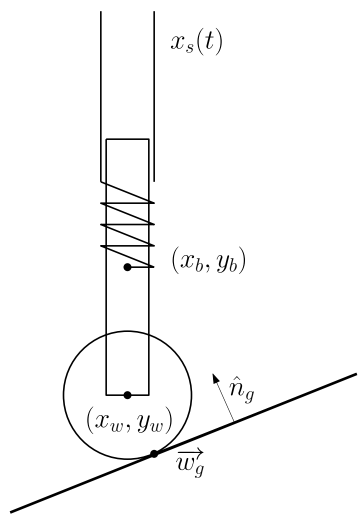
PGS Suspension Solver

December 17, 2021

The suspension can be modeled as two rigid bodies, the bar and the wheel. The bar is connected to a socket with a prismatic constraint, and the socket moves with some specified motion $x_s(t)$. The wheel connects to the bar with positional constraint. The bar is attached to a spring at its center of mass, which acts as an external force on the bar.



Each constraint introduces linear and angular impulses:

$$\overrightarrow{\lambda}_i = \begin{bmatrix} \lambda_{lin} \\ \lambda_{rot} \end{bmatrix}$$

The location of these impulses depends on the constraint, and will change the form of the Jacobian.

The first constraint we consider is the prismatic constraint of the bar/socket system. We have that

$$x_b = x_s(t) \implies u_b = \dot{x}_s(t)$$

$$\omega_b = \omega_s 0$$

The second constraint is the position constraint for the wheel/bar system.

$$x_w - \left(x_b + \frac{L}{2}\sin\theta_b\right) = 0 \implies u_w - u_b - \frac{L}{2}\cos\theta_b\omega_b = 0$$

$$y_w - \left(y_b - \frac{L}{2}\cos\theta_b\right) = 0 \implies v_w - v_b - \frac{L}{2}\sin\theta_b\omega_b = 0$$

We first solve for the constraint impulses:

$$JV_1 = J(V_0 + M^{-1}F_{ext}\Delta t) + JM^{-1}J^T\lambda$$

and then update the velocities:

$$V_1 = V_0 + M^{-1} F_{ext} \Delta t + M^{-1} J^T \lambda$$

All components of JV_1 are zero except where we have elastic collisions or a specified motion, such as that of the bar/socket system. For this constraint, we have that

$$JV_1^{c0} = \begin{bmatrix} u_s(t + \Delta t) \\ 0 \\ 0 \end{bmatrix}$$

We can add Baumgarte stabilization to the bar/wheel constraint by add a term that penalizes constraint violations:

$$JV_1^{c1} = \begin{bmatrix} -\beta \left[x_w - \left(x_b + \frac{L}{2} \sin \theta_b \right) \right] \\ -\beta \left[y_w - \left(y_b - \frac{L}{2} \cos \theta_b \right) \right] \\ 0 \end{bmatrix}$$

The inverse mass matrix is

$$M^{-1} = \begin{bmatrix} \frac{1}{m_b} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_b} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_b} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_w} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_w} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{I_w} \end{bmatrix}$$

The forcing vector is

$$F_{ext} = \begin{bmatrix} f_{sp,x} \\ f_{sp,y} - m_b g \\ 0 \\ 0 \\ -m_w g \\ 0 \end{bmatrix}$$

TODO collision constraint

TODO elastic collision can be handled by not setting to zero