

## 1 Mutability

### Questions

- 1.1 Name two data types that are mutable. What does it mean to be mutable?

Dictionaries, Lists. Being mutable means we can modify them after they've been created.

- 1.2 Name at least two data types that are not mutable.

Tuples, functions, int, float

- 1.3 Will the following code error? If so, why?

```
a = 1
b = 2
dt = {a: 1, b: 2}
```

No -- a and b are both immutable, so we can use them as Dictionary keys.

```
a = [1]
b = [2]
dt = {a: 1, b: 2}
```

Yes -- a and b are mutable, so we can't use them as Dictionary keys.

- 1.4 Fill in the output and draw a box-and-pointer diagram for the following code. If an error occurs, write Error, but include all output displayed before the error.

```
a = [1, [2, 3], 4]
c = a[1]
c
```

[2, 3]

```
a.append(c)
a
```

[1, [2, 3], 4, [2, 3]]

```
c[0] = 0
c
```

```
[0, 3]
```

```
a
```

```
[1, [0, 3], 4, [0, 3]]
```

```
a.extend(c)
```

```
c[1] = 9
```

```
a
```

```
[1, [0, 9], 4, [0, 9], 0, 3]
```

```
list1 = [1, 2, 3]
```

```
list2 = [1, 2, 3]
```

```
list1 == list2
```

```
True
```

```
list1 is list2
```

```
False
```

### 1.5 Check your understanding:

1 What is the difference between the append function, extend function, and the '+' operator?

The append and extend functions both return a value of none. They just mutate the list that we are currently working on. For example, if we did `a = [1,2,3].append(4)`, `a` would evaluate to `None` because the return value of the append function is `None`. However, when we are using the `+` operator, we return the value of the two lists added together. If we did `a = [1,2,3] + [4,5,6]`, we would get that `a` is equal to `[1,2,3,4,5,6]`. The difference between appends and extends is that appends opens up one single space in the list to place the the parameter of appends. This allows for appends to take in both numbers and lists. The extends function takes in a list (that we will refer to as `a`) as its parameter and will open `len(a)` number of boxes in the original list that we are extending.

2 Given the below code, answer the following questions: `a = [1, 2, [3, 4], 5]`

```
b = a[:]
```

```
b[1] = 6
```

```
b[2][0] = 7
```

What does `b` evaluate to?

`b = [1,6, [7, 4], 5]`

What does `a` evaluate to? Are `a` and `b` the same? Please explain your reasoning.

`A = [1,2,[7,4], 5]`.

`A`, `B` are not the same. Because lists are mutable, when you assign `b` to a shallow copy of `a`, you are also copying the pointers to lists within `a`. Thus, that is why nested elements inside a list changed in both arrays, but all the other elements were unaffected by changes to the shallow copy.

## 2 Data Abstraction

### Questions

- 2.1 What are the two types of functions necessary to make an Abstract Data Type? What do they do?

Constructors make the ADT.

Selectors take instances of the ADT and output relevant information stored in it.

- 2.2 Assume that **rational**, **numer**, **denom**, and **gcd** run without error and behave as described below. Can you identify where the abstraction barrier is broken? Come up with a scenario where this code runs without error and a scenario where this code would stop working.

```
def rational(num, den): # Returns a rational number ADT
    #implementation not shown
def numer(x): # Returns the numerator of the given rational
    #implementation not shown
def denom(x): # Returns the denominator of the given rational
    #implementation not shown
def gcd(a, b): # Returns the GCD of two numbers
    #implementation not shown

def simplify(f1): #Simplifies a rational number
    g = gcd(f1[0], f1[1])
    return rational(numer(f1) // g, denom(f1) // g)

def multiply(f1, f2): # Multiplies and simplifies two rational numbers
    r = rational(numer(f1) * numer(f2), denom(f1) * denom(f2))
    return simplify(r)

x = rational(1, 2)
y = rational(2, 3)
multiply(x, y)
```

The abstraction barrier is broken inside `simplify(f1)` when calling `gcd(f1[0], f1[1])`. This assumes `rational` returns a type that can be indexed through. i.e. This would work if `rational` returned a list. However, this would not work if `rational` returned a dictionary.

The correct implementation of `simplify` would be

```
def simplify(f1):
    g = gcd(numer(x), denom(x))
    return rational(numer(f1) // g, denom(f1) // g)
```

- 2.3 Check your understanding

## 1 How do we know what we are breaking an abstraction barrier?

Put simply, a Data Abstraction Violation is when you bypass the constructors and selectors for an ADT, and directly use how its implemented in the rest of your code, thus assuming that its implementation will not change.

We cannot assume we know how the ADT is constructed except by using constructors and likewise, we cannot assume we know how to access details of our ADT except through selectors. The details are supposed to be abstracted away by the constructors and selectors. If we bypass the constructors and selectors and access the details directly, any small change to the implementation of our ADT could break our entire program.

## 2 What are the benefits to Data Abstraction?

With a correct implementation of these data types, it makes for more readable code because:

- You can make constructors and selectors have more informative names.
- Makes collaboration easier
- Other programmers don't have to worry about implementation details.
- Prevents error propagation.
- Fix errors in a single function rather than all over your program.

# 3 Trees

## Questions

3.1 Fill in this implementation of the Tree ADT.

```
def tree(label, branches = []):
    for b in branches:
        assert is_tree(b), 'branches must be trees'
    return [label] + list(branches)

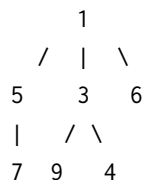
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for b in branches(tree):
        if not is_tree(b):
            return False
    return True

def label(tree):
    return tree[0]

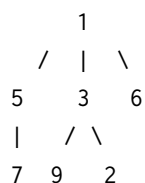
def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return not branches(tree)
```

3.2 A min-heap is a tree with the special property that every nodes value is less than or equal to the values of all of its children. For example, the following tree is a min-heap:



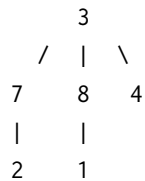
However, the following tree is not a min-heap because the node with value 3 has a value greater than one of its children:



Write a function **is\_min\_heap** that takes a tree and returns True if the tree is a min-heap and False otherwise.

```
def is_min_heap(t):
    for b in branches(t):
        if label(t) > label(b) or not is_min_heap(b):
            return False
    return True
```

- 3.3 Write a function **largest\_product\_path** that finds the largest product path possible. A product path is defined as the product of all nodes between the root and a leaf. The function takes a tree as its parameter. Assume all nodes have a non-negative value.



For example, calling **largest\_product\_path** on the above tree would return 42, since  $3 * 7 * 2$  is the largest product path.

```
def largest_product_path(tree):
    """
    >>> largest_product_path(None)
    0
    >>> largest_product_path(tree(3))
    3
    >>> t = tree(3, [tree(7, [tree(2)]), tree(8, [tree(1)]), tree(4)])
    >>> largest_product_path(t)
    42
    """

    if not tree:
        return 0
    elif is_leaf(tree):
        return label(tree)
    else:
        paths = [largest_product_path(t) for t in branches(tree)]
        return label(tree) * max(paths)
```

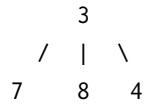
- 3.4 Implement a function `max_tree`, which takes a tree `t`. It returns a new tree with the exact same structure as `t`; at each node in the new tree, the entry is the largest number that is contained in that node's subtrees or the corresponding node in `t`.

```
def max_tree(t):
>>> max_tree(tree(1, [tree(5, [tree(7)]),tree(3,[tree(9),tree(4)]),tree(6)])
tree(9, [tree(7, [tree(7)]),tree(9,[tree(9),tree(4)]),tree(6)])
    if ____:
        return ____
    else:
        new_branches= ____
        new_label = ____
        return ____

def max_tree(t):
    if is_leaf(t):
        return tree(root(t))
    else:
        new_branches = [max_tree(b) for b in branches(t)]
        new_label = max([root(t)] + [root(s) for s in new_branches])
        return tree(new_label, new_branches)
```



- 3.5 Challenge Question: The level-order traversal of a tree is defined as visiting the nodes in each level of a tree before moving onto the nodes in the next level. For example, the level order of the following tree is: 3 7 8 4



Write a function **level\_order** that takes in a tree as the parameter and returns a list of the values of the nodes in level order.

```

def level_order(tree):

    #iterative solution
    def level_order(tree)
        if not tree:
            return []
        current_level, next_level = [label(tree)], [tree]
        while next_level:
            find_next= []
            for b in next_level:
                find_next.extend(branches(b))
            next_level = find_next
            current_level.extend([label(t) for t in next_level])
        return current_level
  
```

- 3.6 Challenge Question: Write a function **all\_paths** which will return a list of lists of all the possible paths of an input tree, t. When the function is called on the same tree as the problem above, the function would return: [[3, 7], [3, 8], [3, 4]]

```

def all_paths(t):
    if _____:
        _____
    else:
        _____
        _____
        _____
        _____
        _____
  
```

```
def all_paths(t):  
    if is_leaf(t):  
        return [[label(t)]]  
    else:  
        total = []  
        for b in branches(t):  
            for path in all_paths(b):  
                total.append([label(t)] + path)  
        return total
```

## 4 Nonlocal Questions

4.1 Draw an environment diagram for the following code:

```
spiderman = 'peter parker'
def spider(man):
    def myster(io):
        nonlocal man
        man = spiderman
        spider = lambda stark: stark(man) + ' ' + io
        return spider
    return myster
truth = spider('quentin is')('the greatest superhero')(lambda x: x)
```

<http://bit.ly/2XZSoEL>

4.2 Draw an environment diagram for the following code:

```
fa = 0
```

```
def fi(fa):  
    def world(cup):  
        nonlocal fa  
        fa = lambda fi: world or fa or fi  
        world = 0  
        if (not cup) or fa:  
            fa(2022)  
            fa, cup = world + 2, fa  
            return cup(fa)  
        return fa(cup)  
    return world
```

```
won = lambda opponent, x: opponent(x)  
us = won(fi(fa), 2019)
```

<http://bit.ly/2G9zxMr>

- 4.3 Write **make\_max\_finder**, which takes in no arguments but returns a function which takes in a list. The function it returns should return the maximum value it's been called on so far, including the current list and any previous list. You can assume that any list this function takes in will be nonempty and contain only non-negative values.

```
def make_max_finder():
    """
    >>> m = make_max_finder()
    >>> m([5, 6, 7])
    7
    >>> m([1, 2, 3])
    7
    >>> m([9])
    9
    >>> m2 = make_max_finder()
    >>> m2([1])
    1
    """

    max_so_far = 0
    def find_max_overall(lst):
        nonlocal max_so_far
        if max(lst) > max_so_far:
            max_so_far = max(lst)
        return max_so_far
    return find_max_overall
```

## 4.4 Check your understanding:

```

x = 5
def f(x):
    def g(s):
        def h(h):
            nonlocal x
            x = x + h
            return x
        nonlocal x
        x = x + x
        return h
    print(x)
    return g
t = f(7)(8)(9)

```

- What is t after the code is executed?
- In the h frame, which x is being referenced? Which frame?
- In the g frame, is a new variable x being created?

<http://bit.ly/2G9zxMr>

- 7
- the x, that is the parameter for f(x) from line 2 ... or frame 1.
- no, g (f2) refers to the x in parent (f1)

## 5 Iterators and Generators

### Questions

- 5.1 What is the definition of an iterable? What is the definition of an iterator? What is the definition of a generator? What built-in functions or keywords are associated with each. Give an example of each.

An iterable is any object that can be passed to the built-in `iter` function. In other words, an iterable is any object that can produce iterators.

An iterator is an object that provides sequential access to values one by one. Its contents can be accessed through the built-in `next` function, and it will signal there are no more values available with a `StopIteration` exception when `next` is called.

A generator object is an iterator, but it is created in a special way – generator functions are defined as a function that yields its values instead of returning them. When generator functions are called, they return a generator object, which can then be used as an iterator.

- 5.2 Evaluate if each line is valid? If not, state the error and how you would fix it.

```
>>> new_list = [2, 3, 6, 8, 8, 3]
>>> next(new_list)
```

```
>>> iter(new_list)[1]
```

```
>>> [x for x in iter(new_list)]
```

```
>>> for i in range(len(iter(new_list))):
...     new_list.append(2)
```

A)

Error: `new_list` is an Iterable not Iterator

Fix:

```
>>> next(iter(new_list))
```

Output:

2

B)

Error, can't use indexing on an iterator

```
>>> new_list[1]
```

3

C)

```
[2, 3, 6, 8, 8, 3]
```

D)

```
Error, cant call len on iterator object  
>>> for i in range(len(new_list)):  
new_list.append(2)
```



5.3 What is the difference between these two statements?

```
a.  def infinity1(start):
        while True:
            start = start + 1
            return start

b.  def infinity2(start):
        while True:
            start = start + 1
            yield start
```

(a) is a function since it uses a return statement. Even tho while True is always true, it will stop after the first iteration when it returns start. On the other hand, (b) is a generator since it uses a yield statement. Since while True is always true, calling next will iterate once and yield start

What would python display?

```
>>> infinity1
```

```
<Function>
```

```
>>> infinity2
```

```
<Function>
```

```
>>> infinity1(2)
```

```
3
```

```
>>> infinity2(2)
```

```
<Generator Instance>
```

```
>>> x = infinity1(2)
```

```
Nothing
```

```
>>> next(x)
```

```
Error, cant call next on integer
```

```
>>> y = infinity2(2)
```

```
Nothing
```

```
>>> next(y)
```

3

```
>>> next(y)
```

4

```
>>> next(infinity2(2))
```

3

- 5.4 They can't stop all of us!!! Write a function **generate\_constant** which, a generator function that repeatedly yields the same value forever.

```
def generate_constant(x):
    """A generator function that repeats the same value x forever.
    >>> area = generate_constant(51)
    >>> next(area)
    51
    >>> next(area)
    51
    >>> sum([next(area) for _ in range(100)])
    5100
    """
```

```
while True:
    yield x
```

- 5.5 4.2 Now implement **black\_hole**, a generator that yields items in seq until one of them matches trap, in which case that value should be repeatedly yielded forever. You may assume that **generate\_constant** works. You may not index into or slice seq.

```
def black_hole(seq, trap):
    """A generator that yields items in SEQ until one of them matches TRAP, in which case that
    value should be repeatedly yielded forever.
    >>> trapped = black_hole([1, 2, 3], 2)
    >>> [next(trapped) for _ in range(6)]
    [1, 2, 2, 2, 2, 2]
    >>> list(black_hole(range(5), 7))
    [0, 1, 2, 3, 4]
    """
```

```
for item in seq:
    if item == trap:
        yield from generate_constant(trap)
    else:
        yield item
```

## 5.6 What Would Python Display?

```
>>> def weird_gen(x):
...     if x % 2 == 0:
...         yield x * 2
>>> wg = weird_gen(2)
>>> next(wg)
>>> next(weird_gen(2))
```

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```
>>> next(wg)
```

StopIteration

```
>>> def greeter(x):
...     while x % 2 != 0:
...         print('hi')
...         yield x
...         print('bye')
>>> greeter(5)
```

&lt;Generator Object&gt;

```
>>> gen = greeter(5)
>>> g = next(gen)
```

hi

```
>>> g = (g, next(gen))
>>> g
```

byehi(5, 5)

```
>>> next(gen)
```

byehi5

```
>>> next(g)
```

Error, tuple is not iterator

An iterator \_\_\_\_\_ a generator

A generator **is** a(n) \_\_\_\_\_ iterator

An iterator is not always represented by a generator  
A generator is a(n) a special type of/user defined iterator

- 5.7 Write a generator function **gen\_inf** that returns a generator which yields all the numbers in the provided list one by one in an infinite loop.

```
>>> t = gen_inf([3, 4, 5])
>>> next(t)
3
>>> next(t)
4
>>> next(t)
5
>>> next(t)
3
>>> next(t)
4
def gen_inf(lst):
```

```
#solution 1
def gen_inf(lst):
    while True:
        for elem in lst:
            yield elem
#solution 2
def gen_inf(lst):
    while True:
        yield from iter(lst)
```

- 5.8 Implement a generator function called `filter(iterable, fn)` that only yields elements of `iterable` for which `fn` returns `True`.

```
def naturals():
    i = 1
    while True:
        yield i
        i += 1

def filter(iterable, fn):
    """
    >>> is_even = lambda x: x % 2 == 0
    >>> list(filter(range(5), is_even))
    [0, 2, 4]
    >>> all_odd = (2*y-1 for y in range (5))
    >>> list(filter(all_odd, is_even))
    []
    >>> s = filter(naturals(), is_even)
    >>> next(s)
    2
    >>> next(s)
    4
    """

    for elem in iterable:
        if fn(elem):
            yield elem
```

- 5.9 What could you use a generator for that you could not use a standard iterator paired with a function for?

Call `next` on an infinite iterator

- 5.10 Define **tree\_sequence**, a generator that iterates through a tree by first yielding the root value and then yielding the values from each branch.

```
def tree_sequence(t):
    """
    >>> t = tree(1, [tree(2, [tree(5)]), tree(3, [tree(4)])])
    >>> print(list(tree_sequence(t)))
    [1, 2, 5, 3, 4]
    """
```

```
yield label(t)
for branch in branches(t):
    for value in tree_sequence(branch):
        yield value
```

Alternate solution:

```
yield label(t)
for branch in branches(t):
    yield from tree_sequence(branch)
```



- 5.11 Write a function **make\_digit\_getter** that, given a positive integer *n*, returns a new function that returns the digits in the integer one by one, starting from the rightmost digit.

Once all digits have been removed, subsequent calls to the function should return the sum of all the digits in the original integer.

```
def make_digit_getter(n):
    """ Returns a function that returns the next digit in n
    each time it is called, and the total value of all the integers
    once all the digits have been returned.
    >>> year = 8102
    >>> get_year_digit = make_digit_getter(year)
    >>> for _ in range(4):
    ... print(get_year_digit())
    2
    0
    1
    8
    >>> get_year_digit()
    11
    """
```

```
def make_digit_getter(n):
    total = 0
    def get_next():
        nonlocal n, total
        if n == 0:
            return total
        val = n % 10
        n = n // 10
        total += val
        return val
    return get_next
```

5.12 Sorry another environment diagram, but it's the last one I promise.

```
def iter(iterable):
    def iterator(msg):
        nonlocal iterable
        if msg == 'next':
            next = iterable[0]
            iterable = iterable[1:]
            return next
        elif msg == 'stop':
            raise StopIteration
    return iterator
def next(iterator):
    return iterator('next')
def stop(iterator):
    iterator('stop')

lst = [1, 2, 3]
iterator = iter(lst)
elem = next(iterator)
```

<https://tinyurl.com/y3xxycgp>