I know that this is overkill but the problem basically lived in my head rent free since our section today and I just *had* to write about it:

For the case with n balls and n indistinguishable bins, what you can basically think of this problem as is asking how many ways are there to divide n into a set of integers that sum to n. Coincidentally, there's this function in number theory called the partition function $p(n)$ (see https://en.wikipedia.org/wiki/Integer_partition) which does exactly that.

In the wikipedia article, it explains that there is no known closed form expression for n, so there's no way to write down $p(n)$ without using infinite sums and other complicated mathematical functions.

If you're satisfied with this as an answer, that's perfectly fine. Me, however, I was interested now in what would the number of ways be if you had n balls into $n-1$ bins now, and I reasoned that this is basically the same as finding the number of ways of partitioning $n-1$, then multiplying by n. You can argue this is the case by first taking away one ball, then the remaining $n-1$ balls into $n-1$ bins gives us $p(n-1)$ ways, and for every partition there's n ways we can throw this last ball into one of the bins giving us $np(n-1)$ in total.

Ok, what about $n-k$ bins now? Well, this would be the same as the previous case, except now we take away k balls, so the partition is now $p(n-k)$. Then, we have k balls left, to distribute into $n-k$ bins, so this is the balls and bins formula we had. Putting these two together, we have:

$$N = \binom{k+(n-k)-1}{k} p(n-k) = \binom{n-1}{k} p(n-k)$$

I think it's safe to say that you won't have to be worried about a problem like this appearing on an exam.