

# Multi-View Relationships for Analytics and Inference

Eric Lei

Thesis Defense  
July 30, 2019

Committee:

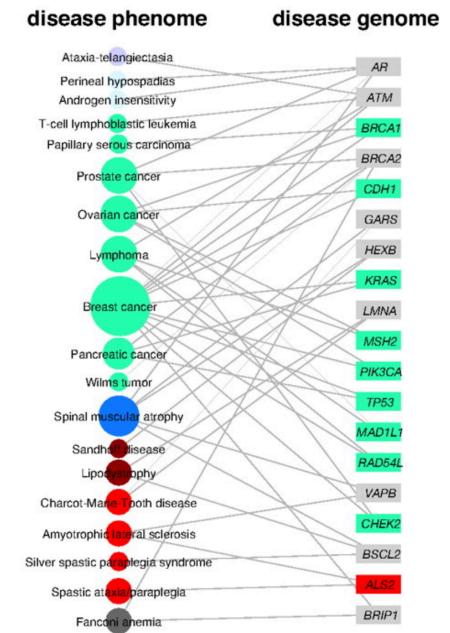
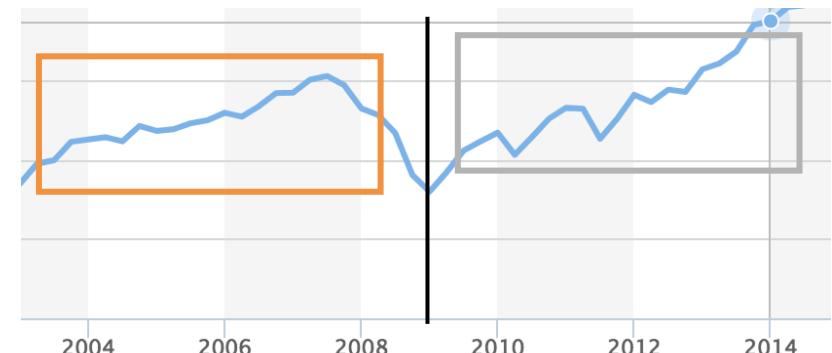
Artur Dubrawski (chair), Barnabas Poczos, Mario Berges, Simon Labov [LLNL]

# Multi-view data

- Features partitioned into multiple sets: *views*
- Latent variables govern the relationship between views
- Multi-view learning finds agreement between views
- Our work explicitly leverages the relationship between views as a unit of analysis

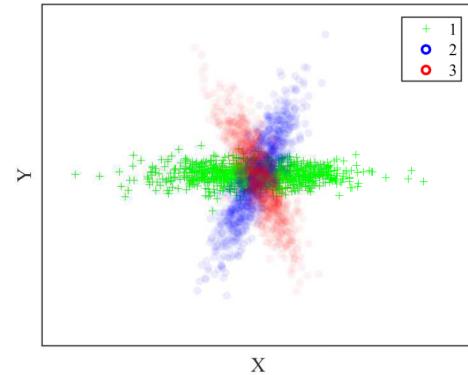


The man at bat readies to swing at the pitch while the umpire looks on.

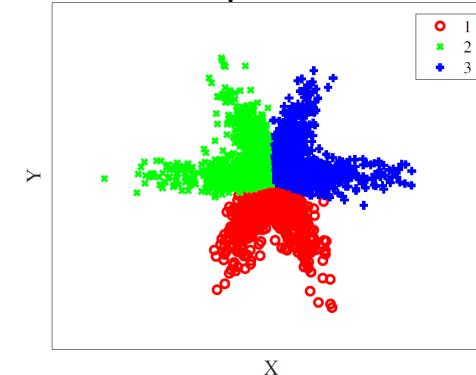


# Characterization of multi-view relationships

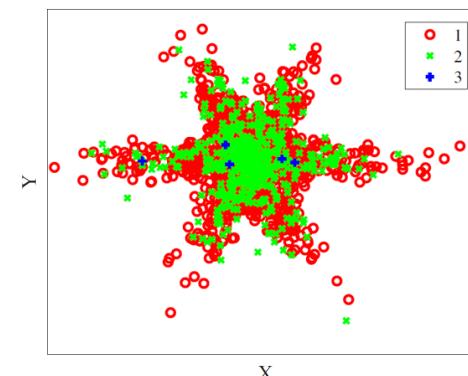
Ground truth



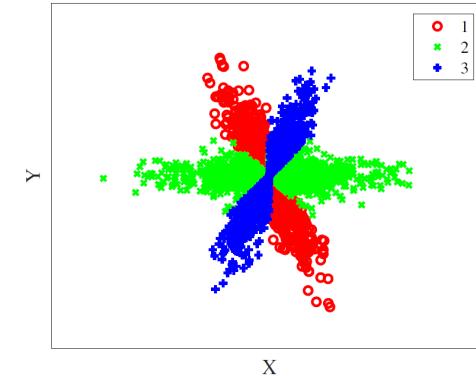
K-means or spectral clustering



Modern multi-view clustering [Zhao 2017]



Clustering aware of multi-view relationships



# Current multi-view learning

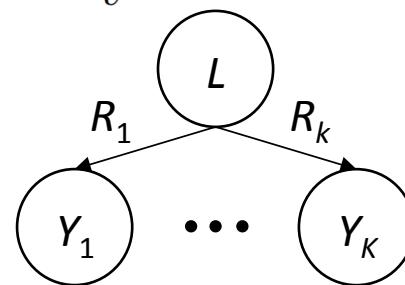
- Regularize single-view solutions  $V^{(v)}$  toward each other [Liu 2013]
- Fuse single-view solutions [Greene 2009]
- Apply single-view algorithms in common subspace by learning shared projection  $Z$  [Gao 2015]
- **This work:** infer latent variables  $L$  that affect all views  $Y_j$ 
  - $L$  often interpretable
  - Analysis of  $L$  and relations  $R_j$

$$\sum_{v=1}^{n_v} \lambda_v \|V^{(v)} - V^*\|_F^2$$

$$\mathbf{X} \approx \mathbf{P}\mathbf{H} \quad \text{such that} \quad \mathbf{P} \geq 0, \mathbf{H} \geq 0$$

Stacked  
solutions  
from views

$$\lambda \sum_v \|Z - Z_v\|$$



Liu et al. (2013). Multi-view clustering via joint nonnegative matrix factorization. ICDM.

Gao et al. (2015). Multi-view subspace clustering. ICCV.

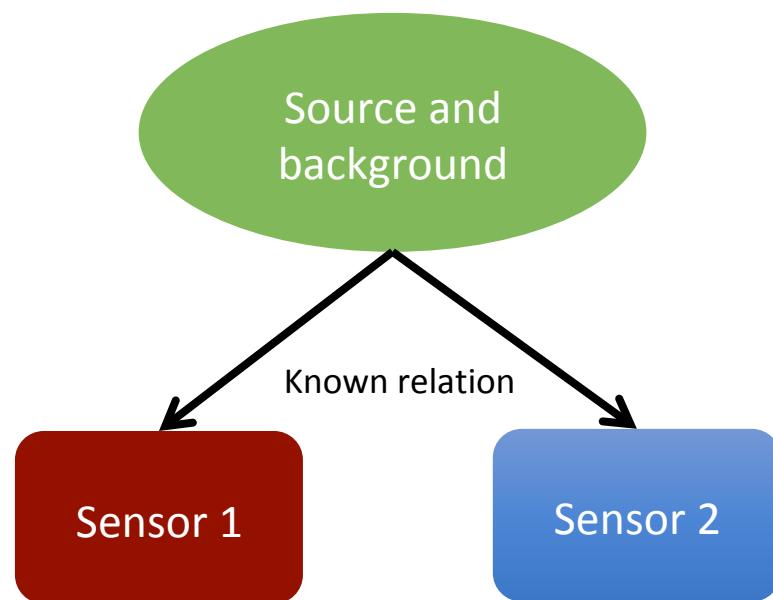
Greene et al. (2009). A matrix factorization approach for integrating multiple data views. ECML PKDD.

# Analysis of multi-view relationships

- **Thesis statement:** It is possible to characterize multi-view relationships and employ them as units of analysis in descriptive analytics and inference
- Present novel methods that characterize multi-view relationships, either using domain knowledge or by learning from data, and employ them as units of analysis
- Reveal structure that alternative methods do not or have competitive empirical performance with the state of the art

# Outline

- **Multi-view filtering**
  - Single sensor method for gamma source detection [NSS 2017]
  - Multiple sensor extension
- Learning multi-view relationships
  - Linear multi-view relationships [NSS 2016]
  - Nonlinear multi-view relationships
    - Clustering [MLHC 2017, ISICEM 2019]
    - Classification [MLHC 2017, ISICEM 2019]



Lei et al. (2016). Radiological threat detection for an unknown energy window by canonical correlation analysis. NSS.  
Lei et al. (2017). Robust detection of radiation threat by simultaneous estimation of source intensity and background. NSS.  
Lei et al. (2017). Bleeding detection by multi-view correlation clustering of central venous pressure. MLHC.  
Lei et al. (2019). Characterization of multi-view hemodynamic data by learning mixtures of multi-output regressors. ISICEM.

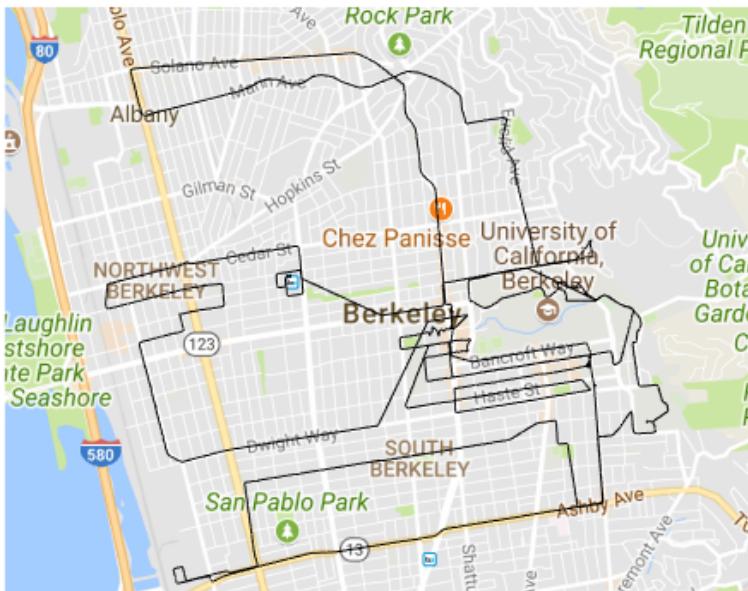
# Multi-view filtering for gamma source detection

- Multiple sensors = multiple views
- How do we leverage multi-view relationships known through domain knowledge?
- Infer latent variables by collectively filtering views
- Reduce dependence on training data

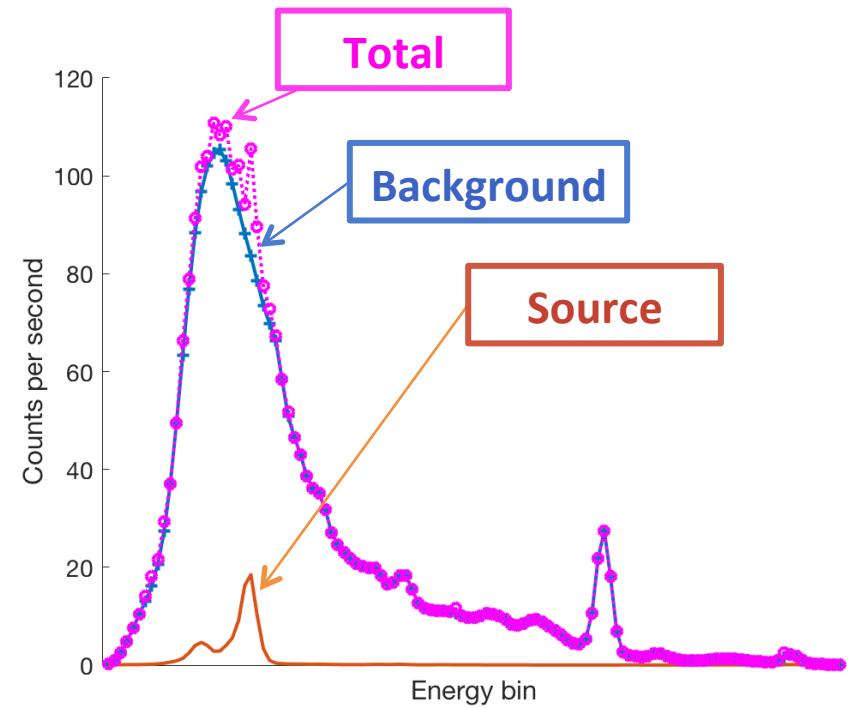


Truck and Pedestrian carry sensors.  
Other Objects are possible source locations.

# Challenges of gamma source detection



Mobile sensor collects photon spectra.



Source affects distribution of photon counts.

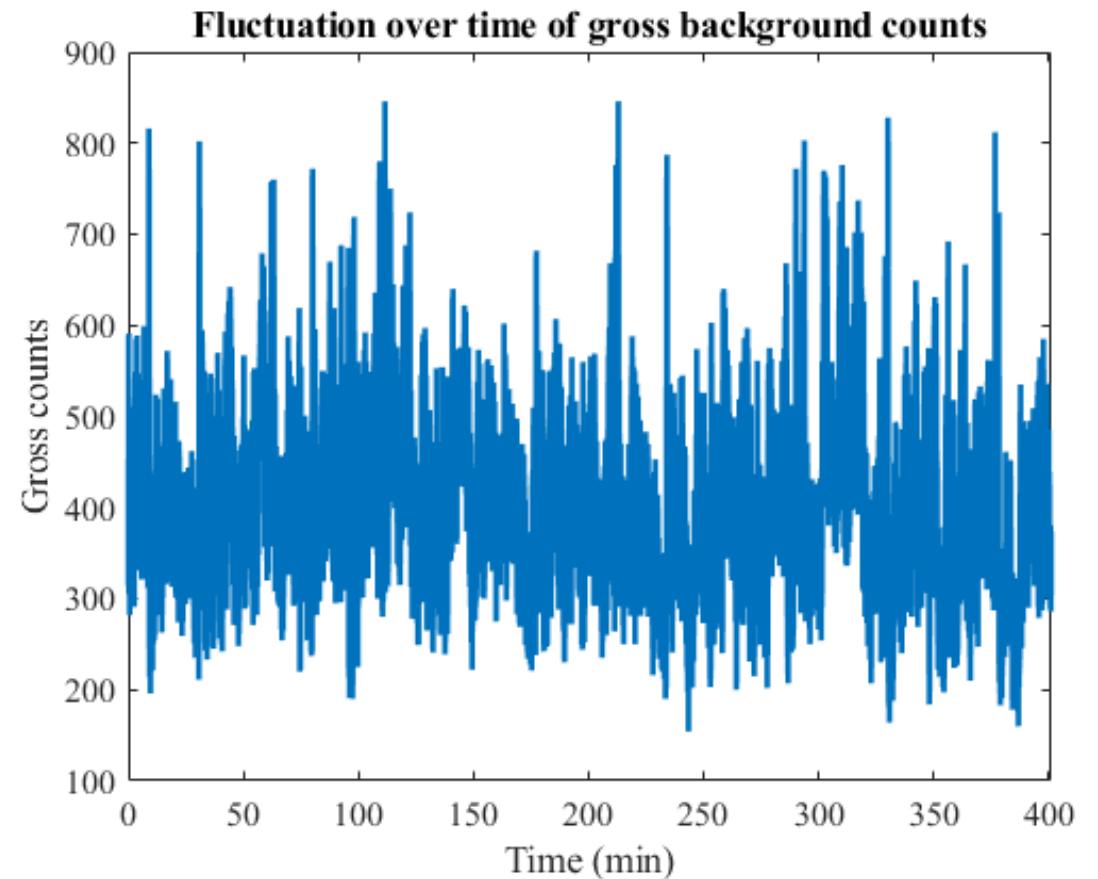
- Unknown background
- Noise
- Hidden sources
- Low signal-to-noise ratio (SNR)

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# Single sensor methods

- Stationary characterizations of background from training
  - Matched Filter [Turin 1960]
  - Spectral Anomaly Detection [Nelson 2012]
  - Gaussian-Poisson MAP [Huggins 2014]
- Assume known source type
- Do not want to depend on training data
  - Robustness to unknown background variation
  - Practical utility



Turin (1960). An introduction to matched filters. IRE Transactions on Information theory.

Nelson and Labov (2012). Aggregation of mobile data. LLNL Technical Report.

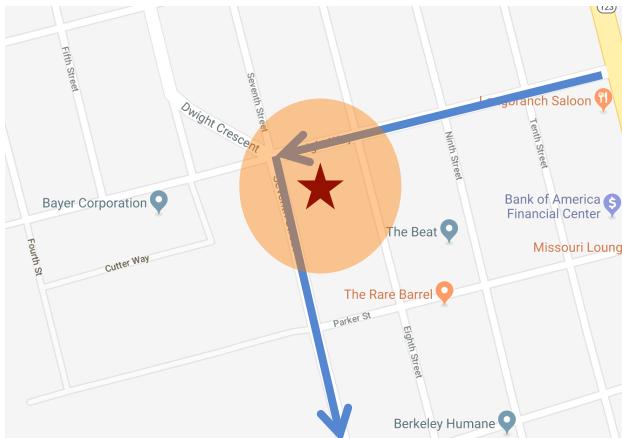
Huggins et al. (2014). Using Gaussian rate priors with Poisson data likelihoods for improved detection of sources of known types in cluttered background scenes. NSS.

# Existing work: State-of-the-art adaptive detector

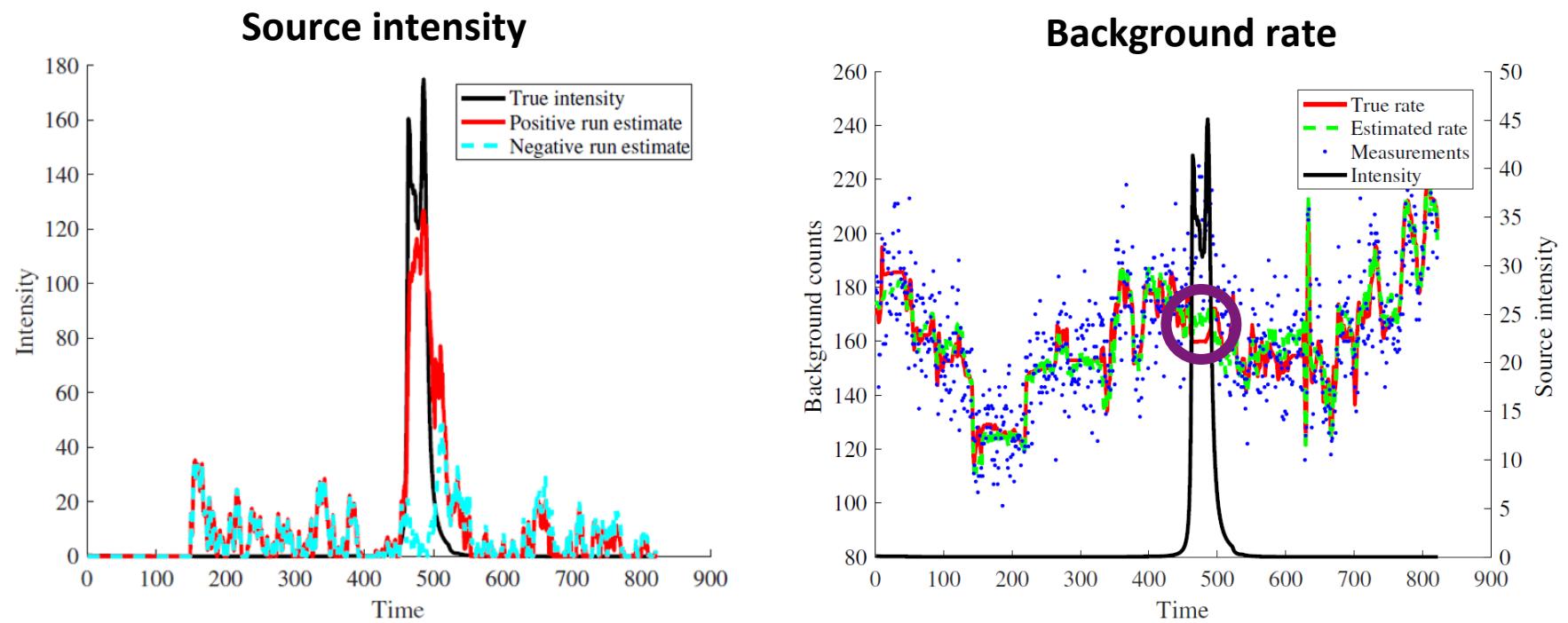
- Orthonormal Subspace Projection Matched Filter with adaptive background basis (RDAK) [Labov 2019]
  - Residual from spectrum  $X$  onto background basis  $B$
  - Similarity with template  $S$
- Warm-up for basis
  - 5 min. minimum
  - 10 min. optimal
- Long warm-up may be infeasible
  - Source contamination
  - No time
  - Warm-up and test mismatch

$$DM(X) = \frac{S^t W(I - B(B^t W B)^{-1} B^t W)X}{k\sqrt{|X|_1}} = \frac{TX}{\sqrt{|X|_1}}$$

# Idea: Simultaneously estimate source intensity and background photon count rates



Sensor passes source.



Adaptive and less dependent on warm-up or training  
Exploits smoothness

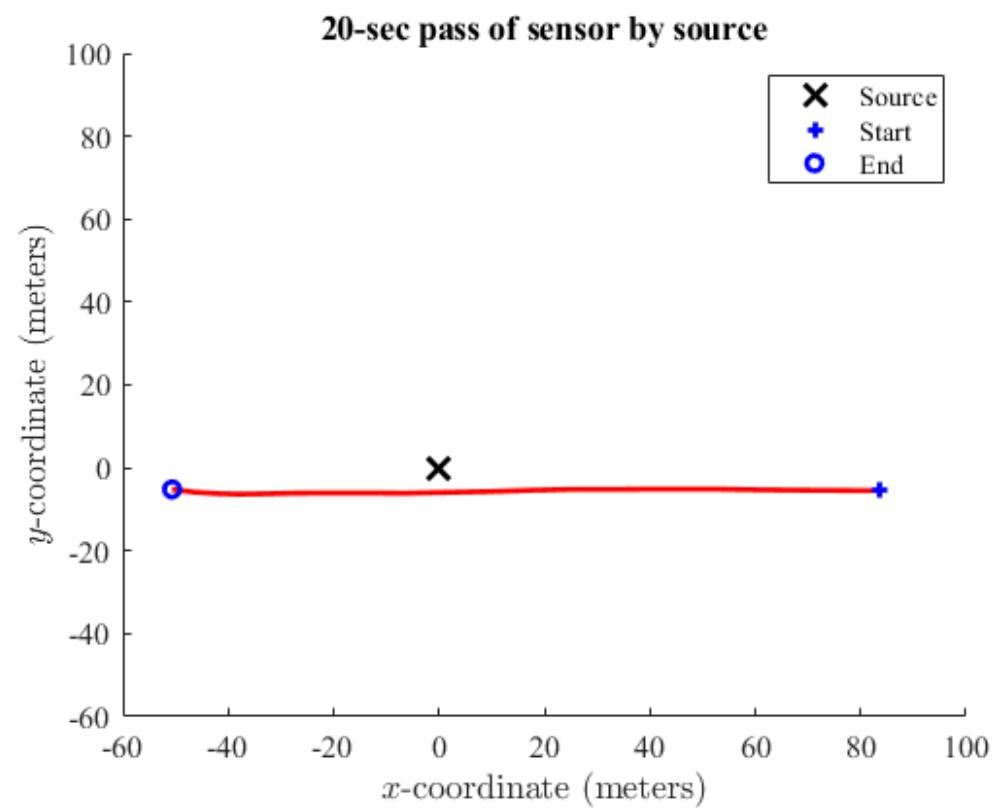
# Reduced dependence on warm-up or training via Kalman Filter

- Radiation as linear dynamical system
  - State  $x = \langle \text{background rates } \lambda, \text{ source intensity } \gamma \rangle$
  - Observation  $y = \langle \text{observed spectrum} \rangle$
- Kalman Filter (KF) estimates mean and covariance of state [Kalman 1960]
- Linear minimum MSE:
$$w_t \stackrel{iid}{\sim} N(0, Q_t) \wedge v_t \stackrel{iid}{\sim} N(0, R_t)$$
$$\implies E[(\hat{x}_{t,KF} - x_t)^2] \leq E[(\hat{x}_{t,Linear} - x_t)^2]$$
  - Often satisfied because  $\text{Poisson}(\lambda) \rightarrow N(\lambda, \lambda)$  as  $\lambda \rightarrow \infty$
  - Warm-up can be about 1 min

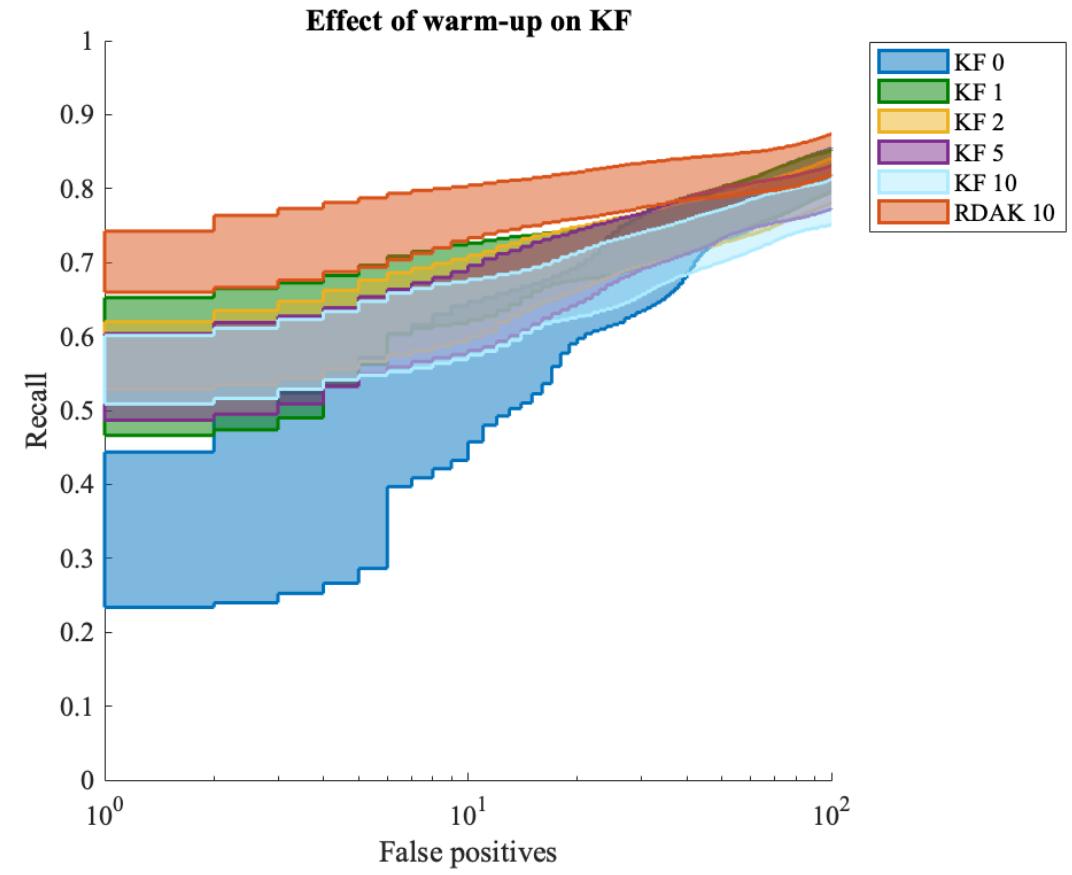
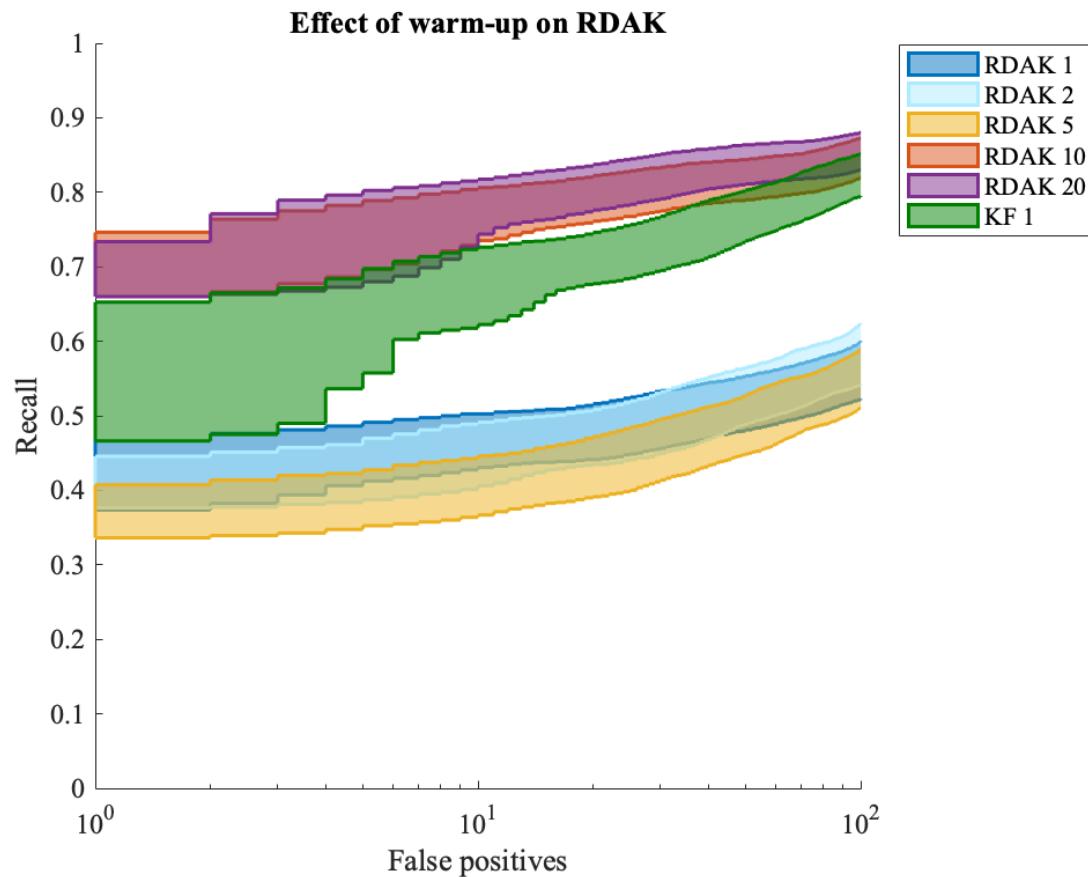
$$x_{t+1} = x_t + w_t$$
$$y_t = \lambda_t + \gamma_t s + v_t$$

# Dataset and experimental design

- Spectroscopic data from a major metropolitan area [not publishable]
- Synthetic injections of industrial isotope at SNR usually in [1, 10]
- Detection of a roadside source in a single pass
- 10 sec before and after
- 20,000 passes
- Disjoint data for warm-up



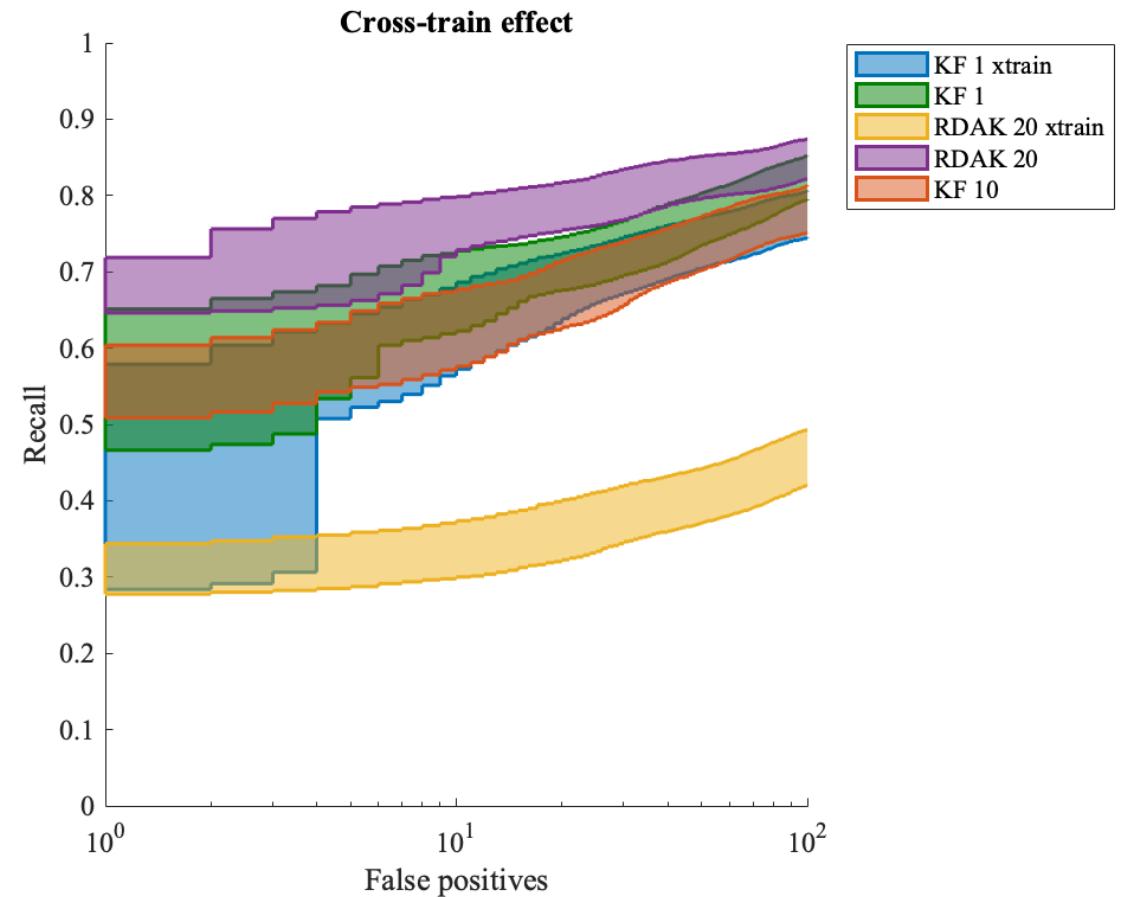
# Our method performs better at short warm-up periods



RDAK degrades with 5 min. or less warm-up. KF is indifferent to the amount of warm-up after 1 min.

# Our method performs better when warm-up differs from test

- Mismatch between warm-up and test
- Warm-up from different sensor in a different area
- RDAK not designed for this and degrades substantially
- Our method is indifferent and performs much better

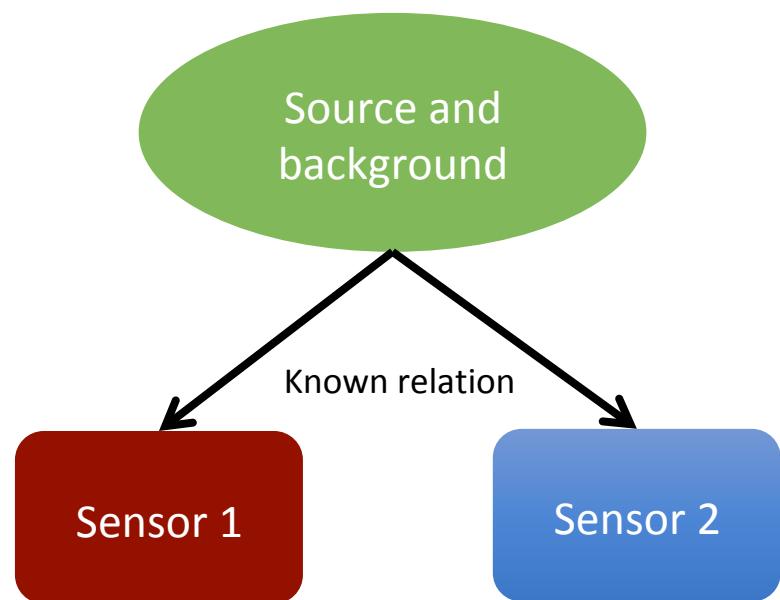


# Key takeaway of single sensor problem

Exploit smoothness to adapt to background with little training, which is useful in many practical scenarios

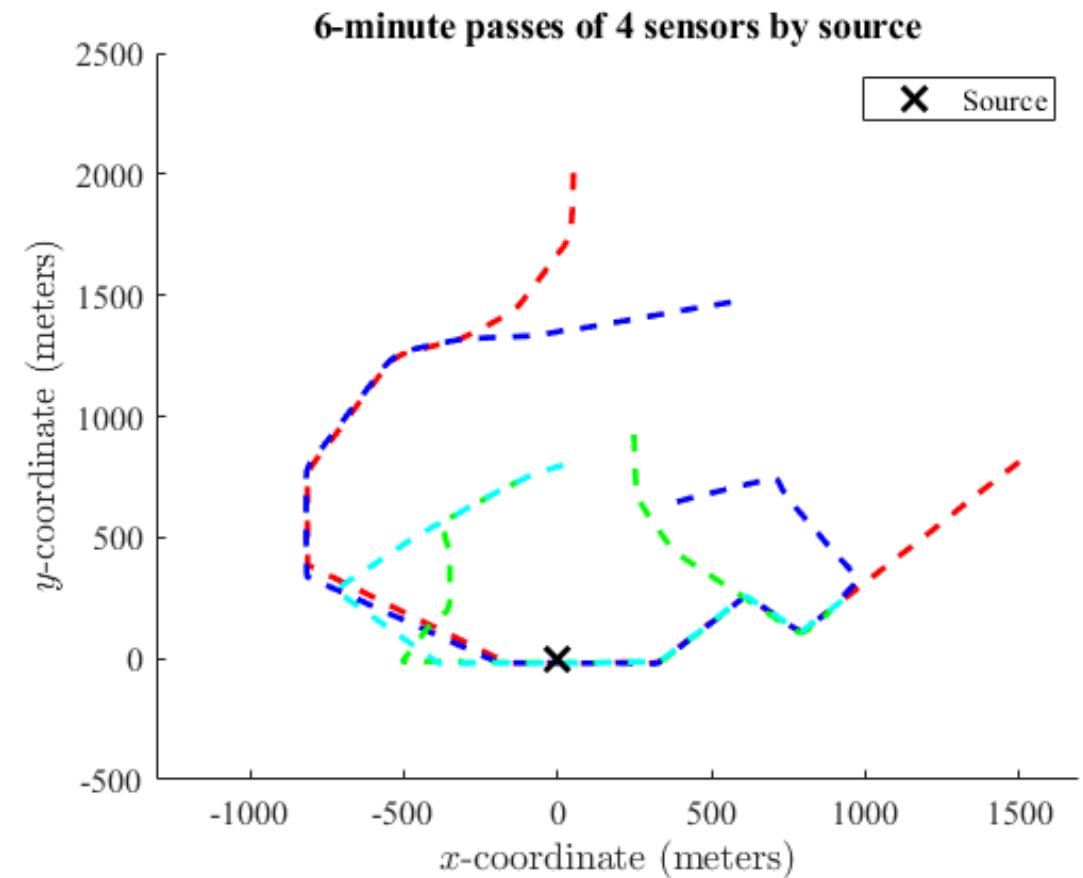
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# Multiple sensors

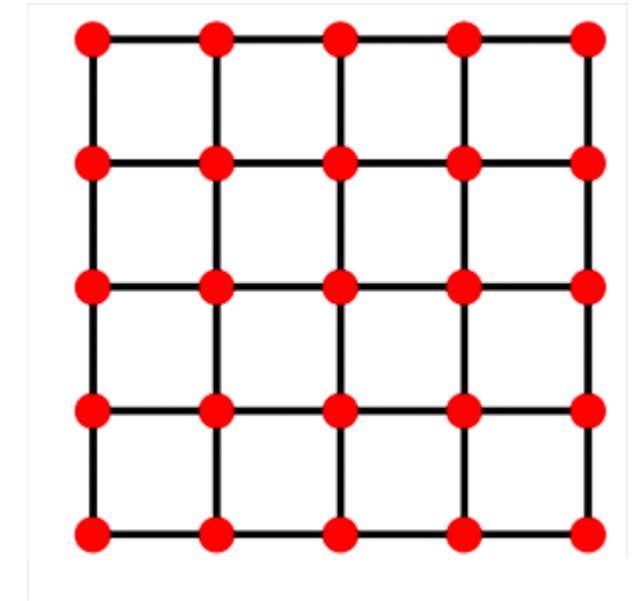
- More than one sensor can be near the source simultaneously
- Simultaneous sensors are related through background and possible source
- How can this contemporaneous multi-view relationship improve inferences?



# Related work: Bayesian Aggregation

- Bayesian Aggregation (BA): State-of-the-art method for multiple sensors [Tandon 2016]
- Tracks probability of  $H_0$  and  $H_{k,L}$ 
  - $H_0$ : No source
  - $H_{k,L}$ : Source is present with intensity  $I_k$  at location  $L$  (and possibly other characteristics)
- Requires reference data to train detector to get scores  $x_i$  and to fit  $\Pr(x_i | H_{k,L})$
- Assumes independence; does not leverage contemporaneous multi-view structure

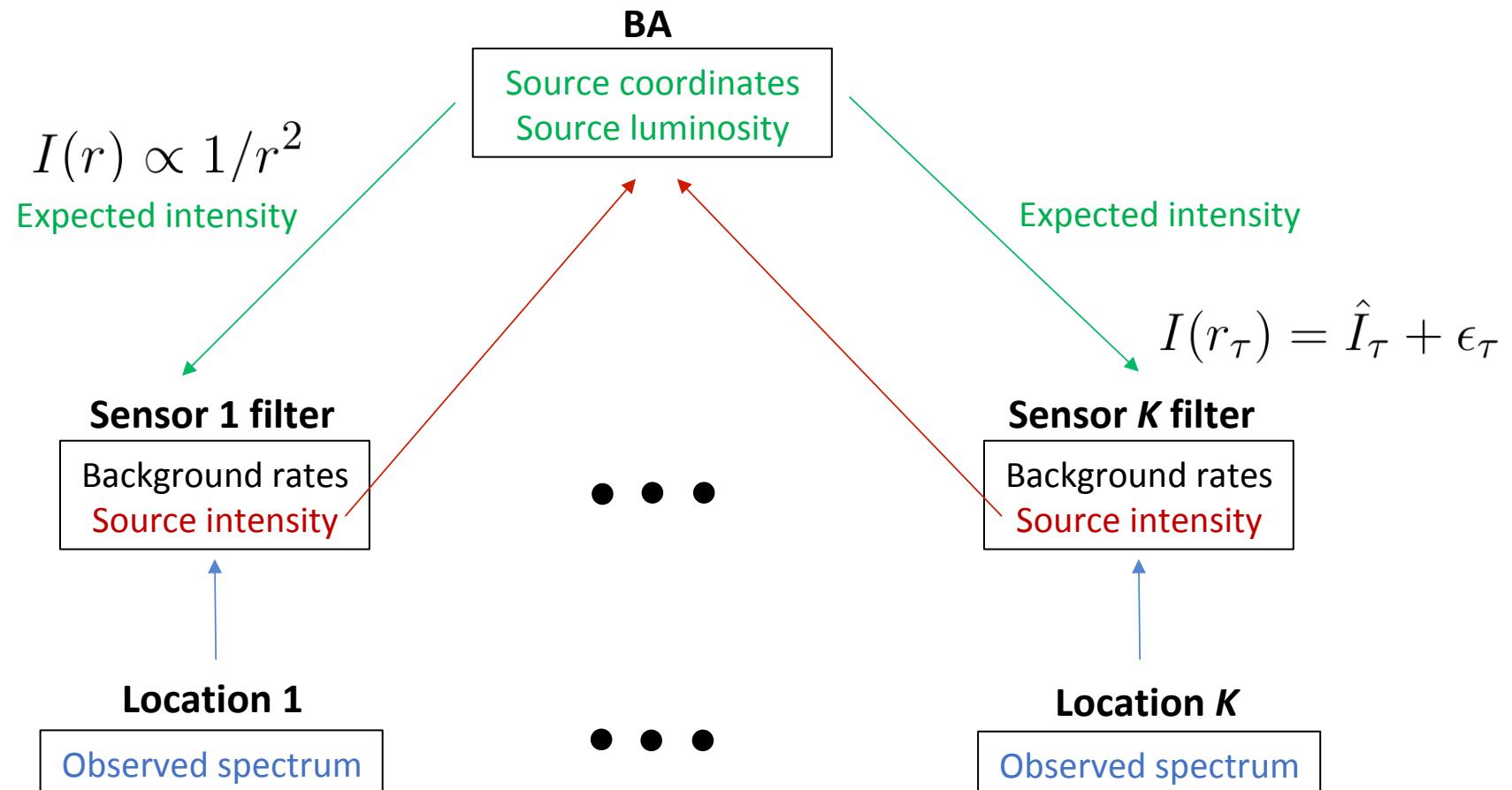
$$\max_{k,L} \frac{\prod_{i=1}^T \Pr(x_i | H_{k,L})}{\prod_{i=1}^T \Pr(x_i | H_{0,L})}$$



# Our approach: Multi-view filtering

- Collectively filters the inferences from individual sensors in the Bayesian Aggregation Filter (BAF)
- **Reduces dependence on training:** Kalman Filter (KF) at each sensor to bypass detector training
- **Improves detection power over BA:** Share information between sensors using the collective inference of all sensors from the previous time step

# Main idea: BA is hub for filters



# Dataset and experimental design

## Previous experiment

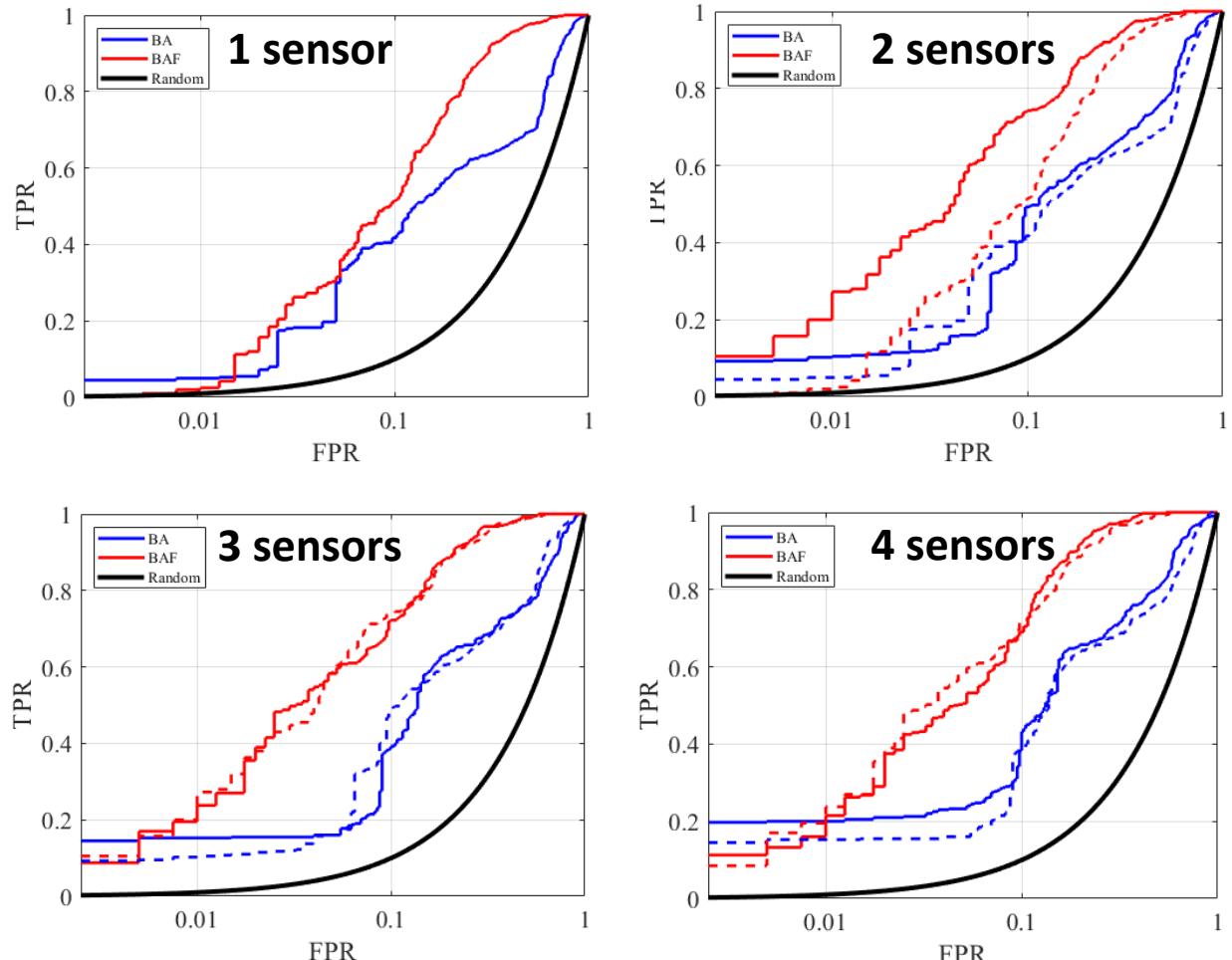
- Spectroscopic data from a major metropolitan area [not publishable]
- Synthetic injections of industrial isotope
- Detection of a roadside source
- Baselines trained on disjoint data

## Current experiment

- 1, 2, 3, or 4 passes as sensors
- No warm-up for KF
- Fix SNR at 4
- 3 min. before and after source
- 400 trials

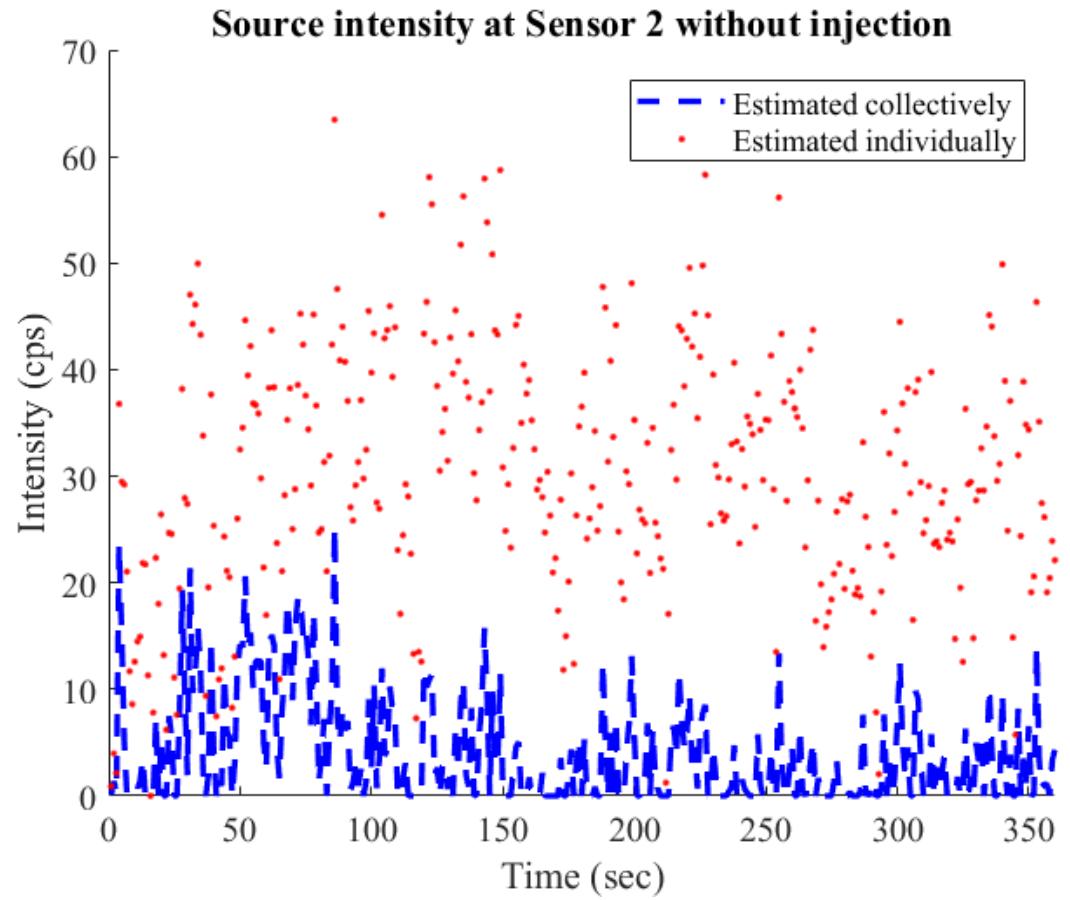
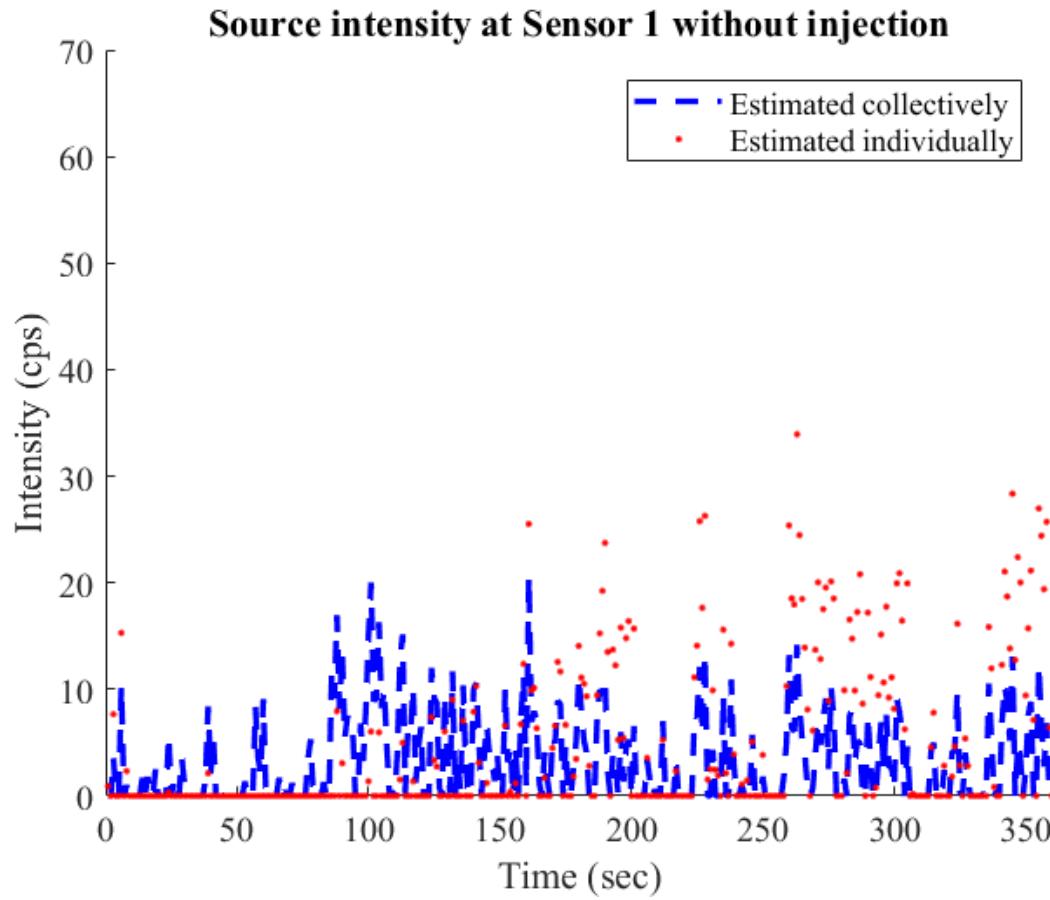
# Our method gains more from multiple sensors

- Our method, **BAF**, performs much better at 2 sensors than 1
- No gain after 2 sensors
- **BA** (with classical MF) always weaker than **BAF** but always benefits from more sensors



Sensors ordered from closest to farthest from source.  
Dashed lines are from the previous number of sensors.

# Effect of collective filtering



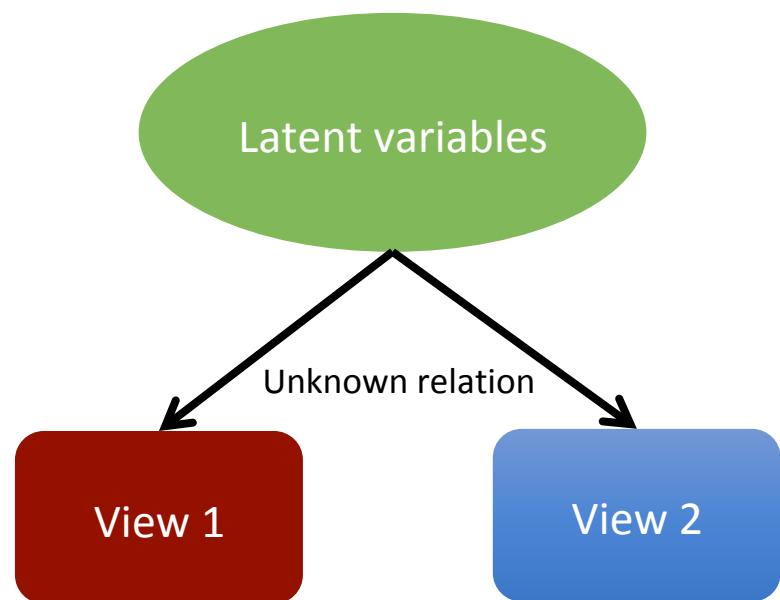
Multi-view filtering greatly decreases false positives from KF

# Key takeaway of multiple sensors problem

Exploit smoothness and contemporaneous multiple views to achieve better performance than state of the art in multi-sensor settings with less training data

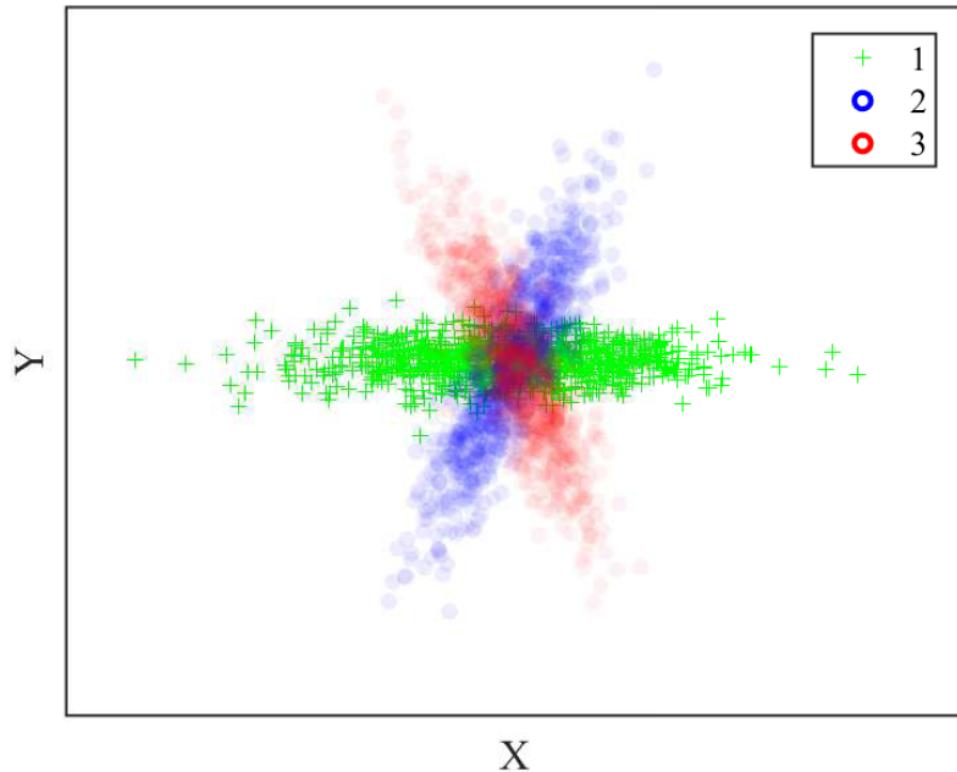
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# Learning multi-view relationships

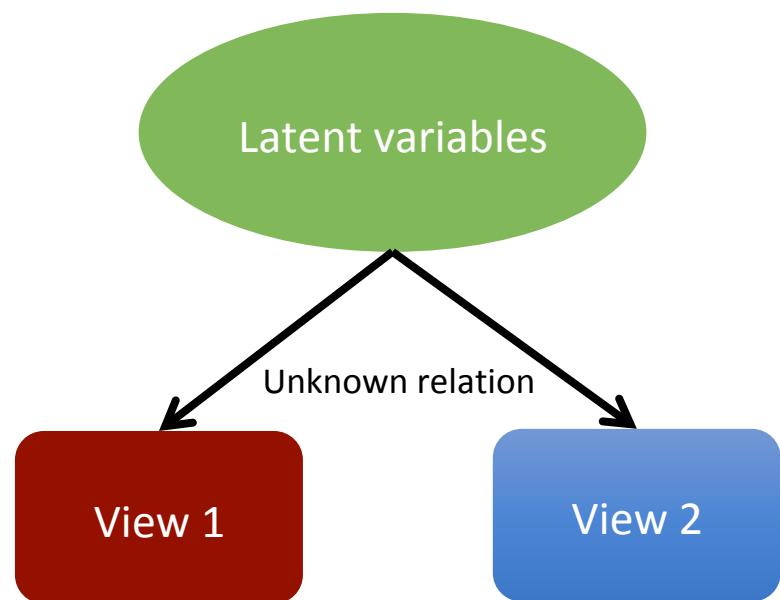
- The filtering work uses domain knowledge about the multi-view relationships
- If we lack this information, can we learn the relationships from data and still utilize them?
  - Linear relationships
  - Nonlinear relationships



Nonlinear multi-view relationship that can be represented by a mixture of 3 linear relationships.

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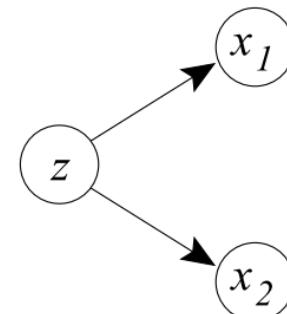


# Canonical Correlation Analysis

- Two-view analogue to Principal Components Analysis
- Learns subspace to maximize correlation between two views [Hotelling 1936]

$$\max_{u,v} \text{corr}(X_1^T u, X_2^T v)$$

- Non-convex optimization with closed-form solution  $A = \Sigma_{XX}^{-1} \Sigma_{XY} \Sigma_{YY}^{-1} \Sigma_{XY}^\top$
- Gaussian model interpretation [Bach 2006]

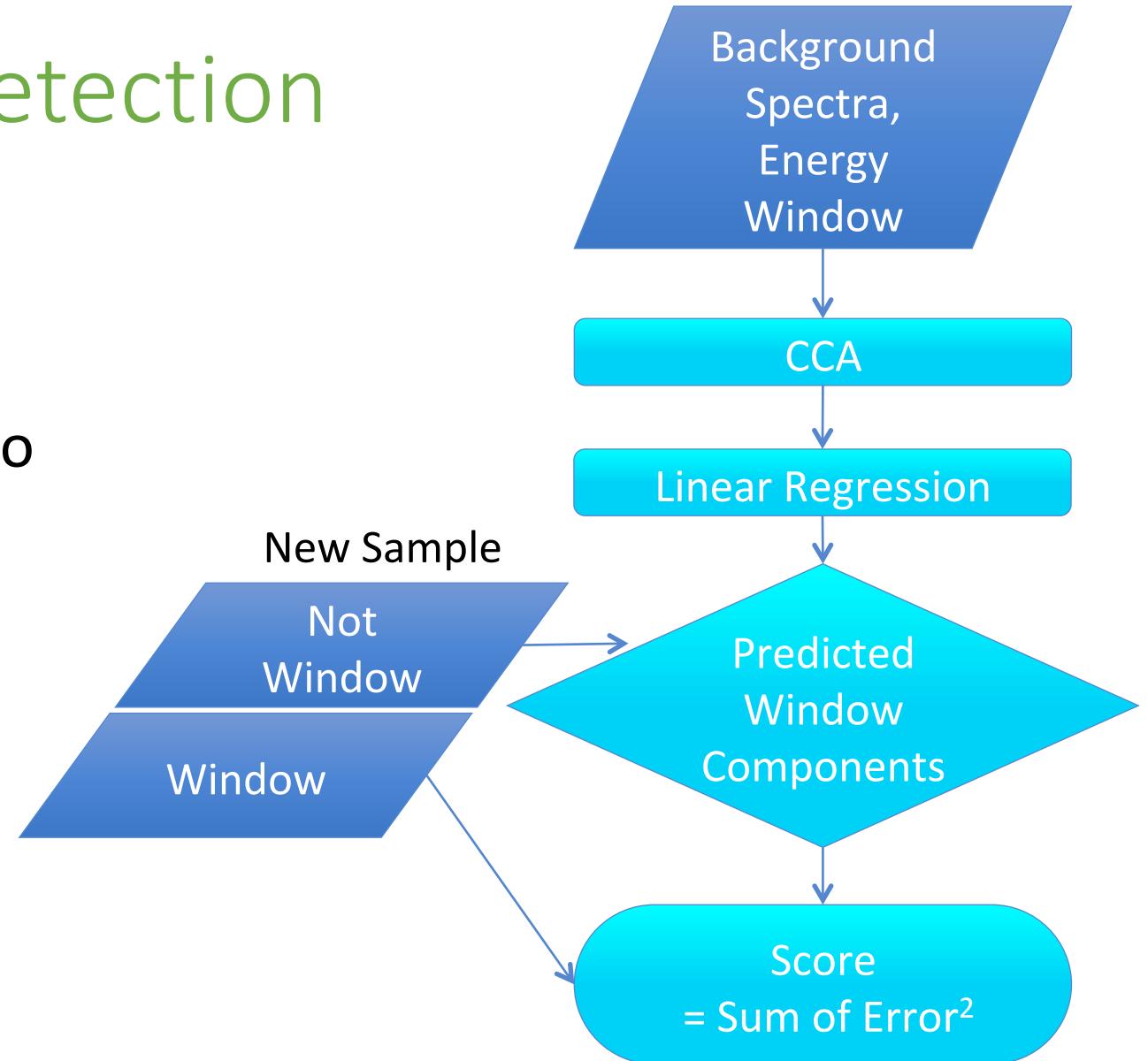


Hotelling (1936). Relations between two sets of variates. Biometrika.

Bach (2006). A probabilistic interpretation of canonical correlation analysis. Berkeley Technical Report.

# Idea: CCA Anomaly Detection

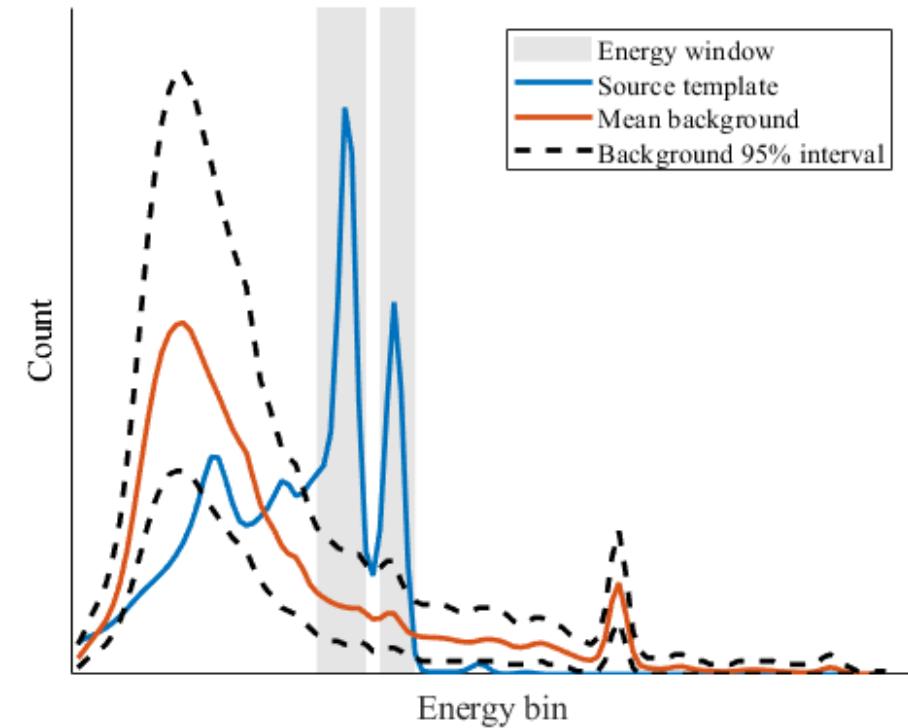
- Characterizes background reference data by CCA
- Checks when new observations do not match this structure
- Apply to imperfect source knowledge
- *Energy windows* as views



# Dataset and experimental design

- 24 hours spectroscopic data in one-second intervals
- Synthetic injections of 67 source templates
- Binary classification of each sample
- Censored Energy Window (CEW): multi-view baseline

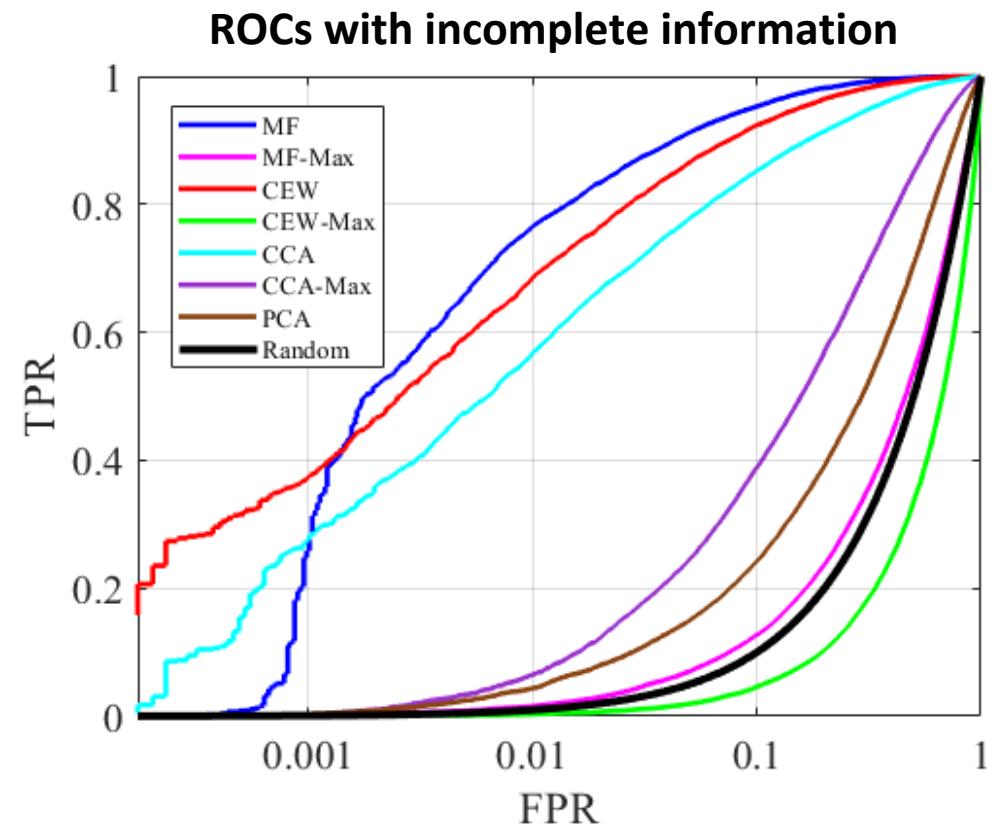
Energy window, source, and background



Expect to see source most clearly in energy window.

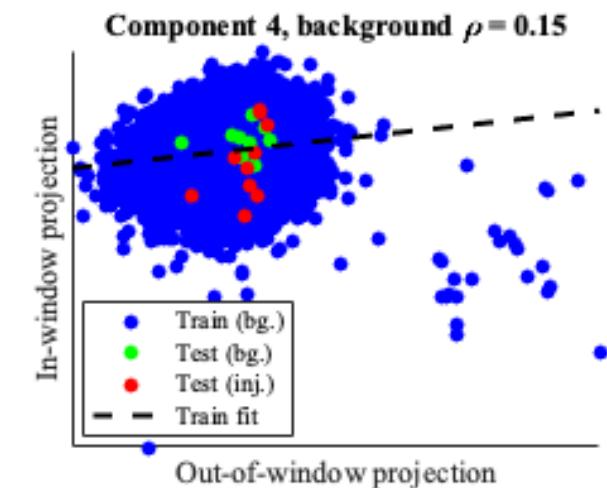
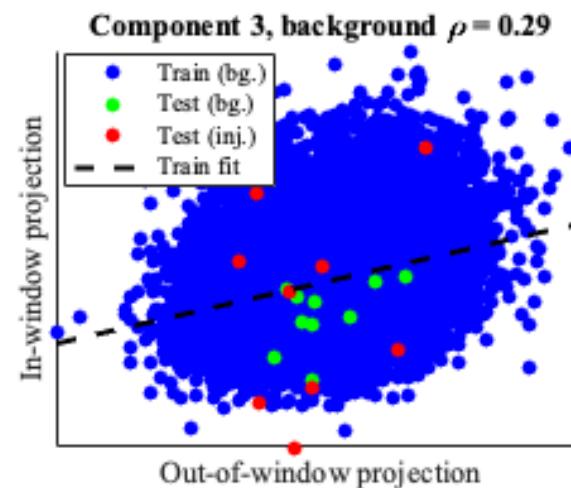
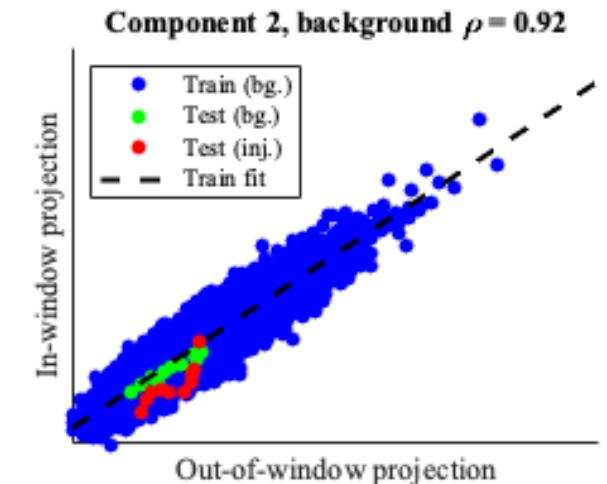
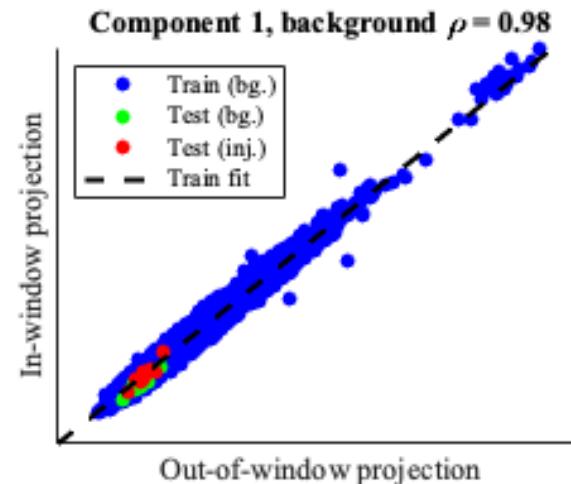
# CCA Detection is more robust

- True source template missing from library
- CCA method outperforms MF-Max, CEW-Max, and PCA alternatives
- We expect to do worse than MF and CEW, which have perfect source information



# Multiple components aid detection

- CCA Detection score is based on residuals
- With injection, small residuals individually, but noticeable combined
- Weaker correlations can even be more salient

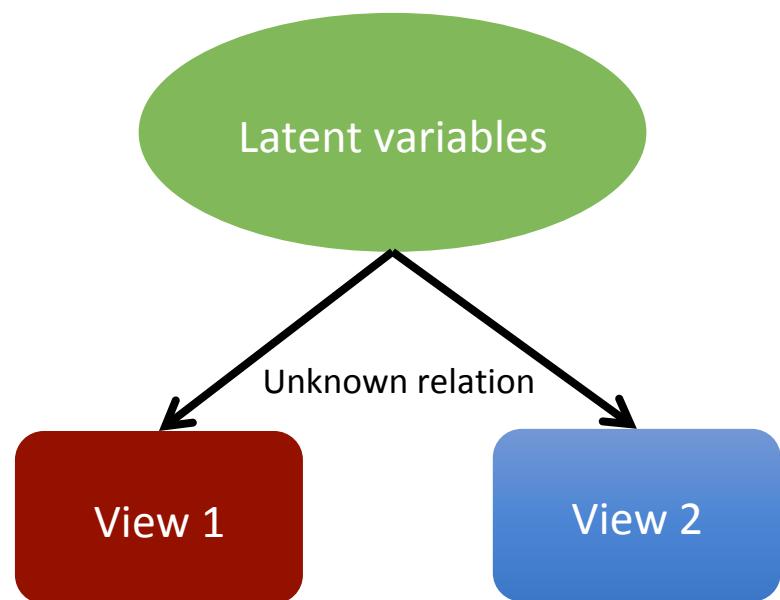


# Key takeaway of imperfect source information problem

Leverage multiple linear correlations between views to make detection robust against imperfect information, a practical scenario

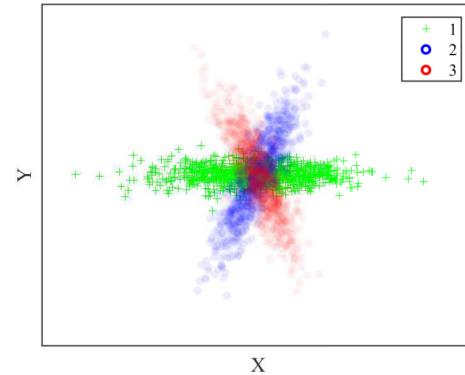
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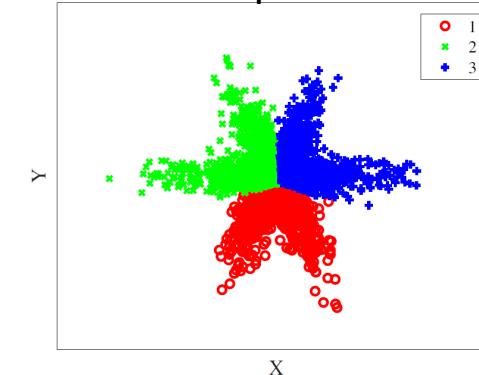


# Clustering methods compared

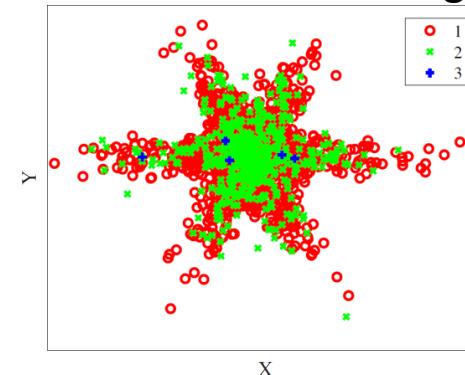
Ground truth



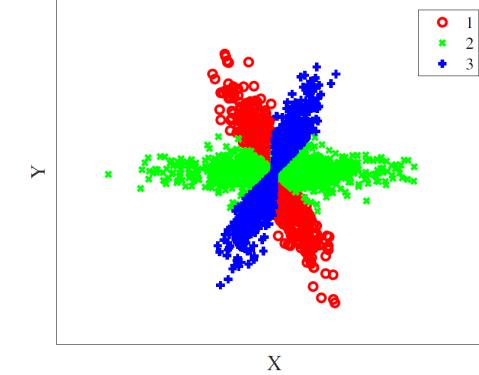
K-means or spectral clustering



Modern multi-view clustering [Zhao 2017]

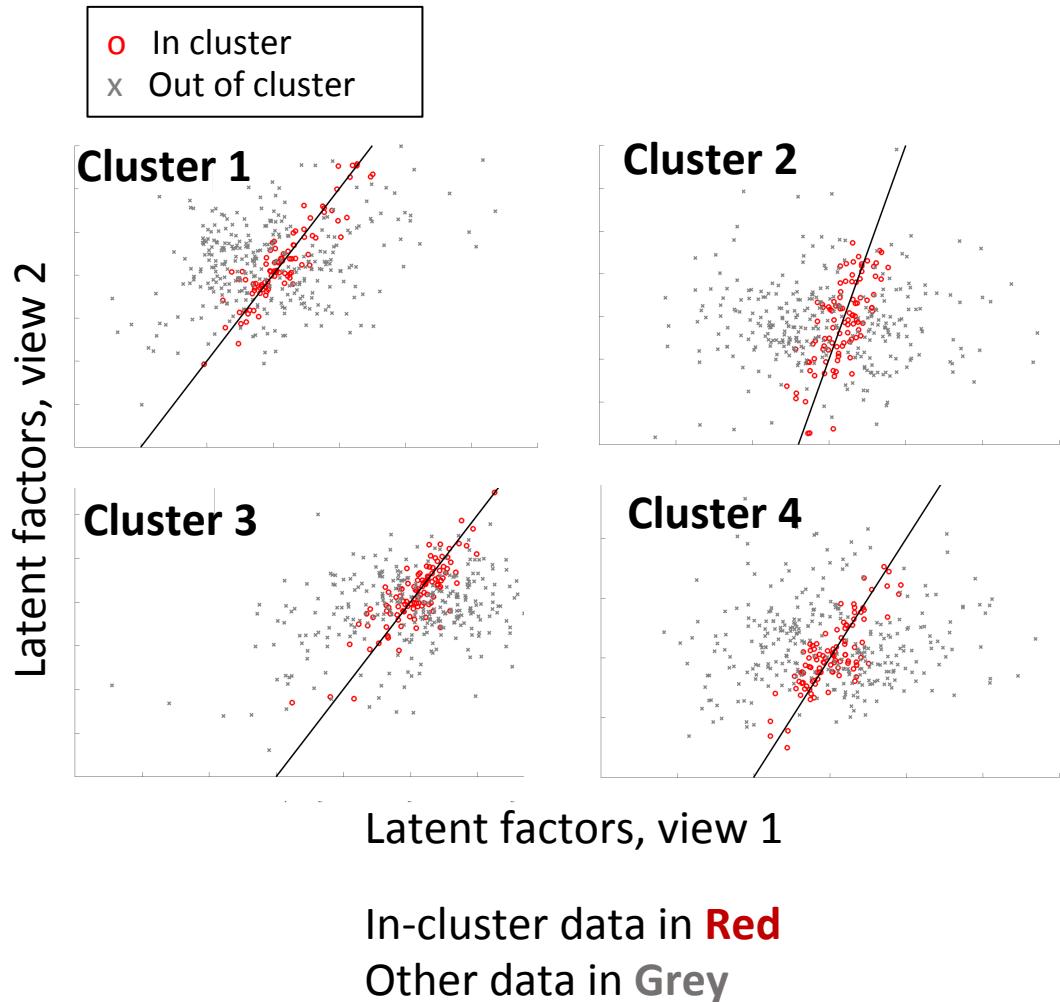


Multi-view relationship clustering



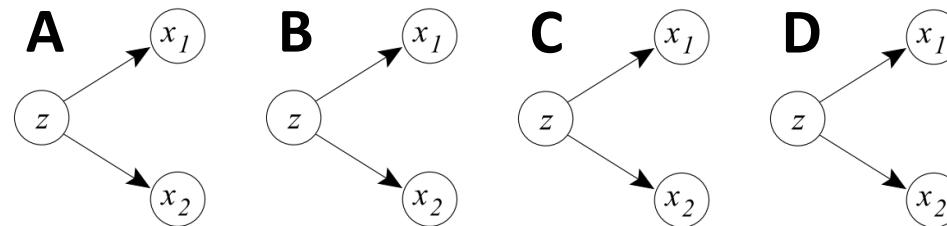
# Our approach

- Clusters observations according to multi-view relationships
- Fits cluster-wise linear relationships
  - Relevant latent factors are discovered in the process
  - Latent factors vary between clusters



# Mixture of Canonical Correlations

- How to model different subsets of observations have different correlations?
- Generative model:



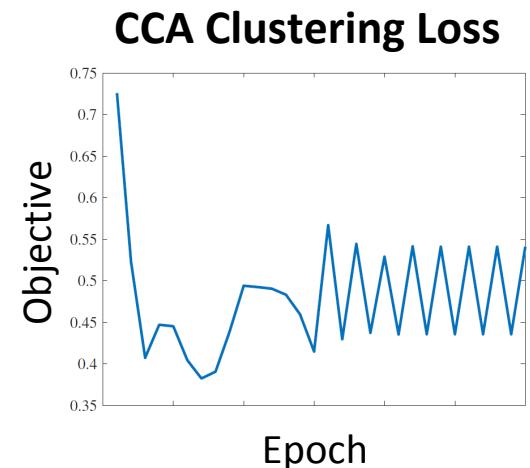
- Each cluster has a conditionally independent CCA structure
- Want to learn both cluster labels and correlations

$$\min_{u,v,R} \quad \sum_j \|X^{(j)}u^{(j)} - Y^{(j)}v^{(j)}\|_2^2$$

$$\text{s.t.} \quad u^{(j)\top} X^{(j)\top} X^{(j)} u^{(j)} = v^{(j)\top} Y^{(j)\top} Y^{(j)} v^{(j)} = 1$$

# Canonical Least Squares Clustering (CLS)

- Expectation Maximization-like iterative algorithm for CCA clusters [Fern 2005]
- Theoretical and empirical convergence problems
- Replace CCA with novel optimization problem, Canonical Least Squares (CLS)
- Non-convex optimization
  - Closed-form solution in first component
  - Greedy solution performs well empirically



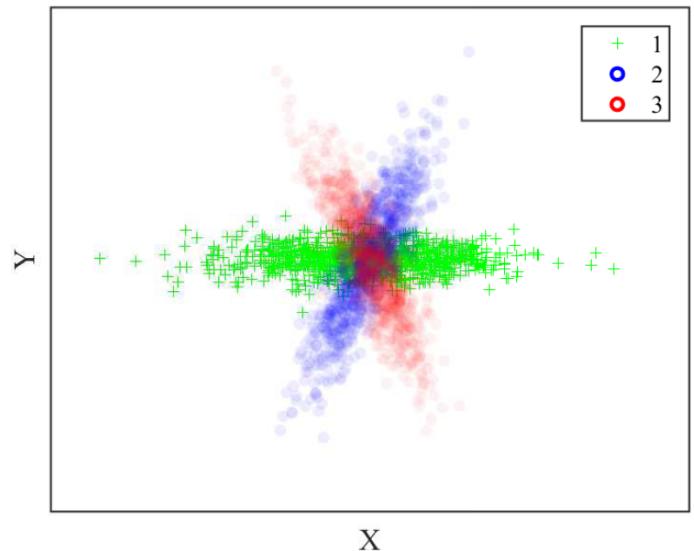
**Canonical Least Squares**

$$\min_{u \in \mathbb{R}^{d_X}, v \in \mathbb{R}^{d_Y}} \|Xu - Yv\|_2^2$$

subject to  $v^\top v = 1$ .

# Experiment on synthetic data

- 10 clusters of 1,000 points each in  $\mathbf{R}^{100}$ 
  - Gaussian
  - All overlap at origin
  - Features partitioned in half to make two views



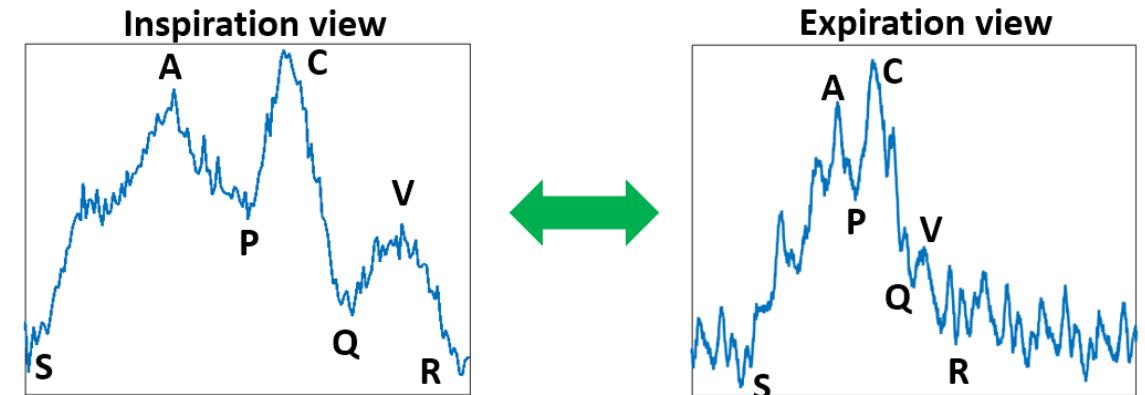
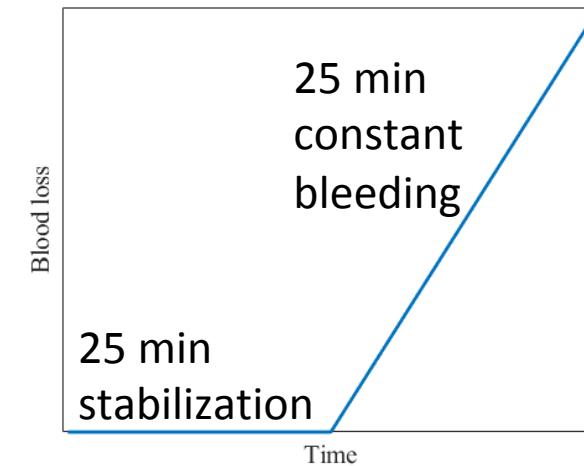
High-dimensional version  
of this with 10 clusters

	CLS	CCA	k-means	Spectral
Adjusted Rand Index	.99 ± .01	.94 ± .02	.005 ± .003	.000 ± .000

CLS Clustering performs best

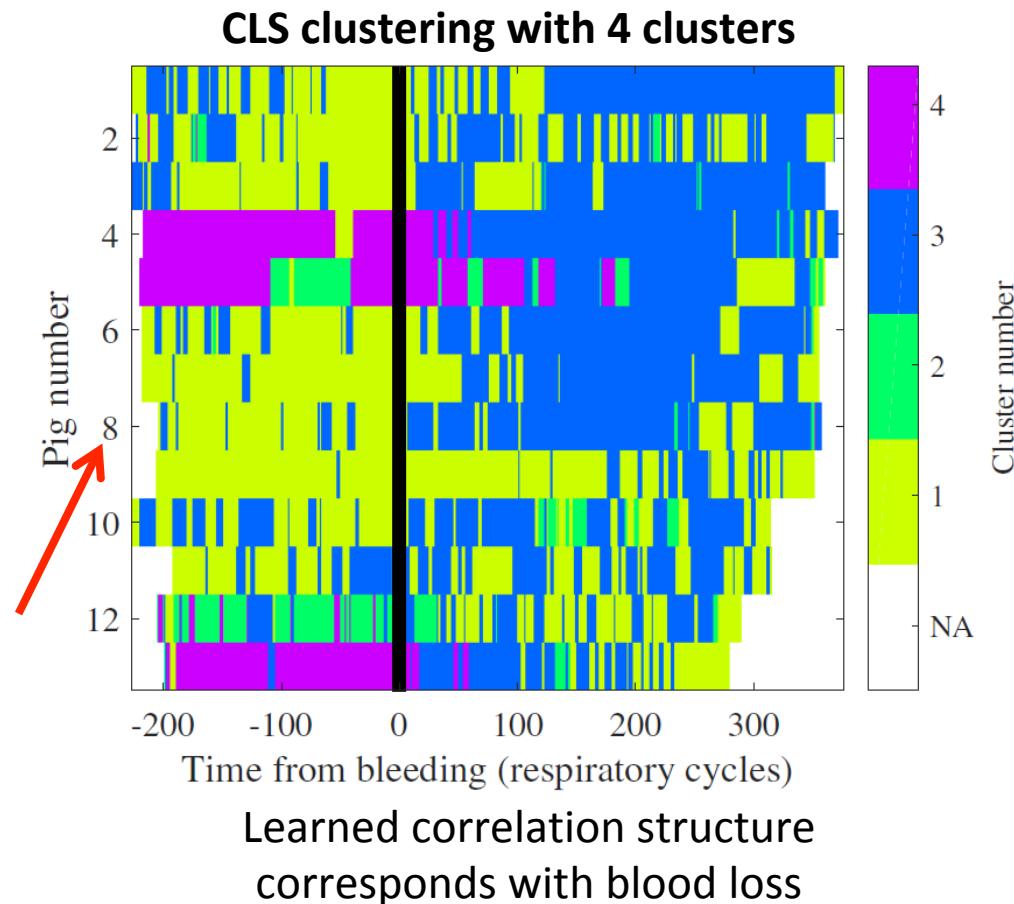
# Experiment on induced bleeding

- 38 anesthetized pigs
- Time series of each subject's *central venous pressure (CVP)*, sampled at 250Hz and featurized using domain knowledge
- Views correspond to CVP waveforms at the top of inspiration and bottom of expiration of the breathing cycle



# Clusters across time and subjects

- No explicit knowledge of bleeding
- Variation across time reflects stages of bleeding response
- Variation across subjects reflects different phenotypes
- Our work may be first to characterize structure of response [Pinsky 2005, Boyd 2011, Marik 2013]

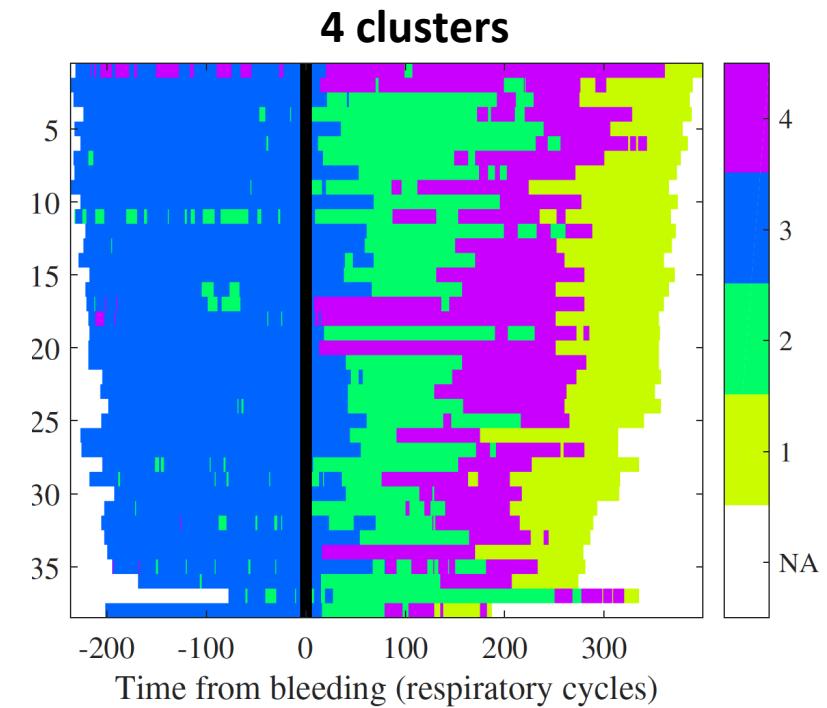
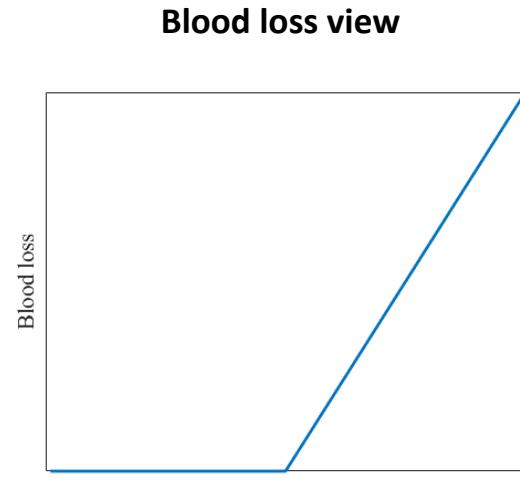
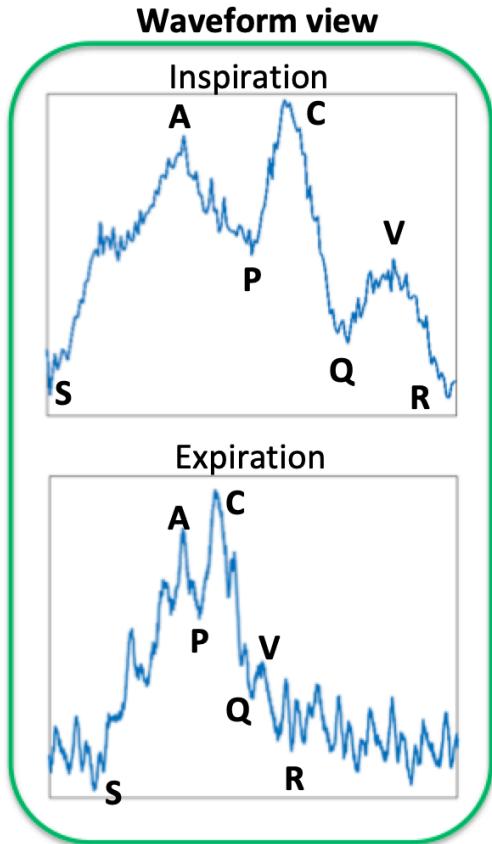


Pinsky and Payen (2005). Functional hemodynamic monitoring. Critical Care.

Boyd et al. (2011). Fluid resuscitation in septic shock. Critical Care Medicine.

Marik and Cavallazzi (2013). Does the central venous pressure predict fluid responsiveness? Critical Care Medicine.

# Fully supervised clusters



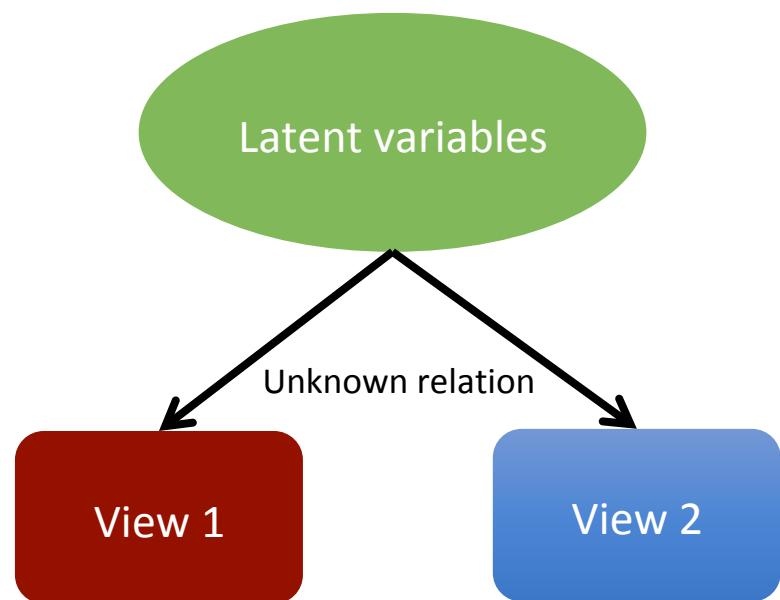
Refined clusters using bleeding information.  
Not same clusters as previous slide.

# Key takeaway of approach to multi-view clustering

- Novel algorithm identifies clusters based on multi-view relationships
- Performs well quantitatively on synthetic dataset and qualitatively on authentic bleeding dataset

# Outline

- Multi-view filtering
  - Single sensor method for gamma source detection [NSS 2017]
  - Multiple sensor extension
- Learning multi-view relationships
  - Linear multi-view relationships [NSS 2016]
  - Nonlinear multi-view relationships
    - Clustering [MLHC 2017, ISICEM 2019]
    - Classification [MLHC 2017, ISICEM 2019]



# CLS classification

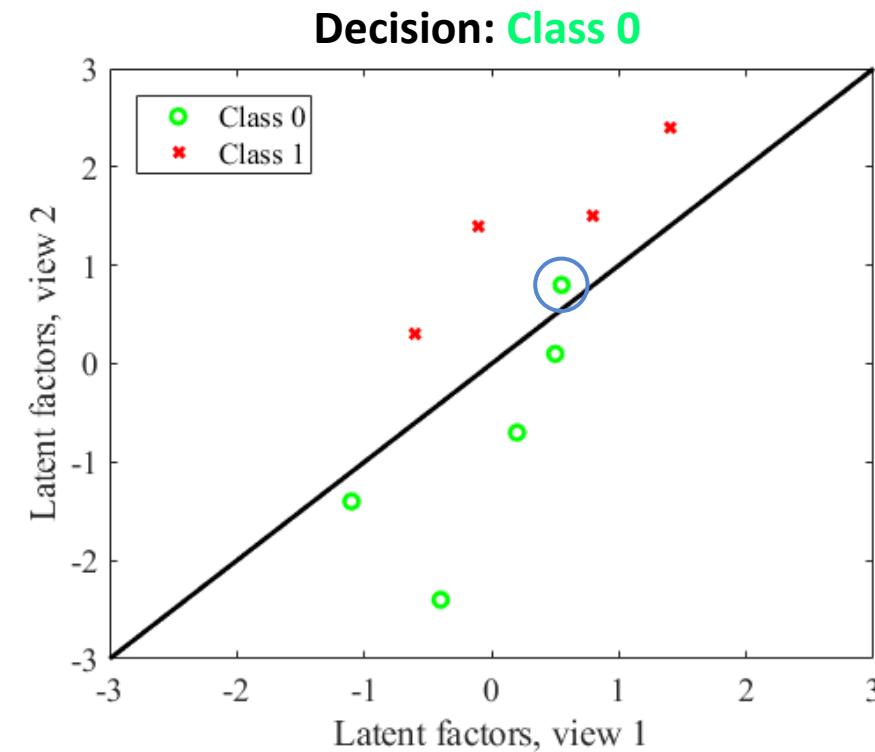
1. Learns CLS clusters independently on each class
2. Checks new point's fit in each cluster
3. Classify according to best fitting cluster

**Nonlinear multiclass generalization of CCA anomaly detection**

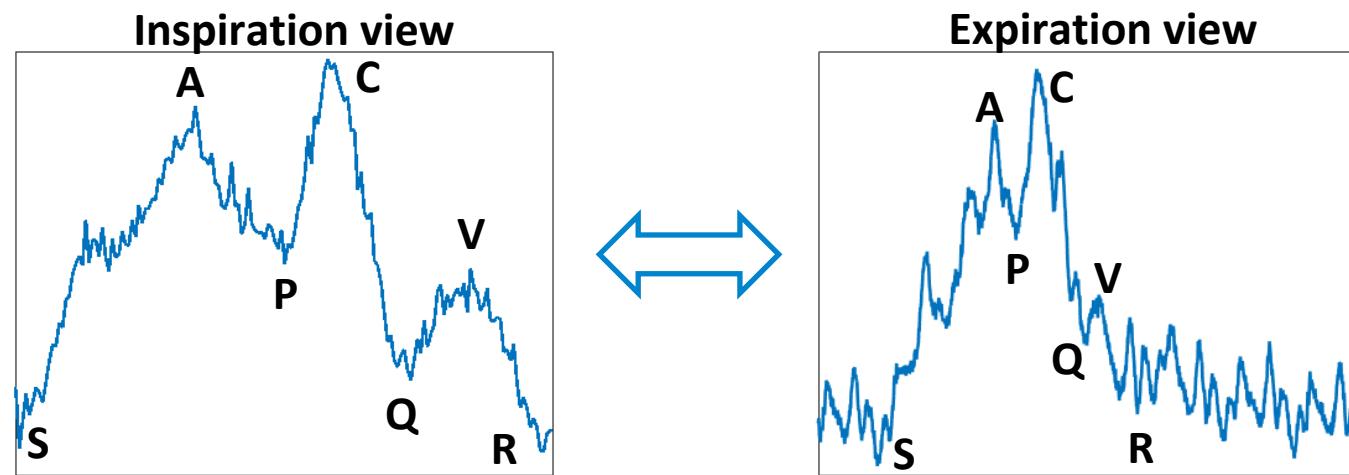
For Class  $C$  with clusters  $j$ , example with views  $x$  and  $y$  is scored as

$$Score_C((x, y)) = - \min_{j \in C} \|U_j x - V_j y\|_2^2$$

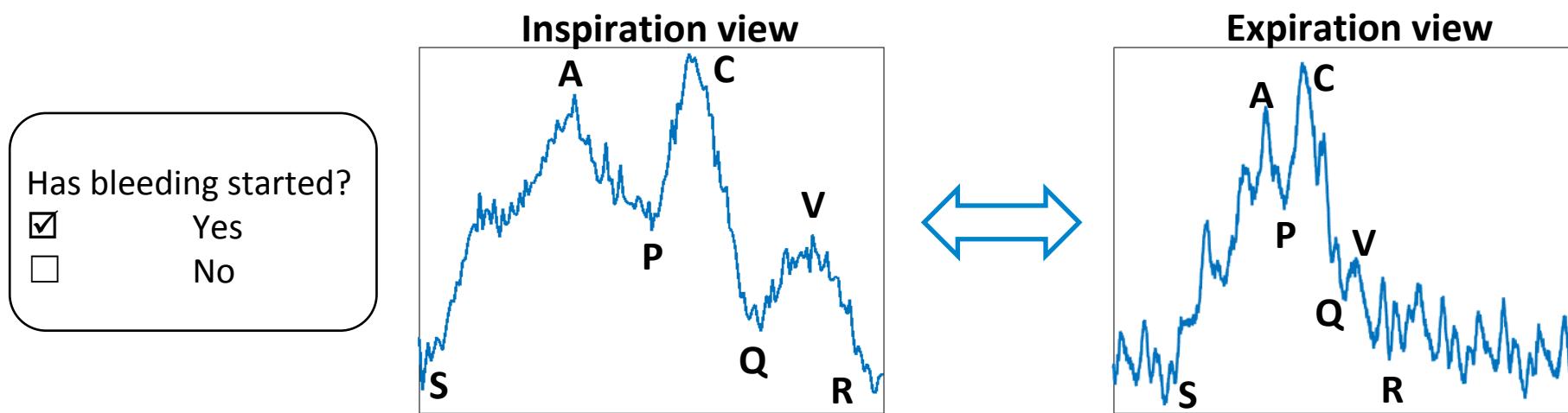
where  $U_j$  and  $V_j$  are CLS loading matrices in cluster  $j$



# Moving CLS to supervised classification setting



# Moving CLS to supervised classification setting



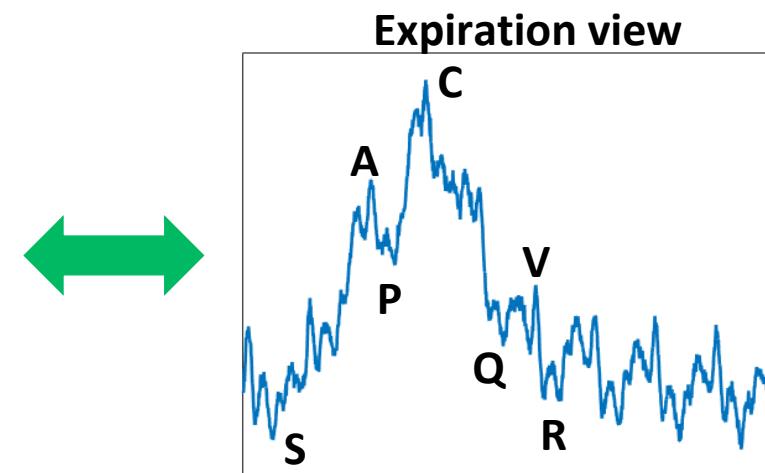
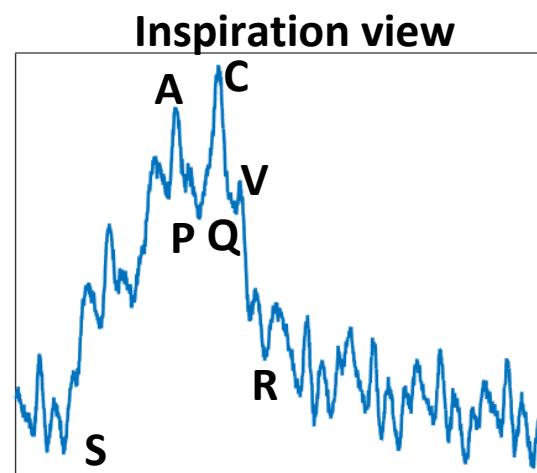
# Moving CLS to supervised classification setting

Decide whether a pair of waveforms came from **before** or **during** bleeding

Has bleeding started?

Yes

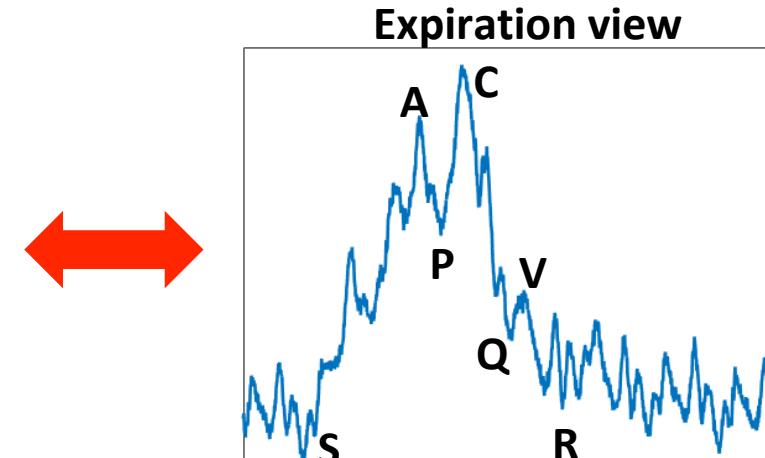
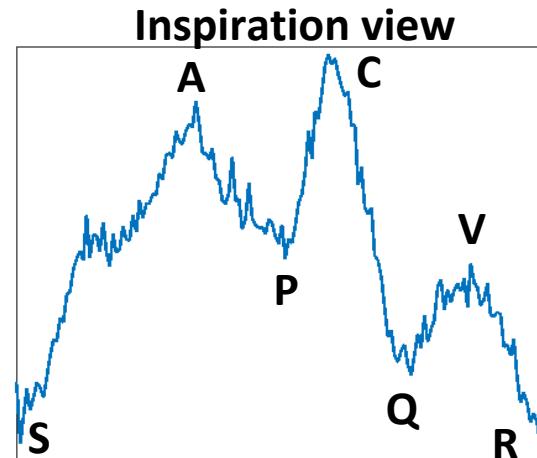
No



Has bleeding started?

Yes

No



# Classifier performance

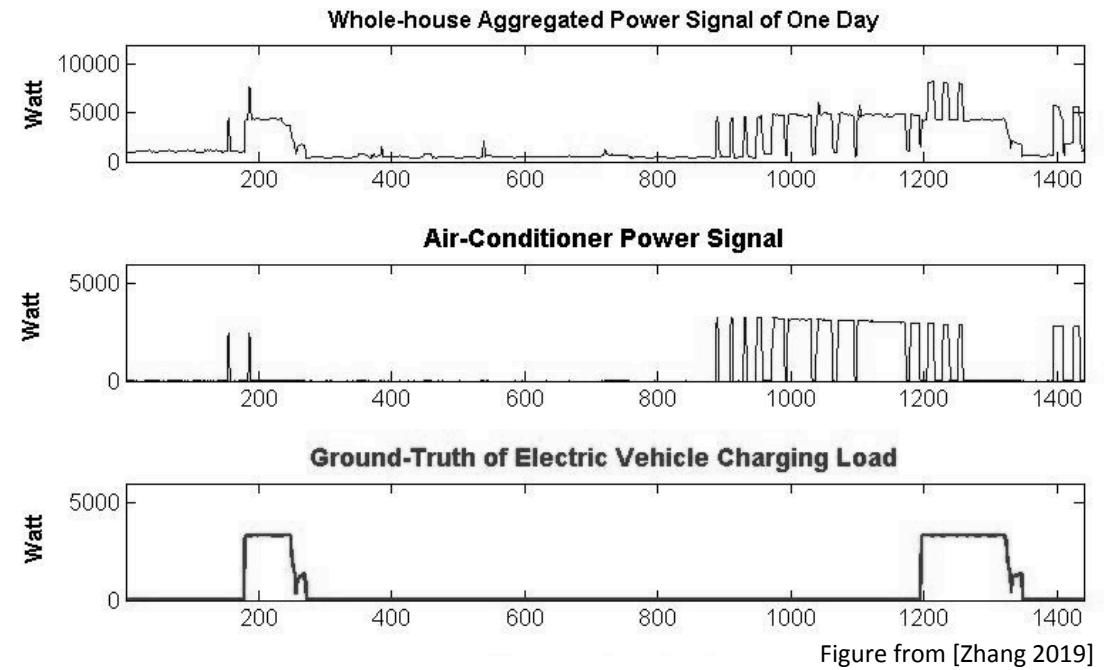
	Single cluster CLS	Final CLS	Random forest
AUC	.701 ± .128	.862 ± .064	.891 ± .075
TPR @ .10 FPR	.468 ± .185	.674 ± .145	.762 ± .167
TPR @ .01 FPR	.222 ± .134	.501 ± .185	.610 ± .210
FPR @ .50 TPR	.239 ± .152	.064 ± .055	.073 ± .075

(95% C.I.s)

- Single cluster CLS: CLS classification with one cluster
- Final CLS: CLS classification with multiple clusters
- Random forest: best single-view classifier

# Different application: Non-intrusive load monitoring (NILM)

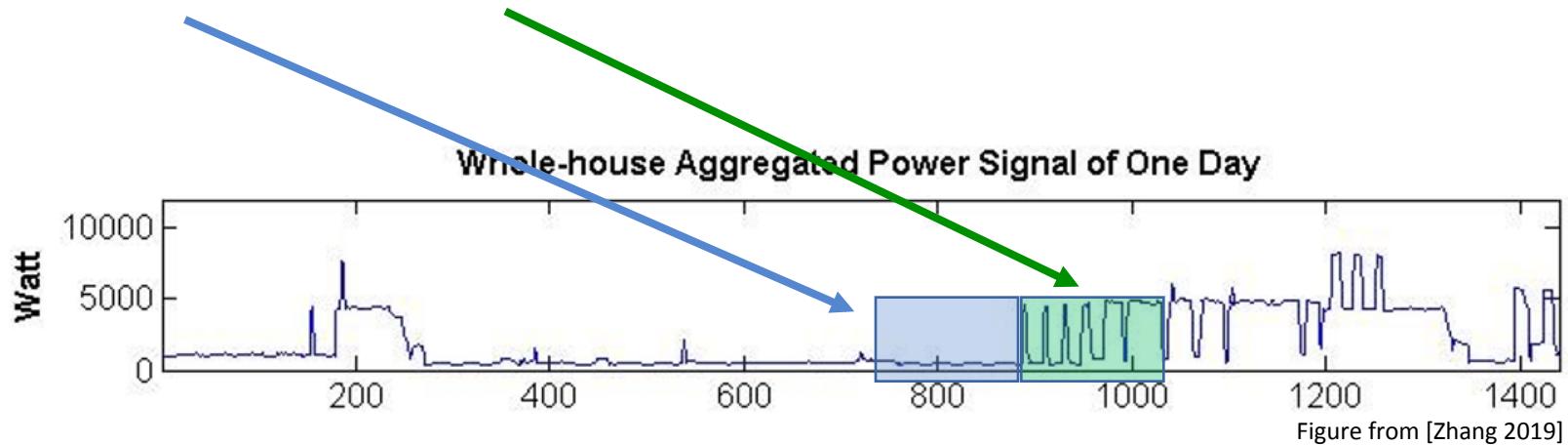
- Pipeline of event-based NILM
- Event detection to identify when appliances are switched on/off; binary classification
- Event classification to identify which appliances are switched; multiclass classification



Disaggregation of power signals.

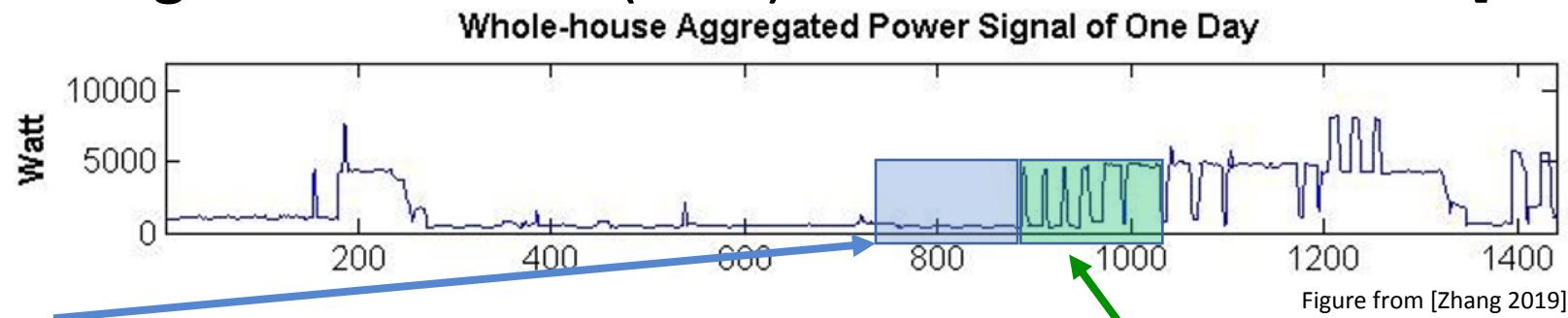
# Approach: Multi-view characterization of change over time

- Transient changes in appliance state carry temporal correlation structures
- Fingerprints of different appliances potentially discovered by clusters
- Views are Past and Present



# Baseline: Goodness-of-fit method

- Chi-squared goodness-of-fit (GOF) test is state-of-the-art [Jin 2011]



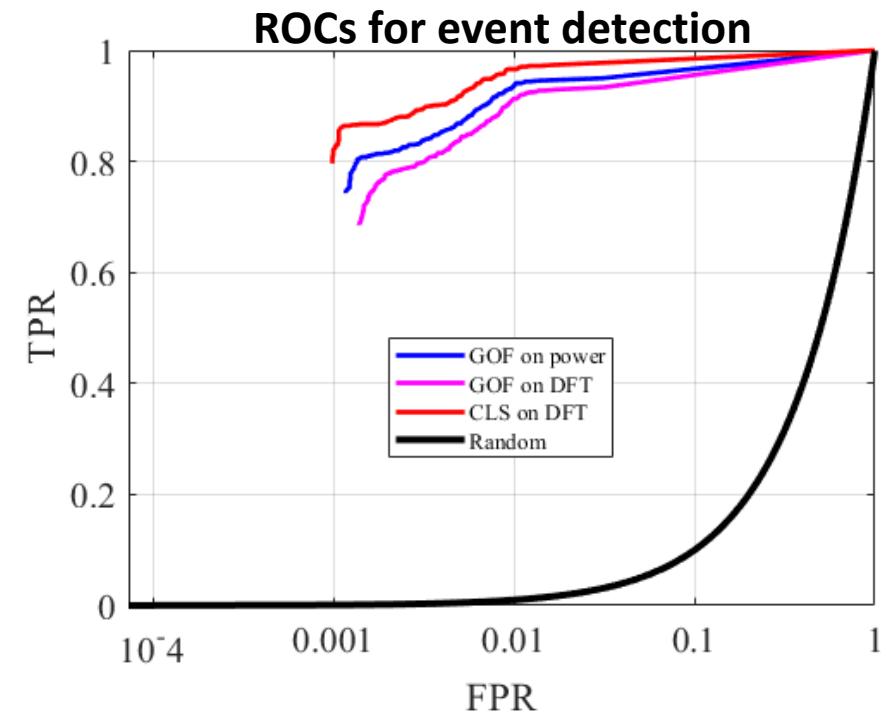
- $H_0$ : Past was drawn from same distribution as Present
- $H_1$ : Distributions differ
- Test statistic has chi-squared distribution
  - Paired in order
  - Gaussian assumption

$$\sum_i \frac{(p_i - q_i)^2}{p_i}$$

# Event detection experiment

- BLUED [Filip 2011]
  - Power from houses with multiple appliances
  - Frequently used in benchmarking
  - 12kHz power over 7 days with labeled events
- Featurization by discrete Fourier transform of each window; top 5 principal components

Results statistically significant.

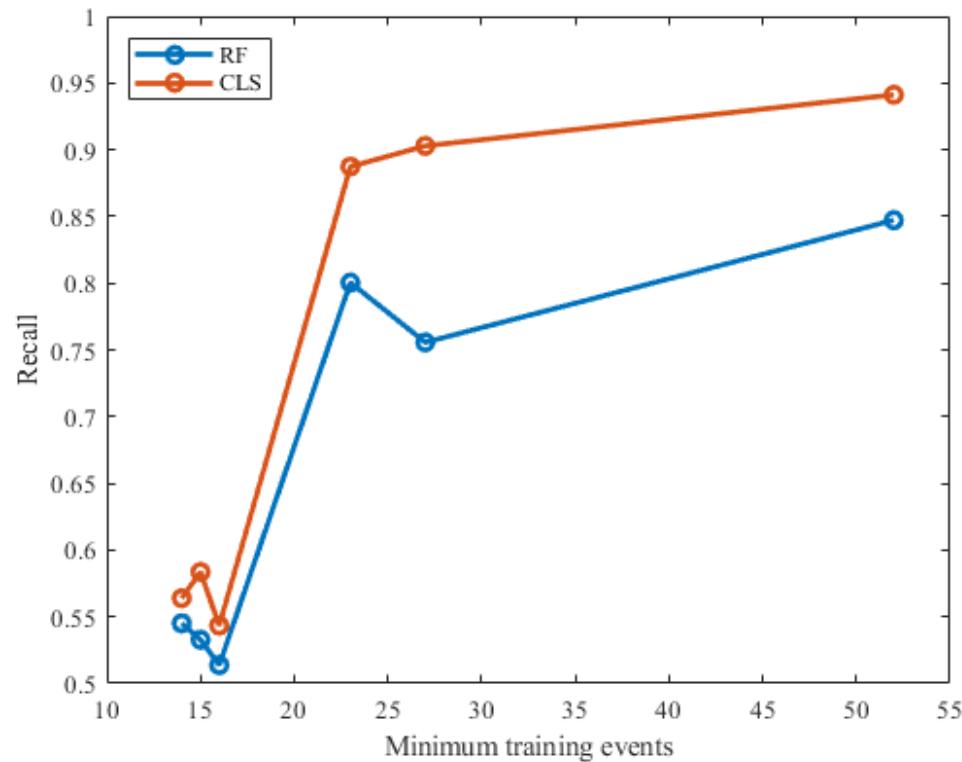
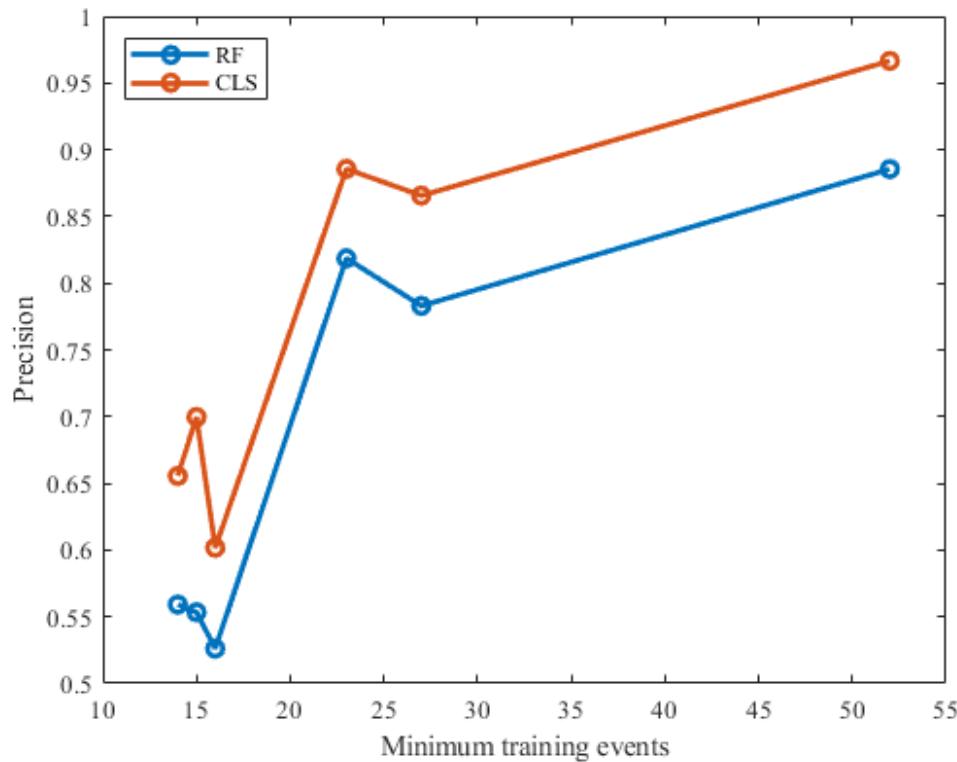


FPR at certain levels of TPR

TPR	80%	85%	90%	95%	98%
GOF on power	.10%	.15%	.50%	.81%	5.5%
<b>CLS on Fourier</b>	<b>.08%</b>	<b>.09%</b>	<b>.26%</b>	<b>.67%</b>	<b>3.92%</b>

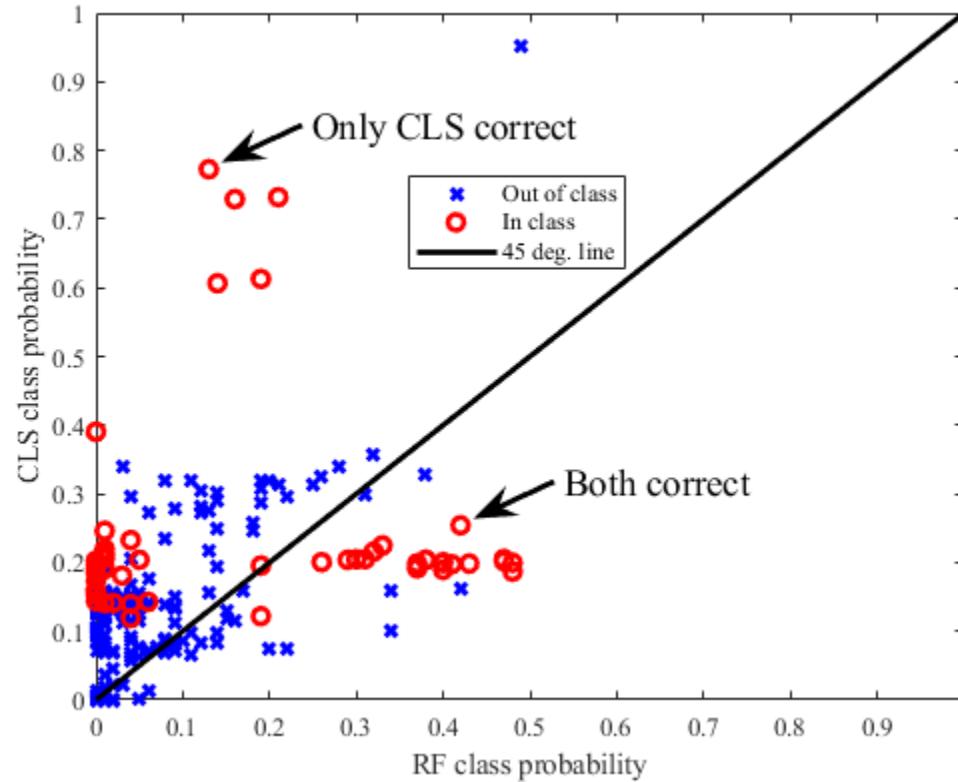
# Event classification experiment

- Varied the threshold of events for a class to be included.
- Compared to **Random Forest (RF)**, the best baseline.

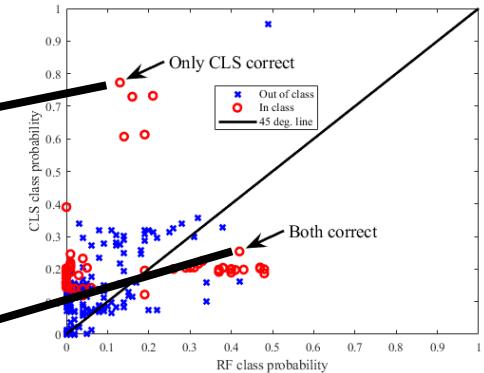
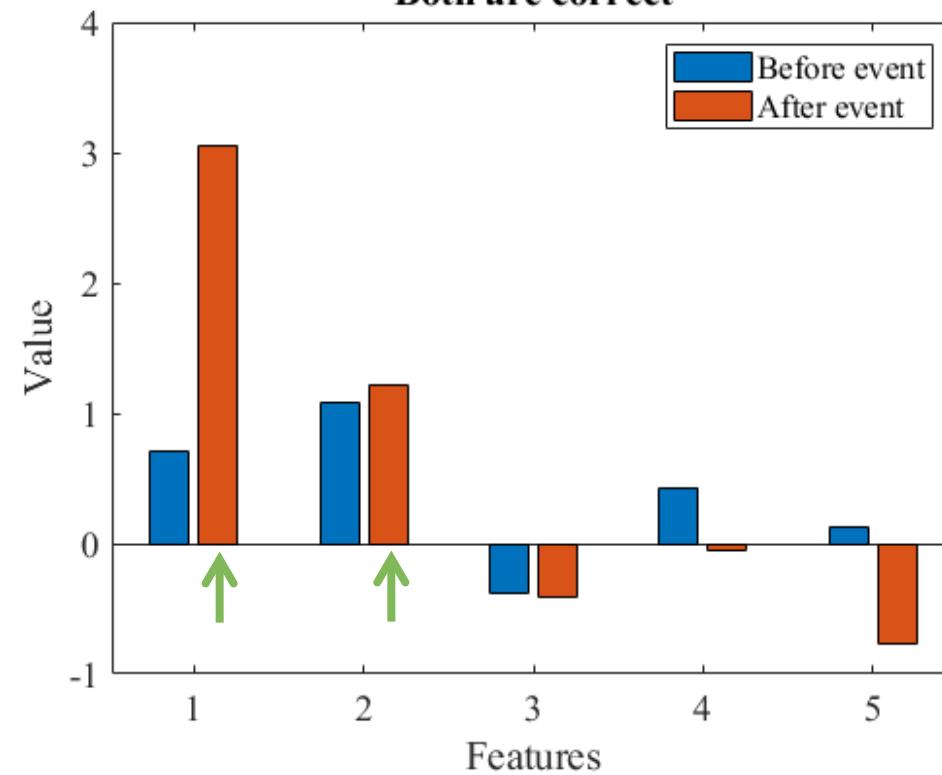
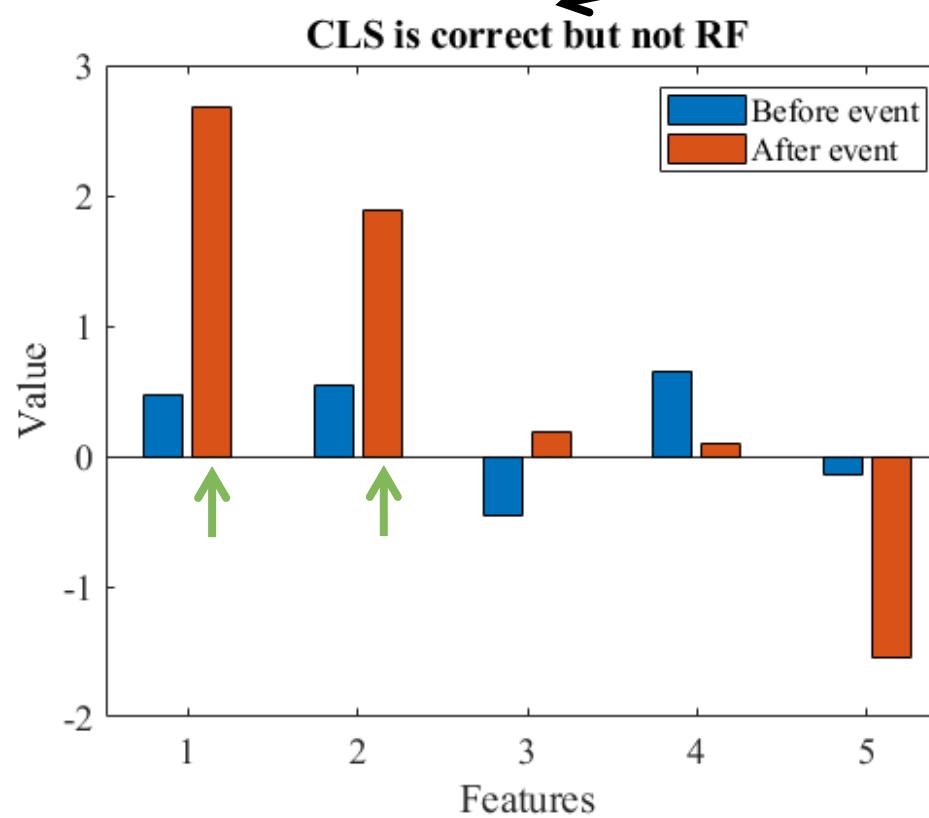


Mostly statistically significant.

# Scores of two examples, one a mistake by random forest

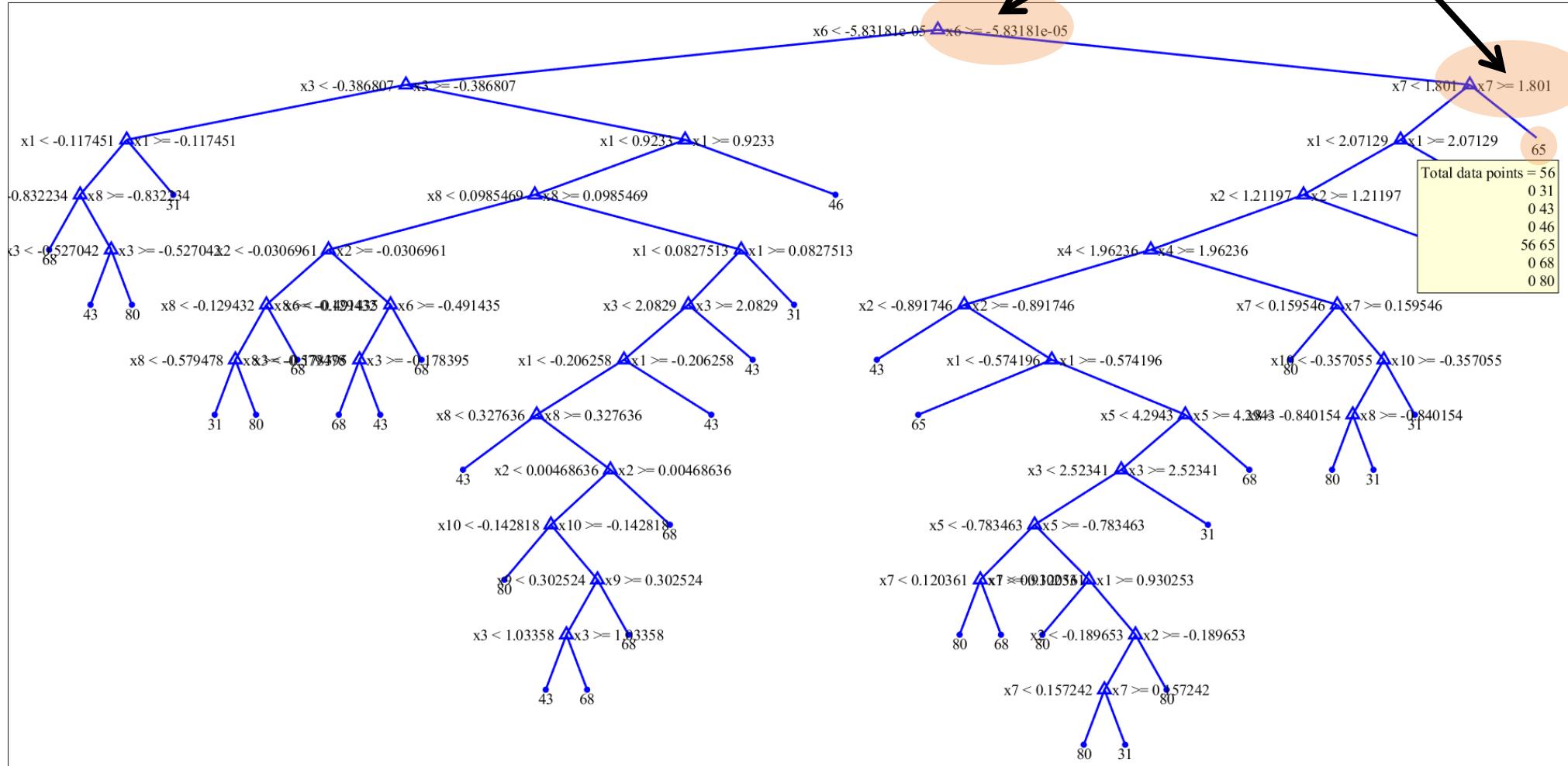


# Features of two examples



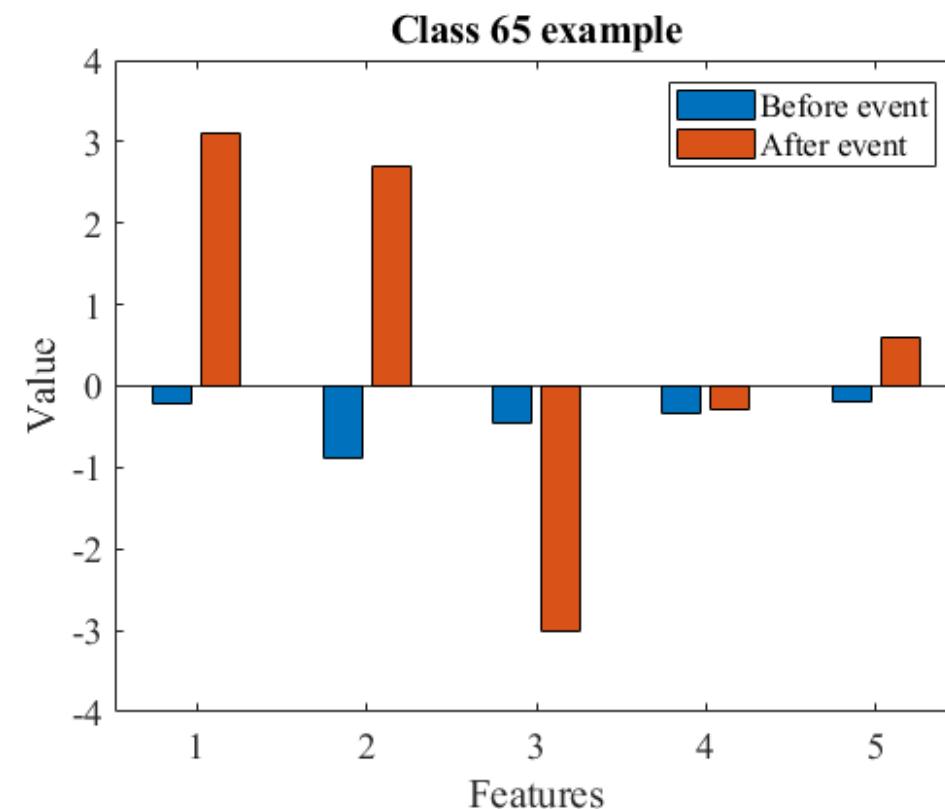
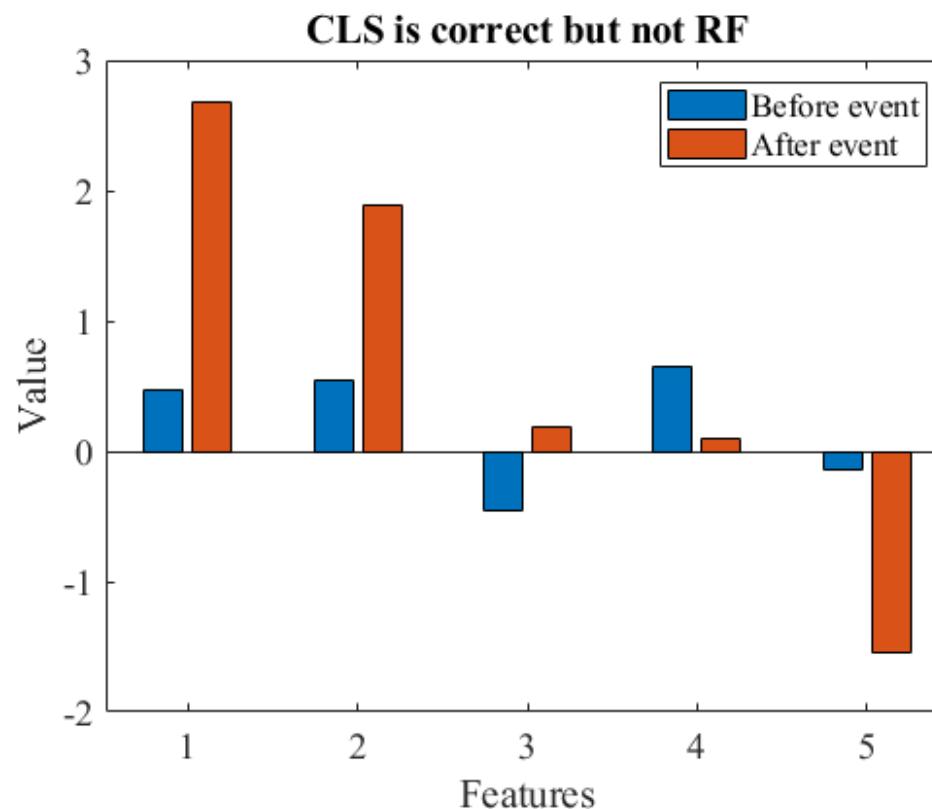
Multi-view relationship is similar between examples.

# A tree that makes a mistake



Each node only uses one view.

# Features of the mistaken class



Tree misclassifies example as Class 65 because it neglects the multi-view relationship.

# Key takeaway of approach to multi-view classification

Leverage multi-view relationships as discriminative factors in classification to perform well on bleeding and load monitoring datasets

# Multi-view relationships for analytics and inference

## Summary

- Operated on multi-view relationships as a unit of analysis, resulting in novel structure and good empirical performance
- Single sensor method for gamma source detection
- Multiple sensor extension with known relationship
- Linear multi-view relationships
- Nonlinear multi-view relationships
  - Clustering
  - Classification
- Multi-view approach to learning on distributions

# Multi-view relationships for analytics and inference

## Future work

- Regression
- Multi-modal data
- Arbitrary nonlinear relationships
  - Mutual information instead of correlation
  - Kernel CCA [Shotaro 2006] and Deep CCA [Galen 2013]
- Theory to explain performance of CLS clustering/classification

Shotaro (2006). A kernel method for canonical correlation analysis. arXiv preprint.  
Galen et al. (2013). Deep canonical correlation analysis. ICML.

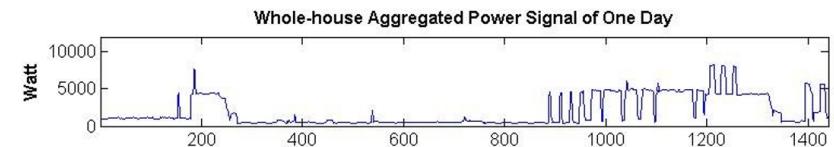
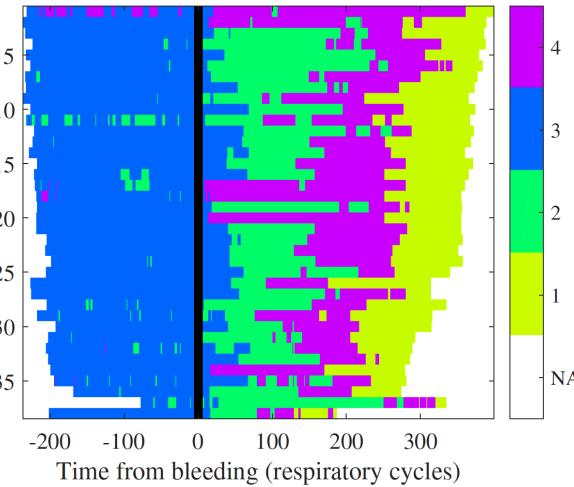
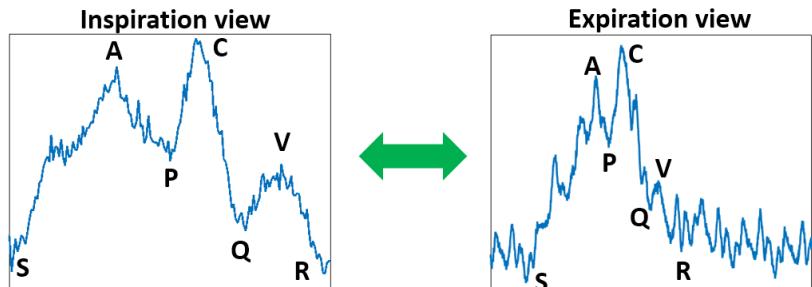
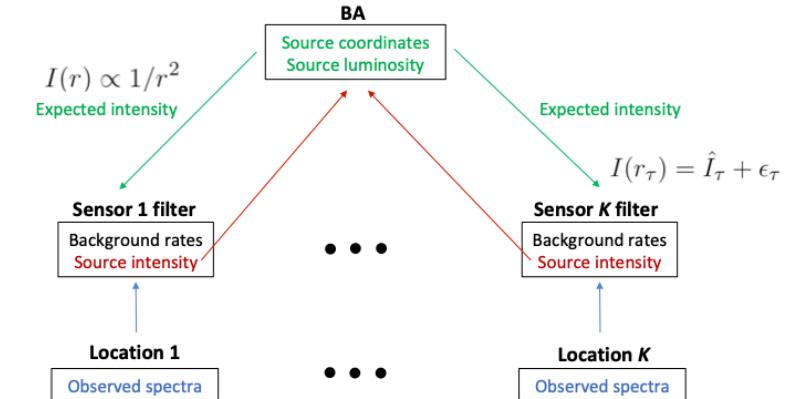
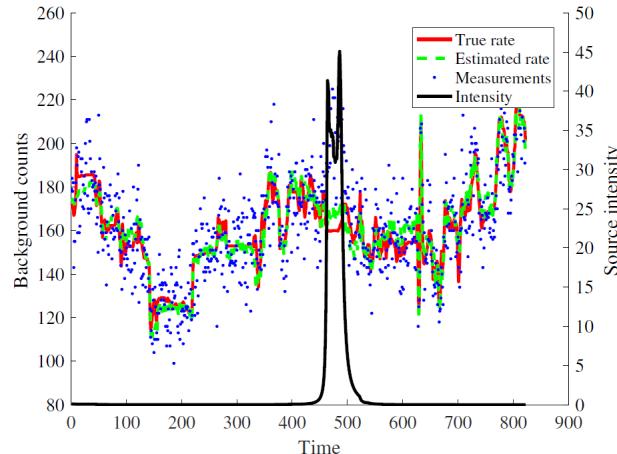
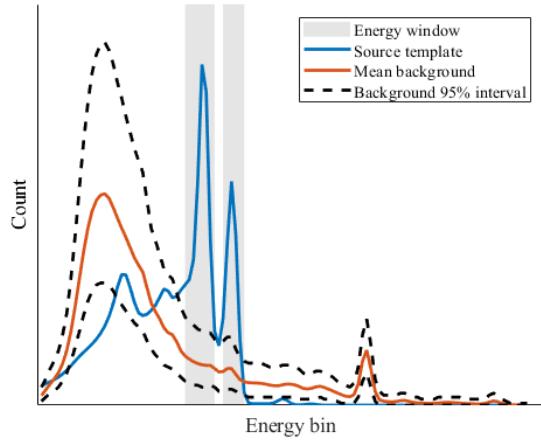
# Thank you



Homeland  
Security



NIH  
National Institutes  
of Health



- Lei et al. (2016). Radiological threat detection for an unknown energy window by canonical correlation analysis. NSS.
- Lei et al. (2017). Robust detection of radiation threat by simultaneous estimation of source intensity and background. NSS.
- Lei et al. (2017). Bleeding detection by multi-view correlation clustering of central venous pressure. MLHC.
- Lei et al. (2019). Characterization of multi-view hemodynamic data by learning mixtures of multi-output regressors. ISICEM.

# Appendix

# More than two views

- Generalized CCA [Horst 1961] finds shared representation  $G$  between all views

$$\begin{aligned} & \underset{U_j \in \mathbb{R}^{d_j \times r}, G \in \mathbb{R}^{r \times N}}{\text{minimize}} \sum_{j=1}^J \|G - U_j^\top X_j\|_F^2 \\ & \text{subject to} \quad GG^\top = I_r \end{aligned}$$

- Other extensions of CCA loss could be sum or minimax over pairwise loss

# Adaptive Filtering to Set Up Kalman Filter

- KF is very sensitive to the hyperparameters and
- An adaptive KF assumes they are non-stationary and estimates them in real-time
  - Bayesian, MLE, covariance matching, correlation
  - Often computationally expensive
- We propose a simple method that functions well for this problem

where  $\hat{x}$  is a Gaussian filter with length  $L$  and variance

- Disadvantage 1: Introduces additional uncertainty
- Disadvantage 2: Requires a short burn-in period (  $L$  measurements)

# Prediction approaches

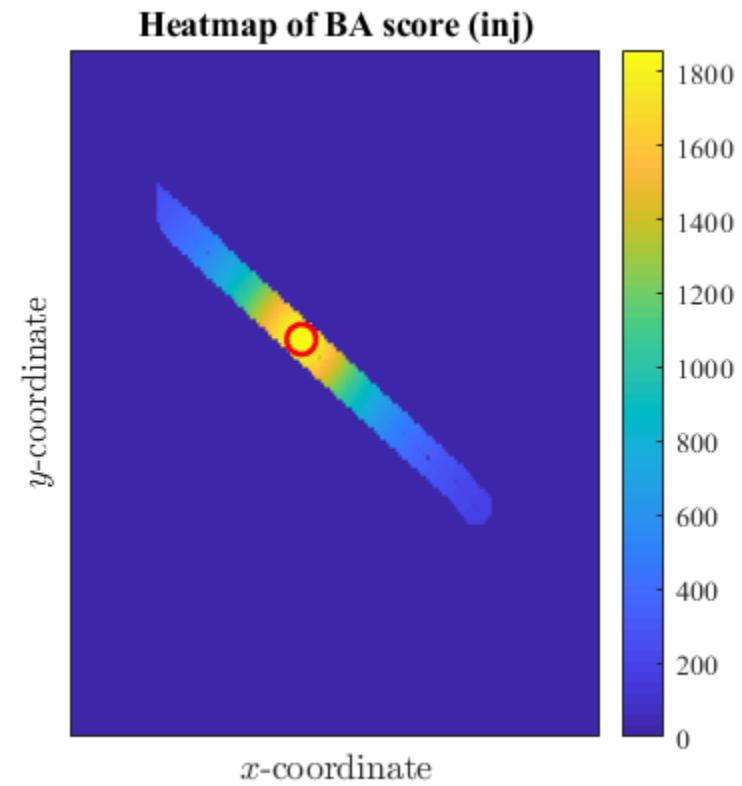
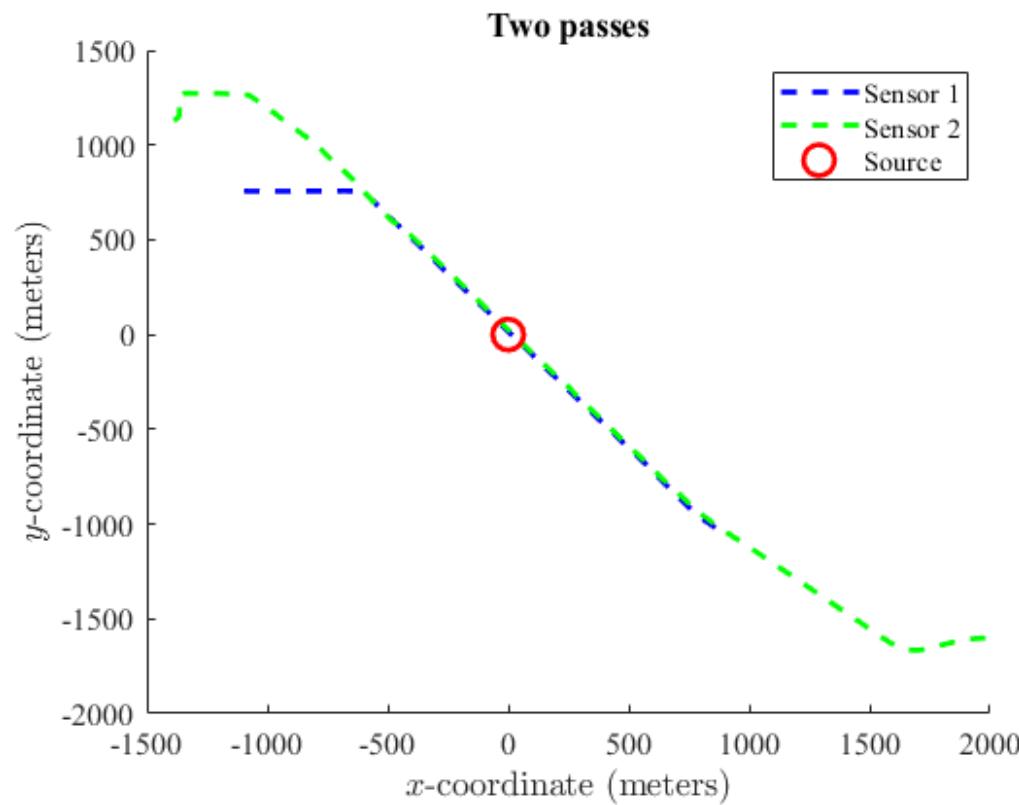
- Estimated background is detection score
- Insert background estimates into other methods
  - Matched Filter maximizes signal-to-noise ratio and is given by:

$$h = \text{cov}(X)^{-1}s$$

for spectra  $X$  and threat template  $s$

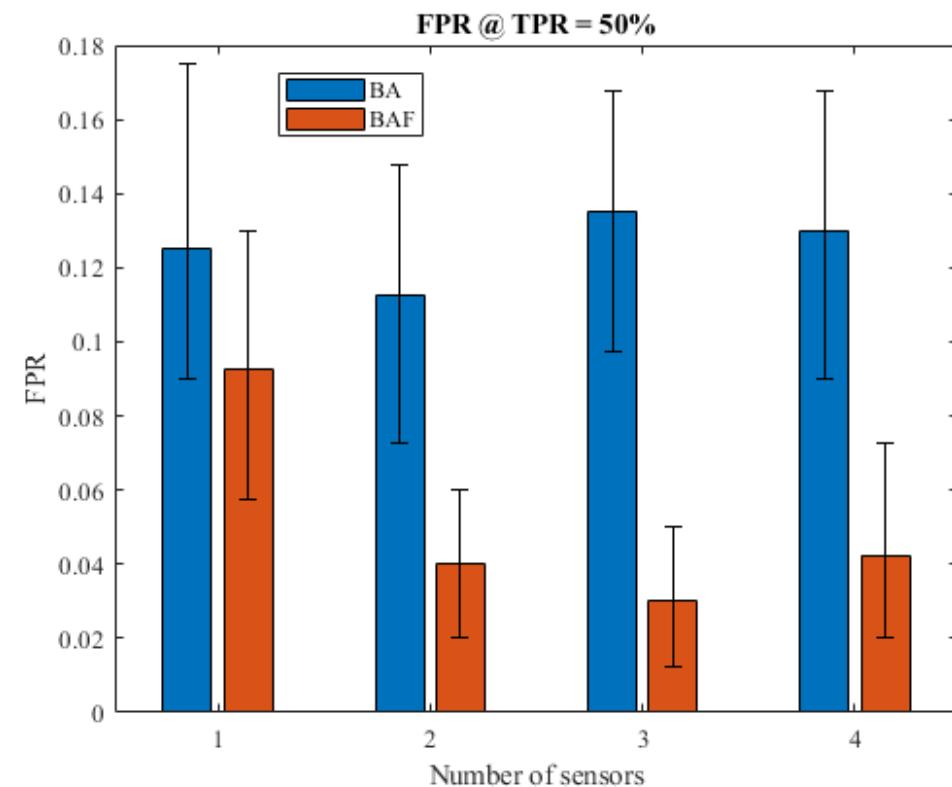
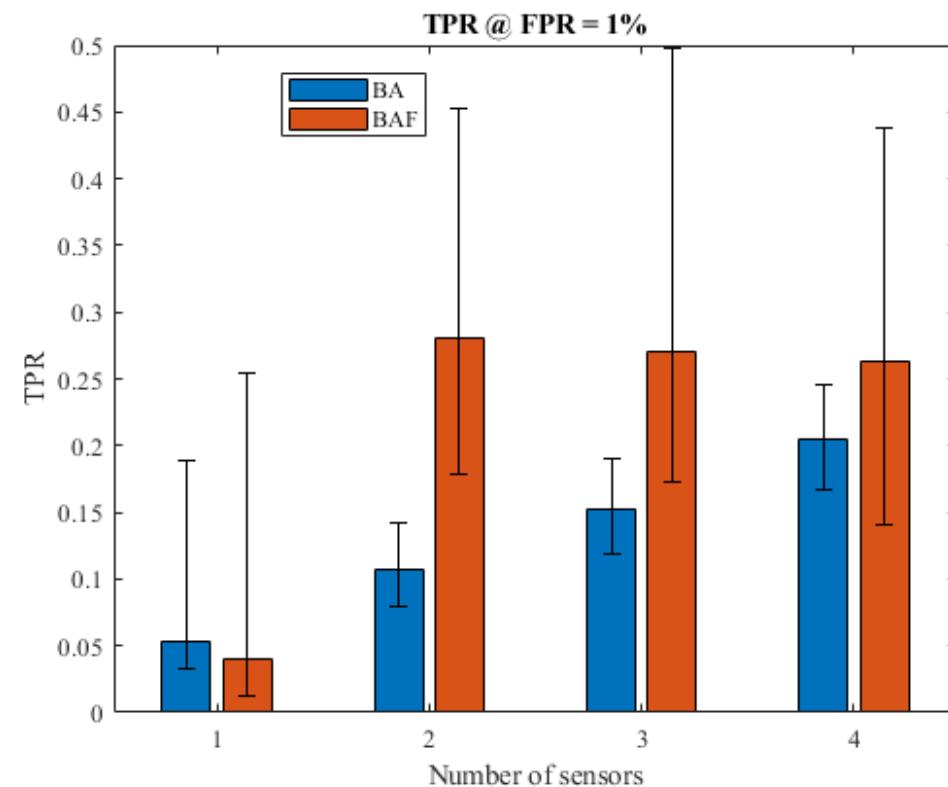
- Use past  $k$  estimates of background to get current covariance? No
- $\text{cov}(X) \rightarrow \text{diag}(x_t)$  where  $x_t$  is current estimated background

# Hypotheses restricted to pass area



BA and BAF look for source in intersection of sensor paths.

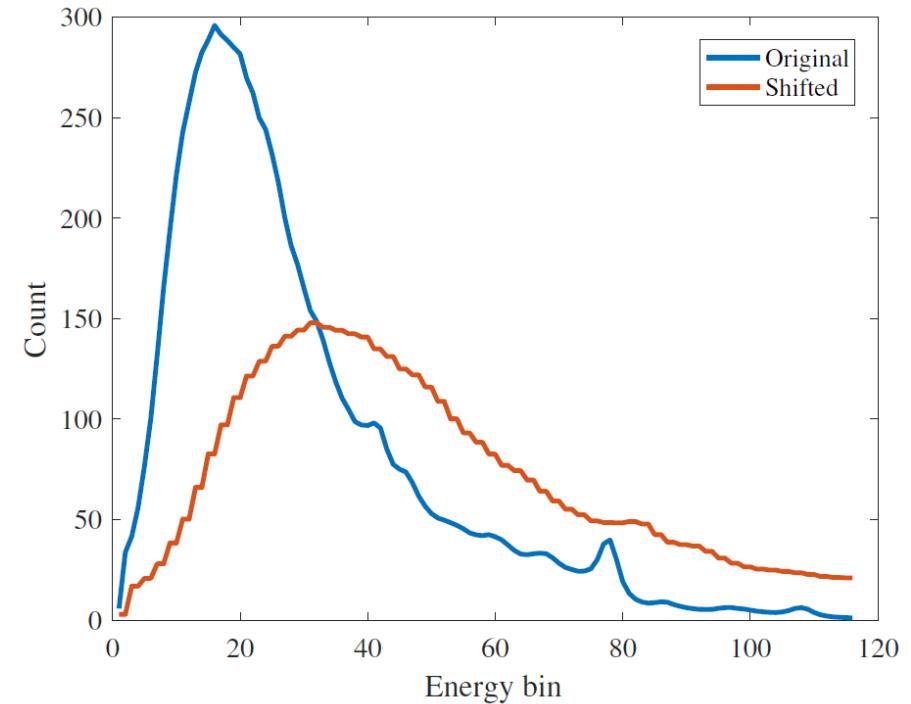
# ROC statistics for multi-view filtering



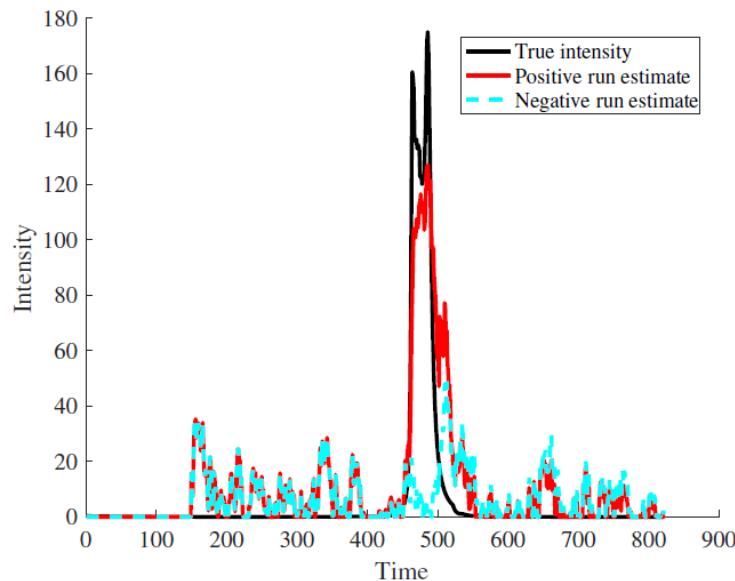
Confidence intervals from bootstrapped passes.

# Evaluating the Kalman Filter with Training-Test Mismatch

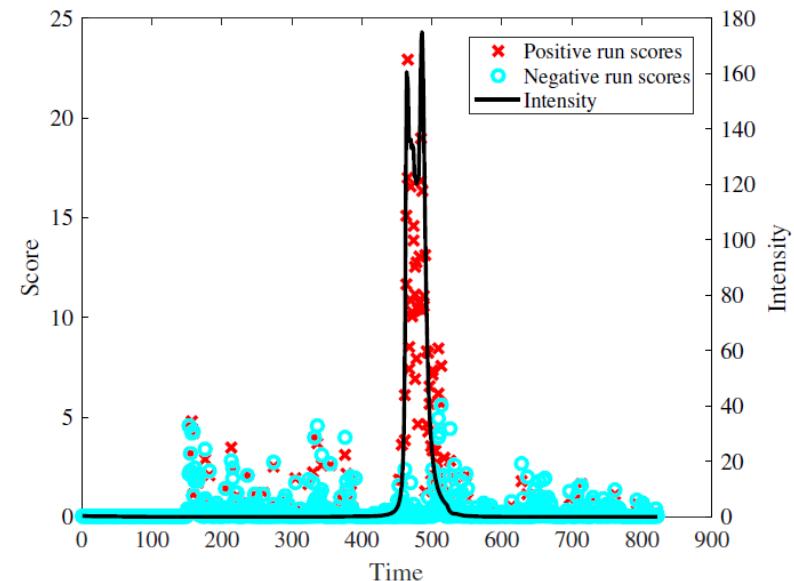
- The Kalman filter does not require training data, but other methods usually do
- It may be naïve to assume that training data match test data
- To induce mismatch between training and test background, test spectra were shifted to higher energy bins, similar to an extreme form of gain drift
- We compared several methods:
  - Oracle: Likelihood ratio using exact background rates and intensity
  - Optimal GP: GP with perfect prior
  - **Kalman GP (KGP)**: GP with prior set by KF
  - Moving Average GP (MA GP): Like KGP but with simple moving average
  - GP: GP method with prior set by training data
  - Naïve KGP: KGP with non-adaptive covariance hyperparameters estimated from training data
  - **Intensity**: Intensity estimated by KF



# Examining Estimated Intensity and Scores



(a) Kalman filter intensity estimates.

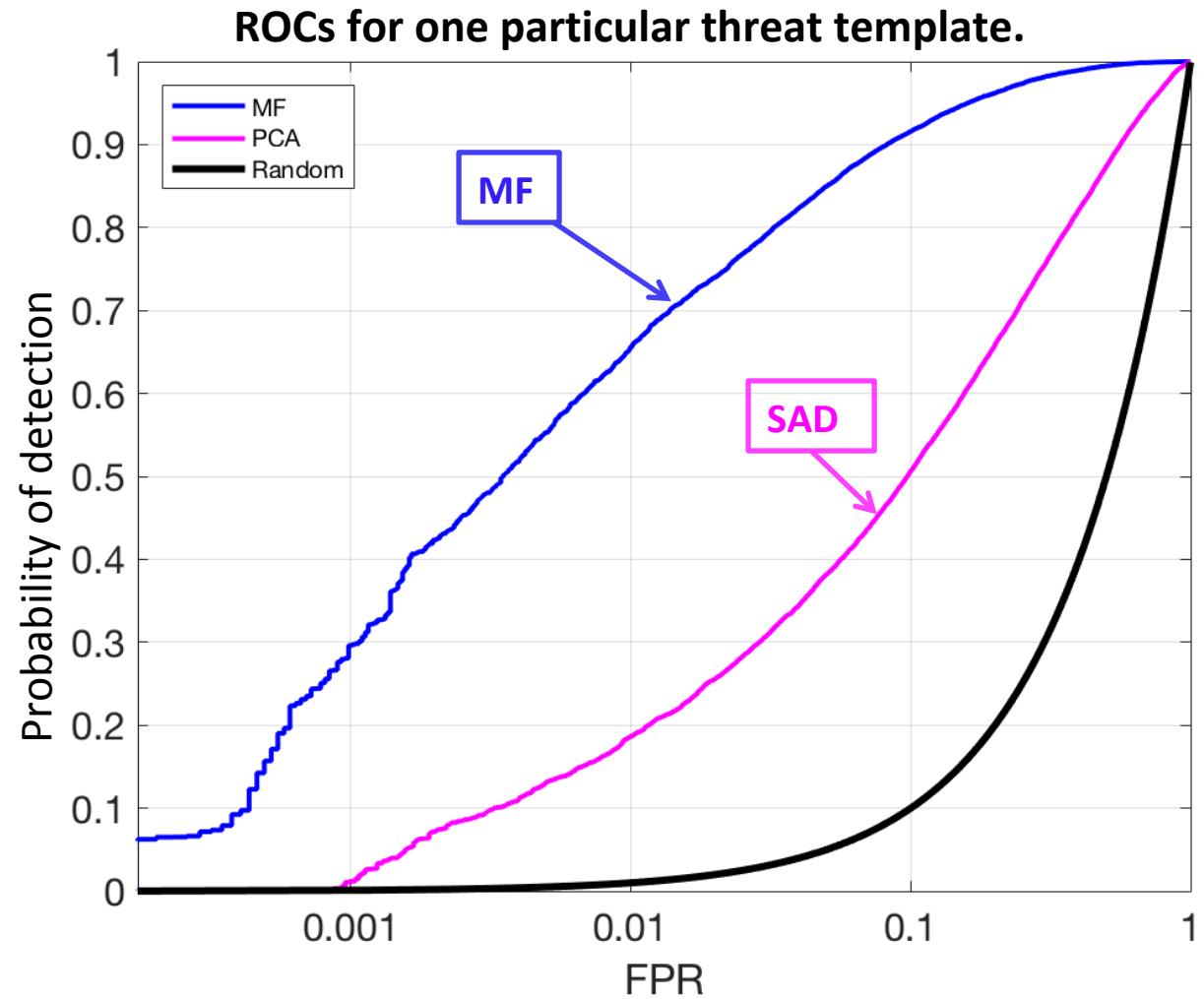


(b) KGP scores.

- Estimates of intensity were compared to the true intensity, which spiked when the detector moved near the source
- Intensity and KGP scores tracked the true intensity well with low lag
- There were not large spikes when there was no source

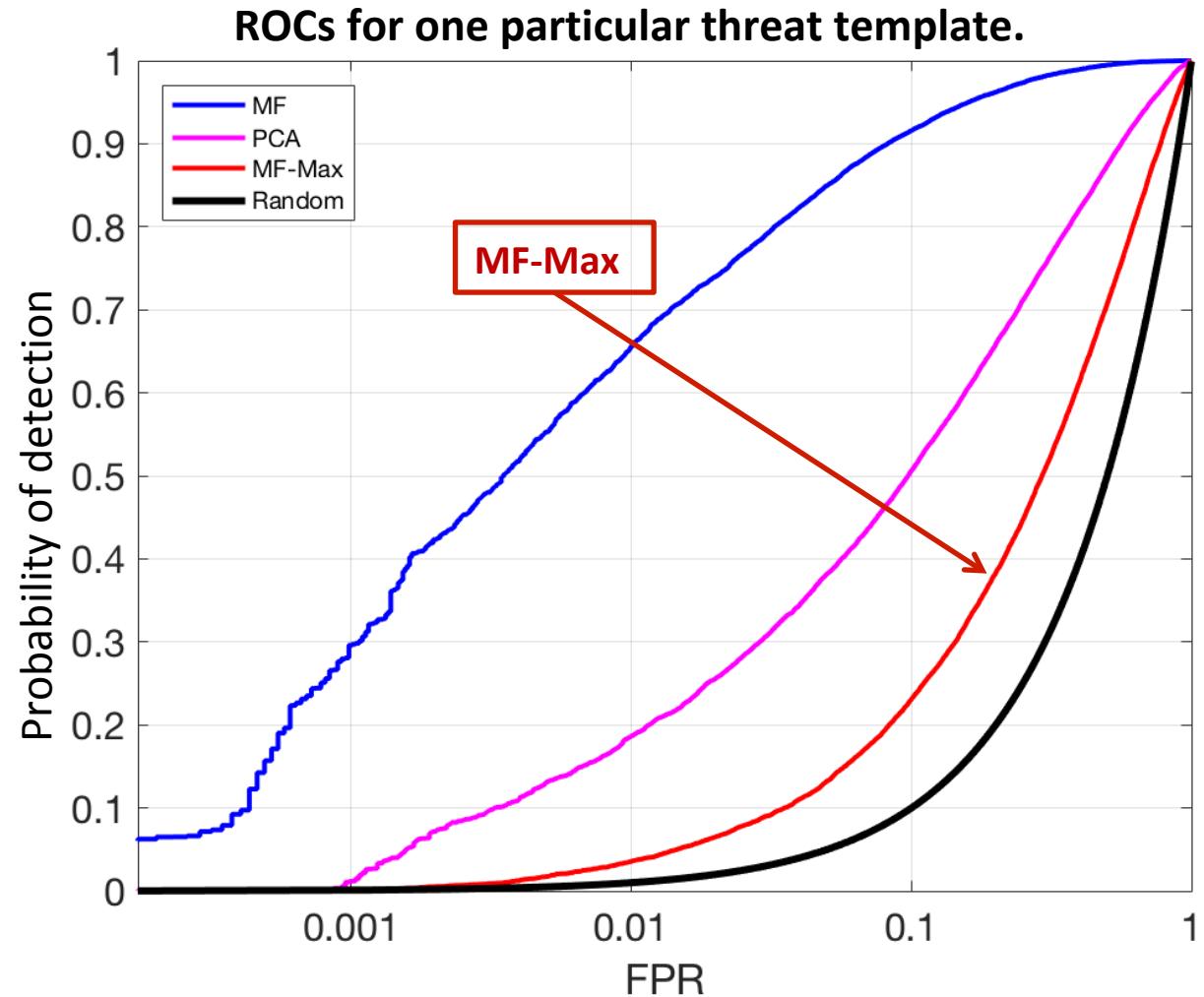
# Alternative Circumstances of Source Detection

- If we do not know what threat design to expect, we can use **Spectral Anomaly Detection (SAD) or PCA** (Tandon 2016).
- If we have perfect knowledge of the shape of threat spectrum, we can use a **Matched Filter (MF)** (Tandon 2016)
- In practice, we often have an idea of what threat to expect, but our knowledge of it is usually imperfect



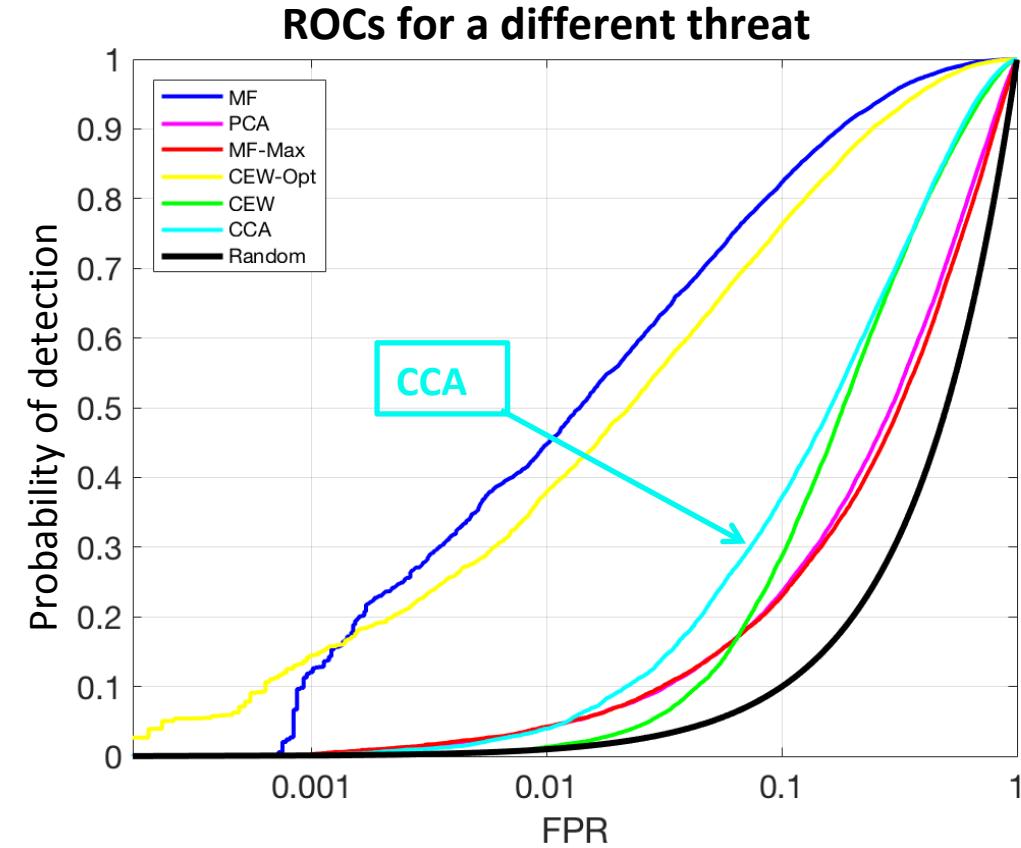
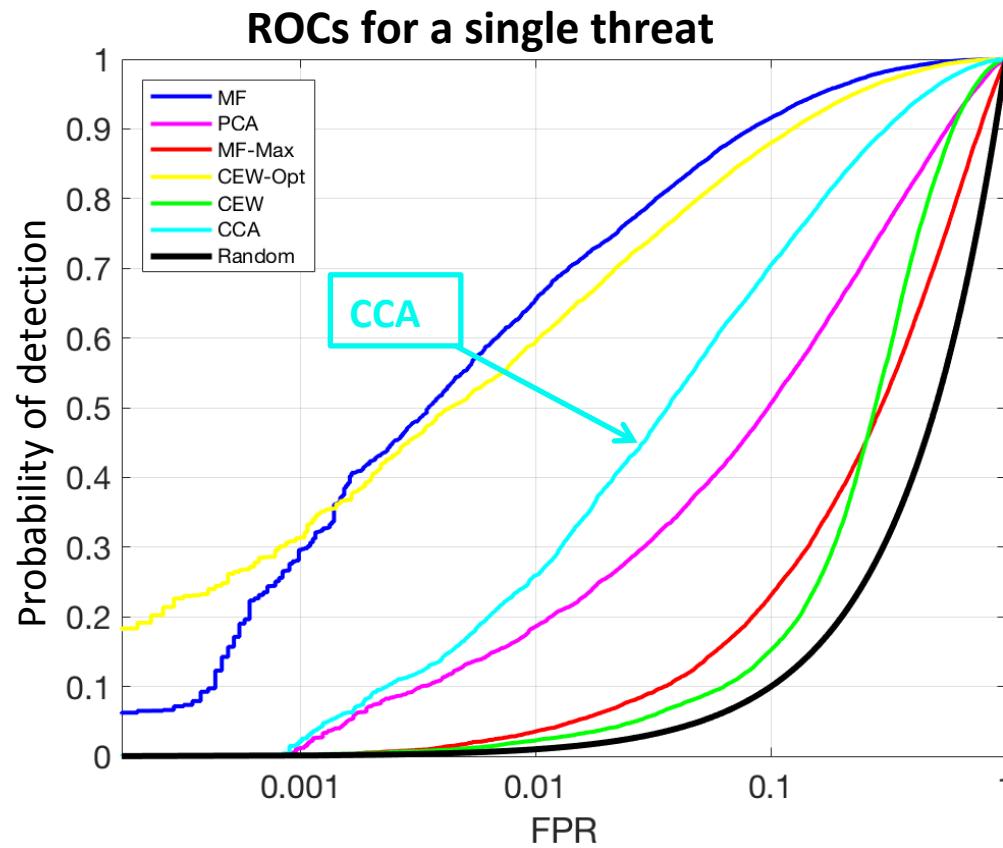
# Alternative Circumstances of Source Detection

- If we can predict the variety of possible threat templates and form a library of threat templates, we can use marginalized version of Matched Filter, i.e., **MF-Max**
  - It would work as well as MF if marginalization always correctly picked the right threat template to use



# Simulations with Imperfect Information

- We compare **MF-Max**, **CEW**, and **CCA** where we marginalize over a threat library that does not contain the actual threat.
- Our **CCA** method yields improved performance closer to the optimal information case.



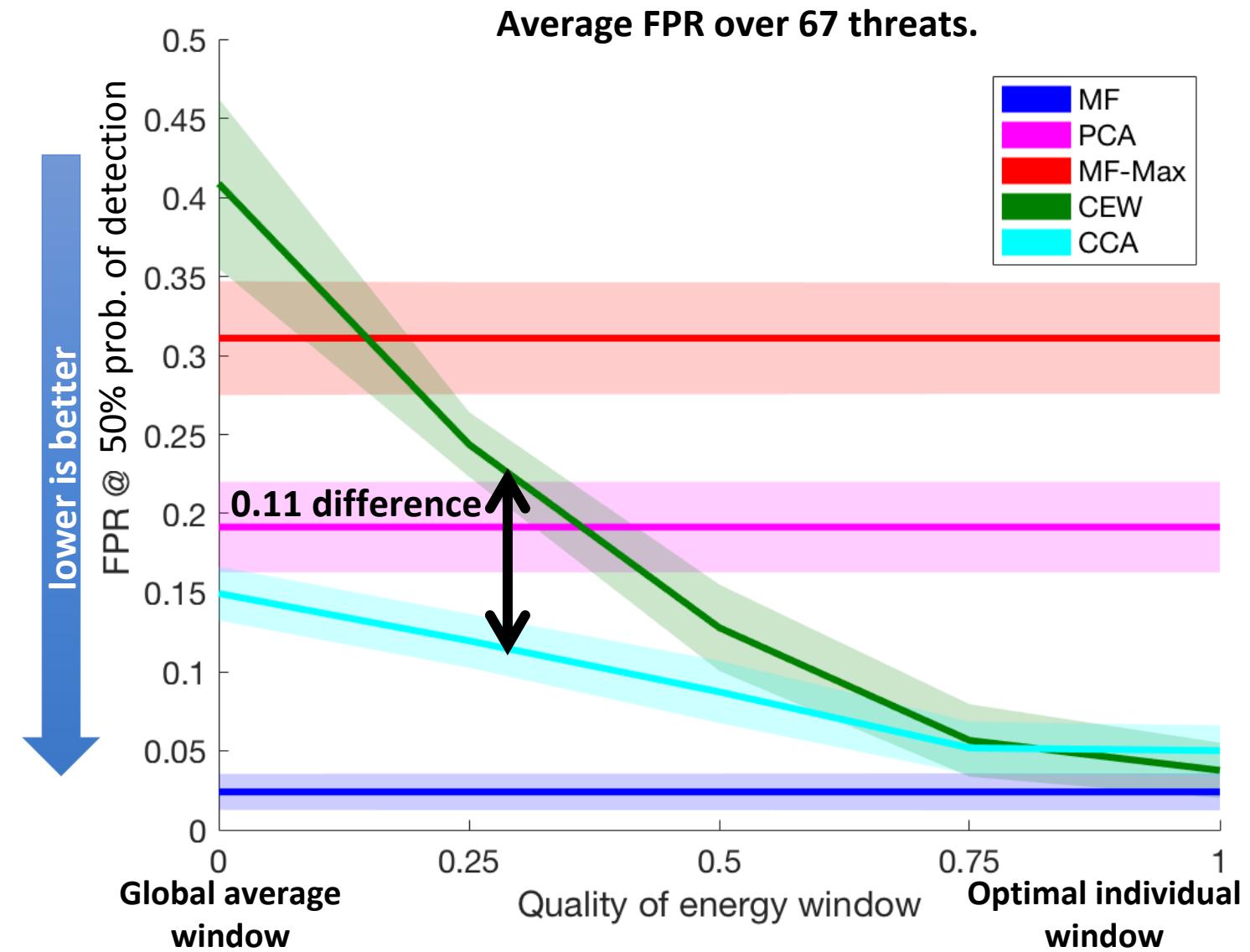
# Changing the Energy Window Quality

1. Compute global average threat template
2. Compute convex combinations of average template and actual template
3. Find energy windows of combination templates and pass to **CEW** and **CCA**



# Changing the Energy Window Quality

- FPR for each method as the window changes from low-quality to optimal
- As information about the threat spectrum decreases, the performance of **CEW** degrades and becomes much worse than **CCA**
- (Other methods do not use a window)



# Existing approaches include CCA

- A common approach is component analysis, such as Canonical Correlation Analysis (CCA), which fits a linear correlation model between two views

View 1

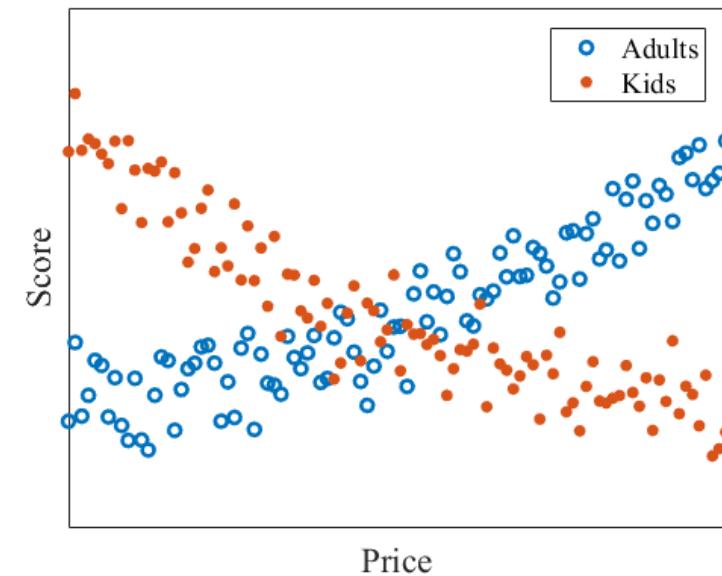
A	B	C	D	E	F	G	H	I
row	range	center_lat	center_lo	plot_id	pi	SF17_VGI	SF17_VGI	
2	0	1	34.62283	-82.733	SF17-BE-p	Top76	-1	-1
3	1	5	1	34.62281	-82.733	SF17-BE-p	P1_569459	60 -0.01176
4	2	9	1	34.62278	-82.733	SF17-BE-p	P1_569244	66 -0.01835
5	3	13	1	34.62275	-82.733	SF17-BE-p	P1_656035	61 -0.01462
6	4	17	1	34.62273	-82.733	SF17-BE-p	P1_329300	62 0.02038
7	5	21	1	34.6227	-82.733	SF17-BE-p	P1_156178	7 0.012624
8	6	25	1	34.62267	-82.733	SF17-BE-p	P1_329646	75 -0.00574
9	7	29	1	34.62265	-82.733	SF17-BE-p	P1_643016	5 0.077219
10	8	33	1	34.6226	-82.733	SF17-BE-p	P1_255744	-1 -1
11	9	37	1	34.6226	-82.733	SF17-BE-p	P1_152651	-1 -1
12	10	41	1	34.62257	-82.733	SF17-BE-p	P1_	134 -0.0129
13	11	45	1	34.62254	-82.733	SF17-BE-p	P1_569459	-1 -1
14	12	49	1	34.62252	-82.733	SF17-BE-p	P1_569244	30 0.024037
15	13	53	1	34.62249	-82.733	SF17-BE-p	P1_656035	55 -0.01009
16	14	57	1	34.62247	-82.733	SF17-BE-p	P1_329300	-1 -1
17	15	61	1	34.62244	-82.733	SF17-BE-p	P1_156178	-1 -1
18	16	65	1	34.62241	-82.733	SF17-BE-p	P1_329646	77 0.032889
19	17	69	1	34.62239	-82.733	SF17-BE-p	P1_643016	-1 -1
20	18	73	1	34.62236	-82.729	SF17-BE-p	P1_255744	-1 -1
21	19	77	1	34.62233	-82.733	SF17-BE-p	P1_152651	-1 -1

View 2

A	B	C	D	E	F	G	H	I
phenotyp	date	northing	easting	gridnum	gridletter	row	range	reading
1	SF17_VGI	6/28/2017	341136.2	3832526	17 S	81	1	0.050488
2	SF17_VGI	6/28/2017	341136.1	3832526	17 S	81	1	0.051184
3	SF17_VGI	6/28/2017	341136.1	3832526	17 S	81	1	0.053378
4	SF17_VGI	6/28/2017	341136.1	3832526	17 S	81	1	0.054087
5	SF17_VGI	6/28/2017	341136.1	3832526	17 S	81	1	0.053768
6	SF17_VGI	6/28/2017	341136	3832526	17 S	81	1	0.053768
7	SF17_VGI	6/28/2017	341136	3832526	17 S	81	1	0.055059
8	SF17_VGI	6/28/2017	341136	3832526	17 S	81	1	0.056373
9	SF17_VGI	6/28/2017	341136	3832526	17 S	81	1	0.056966
10	SF17_VGI	6/28/2017	341135.9	3832526	17 S	81	1	0.058119
11	SF17_VGI	6/28/2017	341135.9	3832526	17 S	81	1	0.058122
12	SF17_VGI	6/28/2017	341135.9	3832526	17 S	81	1	0.05825
13	SF17_VGI	6/28/2017	341135.8	3832526	17 S	81	1	0.059209
14	SF17_VGI	6/28/2017	341135.8	3832526	17 S	81	1	0.058273
15	SF17_VGI	6/28/2017	341135.8	3832526	17 S	81	1	0.058624
16	SF17_VGI	6/28/2017	341135.8	3832526	17 S	81	1	0.058834
17	SF17_VGI	6/28/2017	341135.7	3832526	17 S	81	1	0.0584
18	SF17_VGI	6/28/2017	341135.6	3832526	17 S	81	1	0.0519
19	SF17_VGI	6/28/2017	341135.6	3832526	17 S	81	1	0.05027
20	SF17_VGI	6/28/2017	341135.5	3832526	17 S	81	1	0.050918
21	SF17_VGI	6/28/2017	341135.5	3832526	17 S	81	1	0.052112

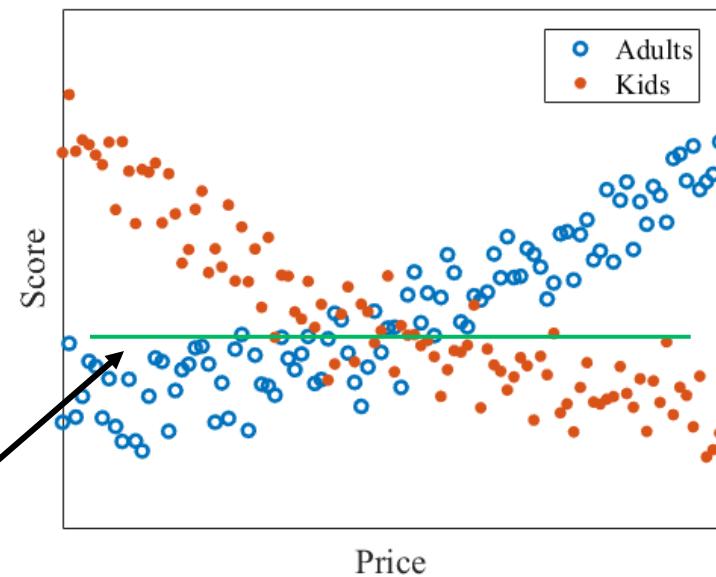
# Restaurant example

- Restaurant characterized only by price
- Review characterized only by score
- Two kinds of reviewers: **adults** and **kids**
- A reviewer's score is a monotonic function of price
  - Increasing in price for **adults**
  - Decreasing in price for **kids**
- We observe restaurant price and review score but not the type of reviewer
- (One variable per view in this example, but usually multivariate)

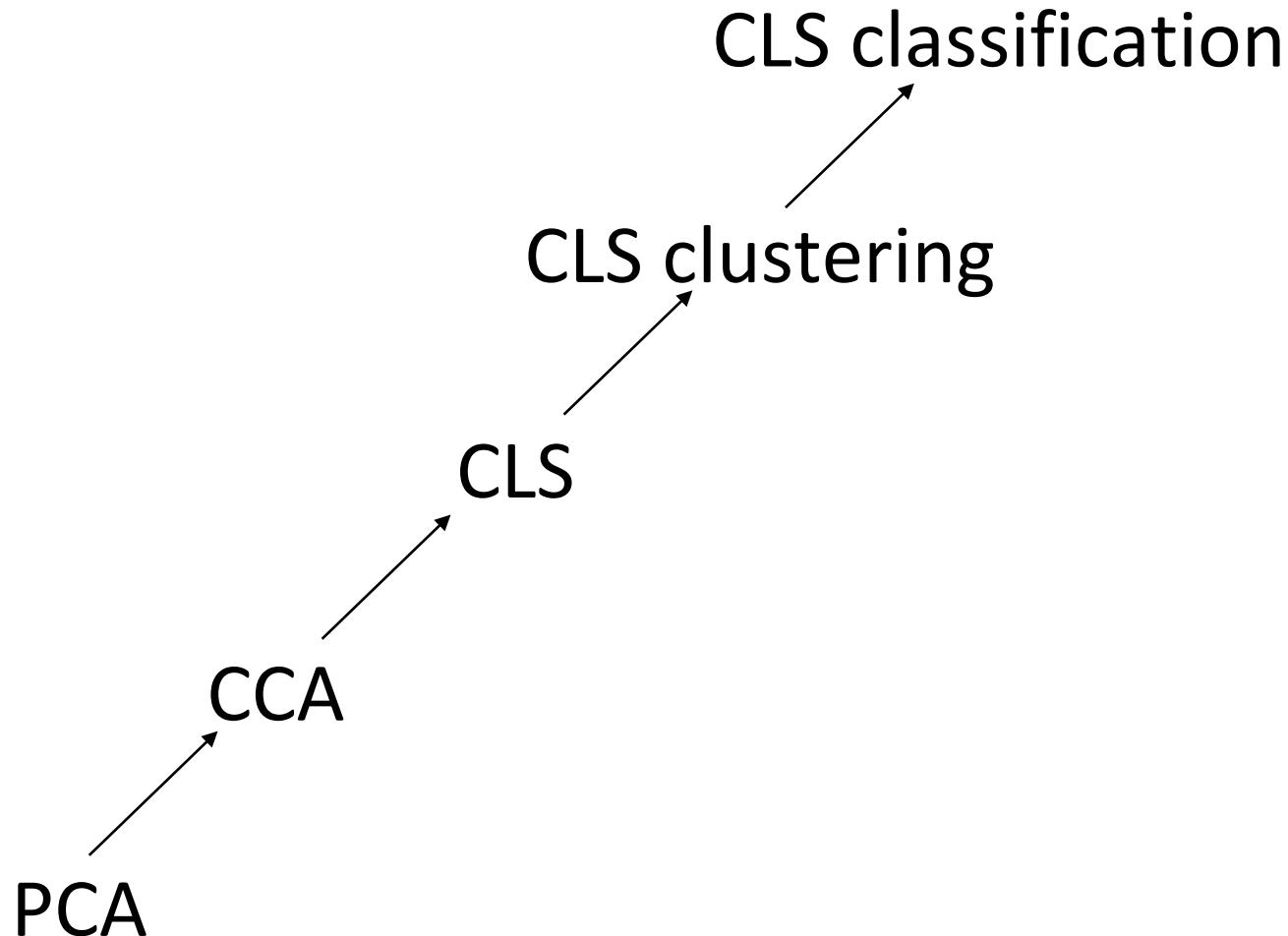


# Restaurant example (cont.)

- Given restaurants and reviews, task is to identify the relationship between them
- Since there are two populations of reviewers, this relationship varies
- Existing approaches like CCA may struggle because they only search for **global linear structure**



# Roadmap to our approach



# Principal components analysis and canonical correlation analysis

## PCA

- Analyzes directions of maximum variance in a single view
- Decomposes view into linear combinations of variables
- Finds multiple orthogonal loadings
- Components are ranked by contribution to variance

## CCA

- Analyzes directions of maximum correlation between two views
- Decomposes each view into linear combinations of variables
- Finds multiple orthogonal components
- Components are ranked by contribution to covariance

# Formal statement of CCA

- Non-convex optimization with closed-form solution:

$$\max_{u \in \mathbb{R}^{d_X}, v \in \mathbb{R}^{d_Y}} \text{Corr}(X^T u, Y^T v)$$

- $X^T u$  and  $Y^T v$  are latent factors called canonical variables
- The solution for  $u$  is the leading eigenvector of

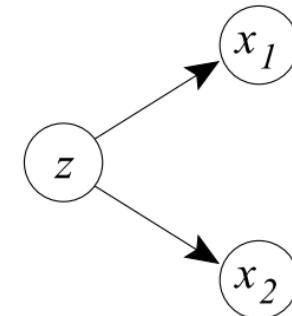
$$A = \Sigma_{XX}^{-1} \Sigma_{XY} \Sigma_{YY}^{-1} \Sigma_{XY}^T$$

and similarly for  $v$

- There are multiple components: the  $m$ th factor can be found by solving

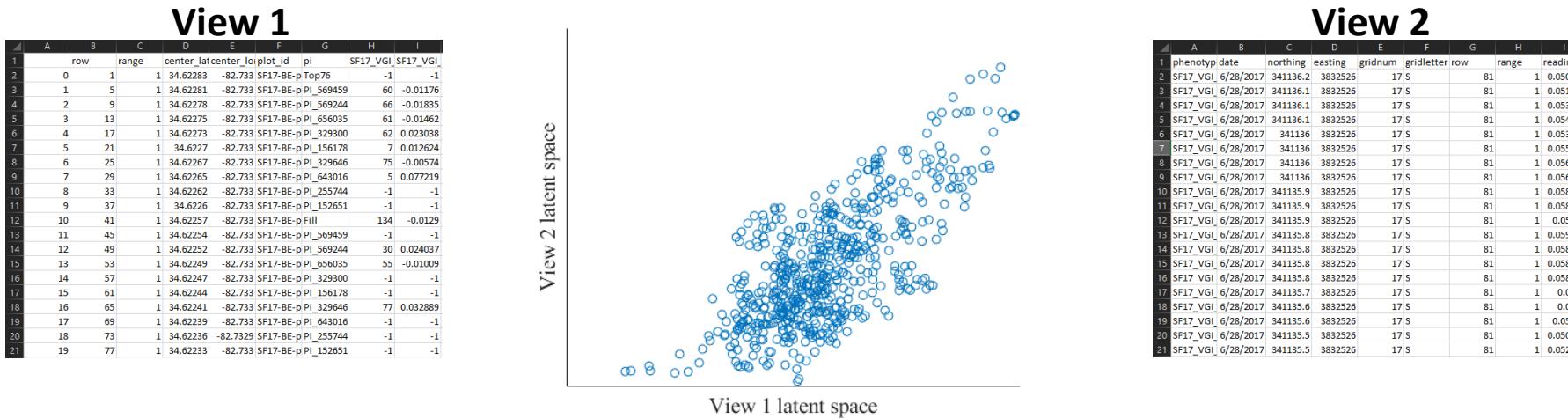
$$\max_{u \in \mathbb{R}^{d_X}, v \in \mathbb{R}^{d_Y}} \text{Corr}(X^T u, Y^T v)$$

subject to  $\text{Cov}(Xu, Xu_i) = \text{Cov}(Yv, Yv_i) = 0,$   
 $i = 1, \dots, m - 1.$



# Recap of the linear approach

- A common approach is component analysis, such as Canonical Correlation Analysis (CCA), which fits a linear correlation model between two views



- However, CCA might struggle if the correlations are nonlinear or non-global

# Mixture of CCA optimization

- This (simplified) optimization problem looks like

$$\min_{u,v,R} \quad \sum_j \|X^{(j)}u^{(j)} - Y^{(j)}v^{(j)}\|_2^2$$

$$\text{s.t.} \quad u^{(j)\top} X^{(j)\top} X^{(j)} u^{(j)} = v^{(j)\top} Y^{(j)\top} Y^{(j)} v^{(j)} = 1$$

- $R$  is cluster labels
- $X^{(j)}$  and  $Y^{(j)}$  are subsampled data matrices of cluster  $j$
- PSD objective in  $u,v$  but quadratic constraints
- Alternative way of writing CCA that looks like least-squares

# Canonical Least Squares

- CLS is our alternative to CCA that is better for clustering
- Minimizes squared error in latent space:

$$\min_{u \in \mathbb{R}^{d_X}, v \in \mathbb{R}^{d_Y}} \|Xu - Yv\|_2^2$$

subject to  $v^\top v = 1$ .

- PSD objective and quadratic constraint
  - data-agnostic  $v^\top v = 1$  inst  $u^\top X^\top X u = v^\top Y^\top Y v = 1$
- Still has closed-form solution:  $v$  is the lowest eigenvector of  $Y^\top H Y$  and  $u = (X^\top X)^{-1} X^\top Y v$  where  $H = I - X(X^\top X)^{-1} X^\top$
- Lowest eigenvector corresponds to minimizing variance of a regression residual

# Multiple CLS components

- Like CCA, CLS can find multiple components
- PSD objective and quadratic constraints:

$$\begin{array}{ll} \min_{\substack{U \in \mathbb{R}^{d_X \times m} \\ V \in \mathbb{R}^{d_Y \times m}}} & \|XU - YV\|_{\mathcal{F}}^2 \\ \text{subject to} & V^T V = I. \end{array}$$

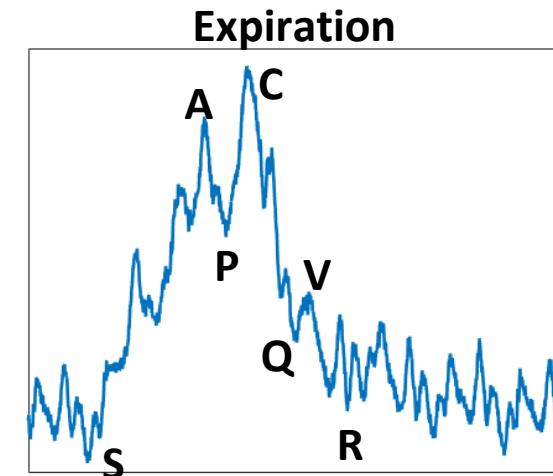
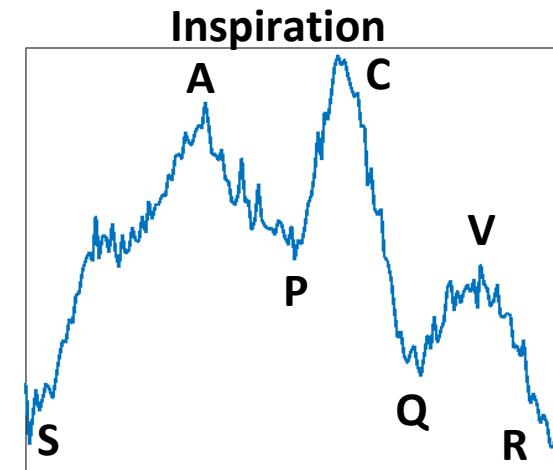
- No known closed-form solution
  - Constraint prevents 0 solution, does not normalize scale
  - Sensitive to scale, so usually standardize data
- We solve for components  $V$  via a greedy sequential approximation by taking  $V$  to be the lowest eigenvectors  $CY^T H Y$
- CCA was a multiple correlation problem, while CLS is a multi-output regression problem

# Convergence

- When number of components  $m = 1$ , algorithm is guaranteed to converge
  - Objective decreases at every step
  - For some problems, first component is most meaningful
- For higher  $m$ , the algorithm does not necessarily converge because of greedy approximation
  - Empirically not a problem

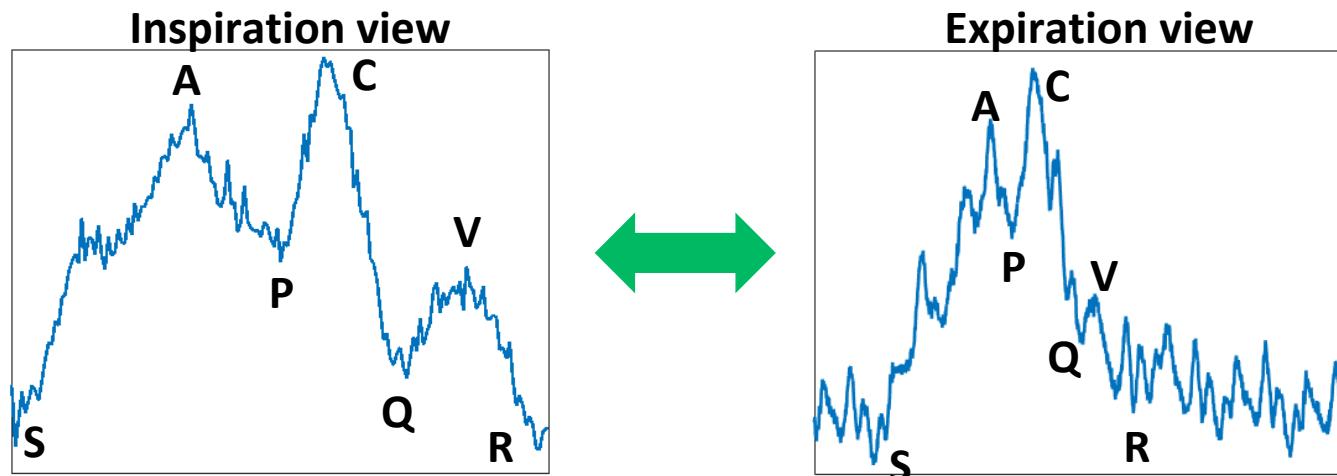
# Analyzing blood loss with CVP

- Dataset contains time series of each pig's **central venous pressure** (CVP), blood pressure near right atrium of heart
- Each waveform is from either inspiration or expiration phase of respiration
- Thirteen features were extracted from each waveform as averages and ratios between different points of the CVP waveform



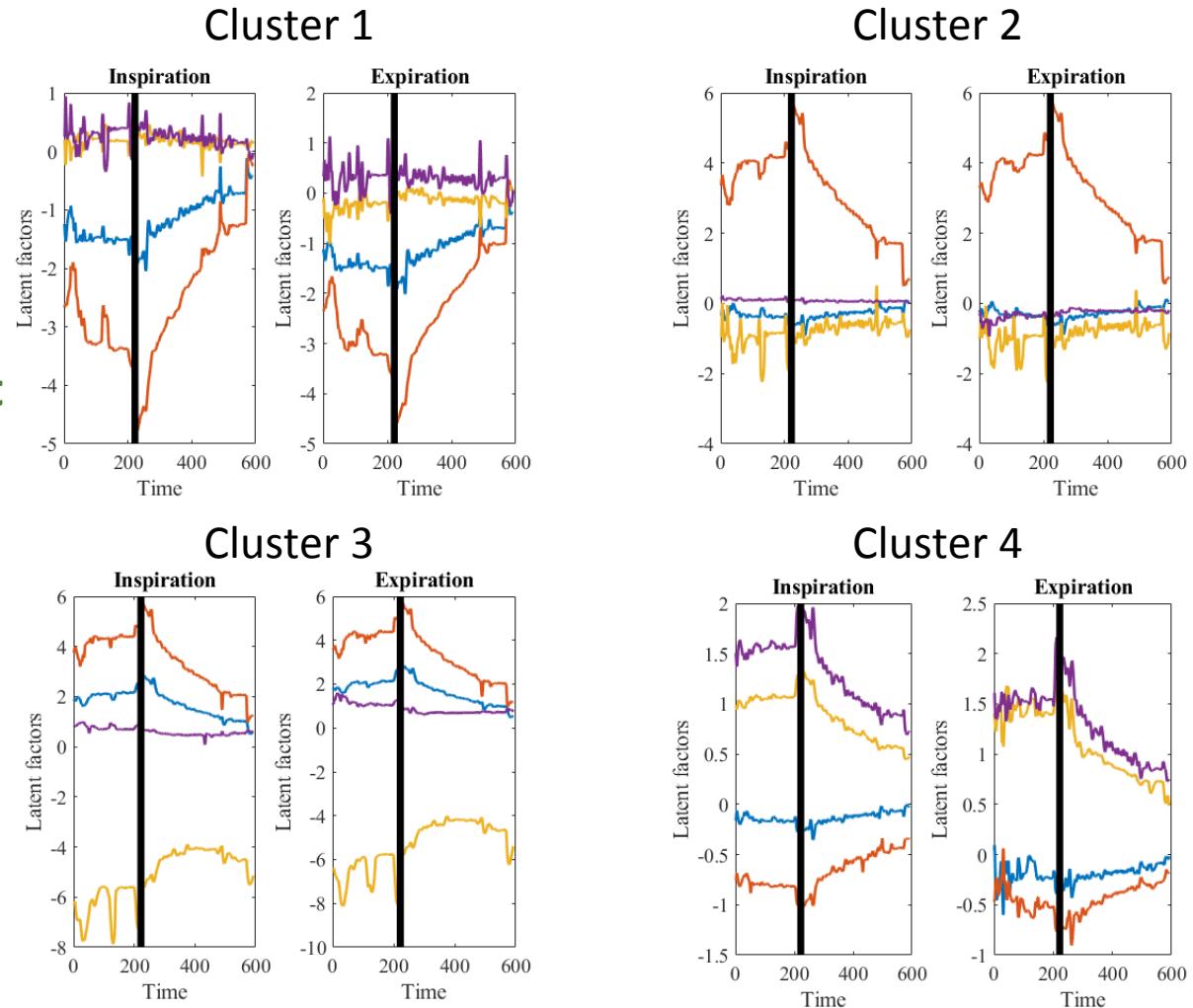
# Unsupervised setting: correlating inspiration and expiration waveforms

- We consider inspiration and expiration as the views
- We expect the correlations to still depend on bleeding, which is unobserved



# Unsupervised latent variables

- First component
- Second component
- Third component
- Fourth component

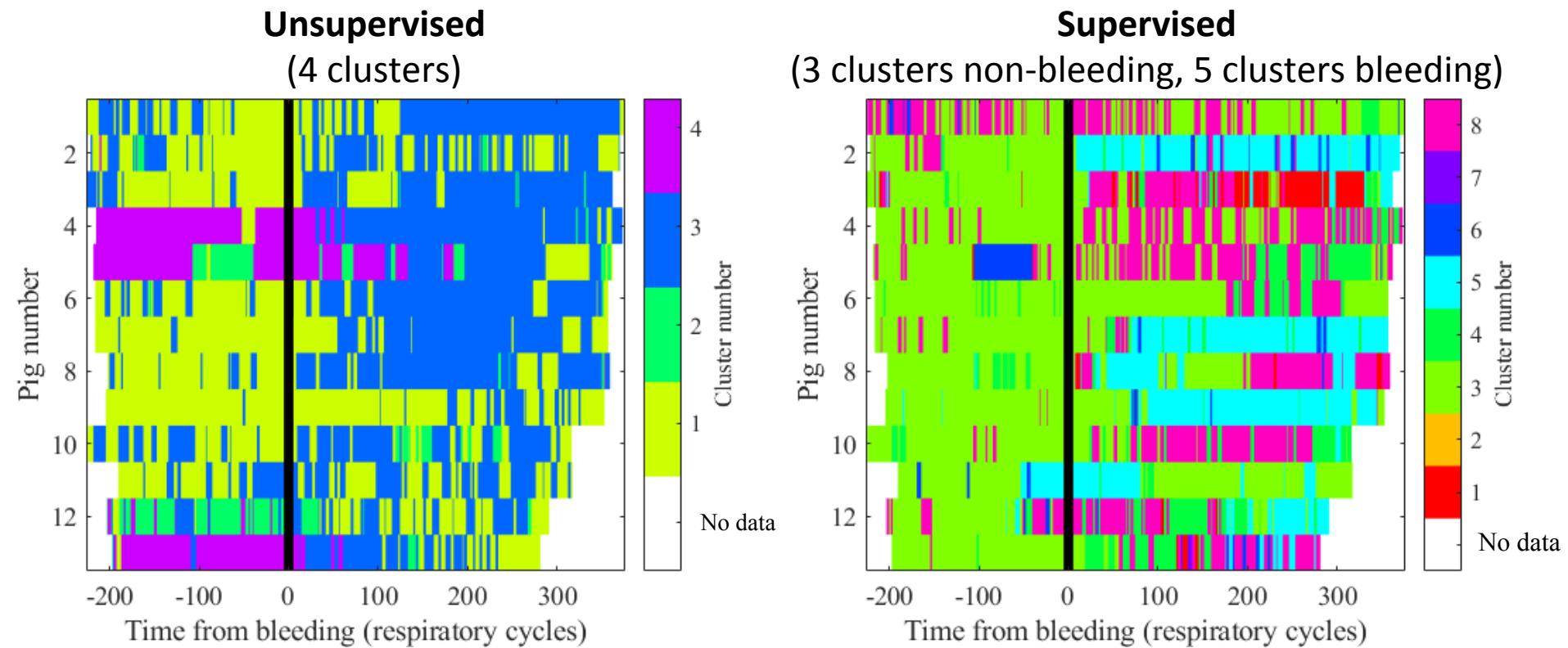


The latent variables resemble blood loss.

Supervised classification setting: correlating inspiration and expiration with knowing whether bleeding had started

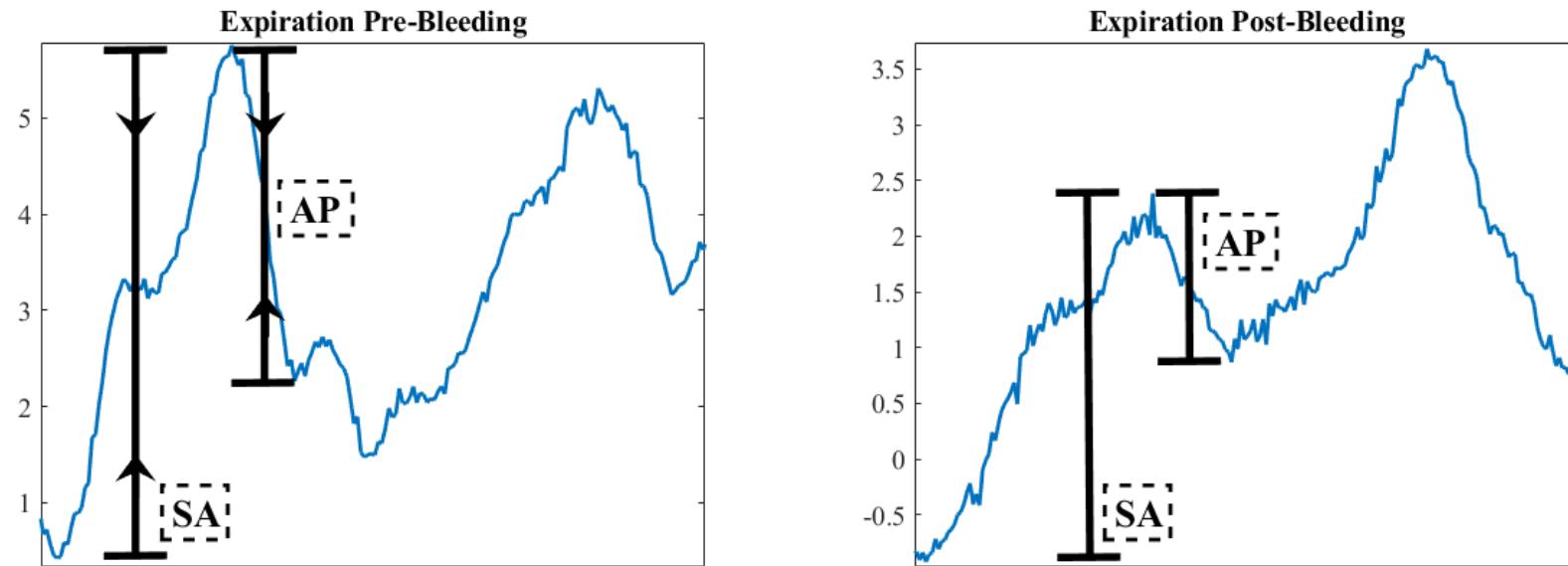
- Task: decide whether a pair of waveforms came from **before** or **after** onset of bleeding

# Cluster assignments for test pigs



We identified pre-bleeding and post-bleeding clusters, which are usually distinct. The predominant pre-bleeding cluster is light green.

# Waveform analysis



- We analyze the impact of waveform features by analyzing the gradient of classification score
- **Left:** Our model predicts that shrinking the marked lengths is correlated with bleeding in a pre-bleeding observation.
- **Right:** The corresponding lengths have shrunk in a post-bleeding observation.

# Global regression model for single-view data

- Reduced-rank regression (RRR) is a multi-output regression method

$$\min_B \mathcal{L} = \|Y - XB\|_{\mathcal{F}}^2 \quad \text{rank}(B) \leq r$$

- Has closed-form solution using eigendecomposition of  $XB_{\text{OLS}}$
- Also can be written  $\min_{U,V} \|Y - XUV^T\|_{\mathcal{F}}^2$

$$X \xrightarrow{U} Z \xrightarrow[V]{\text{rank } r} Y$$

Low-rank latent space links inputs and outputs

# Global regression model for multi-view data

- Inputs and coefficients are partitioned into  $G$  views  $B = (B_1^\top, \dots, B_G^\top)^\top$
- Want each  $B_g$  to be “low rank” – group-wise low rank constraint

$$\min_B \frac{1}{2n} \|Y - XB\|_{\mathcal{F}}^2 + \lambda \sum_{g=1}^G w_g \|B_g\|_*$$

- $\|M\|_*$  is nuclear norm, the sum of singular values, a convex relaxation of rank
- Exploits multi-view structure because each view connects to a separate low-rank latent space

# MoE with iRRR

- Nuclear norm regularization corresponds to a prior on expert parameters
- However, we are unsure if there is a valid probability distribution that leads to the nuclear norm penalty
- We use a pseudo-distribution that suffices mathematically

$$\begin{aligned}\Pr(B) &= \prod_g \Pr(B_g) \\ &\propto \prod_g \exp(-\lambda w_g \|B_g\|_*) = \exp(-\lambda \sum_g w_g \|B_g\|_*)\end{aligned}$$

- From here, EM is straightforward

# Multi-view relationships in sets of points

- iRRR does not use relationships between views
- We propose to use these relationships by weighting experts based on correlation structure
- Correlations are only defined on sets of points, so we assume the observations are already partitioned
- All points in a partition are given the same expert weights