PGS Suspension Solver

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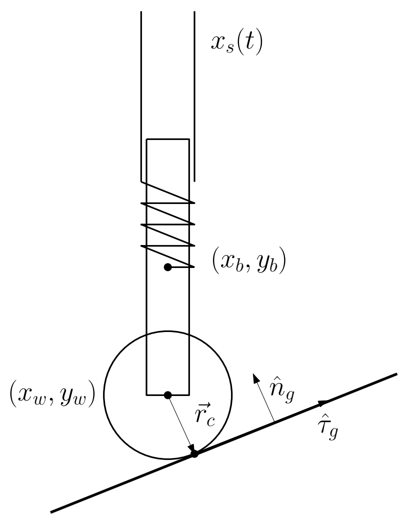
The suspension can be modeled as two rigid bodies, the bar and the wheel. The bar is connected to a socket with a prismatic constraint, and the socket moves with some specified motion $x_s(t)$. The wheel connects to the bar with positional constraint. The bar is attached to a spring at its center of mass, which acts as an external force on the bar.



The velocity state vector contains the linear and angular velocities for the bar and wheel:

$$V = \begin{bmatrix} u_b \\ v_b \\ \omega_b \\ u_w \\ v_w \\ \omega_w \end{bmatrix}$$

The first constraint we consider is the prismatic constraint of the bar/socket system. We have that

$$x_b = x_s(t) \implies u_b = \dot{x}_s(t)$$

$$\omega_b = \omega_s = 0$$

The second constraint is the position constraint for the wheel/bar system.

$$x_w - \left(x_b + \frac{L}{2} \sin\theta_b \right) = 0 \implies u_w - u_b - \frac{L}{2} \cos\theta_b \omega_b = 0$$

$$y_w - \left(y_b - \frac{L}{2} \cos\theta_b \right) = 0 \implies v_w - v_b - \frac{L}{2} \sin\theta_b \omega_b = 0$$

The third constraint is the contact constraint between the wheel and ground, and is only active if the wheel and ground are in contact:

$$(\vec{u}_w + \vec{\omega}_w \times \vec{r}_c) \cdot \hat{n}_g = 0$$

Since the wheel is a perfect circle, the contact vector $\vec{r}_c$ is always orthogonal to the ground normal, so $(\vec{\omega}_w \times \vec{r}_c) \cdot \hat{n}_g = 0$. Since the contact impulse can only push the wheel away, we make sure that the corresponding component of λ is greater than zero.

The final constraint is the friction constraint:

$$(\vec{u}_w + \vec{\omega}_w \times \vec{r}_c) \cdot \hat{\tau}_g = 0$$

The component of λ corresponding to this constraint is bounded by the normal impulse multiplied by the coefficient of friction, but that is ignored for simplicity.

We first solve for the constraint impulses λ by solving the system:

$$JV_1 = J(V_0 + M^{-1}F_{ext}\Delta t) + JM^{-1}J^T\lambda$$

and then update the velocities:

$$V_1 = V_0 + M^{-1}F_{ext}\Delta t + M^{-1}J^T\lambda$$

The inverse mass matrix is

$$M^{-1} = \begin{bmatrix} \frac{1}{m_b} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_b} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_b} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_w} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_w} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{I_w} \end{bmatrix}$$

The forcing vector is

$$F_{ext} = \begin{bmatrix} f_{sp,x} \\ f_{sp,y} - m_b g \\ 0 \\ 0 \\ -m_w g \\ 0 \end{bmatrix}$$

All components of JV_1 are zero except where we have elastic collisions or a specified motion, such as that of the bar/socket system. For this constraint, we have that

$$JV_1^{c0} = \begin{bmatrix} u_s(t + \Delta t) \end{bmatrix}$$

We can add Baumgarte stabilization to the bar/wheel constraint by add a term that penalizes constraint violations:

$$JV_1^{c1} = \begin{bmatrix} -\beta \left[x_w - \left(x_b + \frac{L}{2}\sin\theta_b\right)\right] \\ -\beta \left[y_w - \left(y_b - \frac{L}{2}\cos\theta_b\right)\right] \end{bmatrix}$$

We can also add Baumgarte stabilization to the wheel contact constraint by adding a term proportional to the penetration depth of the wheel into the ground d_{wg}:

$$JV_1^{c2} = \begin{bmatrix} -\beta d_{wg} \end{bmatrix}$$

TODO elastic collision can be handled by not setting JV_1 to zero