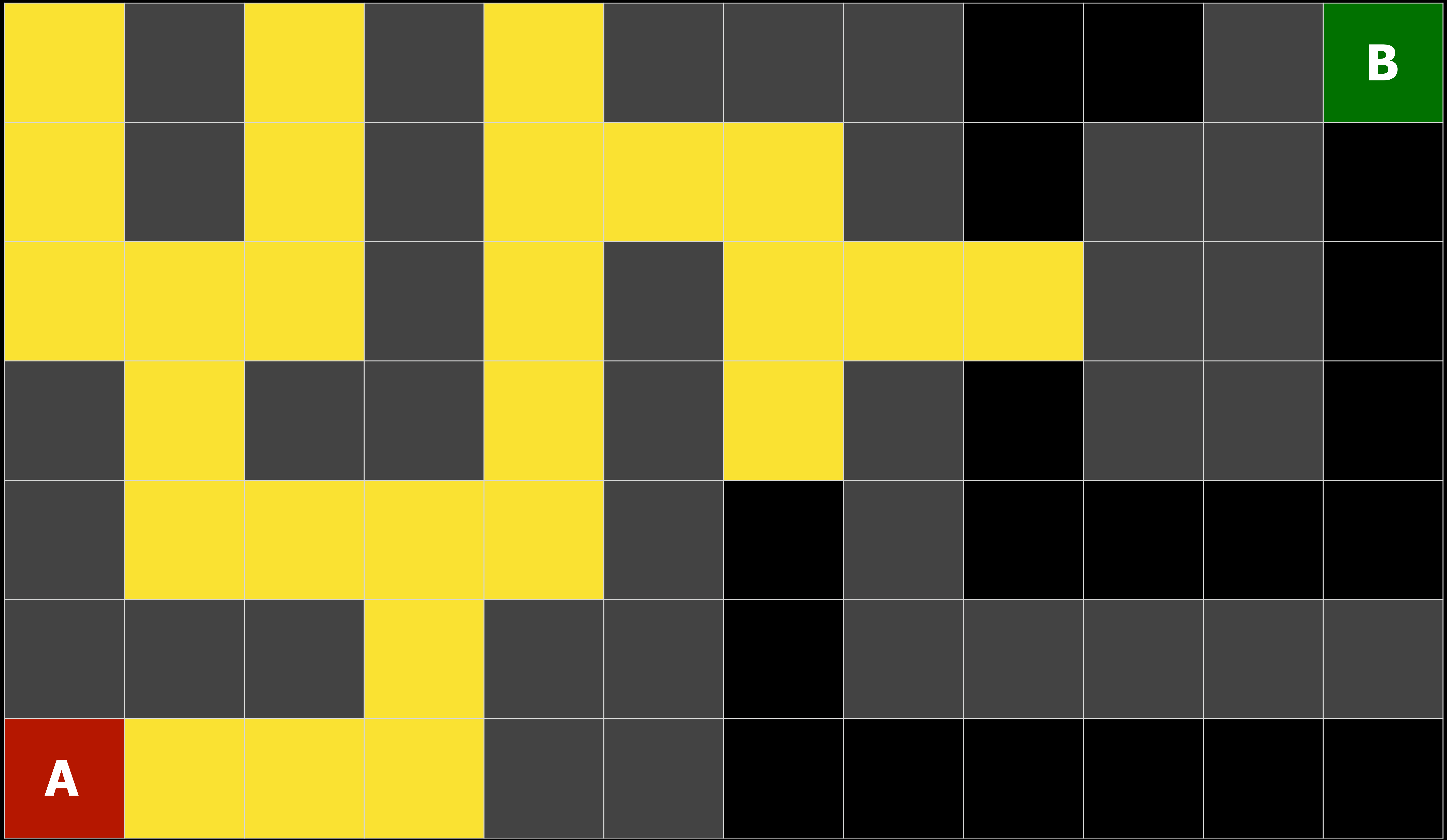
Optimization

optimization

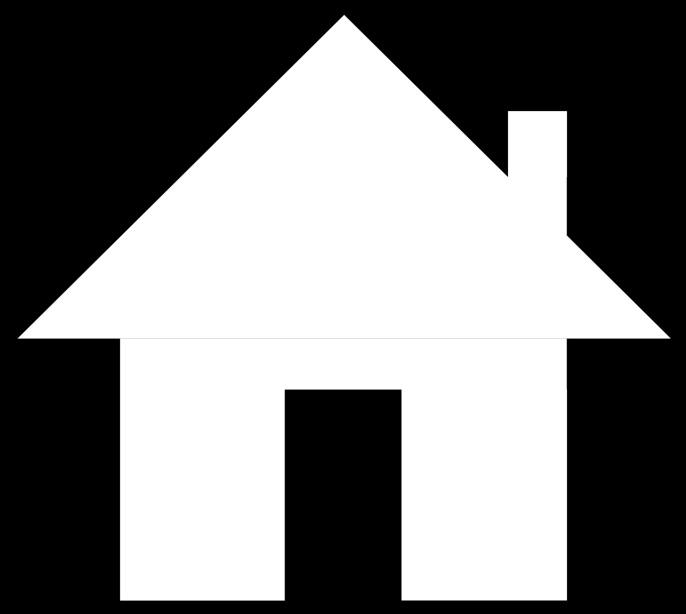
choosing the best option from a set of options

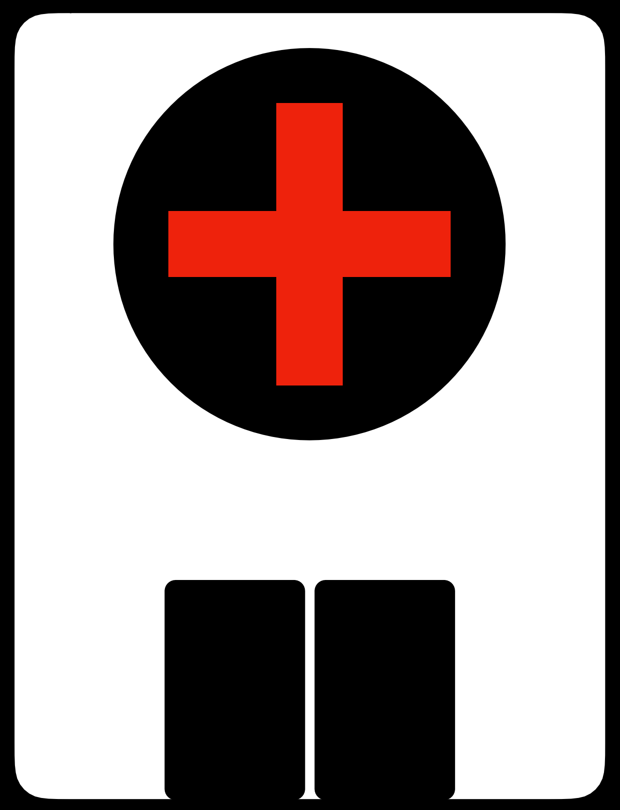
local search

search algorithms that maintain a single node and searches by moving to a neighboring node

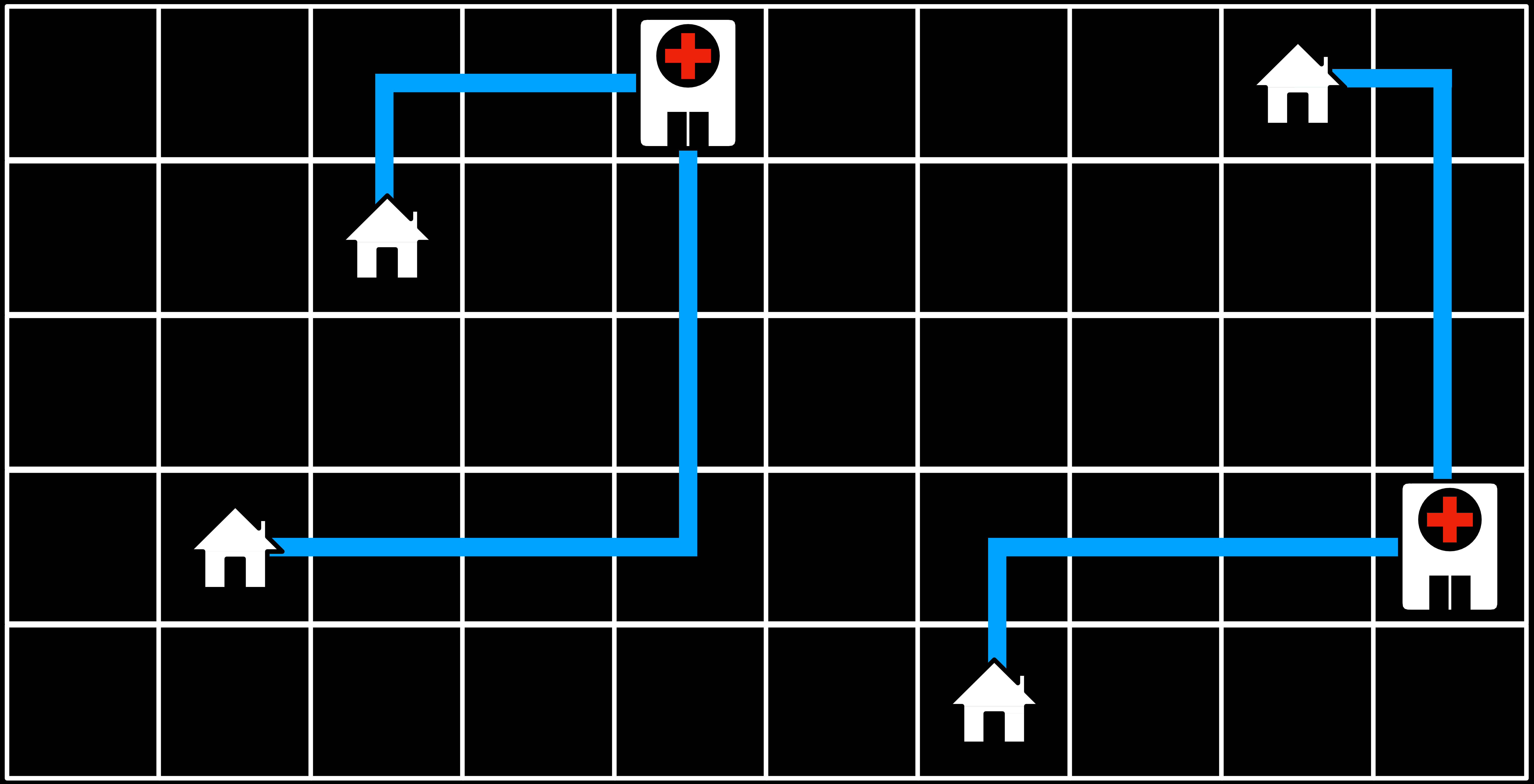

B
A

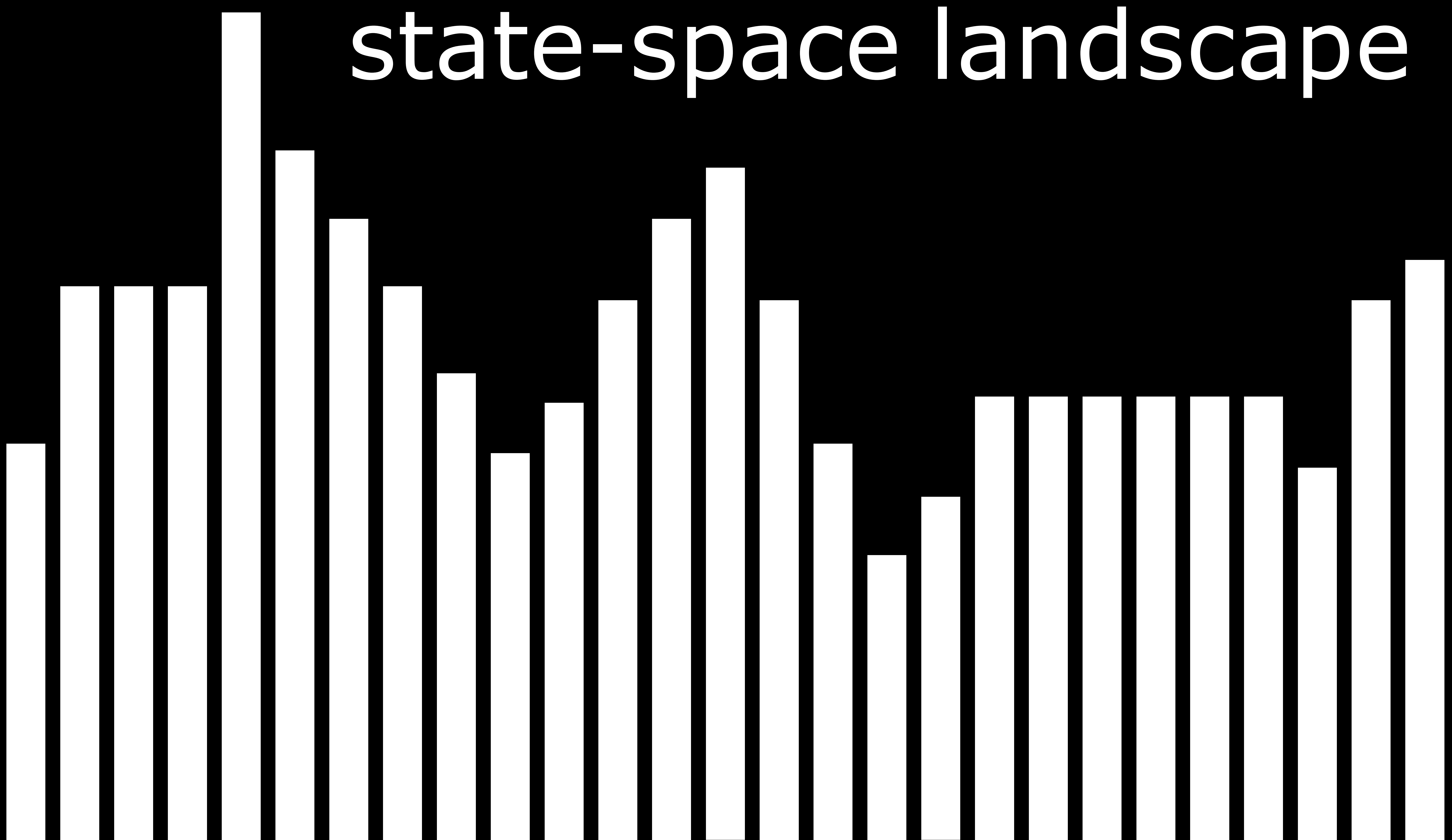
											B



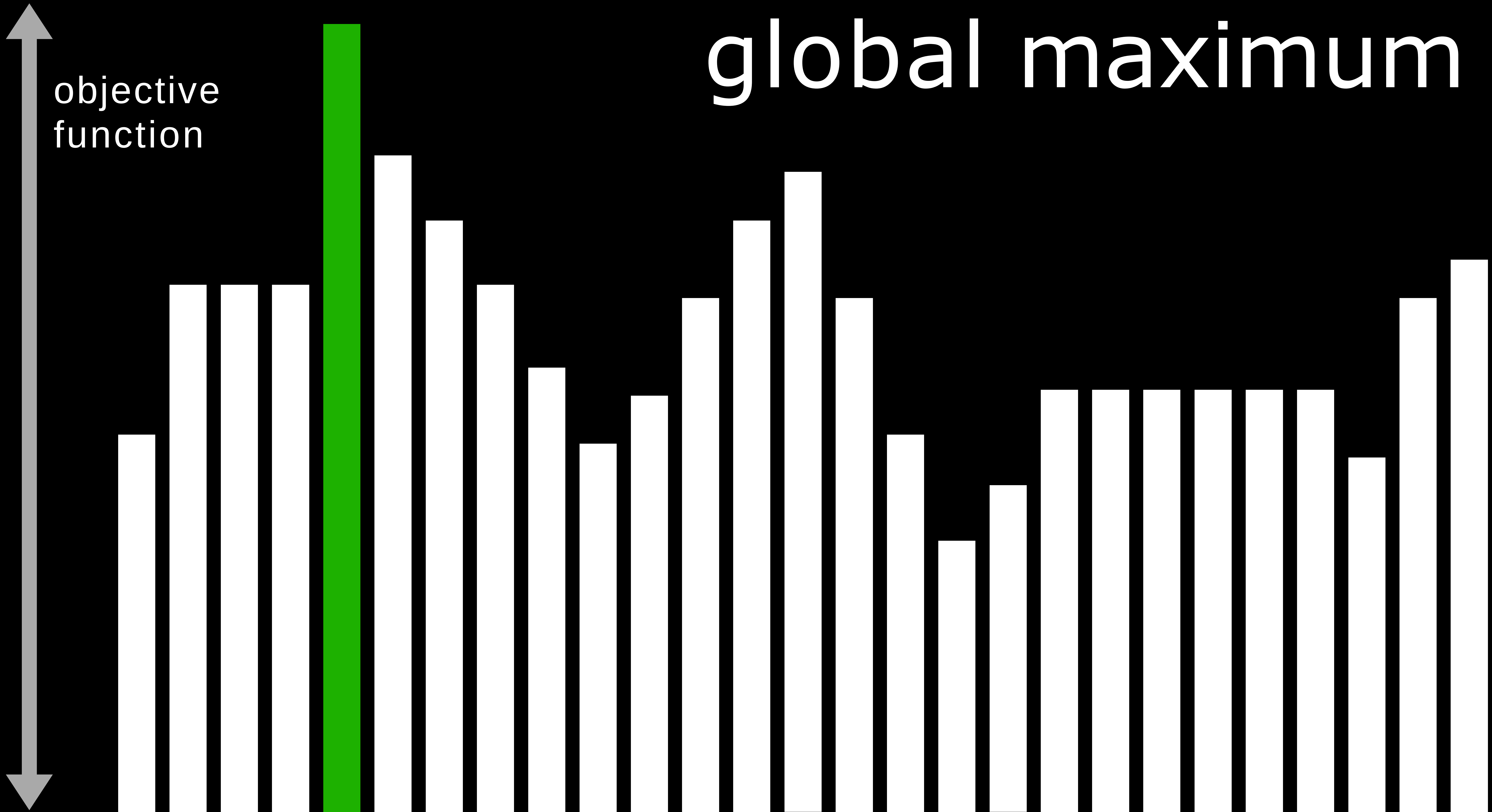


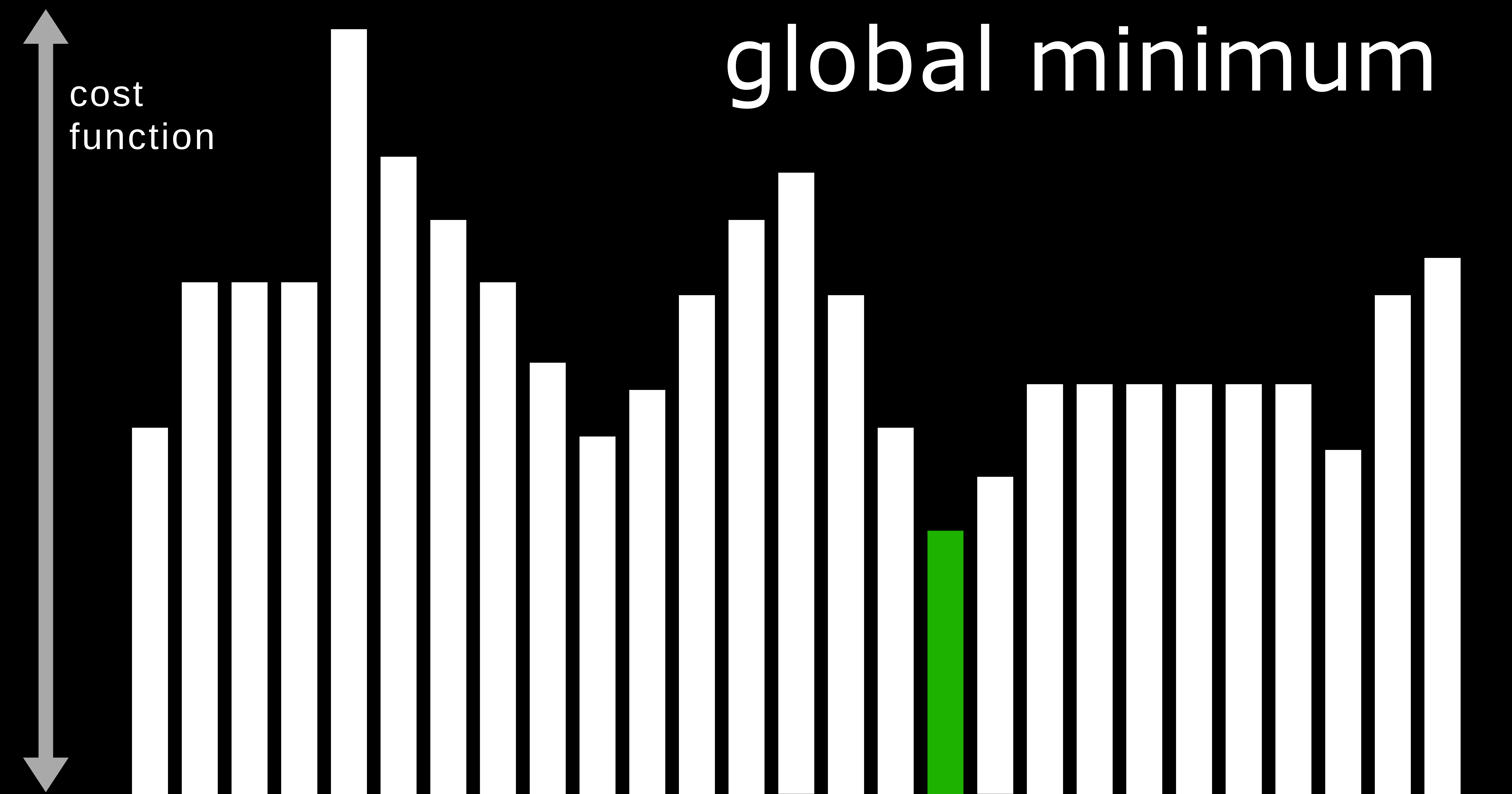
Cost: 17

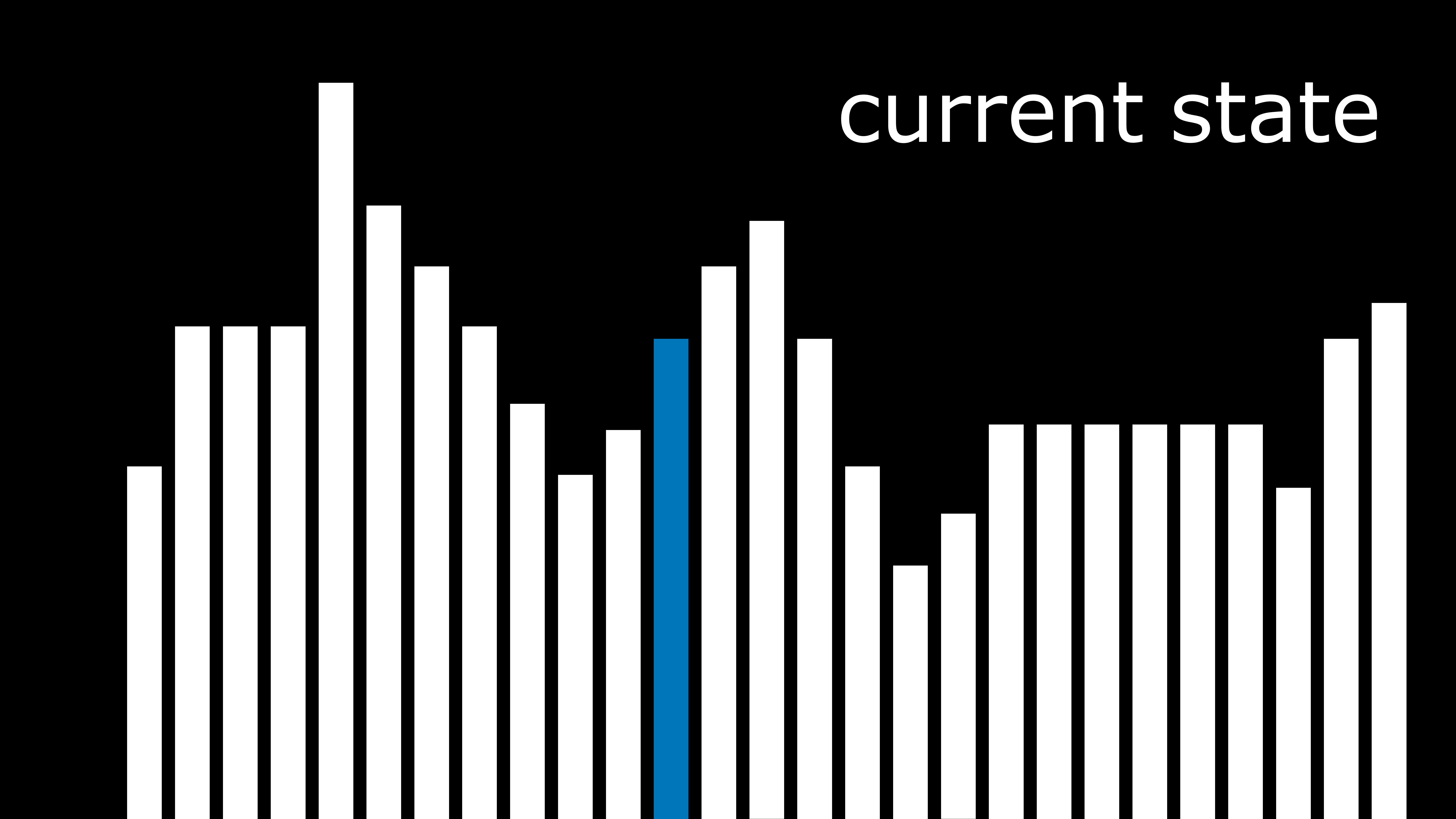



state-space landscape

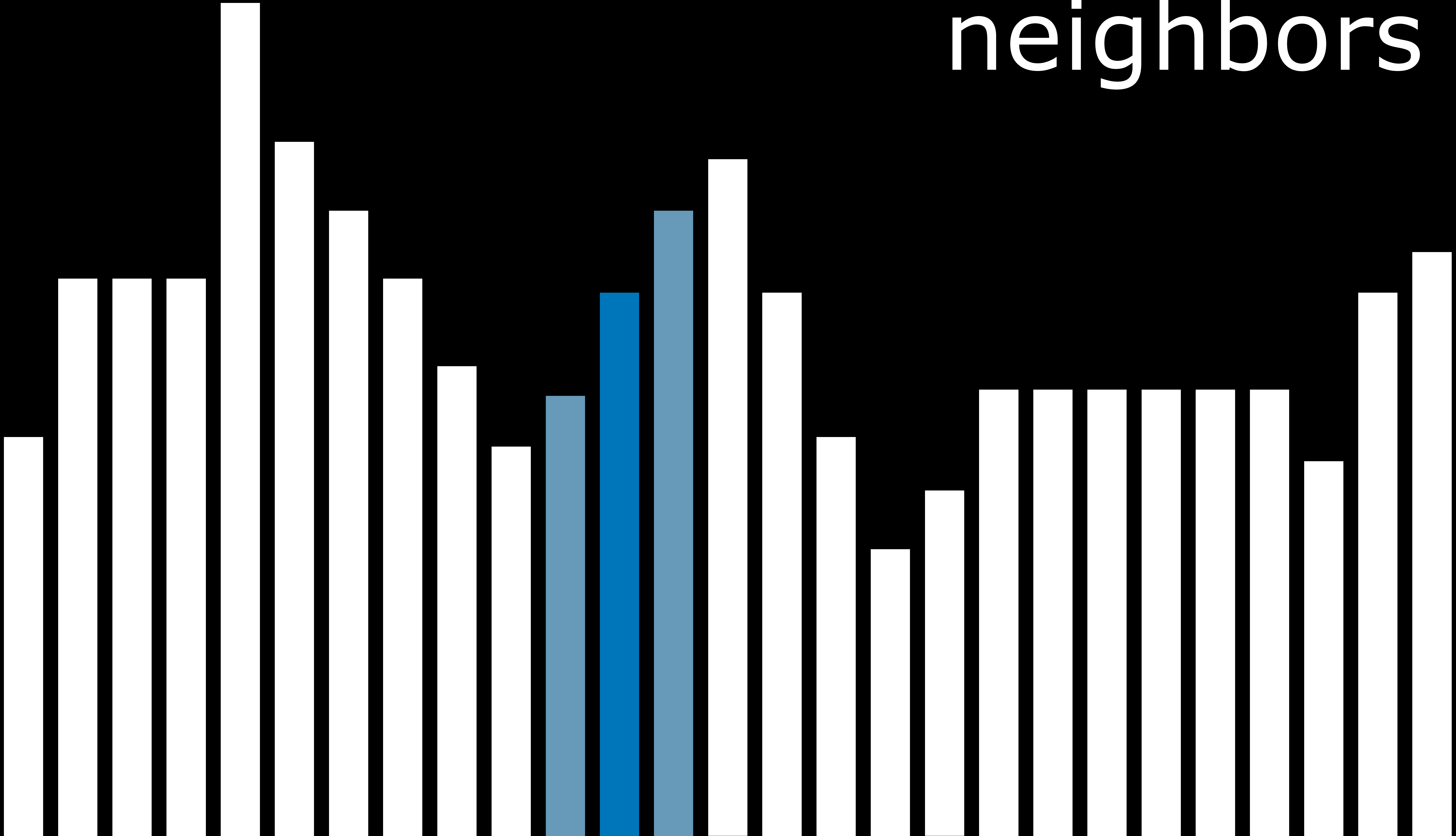
global maximum



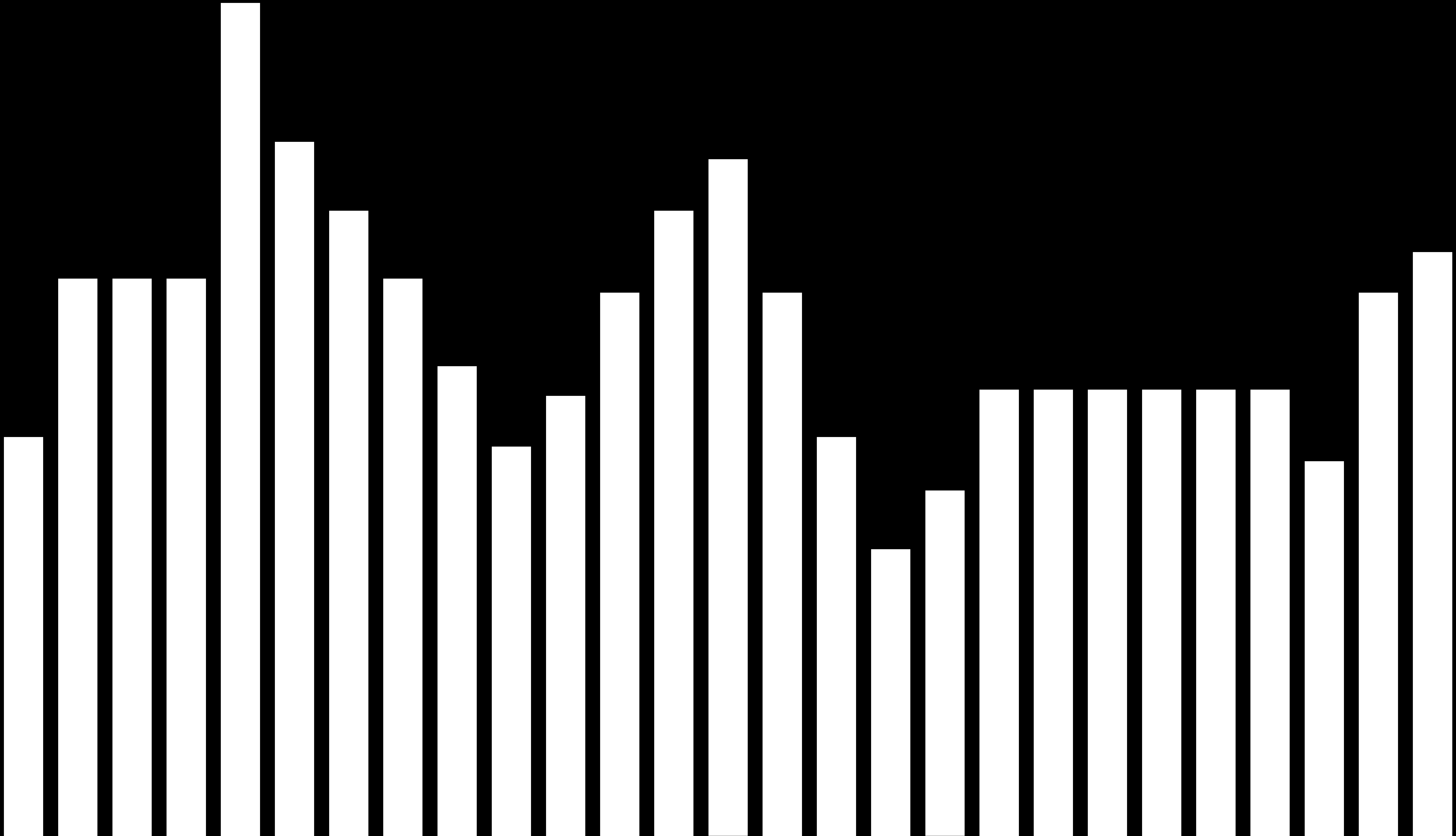

global minimum
cost
function

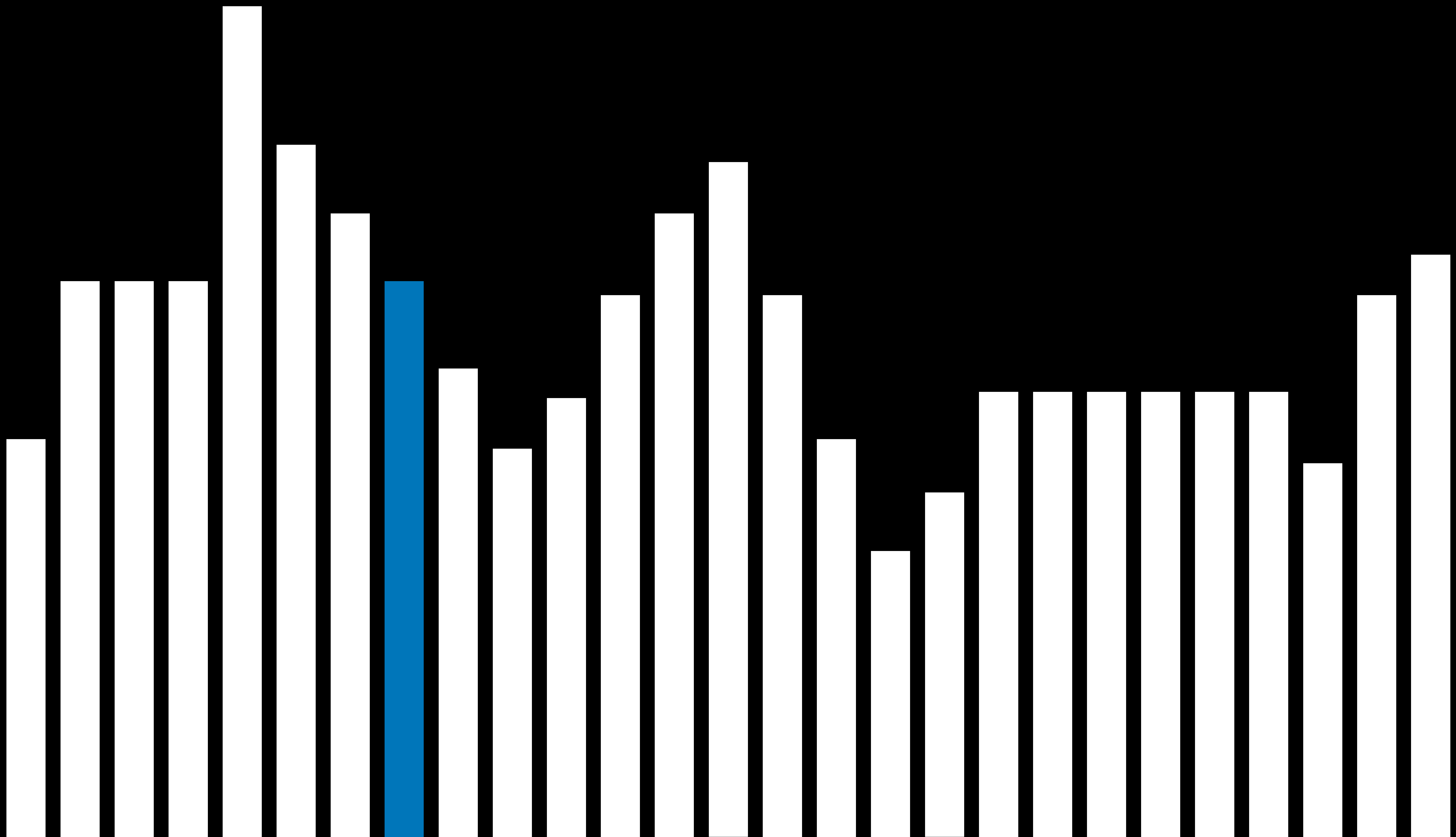

current state

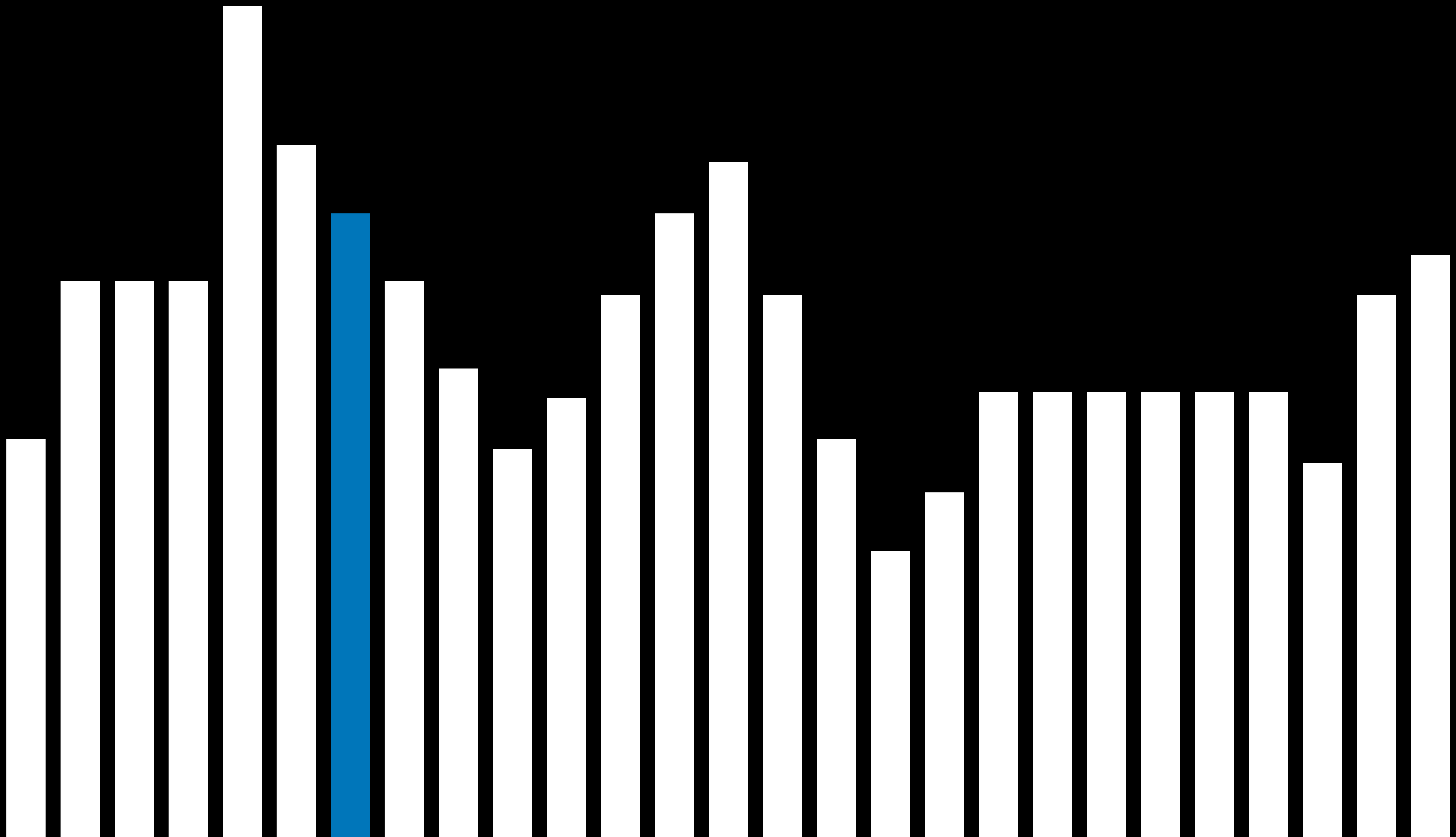
neighbors

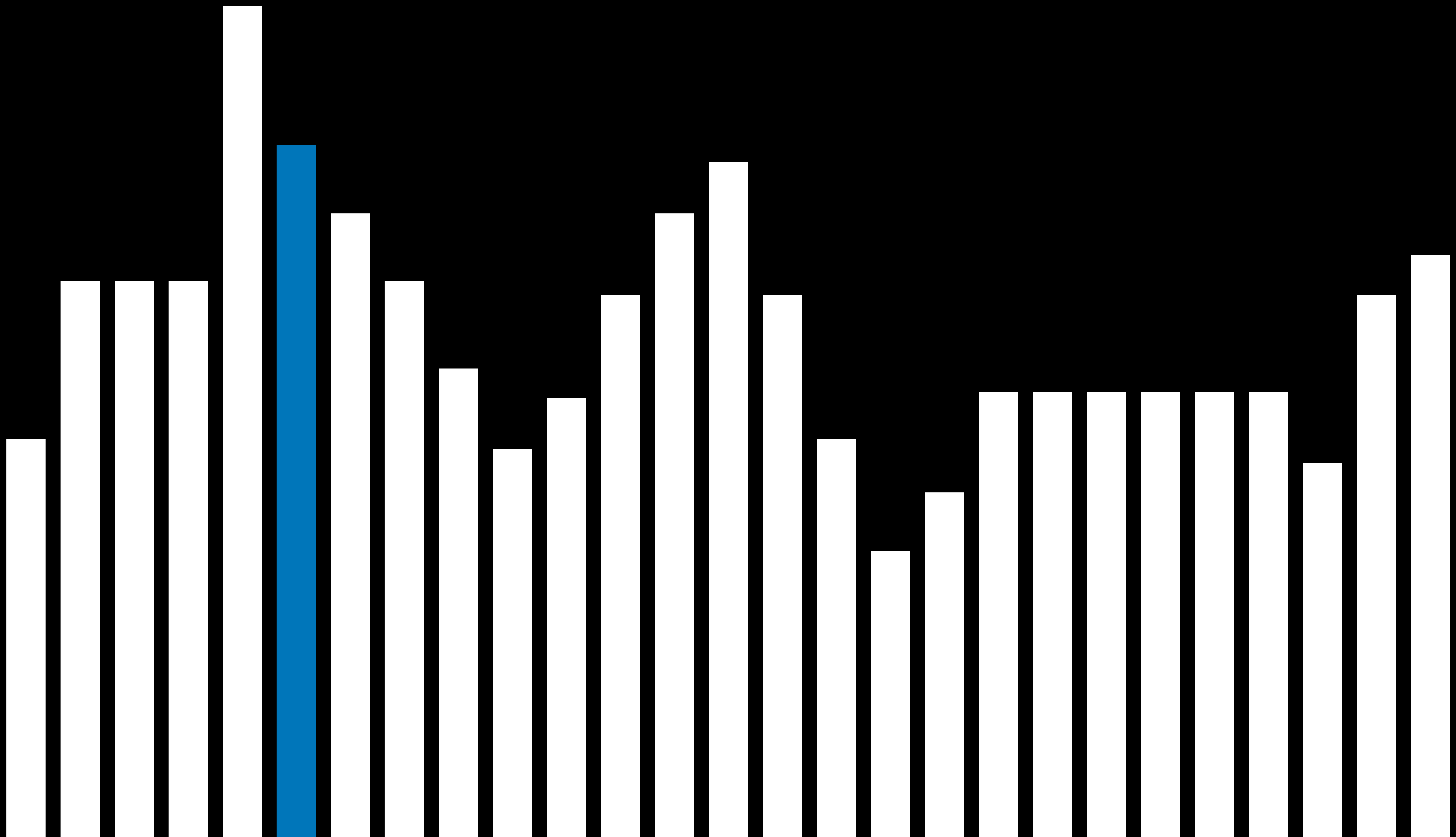


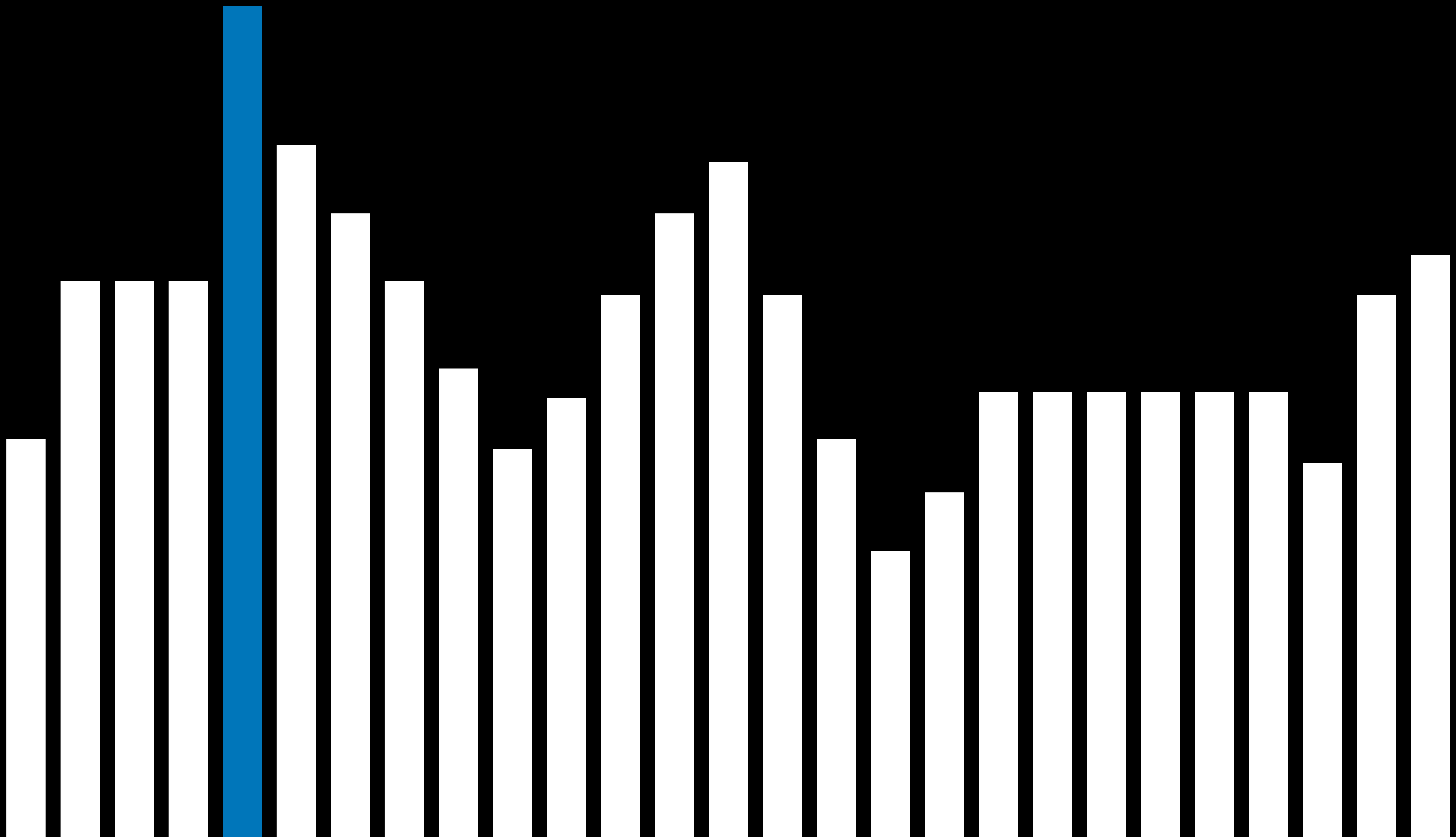
Hill Climbing

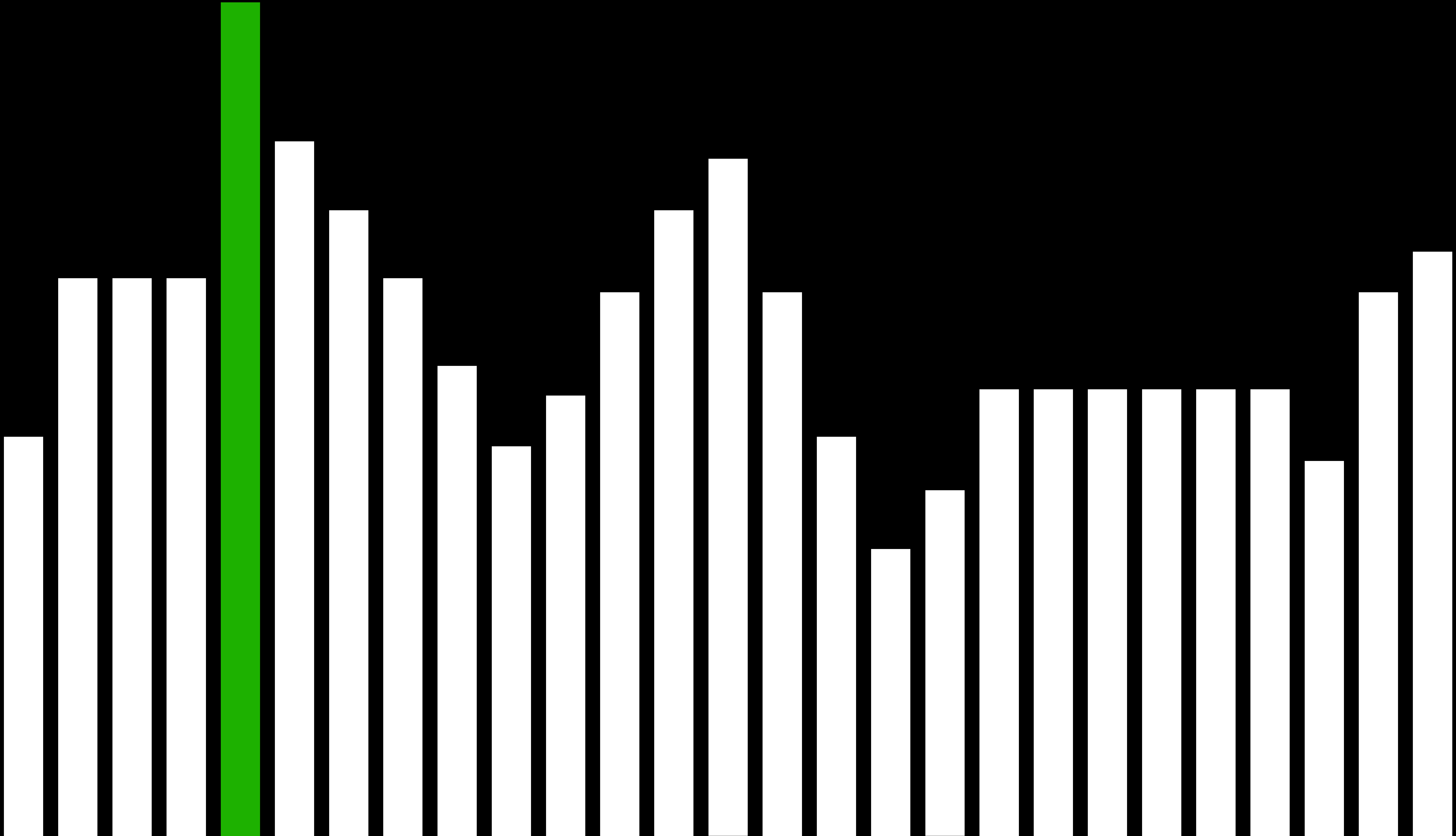


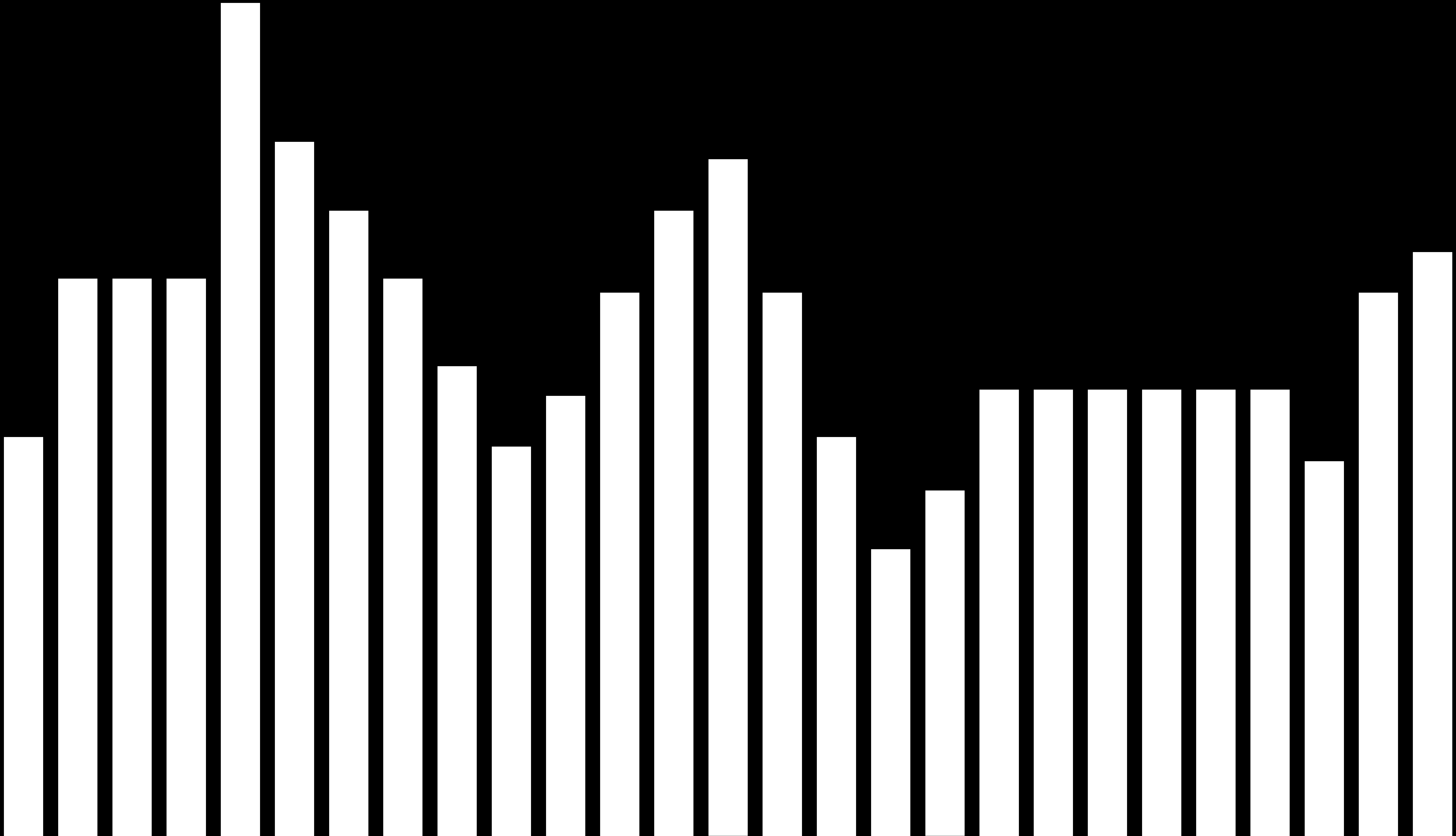


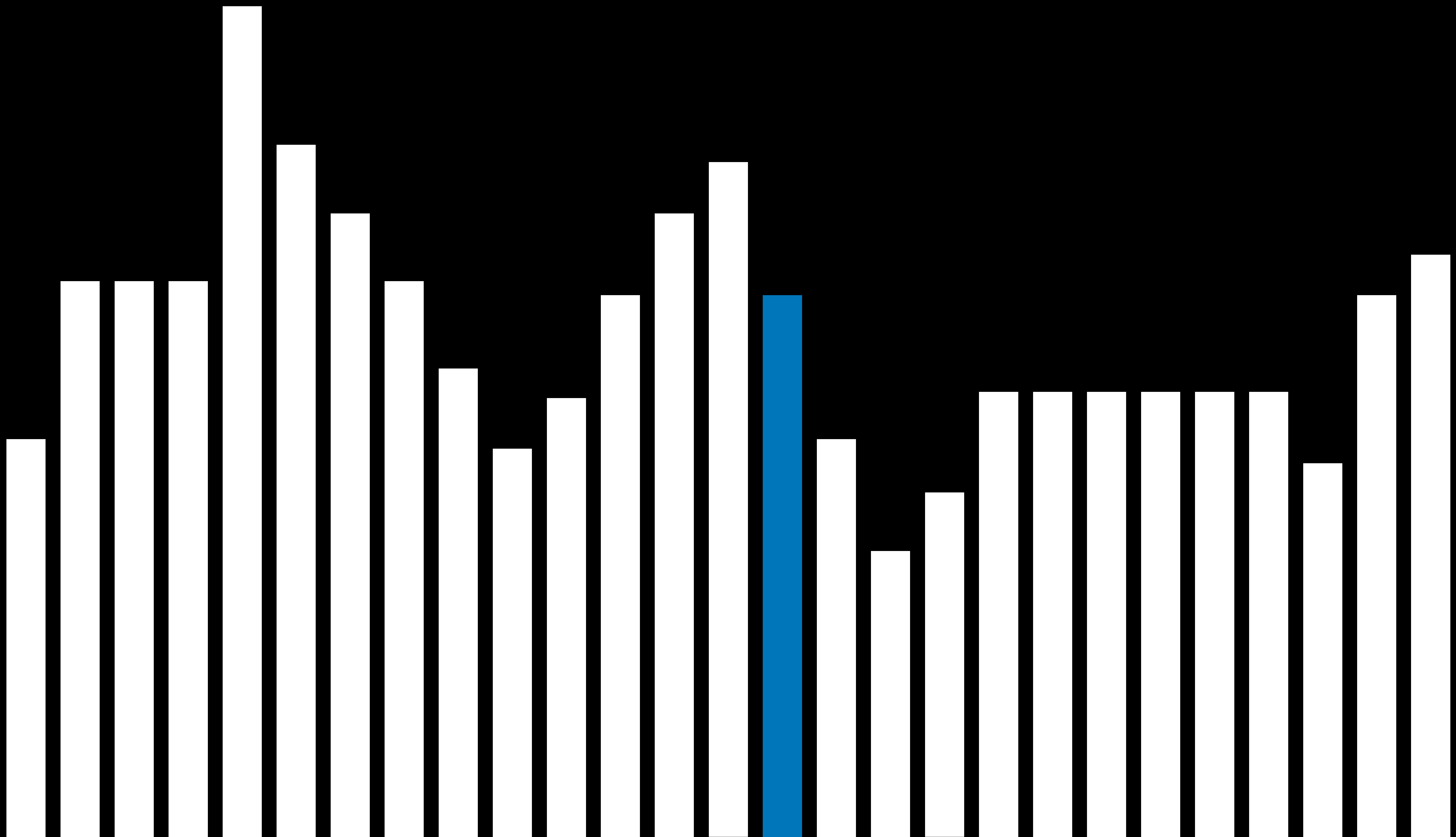


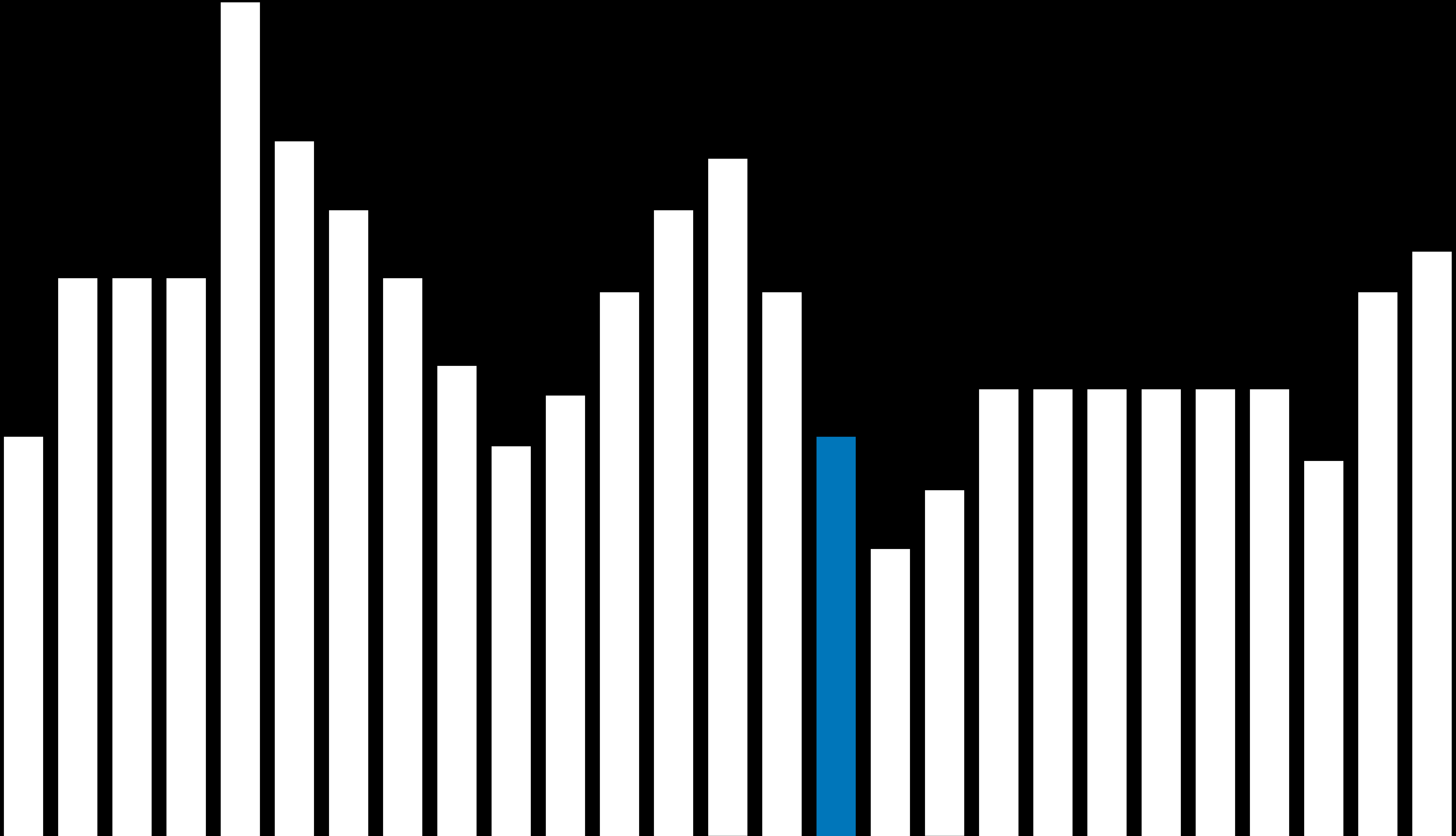


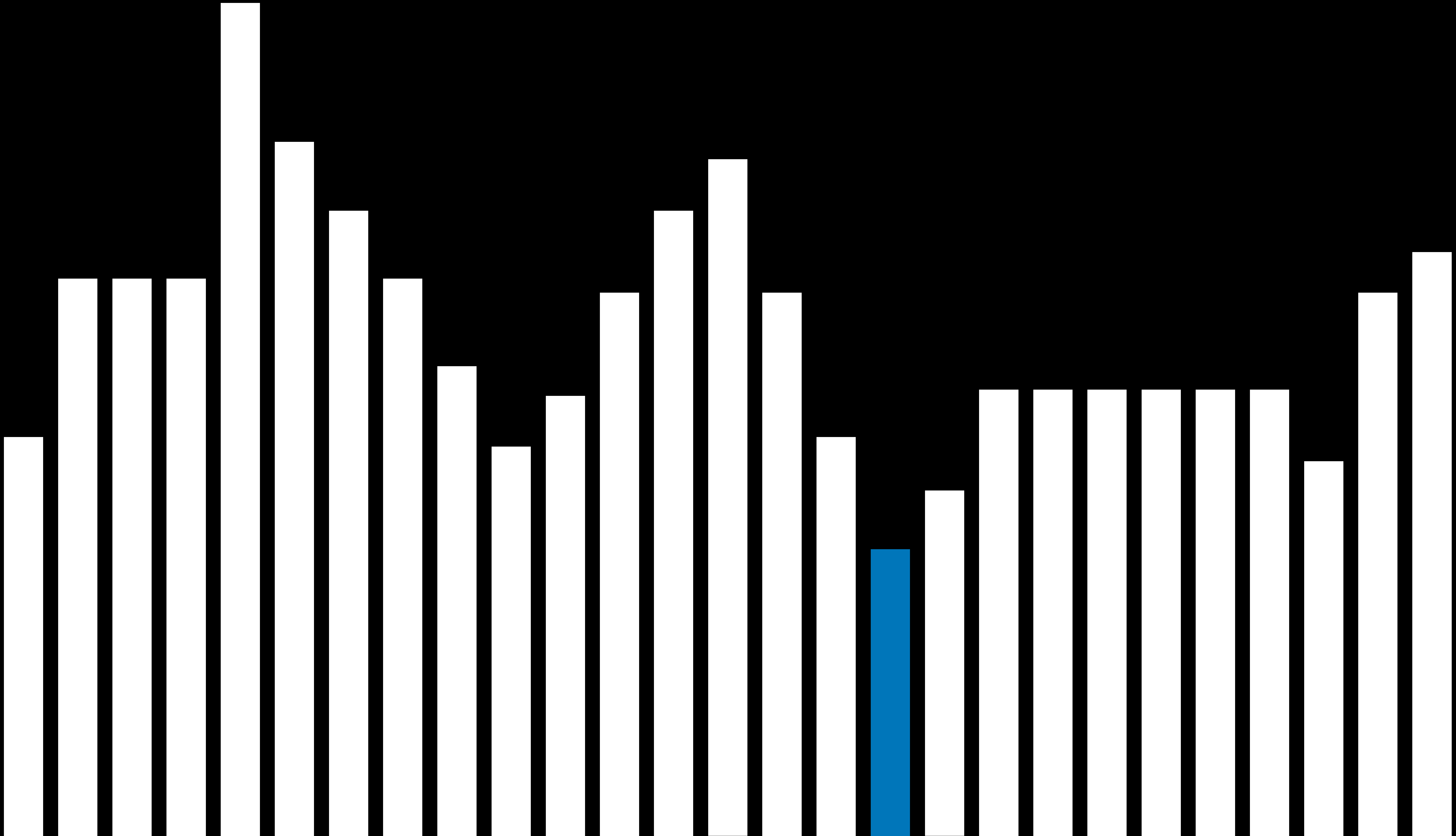


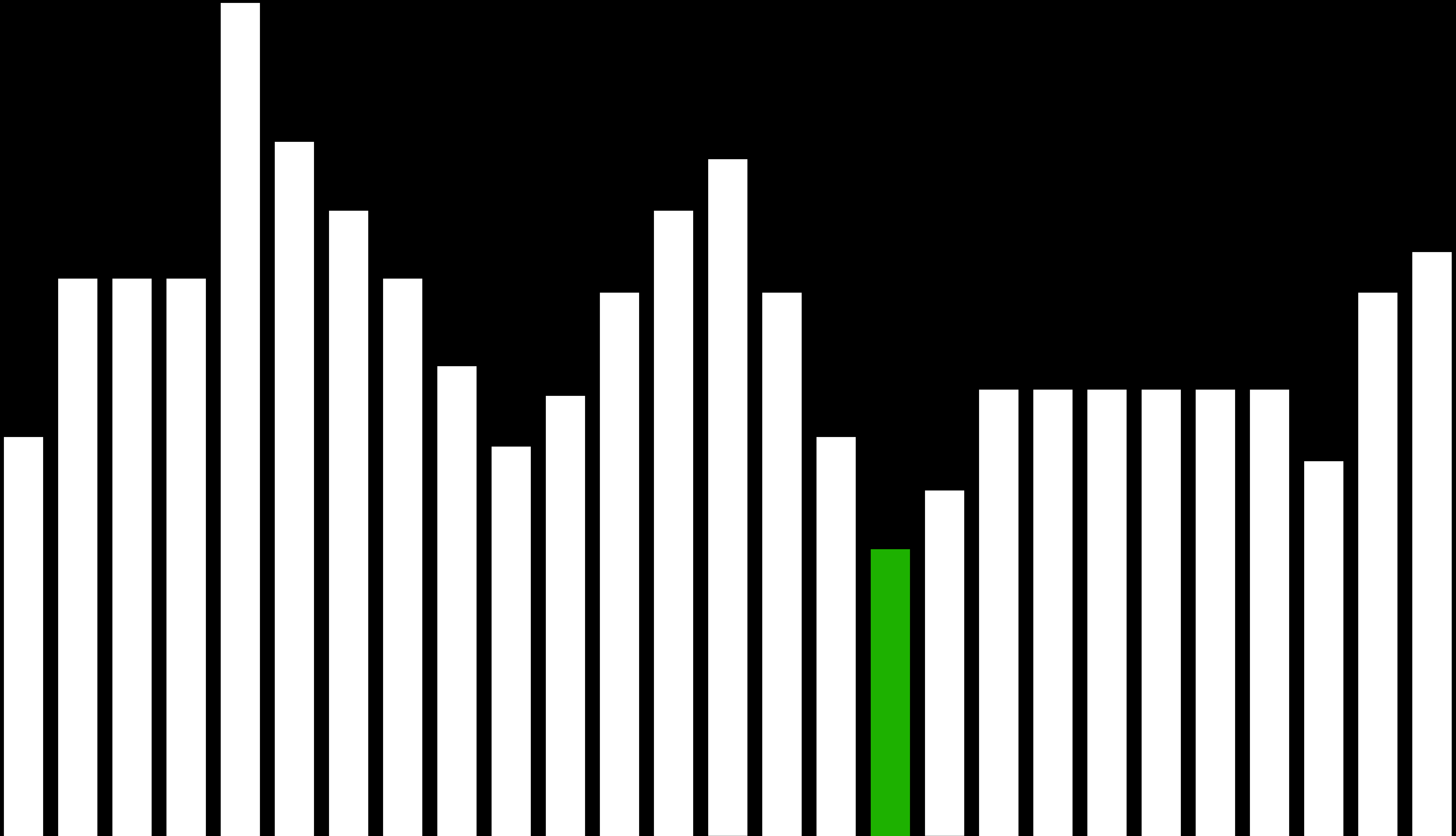








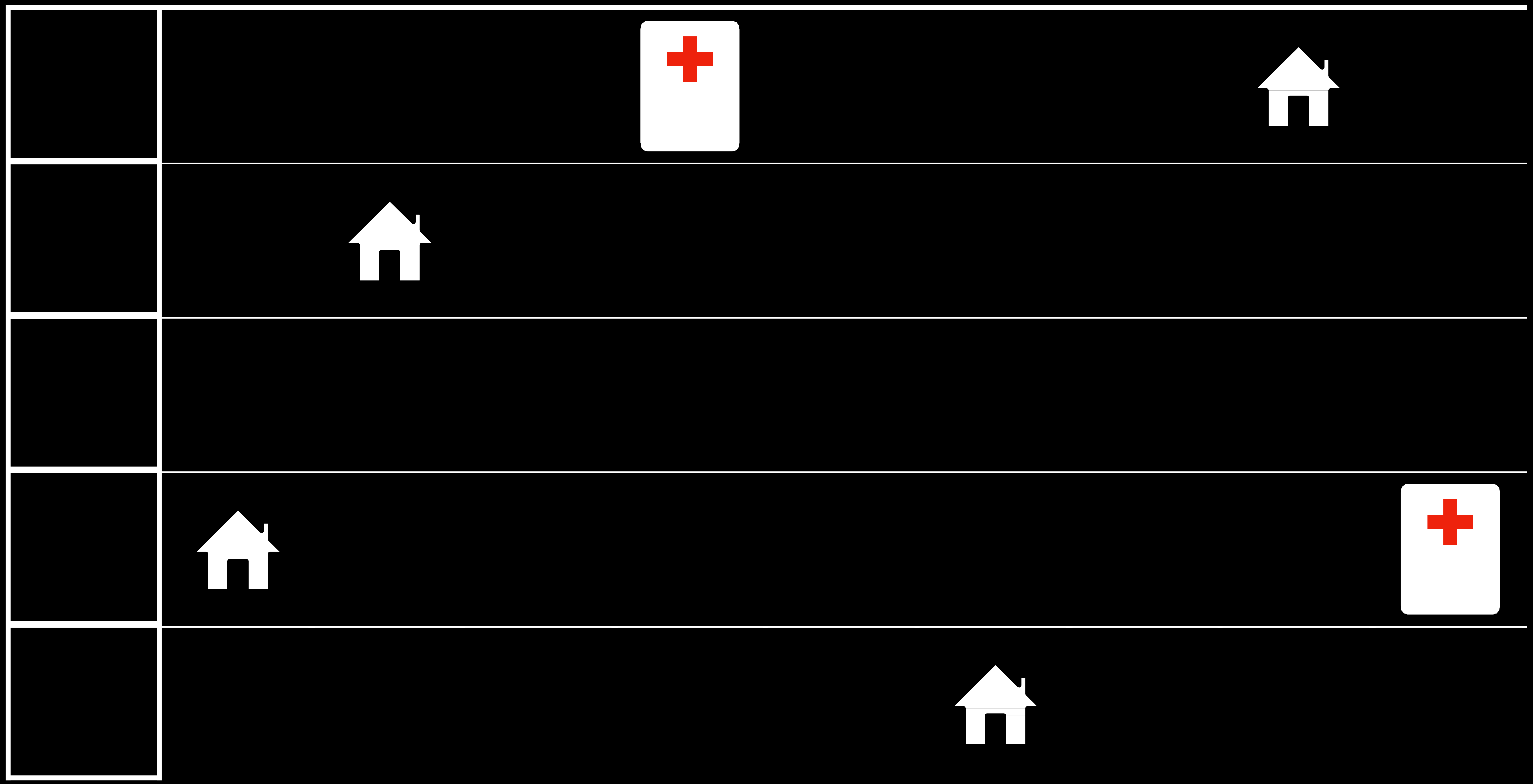




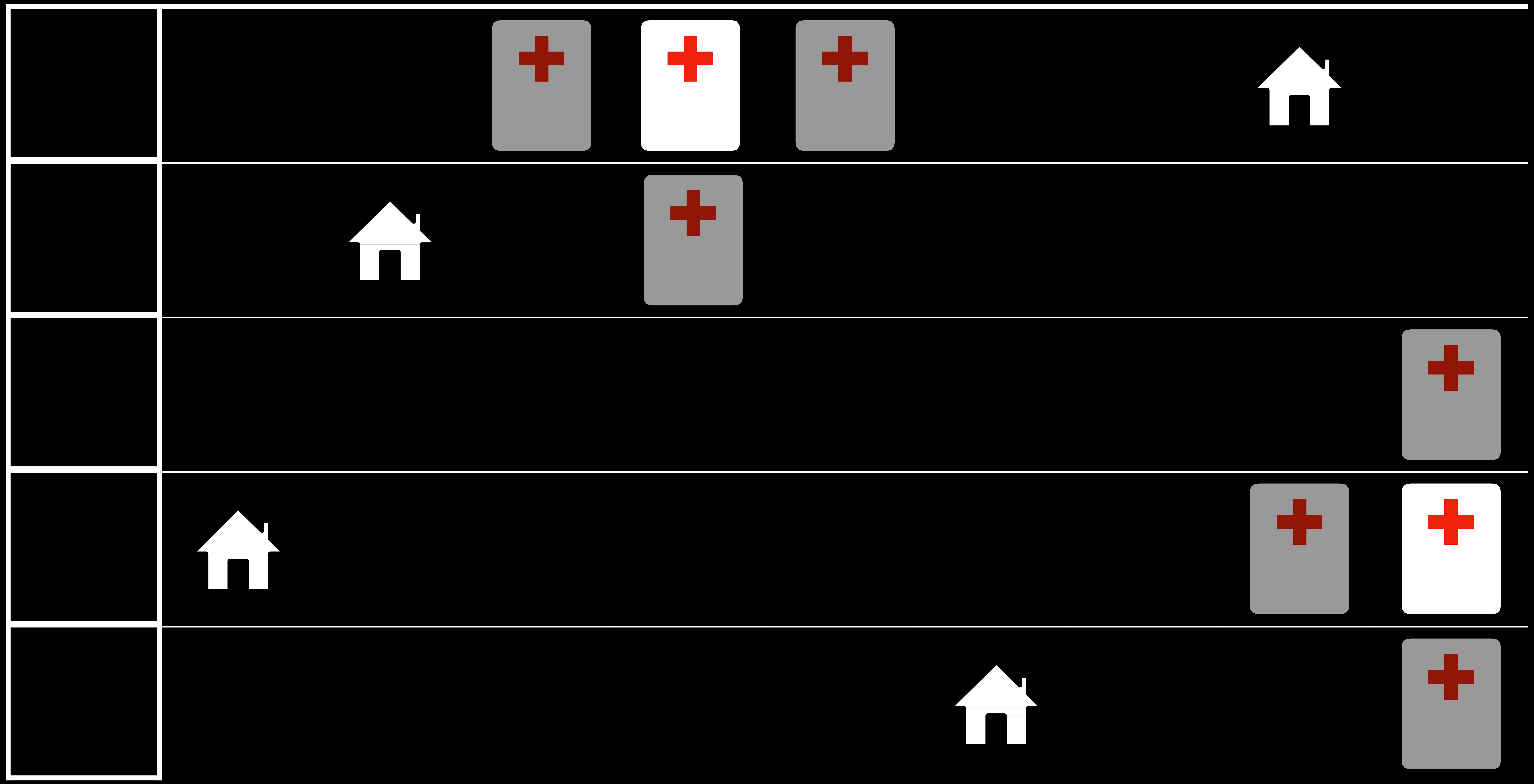
Hill Climbing

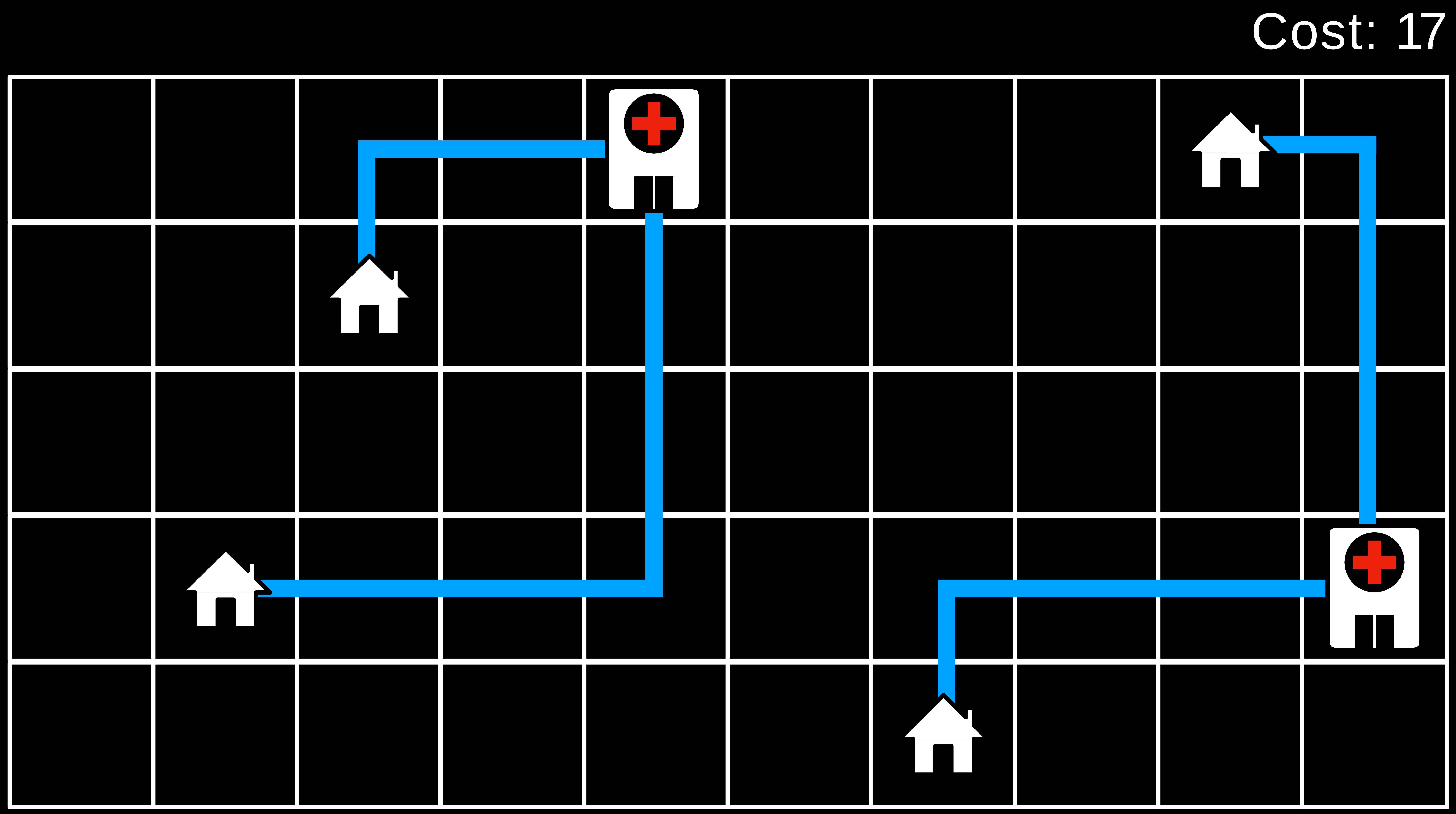
```
function HILL-CLIMB(problem):
    current = initial state of problem
    repeat:
        neighbor = highest valued neighbor of current
        if neighbor not better than current:
            return current
        current = neighbor
```

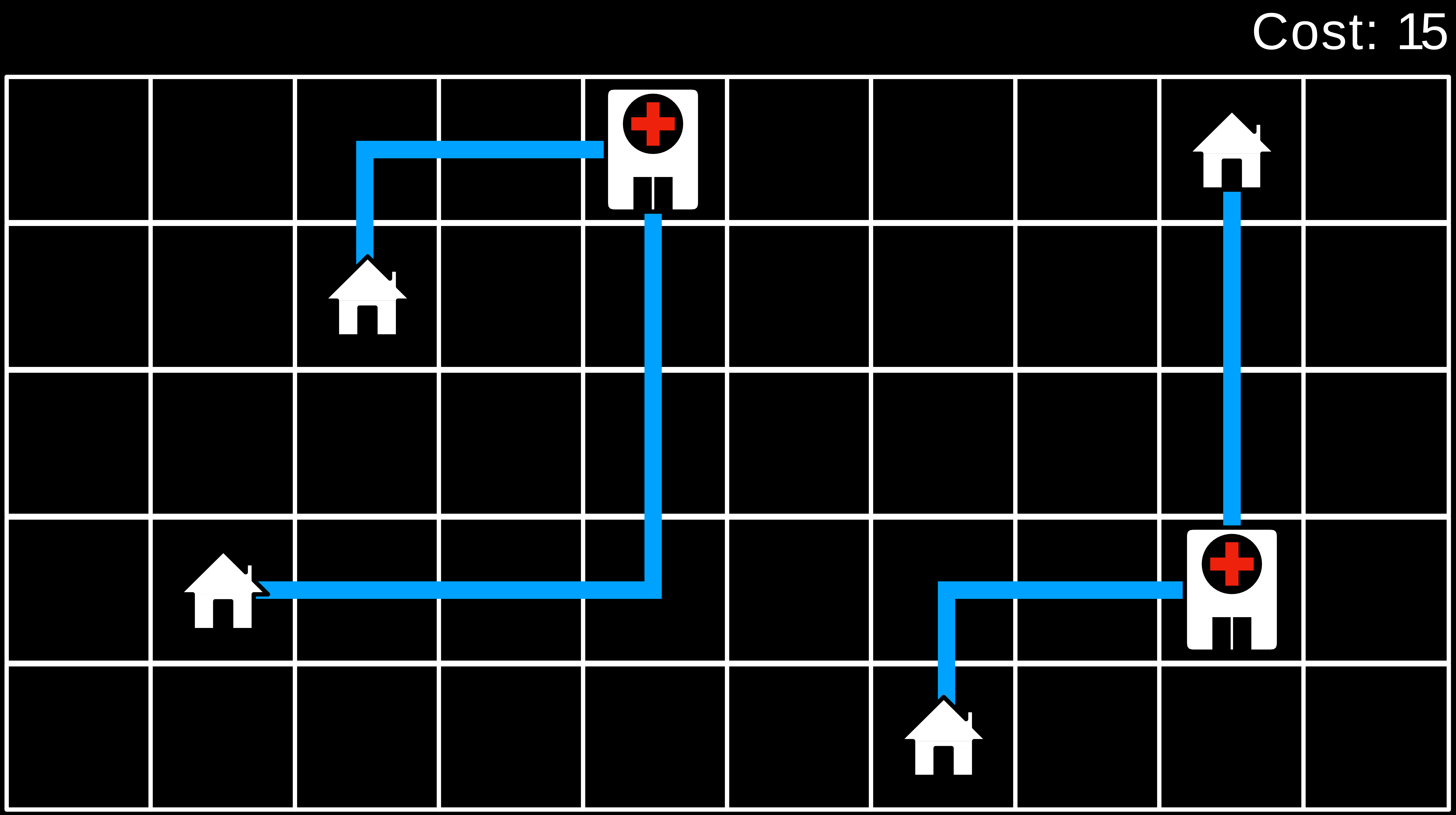
Cost: 17



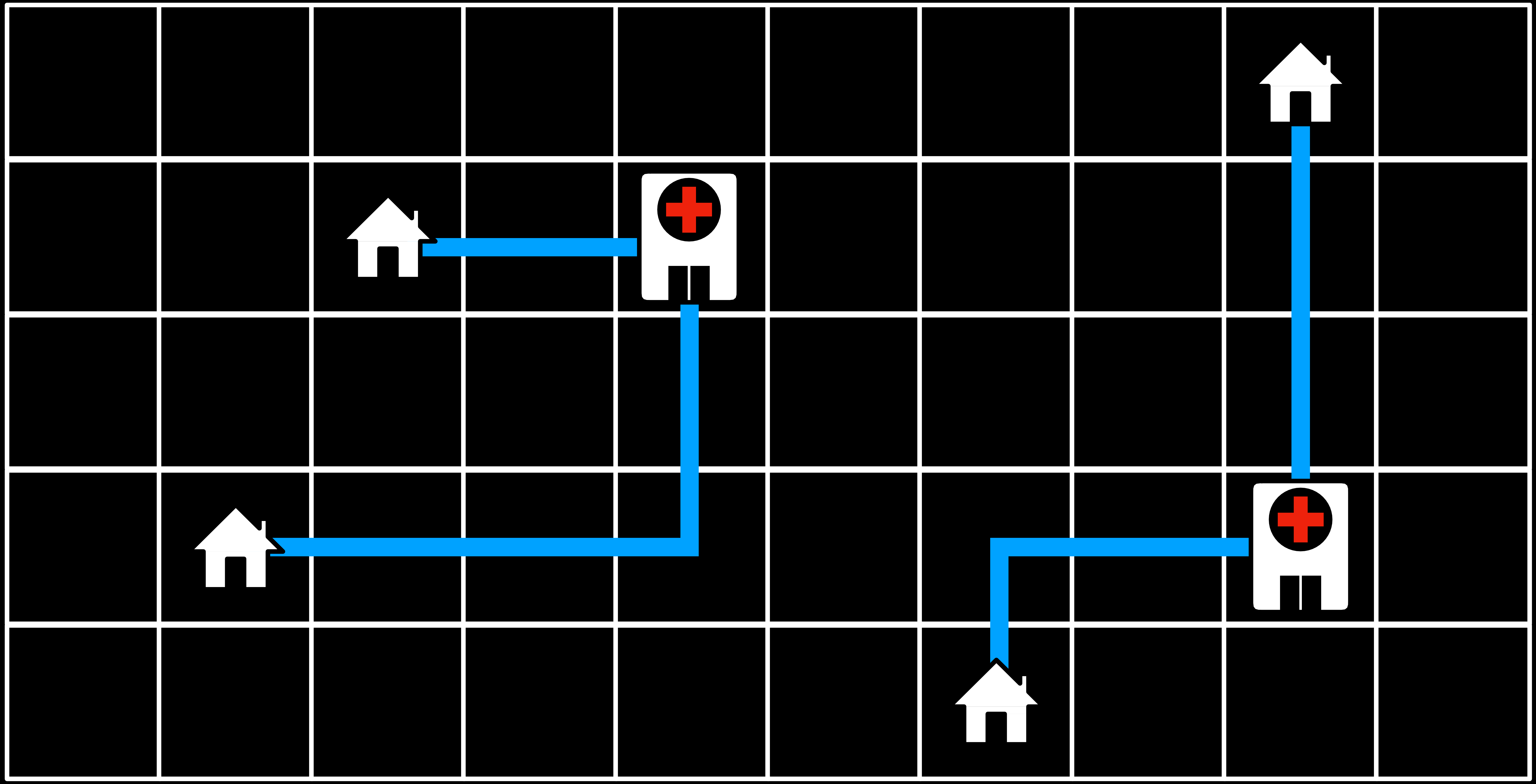
Cost: 17



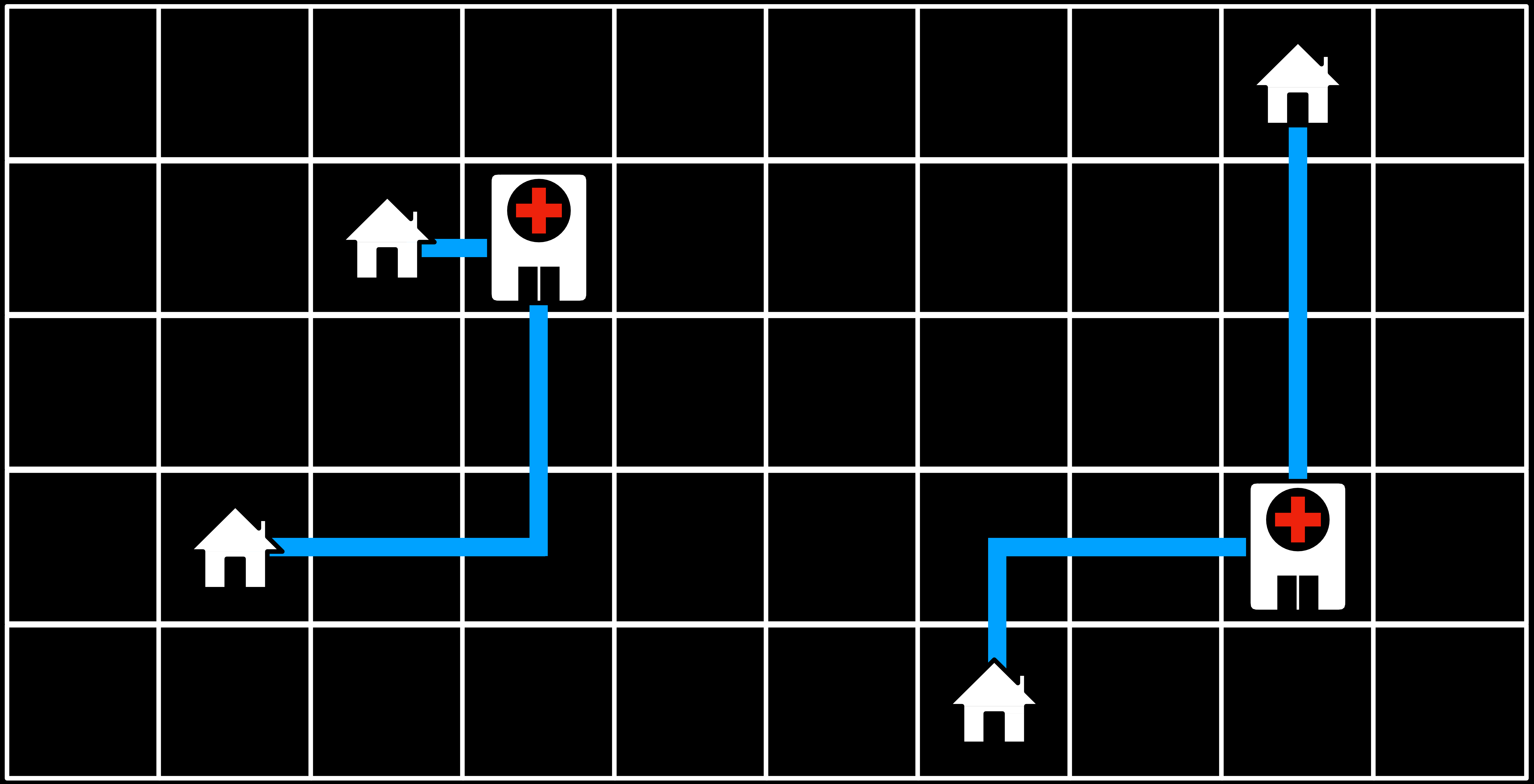

Cost: 17


Cost: 15

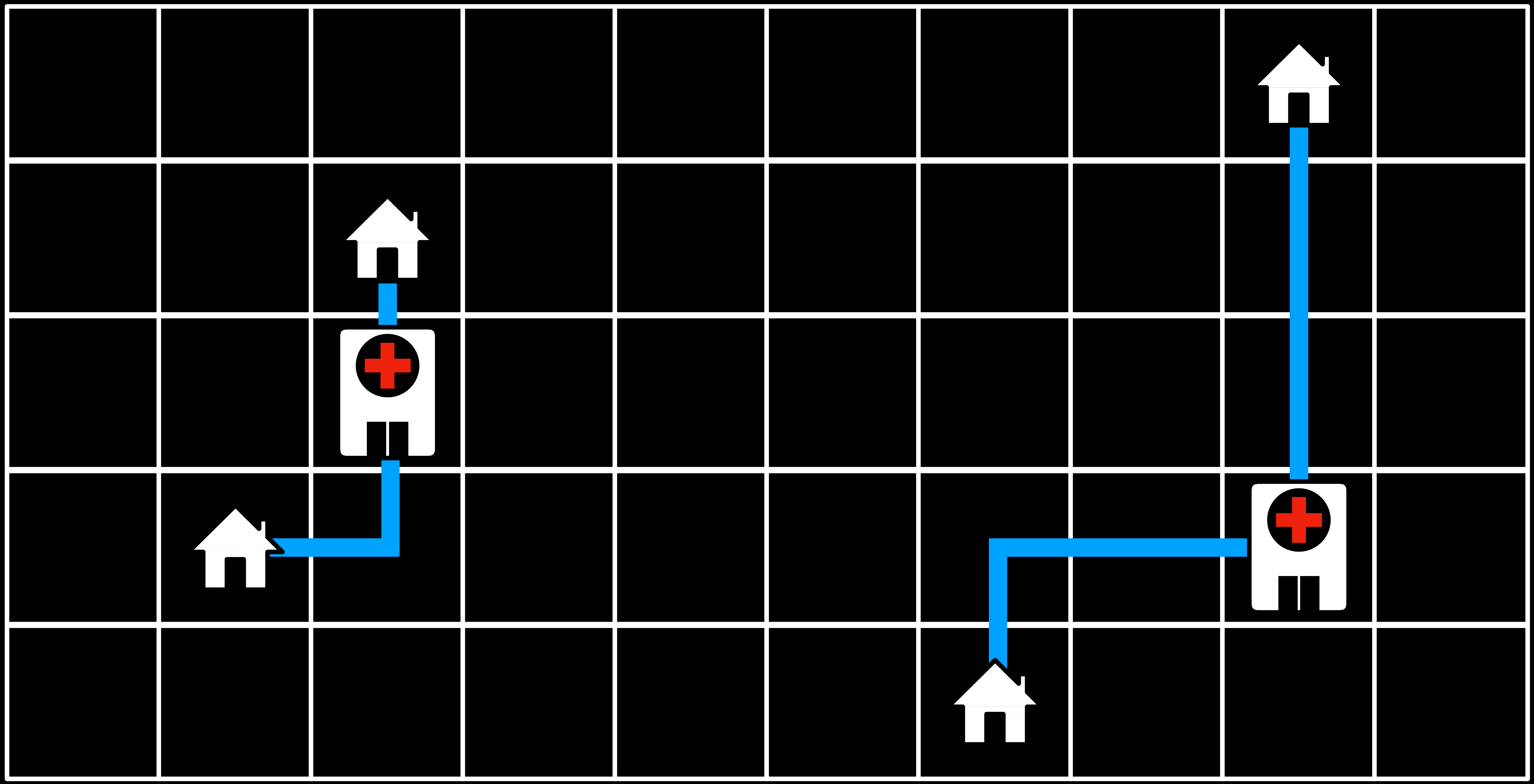
Cost: 13

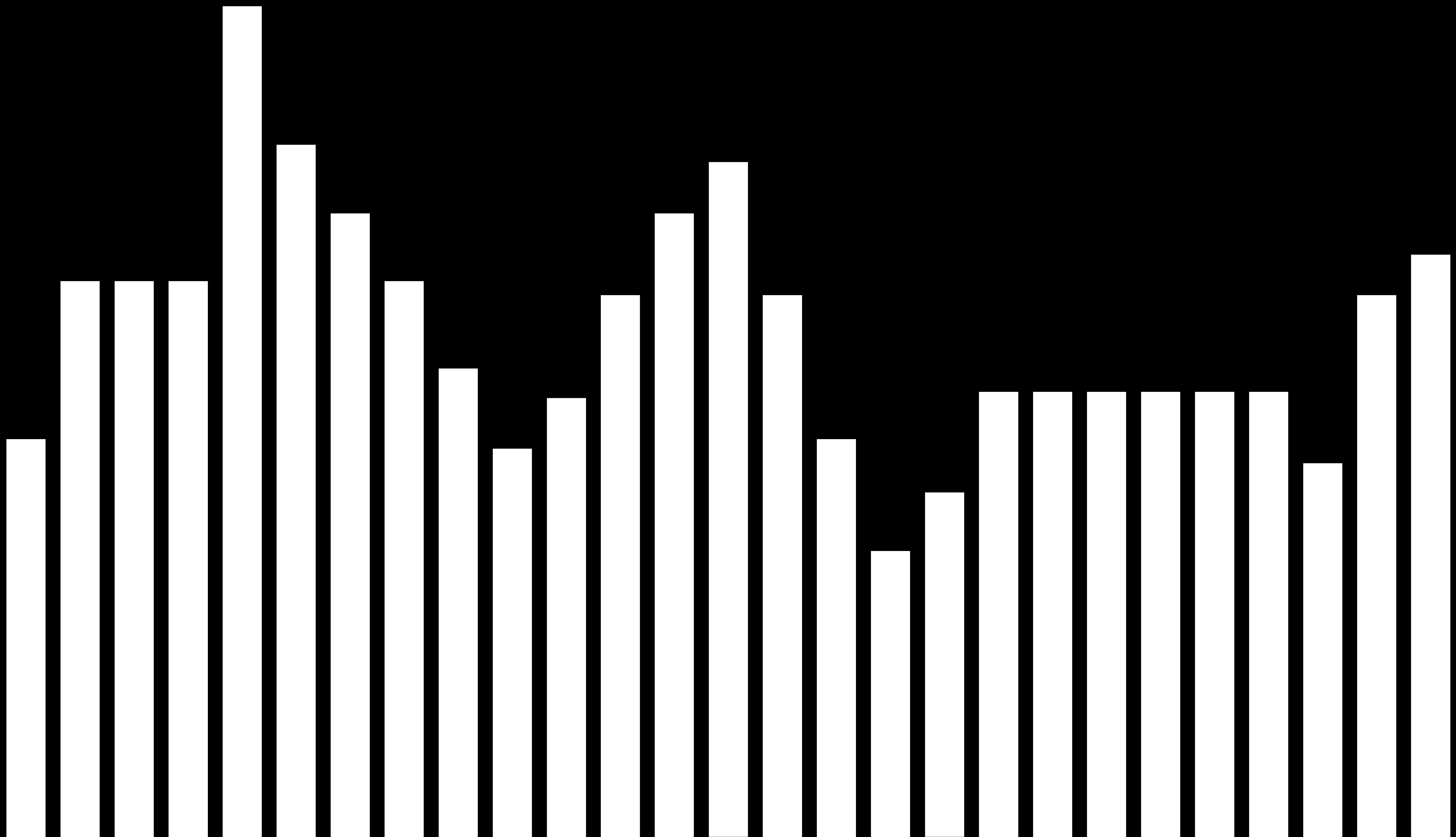


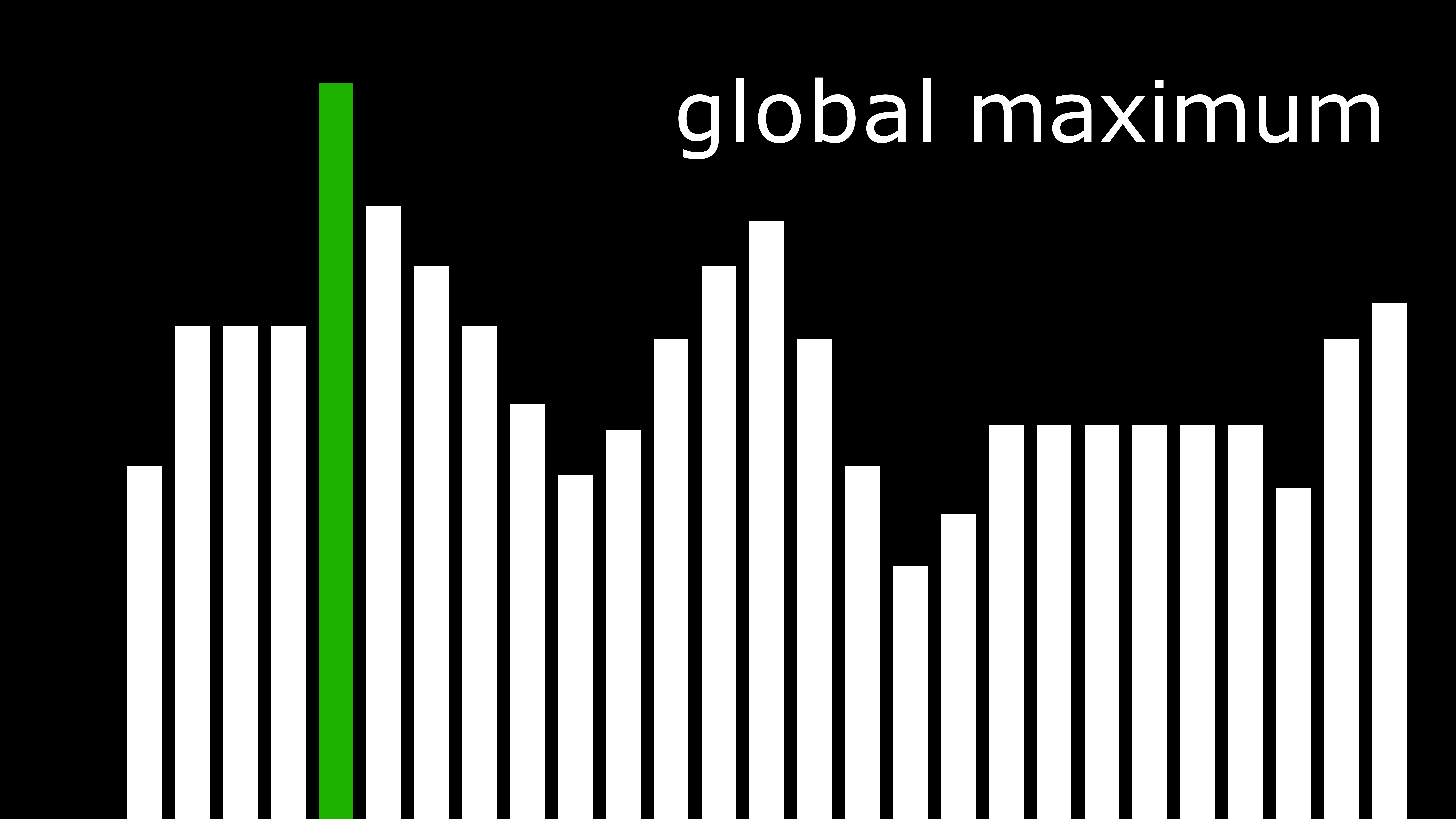
Cost: 11



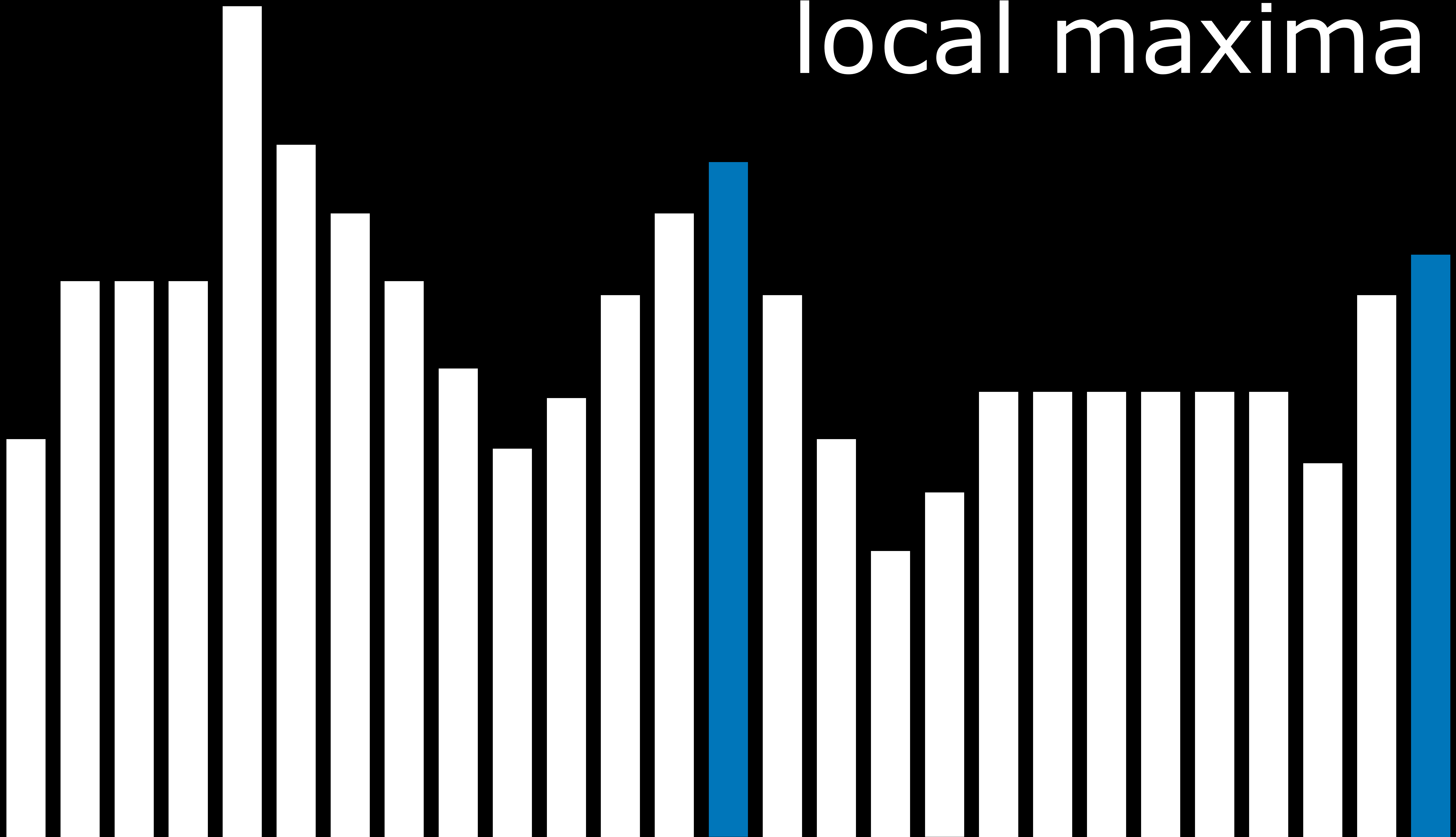
Cost: 9

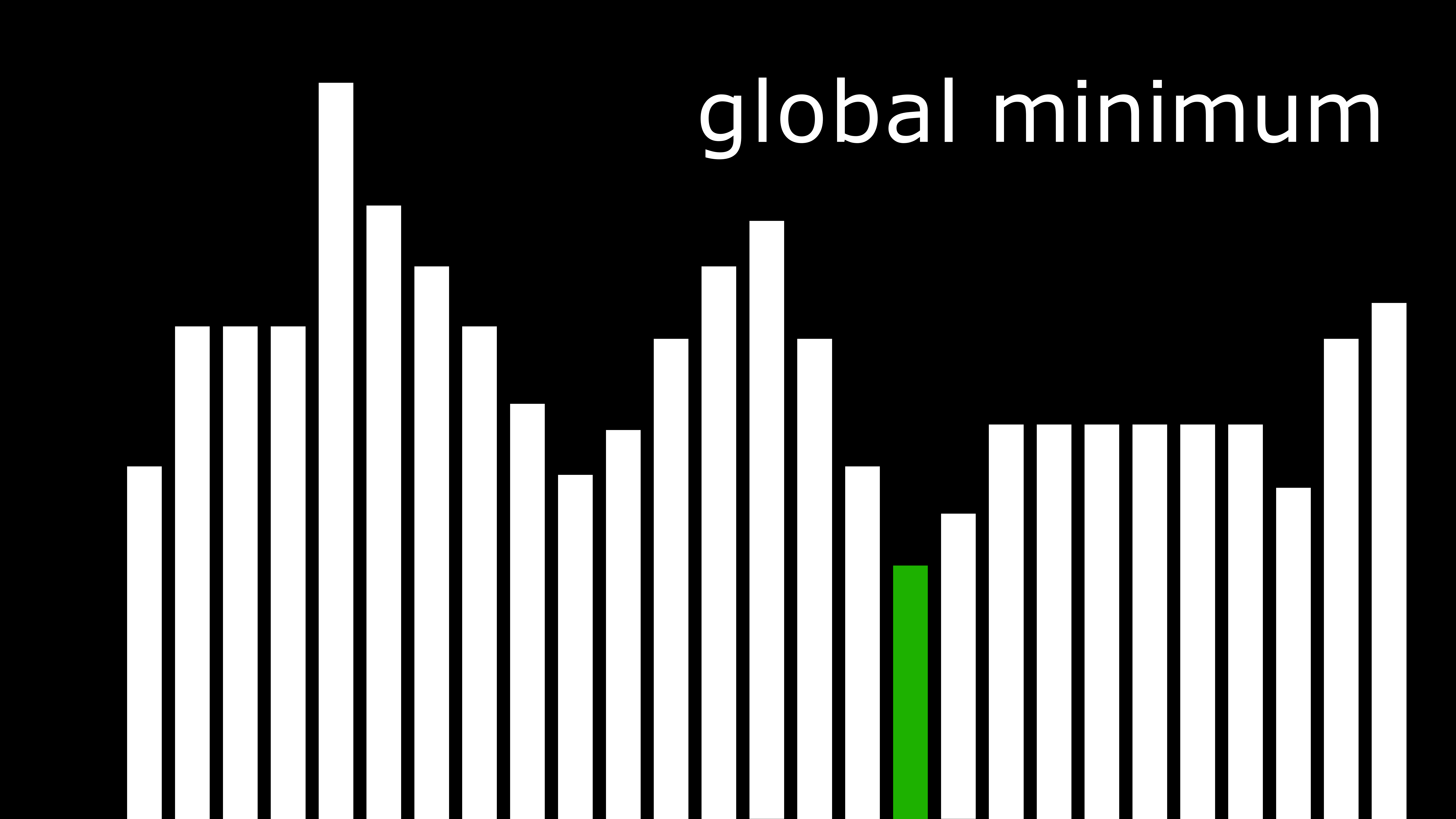


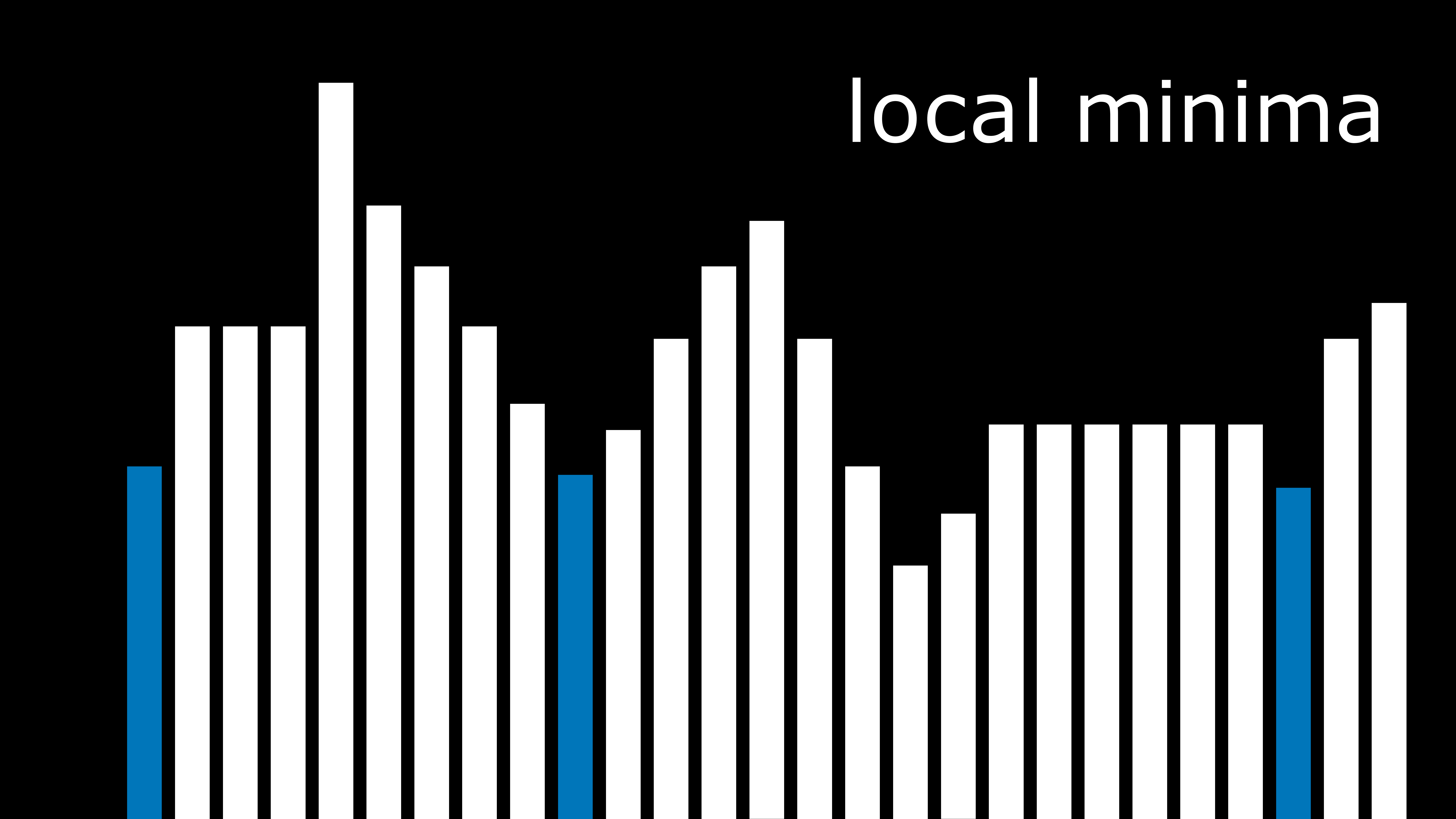


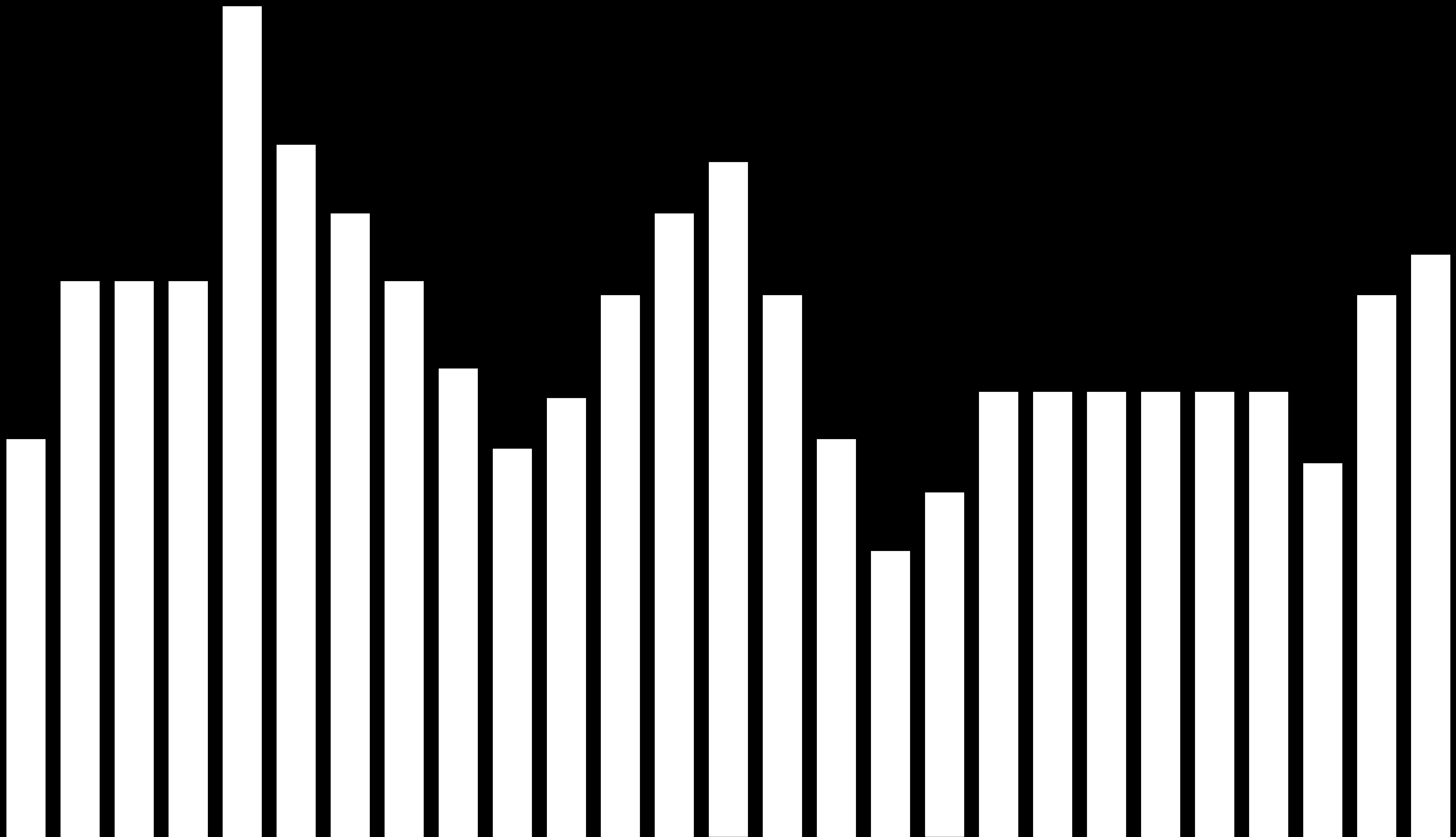

global maximum

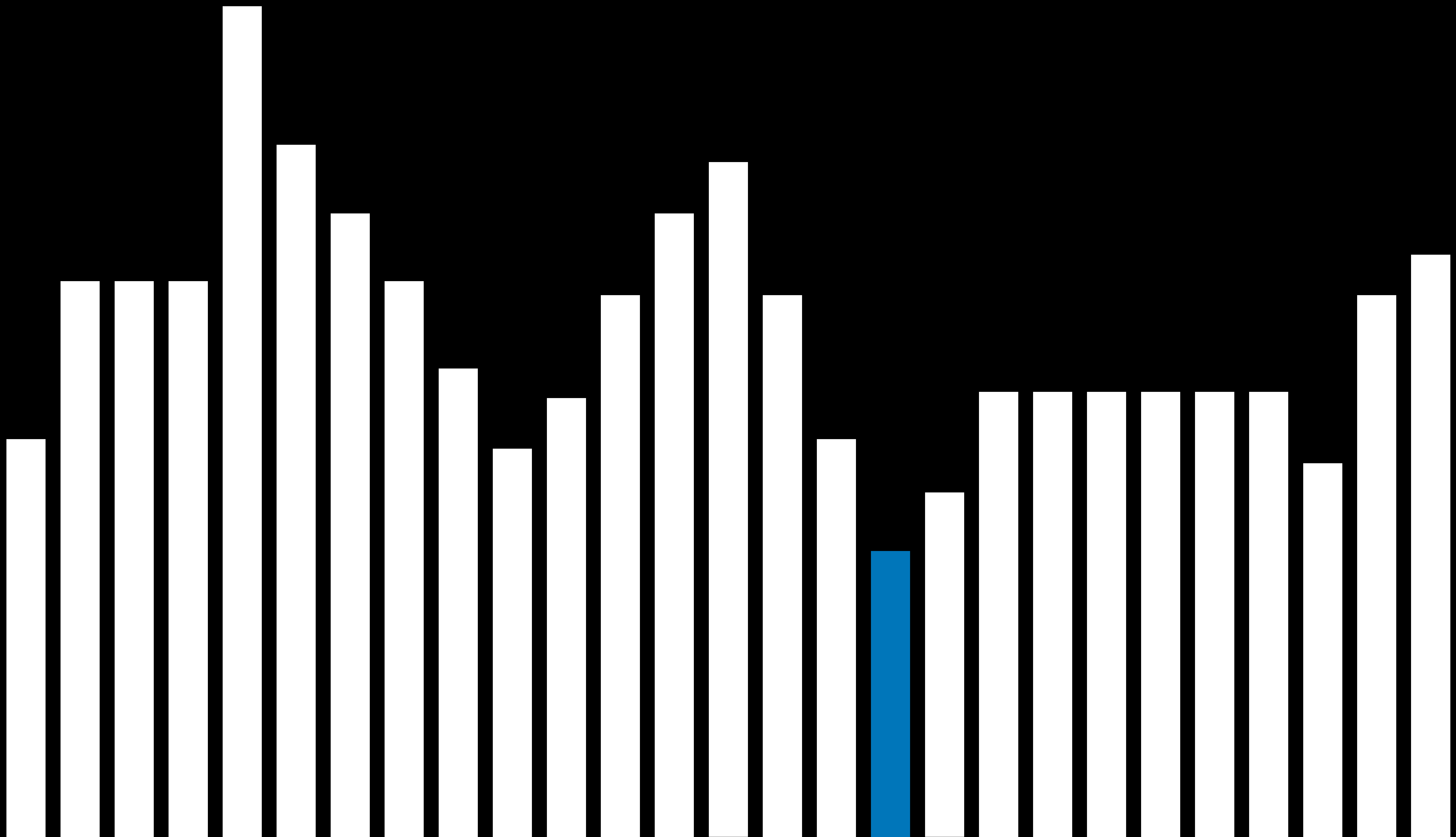
local maxima

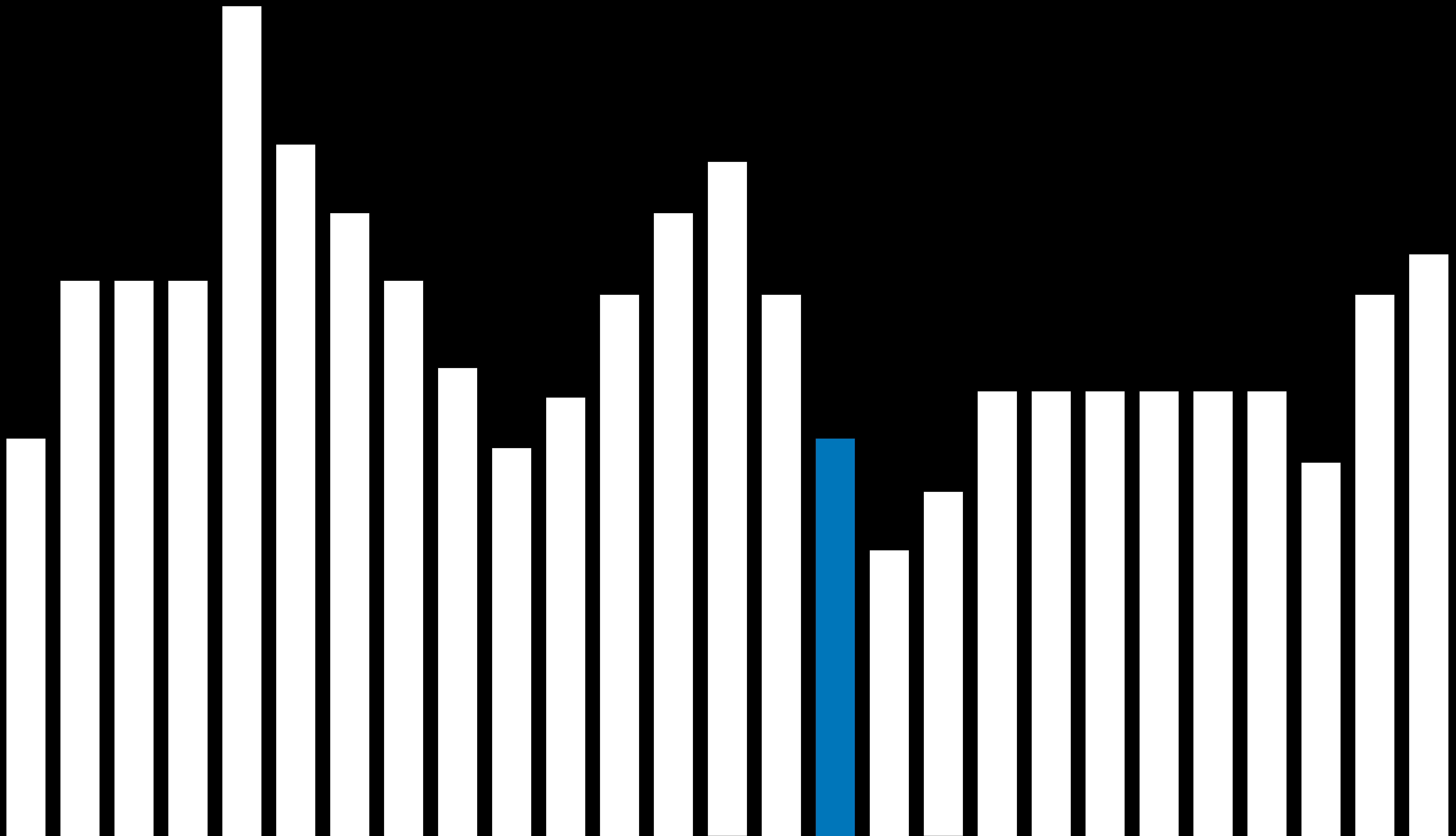


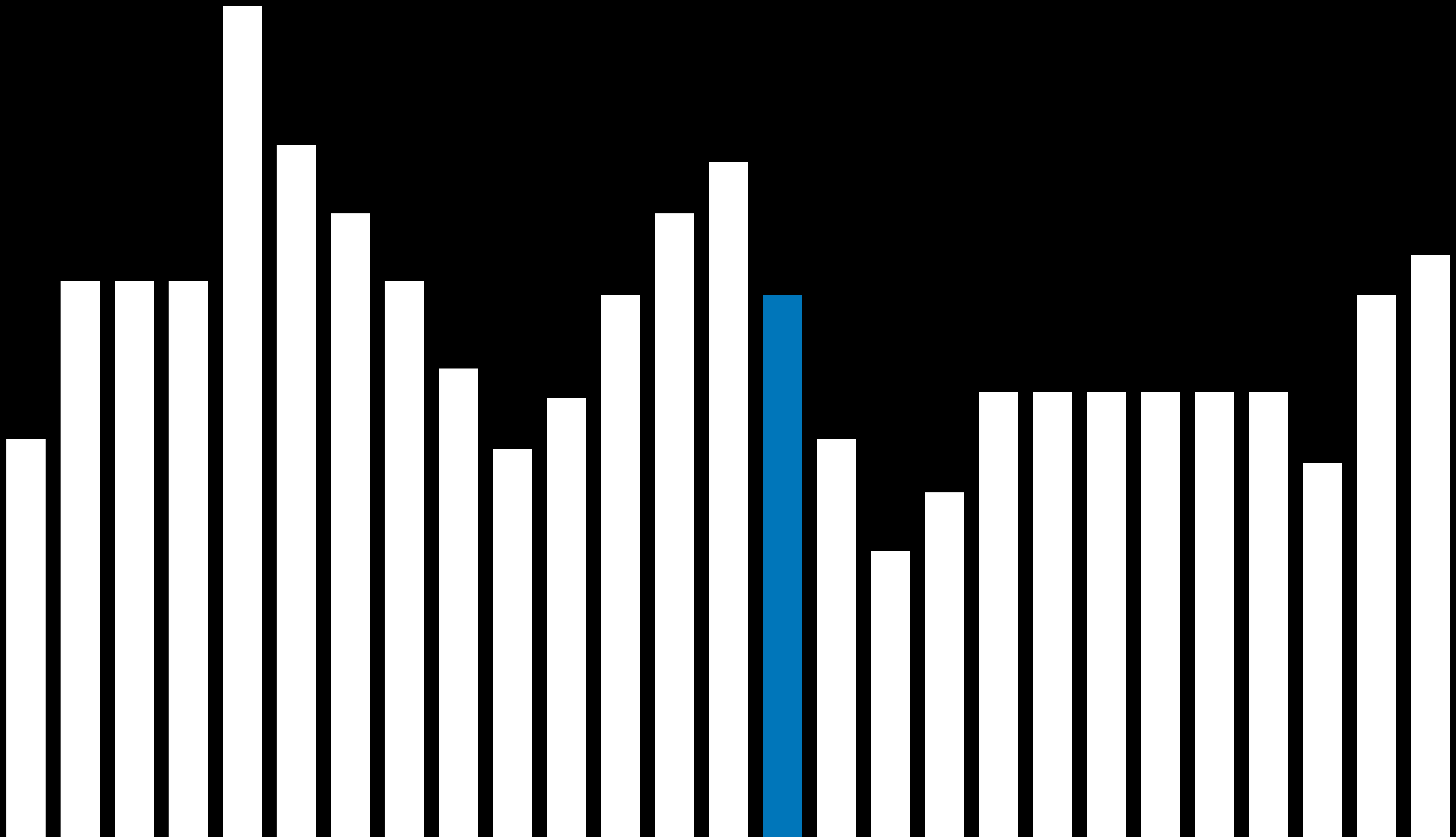

global minimum

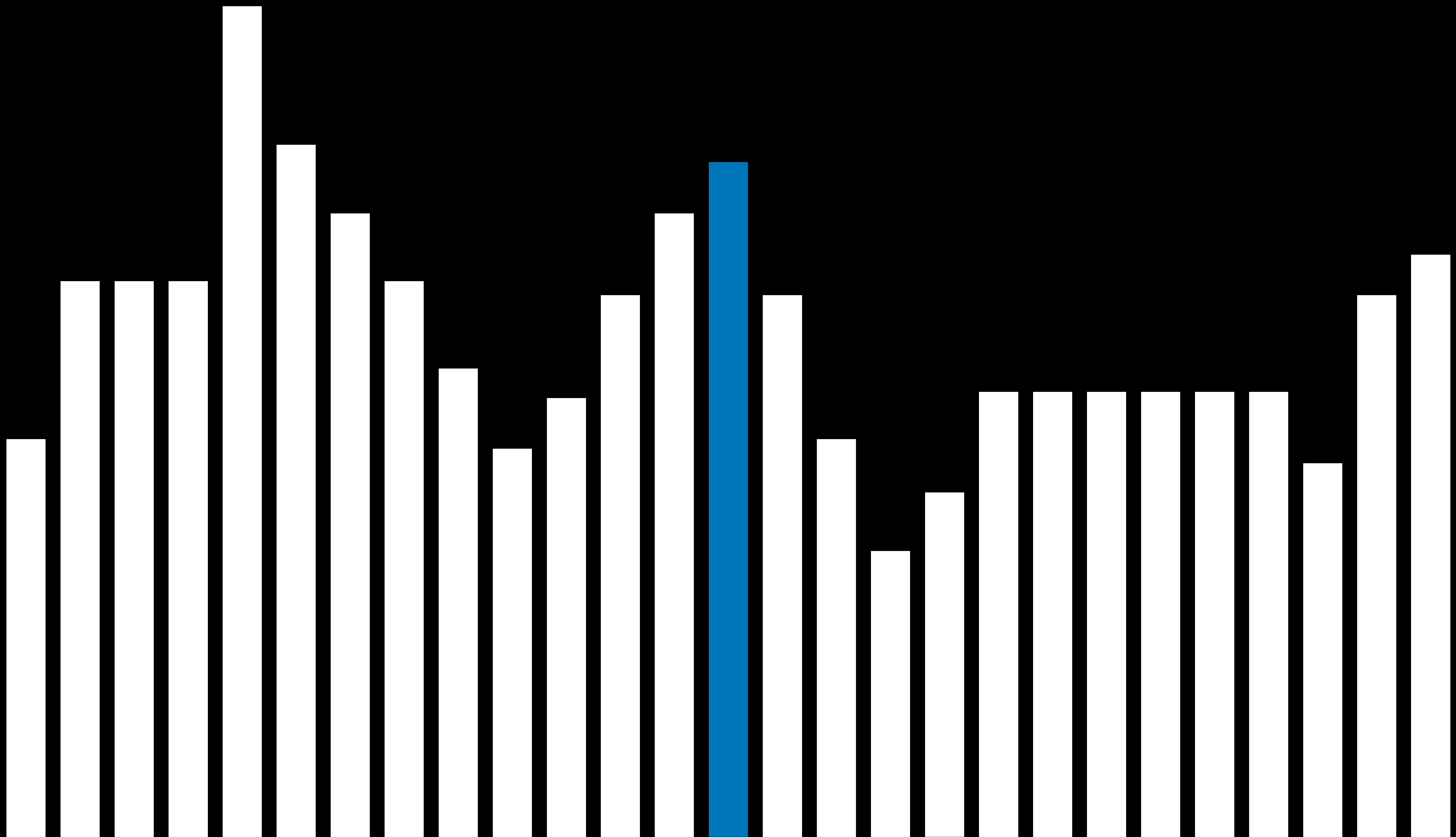

local minima



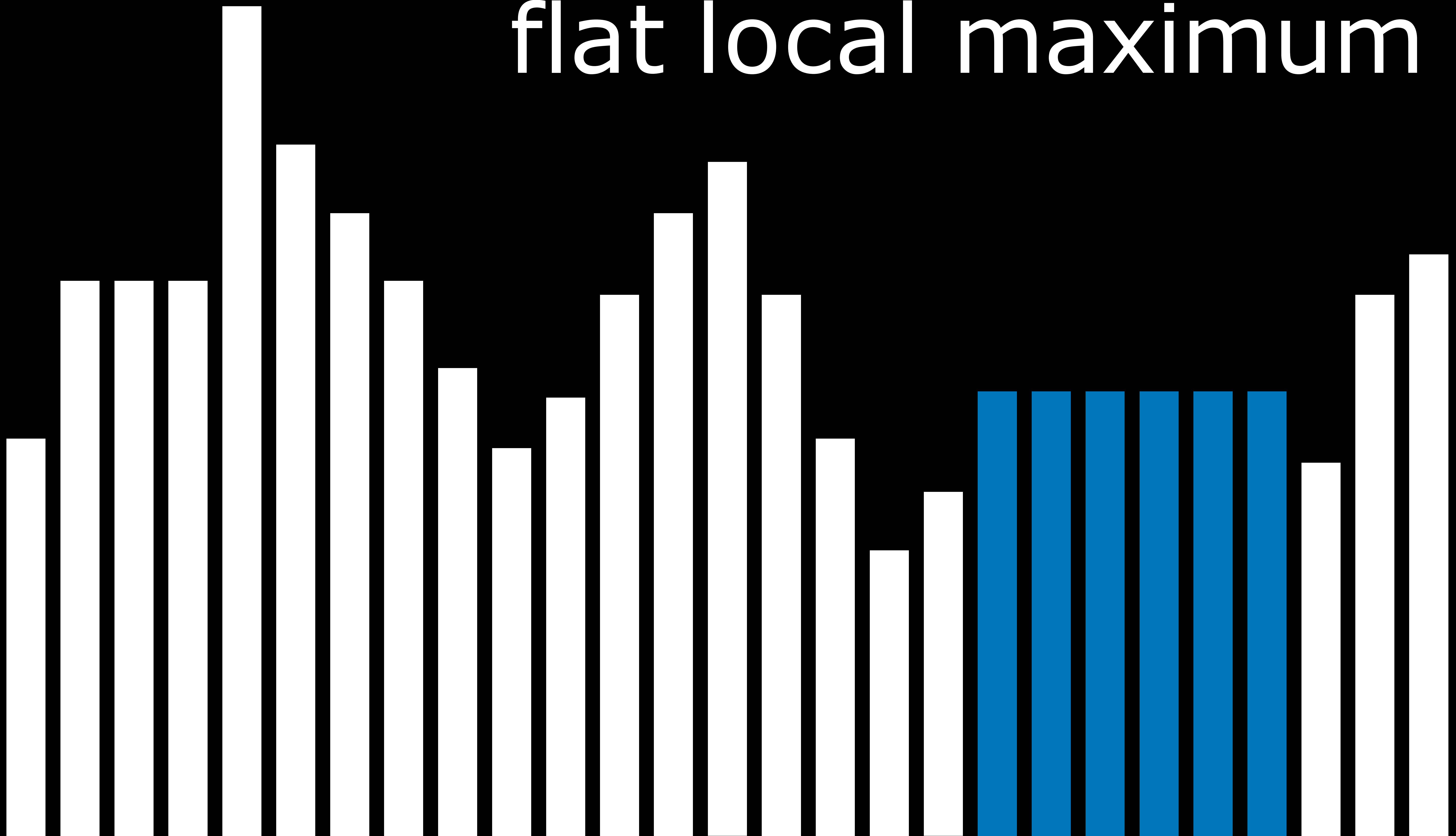








flat local maximum

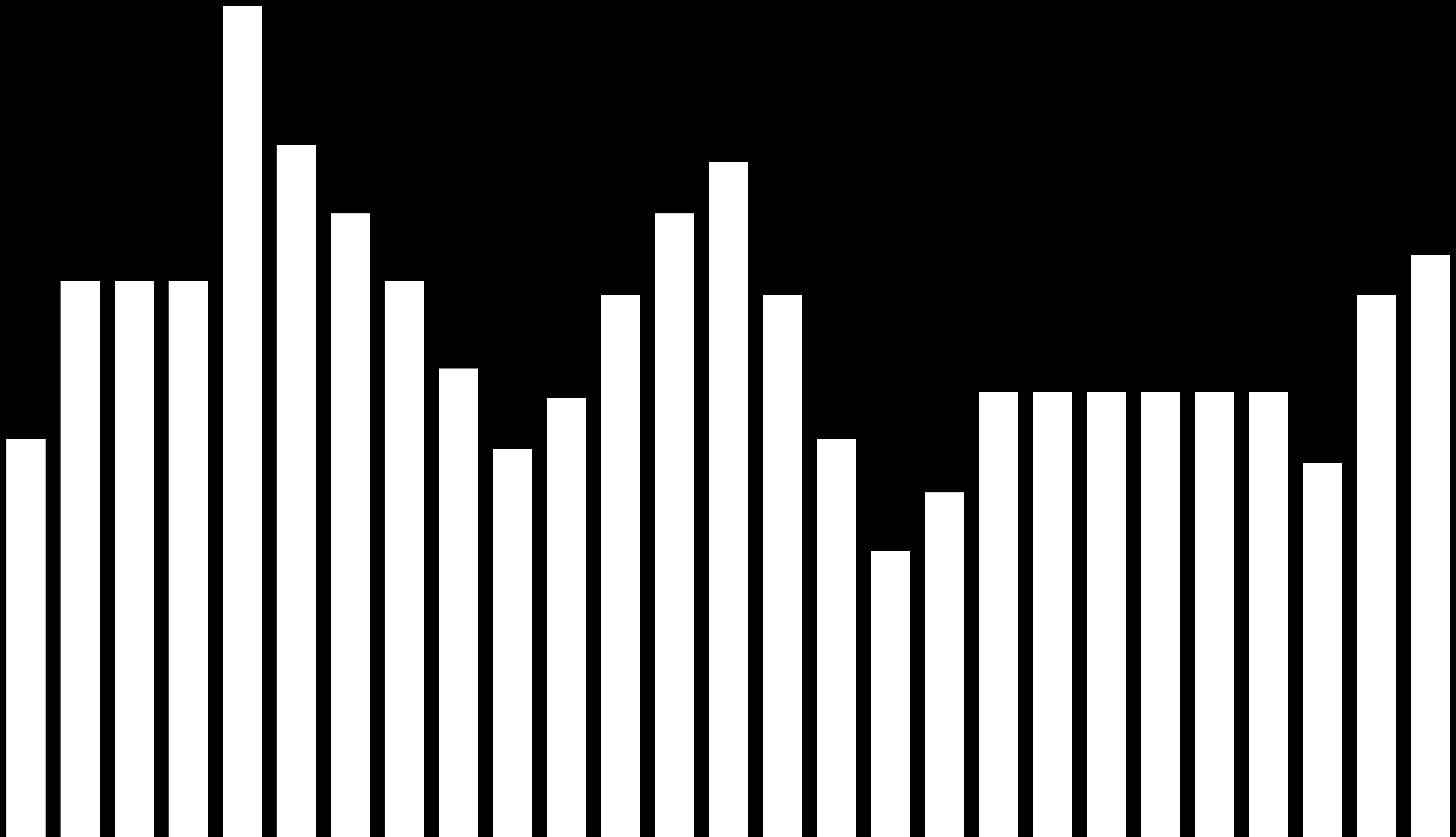


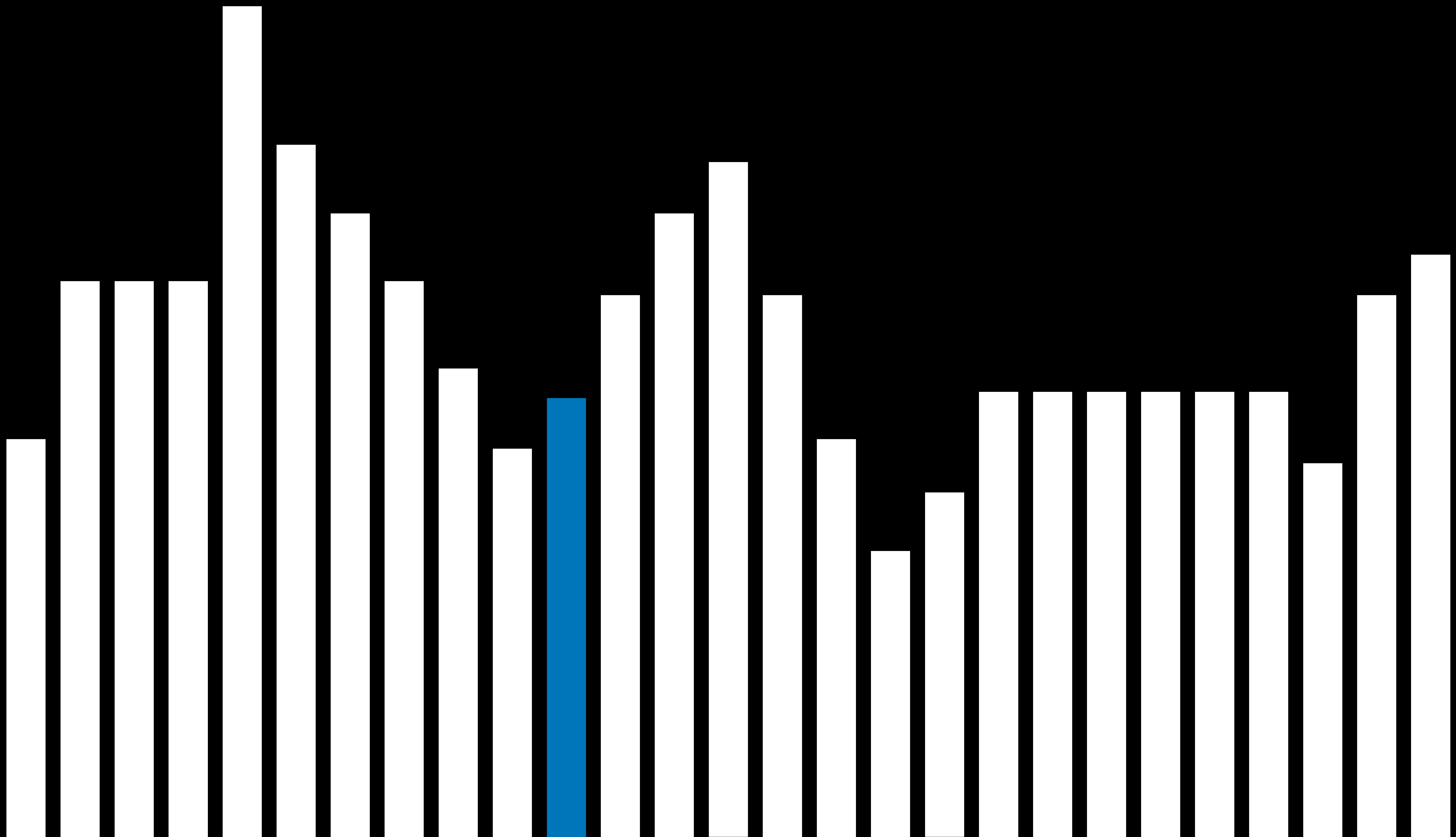

shoulder

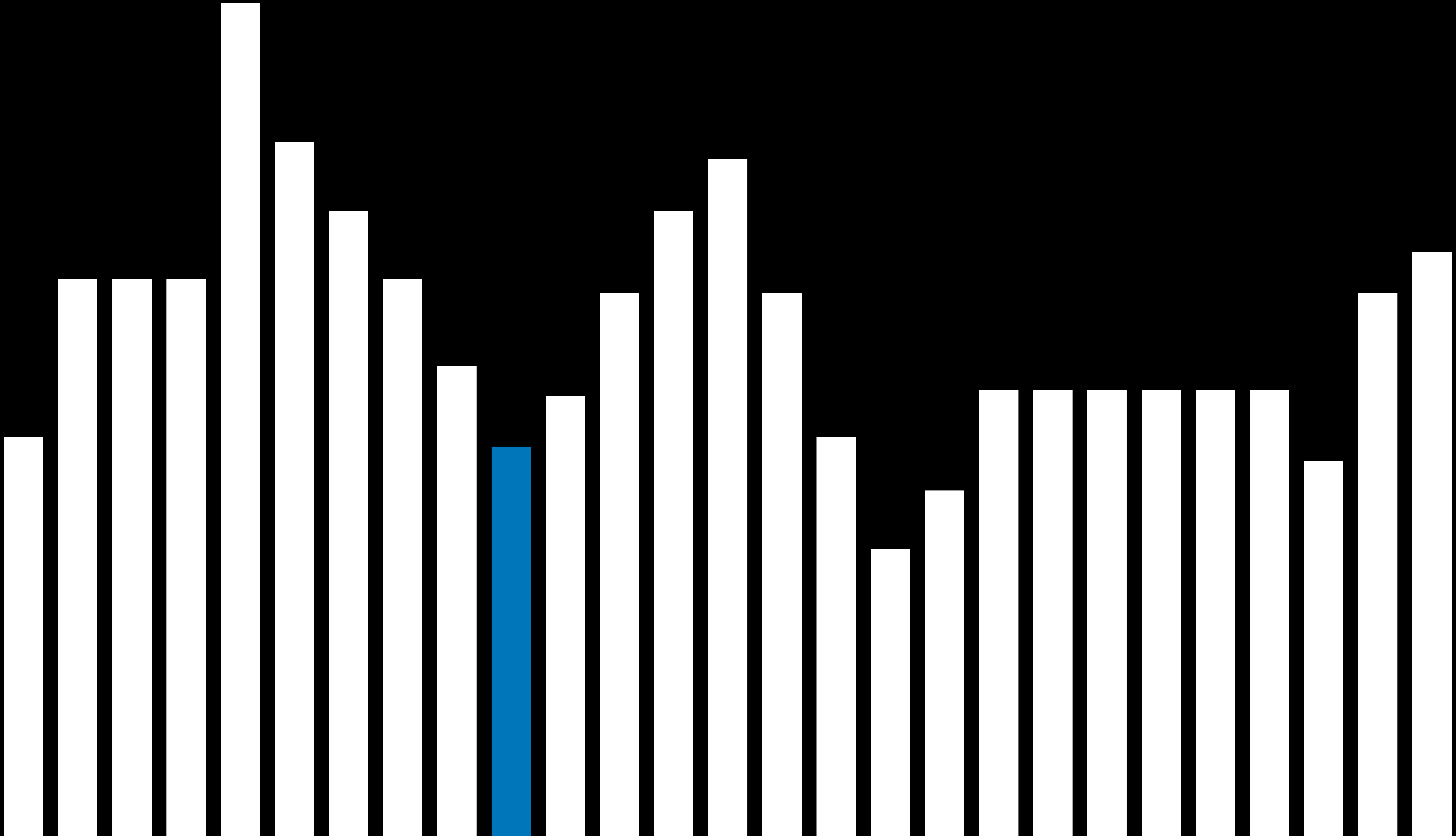
Hill Climbing Variants

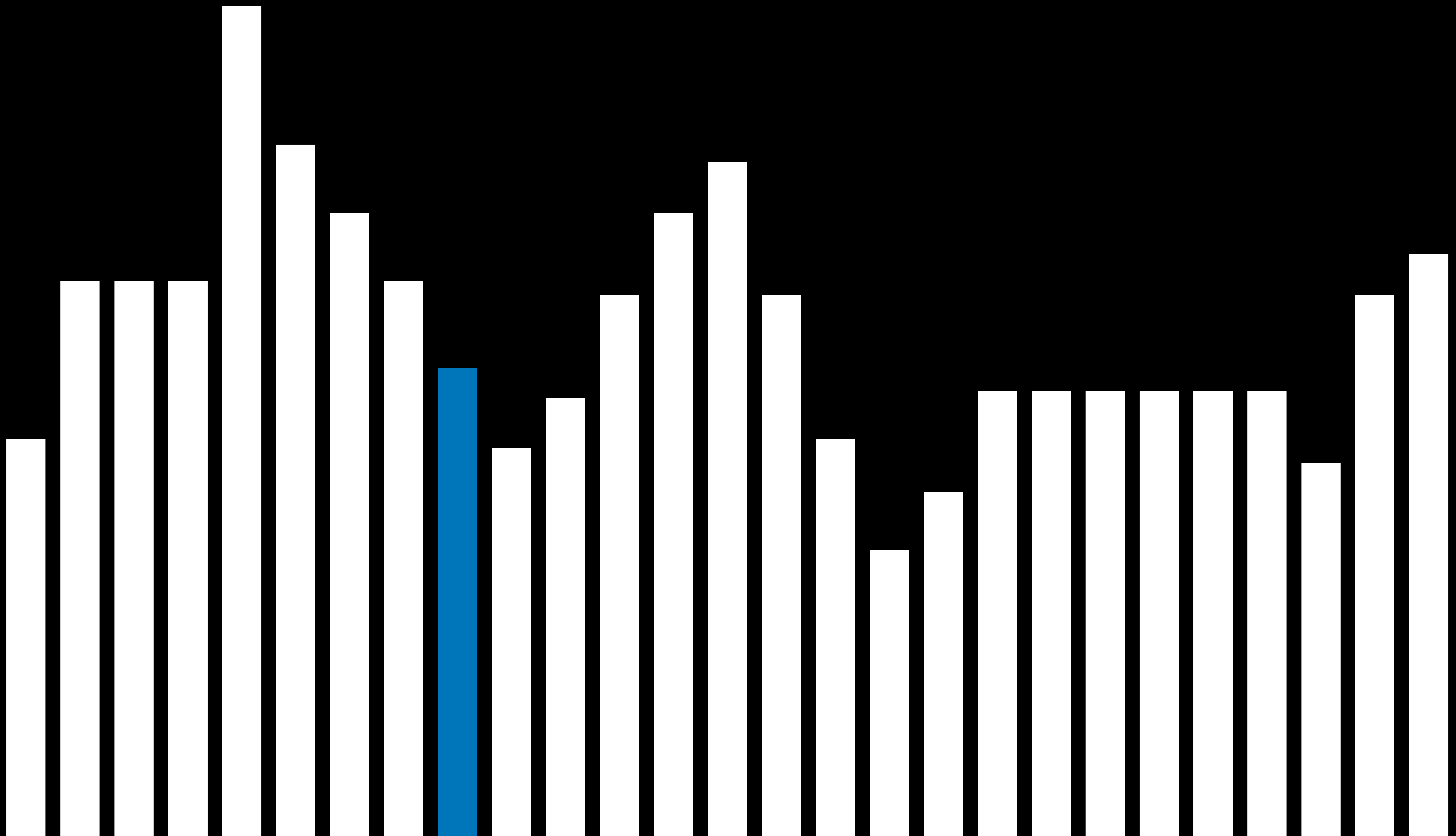
Variant	Definition
steepest-ascent	choose the highest-valued neighbor
stochastic	choose randomly from higher-valued neighbors
first-choice	choose the first higher-valued neighbor
random-restart	conduct hill climbing multiple times
local beam search	chooses the *k* highest-valued neighbors

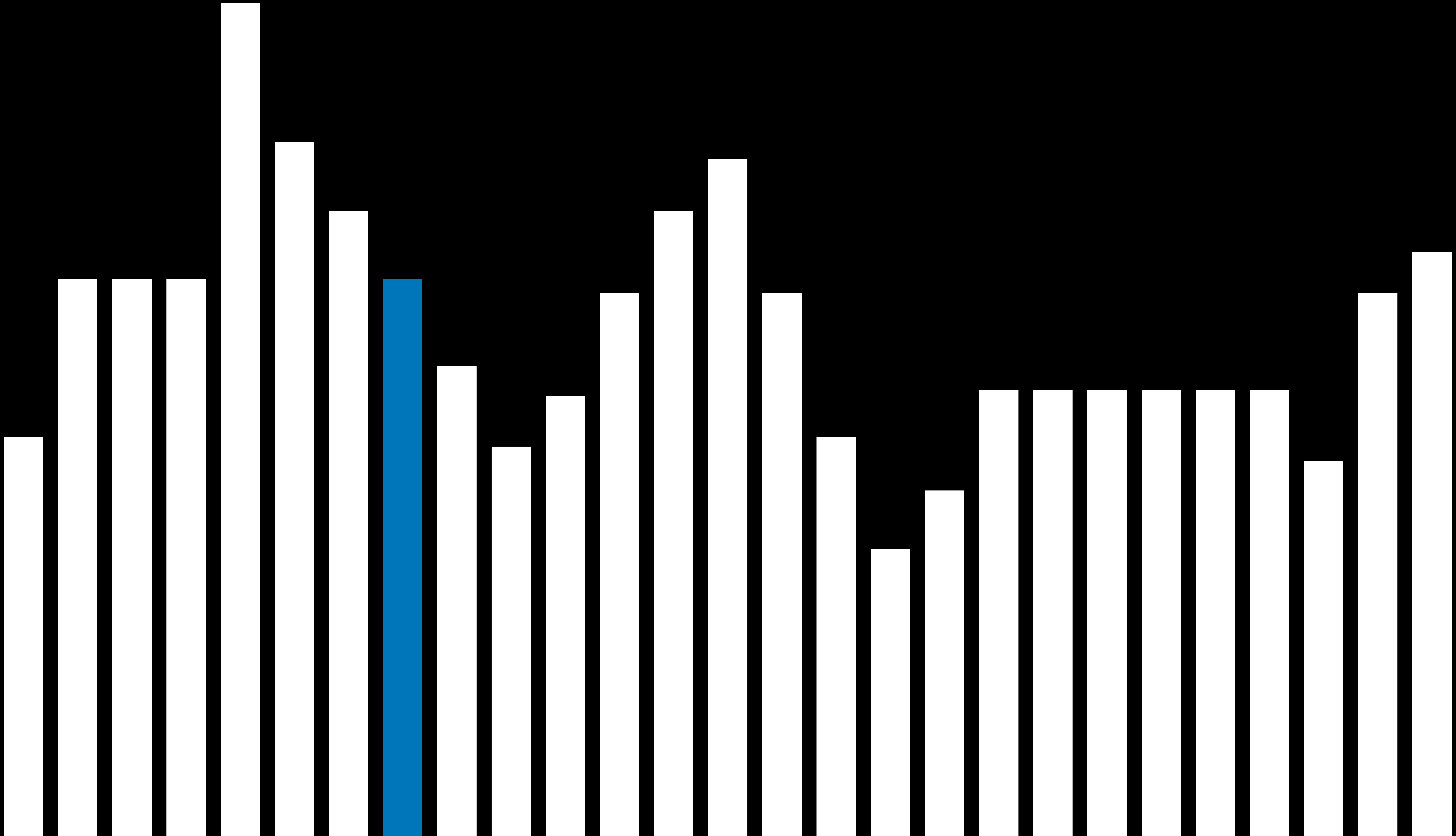
Simulated Annealing

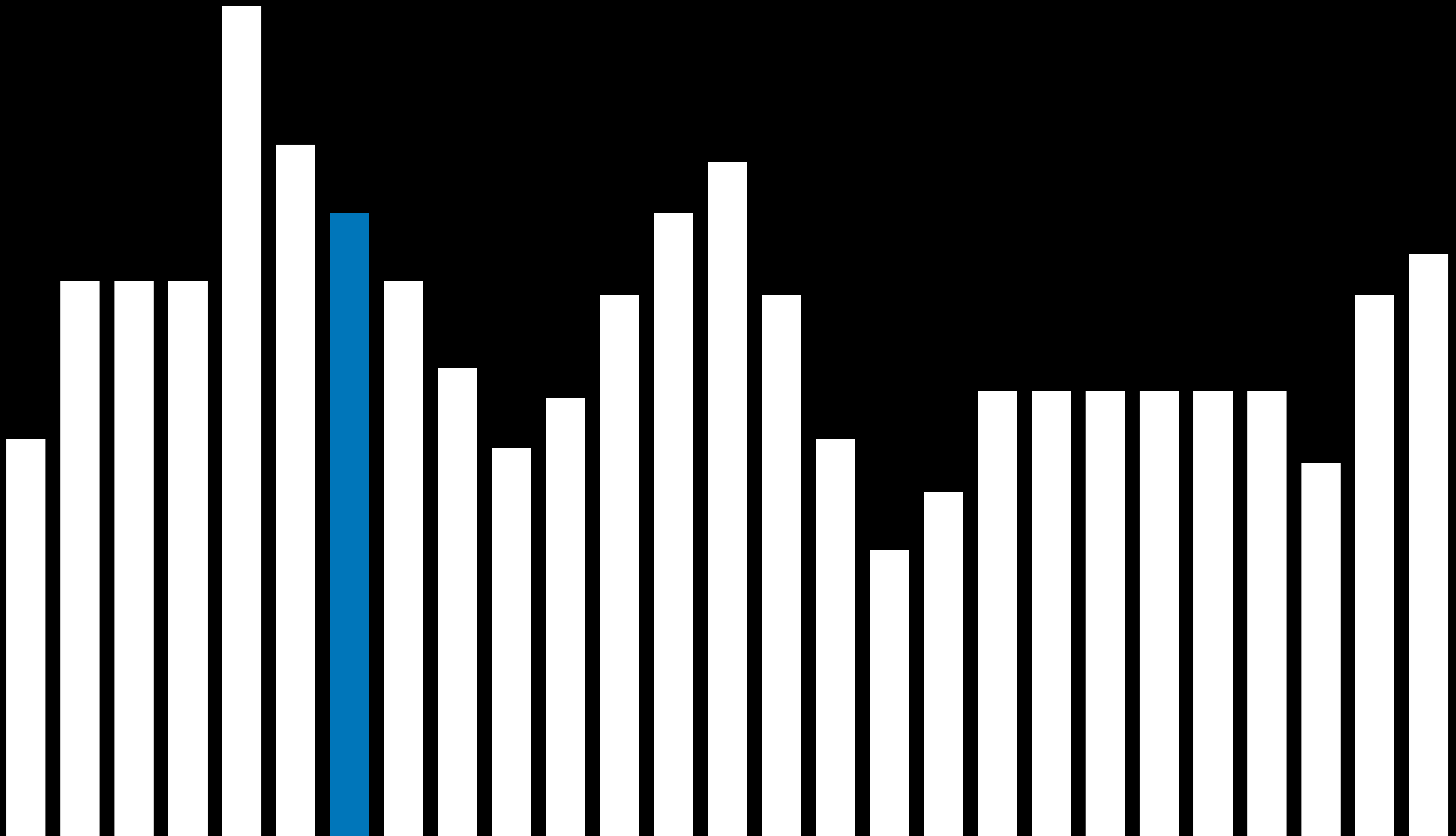


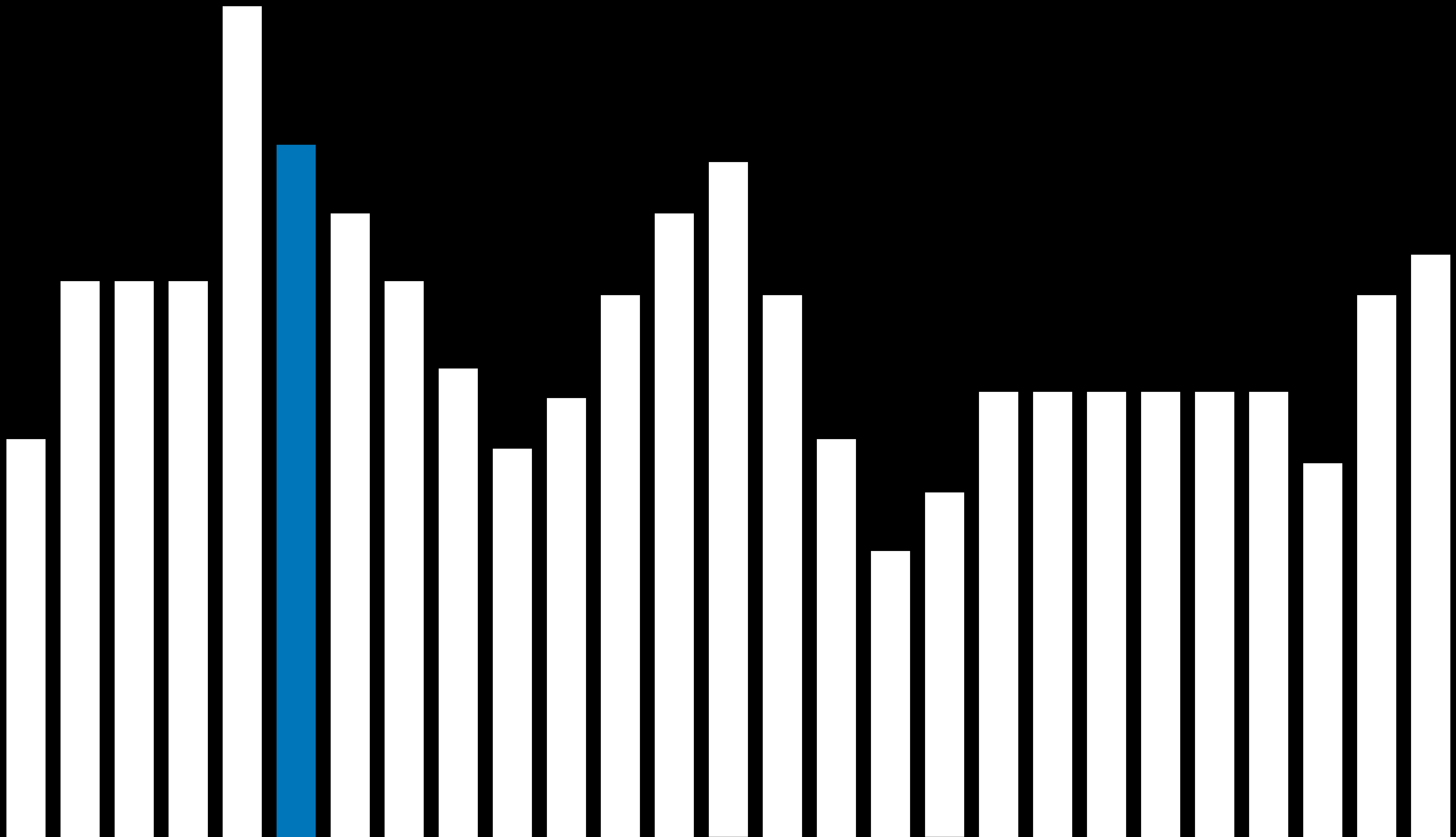


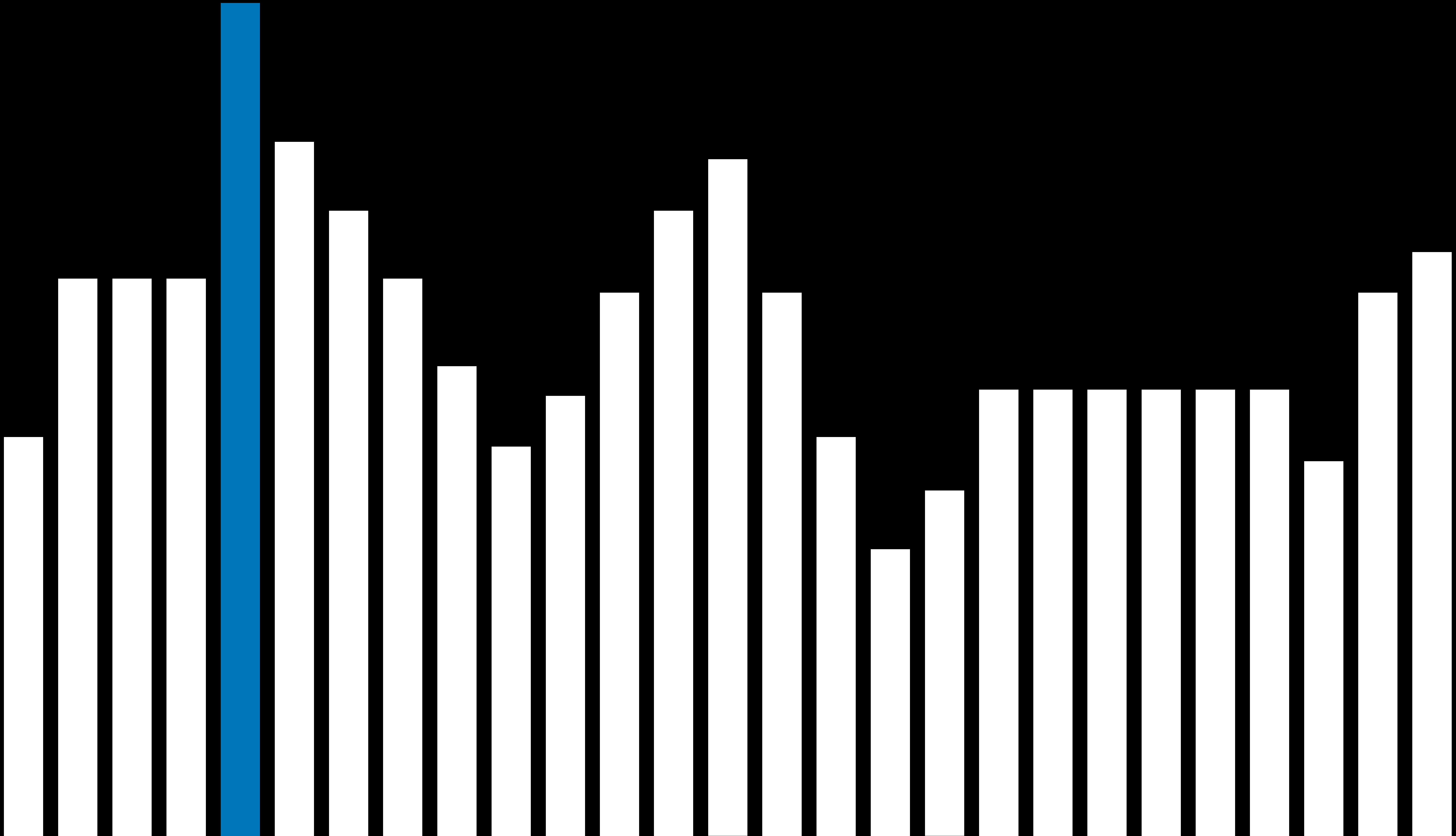












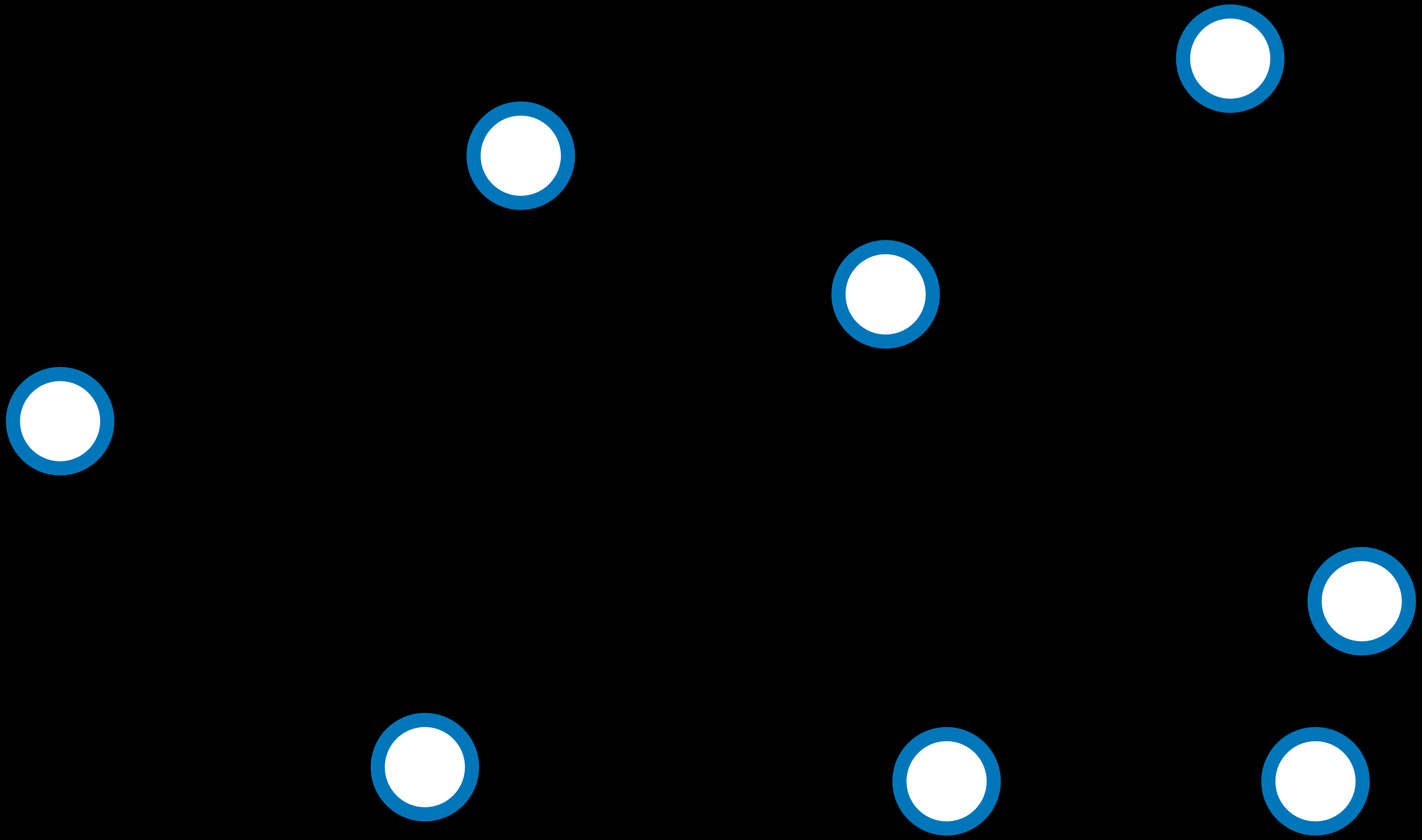
Simulated Annealing

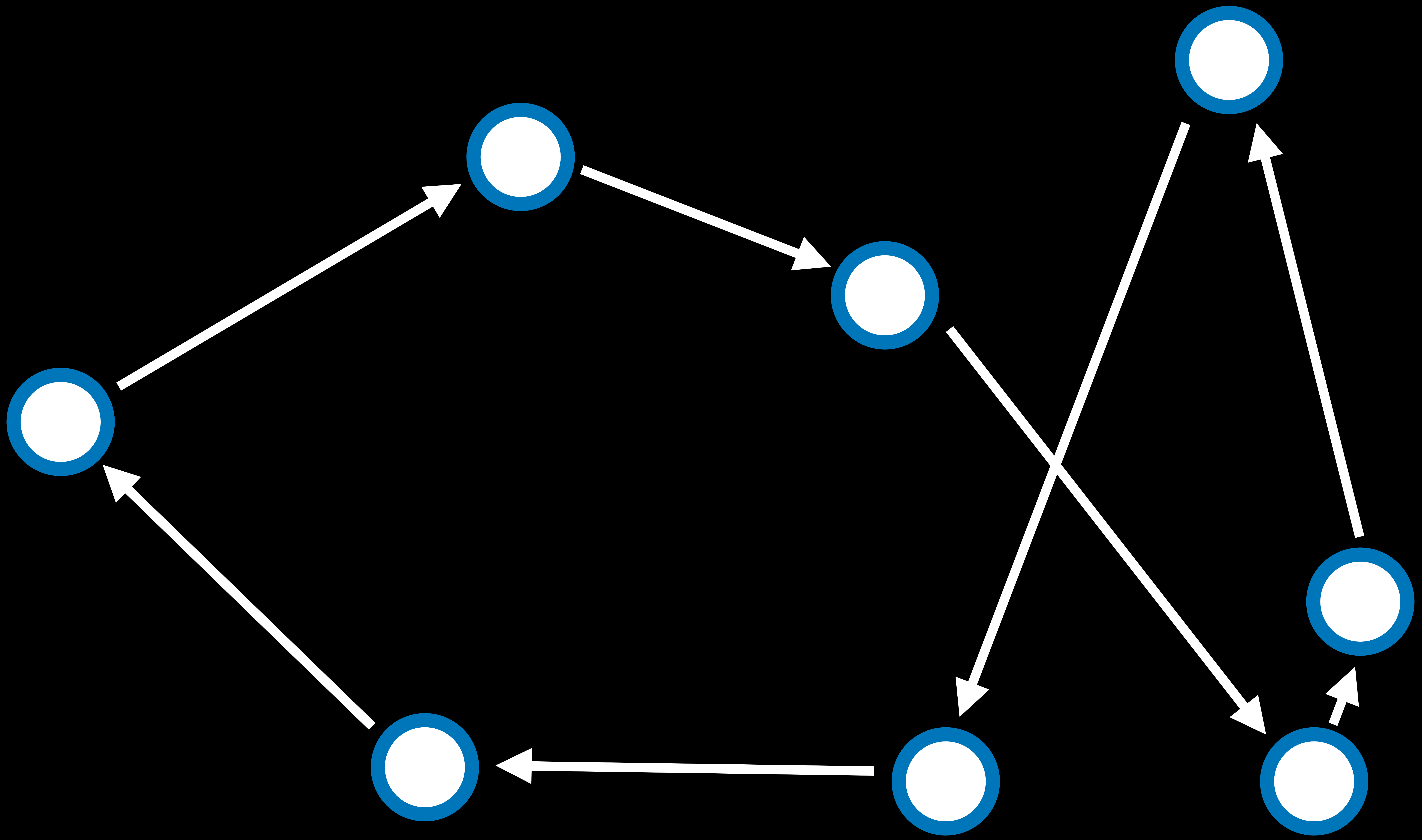
- Early on, higher "temperature": more likely to accept neighbors that are worse than current state
- Later on, lower "temperature": less likely to accept neighbors that are worse than current state

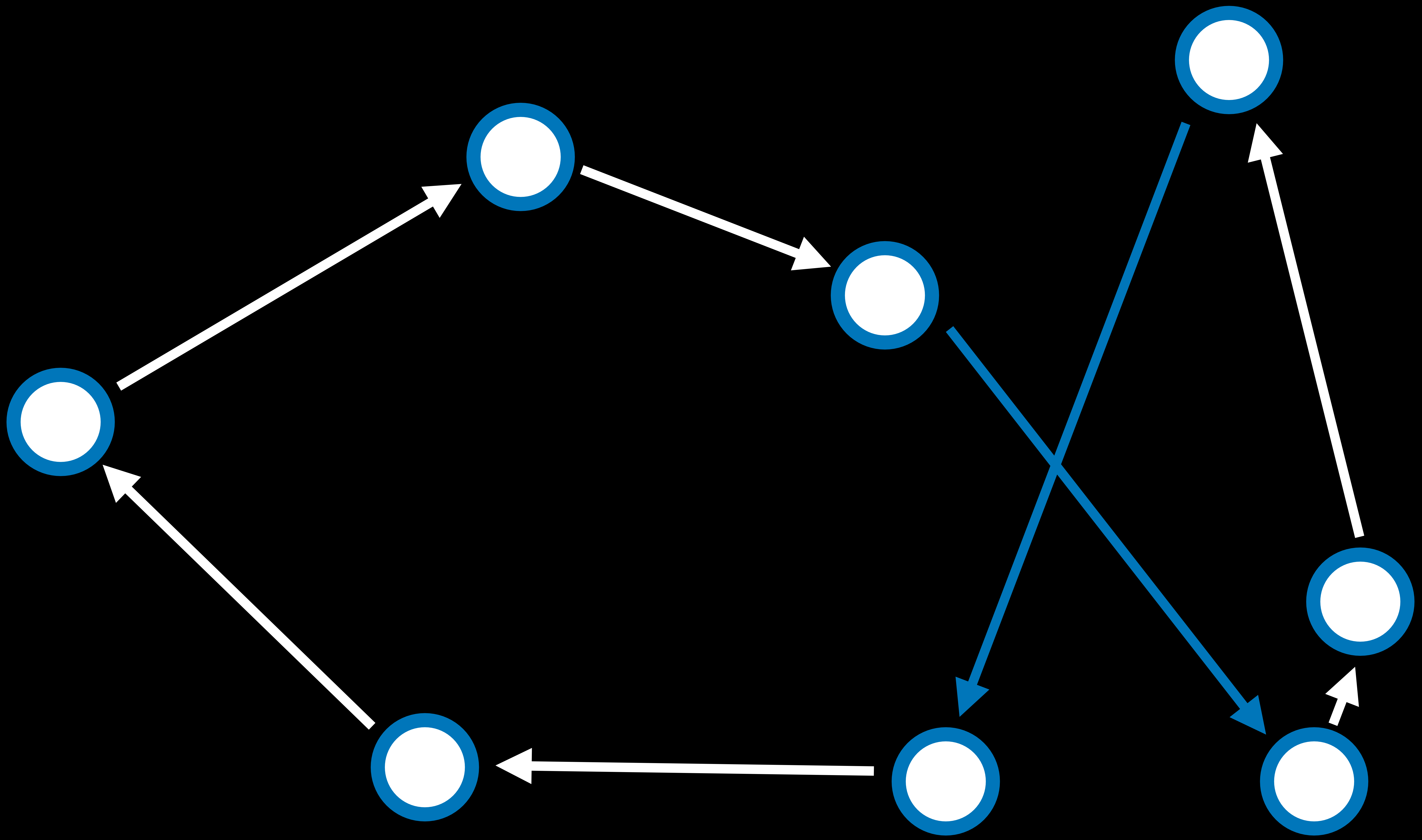
Simulated Annealing

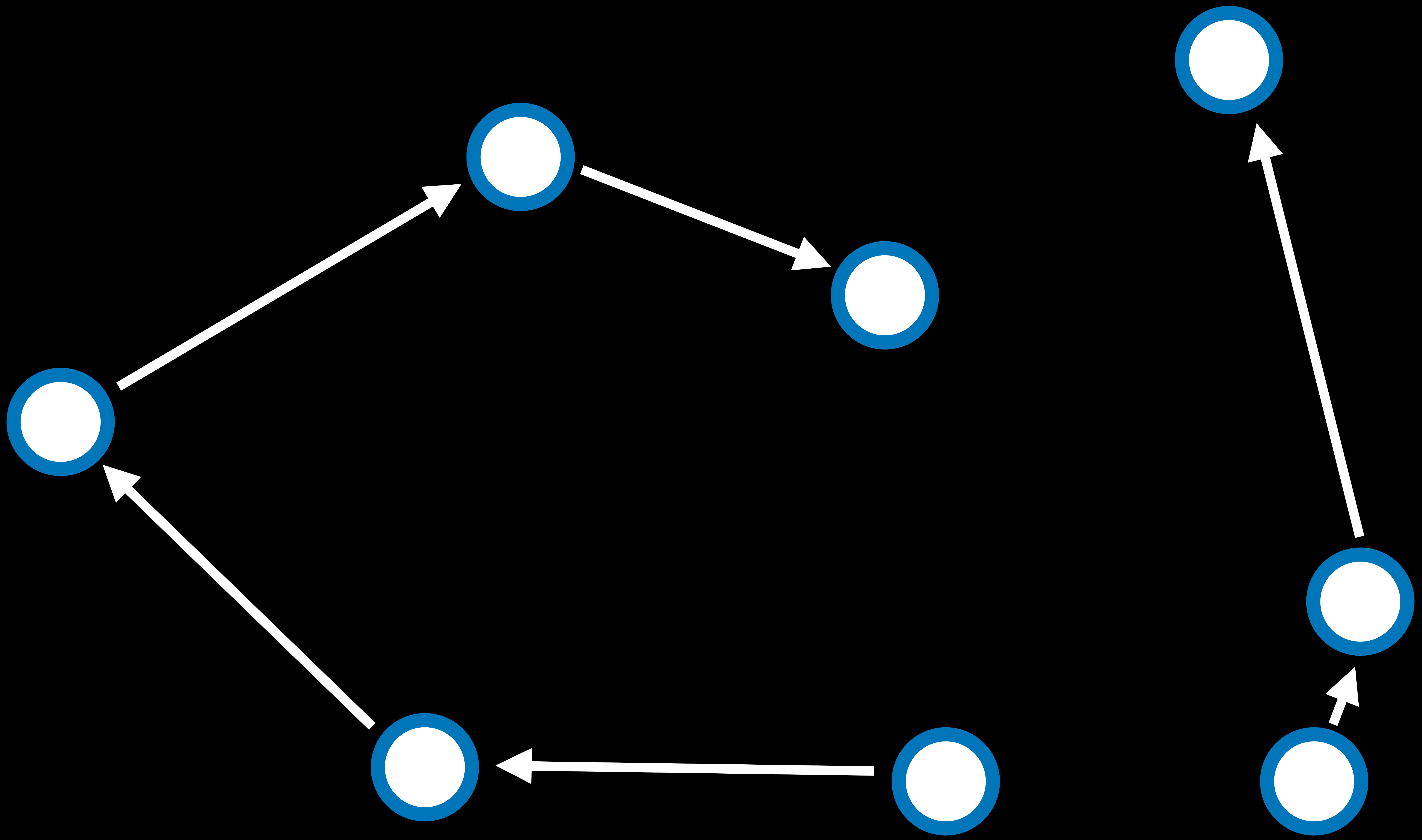
```
function SIMULATED-ANNEALING(problem, max):
    current = initial state of problem
    for t = 1 to max:
        T = TEMPERATURE(t)
        neighbor = random neighbor of current
        ΔE = how much better neighbor is than current
        if ΔE > 0:
            current = neighbor
        with probability e^(ΔE/T) set current = neighbor
    return current
```

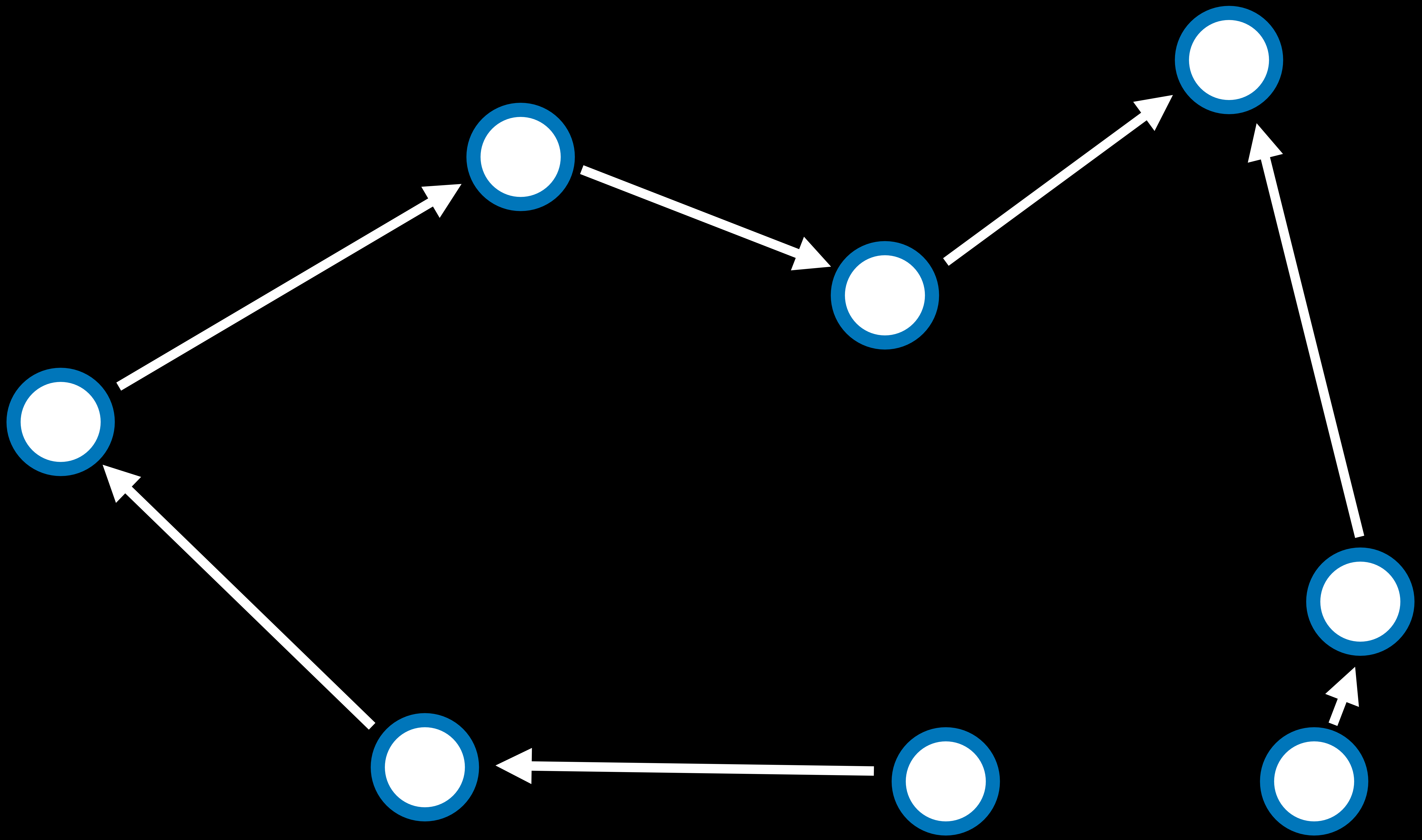
Traveling Salesman Problem

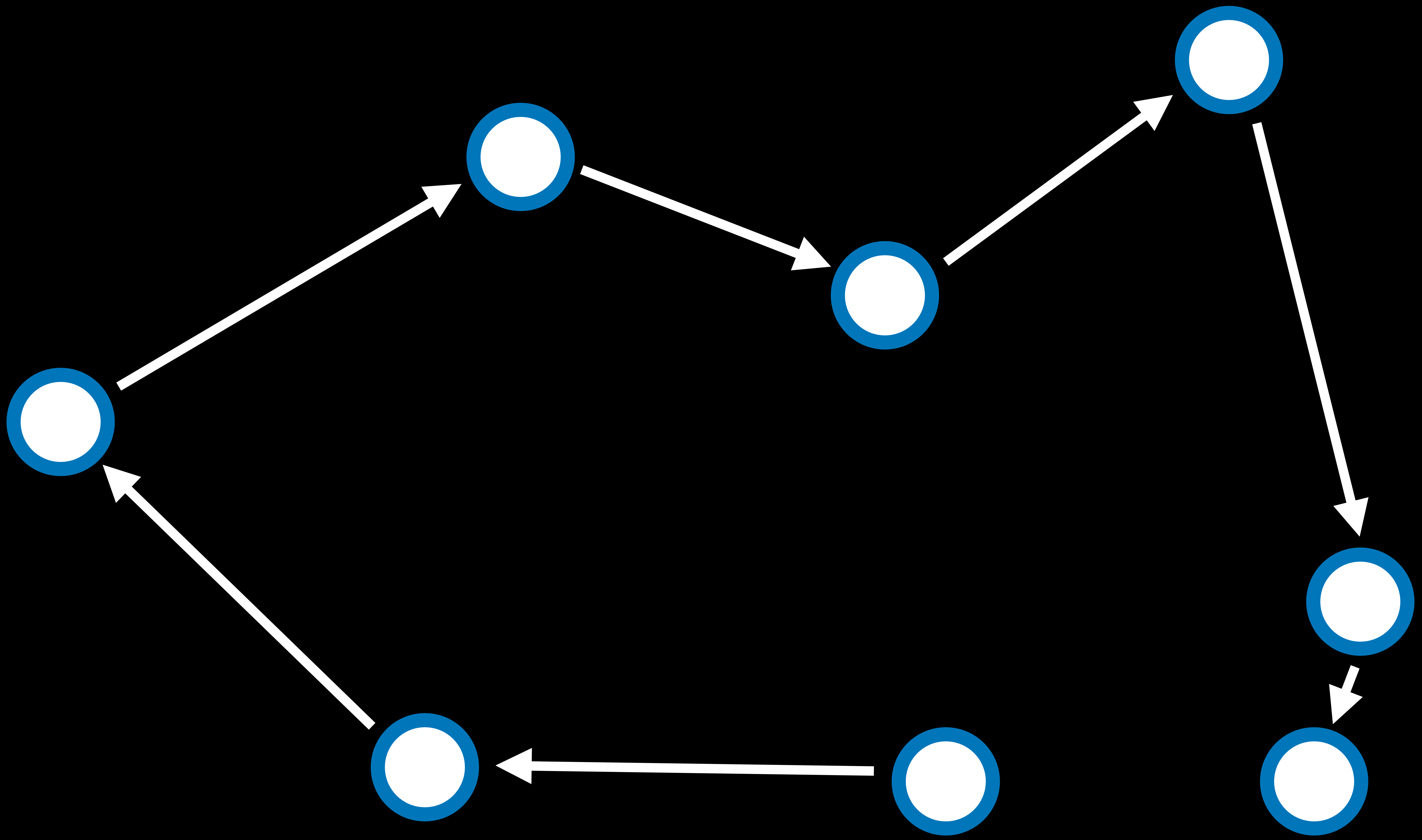


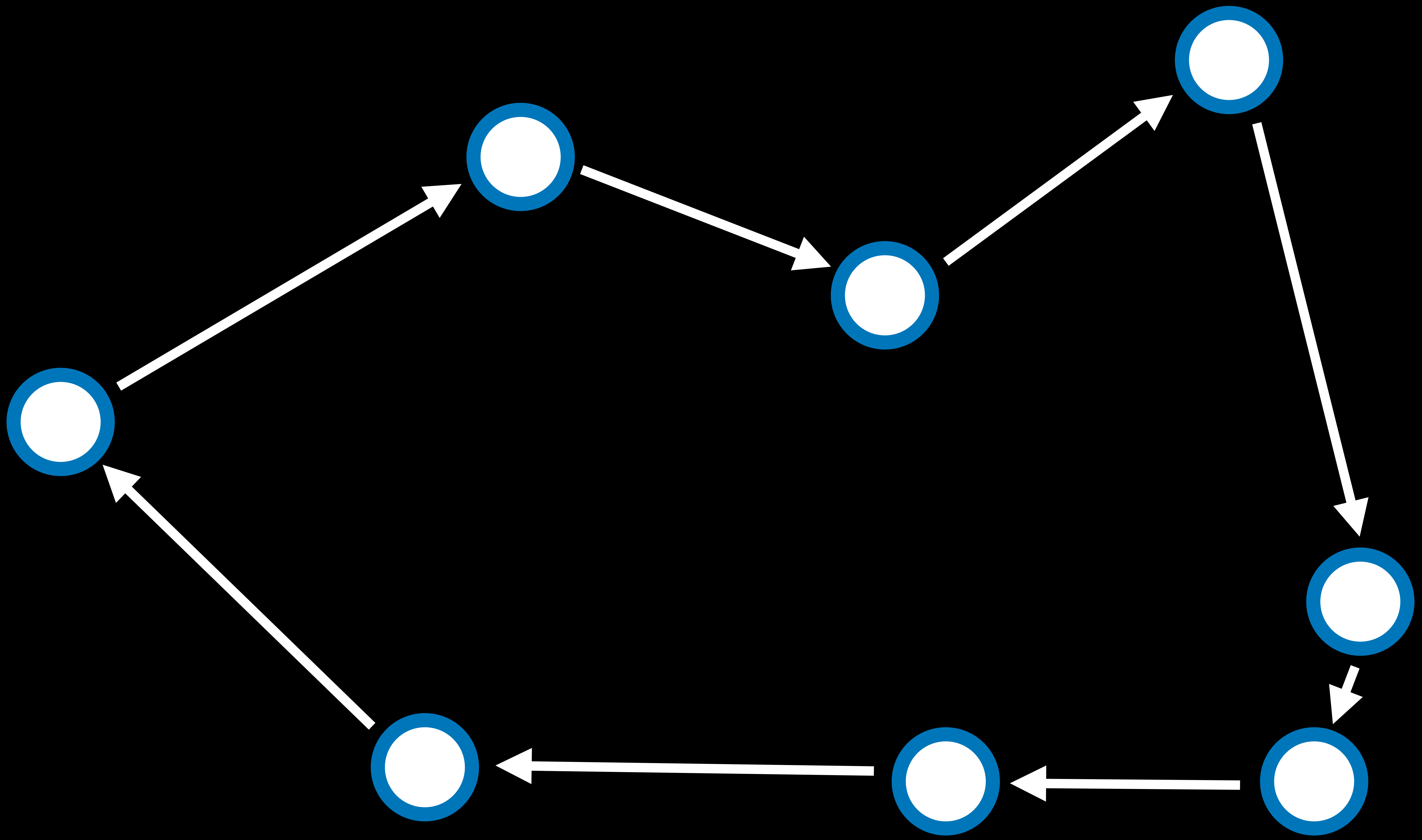












Linear Programming

Linear Programming

- Minimize a cost function $c_1x_1 + c_2x_2 + \ldots + c_nx_n$
- With constraints of form $a_1x_1 + a_2x_2 + \ldots + a_nx_n \leq b$
  or of form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- With bounds for each variable $l_i \leq x_i \leq u_i$

Linear Programming Example

- Two machines X_1 and X_2. X_1 costs $50/hour to run, X_2 costs $80/hour to run. Goal is to minimize cost.
- X_1 requires 5 units of labor per hour. X_2 requires 2 units of labor per hour. Total of 20 units of labor to spend.
- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

- X_1 requires 5 units of labor per hour. X_2 requires 2 units of labor per hour. Total of 20 units of labor to spend.
- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

Constraint: $5x_1 + 2x_2 \leq 20$

- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

Constraint: $5x_1 + 2x_2 \leq 20$

Constraint: $10x_1 + 12x_2 \geq 90$

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

Constraint: $5x_1 + 2x_2 \leq 20$

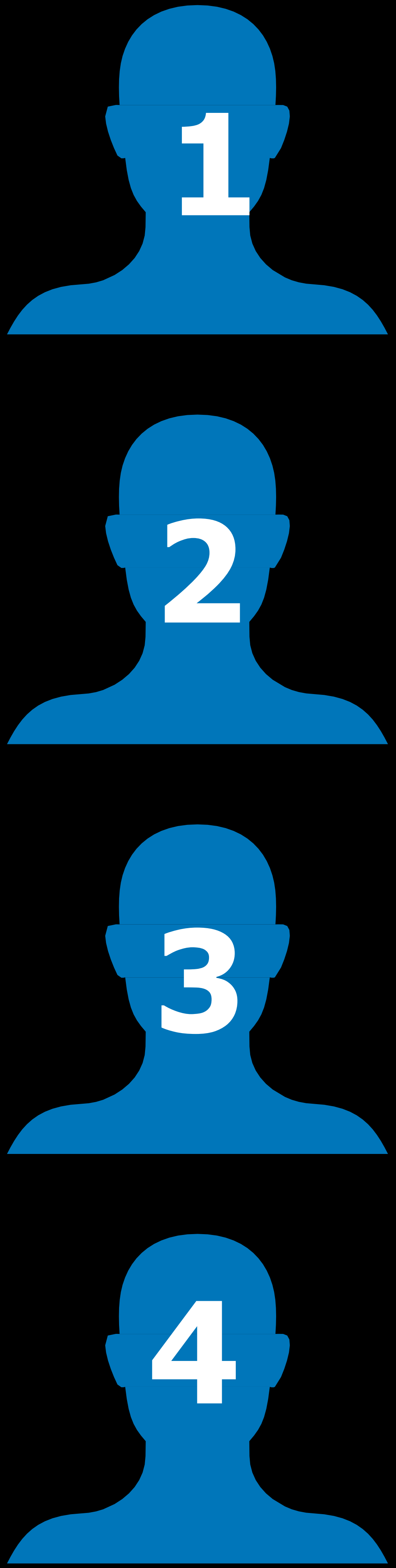
Constraint: $(-10x_1) + (-12x_2) \leq -90$

Linear Programming Algorithms

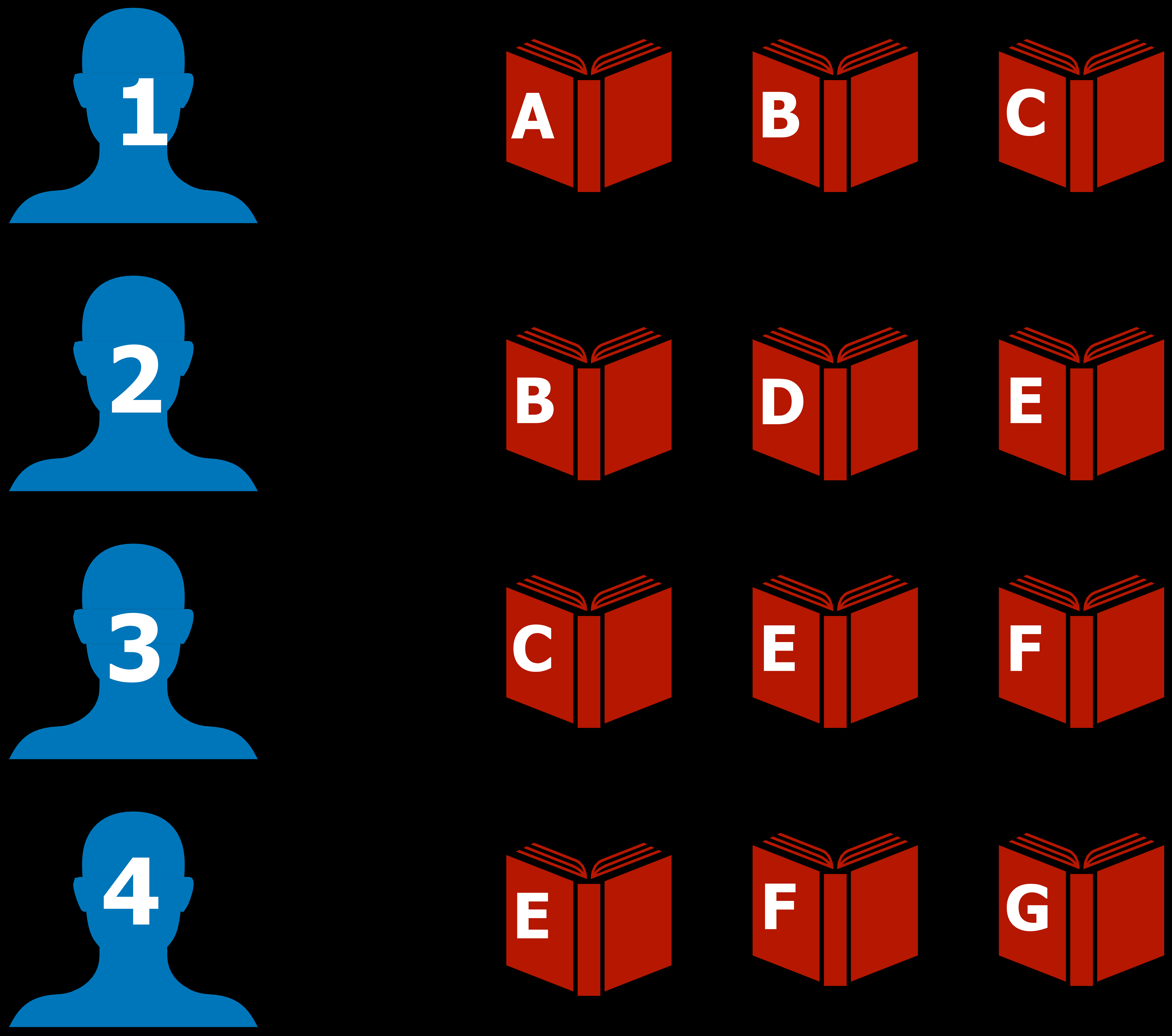
- Simplex
- Interior-Point

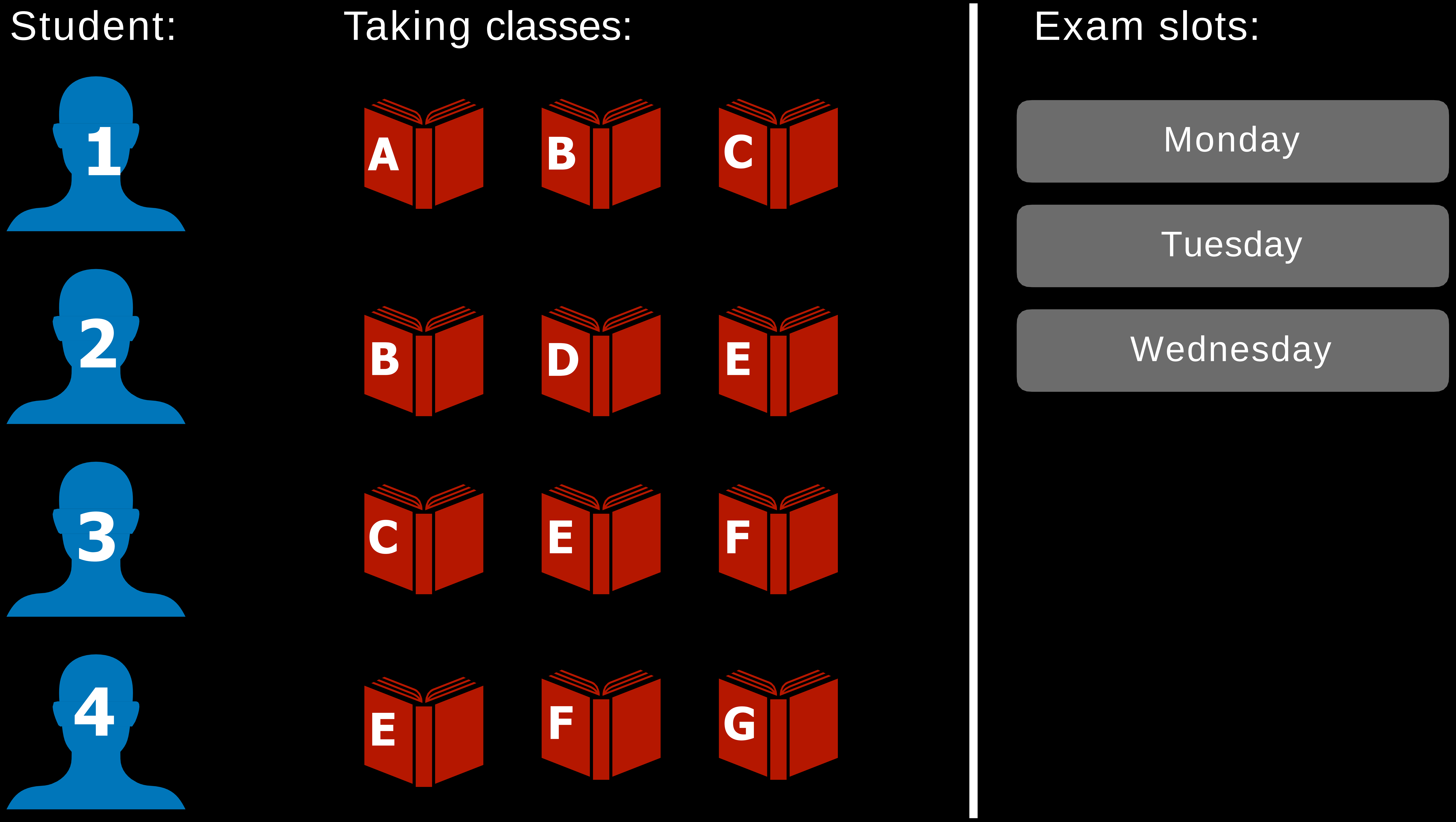
Constraint Satisfaction

Student:



Student: Taking classes:




Student:
Taking classes:
Exam slots:
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4
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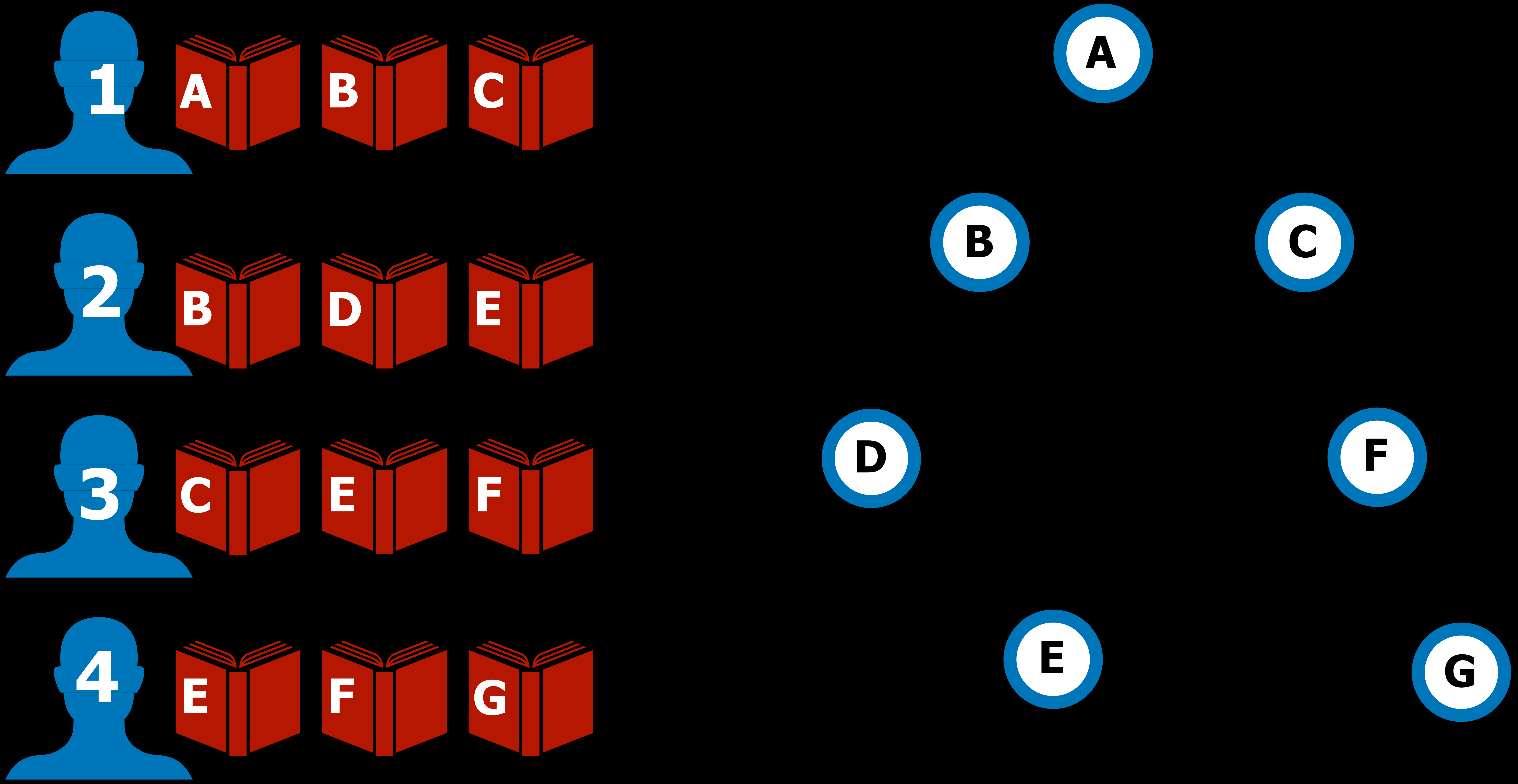
Monday
Tuesday
Wednesday



1 A B C
2 B D E
3 C E F
4 E F G
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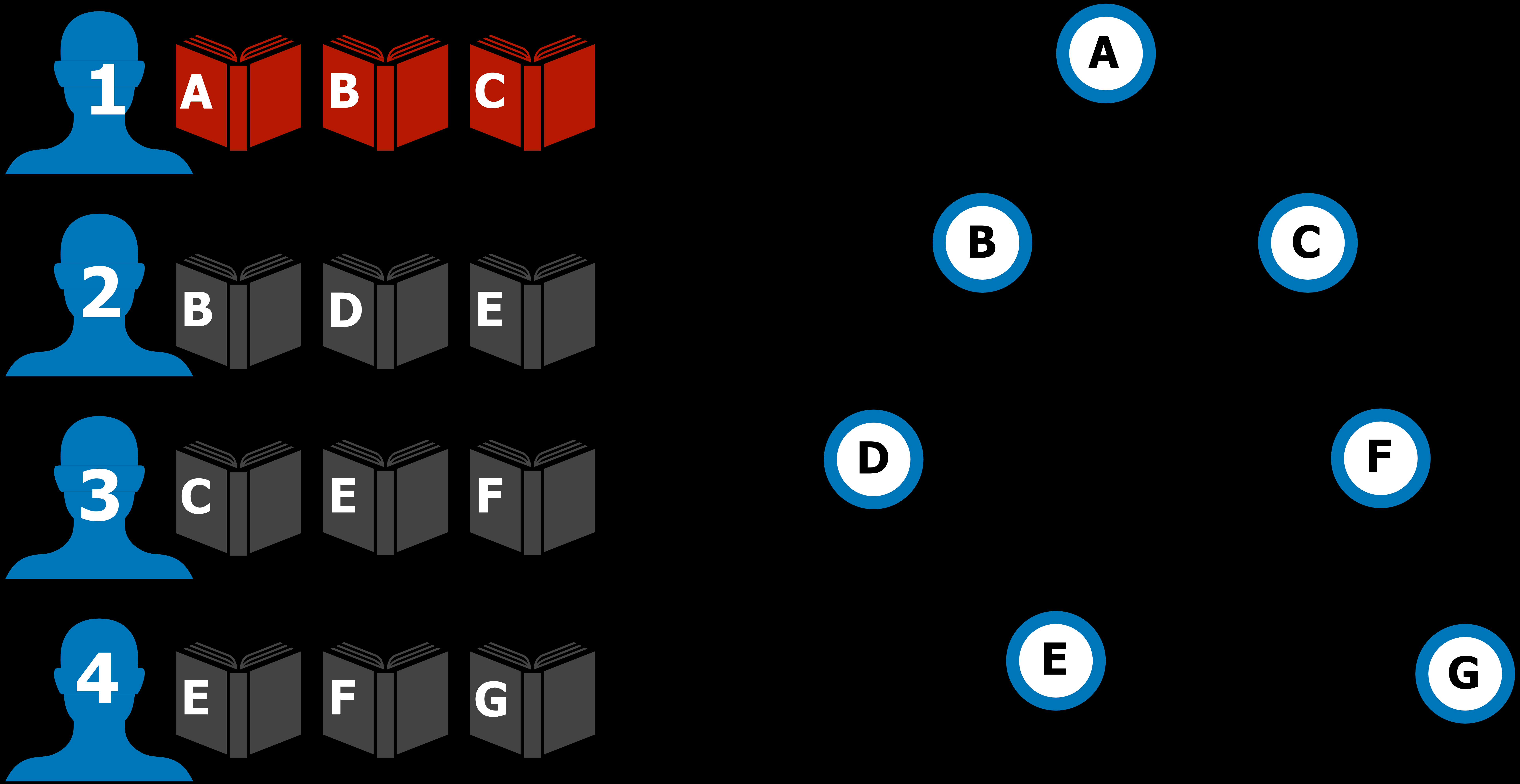

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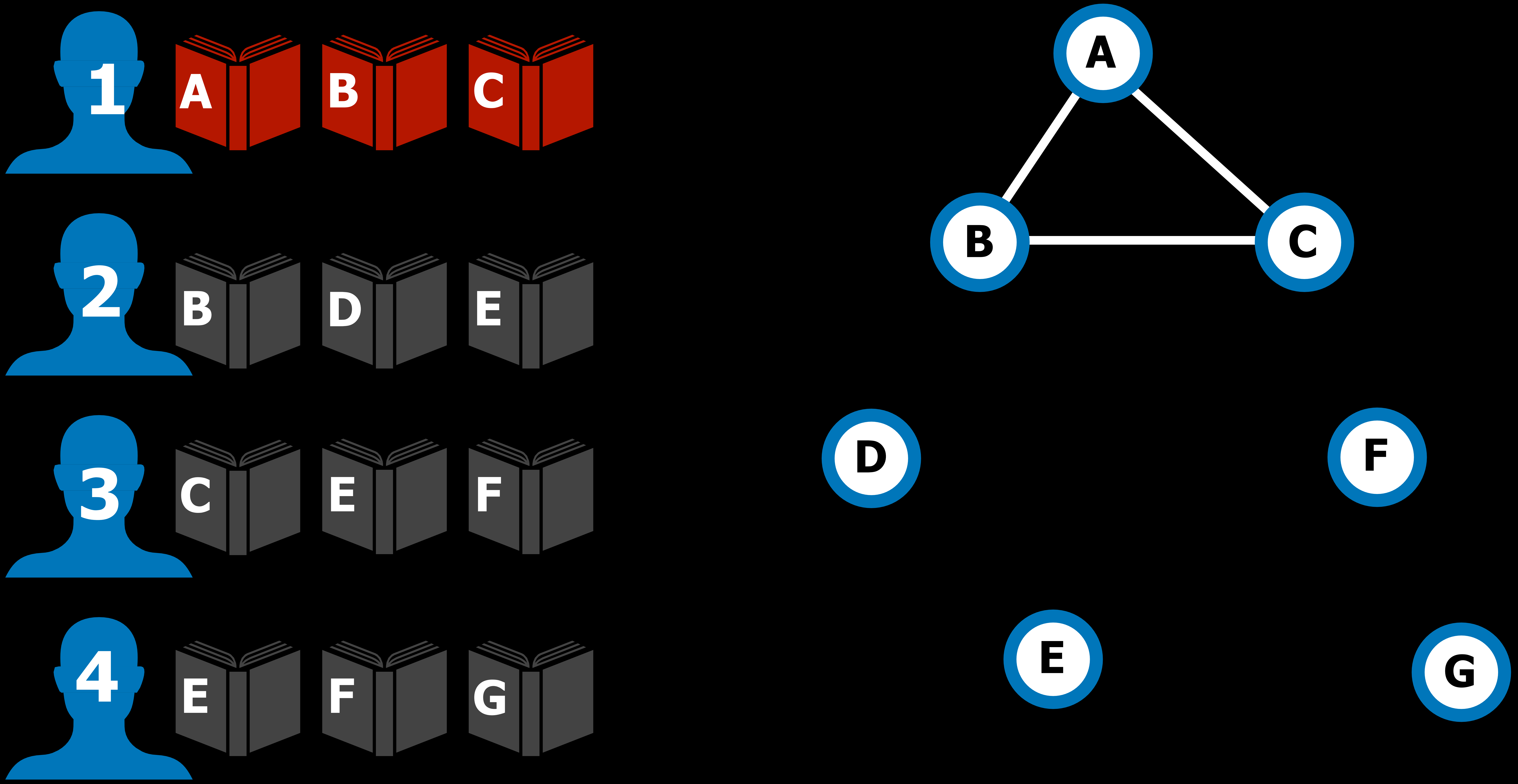
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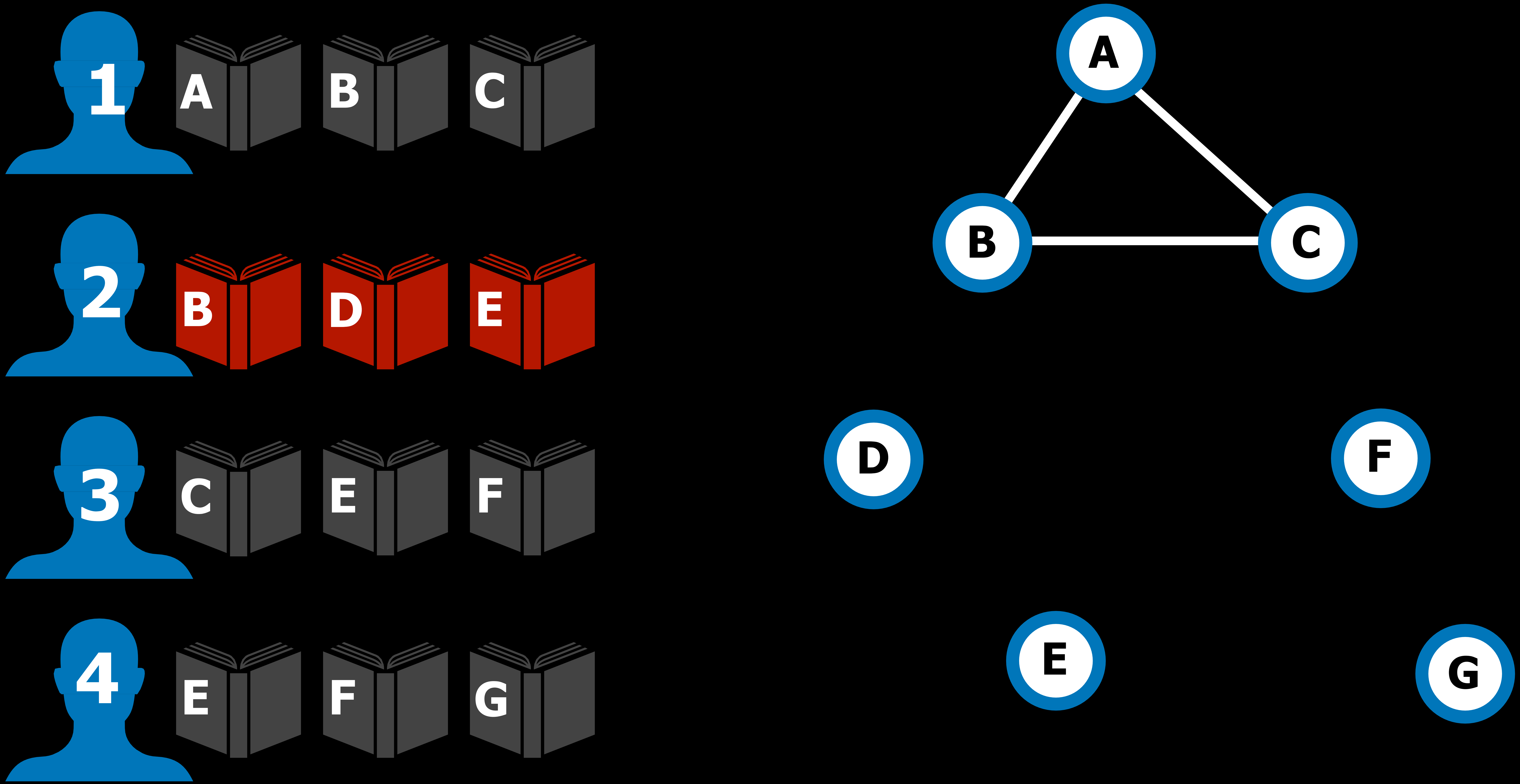
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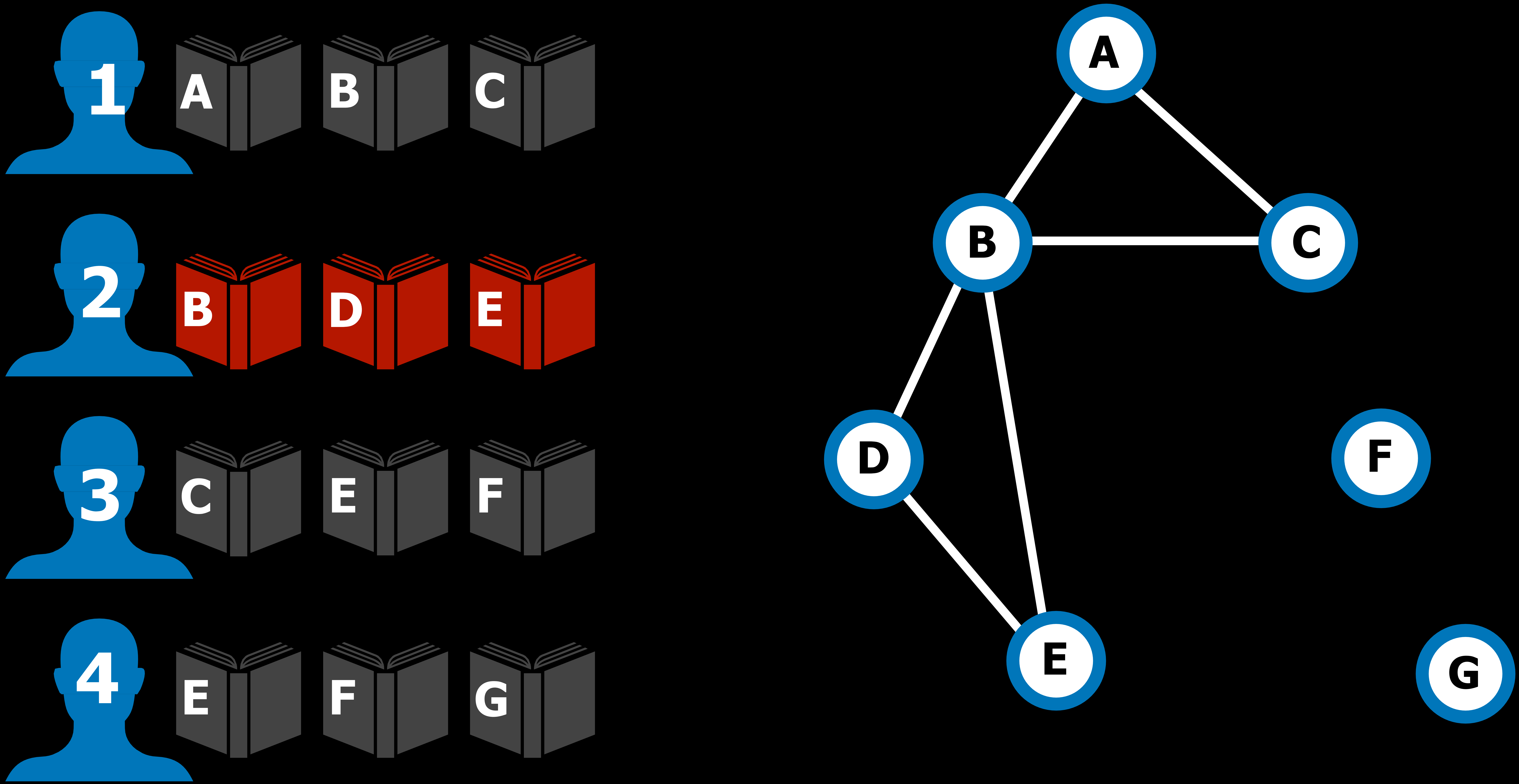
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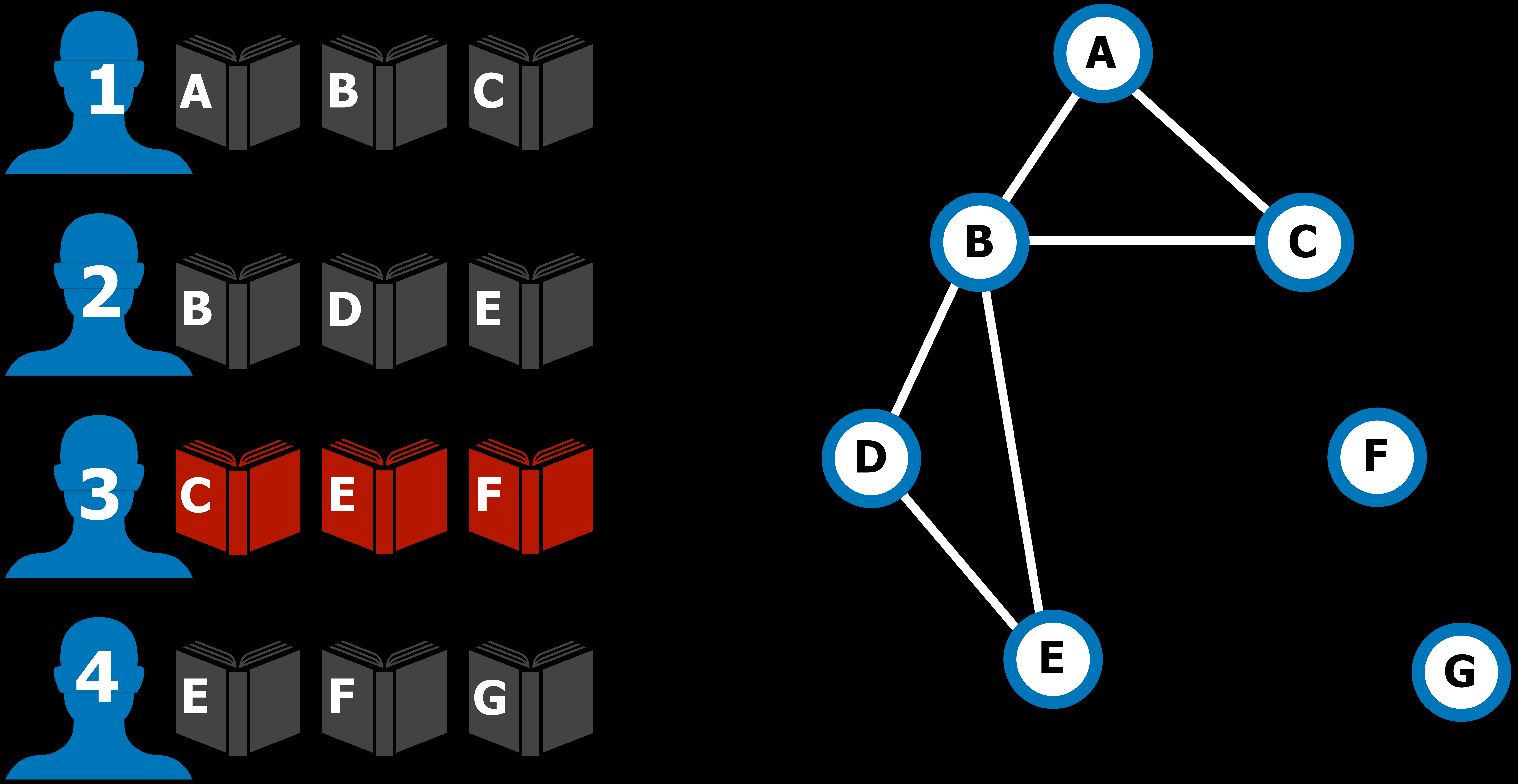
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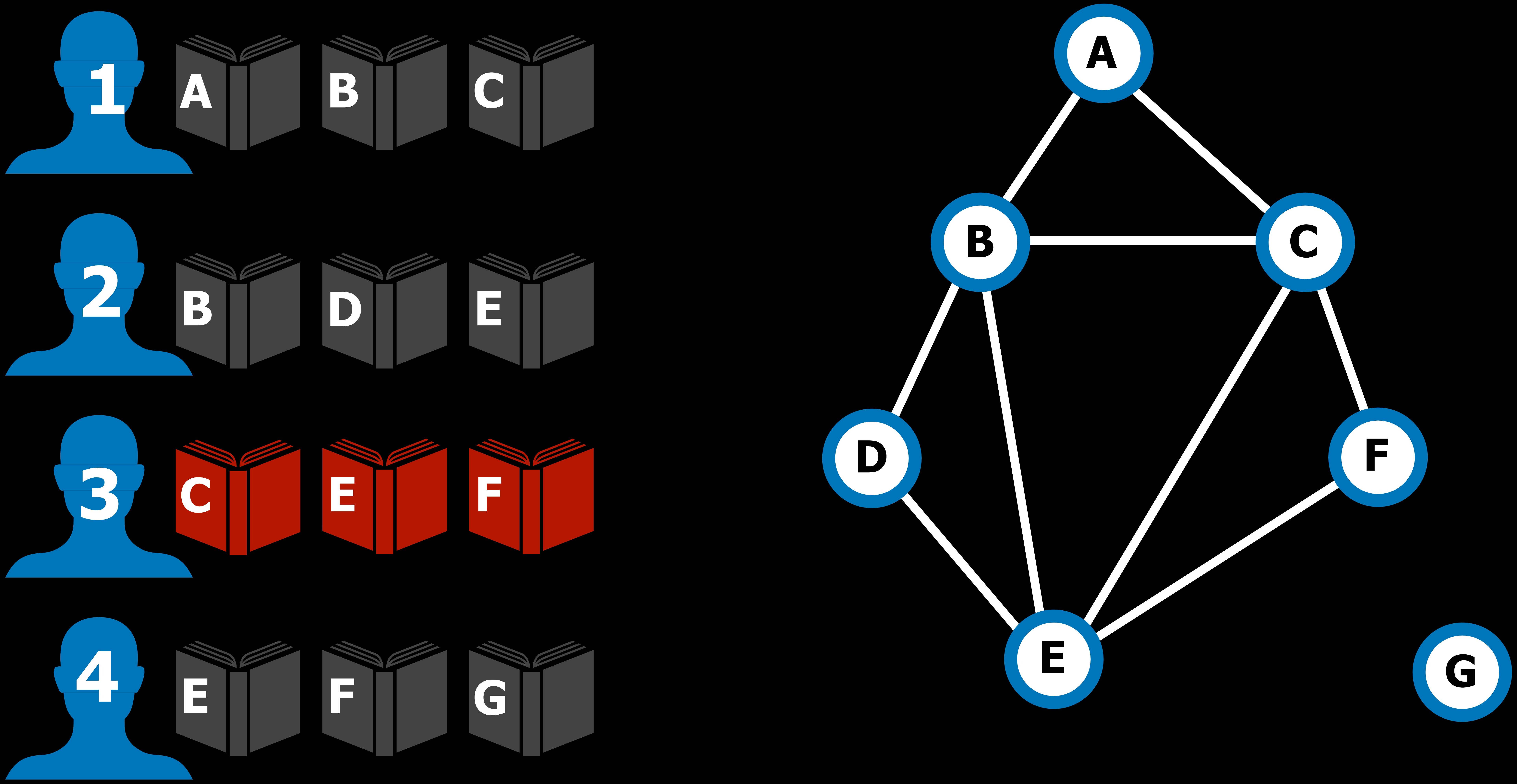
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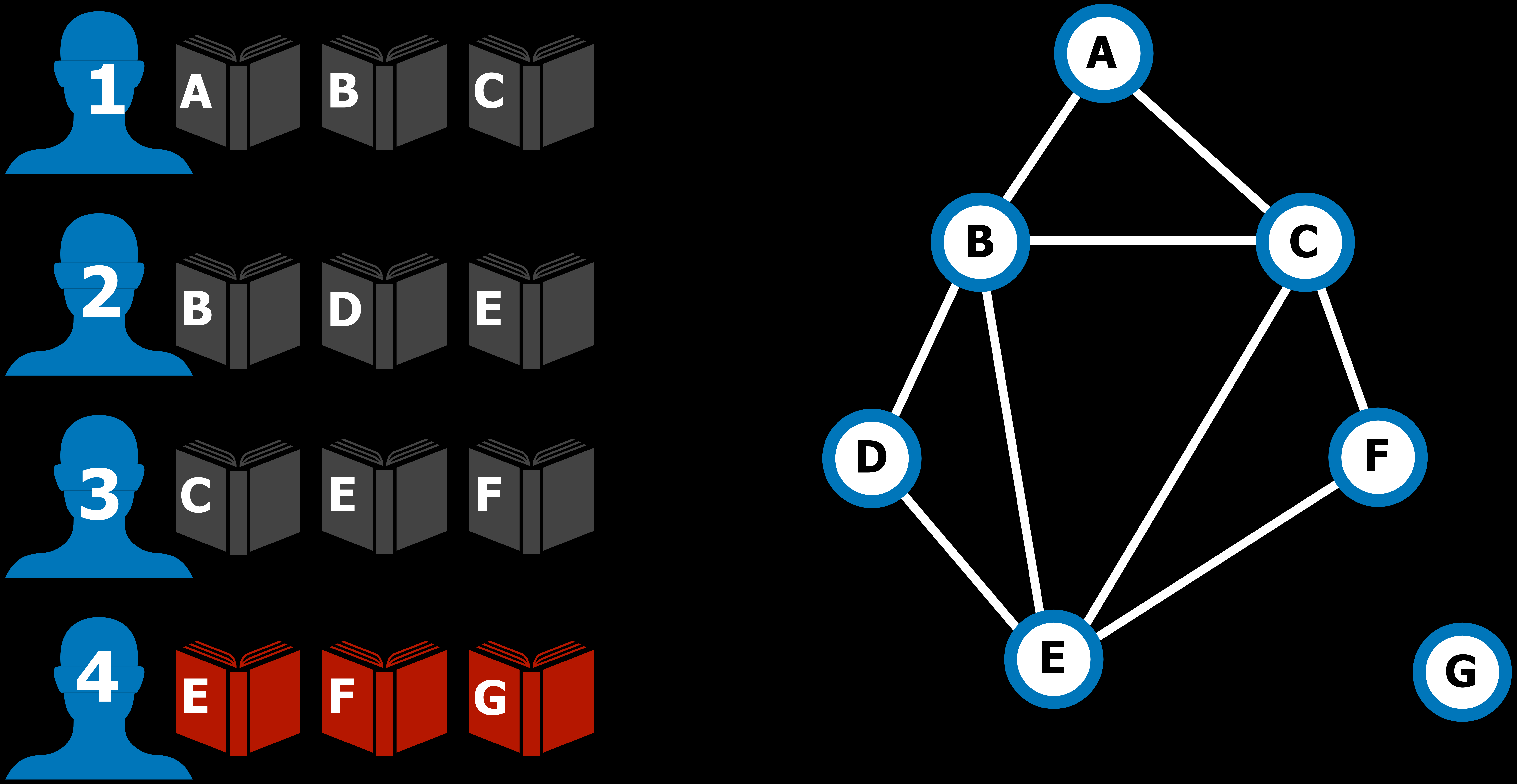
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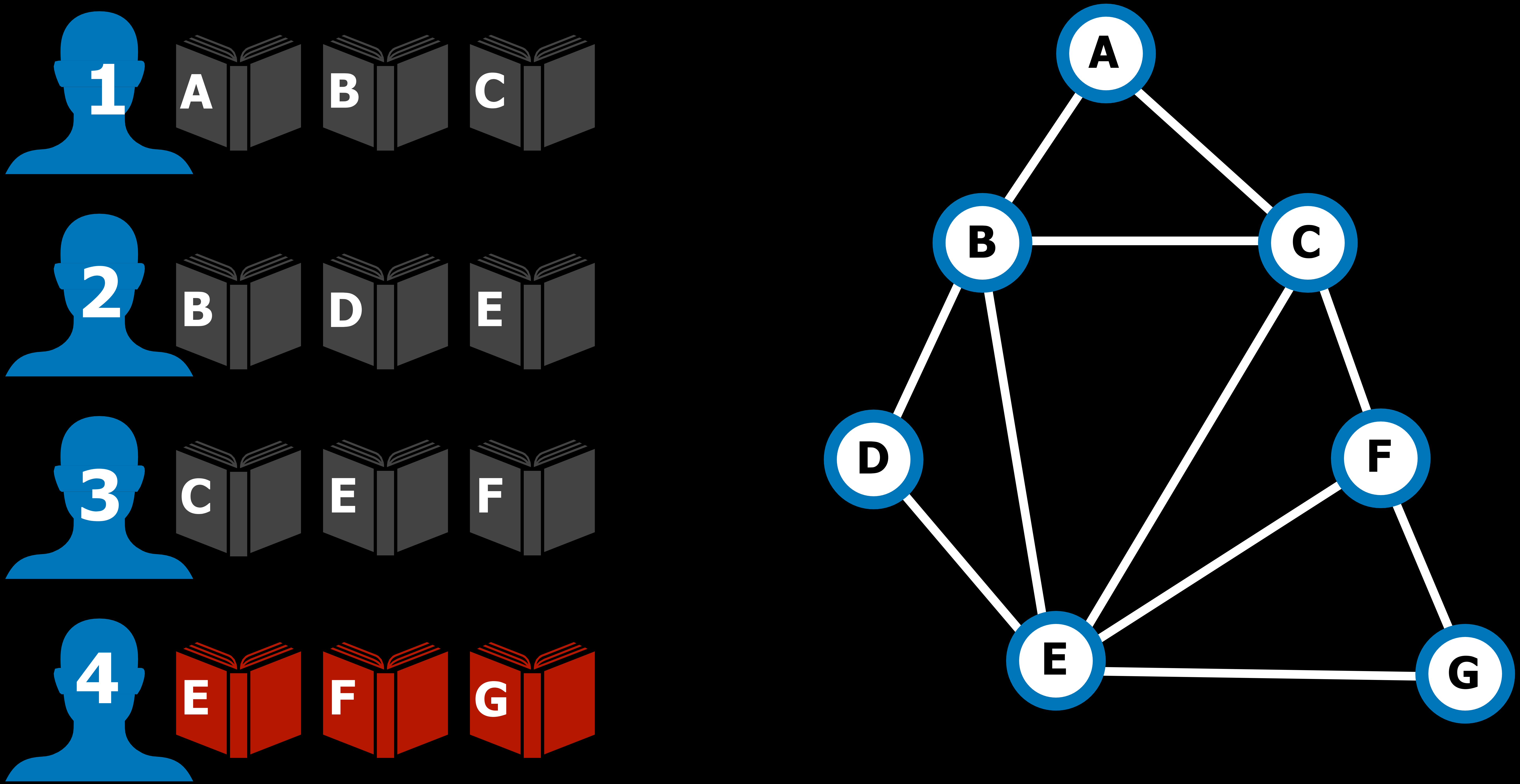
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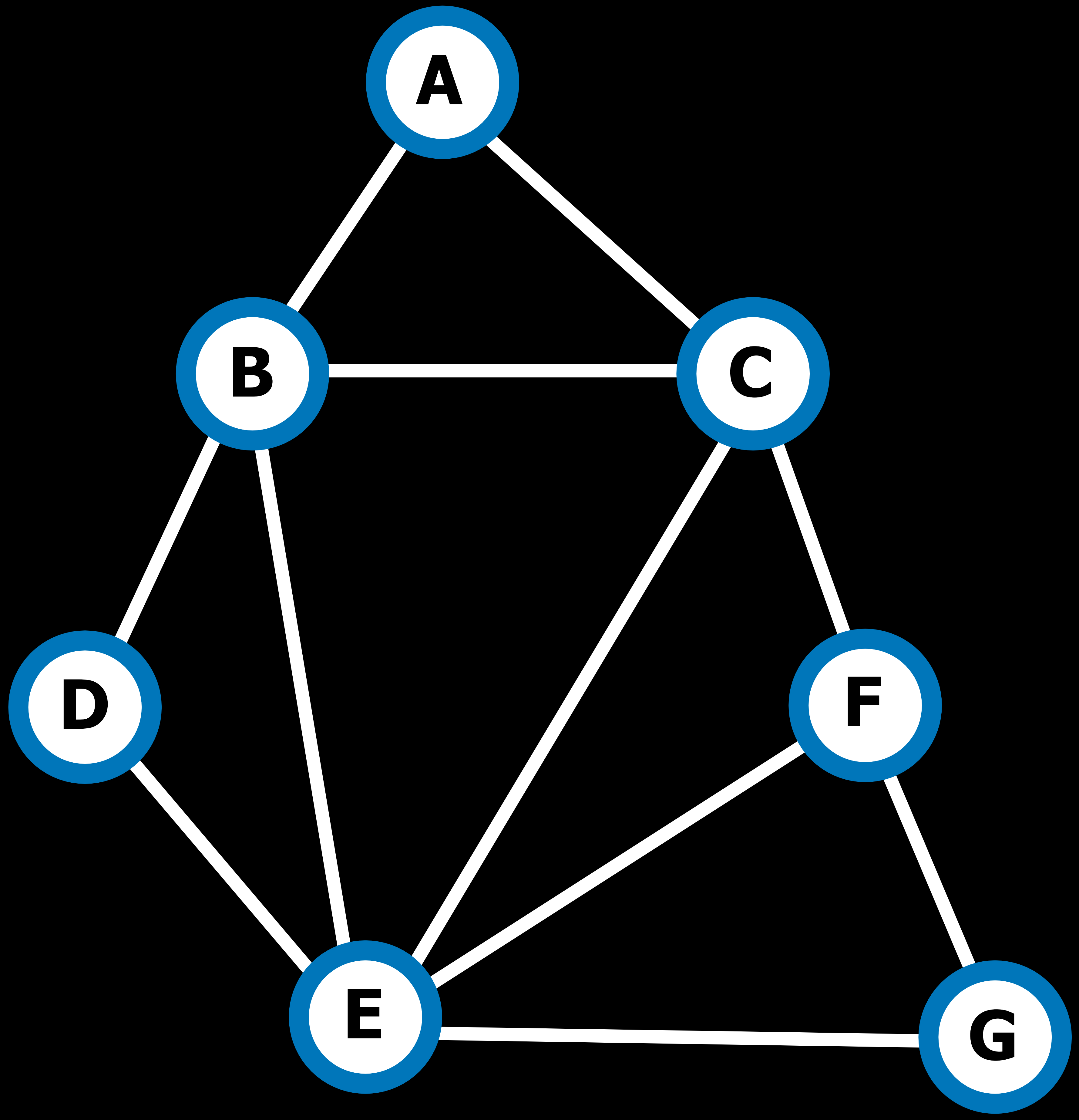
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Constraint Satisfaction Problem

- Set of variables $\{X_1, X_2, ..., X_n\}$
- Set of domains for each variable $\{D_1, D_2, ..., D_n\}$
- Set of constraints C

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Variables

$$\{(0, 2), (1, 1), (1, 2), (2, 0), \ldots\}$$

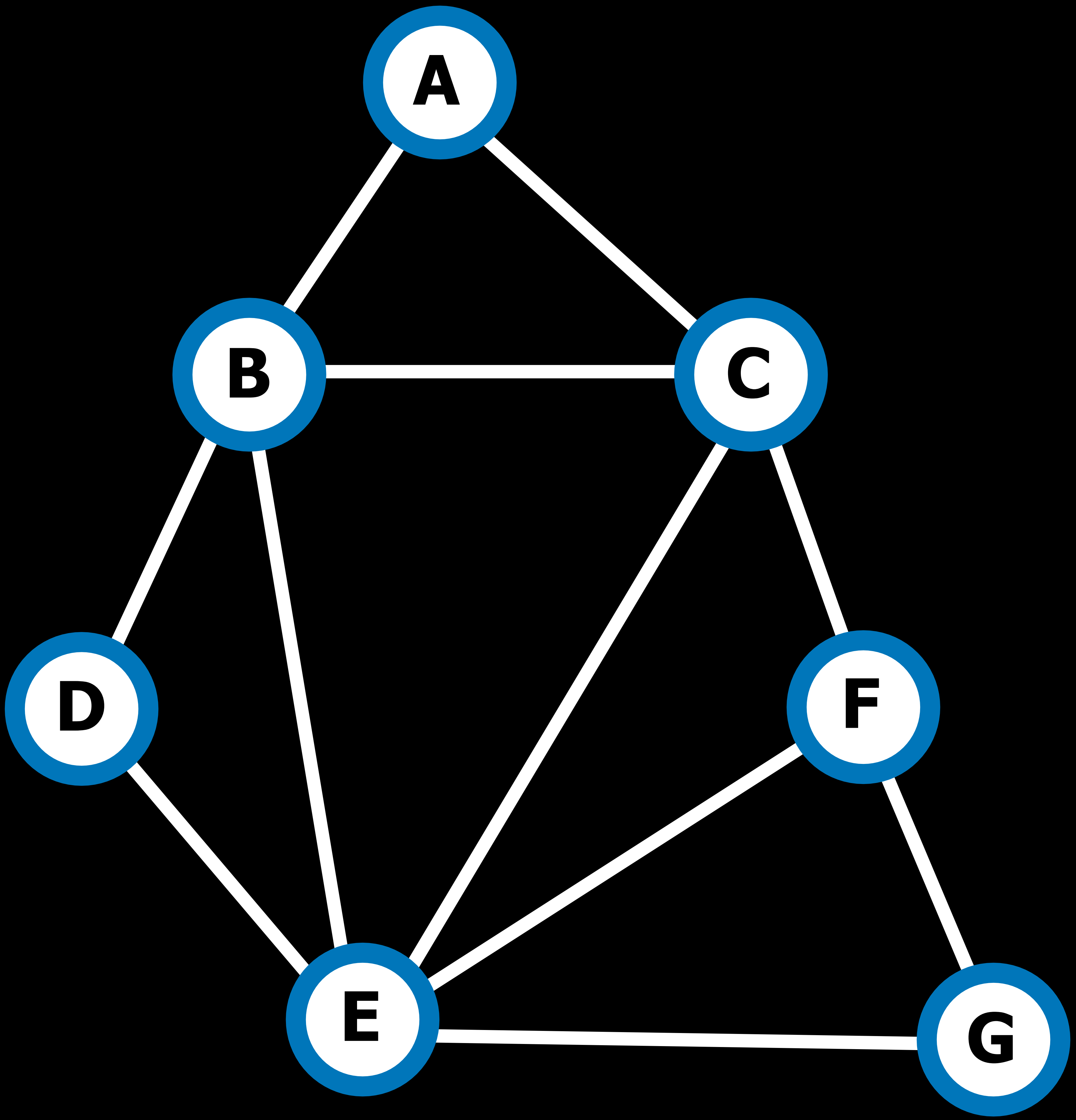
Domains

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

for each variable

Constraints

$$\{(0, 2) \neq (1, 1) \neq (1, 2) \neq (2, 0), \ldots\}$$



Variables

$\{A, B, C, D, E, F, G\}$

Domains

$\{Monday, Tuesday, Wednesday\}$

for each variable

Constraints

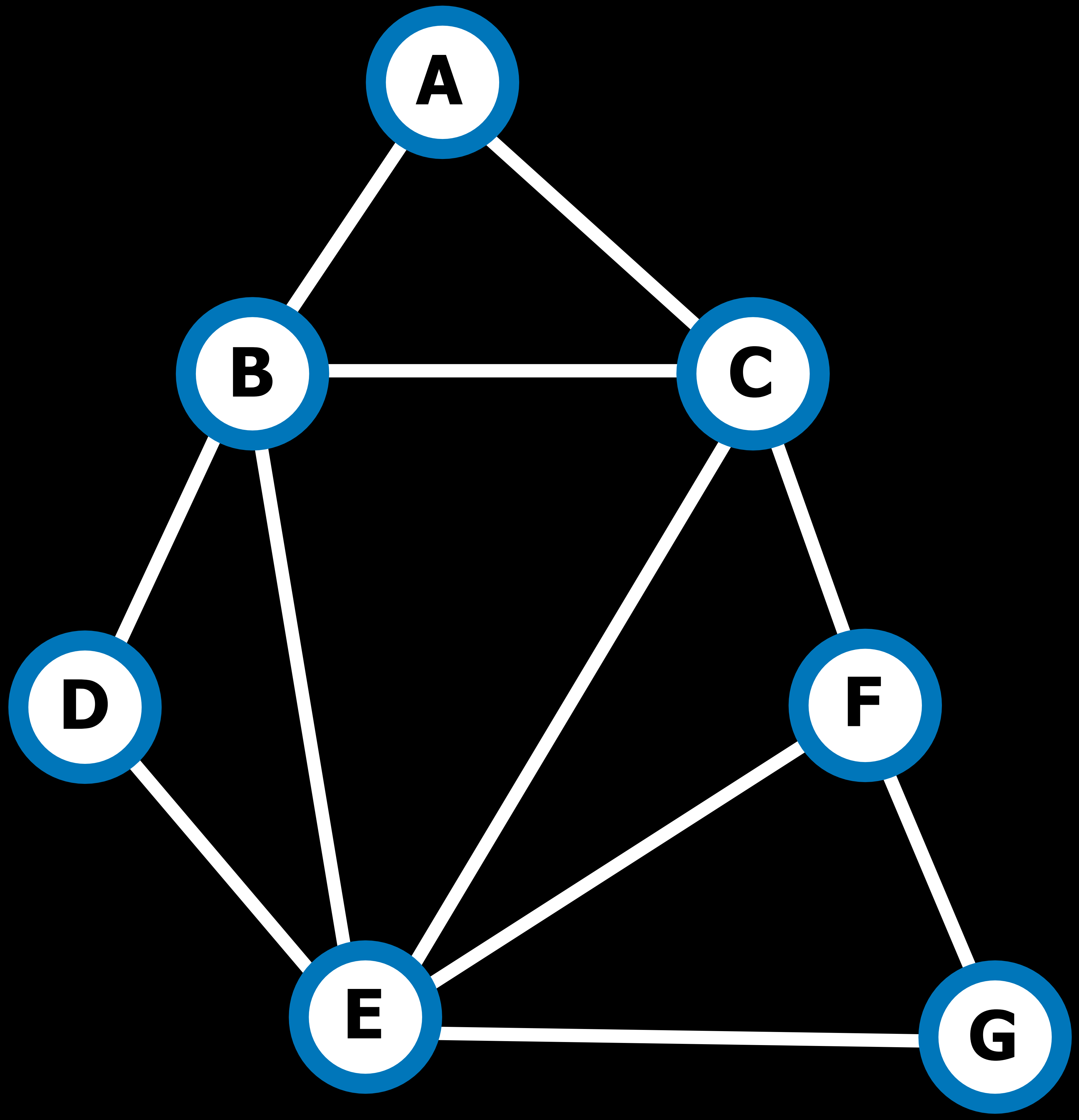
$\{A \neq B, A \neq C, B \neq C, B \neq D, B \neq E, C \neq E, C \neq F, D \neq E, E \neq F, E \neq G, F \neq G\}$

hard constraints

constraints that must be satisfied in a correct solution

soft constraints

constraints that express some notion of which solutions are preferred over others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unary constraint

constraint involving only one variable

unary constraint

$$\{A \neq Monday\}$$

binary constraint

constraint involving two variables

binary constraint

$\{A \neq B\}$

node consistency

when all the values in a variable's domain satisfy the variable's unary constraints

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

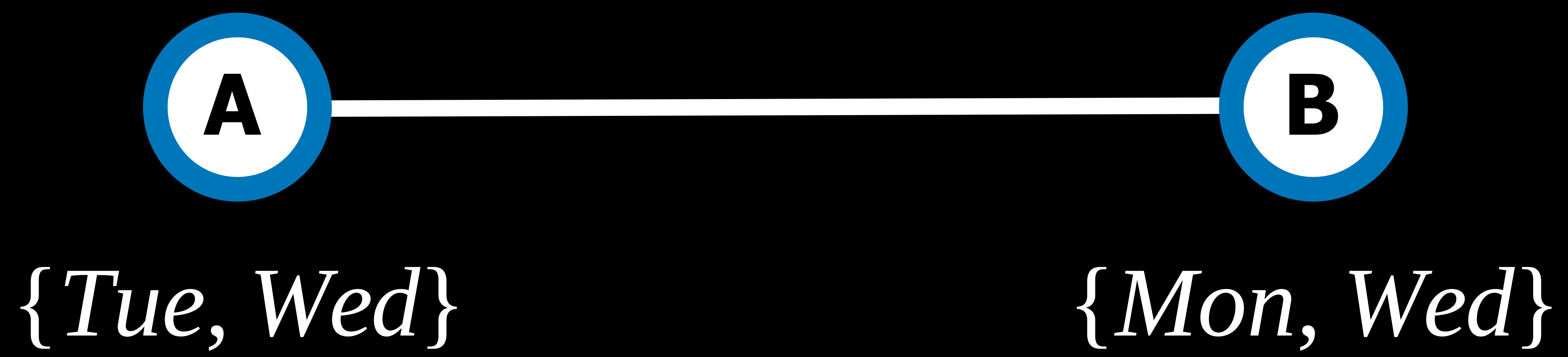
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Tue, Wed\}$

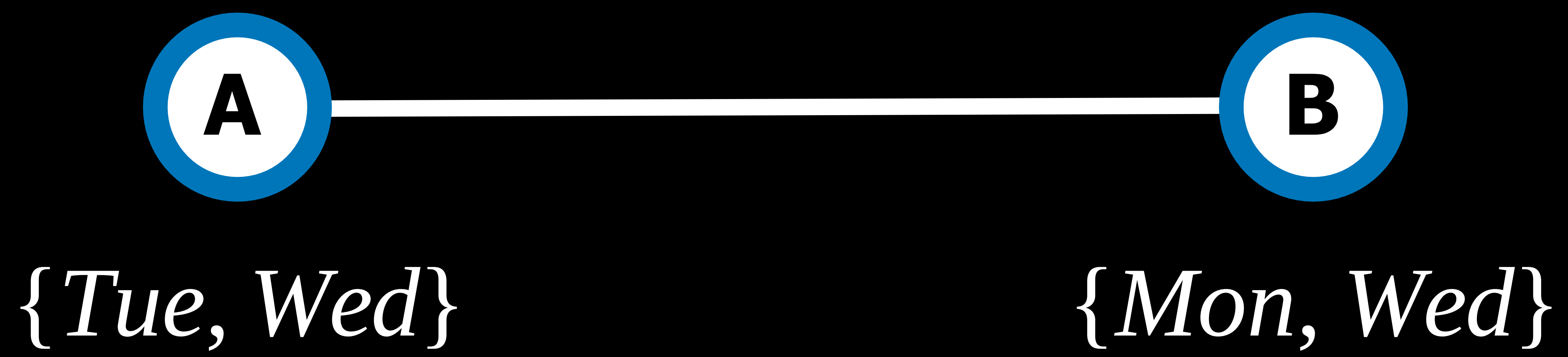
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Tue, Wed\}$

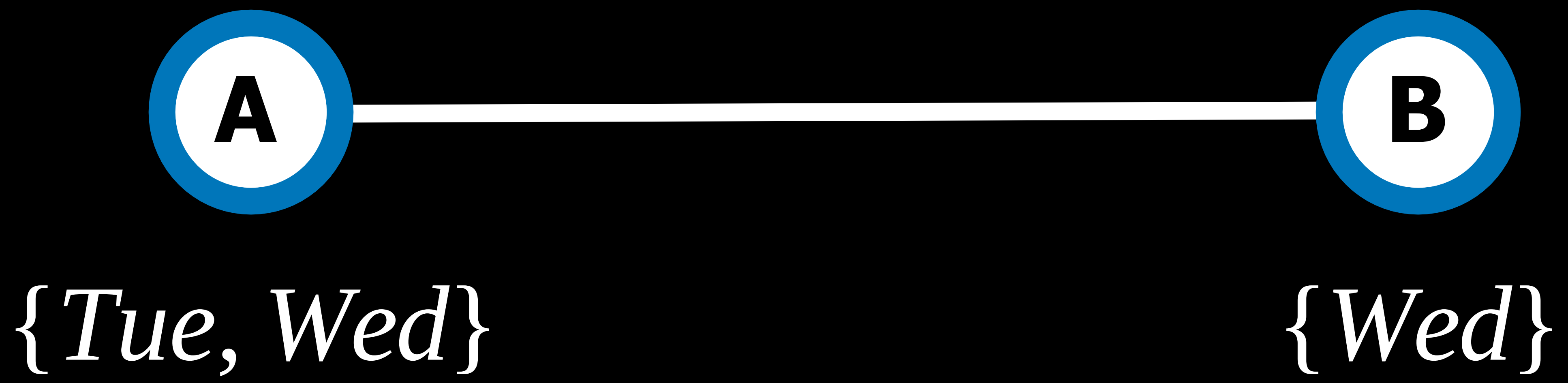
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



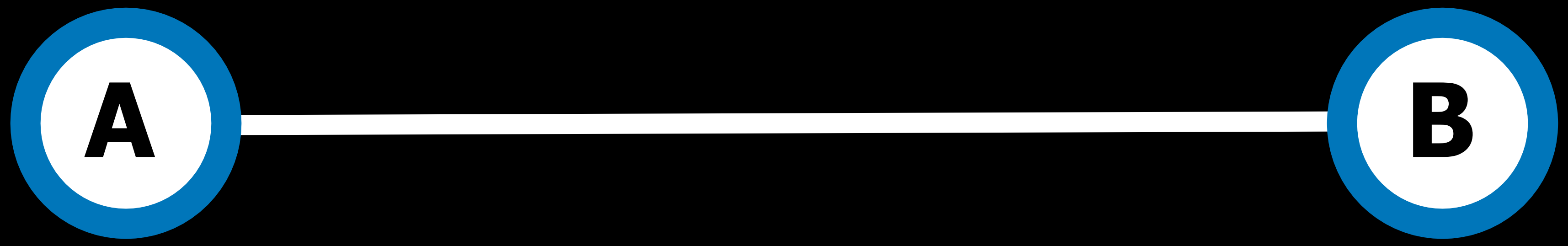
$$\{A \neq Mon,\ B \neq Tue,\ B \neq Mon,\ A \neq B\}$$



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon,\ B \neq Tue,\ B \neq Mon,\ A \neq B\}$$



$\{Tue, Wed\}$ $\{Wed\}$

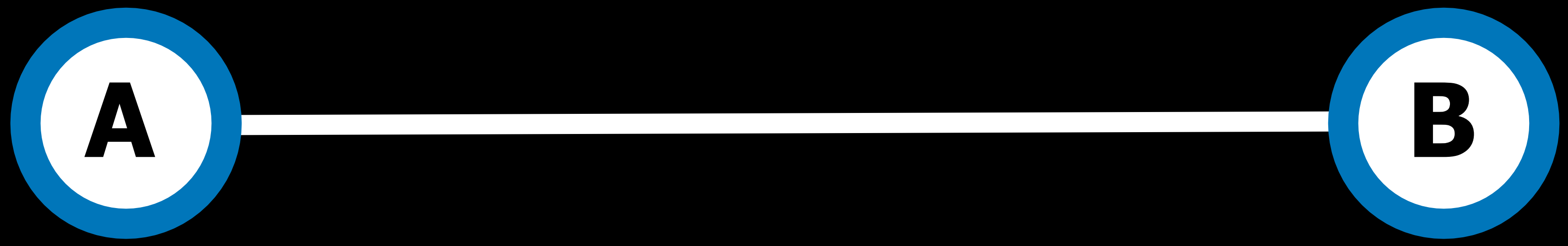
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

arc consistency

when all the values in a variable's domain satisfy the variable's binary constraints

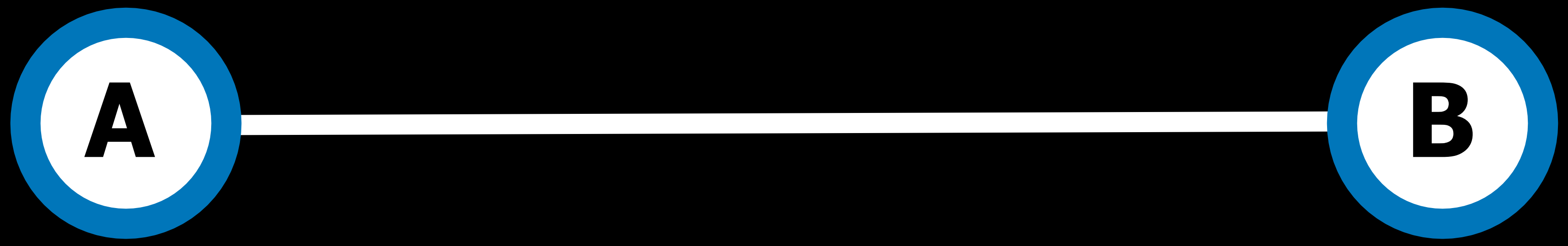
arc consistency

To make X arc-consistent with respect to Y, remove elements from X's domain until every choice for X has a possible choice for Y



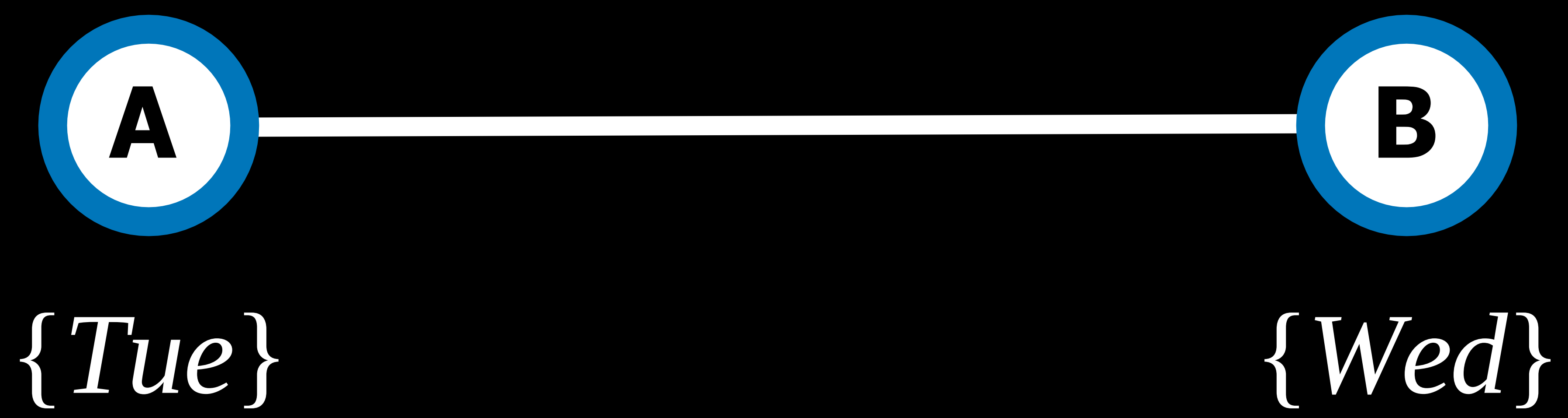
$\{Tue, Wed\}$ $\{Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

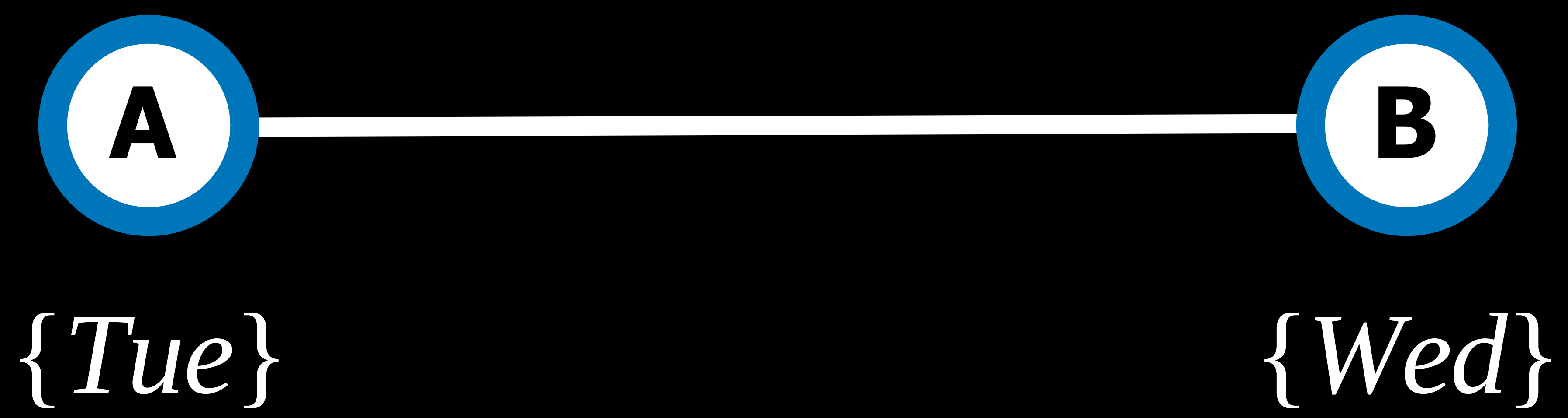


$\{Tue, Wed\}$ $\{Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



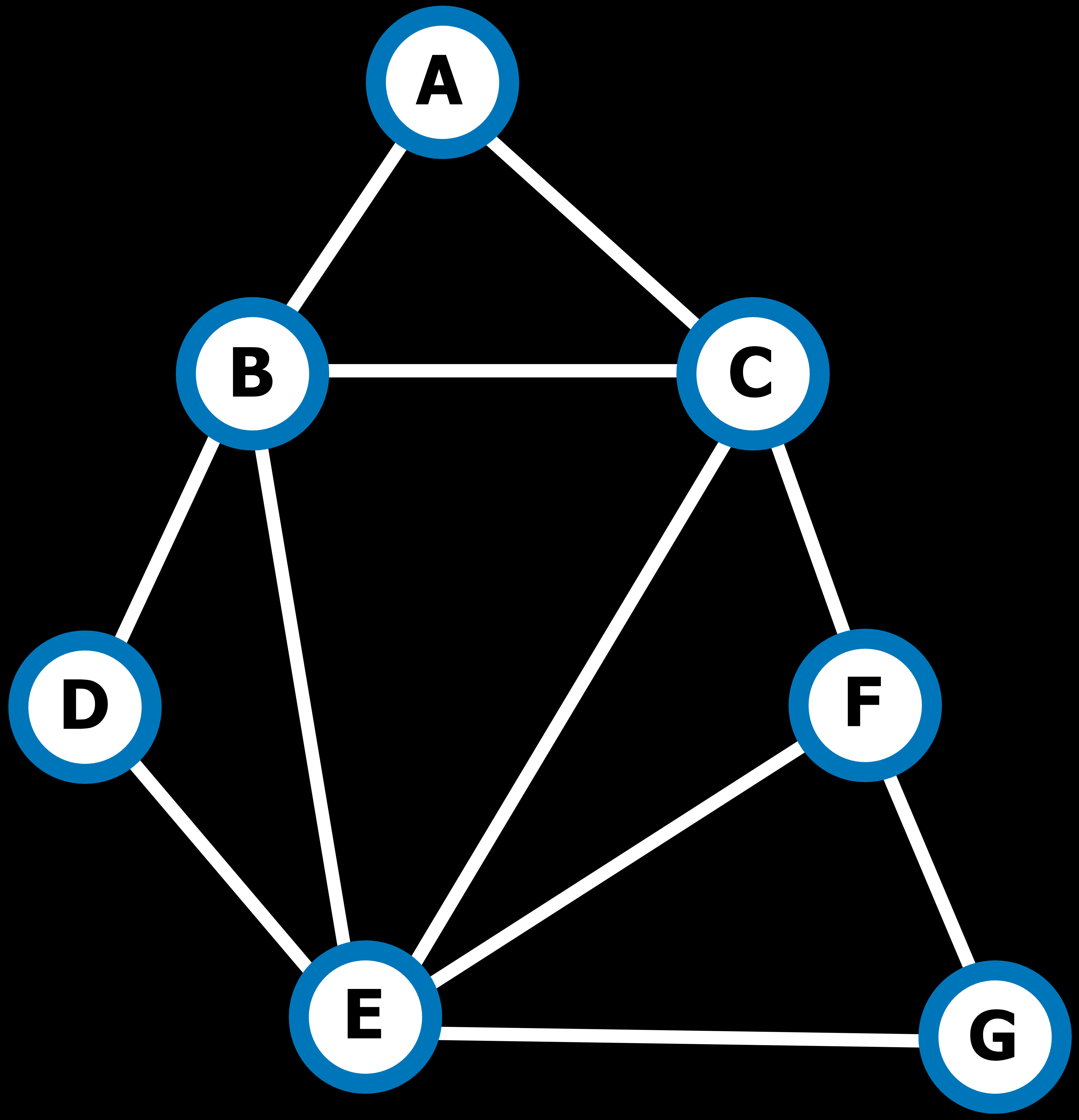
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

Arc Consistency

```
function REVISE(csp, X, Y):
    revised = false
    for x in X.domain:
        if no y in Y.domain satisfies constraint for (X, Y):
            delete x from X.domain
            revised = true
    return revised
```

Arc Consistency

```
function AC-3(csp):
    queue = all arcs in csp
    while queue non-empty:
        (X, Y) = DEQUEUE(queue)
        if REVISE(csp, X, Y):
            if size of X.domain == 0:
                return false
            for each Z in X.neighbors - {Y}:
                ENQUEUE(queue, (Z, X))
    return true
```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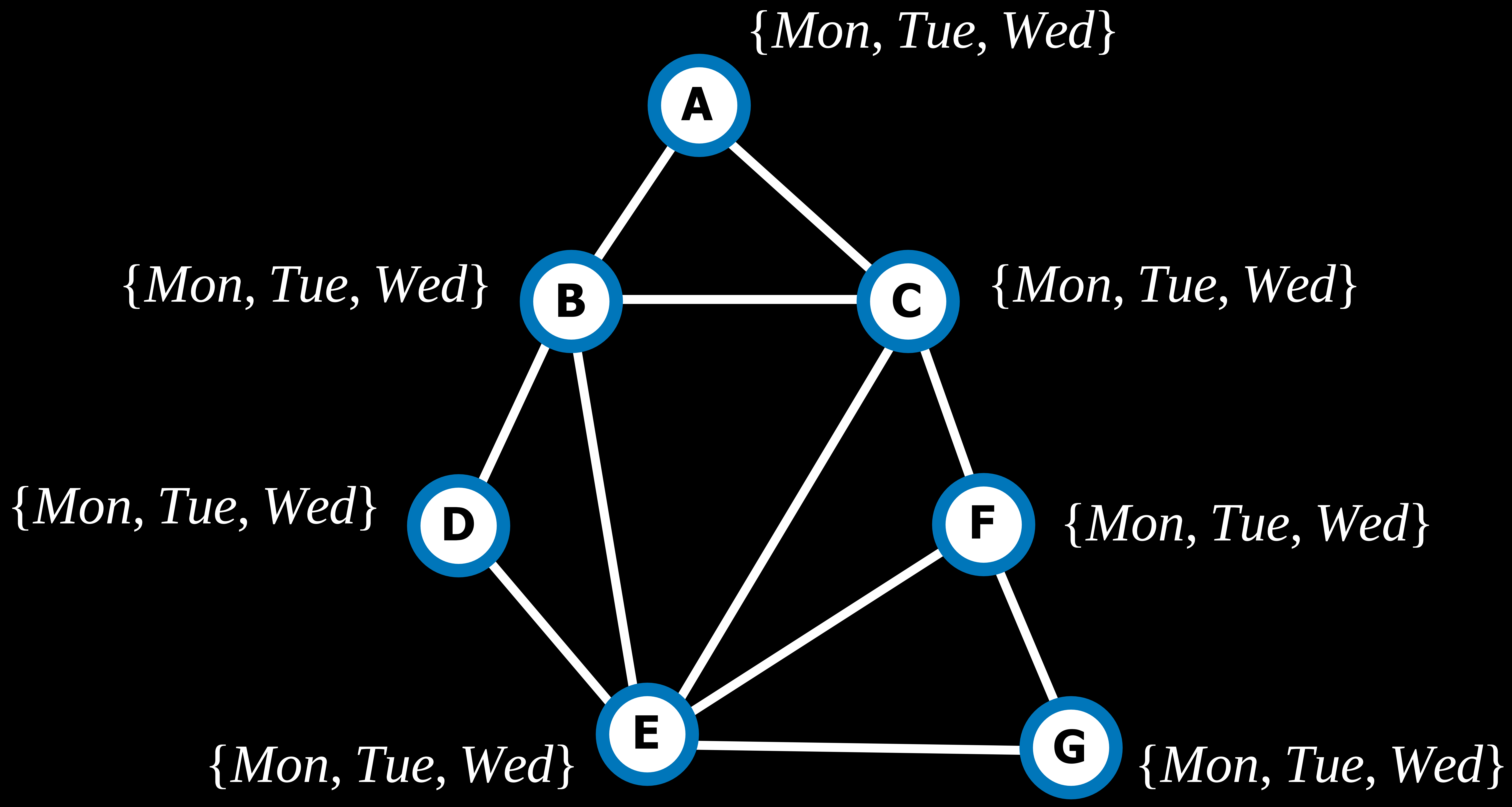
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Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

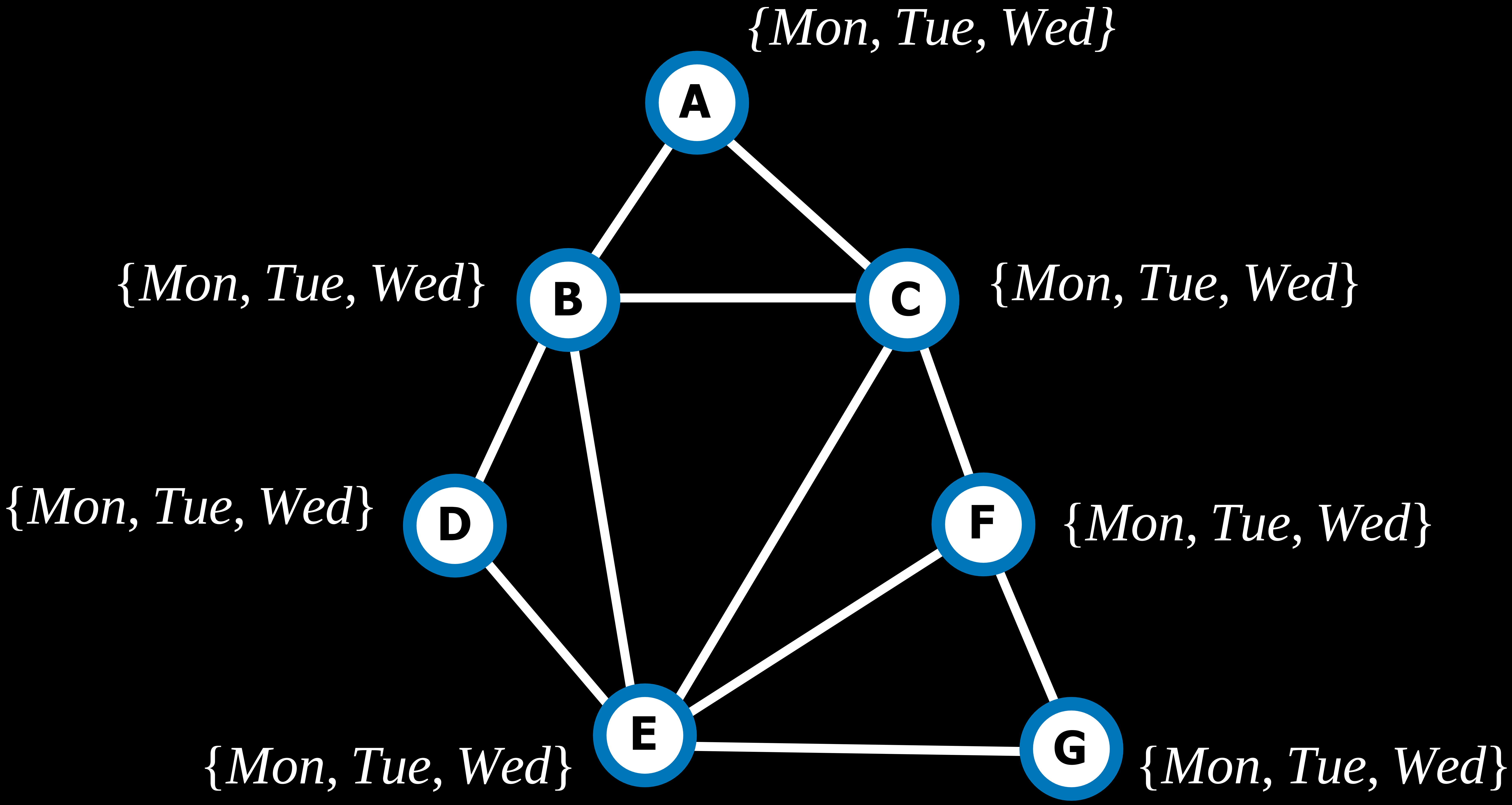
CSPs as Search Problems

- initial state: empty assignment (no variables)
- actions: add a $\{variable = value\}$ to assignment
- transition model: shows how adding an assignment changes the assignment
- goal test: check if all variables assigned and constraints all satisfied
- path cost function: all paths have same cost

Backtracking Search

Backtracking Search

function BACKTRACK(*assignment*, *csp*):
    if *assignment* complete: return *assignment*
    var = SELECT-UNASSIGNED-VAR(*assignment*, *csp*)
    for *value* in DOMAIN-VALUES(*var*, *assignment*, *csp*):
        if *value* consistent with *assignment*:
            add {*var* = *value*} to *assignment*
            result = BACKTRACK(*assignment*, *csp*)
            if *result* ≠ *failure*: return *result*
        remove {*var* = *value*} from *assignment*
    return *failure*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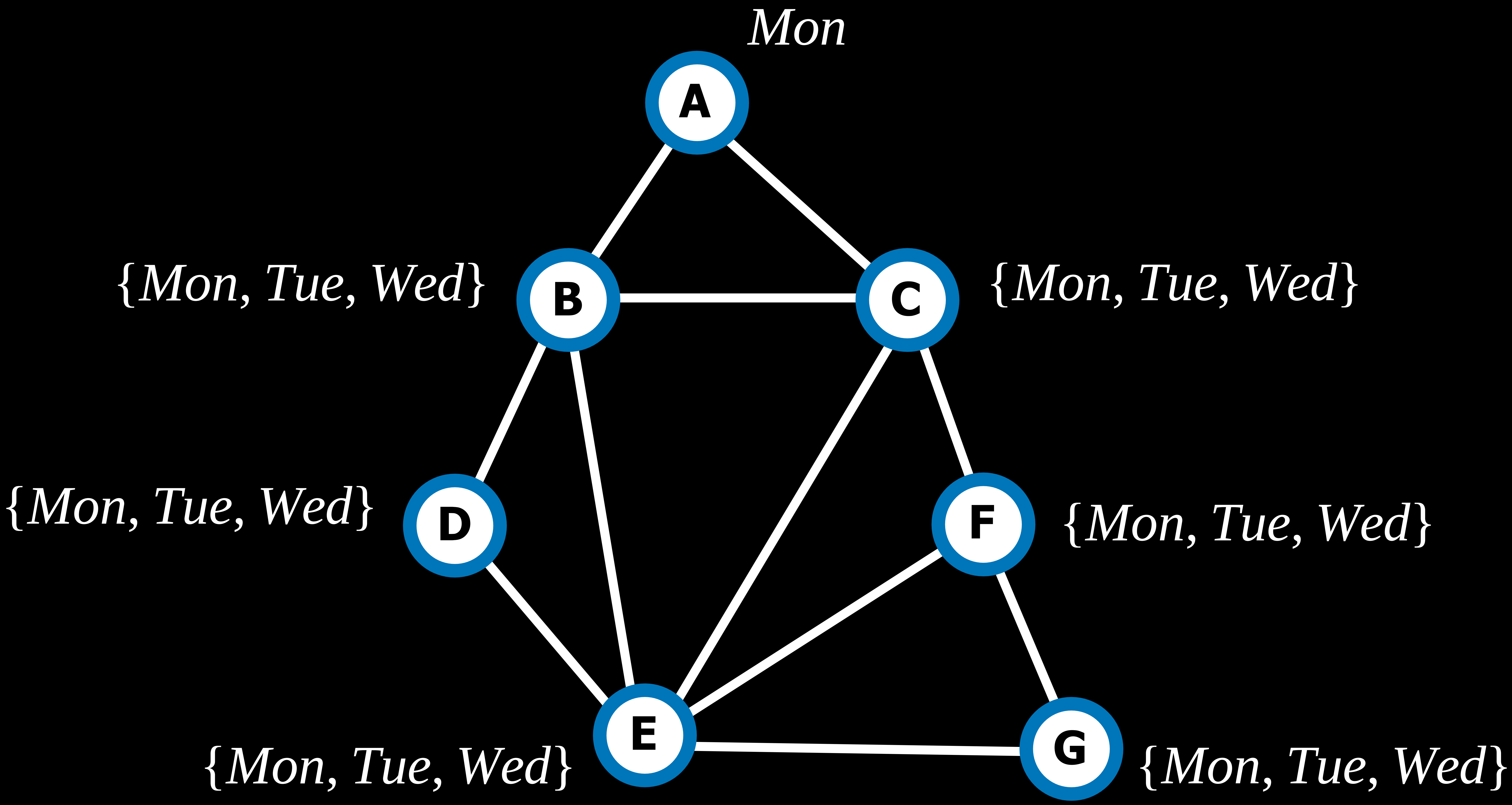
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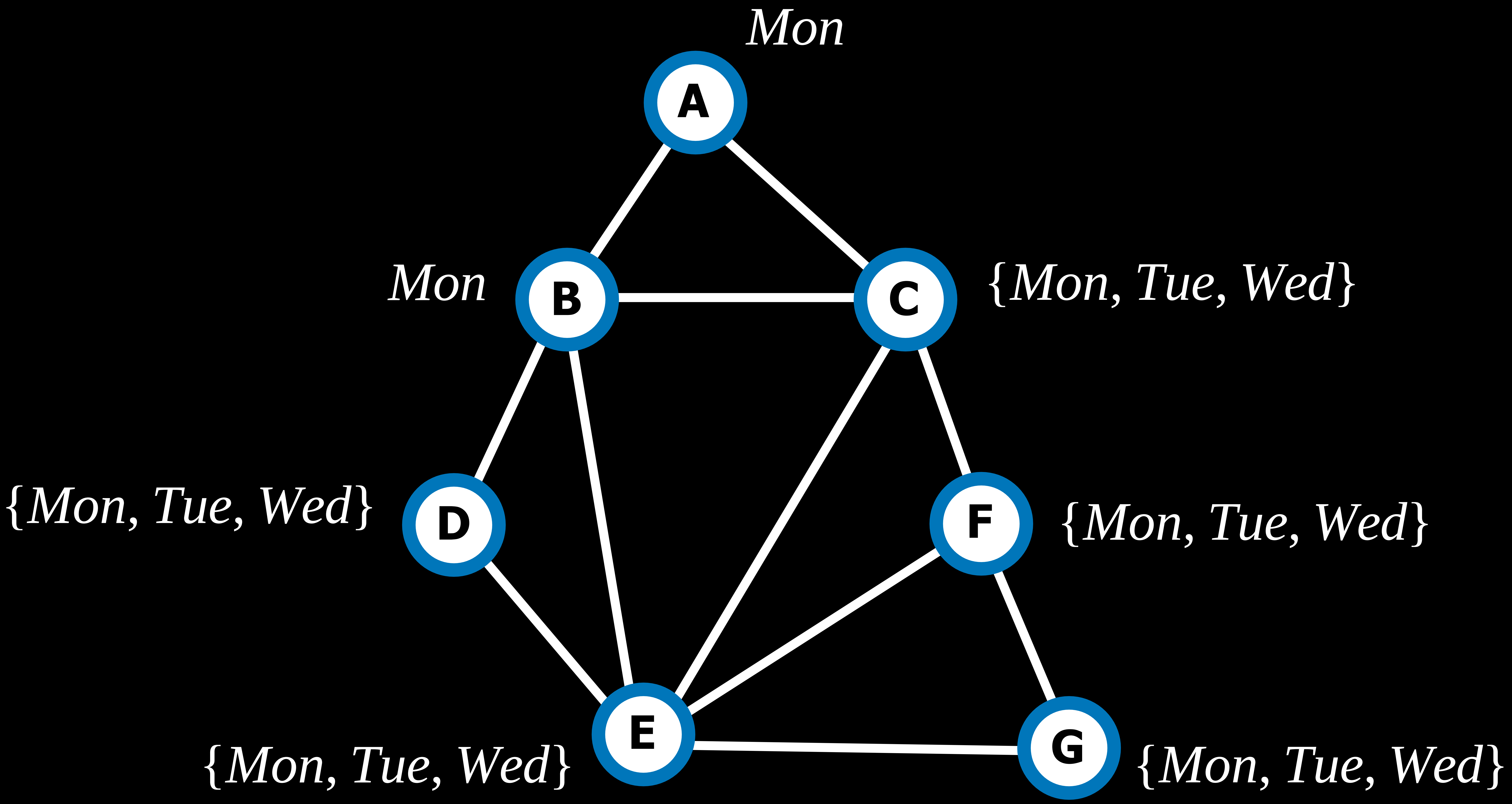
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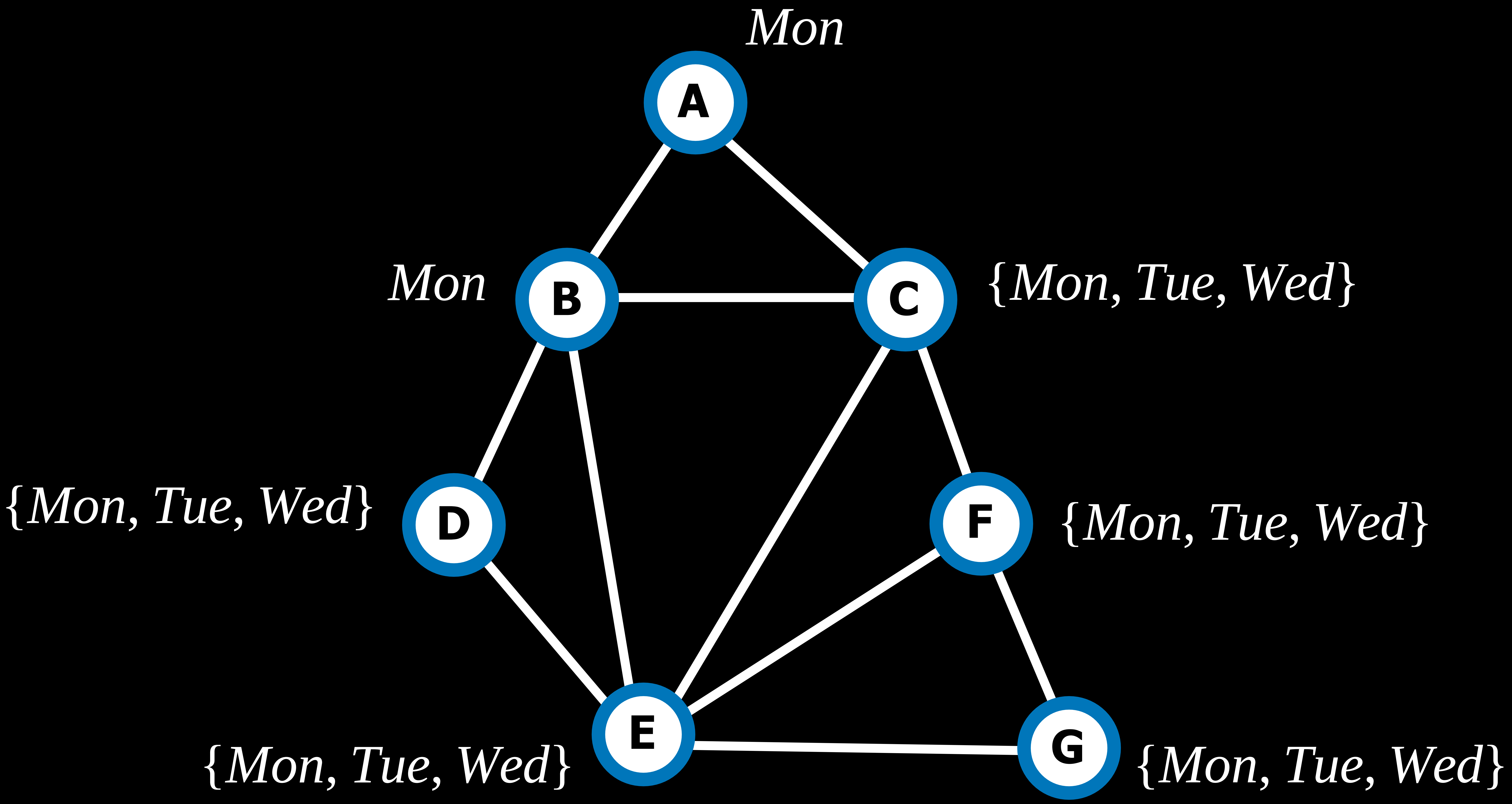
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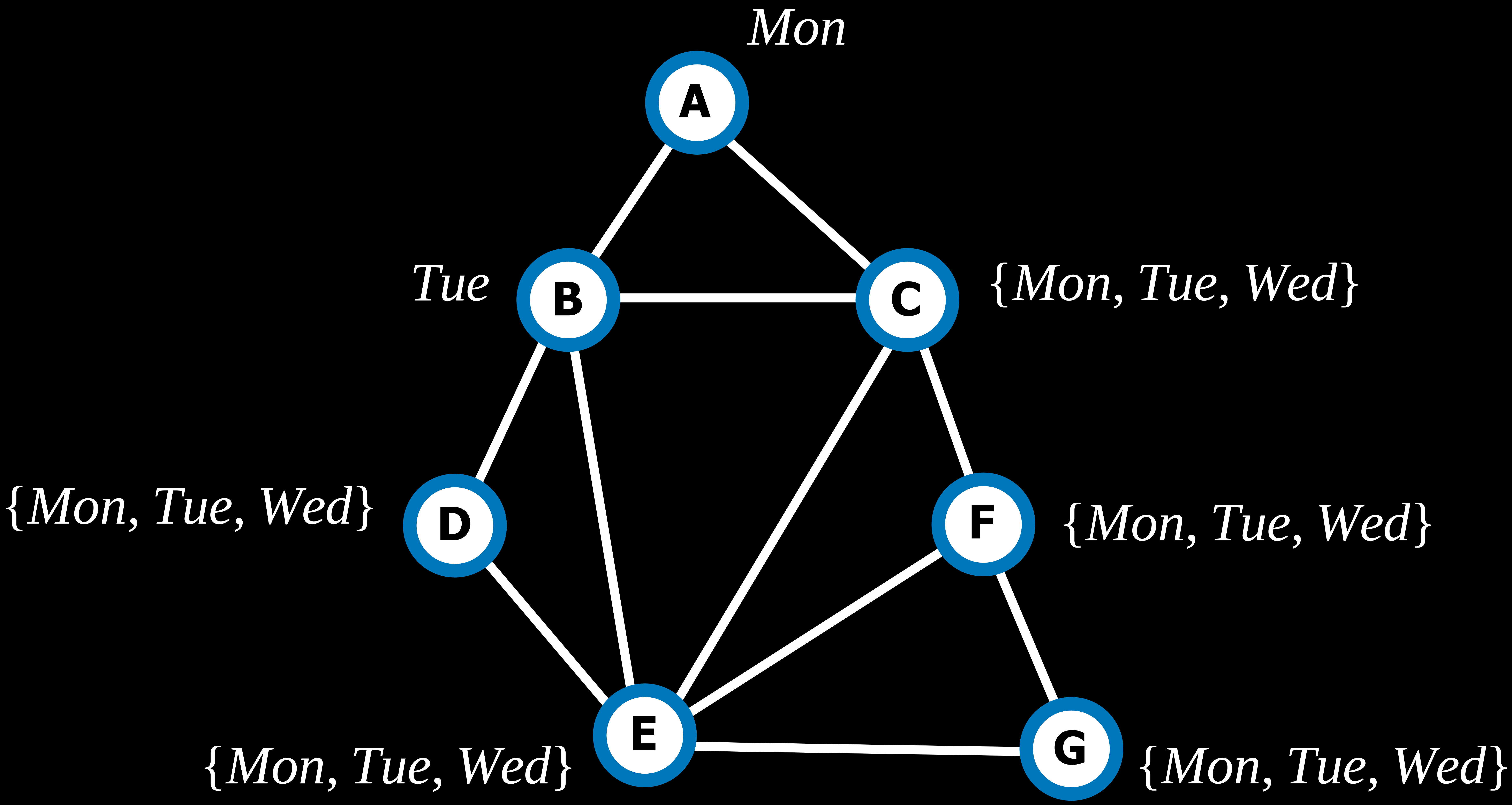
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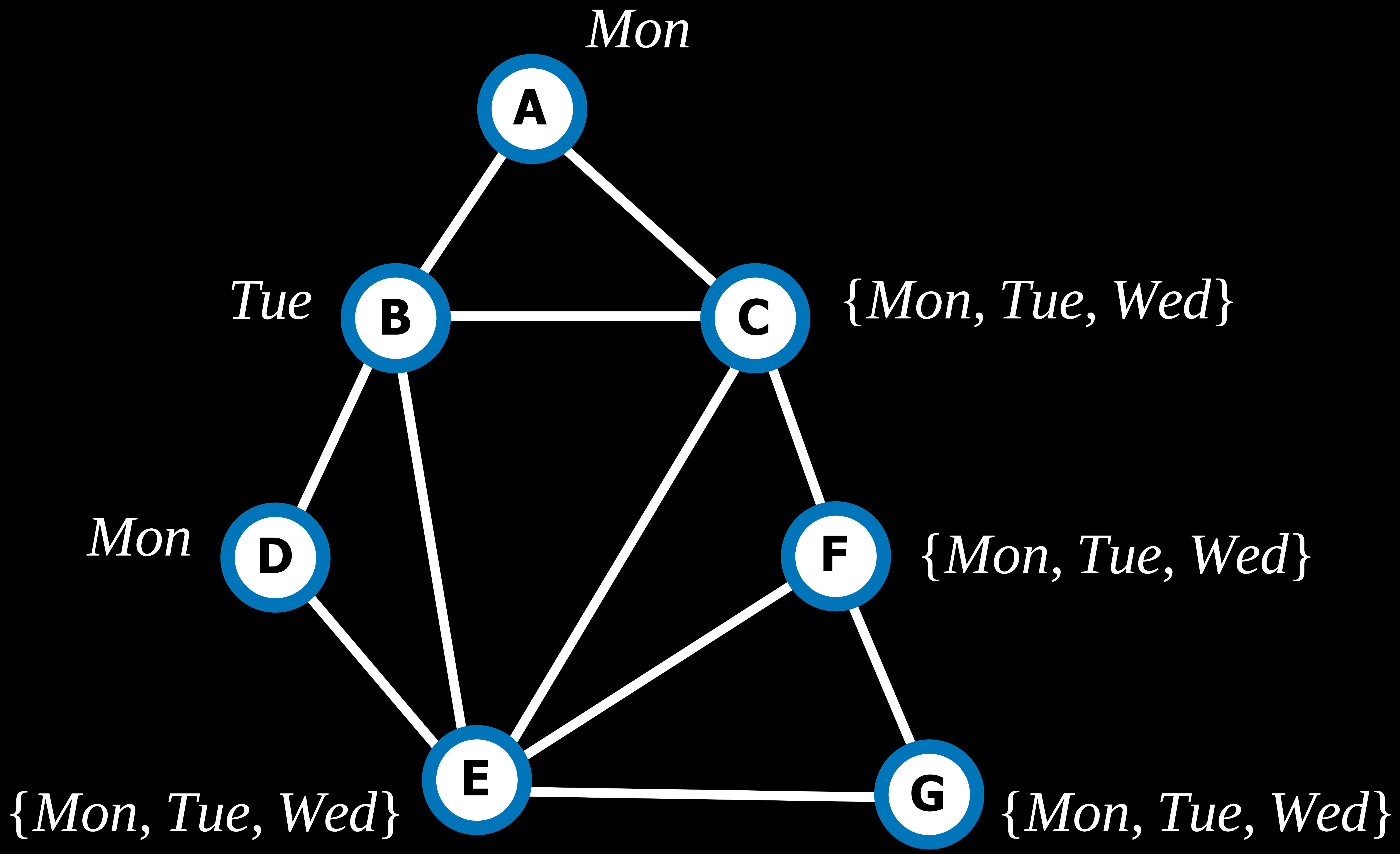
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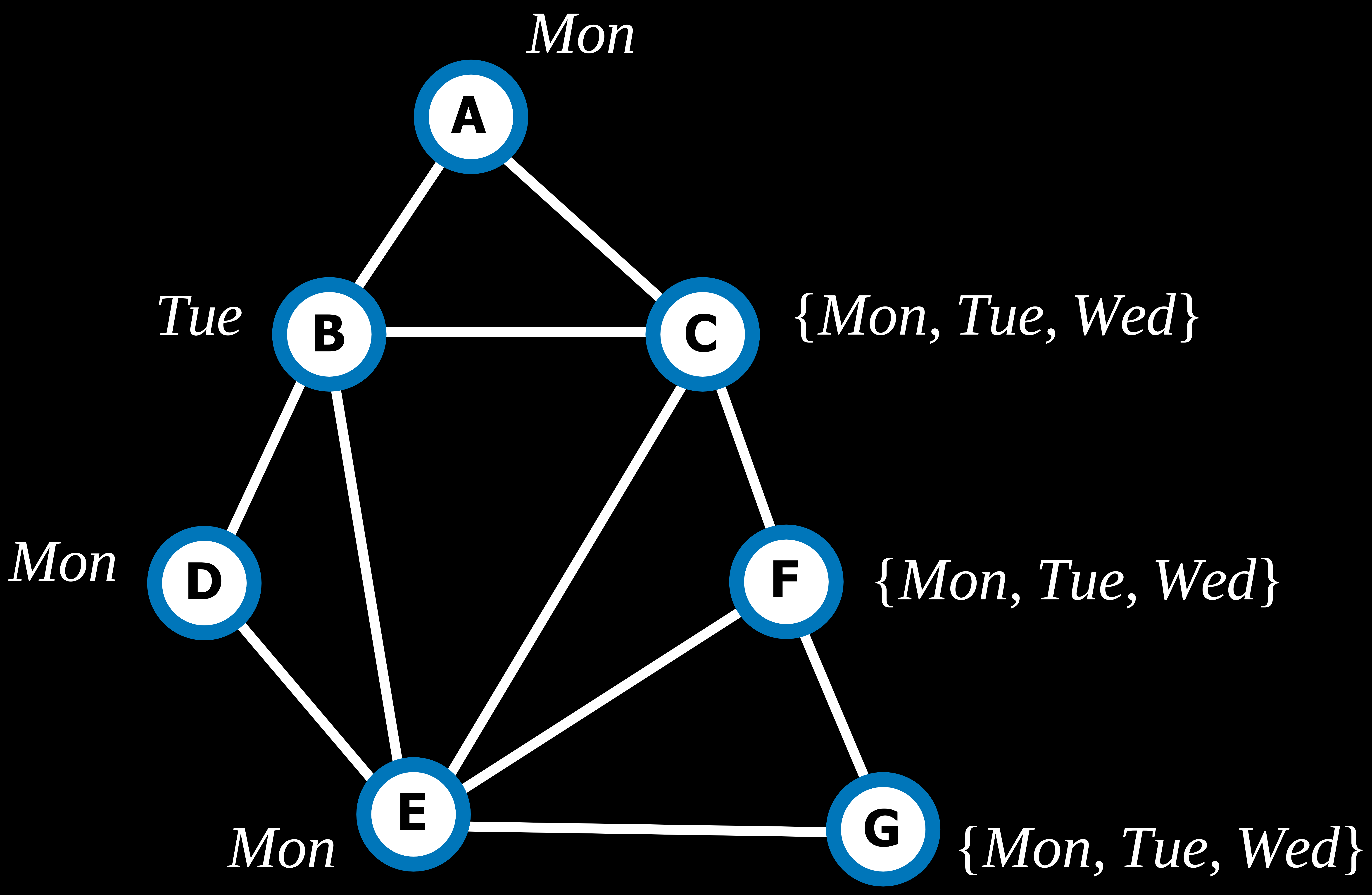
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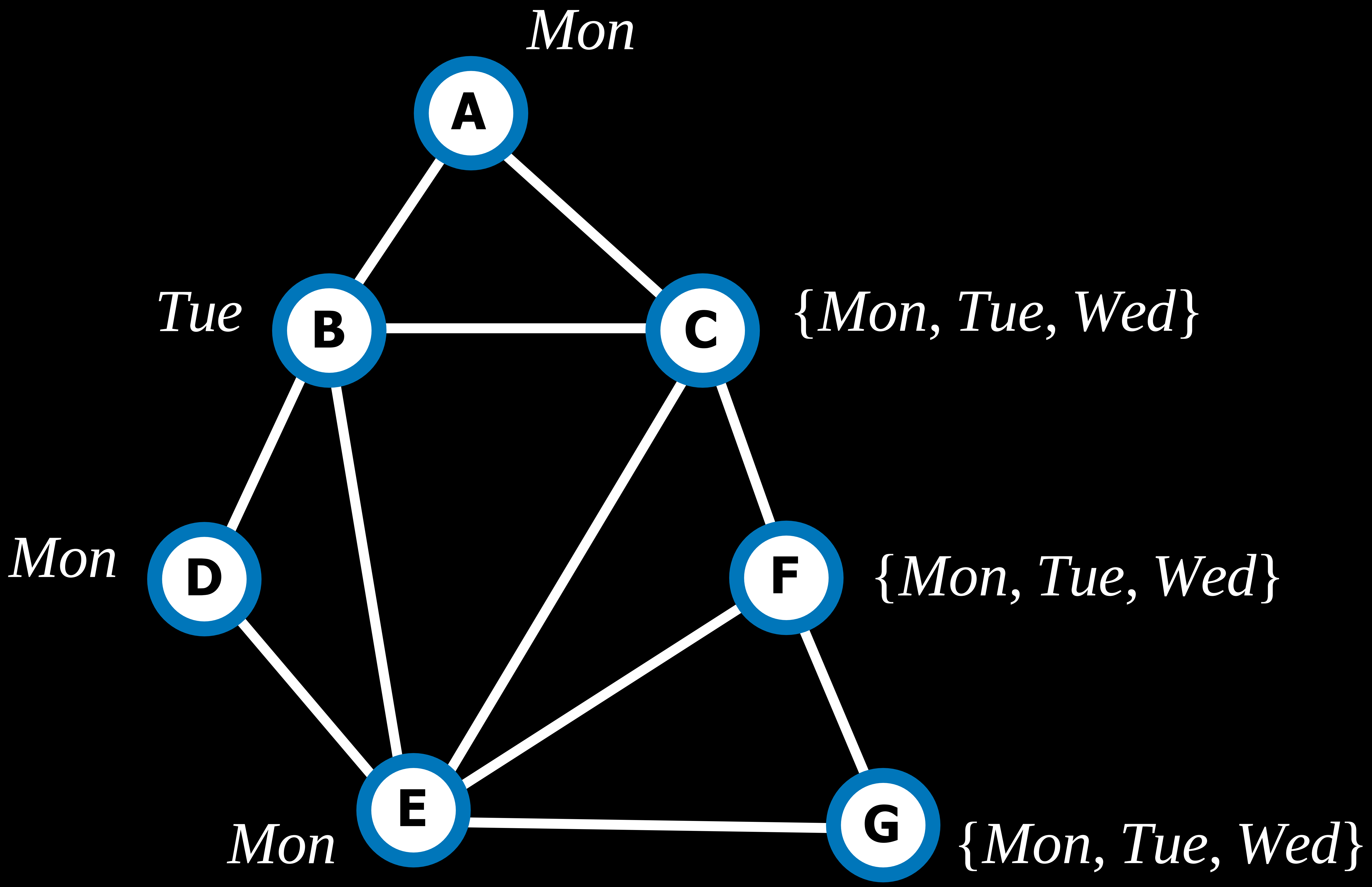
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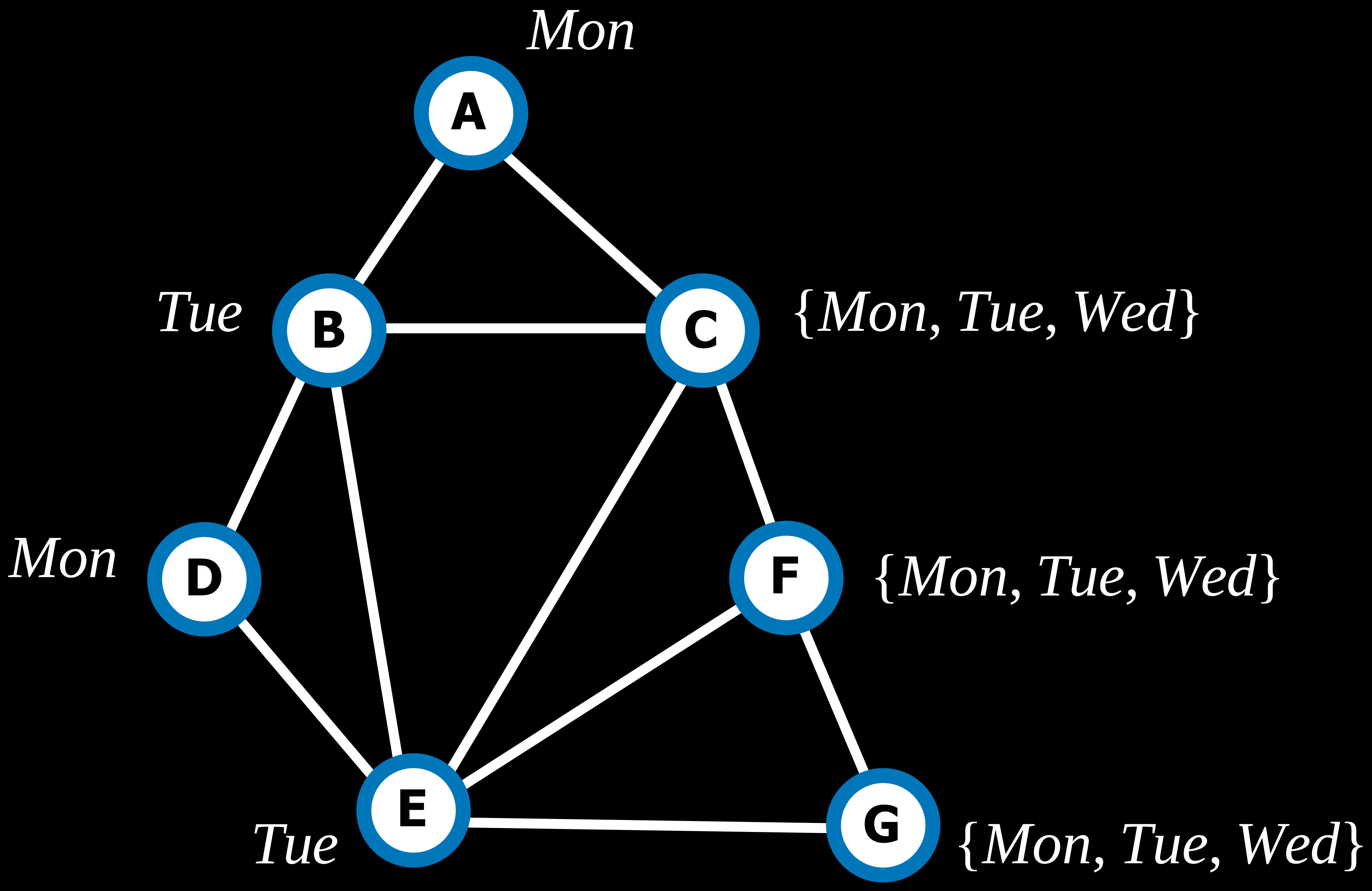
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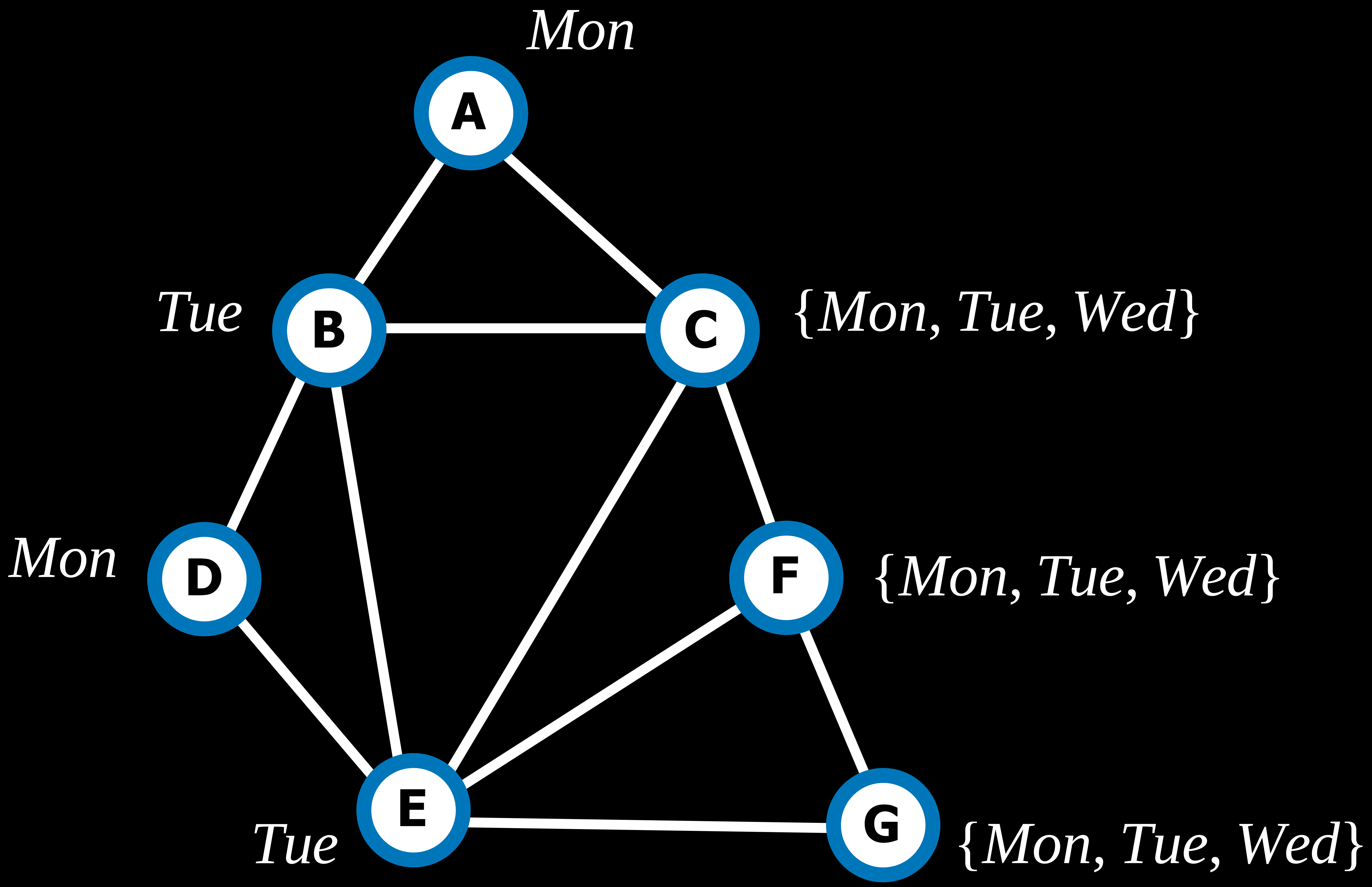
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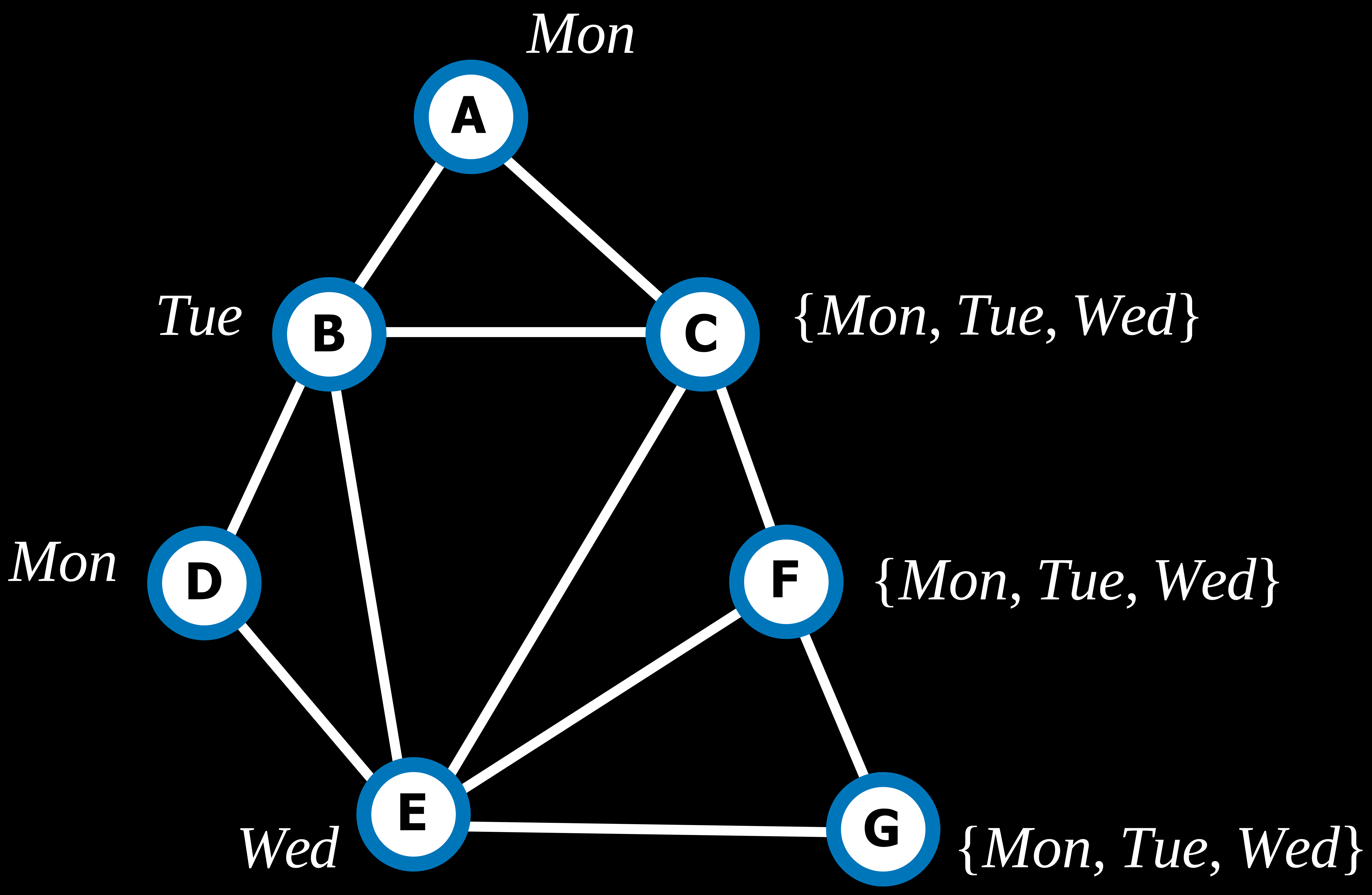
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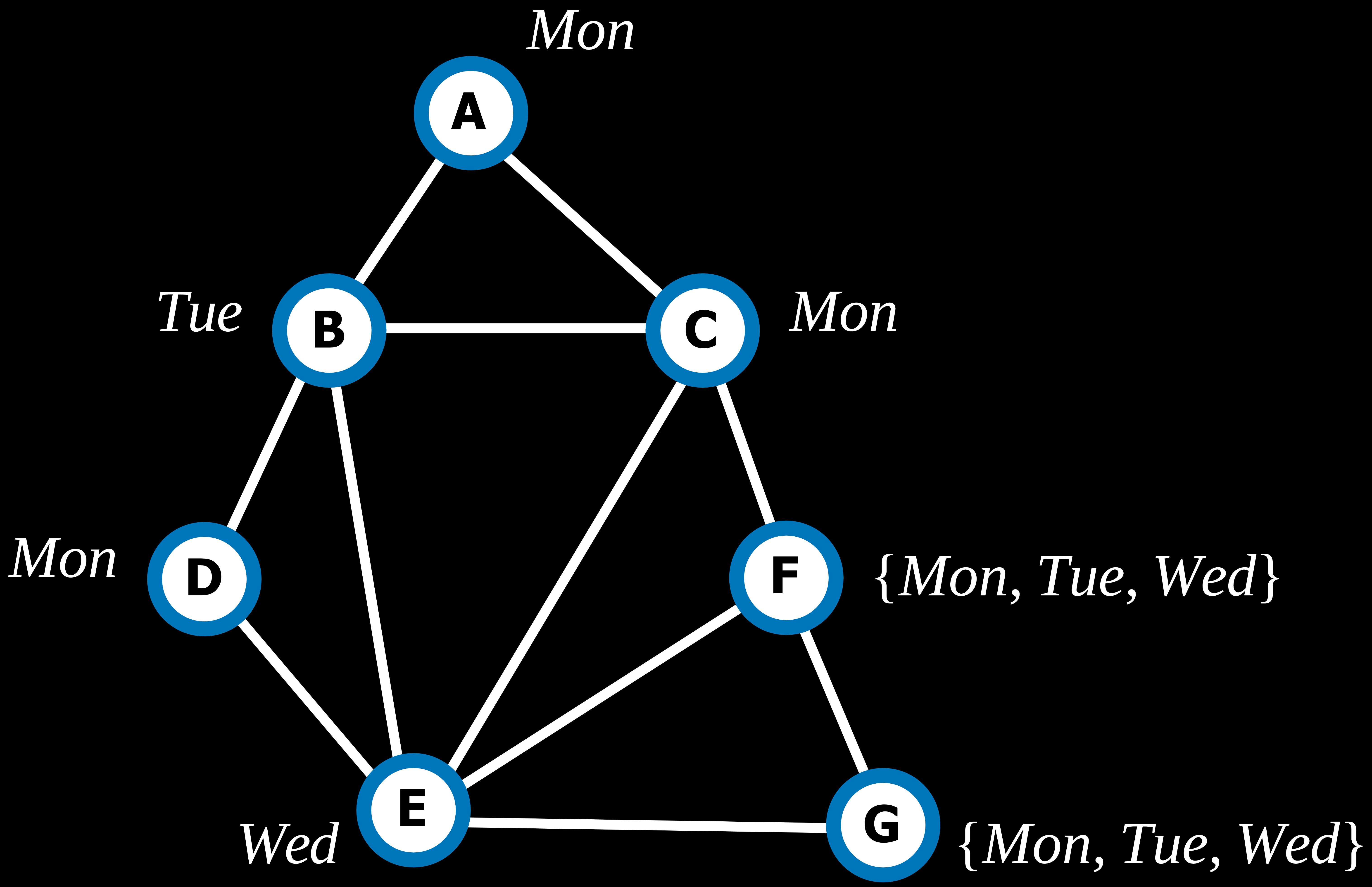
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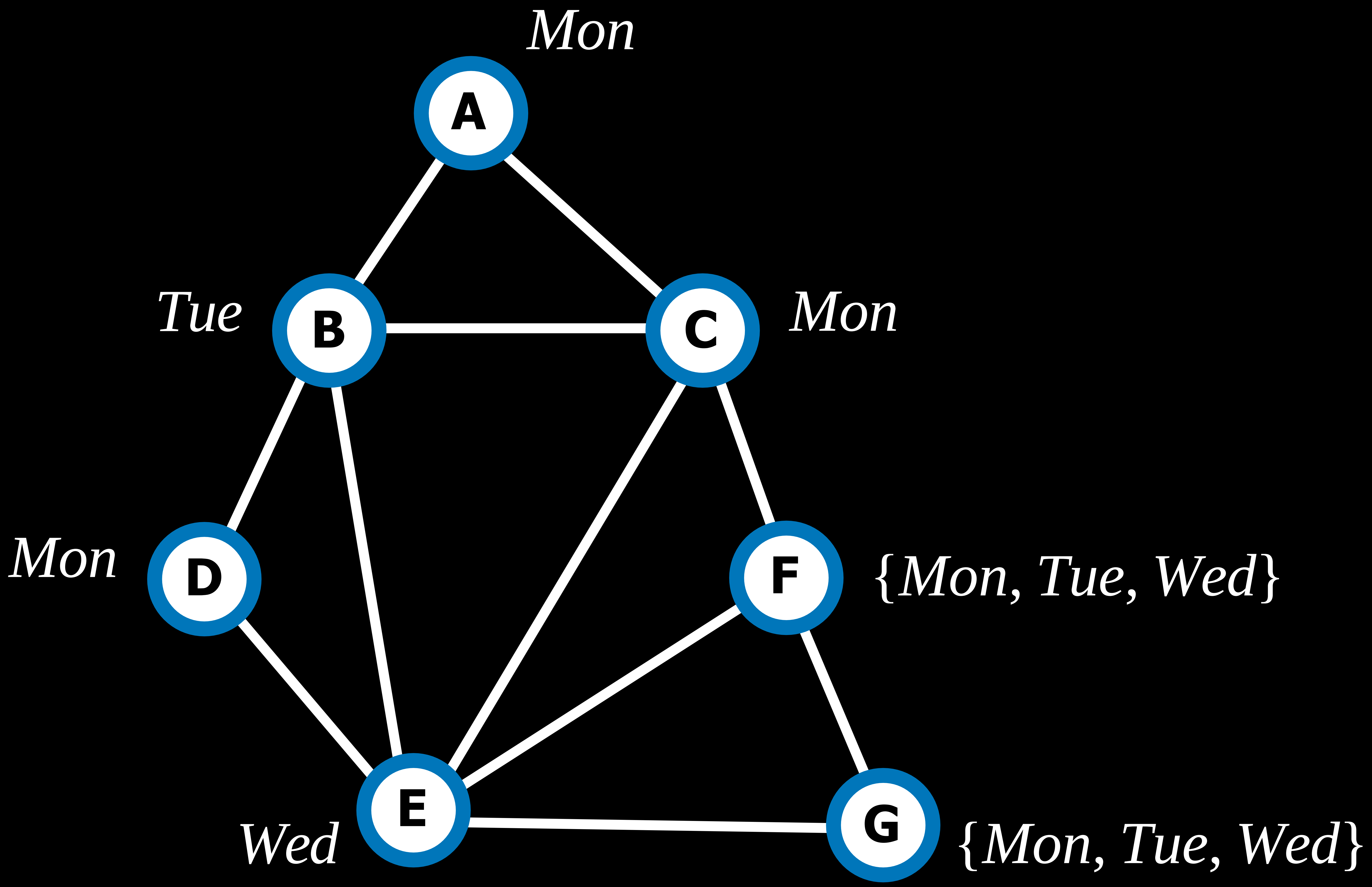
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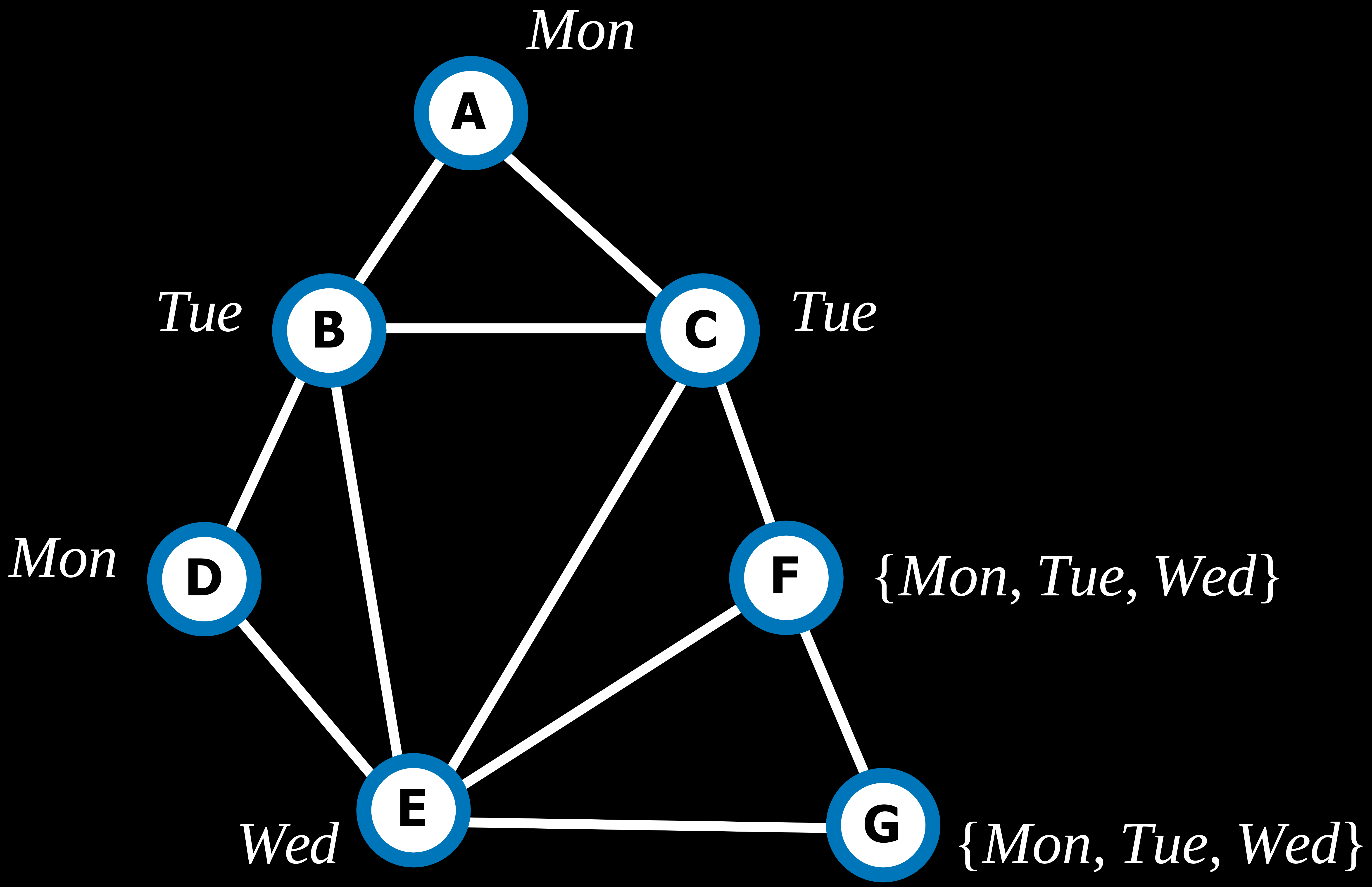
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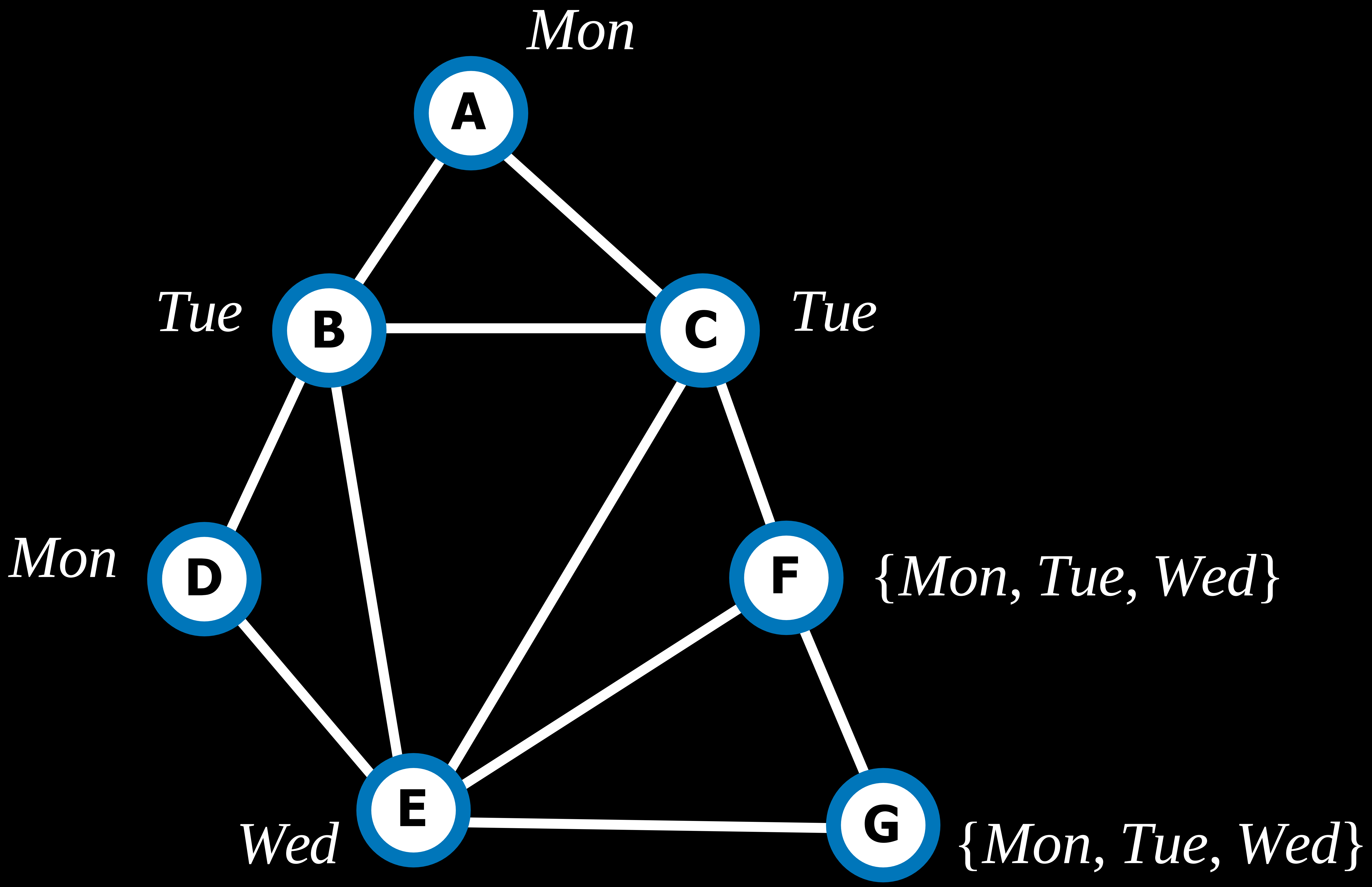
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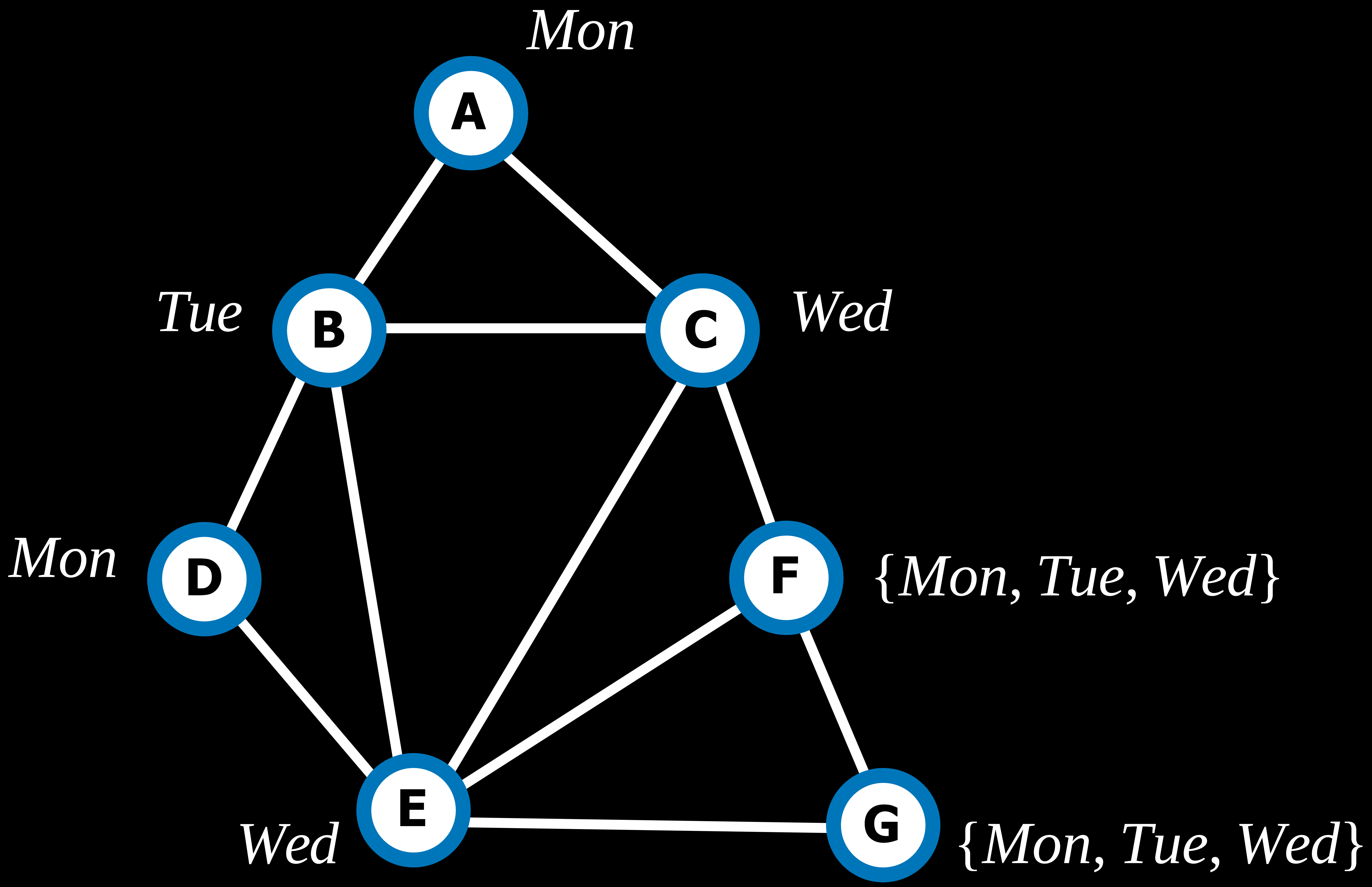

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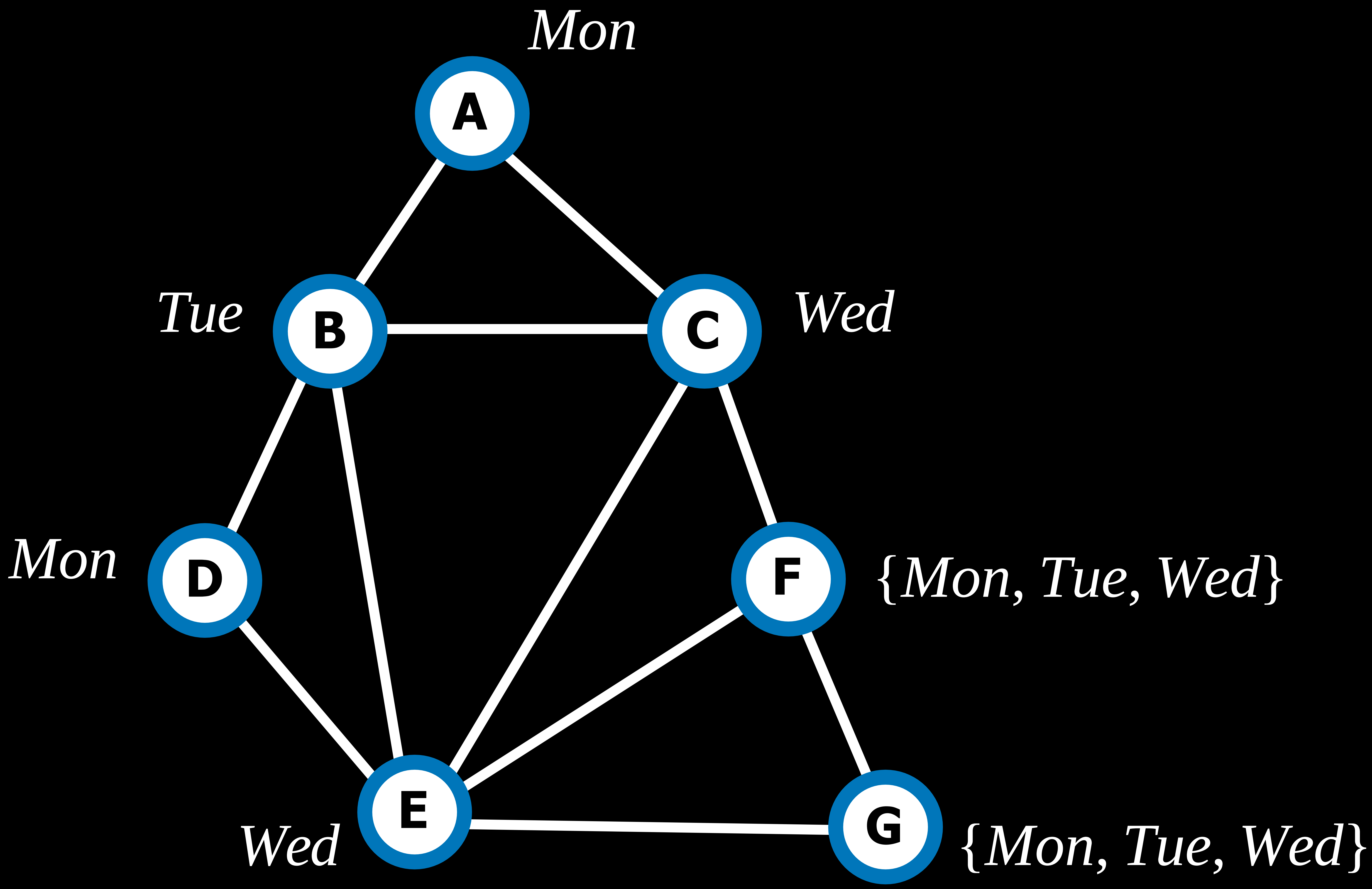
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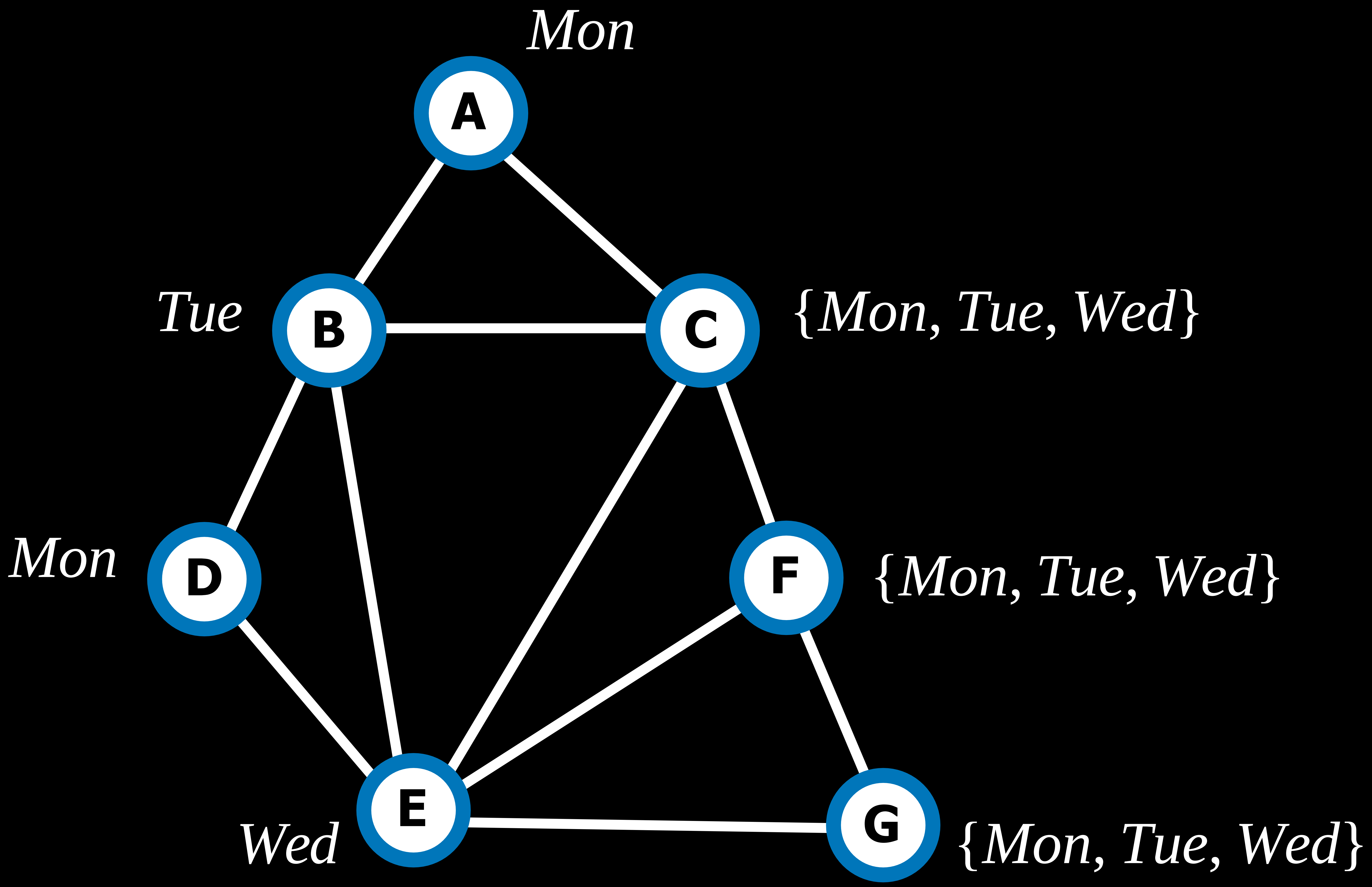

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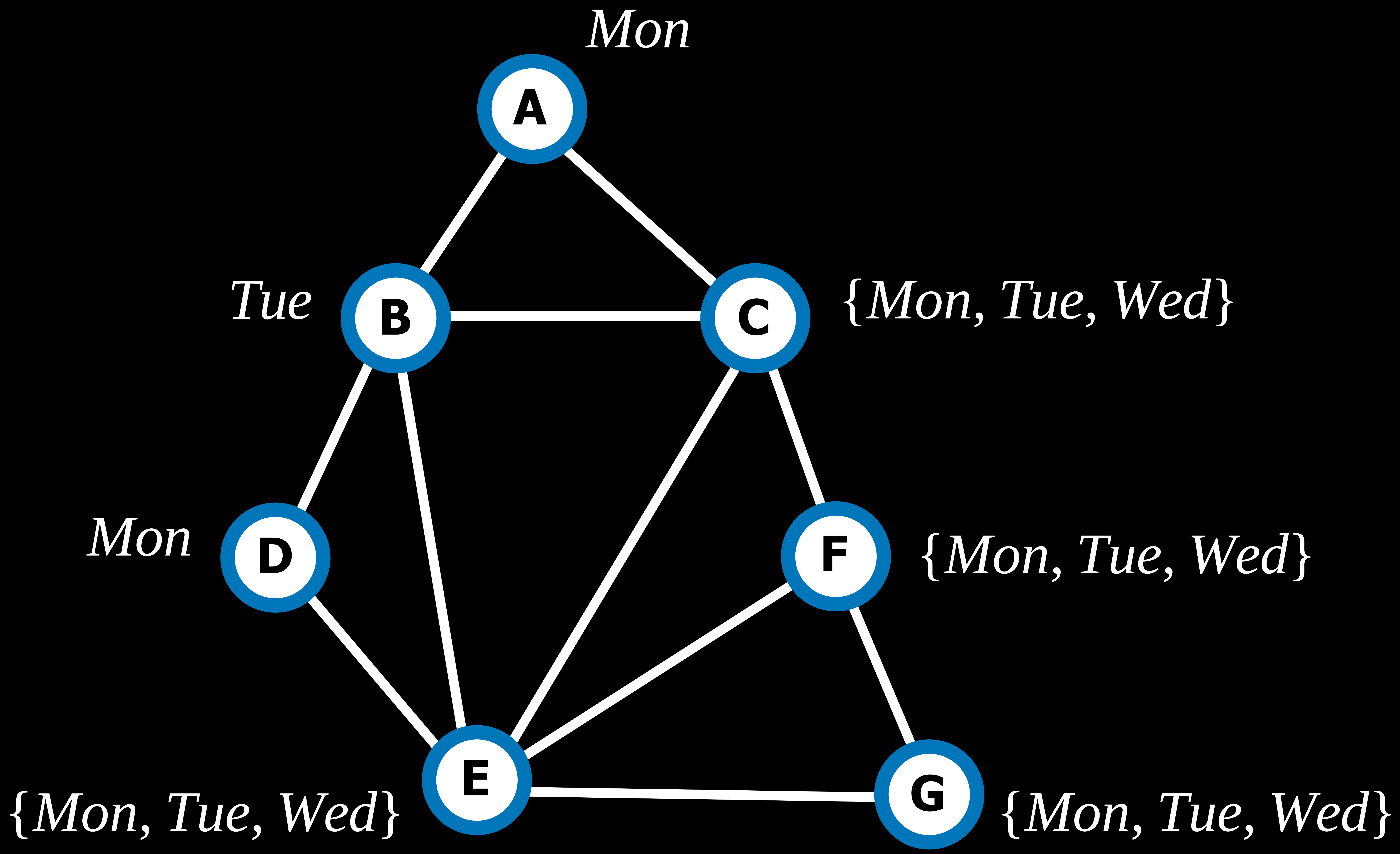
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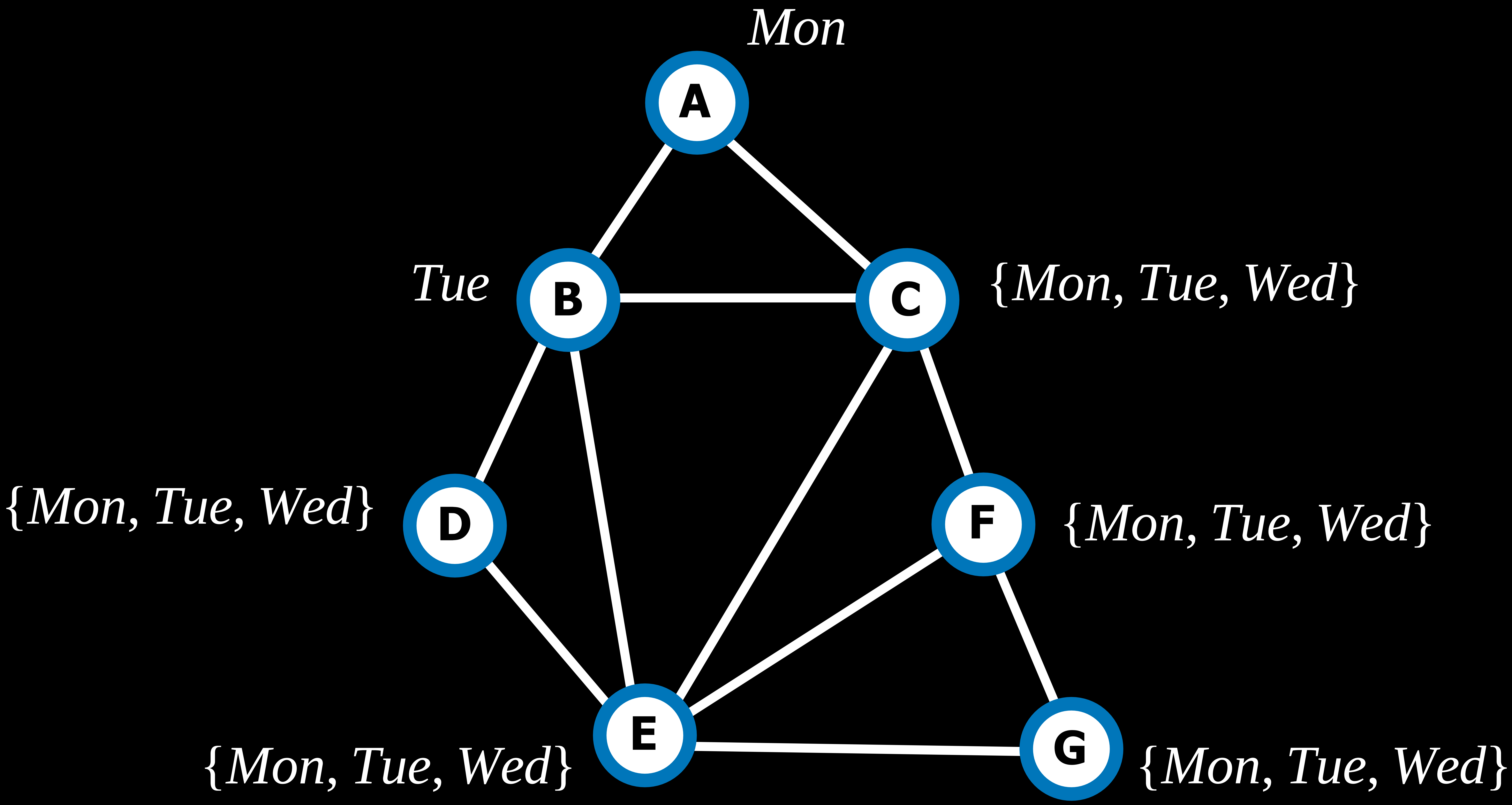
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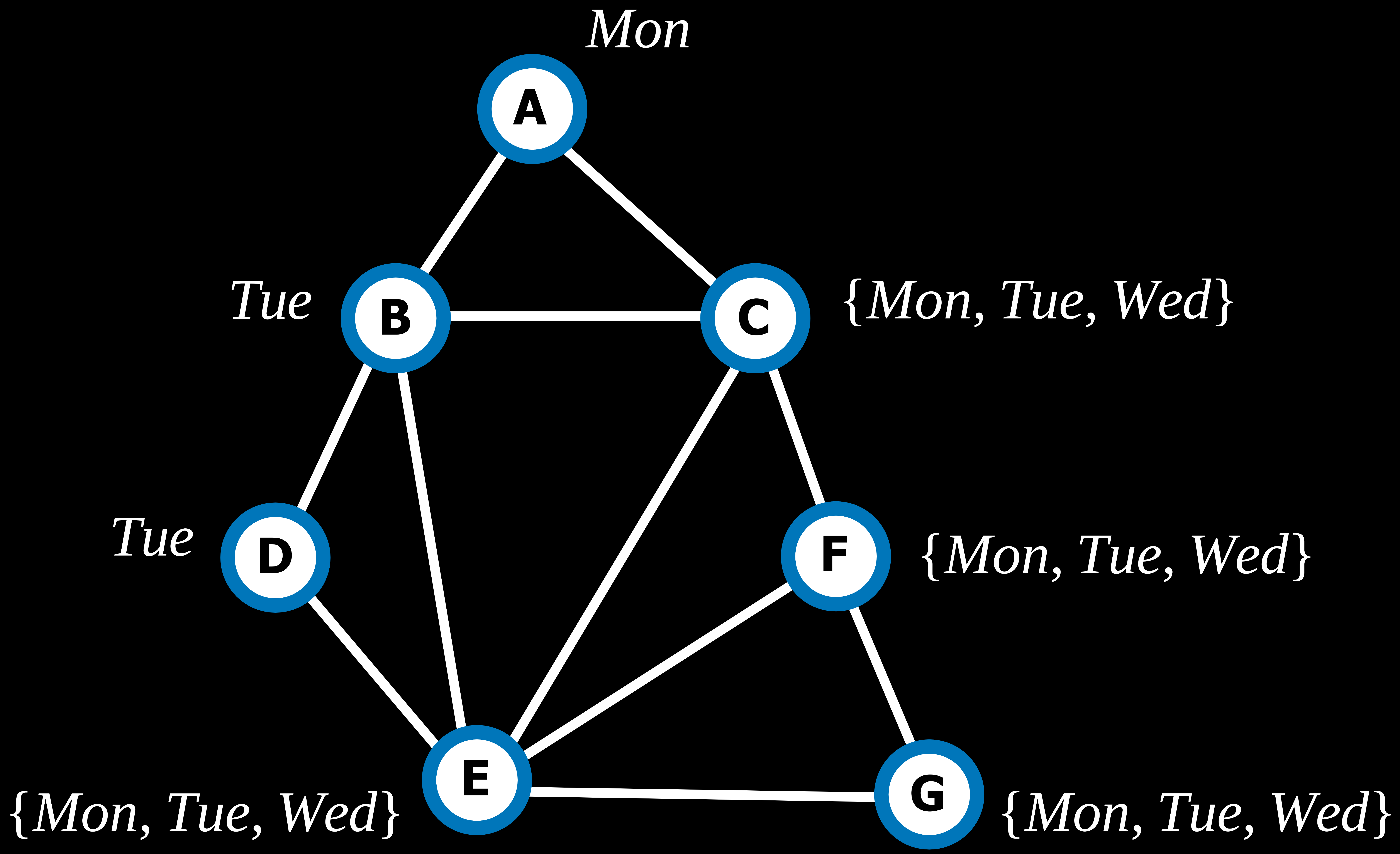
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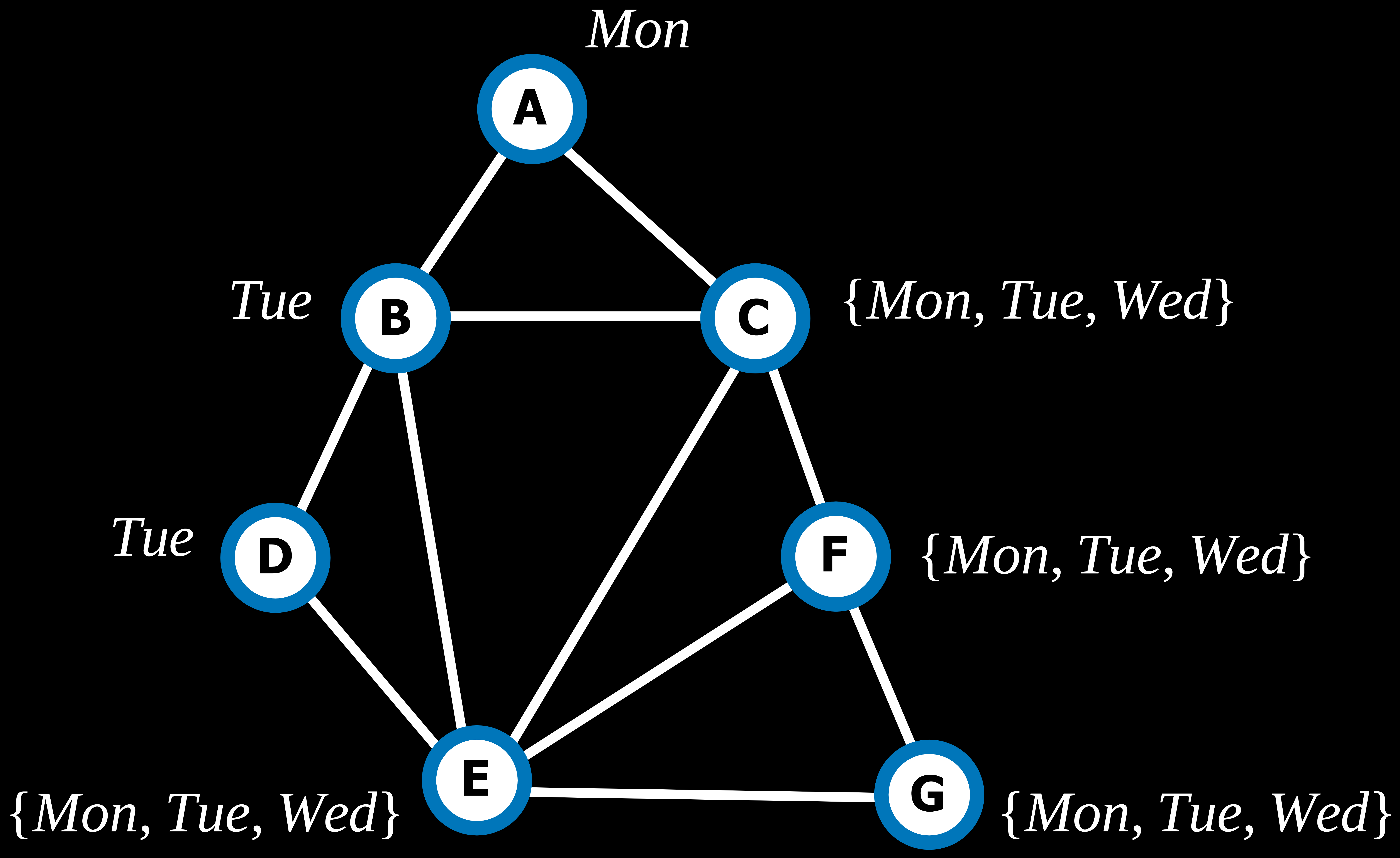

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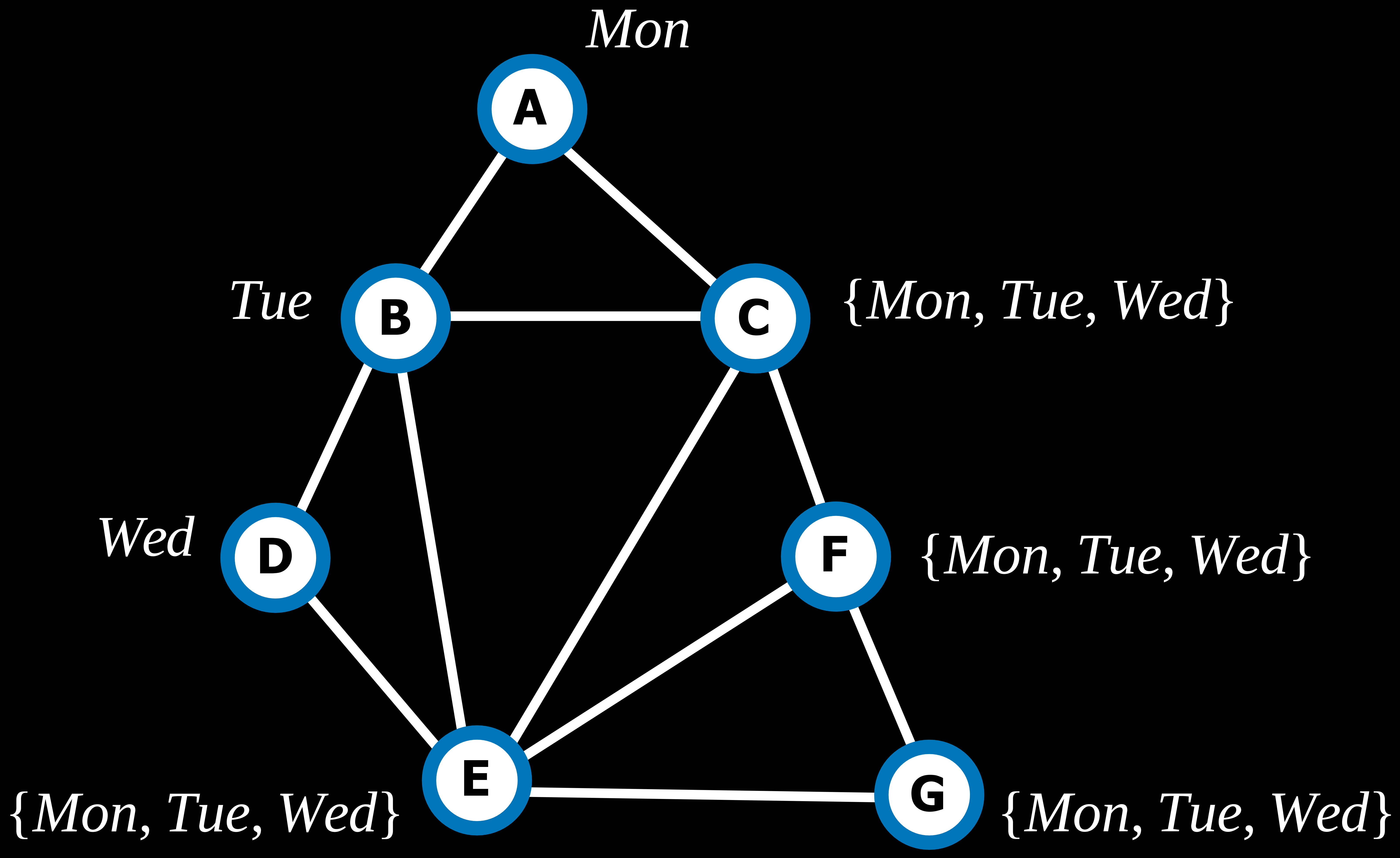
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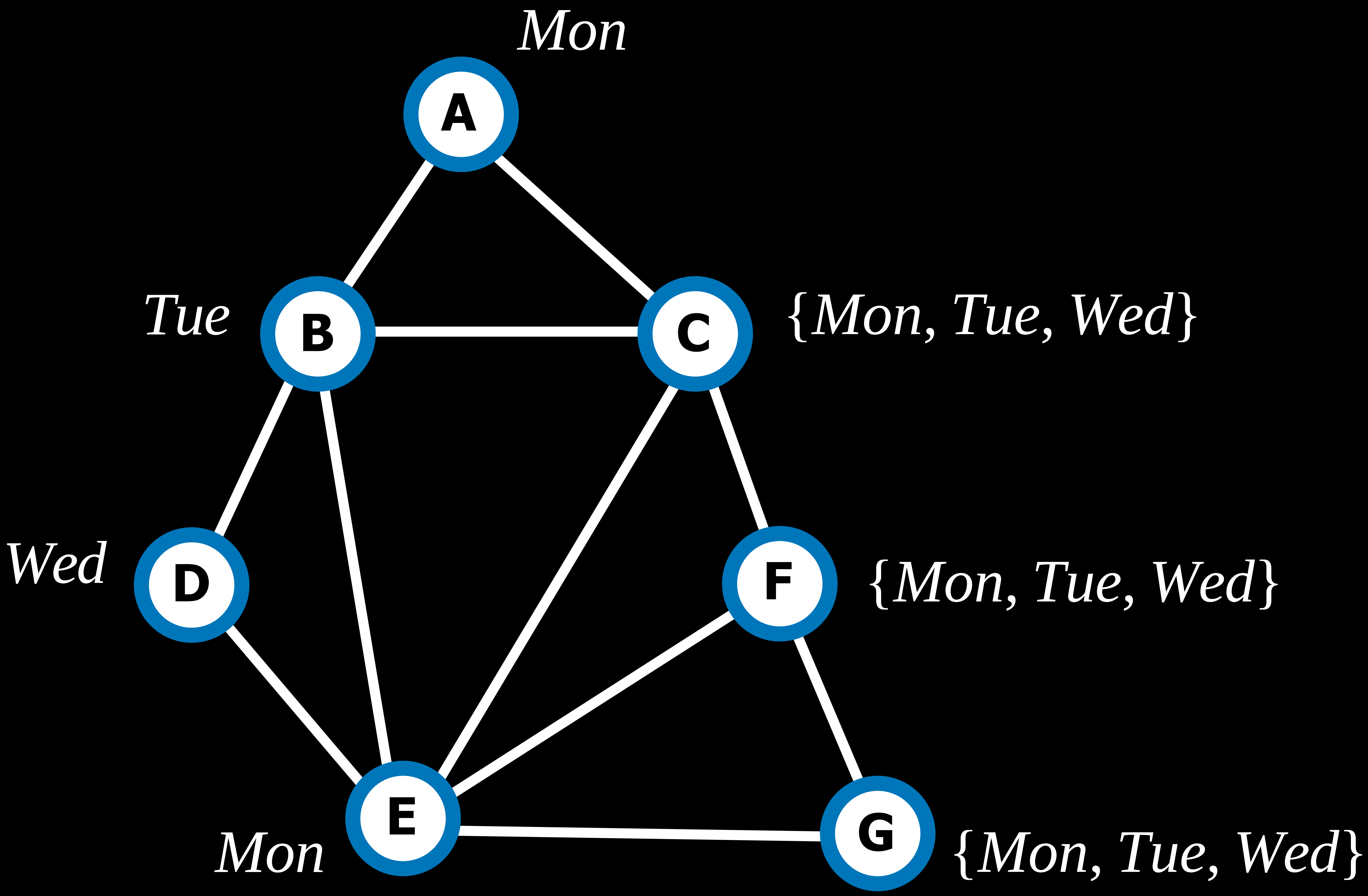
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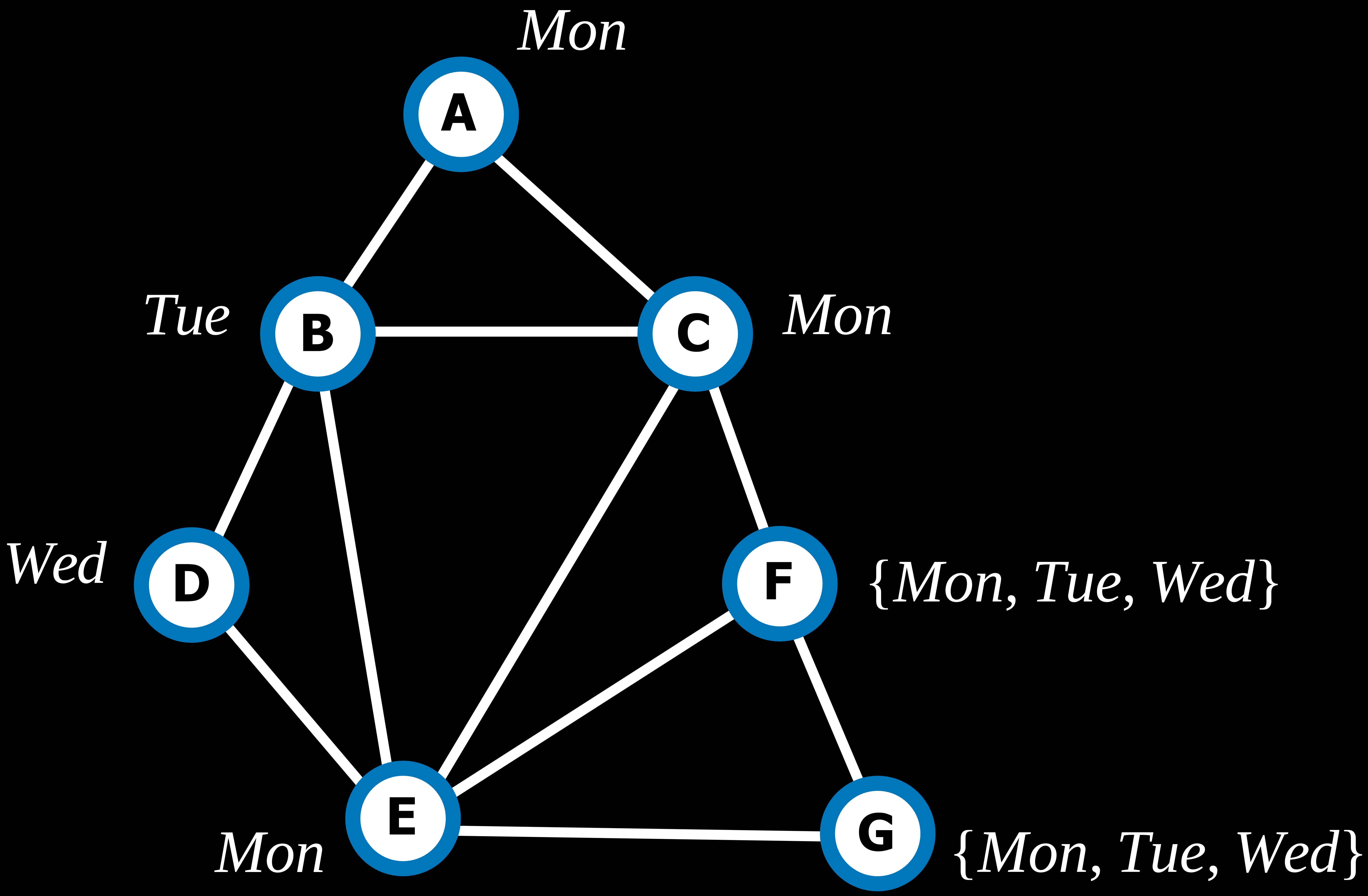
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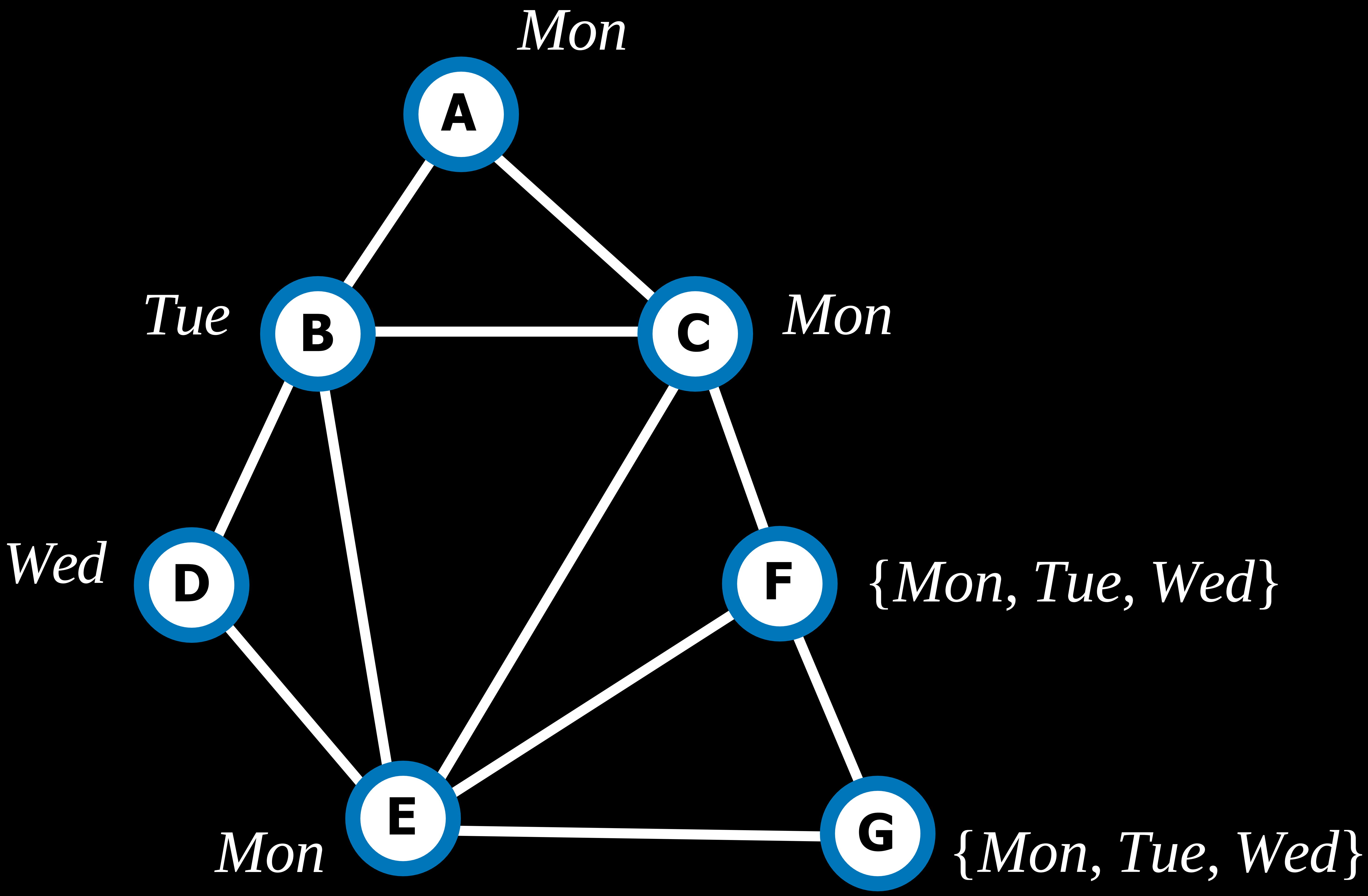

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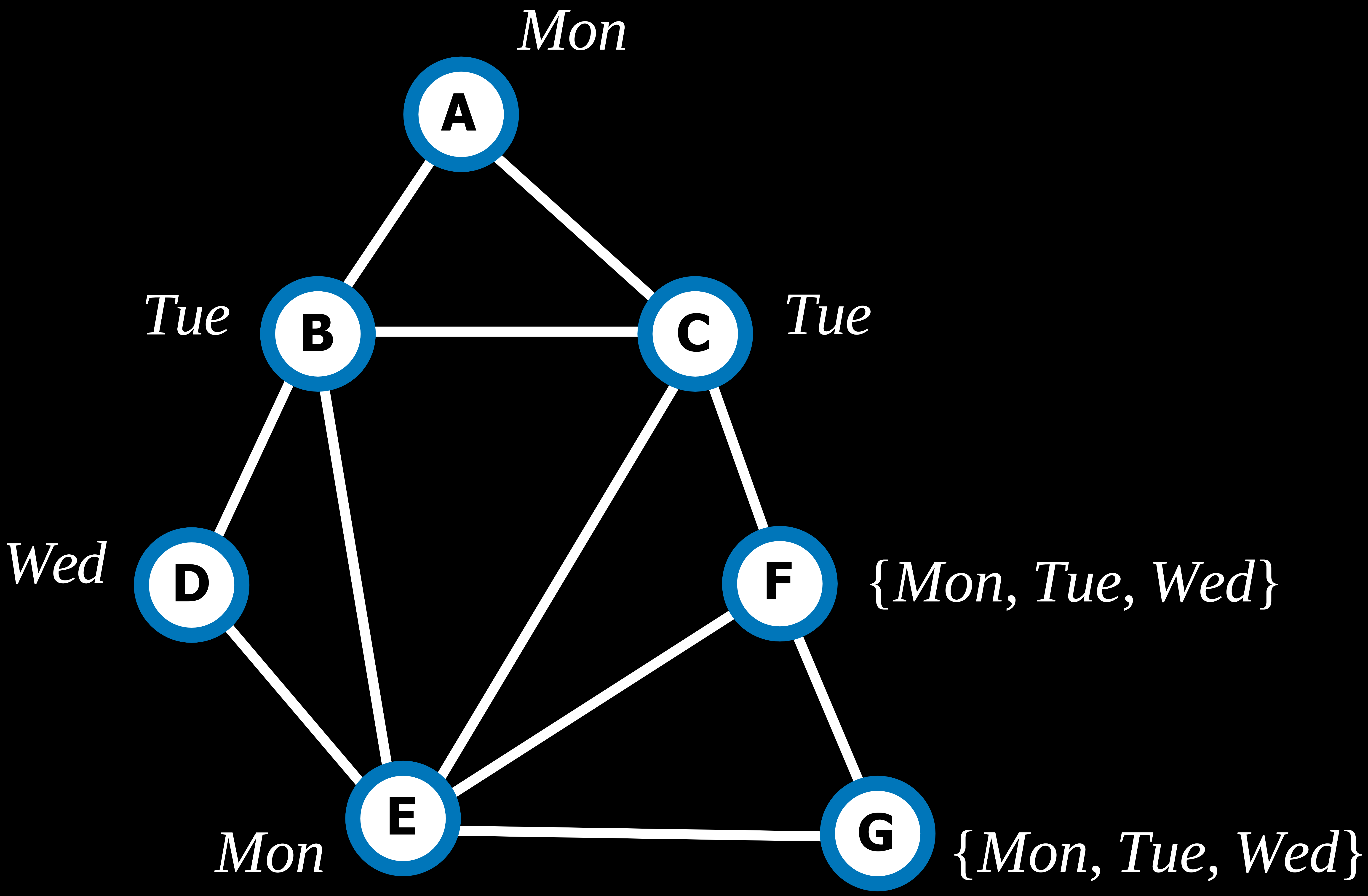
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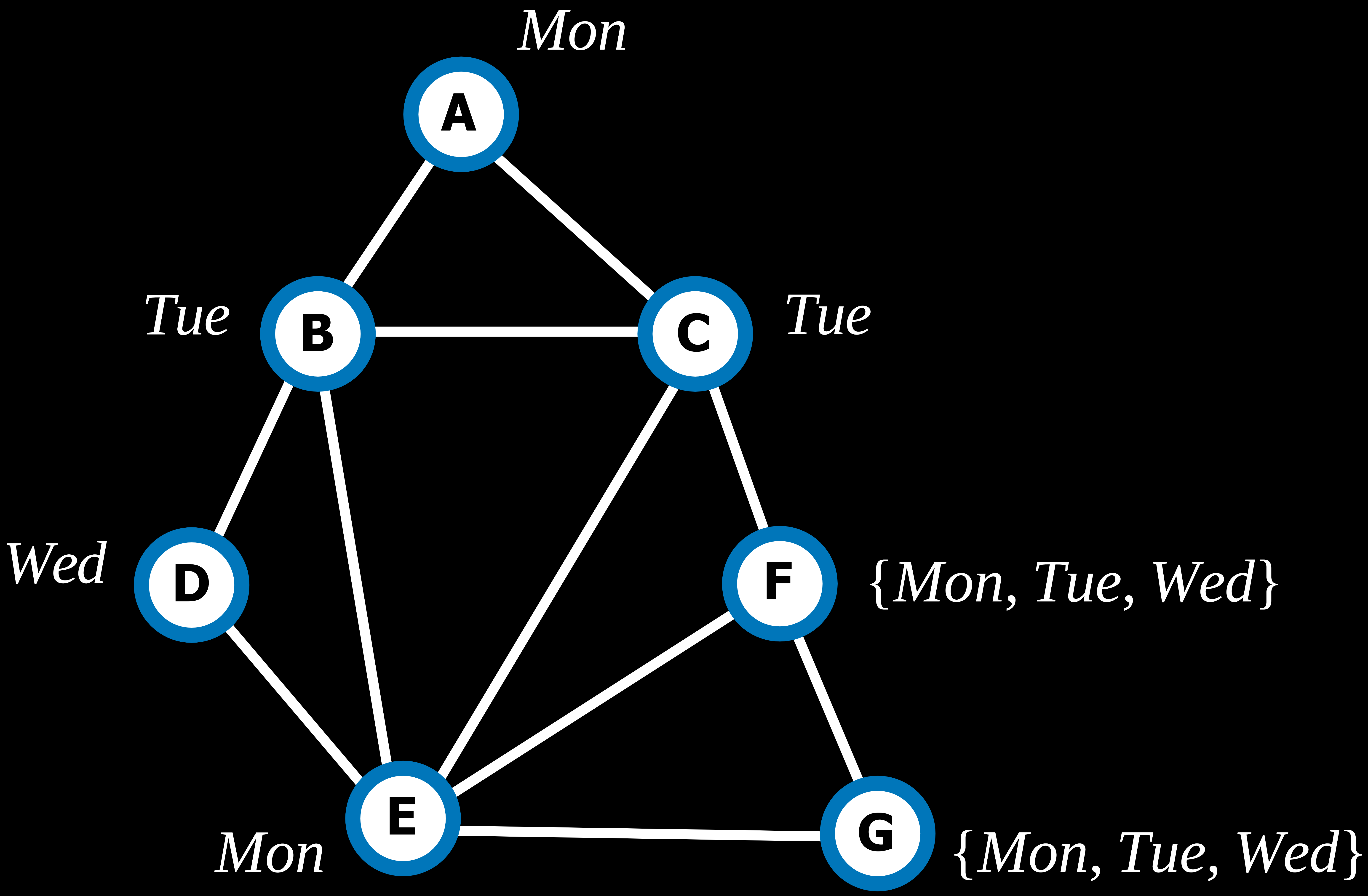
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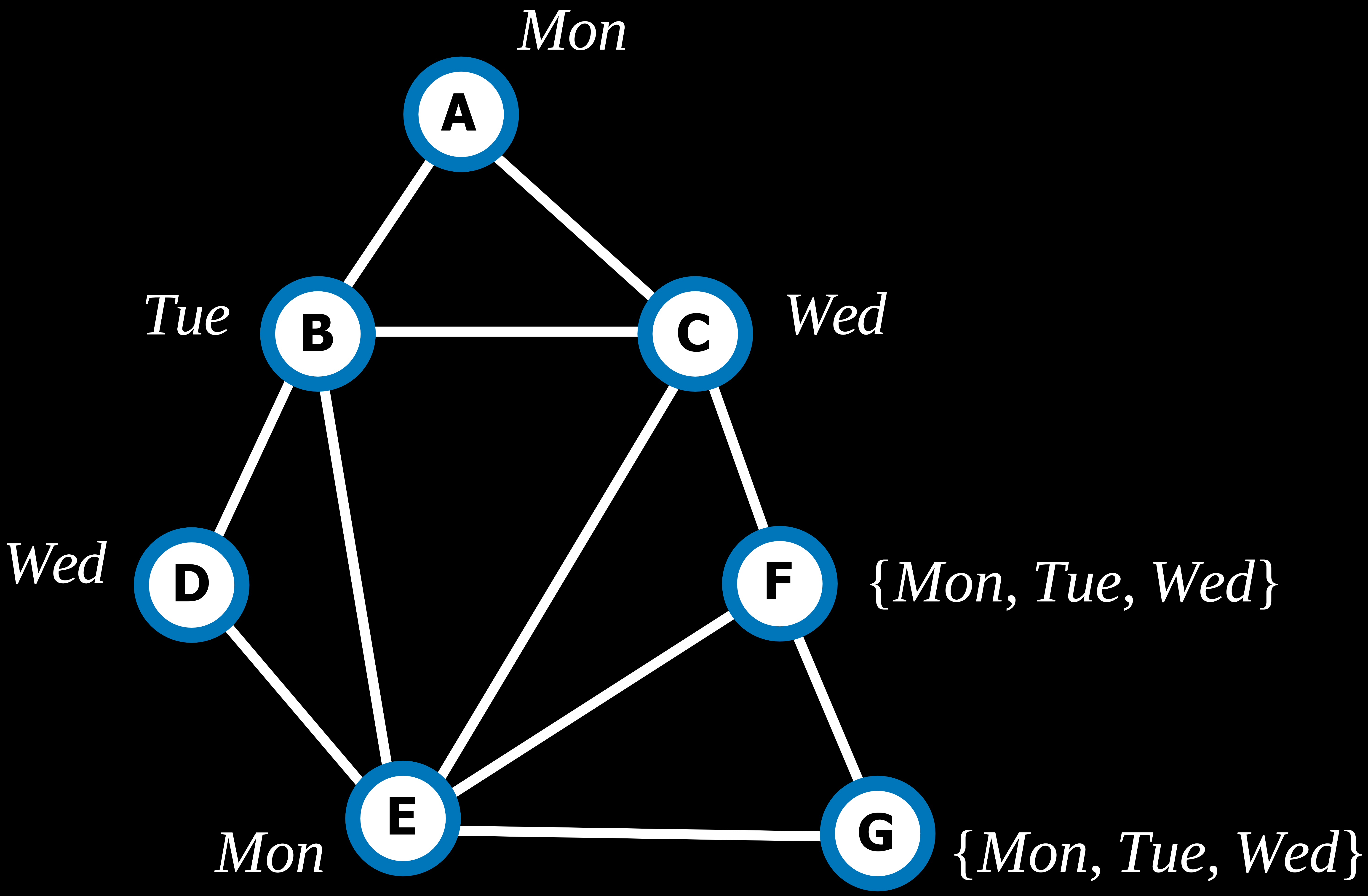
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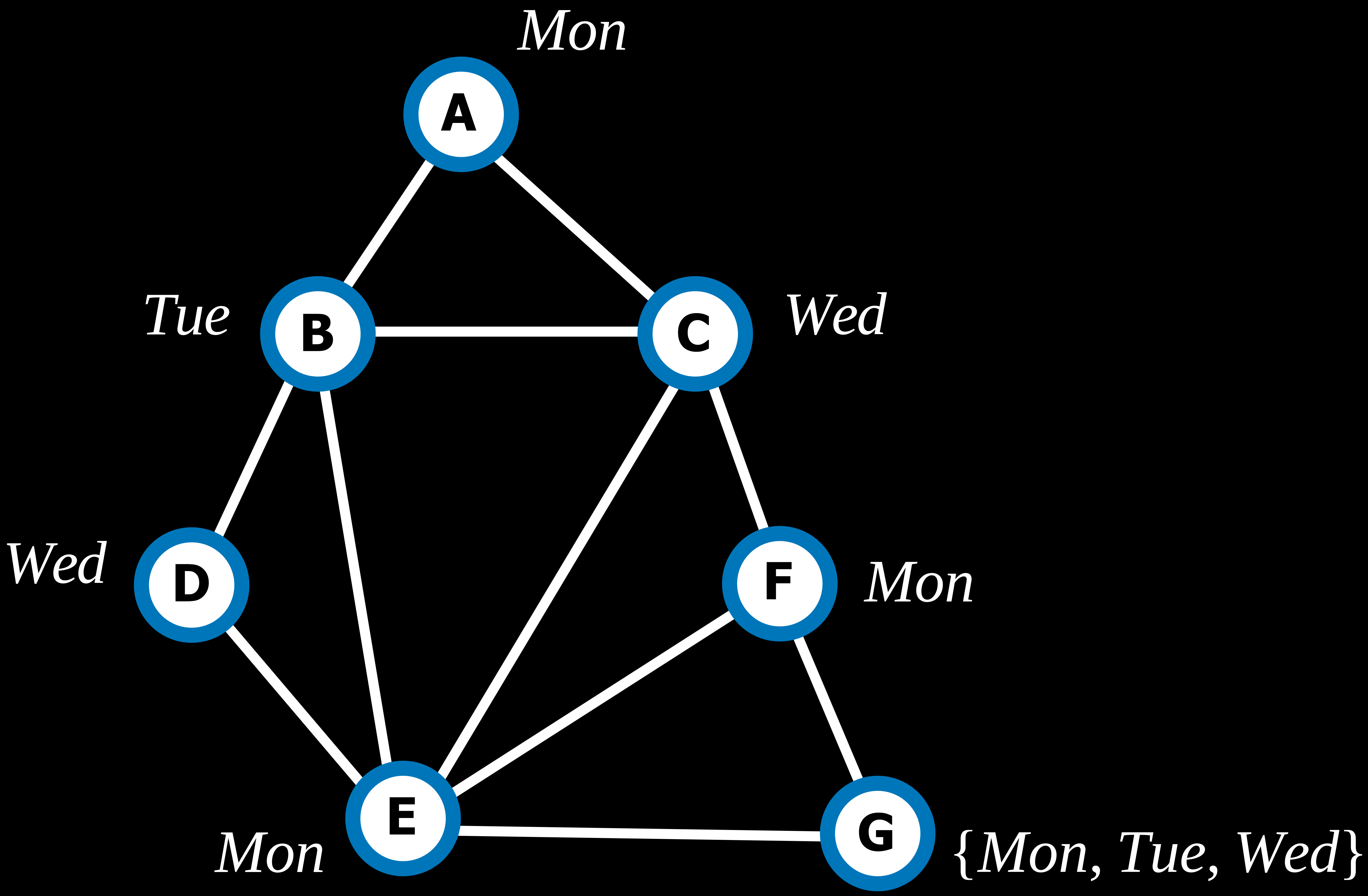
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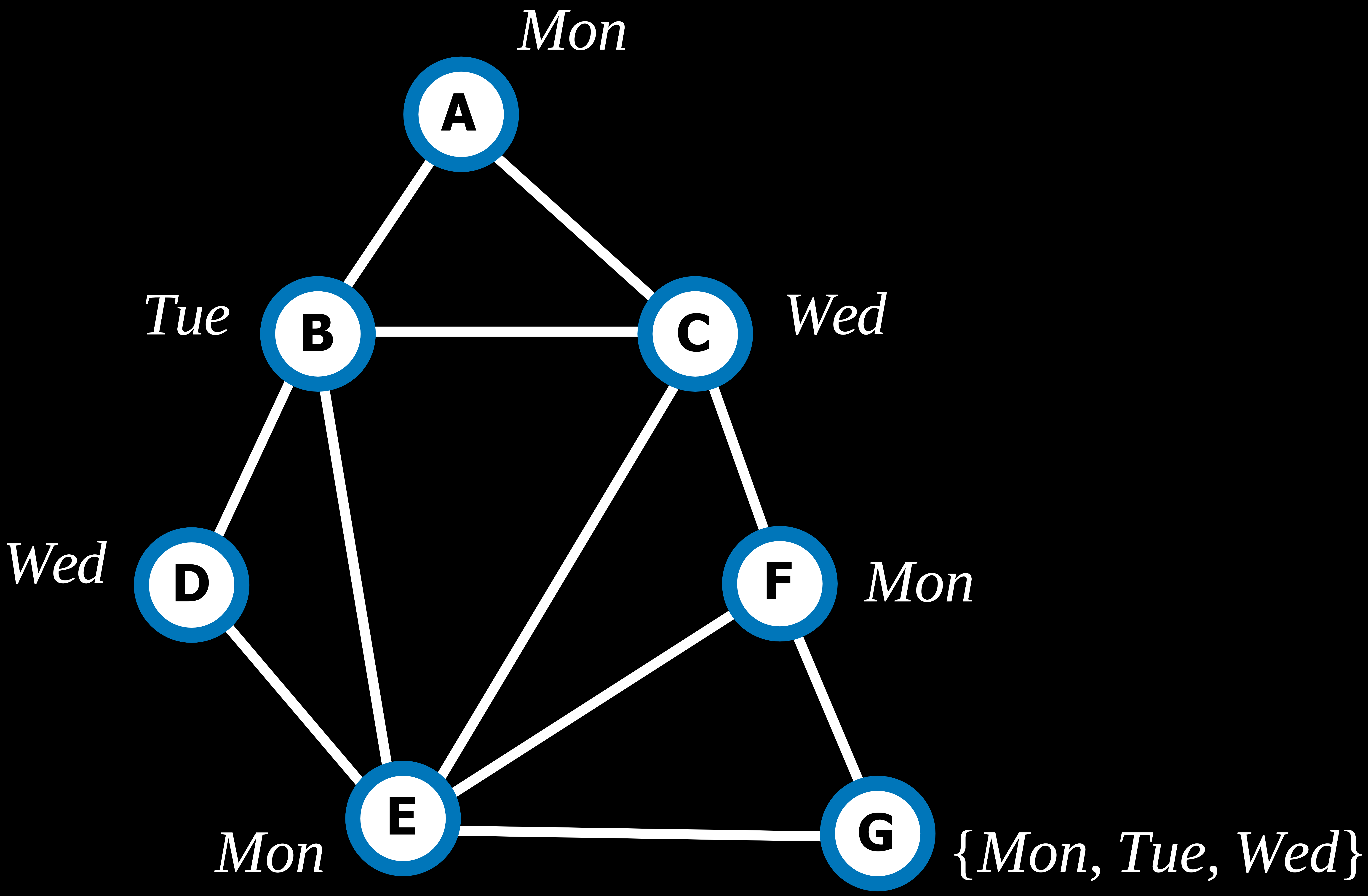
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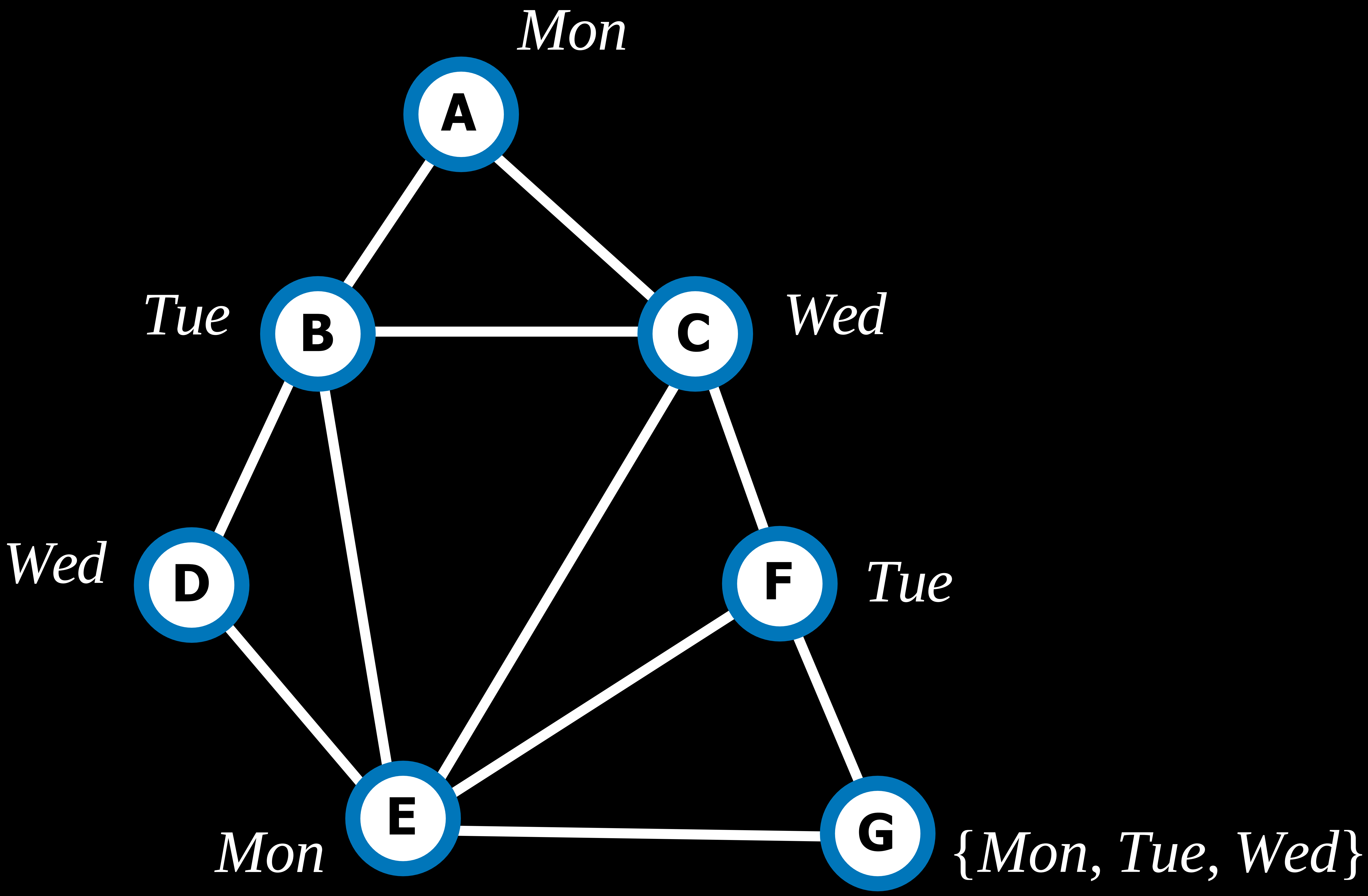

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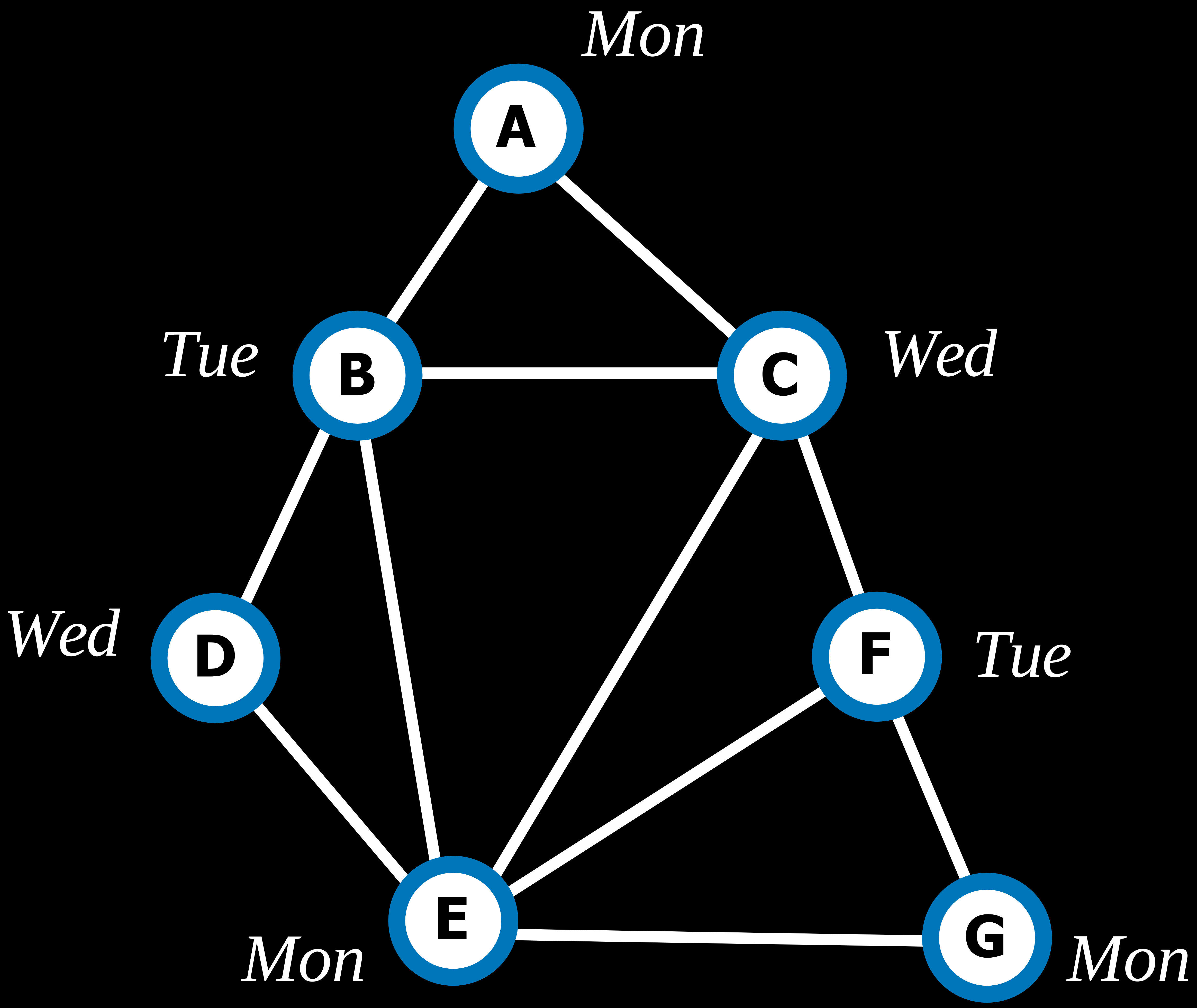
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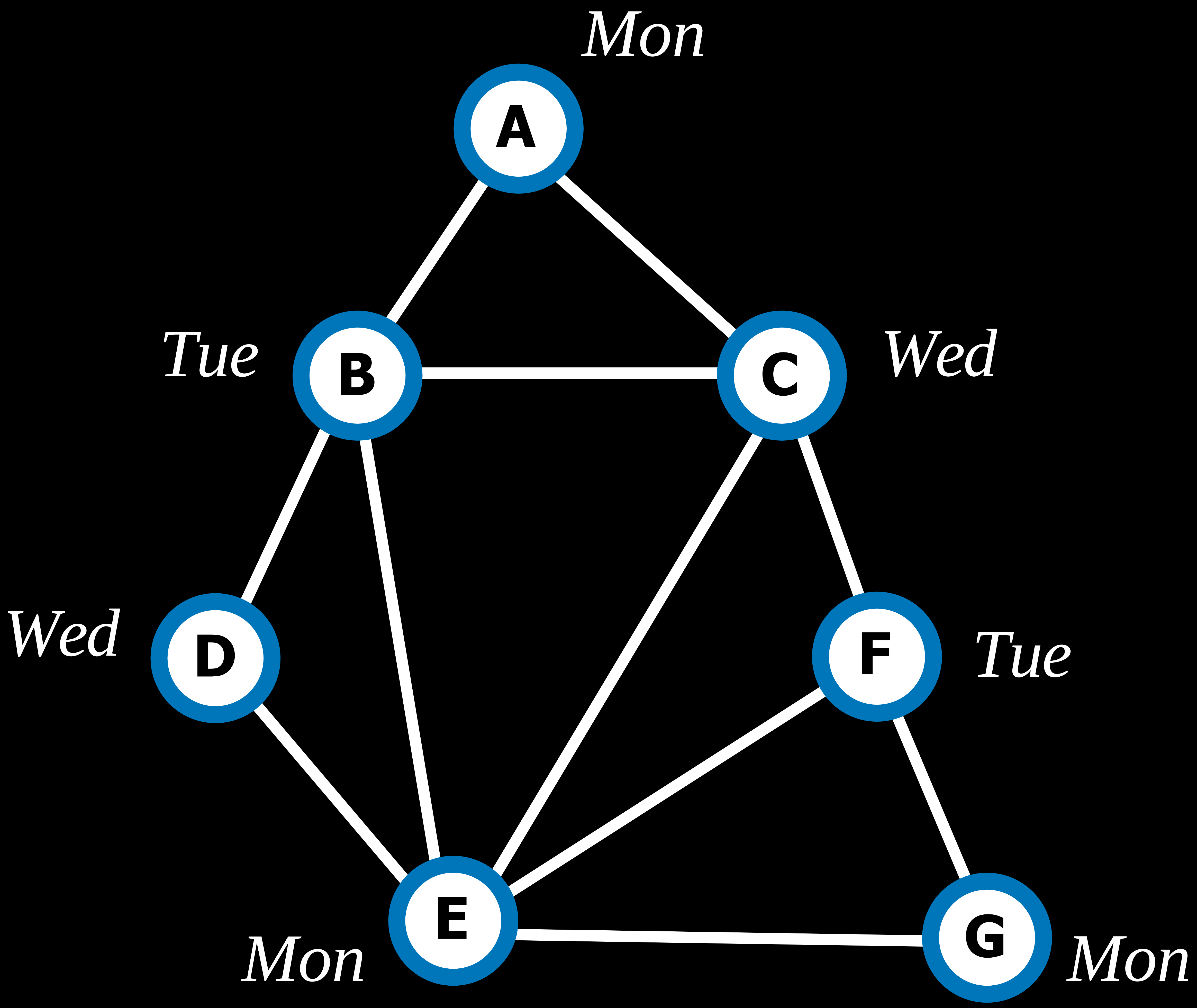
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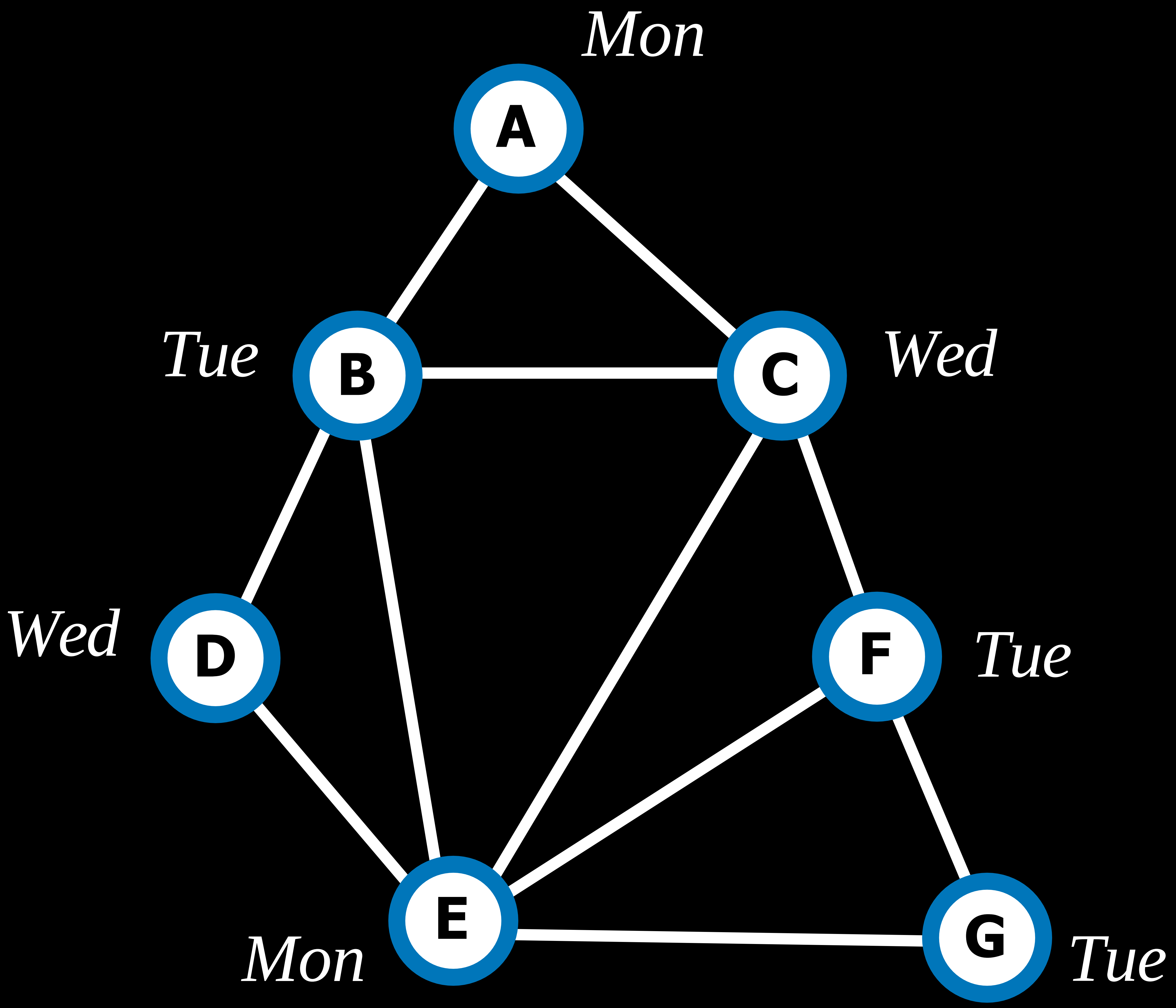
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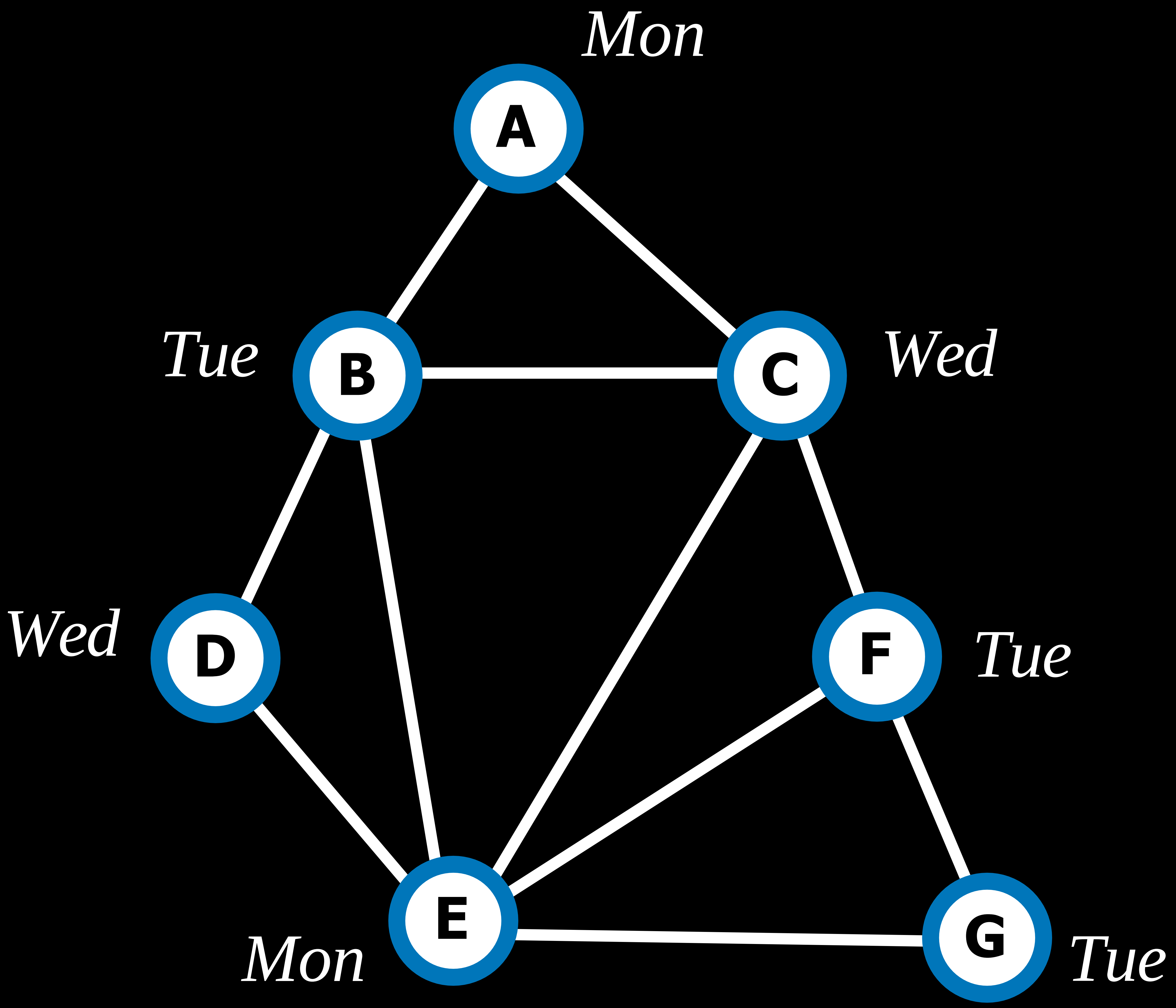
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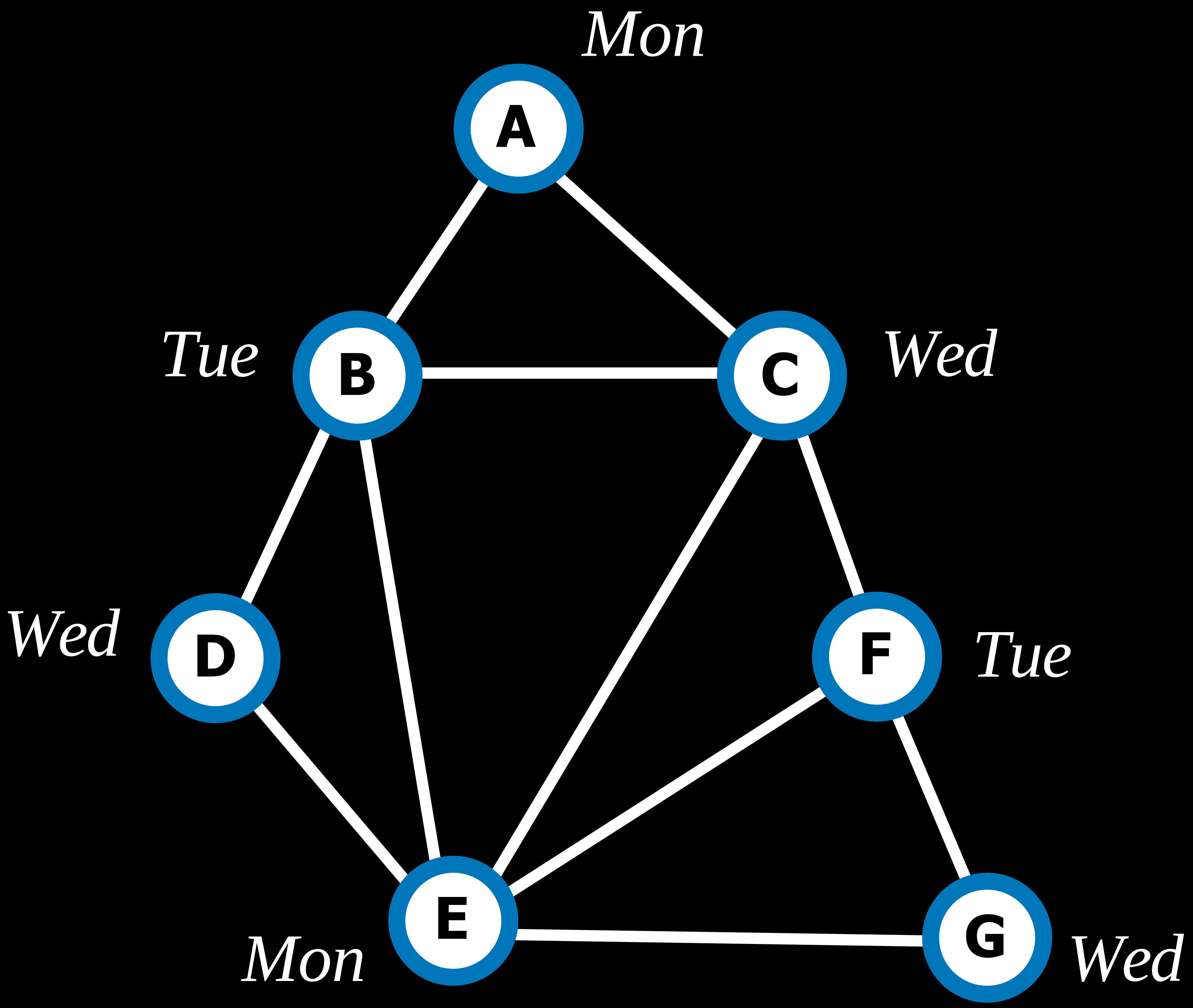
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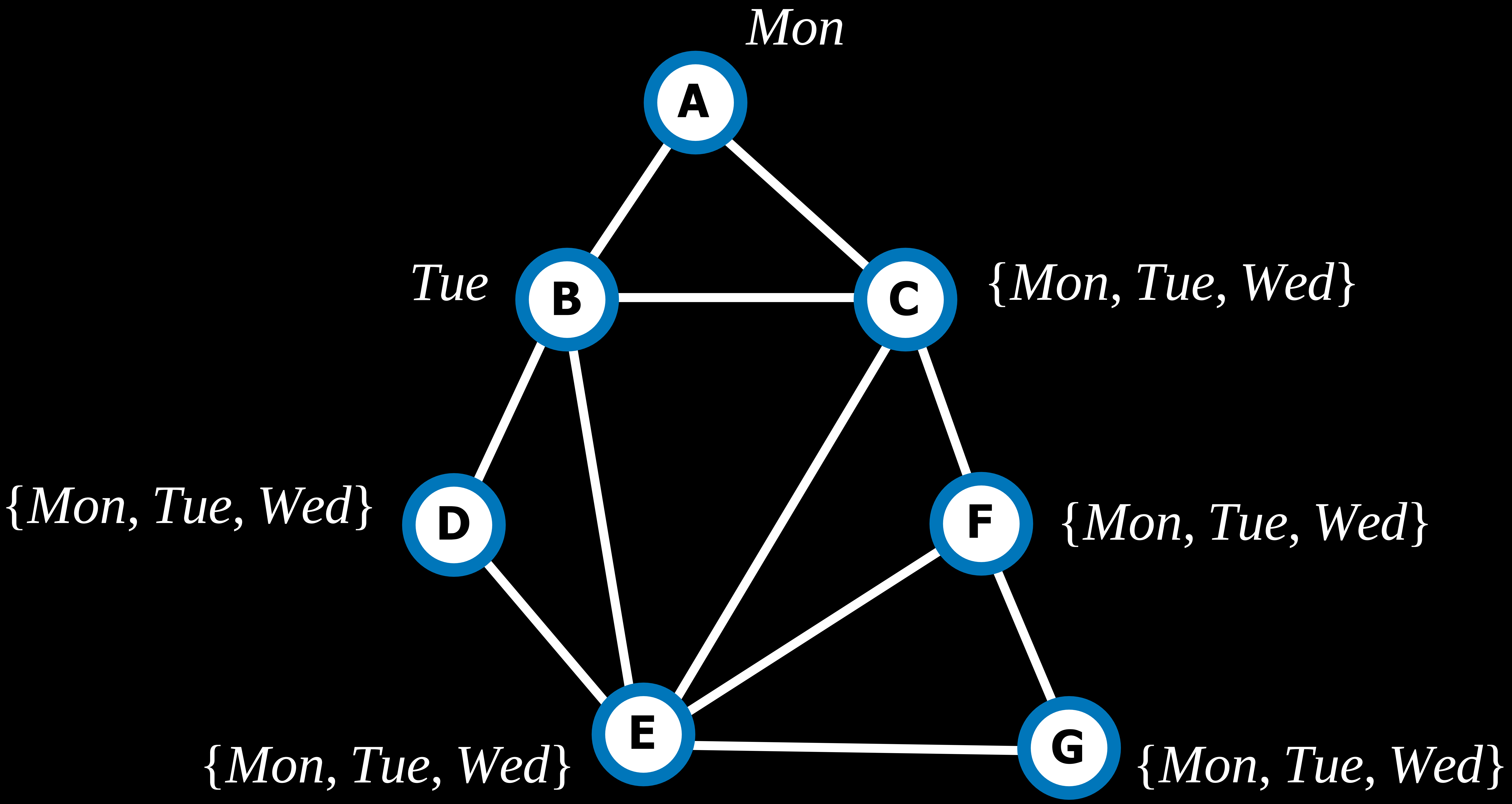
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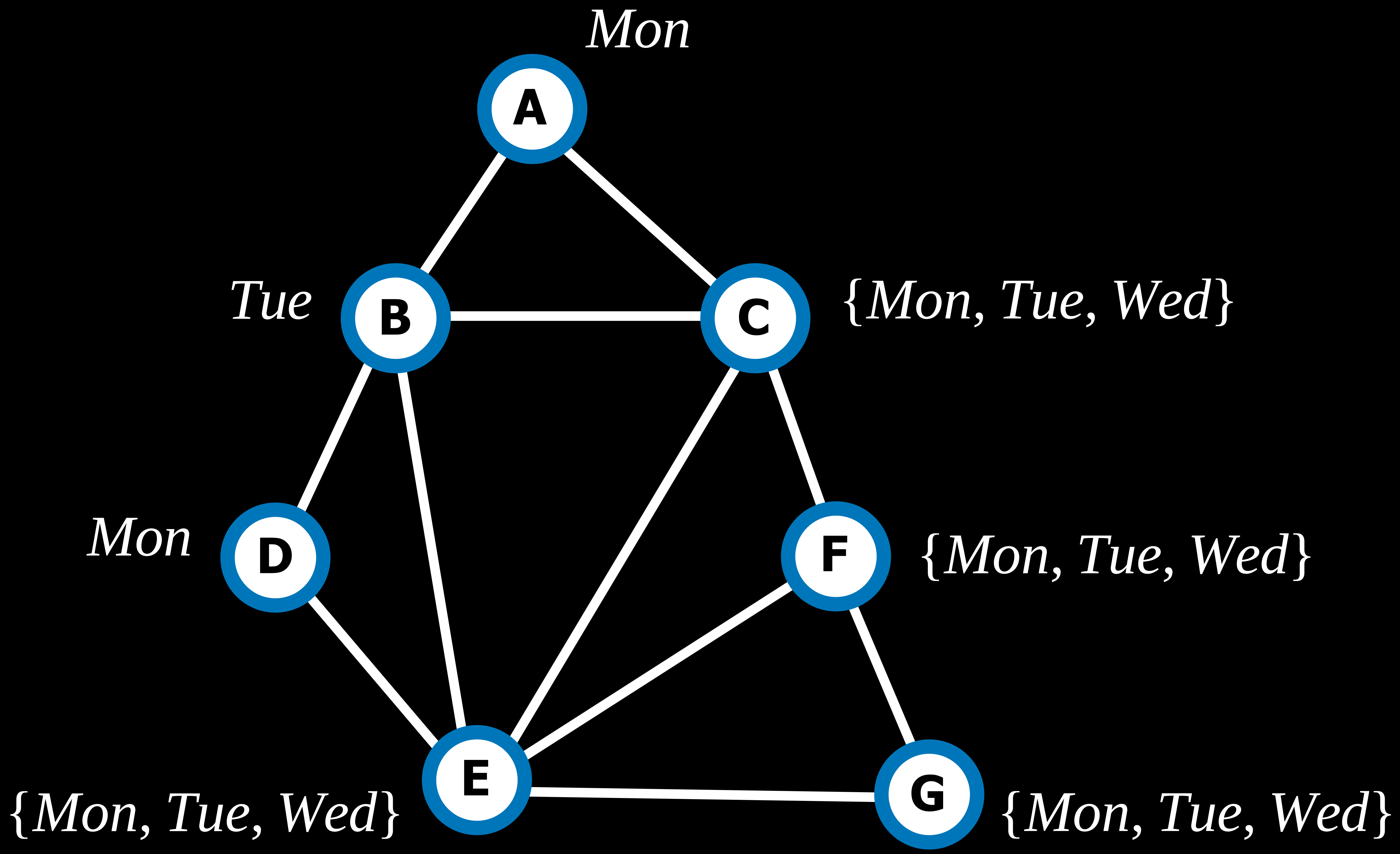
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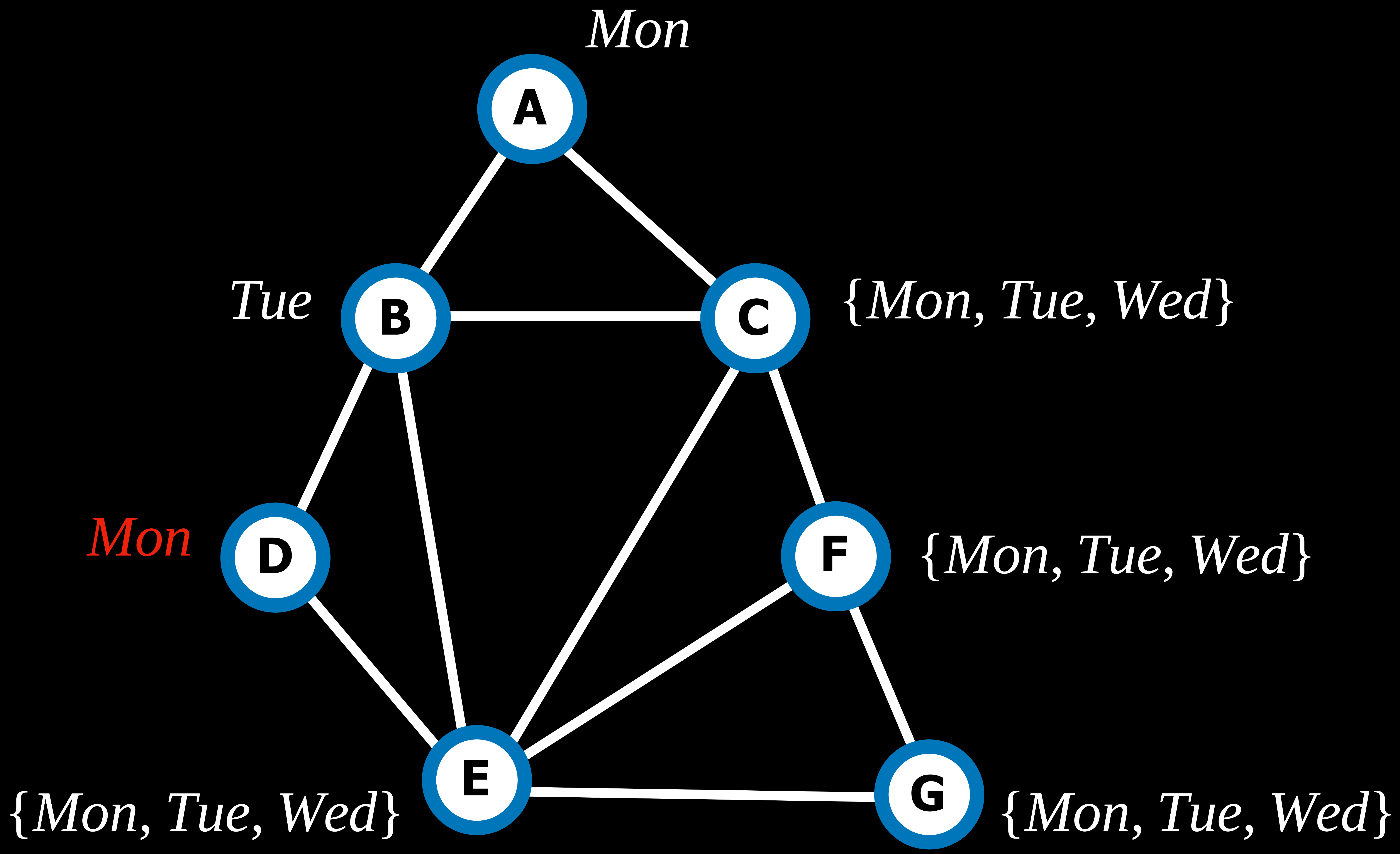
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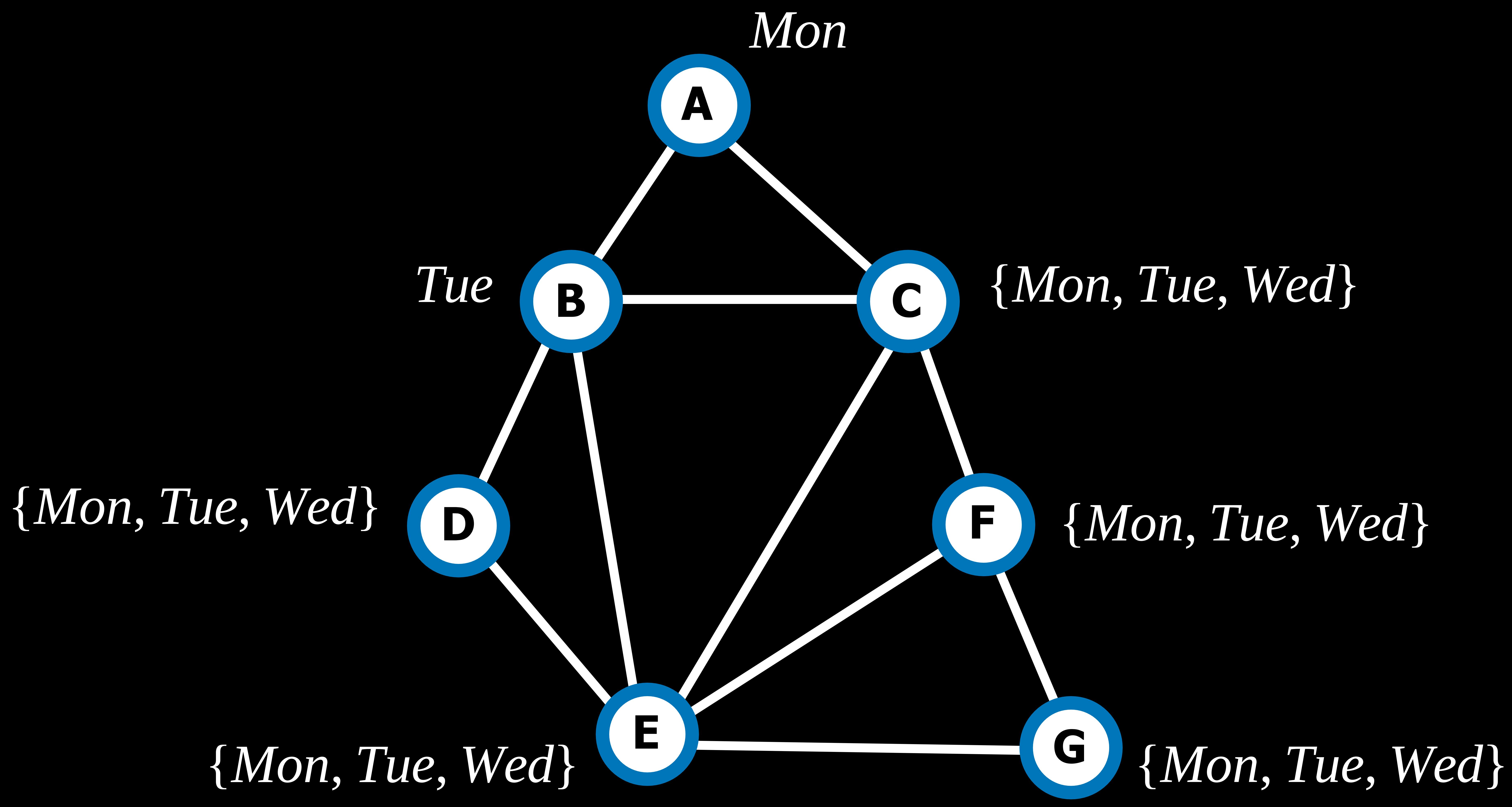
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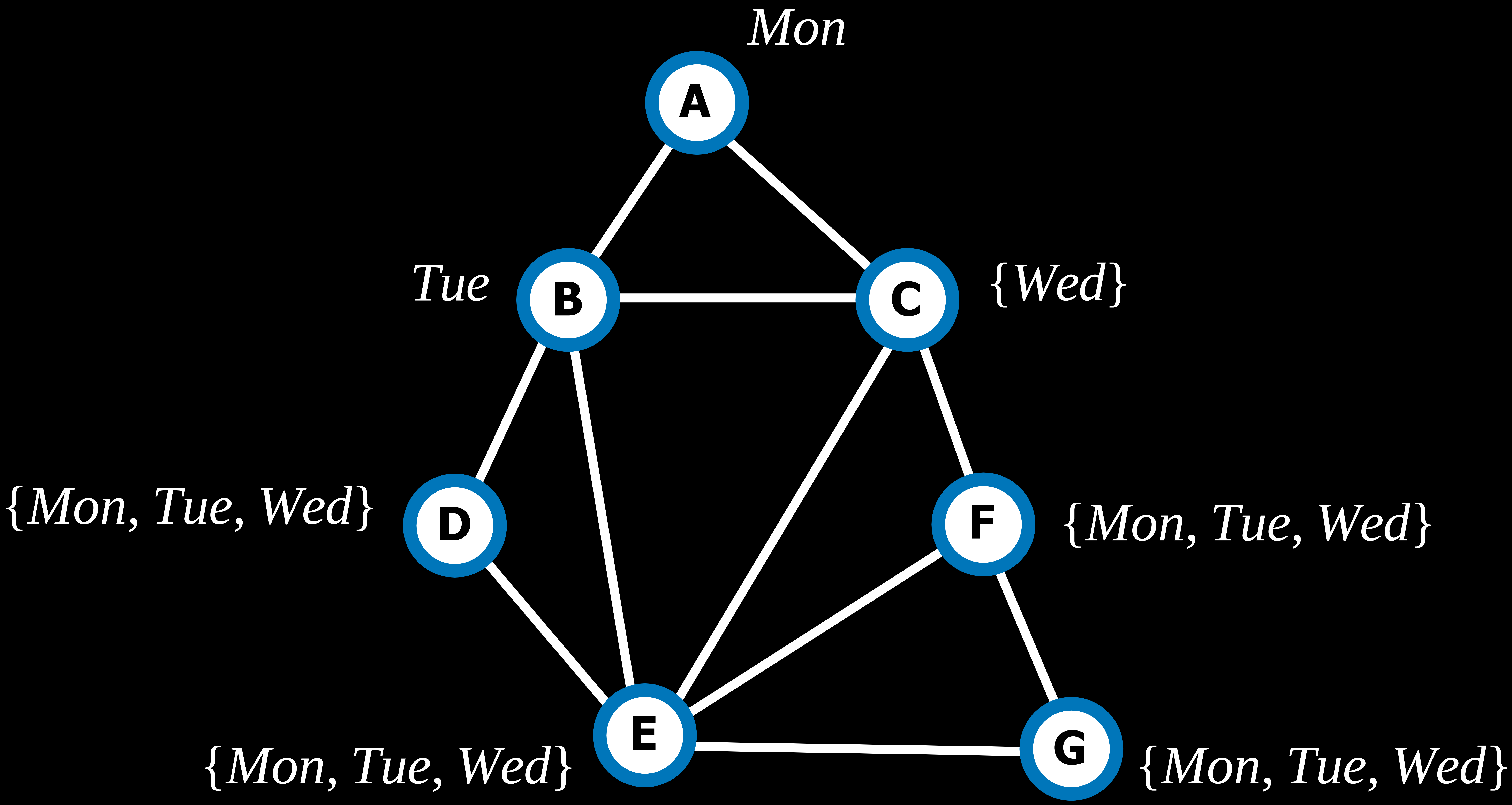
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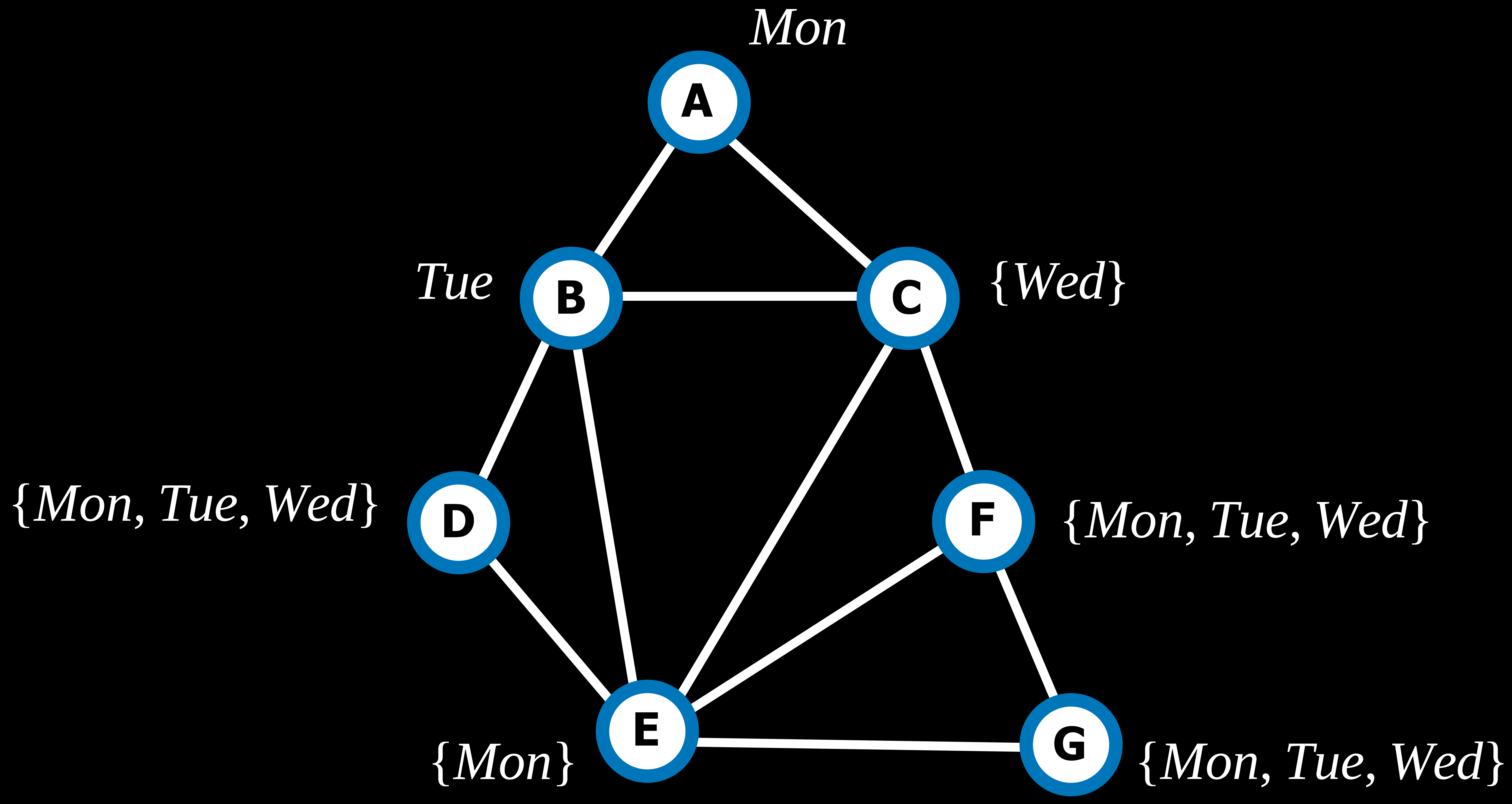

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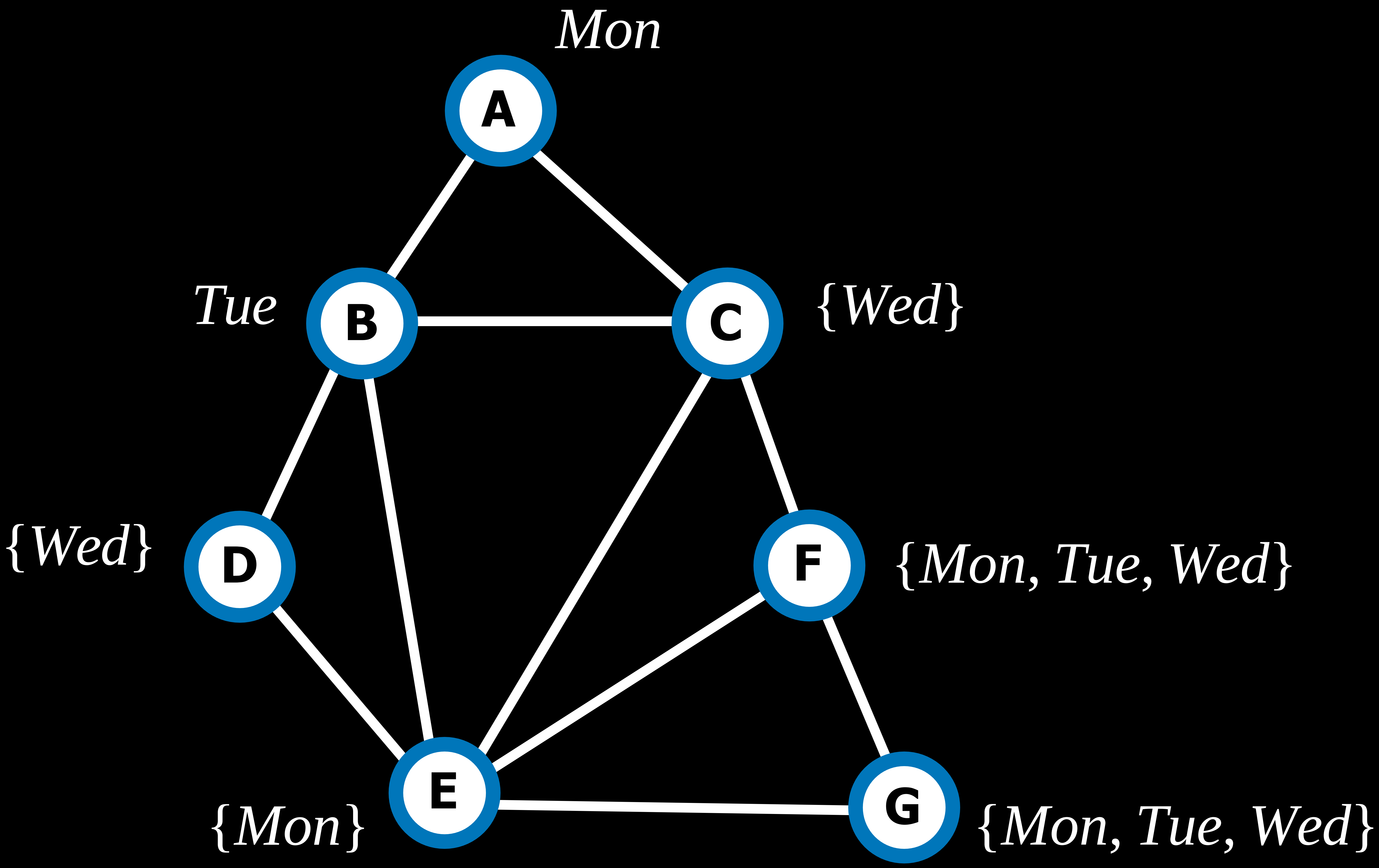
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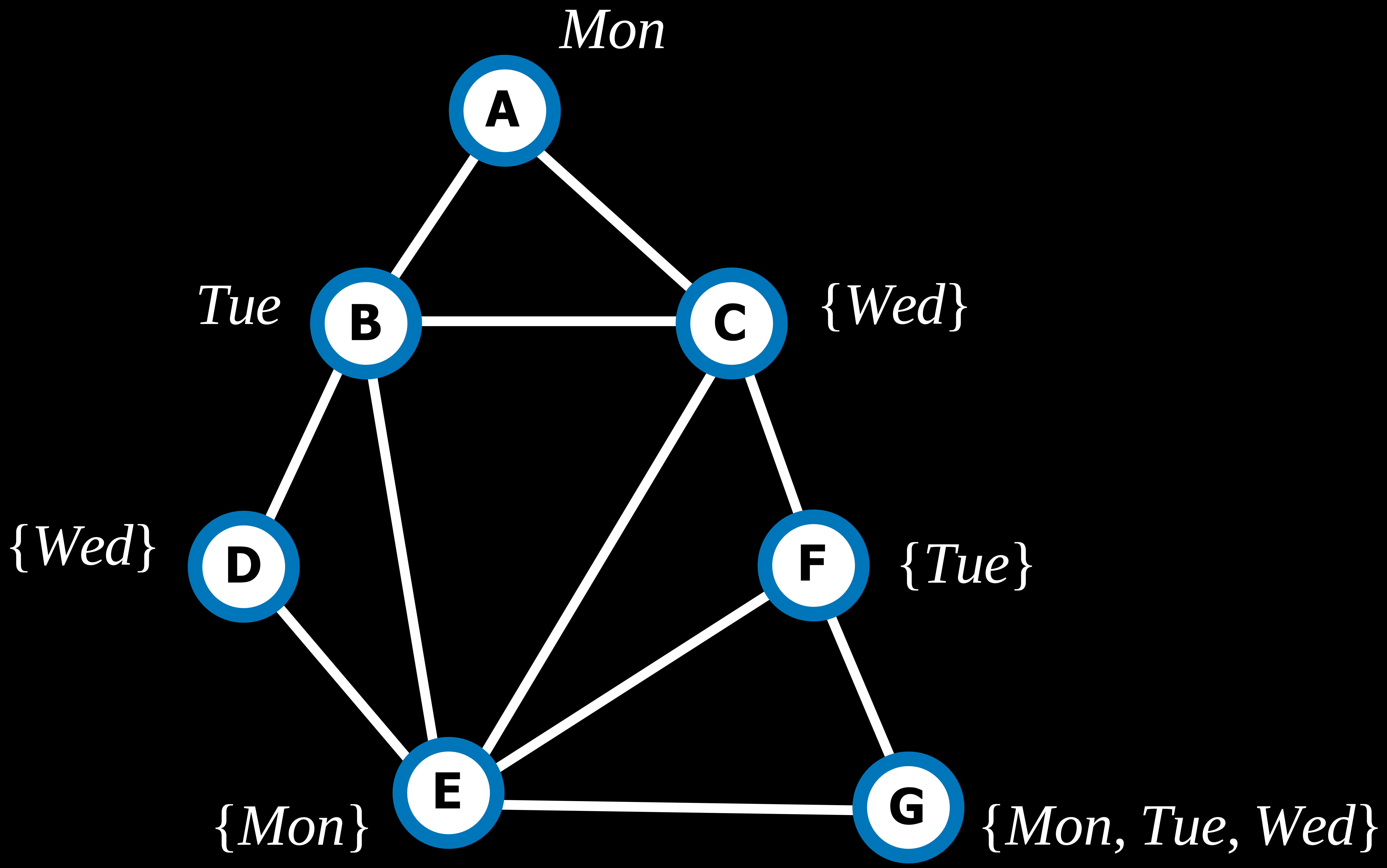
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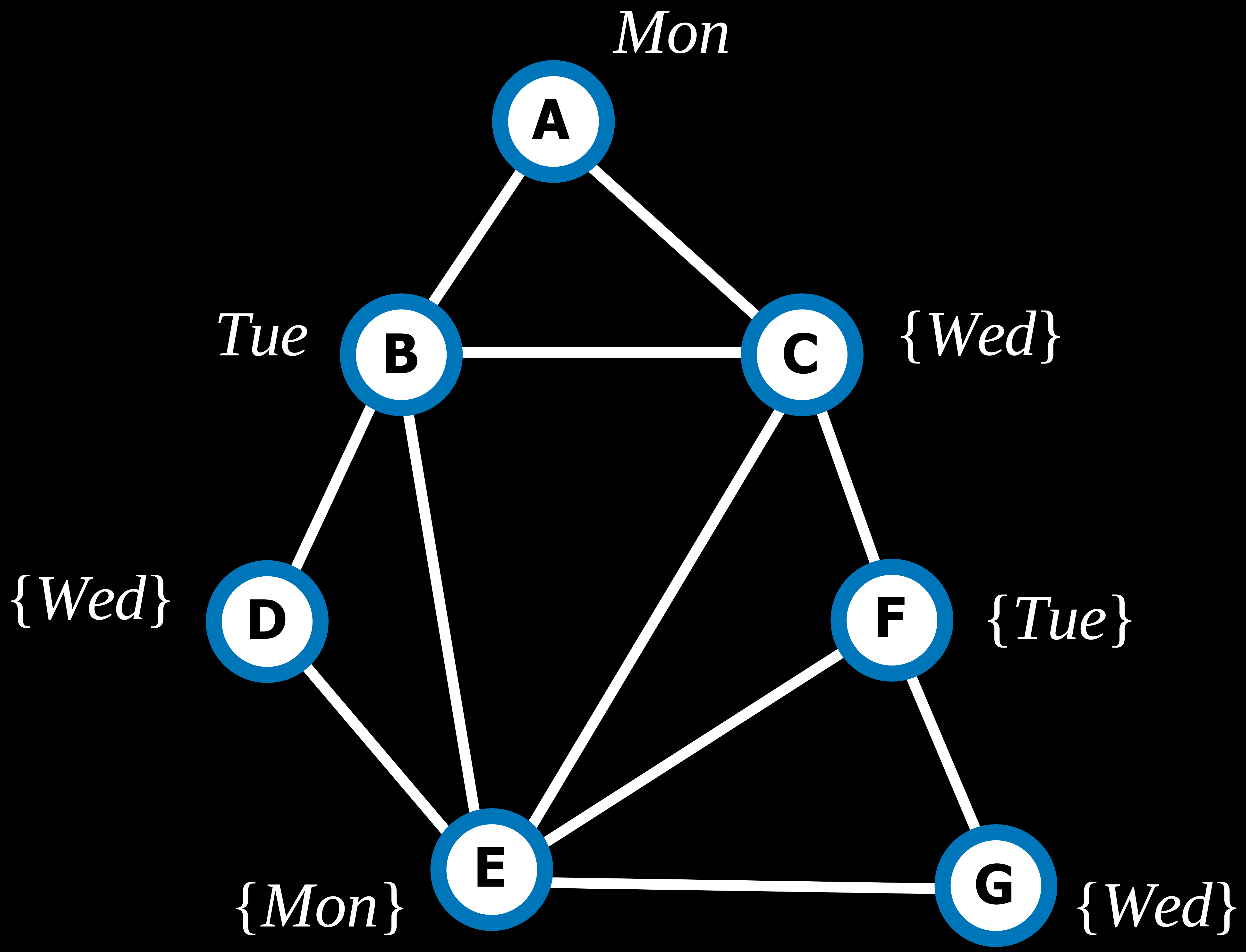
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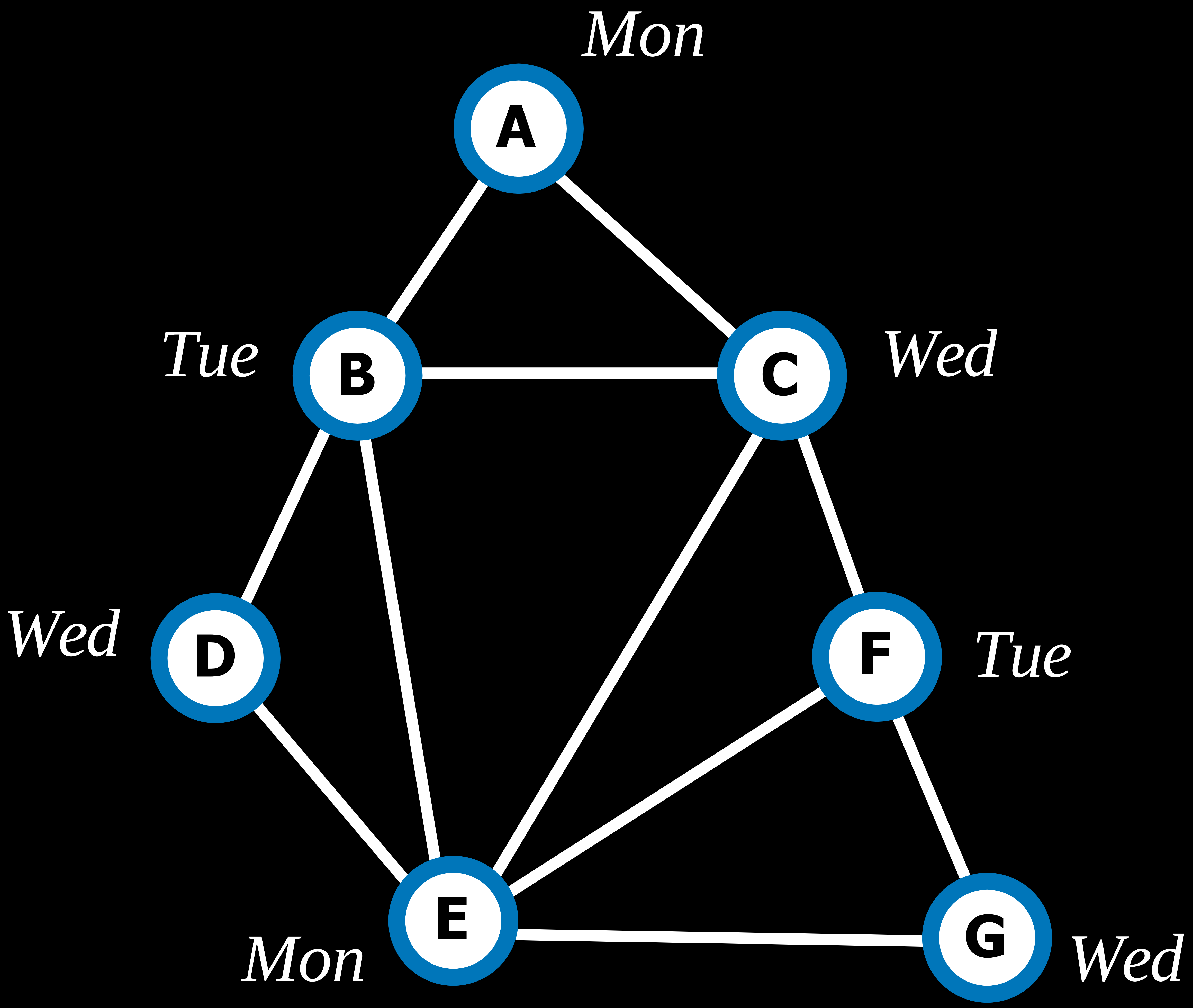
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maintaining arc-consistency

algorithm for enforcing arc-consistency every time we make a new assignment

maintaining arc-consistency

When we make a new assignment to X, calls $\mathrm{AC\text{-}3}$, starting with a queue of all arcs (Y, X) where Y is a neighbor of X

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
    return failure
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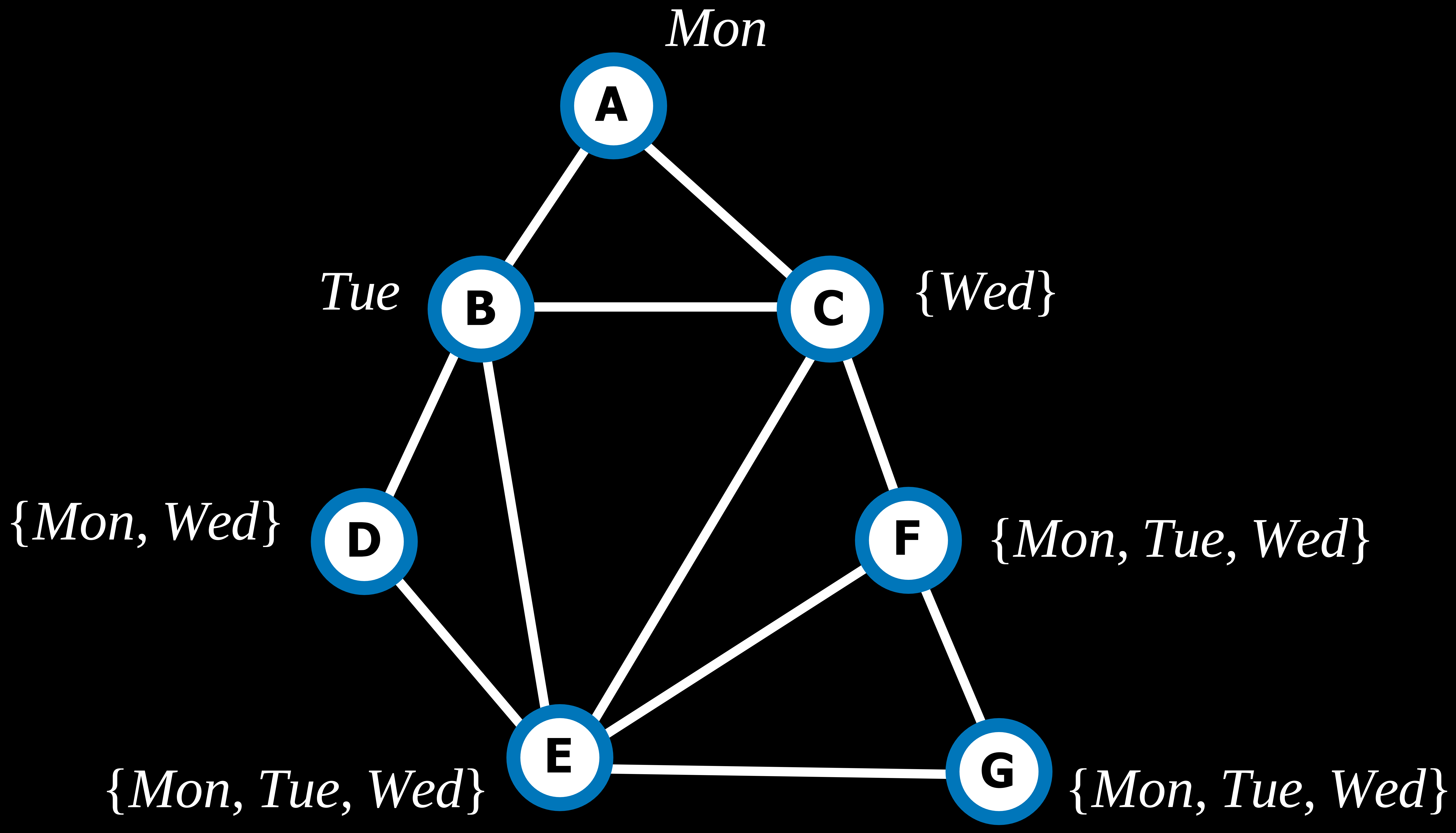
SELECT-UNASSIGNED-VAR

- **minimum remaining values (MRV)** heuristic: select the variable that has the smallest domain
- **degree** heuristic: select the variable that has the highest degree



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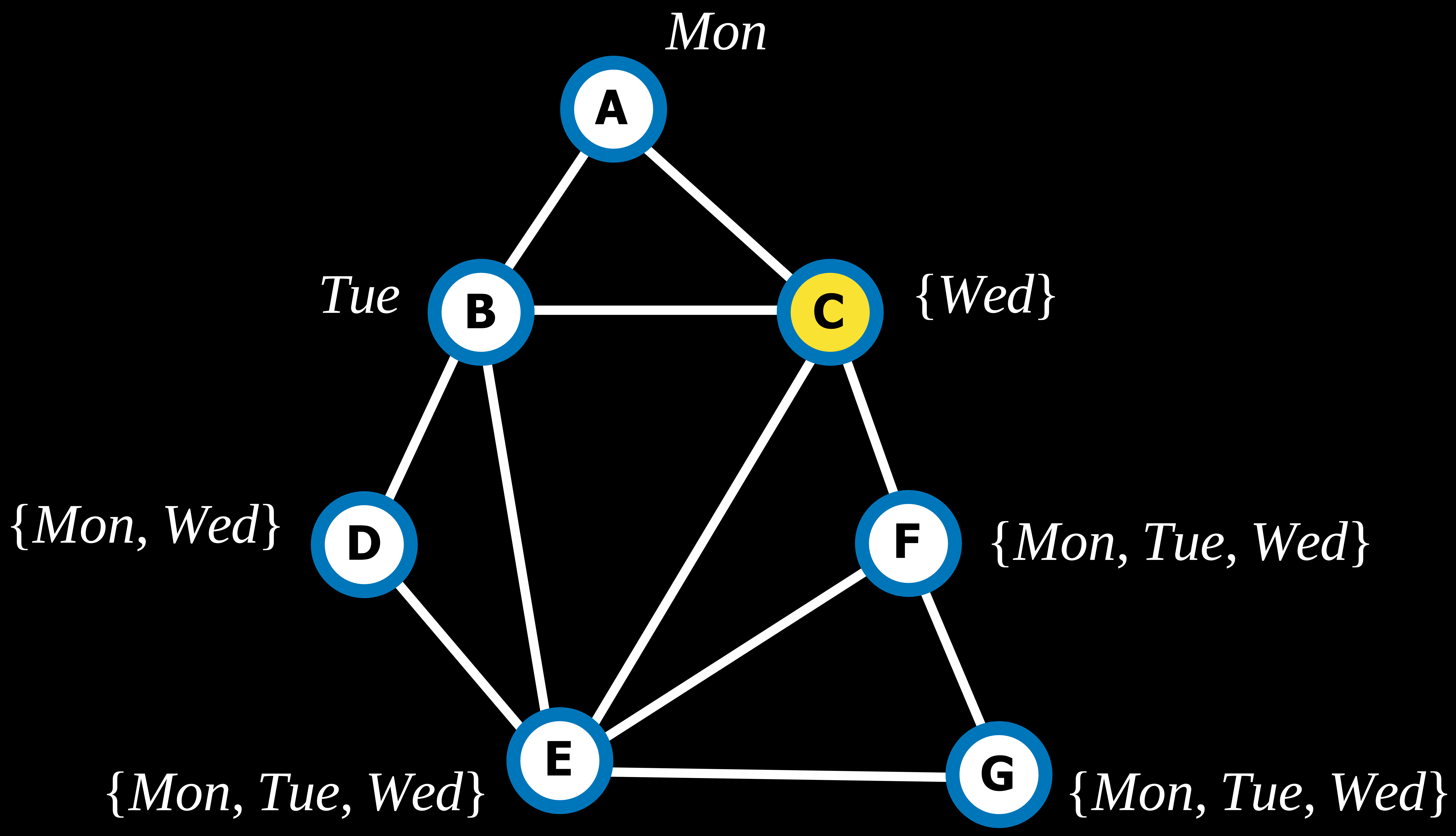
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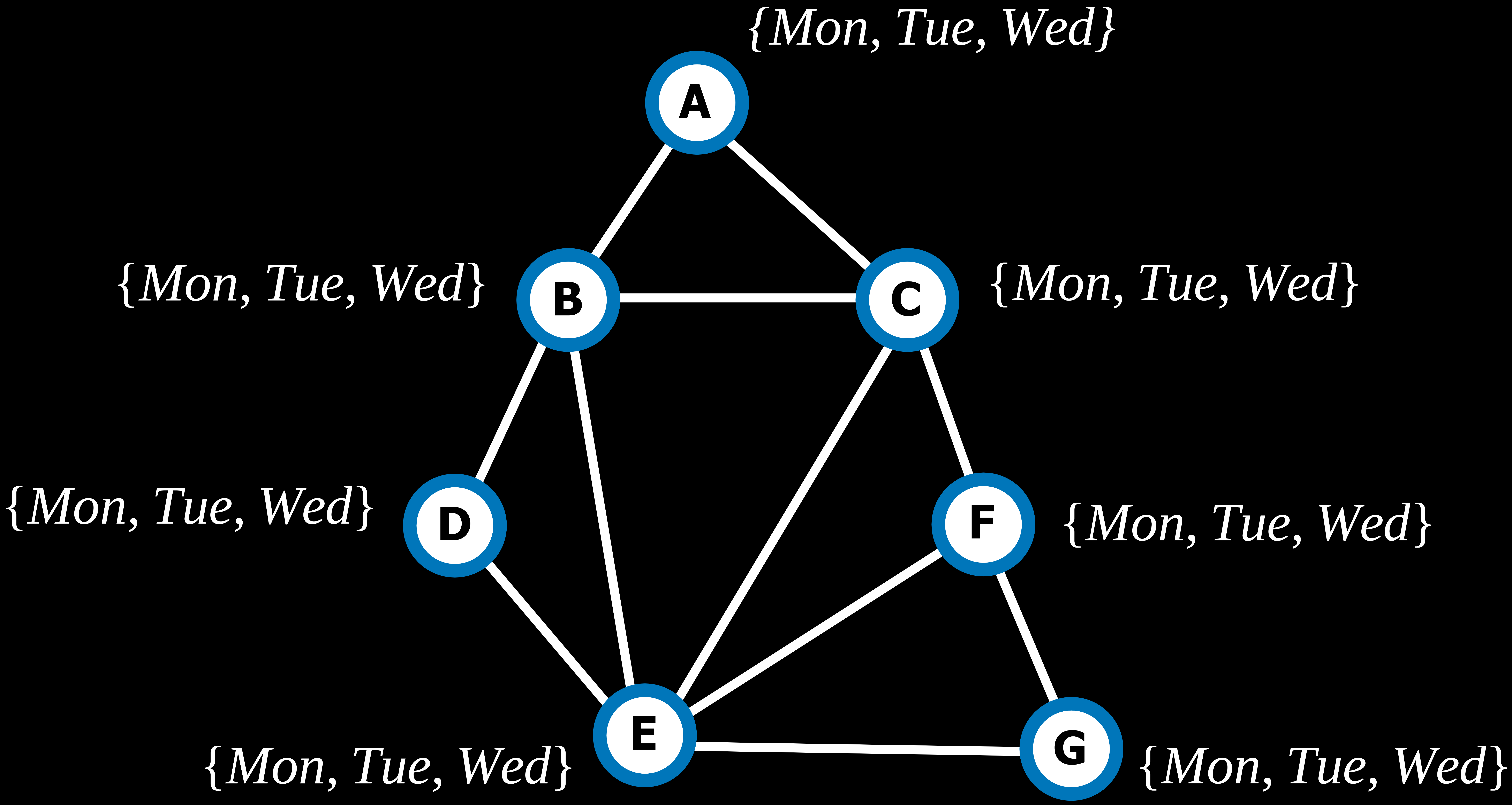
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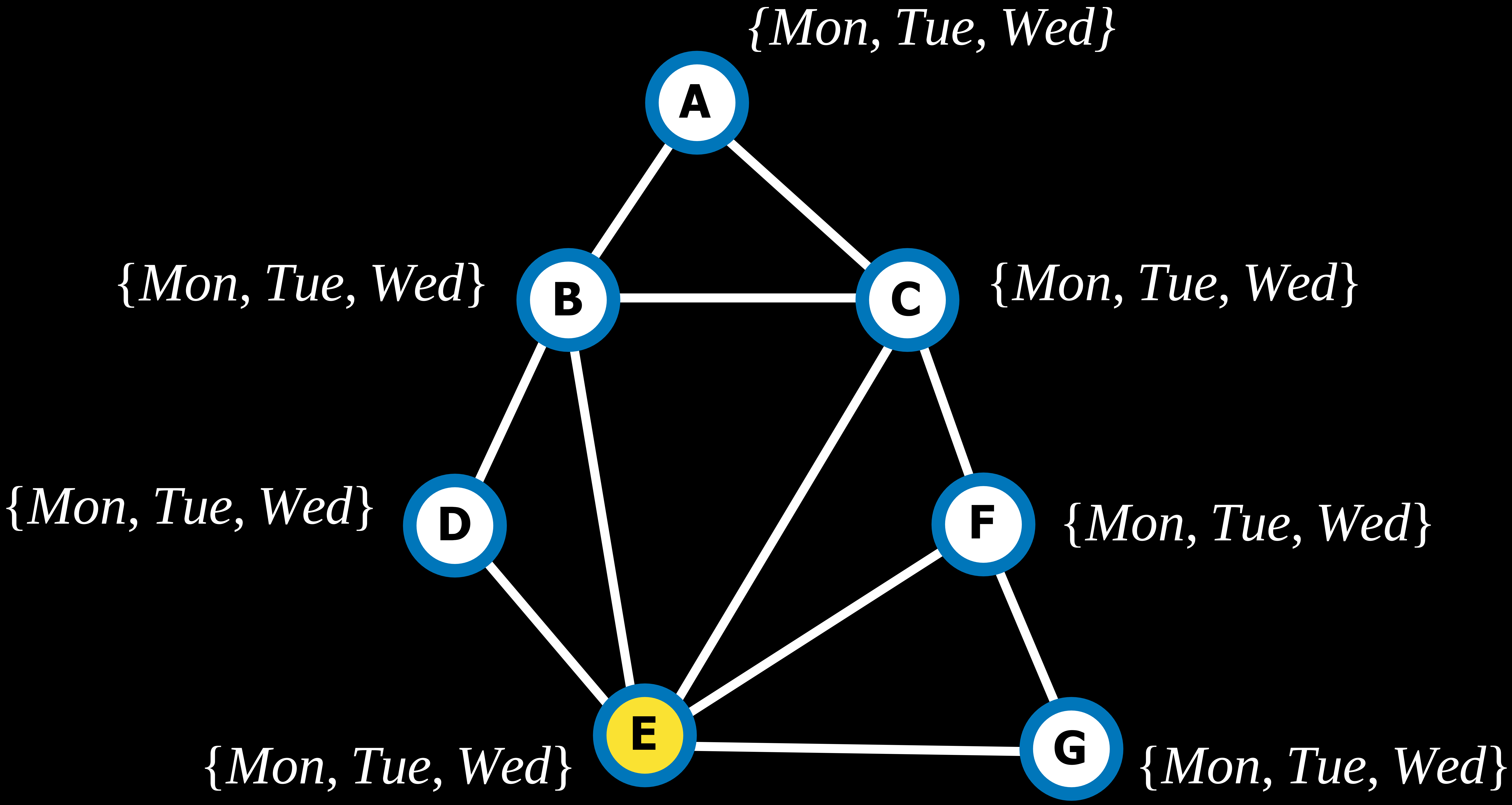
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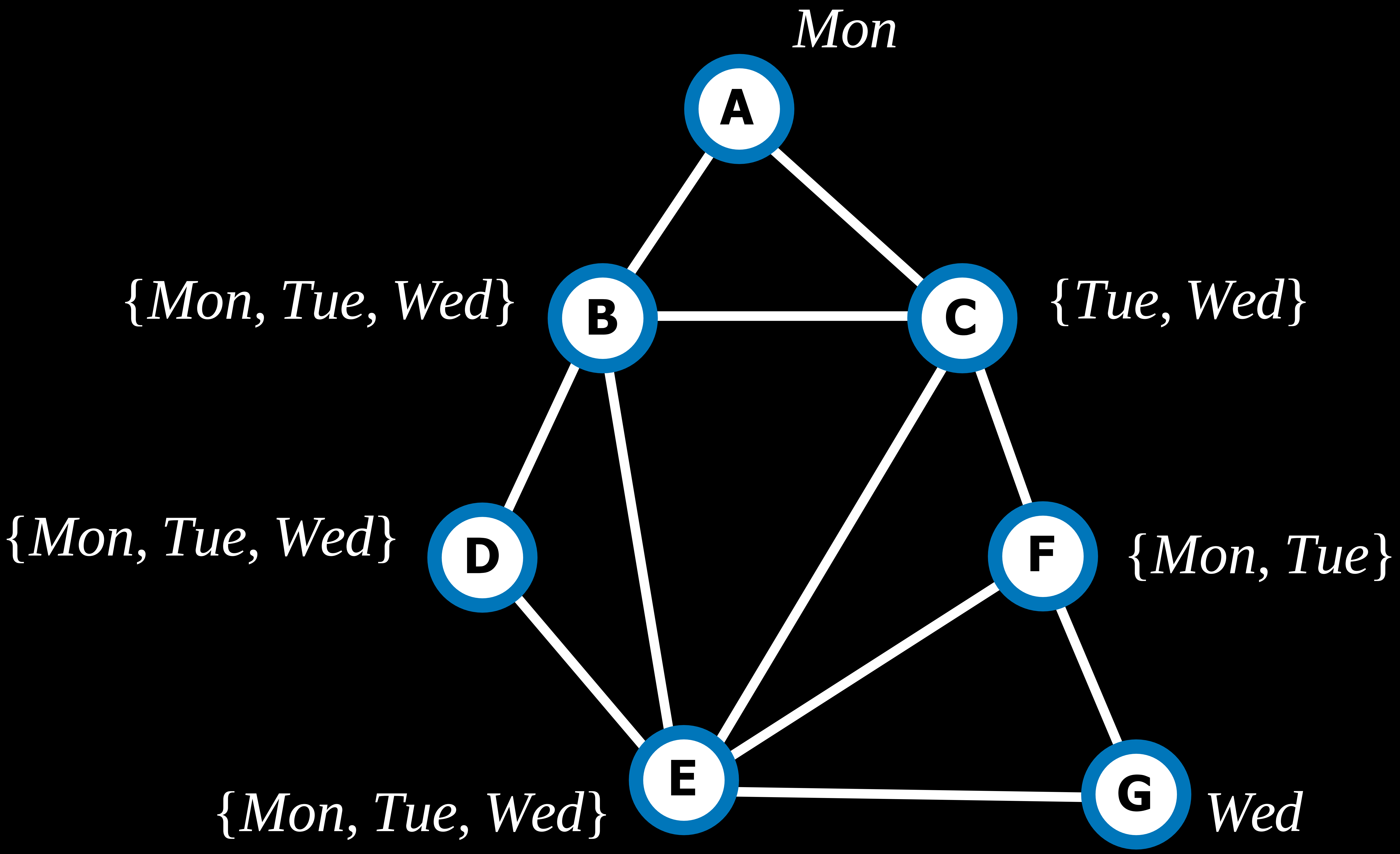
DOMAIN-VALUES

- **least-constraining values** heuristic: return variables in order by number of choices that are ruled out for neighboring variables
    - try least-constraining values first



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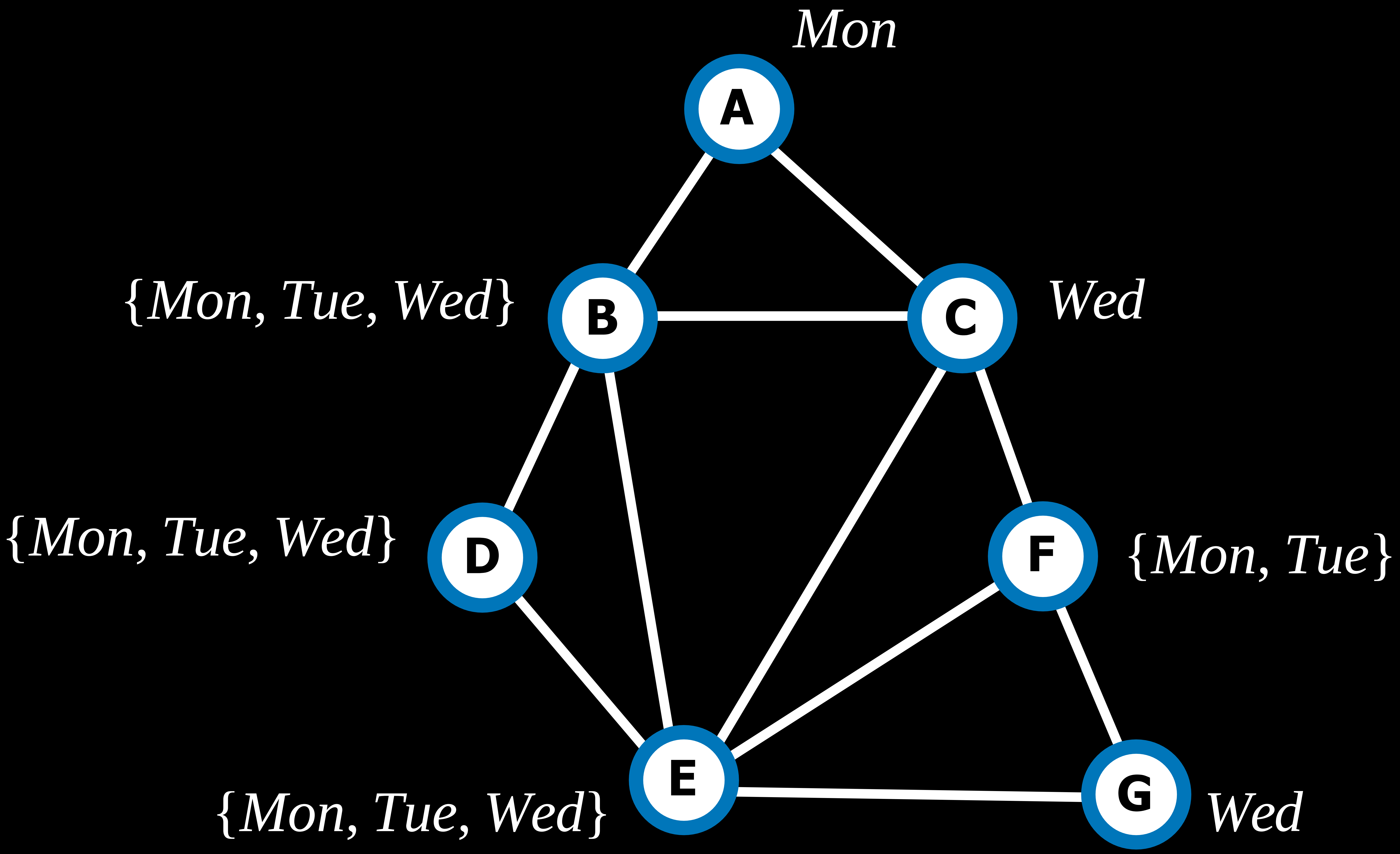
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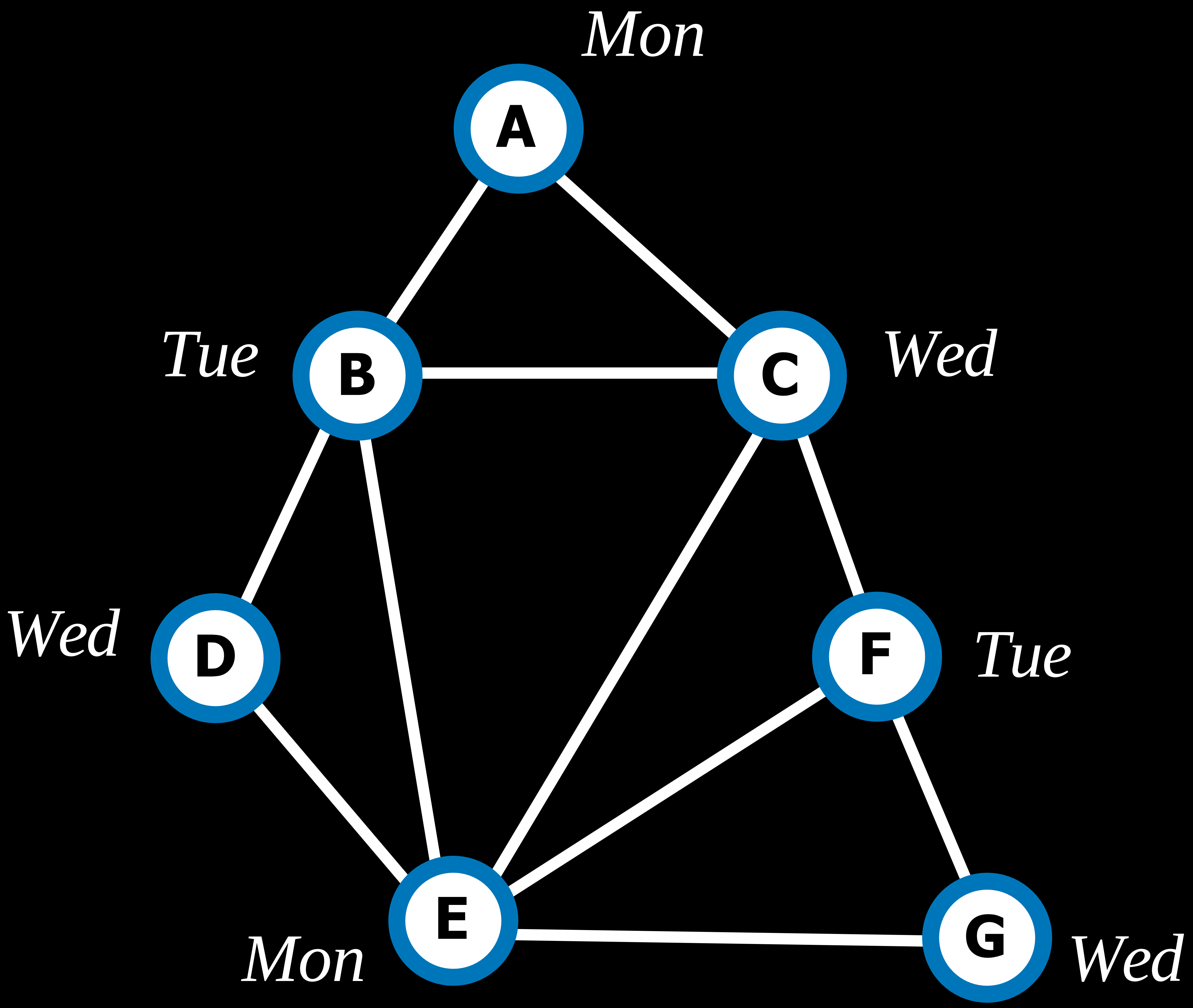
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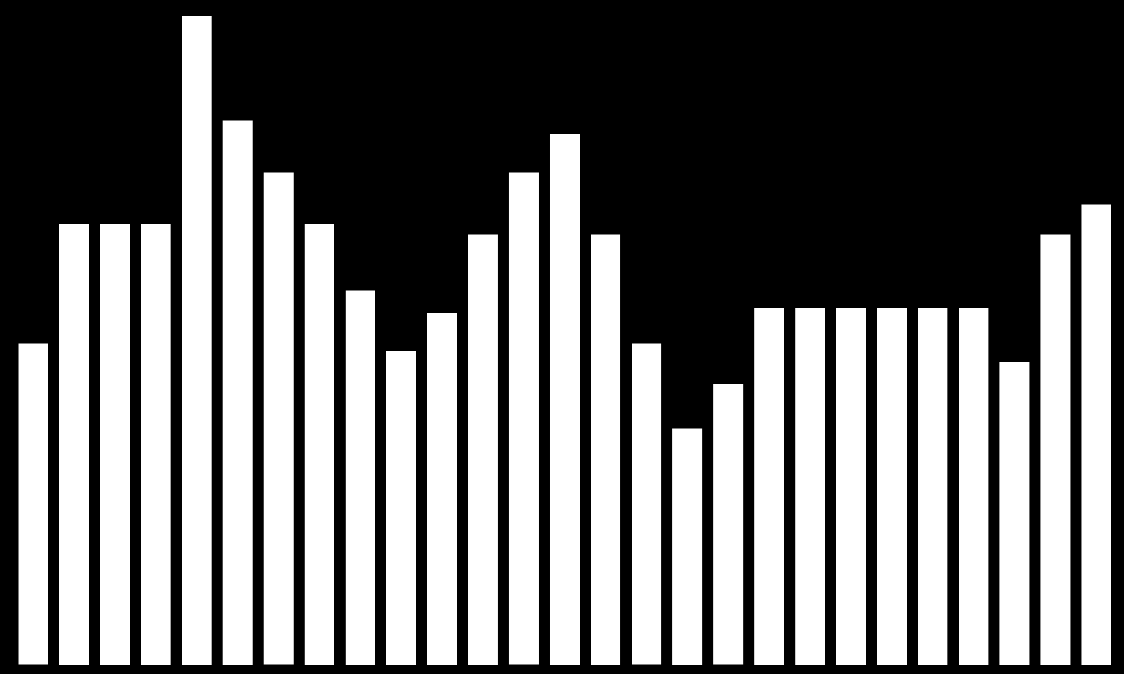
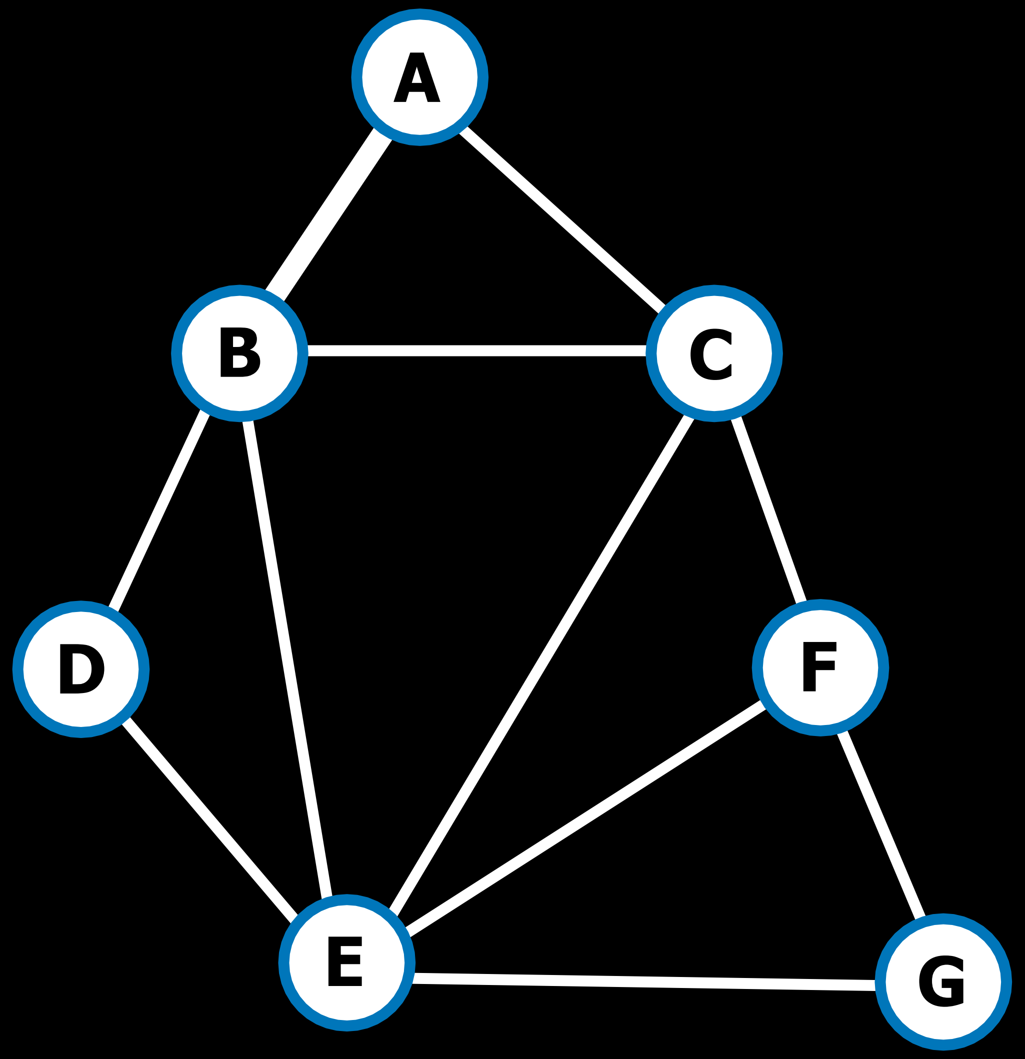
Problem Formulation



Local Search

$$50x_1 + 80x_2$$
$$5x_1 + 2x_2 \leq 20$$
$$(-10x_1) + (-12x_2) \leq -90$$

Linear Programming



Constraint Satisfaction