Physics 333 – Problem Set 4

Question 1: Flight

a) Estimate how much more fuel a Boeing 747-300 has to carry on a flight from Vancouver International Airport to London Heathrow Airport to accommodate for laptop use in flight. Assume that everyone on the plane has a laptop and that the jet fuel is converted to electricity with an efficiency of 0.33.

Distance: According to Google, YVR to LHR is 7574 km.
747-300 Max Passengers: 608 passengers.
Laptop: 2021 M1 Macbook Air
Laptop Power Consumption: 30 W
Jet Fuel Efficiency: 0.33
Laptop On-time Duration: Equivalent to flight duration of 9hr20mins = 33600 seconds
Energy of Jet Fuel: 35 MJ per litre

Thus, we can now calculate the energy used per passenger and then convert it to Jet Fuel used per passenger.

$$\text{Energy} = (30J/second)(33600seconds) = 1.008 \text{ MJ}$$
$$\text{Jet Fuel} = \frac{1.008MJ}{(0.333)(35MJ/L)} = 0.086 \text{ litre}$$

With 608 passengers, we can observe that each passenger with a laptop plugged in will use 0.086 litres of additional jet fuel. Thus, 608 passengers ALL with laptops plugged in will use 52.288 L of jet fuel.

b) How does the cost of transport of a Boeing 747-300 compare to that of a 2010 Toyota Prius, which has a fuel economy of 4.7 L/100 km? Compare is terms of L/100 km/passenger. Assuming the average number of passengers in Toyota to be 1.5.

According to Google, a Boeing 747-300 burns 12 L/km which translates to 1200 L/100 km.

Thus, per person we will have:

$$\text{Boeing 747-300: } \frac{1200L/100km}{608passengers} = 1.97 \text{ L/100 km per passenger}$$

$$\text{2010 Toyota Prius: } \frac{4.7L/100km}{1.5passengers} = 3.1 \text{ L/100km per passenger}$$

Surprisingly, the Boeing 747-300 is technically more fuel efficient which is why airplanes rarely ever fly less than full.

c) How much fuel does a plane require to go from rest on the runway to the cruising speed at their typical altitude? Hint: Consider the potential energy and kinetic energy required to get the plane to altitude and to speed. Assume that the plane gets to cruising speed almost immediately and spends 20 mins gaining altitude. Assume that it takes 20 min for the plane to reach cruising altitude. Remember to account for drag. Note that the drag force acts on the airplane wings, not on its cross-sectional area. Include engine's efficiency.

We'll first need to find some facts about the Boeing 747-300.

According to Google...
Boeing 747-300 Takeoff Mass: 340,100 kg Boeing 747-300 Cruise Speed: 933 km/h = 259 m/s.

Boeing 747-300 Cruise Altitude: 35,105 ft = 10700 meters.
Boeing 747-300 Drag Coefficient: 0.031
Boeing 747-300 Cross Sectional Area: 525 m^2

First we will need to account for potential and kinetic energy.

$$\text{Energy} = mgh + \frac{1}{2}mv^2$$
$$= (340,100kg)(9.8m/s^2)(10,700) + \frac{1}{2}(340,100kg)(259m/s)^2$$
$$= 47070010050 \text{ J}$$
$$= 47 \text{ GJ}$$
$$= \text{Engine efficiency is 1/3, so it will actually be...}$$
$$= 141 \text{ GJ}$$

Now we account for the drag force.

$$F_D = \frac{1}{2}pv^2C_DA$$
$$= \frac{1}{2}(0.8kg/m^3)(259m/s)^2(0.031)(525m^2)$$
$$= 436697.31 \text{ N}$$

As it takes 20 minutes and we are traveling at 259 m/s, the plane will travel 310,800 m in 20 minutes. Now we can calculate the work. As the engines are 1/3 efficient, we will need to do 3 times the work.

$$\text{Work} = (436697.31N)(310,800)(3)$$
$$= 407176571844 \text{ J}$$
$$= 407 \text{ GJ}$$

Thus, we will have a total of 141 GJ + 407 GJ = 548 GJ of energy used. Now we simply need to calculate how much fuel will be used.

$$\text{Jet Fuel Consumption} = (454,000,000,000J)(\frac{1}{35x10^6J}) = 15675 \text{ L of fuel}$$

It will take 15657 L of fuel for the plane to go from rest on the run way to cruising speed at cruising altitude.

d) How much fuel does it cost the plane to fly for 20 minutes at cruising altitude and cruising speed? Assume air density at cruising altitude to be 0.4 kg/m3. Compare your answer to part c). Should planes fly at higher or lower altitudes?

As the plane is flying at cruising altitude and cruising speed, we do not need to account for potential or kinetic energy. The only energy lost we need to account for is drag. Thus, we need to just find the cruise speed and altitude of a Boeing 747-300.

According to Google... Boeing 747-300 Cruise Speed: 933 km/h = 259 m/s.
Boeing 747-300 Cruise Altitude: 35,105 ft = 10700 meters.
Boeing 747-300 Drag Coefficient: 0.031
Boeing 747-300 Cross Sectional Area: 525 m^2

$$F_D = \frac{1}{2}pv^2C_DA$$
$$= \frac{1}{2}(0.4kg/m^3)(259m/s)^2(0.031)(525m^2)$$
$$= 218348.655 \text{ N}$$

Now we are able to calculate the work required for 20 minutes. 20 minutes of flight at 259 m/s = 310,800 m. We also need to account that the engine is 1/3 efficient, so 3 times more work.

$$\text{Work} = (218348.655N)(310,800)(3) = 203.6 \text{ GJ}$$

$$\text{Jet Fuel Consumption} = (203,588,285,922J)(\frac{1}{35x10^6J}) = 5816 \text{ L of fuel to cruise for 20 minutes.}$$

It will cost 5816 L of fuel to cruise at cruising altitude and speed for 20 minutes. Quite evidently, it is much better for planes to fly at higher altitudes with lower air density.

Question 2: BC Ferries

Over the Summer I took a ferry to Gabriola Island. Actually, I took two. The first, which goes from Vancouver to Naniamo, was the giant super ferry called the Coastal Celebration. The second, which goes from Naniamo to Gabriola was the Quinsam.

a) Calculate the hull speed for each of these ferries. You can Google the lengths.

The Coastal Celebration has a hull length of 160m.
The Quinsam has a hull length of 90m.

Thus, the hull speed for these ferries are:

$$V_{CoastalCelebration} = 3.4\sqrt{160m} = 43 \text{ knots.}$$
$$V_{Quinsam} = 3.4\sqrt{90m} = 32 \text{ knots.}$$

b) Calculate the Froude number at hull velocity for each of these ferries. Is this what you expect? Explain.

43 knots = 22.1211 meters per second.
32 knots = 16.4622 meters per second.

Using the equation for Froude number, we get:

$$FR_{CoastalCelebration} = \frac{22.12m/s}{\sqrt{g(160m)}} = 0.56$$
$$FR_{Quinsam} = \frac{16.46m/s}{\sqrt{g(90m)}} = 0.55$$

Expectantly, they are practically the same number. This is because this aligns with the critical Froude Number of 0.56.

c) Using the graph below, estimate the speed at which you expect each ship to travel. This is marked the "design speed" in the figure below, and the Fr where the drag starts increasing exponentially. Compare your estimate to the actual maximum speed of the ferries. Comment on why your estimate is above or below the actual travel speed. Think of the purpose of the ferries.

Real Maximum Velocity of Coastal Celebration = 23 knots.
Real Maximum Velocity of Quinsam = 12 knots.

$$V_{CoastalCelebration} = 0.267\sqrt{g(160m)} = 21 \text{ knots.}$$
$$V_{Quinsam} = 0.267\sqrt{g(90m)} = 15 \text{ knots.}$$

After using the Froude number of 0.267 to calculate the velocity, we can observe they are relatively close to the maximum velocities of the Coastal Celebration and Quinsam. In the case of the Coastal Celebration, it likely has a higher maximum velocity of 23 knots in comparison to it's optimal velocity of 21 knots by design for emergencies. It would likely travel at 21 knots most of the time and go up to 23 knots if it has to. However, this is not the case in the Quinsam where the optimal velocity is 15 knots, but it has a maximum velocity of 12 knots. As the Quinsam travels shorter distances, the engineers probably found a net positive gain by having a smaller engine that is close to the optimal velocity of 15 knots, instead of having a larger engine that consumes more fuel to achieve 15 knots.

Question 3: Bikes vs. Cars

It turns out that people, just like engines, are about 25 percent efficient at turning chemical energy into mechanical energy. We're going to assume that during our commute cars and bikes have the same average speed of 20 km/hr. Choose appropriate values for the CD and μRR for the car and the bike.

a) Calculate the energy cost of transport for a each mode of transport in terms of kJ/km. Assume each mode is travelling a constant speed.

Car (Audi A4):
Drag Coefficient C_D: 0.29
Frontal Area: $2.03m^2$
Rolling resistance μRR: 0.015
Total mass with one person: 1600 kg + 70 kg = 1670 kg

Bicycle (Carbon Fiber):
Drag Coefficient C_D: 0.9
Frontal Area: $1.5m^2$
Rolling resistance μRR: 0.004
Total mass with one person: 7 kg + 70 kg = 77 kg

First, we need to calculate the work done by using the equation:

$$W = \frac{1}{2}pAC_Dv^2d + \mu RRmg$$

Work done for Car per kJ/km

$$= \frac{1}{2}(1.23kg/m^3)(2.03m^2)(0.29)(1000m)(5.6m/s)^2 + (0.015)(1670kg)(9.81m/s^2)(1000m)$$
$$= 11.30kJ + 245.74kJ = 257.04kJ$$

Energy cost of transport for Car = $\frac{257.04kJ}{0.25}$ = 1028.16 kJ per km

Work done for Bicycle per kJ/km

$$= \frac{1}{2}(1.23kg/m^3)(1.5m^2)(0.9)(1000m)(5.6m/s)^2 + (0.004)(77kg)(9.81m/s^2)(1000m)$$
$$= 26.03kJ + 3.02kJ = 29.05kJ$$

Energy cost of transport for Bicycle = $\frac{29.05kJ}{0.25}$ = 116.2 kJ per km

As we can see, the energy cost of transport is approximately 8.8 times higher than that of the bike.

b) For each mode of transportation calculate dollar cost of transportation in dollar/km if we used gasoline to power both.

Current gasoline price in Burnaby: 1.56 dollars per litre.
First google result for gasoline's energy content in joules: 31536 kJ per litre

$$\text{Dollar cost of transportation for Car}$$
$$= \frac{1028.16 kJ/km}{31536 kJ/L}(1.56\text{ dollars/L})$$
$$= 0.05 \text{ dollars per km}$$

$$\text{Dollar cost of transportation for Bicycle}$$
$$= \frac{116.2 kJ/km}{31536 kJ/L}(1.56\text{ dollars/L})$$
$$= 0.005 \text{ dollars per km}$$

c) For each mode of transportation calculate dollar cost of transportation in dollar/km if we used food to power both. Hint: Lesson 6 has some information about the cost of different diets that you might find useful.

I spend approximately 25 dollars per day on food and will use this value for cost. This converts to approximately 0.0026 dollars per kJ. Thus, if we are powering with food, then:

$$\text{Dollar Cost of Transportation for Car}$$
$$= 1028.16 kJ/km(0.0026\text{ dollars/kJ})$$
$$= 2.67 \text{ dollars per km}$$

$$\text{Dollar Cost of Transportation for Bicycle}$$
$$= 116.2 kJ/km(0.0026\text{ dollars/kJ})$$
$$= 0.30 \text{ dollars per km}$$

d) Rank the four costs that you found in b) and c) (even though we don't fuel people with gasoline, and cars with food). Which, money-wise, is the best? Which is the worst? Explain.

1. Bicycle (Gasoline): 0.005 dollars per km
2. Car (Gasoline): 0.05 dollars per km
3. Bicycle (Food): 0.30 dollars per km
4. Car (Food): 2.67 dollars per km

Evidently, we can see that powering a bicycle with gasoline is by far the best bang for your buck and that powering a car with food is by far the worst. Food is significantly more expensive for energy in comparison to gasoline. It's evident that gasoline is extremely good at what it does and is very cost-efficient. In reality (no Car (Food) or Bicycle (Gasoline), if we are thinking about only the perspective of cost per km, a car with gasoline is much better than a bicycle with food. However, humans obviously need to eat on a daily basis so a bicycle, although on paper does not look more cost-efficient, really is due to the fact that you are not buying food just to ride a bicycle.