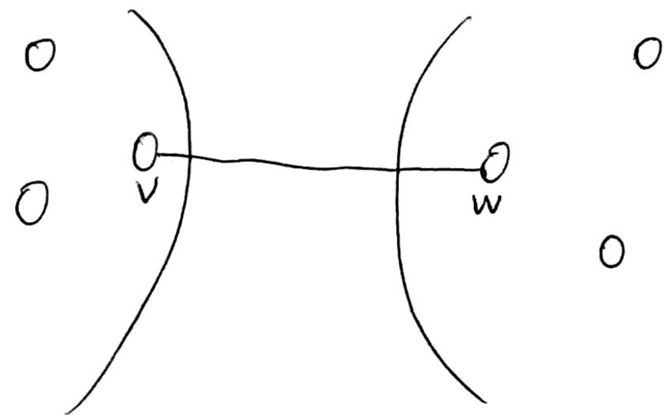
2. (a)

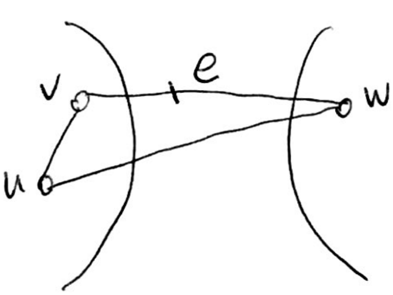


Since edge e is unique, the two nodes of e is (v, w), then we can partition the graph G into two parts.

Because e is the only edge connect the two parts, if e $\notin$ MST, then the MST is not connected and contradictory to the definition of MST must be a tree.

(b). statement: If G has a cycle that has a unique heaviest edge e, then e cannot be part of any MST.

proof: Assume the statement is false, there's a MST that has e. Since e is unque, removing e from MST would cause disconnection of MST, and ~~before~~ be cut into two parts. Because e is in a cycle, we can find another edge in the cycle adding into the MST to be reconnected, the new tree now has less total weight, contradiction.



(c). statement:

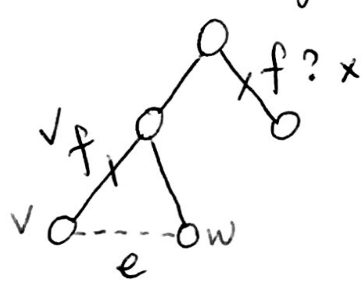
Let e be any edge of minimum weight in G, then e must be part of some MST.

proof: prove by kruskal Algorithm, that alway process edges in the order of their costs (starting from the least).

(d). Statement:

If the lightest edge in a graph is unique, then it must be part of every MST.

proof: suppose there is a MST that not inculde the lightest edge e:(v, w).



if we add e into this MST. (connect the node v and w). there definetly would form a cycle.

We want to ~~find~~ remove anothe edge in MST to be replaced by e, can we replace arbitrary edge f? NO, only the edge in the cycle can be replaced, otherwise will break the MST, then contradictary to that MST must be connected. Now we have a new tree with less cost, contradictary to assumption. prove done