiv \int_{-\infty}^{\infty} f(t)^2 dt \quad (3.3)$$

From the Parseval's identity this may be written as:

$$E_{\infty} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \quad (3.4)$$

The **time dispersion** of the signal is given by

$$(\Delta t)^2 \equiv \frac{1}{E_{\infty}} \int_{-\infty}^{\infty} (t - \bar{t})^2 f(t)^2 dt \quad (3.5)$$

where if we shift the **center of gravity** of the signal to the origin  $\bar{t} = 0$ , then

$$(\Delta t)^2 = \frac{1}{E_{\infty}} \int_{-\infty}^{\infty} t^2 f(t)^2 dt \quad (3.6)$$

In an analogous way, the **spectral bandwidth** of the signal is given by

$$(\Delta \omega)^2 \equiv \frac{1}{2\pi E_{\infty}} \int_{-\infty}^{\infty} (\omega - \bar{\omega})^2 |F(\omega)|^2 d\omega \quad (3.7)$$

where if we consider an **spectral center of gravity**,  $\bar{\omega} = 0$

$$(\Delta \omega)^2 = \frac{1}{2\pi E_{\infty}} \int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega \quad (3.8)$$

If  $f'(t)$  is the derivative of the function, its Fourier transform is  $j\omega F(\omega)$ . By applying the Parseval's theorem to the Fourier pair  $f'(t) \longleftrightarrow j\omega F(\omega)$  we obtain:

$$\int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} f'(t)^2 dt \quad (3.9)$$

By substituting in equation ((3.8)), we have:

$$(\Delta\omega)^2 = \frac{1}{E_\infty} \int_{-\infty}^{\infty} f'(t)^2 dt \quad (3.10)$$

We use equations ((3.6)) and (3.10) to calculate:

$$(\Delta t)^2(\Delta\omega)^2 = \frac{1}{E_\infty^2} \int_{-\infty}^{\infty} t^2 f(t)^2 dt \int_{-\infty}^{\infty} f'(t)^2 dt \quad (3.11)$$

Applying the Schwartz's inequality for the integrals on the right-hand side of (3.11):

$$\int_{-\infty}^{\infty} t f(t)^2 dt \int_{-\infty}^{\infty} f'(t)^2 dt \geq \left| \int_{-\infty}^{\infty} t f(t) f'(t)^2 dt \right|^2 \quad (3.12)$$

We may integrate by parts the integral on the right-hand side of (3.12)

$$\int_{-\infty}^{\infty} t f(t) f'(t)^2 dt = \frac{1}{2} t f(t)^2 \Big|_{-\infty}^{\infty} - \frac{1}{2} \int_{-\infty}^{\infty} f(t)^2 dt \quad (3.13)$$

If  $\lim_{t \rightarrow \infty} t f(t)^2 = 0$ , the first term on the right-hand side of (3.13) vanishes and from equation (3.3) we have

$$\int_{-\infty}^{\infty} t f(t) f'(t)^2 dt = -\frac{1}{2} E_\infty \quad (3.14)$$

If we use this into (3.12) and then into (3.11) we obtain:

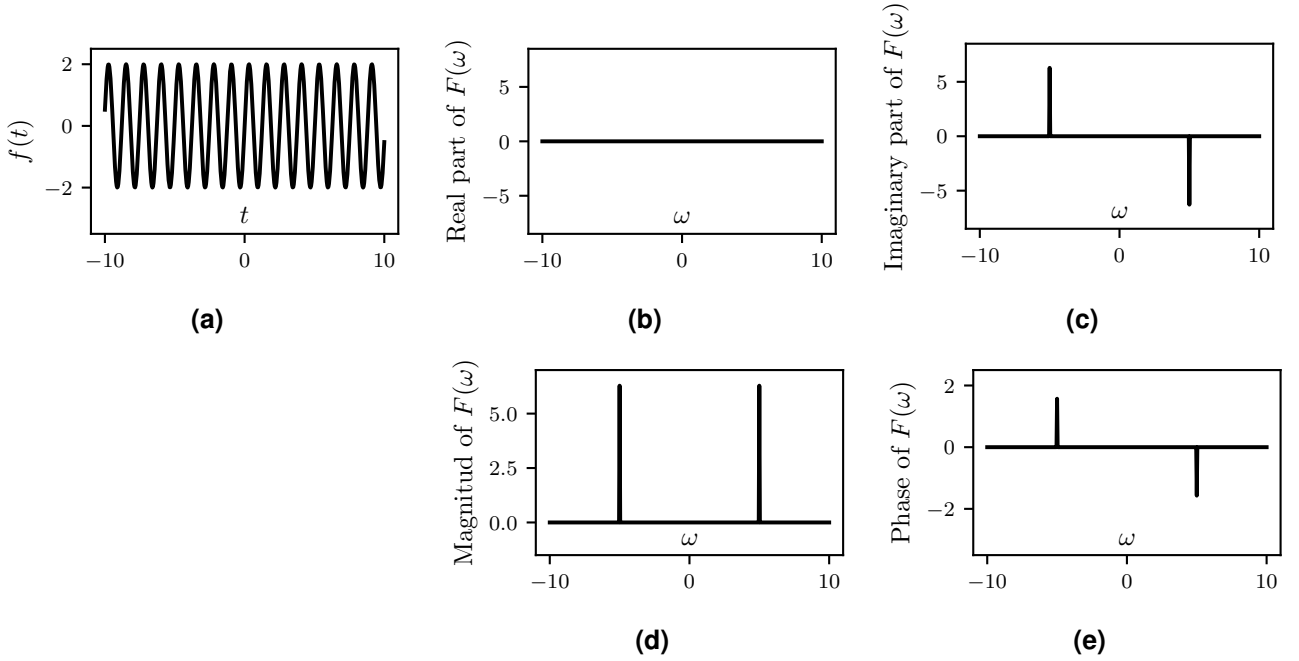
$$(\Delta t)^2(\Delta\omega)^2 \geq \frac{1}{4} \quad (3.15)$$

This is the mathematical statement of the uncertainty principle in signal processing.

### 3.1.1 Examples

The uncertainty principle shows that the size, the shape and the shift of the window through which we make measurements affects the accuracy of what we compute. For example, let us consider a signal  $f(t)$  with Fourier transform  $F(\omega)$ . Let us assume that we observe only a part of the signal through a window  $w(t)$ , with Fourier transform  $W(\omega)$  centered at  $t_0$

$$h(t) = f(t)w(t - t_0) \quad (3.16)$$



**Figure 3.1:** A continuous function (a), and the real part (b), imaginary part (c), magnitude (d) and phase (e) of its Fourier transform.

Due to the shifting property of the Fourier transform, the Fourier transform of the window is  $e^{-j\omega t_0} W(\omega)$ . Since the window multiplies the signal, the Fourier transform of the window is convolved with the Fourier transform of the signal. Therefore, the Fourier transform of what we observe is given by:

$$H(\omega) = \int_{-\infty}^{\infty} F(\omega - u) e^{-j\omega t_0} W(u) du \quad (3.17)$$

In general  $H(\omega)$  is different from  $G(\omega)$  and depends on the locality of the window  $t_0$ .

To see this behavior, consider a signal  $f(t) = A \sin \omega_0 t$ , where  $A$  is a positive constant, and a window  $w(t)$  defined by a Gaussian function.

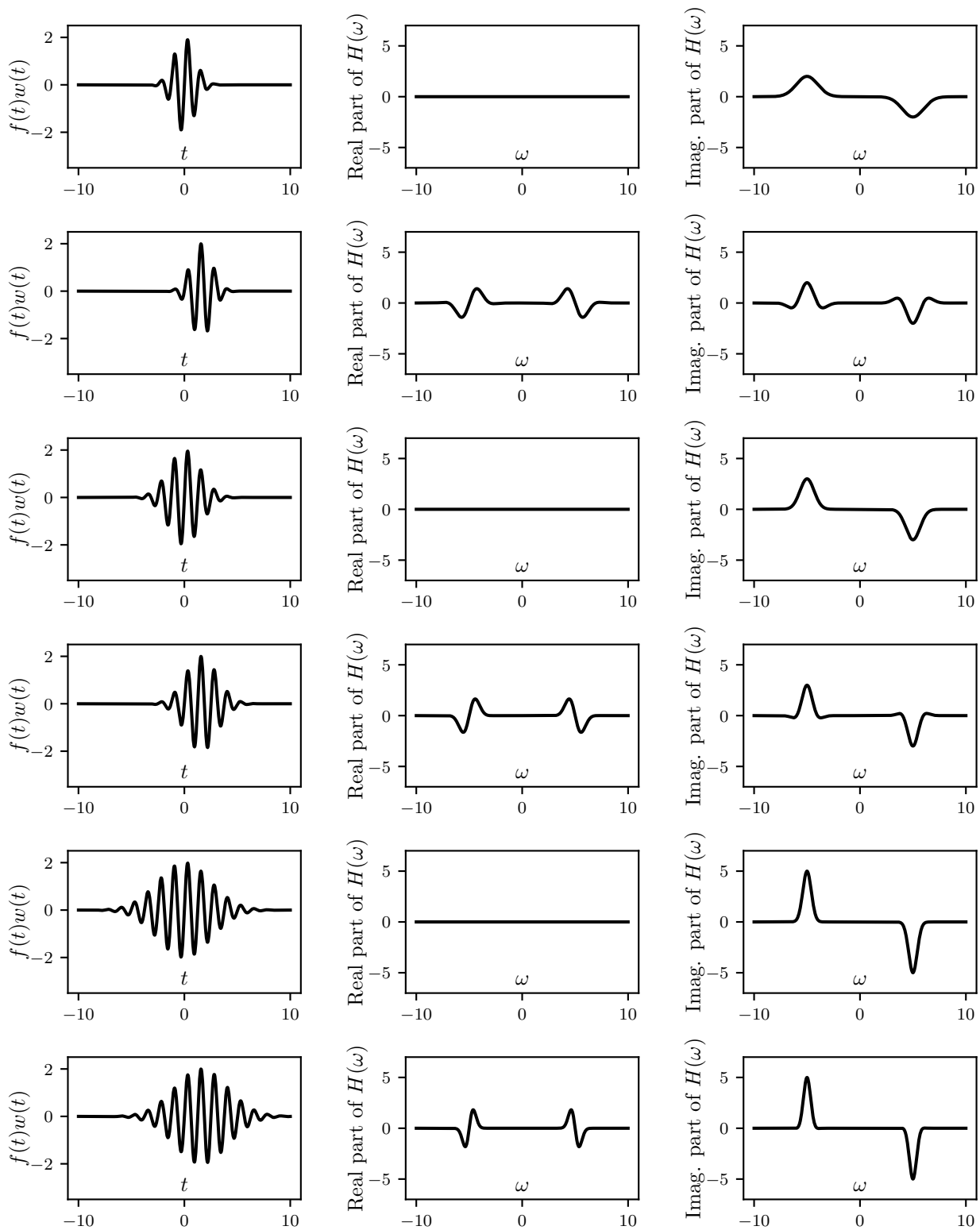
$$w(t) = e^{-\frac{(t-t_0)^2}{2\sigma^2}} \quad (3.18)$$

A Gaussian window is infinite in extent, so it is characterized by its locality  $t_0$  and its standard deviation, which in this context is also called *spread* and is denoted by  $\sigma$ .

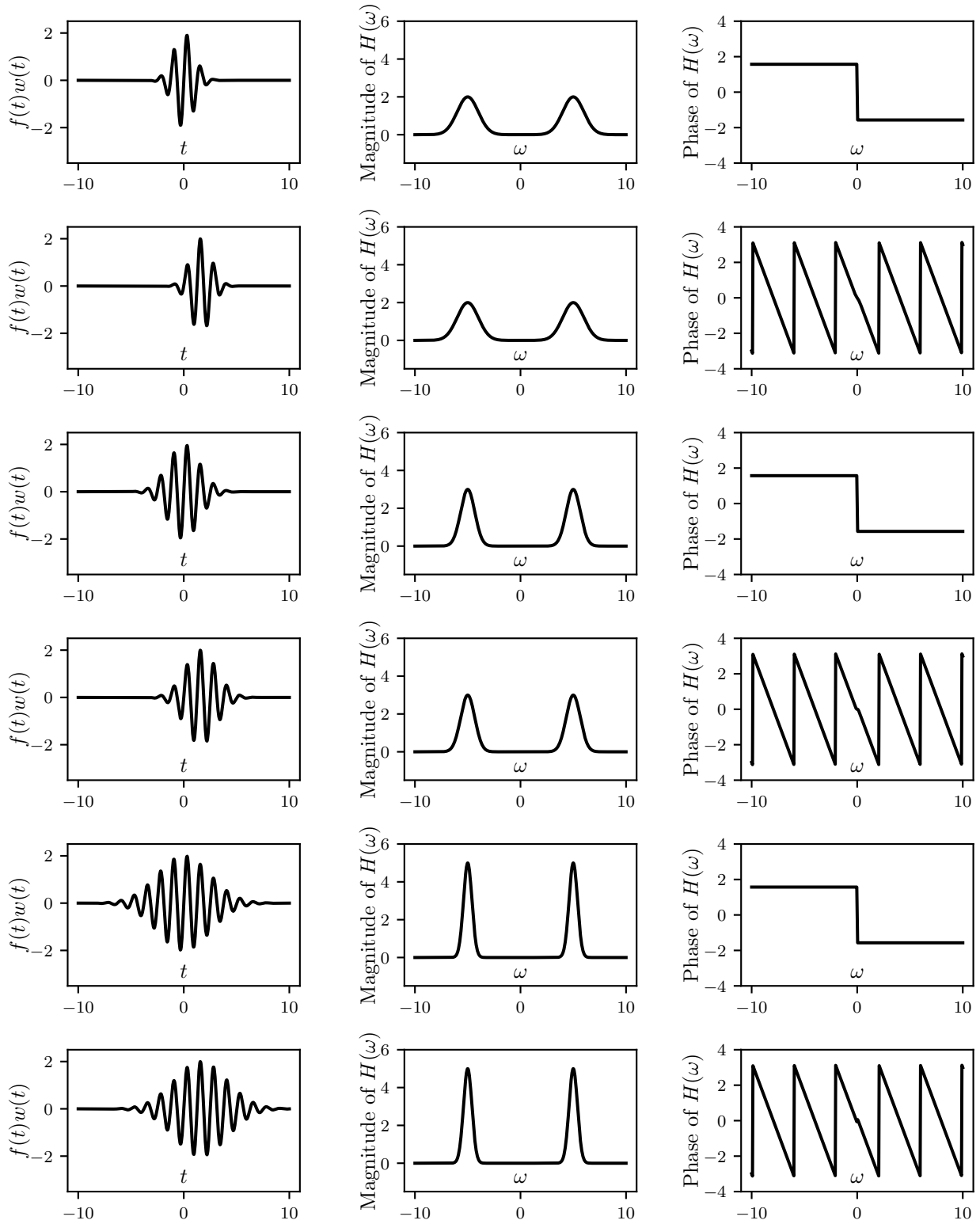
Figures demonstrate the result for a signal with  $\omega_0 = 5$  and  $A = 2$ . Figure shows the continuous signal and the real and imaginary parts and the magnitude and phase of its Fourier transform. Figure shows various windowed parts of the signal ( $h(t)$ ) and the real and imaginary parts of their corresponding Fourier transforms. Figure is the same as figure , but it shows the magnitude and phase of each Fourier transform.

These Fourier transforms should be compared with their counterparts in figure in order to appreciate the effect of both the size of the window and the locality of the window (Gaussian function). In all cases the main peaks of the Fourier transform's magnitude, which correspond to delta function impulses at  $\omega = \pm 5$  in the continuous case, are preserved, but they become less sharp and the recovered value starts to move away from the real value as soon as the size of the window decreases.

For the analysis of discrete signals it is possible to estimate the uncertainty principle. The easiest way is to consider signal segments and calculate the discrete Fourier transform (DFT) of each segment. This is the so-called Short Time Fourier Transform (STFT). If we consider an odd-sized window and associate the DFT that we calculate within it with the sample in the center of the window, we will be associating each sample of the signal with a "small" Fourier transform. In this context, how "small" the DTF is depends on the size of the window.



**Figure 3.2:** The effect of Gaussian window. Sinusoidal signal bounded by a Gaussian window (first column) and the real and imaginary part of its corresponding Fourier transform (second and third columns). From top to bottom:  $[\sigma = 1, t_0 = 0]$ ,  $[\sigma = 1, t_0 = 1.6]$ ,  $[\sigma = 1.5, t_0 = 0]$ ,  $[\sigma = 1.5, t_0 = 1.6]$ ,  $[\sigma = 2.5, t_0 = 0]$ ,  $[\sigma = 2.5, t_0 = 1.6]$ .



**Figure 3.3:** The effect of Gaussian window. Sinusoidal signal bounded by a Gaussian window (first column) and the magnitude and phase of its corresponding Fourier transform (second and third columns). From top to bottom:  $[\sigma = 1, t_0 = 0]$ ,  $[\sigma = 1, t_0 = 1.6]$ ,  $[\sigma = 1.5, t_0 = 0]$ ,  $[\sigma = 1.5, t_0 = 1.6]$ ,  $[\sigma = 2.5, t_0 = 0]$ ,  $[\sigma = 2.5, t_0 = 1.6]$ .



## Chapter 4

# One Last Part

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### 4.1 First Section

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## Chapter 5

# Conclusion

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### 5.1 First Section

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## 5.2 Second Section

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## **Appendix A**

### **First Appendix**



## **Appendix B**

### **Second Appendix**



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## Résumé

Cuius acerbitati uxor grave accesserat incentivum, germanitate Augusti turgida supra modum, quam Hannibaliano regi fratris filio antehac Constantinus iunxerat pater, Megaera quaedam mortalis, inflammatrix saevientis adsidua, humani cruoris avida nihil mitius quam maritus; qui paulatim eruditiores facti processu temporis ad nocendum per clandestinos versutosque rumigerulos conpertis leviter addere quaedam male suetos falsa et placentia sibi discentes, adfectati regni vel artium nefandarum calumnias insonatibus adfligebant.

Saraceni tamen nec amici nobis umquam nec hostes optandi, ultro citroque discursantes quicquid inveniri poterat momento temporis parvi vastabant milvorum rapacium similes, qui si praedam dispexerint celsius, volatu rapiunt celeri, aut nisi impetraverint, non inmorantur.

Vita est illis semper in fuga uxoresque mercenariae conductae ad tempus ex pacto atque, ut sit species matrimonii, dotis nomine futura coniunx hastam et tabernaculum offert marito, post statum diem si id elegerit discessura, et incredibile est quo ardore apud eos in venerem uterque solvitur sexus.

Sed tamen haec cum ita tutius observentur, quidam vigore artuum imminuto rogati ad nuptias ubi aurum dextris manibus cavatis offertur, inpigre vel usque Spoletium pergunt. haec nobilium sunt instituta.

## Mots Clés

Caesar licentia post honoratis haec adhibens urbium honoratis nullum Caesar.

## Abstract

Verum ad istam omnem orationem brevis est defensio. Nam quoad aetas M. Caeli dare potuit isti suspicioni locum, fuit primum ipsius pudore, deinde etiam patris diligentia disciplinaque munita. Qui ut huic virilem togam dedit, nihil dicam hoc loco de me; tantum sit, quantum vos existimatis; hoc dicam, hunc a patre continuo ad me esse deductum; nemo hunc M. Caelium in illo aetatis flore vidit nisi aut cum patre aut mecum aut in M. Crassi castissima domo, cum artibus honestissimis erudiretur. Et eodem impetu Domitianum praecipitem per scalas itidem funibus constrinxerunt, eosque coniunctos per ampla spatia civitatis acri raptare discursu. iamque artuum et membrorum divulsa conpage supercandentes corpora mortuorum ad ultimam truncata deformitatem velut exsaturati mox abiecerunt in flumen. Erat autem diritatis eius hoc quoque indicium nec obscurum nec latens, quod ludicris cruentis delectabatur et in circo sex vel septem aliquoties vetitis certaminibus pugilum vicissim se concidentium perfusorumque sanguine specie ut lucratus ingentia laetabatur.

Ego vero sic intellego, Patres conscripti, nos hoc tempore in provinciis decernendis perpetuae pacis habere oportere rationem. Nam quis hoc non sentit omnia alia esse nobis vacua ab omni periculo atque etiam suspitione belli?

## Keywords

Delatus delatus nominatus onere aut trahebatur quod tenus et bonorum.