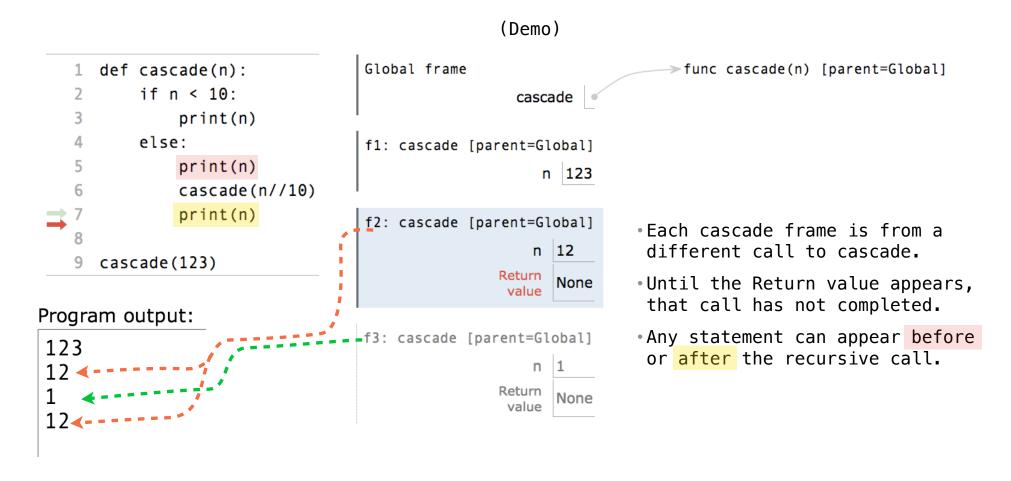


## The Cascade Function



## Two Definitions of Cascade

(Demo)

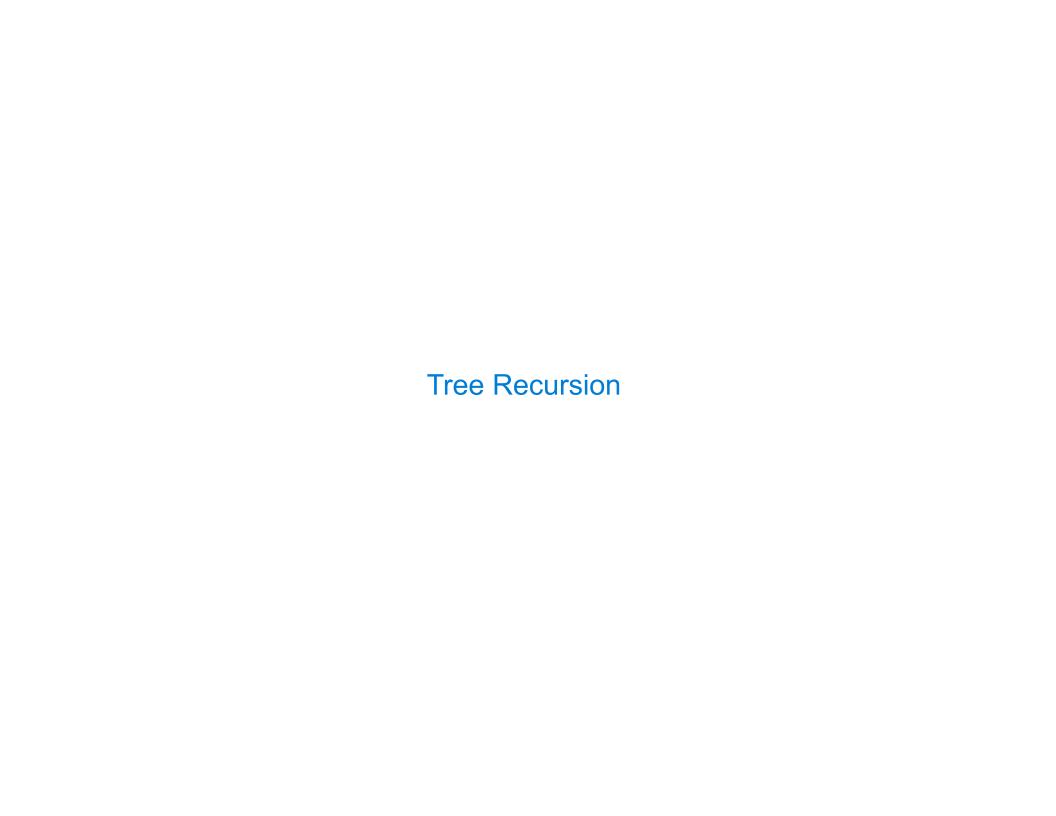
- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

## **Inverse Cascade**

```
Write a function that prints an inverse cascade:
```

/



## Tree Recursion

Tree—shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

```
n: 0, 1, 2, 3, 4, 5, 6, 7, 8, ..., 35

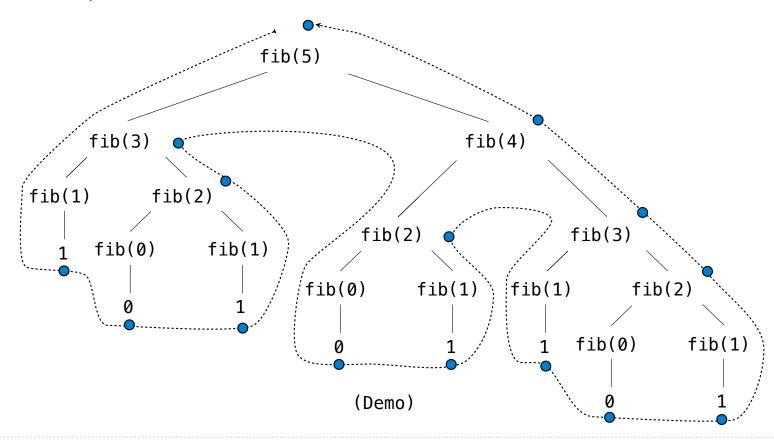
fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21, ..., 9,227,465
```

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



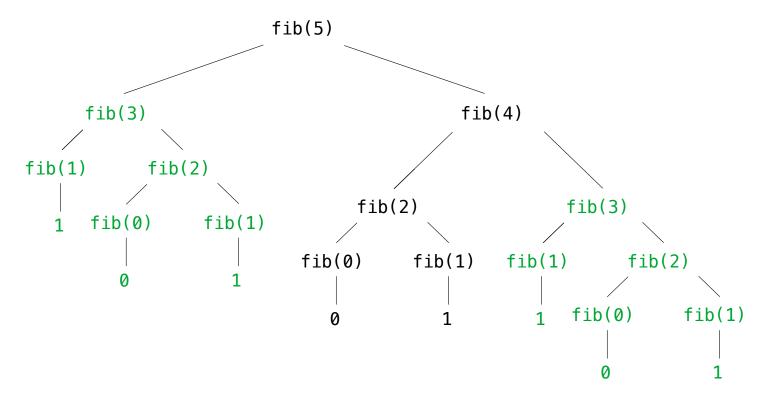
# A Tree-Recursive Process

The computational process of fib evolves into a tree structure



# Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times



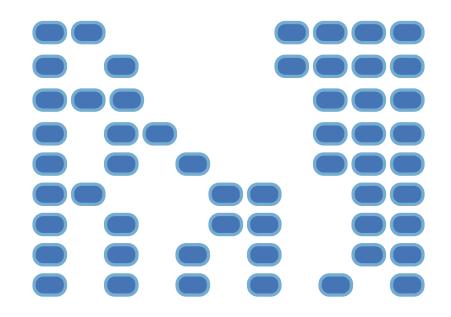
(We will speed up this computation dramatically in a few weeks by remembering results)

**Example: Counting Partitions** 

# **Counting Partitions**

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

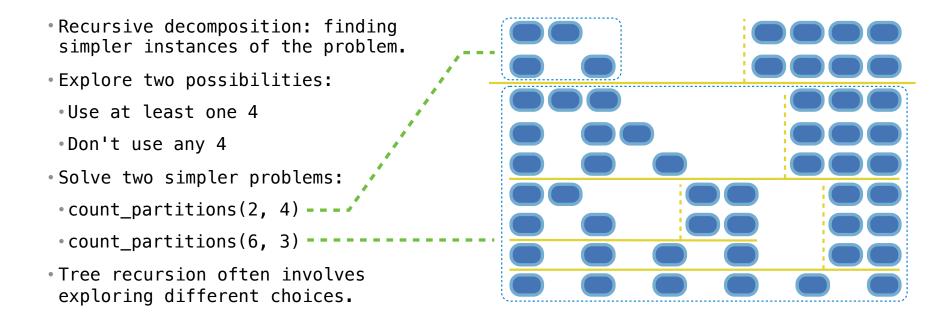
#### count\_partitions(6, 4)



# **Counting Partitions**

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in non-decreasing order.

count\_partitions(6, 4)



## **Counting Partitions**

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
def count partitions(n, m):
Recursive decomposition: finding
                                              if n == 0:
 simpler instances of the problem.
                                                  return 1
Explore two possibilities:
                                              elif n < 0:
                                                   return 0
•Use at least one 4
                                              elif m == 0:
•Don't use any 4
                                                  return 0
• Solve two simpler problems:
                                               else:
                                               with m = count partitions(n-m, m)
 count partitions(2, 4) ---
                                                   without m = count partitions(n, m-1)
 count partitions(6, 3) -----
                                                   return with m + without m

    Tree recursion often involves

 exploring different choices.
                                           (Demo)
```