

# 1 Efficiency

When we talk about the efficiency of a function, we are often interested in the following: as the size of the input grows, how does the runtime of the function change? And what do we mean by “runtime”?

- `square(1)` requires one primitive operation: `*` (multiplication). `square(100)` also requires one. No matter what input `n` we pass into `square`, it always takes one operation.

input	function call	return value	number of operations
1	<code>square(1)</code>	$1 \cdot 1$	1
2	<code>square(2)</code>	$2 \cdot 2$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
100	<code>square(100)</code>	$100 \cdot 100$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n$	<code>square(<math>n</math>)</code>	$n \cdot n$	1

- `factorial(1)` requires one multiplication, but `factorial(100)` requires 100 multiplications. As we increase the input size of `n`, the runtime (number of operations) increases linearly proportional to the input.

input	function call	return value	number of operations
1	<code>factorial(1)</code>	$1 \cdot 1$	1
2	<code>factorial(2)</code>	$2 \cdot 1 \cdot 1$	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
100	<code>factorial(100)</code>	$100 \cdot 99 \cdots 1 \cdot 1$	100
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n$	<code>factorial(<math>n</math>)</code>	$n \cdot (n - 1) \cdots 1 \cdot 1$	$n$

Here are some general guidelines for finding the order of growth for the runtime of a function:

- If the function is recursive or iterative, you can subdivide the problem as seen above:
  - Count the number of recursive calls/iterations that will be made in terms of input size  $n$ .
  - Find how much work is done per recursive call or iteration in terms of input size  $n$ .

The answer is usually the product of the above two, but be sure to pay attention to control flow!

## 2 Efficiency

- If the function calls helper functions that are not constant-time, you need to take the runtime of the helper functions into consideration.
- We can ignore constant factors. For example  $1000000n$  and  $n$  steps are both linear.
- We can also ignore smaller factors. For example if  $h$  calls  $f$  and  $g$ , and  $f$  is Quadratic while  $g$  is linear, then  $h$  is Quadratic.
- For the purposes of this class, we take a fairly coarse view of efficiency. All the problems we cover in this course can be grouped as one of the following
  - Constant: the amount of time does not change based on the input size. Rule:  $n \rightarrow 2n$  means  $t \rightarrow t$ .
  - Logarithmic: the amount of time changes based on the logarithm of the input size. Rule:  $n \rightarrow 2n$  means  $t \rightarrow t + k$ .
  - Linear: the amount of time changes based on the logarithm of the input size. Rule:  $n \rightarrow 2n$  means  $t \rightarrow 2t$ .
  - Quadratic: the amount of time changes based on the logarithm of the input size. Rule:  $n \rightarrow 2n$  means  $t \rightarrow 4t$ .
  - Exponential: the amount of time changes based on the logarithm of the input size. Rule:  $n \rightarrow n + 1$  means  $t \rightarrow 2t$ .

## Questions

1.1 What is the efficiency of `bonk`?

```
def bonk(n):  
    total = 0  
    while n >= 2:  
        total += n  
        n = n / 2  
    return total
```

Logarithmic, because our while loop iterates at most  $\log(n)$  times, due to  $n$  being halved in every iteration. Another way of looking at this if you duplicate the input, we only add a single iteration to the time, which also indicates logarithmic. [Video walkthrough](#)

1.2 Previously, we looked at the `is_prime` function. Here's the code for it:

```
def is_prime(n):  
    if n == 1:  
        return False  
    k = 2  
    while k < n:  
        if n % k == 0:  
            return False  
        k += 1  
    return True
```

What is the efficiency of `is_prime`?

Linear

1.3 What is the efficiency of `mod_7`?

```
def mod_7(n):  
    if n % 7 == 0:  
        return 0  
    else:  
        return 1 + mod_7(n - 1)
```

Constant, since at worst it will require 6 recursive calls to reach the base case.