Physics 5CL

Heat Flow and Calorimetry

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Distribution of Efforts:

Initials:	Experiment 2	Experiment 3
ED	1	1
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Experiment 2

In Experiment 2, we conducted a mixing experiment to determine the latent heat of fusion of ice, based on the known value of the heat capacity of water. We used the following pieces of equipment:

- Tub of ice
- Shelolab water heater
- 2x 2 nested styrofoam cups
- Mass scale
- Logger Pro software
- Temperature probe
- LabQuest Mini

Background

The temperature T of an object is correlated with its internal energy U. When an amount of heat Q is added to the object, it increases in energy and thus temperature. The amount of heat needed to increase the temperature of an object (of constant volume) by one degree Kelvin is its "heat capacity" C_V:

$$C_V = \frac{dQ}{dT}$$

Heat capacity is a material property, so it is useful to normalize it with respect to mass. The "specific heat capacity" c is the amount of heat needed to raise the temperature of *one mass unit* (normally grams or kilograms) of a material by one degree Kelvin. While the heat capacity of an object is not constant with respect to temperature and pressure, the heat capacities of water and ice change negligibly in the temperature ranges we'll be using them in, so we can treat them as constant.

The specific heat capacity equation relates the mass of a sample m, the amount of heat provided to it Q, and its change in temperature ΔT via its specific heat capacity c:

$$Q = mc\Delta T$$

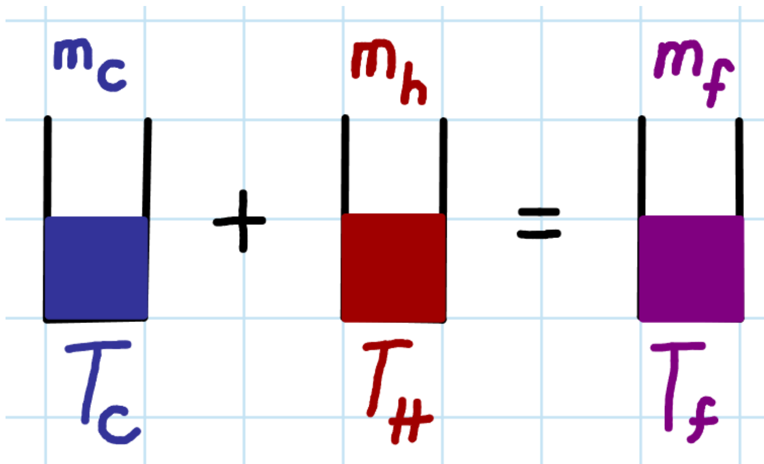
During a phase change, an object gains energy as it is heated, but it has no change in temperature. Therefore, the heat capacity is undefined, and a different quantity must be used to understand the transformation. The "latent heat" of an object is the amount of heat required to complete the object's phase transition; the "specific latent heat" L of a material is the amount of heat required to phase transition one mass unit of the material from one phase to another. The latent heat required to transition one mass unit of a material from the solid to the liquid phase, i.e. to melt it, is known as the latent heat of fusion.

The latent heat equation relates the mass of a sample m and the heat provided to it Q via its specific latent heat L:

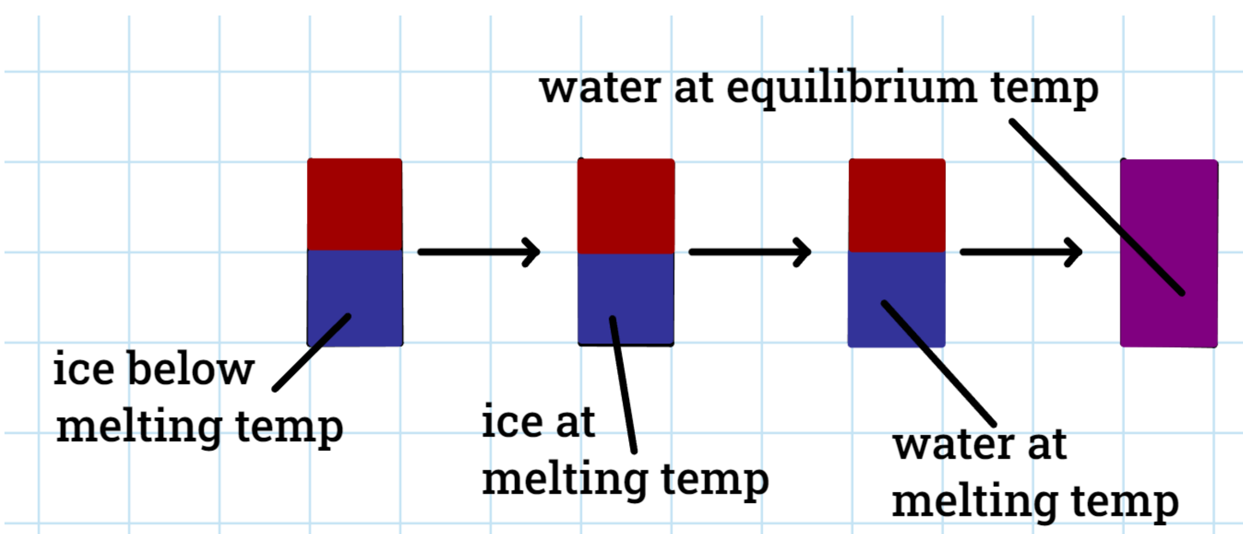
$$Q = mL$$

A mixing experiment allows one to calculate the specific heat capacity or specific latent heat of a substance undergoing temperature change or a phase transition. Two samples of the same material are mixed, and the heat lost from the hotter sample Q_H must match the heat gained by the colder sample Q_C. Thus with two samples of a material (in the same phase), one hot (H) and one cold (C), we have:

$$Q_H = m_H c \Delta T_H$$
$$Q_C = m_C c \Delta T_C$$
$$m_H c \Delta T_H = m_C c \Delta T_C$$



By knowing the masses and temperature changes (final temperature - initial temperature) of the materials, the specific heat can be calculated. Now consider two samples, one hot (H) and one cold (C), of a material which are in different phases; for example, consider water (specific heat c_w) and ice (specific heat c_i). The mixture will go through three states: heating the ice to its melting temperature (temperature change ΔT_{C1}), melting the ice, and heating the resulting cold water to equilibrium with the hot water (temperature change ΔT_{C2}).



The total heat lost by the hot water is equal to the heat gained by the cold ice, the melting ice, and the cold water. Thus we can set up an equation describing this process:

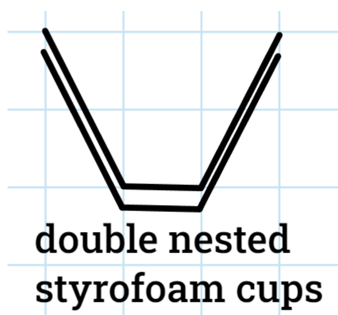
$$m_H c_w \Delta T_H = m_C c_i \Delta T_{C1} + m_C L + m_C c_w \Delta T_{C2}$$

Knowing the values of c_w and c_i, we can calculate the latent heat of fusion of water L.

Procedure

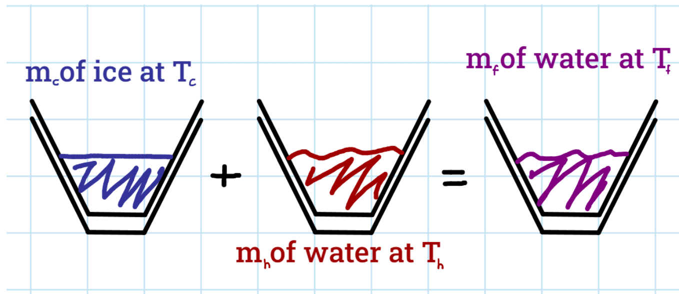
Ultimately, our goal was to conduct an experiment where we mixed a sample of water with a sample of ice while measuring the initial and final conditions. Each such trial gave one value for the latent heat of ice, so to increase our confidence in our results, we conducted four trials. The procedure for each one was identical.

We first created two cups, each formed from two nested styrofoam cups. Styrofoam is a highly insulating material, heat-wise, so by using these cups to contain our mixing ingredients, we limited the amount of heat exchanged with the outside world. This increased the isolation of our system and thus the accuracy of our results. We measured the mass of these combination cups to be 10 ± 1 g.



For each trial, we filled the first of these two cups with ice, and measured the mass of the ice using the mass scale (we subtracted off 10 g from the displayed value to account for the mass of the cup). We then inserted the temperature probe into the ice, and when the probe reached equilibrium we recorded its temperature. At the same time, we filled the other cup with hot water and measured the mass of water in the same way as the ice. We then measured and recorded the water temperature. We did this quickly, with as much parallelization as possible, to limit how much heat exchange occurred with the environment while we set up our measurements.

In total, we measured the initial mass m and temperature T of both the ice and hot water samples, m_c and T_c and m_h and T_h respectively.



With the initial conditions measured, we then poured the cup with water into the cup with ice, placing the cup with the resulting mixture on the mass scale. After measuring the mass of the mixture, we inserted the temperature probe and waited for the mixture to reach equilibrium. After the temperature stopped fluctuating, we recorded the final temperature of the mixture.

The data for our four trials is collected in the following table:

Trial #	m_c (g)	m_h (g)	m_f (g)	T_c (°C)	T_h (°C)	T_f (°C)
1	65 ± 1	156 ± 1	218 ± 1	-1.3 ± 0.1	46.0 ± 0.1	10.7 ± 0.1
2	71 ± 1	175 ± 1	243 ± 1	-1.3 ± 0.1	45.7 ± 0.1	11.9 ± 0.1
3	39 ± 1	303 ± 1	339 ± 1	-1.0 ± 0.1	45.6 ± 0.1	33.0 ± 0.1
4	124 ± 1	148 ± 1	263 ± 1	-1.4 ± 0.1	45.3 ± 0.1	-0.1 ± 0.1

We kept the initial temperatures of the ice and hot water relatively constant for each trial, varying the ratio between the two in order to get a good spread of final temperatures in the hope that this would add robustness to our calculated latent heat value.

Unfortunately, we did not add a sufficient mass of hot water in the fourth trial, so the ice did not fully melt. Since we do not have a way of determining how much ice melted, we cannot use the data from that trial.

Something to note is that our initial temperatures for the ice go below the melting point of distilled water (0°C): in class, we assumed that the ice, being made from tap water, was simply impure (had some amount of minerals dissolved in it) and so its melting point was lower than that of distilled water. We found this to be likely because our ice bucket had some water in it, implying that the ice in it was in the process of melting, and so would be at its melting temperature. However, it is also possible that the ice in contact with the water was melting at 0°C, while the rest of the ice, being somewhat fragmented and separated by air pockets, was able to remain at a lower temperature, as the air prevented it from being in thermal contact with the ice that was melting. We explore the effects of this below.

Analysis

In our mixing experiment, the total heat lost by the hot water is the same as that absorbed by the ice as it melts, added to the amount absorbed as it then heats to the equilibrium point. We created the following equation to describe this behavior:

$$m_H c_w \Delta T_H = m_C L + m_C c_w \Delta T_C$$

We will take the value of $c_w = 4.184 \pm 0.001\, J/g°C$ to be given. With this, we can solve for the latent heat:

$$L = \frac{c_w[m_H(T_H - T_f) - m_C(T_f - T_C)]}{m_C}$$

We now calculate the latent heat of each of our trials:

Trial #	L (g/°C)
1	304 ± 6
2	293 ± 6
3	267 ± 12
Mean	288 ± 11

Note that the error α_L in the latent heat is calculated via standard error propagation, and that the error in the mean is the standard error calculated from the latent heat values, as it was the larger standard error (compared to the standard error calculated from the latent heat errors).

The accepted value for the latent heat of fusion of distilled water is 334 ± 1 J/g. Unfortunately, none of our trials, nor the mean, agree with this value using the quadrature agreement test. However, it's possible that this is due to our assumption that the ice was at its melting temperature. If we instead assume that the ice had a melting temperature of 0°C, i.e. that it was colder than its melting temperature, then we find that the total heat lost by the hot water is the same as that absorbed by the ice as it heats to its melting point, melts, and then heats to the equilibrium point. We can modify our original equation equation to describe this behavior:

$$m_H c_w \Delta T_H = m_C c_i \Delta T_{C1} + m_C L + m_C c_w \Delta T_{C2}$$

We will now also take the value of $c_i = 2.093 \pm 0.001\, J/g°C$ to be given. With this, we can solve for the latent heat:

$$L = \frac{c_w[m_H(T_H - T_f) - m_C T_f] - c_i m_C \cdot (-T_C)}{m_C}$$

We now calculate the new latent heat values for each trial:

Trial #	Old L (g/°C)	New L (g/°C)
1	304 ± 6	307 ± 6
2	293 ± 6	296 ± 6
3	267 ± 12	269 ± 12

Mean	288 ± 11	291 ± 11

As we can see, this correction does improve our results. However, its effect is negligible in that it only changes our result by 3 or so J/g, and this is not sufficient to bring our value into quadrature agreement with the accepted value.

Our latent heat values for each trial are relatively similar to each other, yet are not accurate according to the accepted value; this indicates that we have some systematic error in our experiment which is causing our values to be lower than they should be. This could be due to some problem with our measuring equipment, or alternatively it could be due to less-than-perfect heat isolation of our mixing apparatus. Perhaps the initial temperatures change between measuring them and mixing the two cups: this would cause less heat transfer overall for the same process (melting the ice), which would cause our values to undershoot.

Conclusion
Overall, although we were unable to obtain an accurate value for the latent heat of fusion of ice, this experiment was still a success. We learned about the theory of mixing experiments and specific/latent heat values as well as how to use calorimetry equipment.

Experiment 3

In Experiment 3, we found the heat conductivity of a styrofoam cup in order to determine how good of a thermal insulator it was. We then found the heat conductivity of a metal can to determine its suitability for experiments which require very fast heat transfer. Our equipment included:

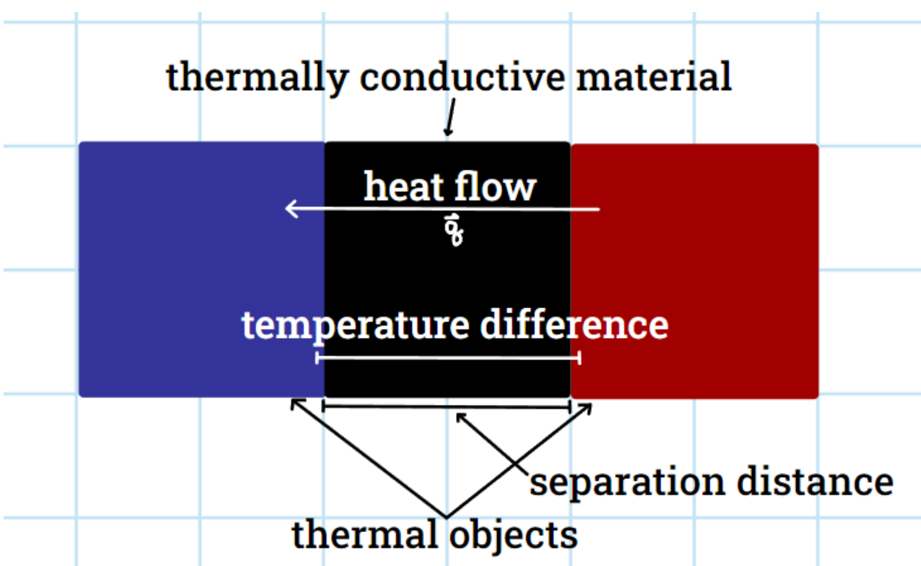
- Shelolab water heater
- 2x 2 nested styrofoam cups
- Mass scale
- Logger Pro software
- Temperature Probe
- LabQuest Mini

Background

From the specific heat capacity equation from Experiment 2, we know how to calculate the net amount of heat added to a material based on its net temperature change. By plotting the temperature of a material over time, we can also determine the rate of heat transfer into the object, assuming that we know its mass and specific heat, by taking the derivative of the time-temperature curve and multiplying by a factor of mass x specific heat.

$$H = m_W c_W \frac{dT}{dt}$$

The units of H are power, i.e. energy per time. This makes sense since heat is just energy in transit. Another interesting quantity is the rate of heat flow through some given material, i.e. how thermally conductive it is.



The thermal conductivity of a material, K, has units of W/m°C. If two thermal reservoirs sandwich a thermally conductive material, the thermal conductivity tells you what the rate of heat transfer is (hence the units of power) through the material per meter of material separating the two thermal objects, per degree of temperature difference between the two thermal objects.

Therefore, we have that

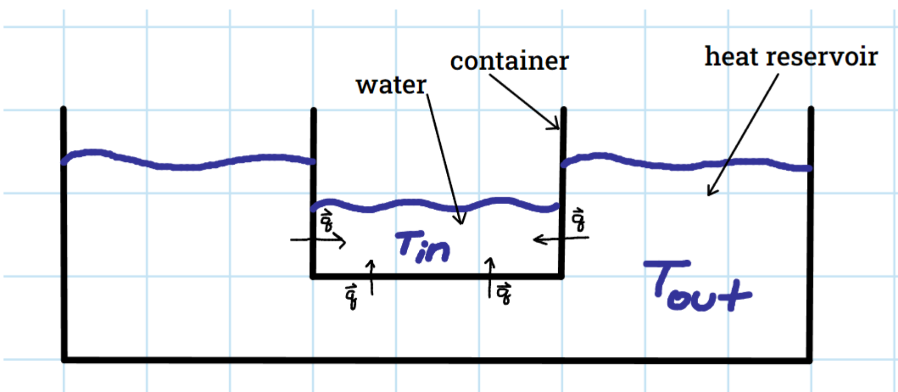
$$H = \frac{K*A}{l}\Delta T$$

Rearranging, we can solve for K:

$$K = \frac{H \cdot l}{A\Delta T}$$

To experimentally determine the thermal conductivity of a material, we place a heat reservoir on one side of the material and cold water on the other side. By tracking the temperature of the cold water over time, we can find the rate of heat flow into the water over time, and if we know the length and surface area of the material, as well as the instantaneous temperature difference between the reservoir and water, we can find K.

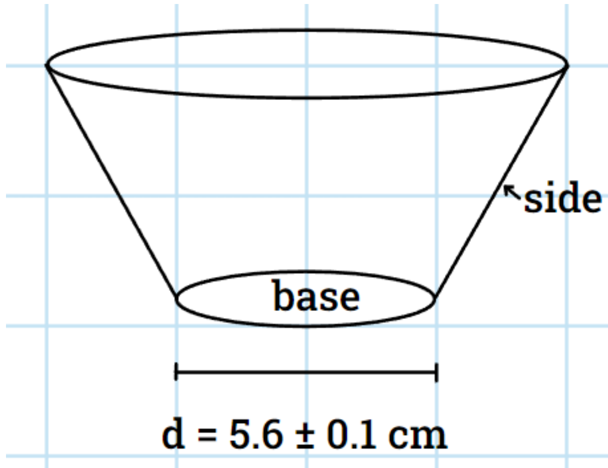
To accomplish this, we will fill a container made of the material with water, then heat the outside of the container by placing it in a water bath heat reservoir; this will cause heat to flow from the bath to the water in the container. Tracking the temperature of the water over time will allow us to find the rate of heat flow into the water over time.



From this, the thermal conductivity of the container can be found, assuming that we know its dimensions and the mass and specific heat of the water.

Procedure

We first measured the dimensions of a chopped up styrofoam cup which was identical to ours.



We measured the base of the styrofoam cup to have a thickness of 0.20 ± 0.01 cm, while the side had a thickness of 0.19 ± 0.01 cm. The base diameter was 5.6 ± 0.1 cm. For the purposes of heat flow, we modeled the styrofoam cup as a cylinder with a base radius of 3.0 ± 0.1 cm, slightly larger than the actual base radius, to account for the fact that the cup sides were very slightly sloped outwards; the surface area of this cylinder is negligibly different from the true surface area of the cup.

Therefore, the thermal conductivity of the styrofoam cup, as calculated in the prelab, is given by the equation:

$$K = \frac{m_W c_W \frac{dT}{dt} \ell}{(2\pi r h + \pi r^2)(T_{out} - T_{in})}$$

where $2\pi r h + \pi r^2$ is the surface area of the styrofoam cup, while l is its thickness.

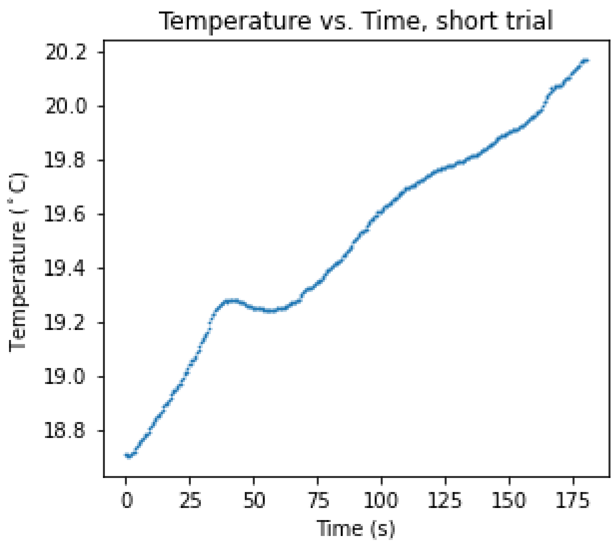
We next prepared the hot water bath (heat reservoir) and filled the styrofoam cup with cold water. We measured the mass of water inside of the cup by placing the cup on the scale and subtracting the cup mass of 10 ± 1 g, and measured its height by holding it up to the light (so we could see the water through the walls) with the ruler by its side and seeing which marking the water level was next to.

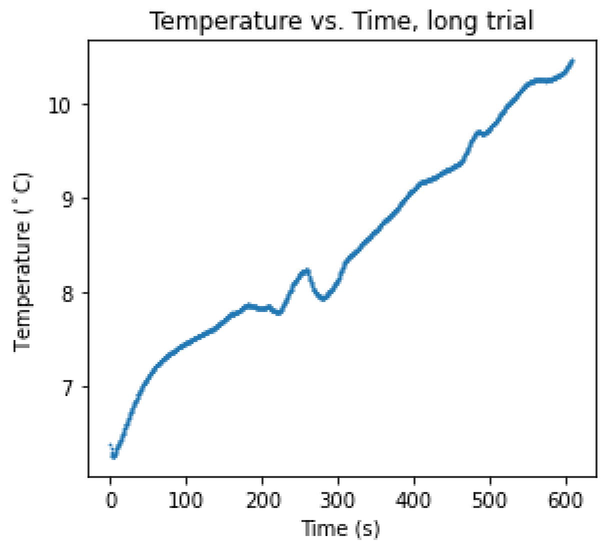
We inserted the temperature probe into the styrofoam cup (so that it did not touch the sides, and only touched the water inside), and then held this cup in the bath while recording the temperature with Logger Pro for three minutes. After exporting our data, we poured out the water and allowed the cup to cool, then performed the experiment a second time, but this time recording the temperature for ten minutes. One challenge we faced was not burning our hands: because we needed the height of water in the bath to be higher than that in the cup, and we filled the cup up too much, we had to push the cup far down into the water.

We then repeated this entire experiment, but with the metal can instead of the styrofoam cup. We first measured its dimensions using calipers, and found its mass to be 45 ± 1 g. For each trial, we measured the height of water in the cup by pouring water in until it reached one of the knuckles on a finger we inserted into the can, then measuring the length from tip to knuckle of that finger. After putting the probe in the cup (again not touching the sides), we held the cup in the bath and took data for 3 minutes and 10 minutes for the two trials respectively. The equation for the heat conductivity of this can is identical to that of the styrofoam cup, because the metal can is actually a cylinder.

Analysis

The raw data was collected and analyzed via a .csv file, so unfortunately we cannot provide a specific table about the data. However, the following are two scatter plots of the water temperature over time:





Qualitatively speaking, this scatter plot makes sense - despite the large temperature difference between the water inside and outside the cup, along with the fact that the walls of the cup were extremely thin, the water only climbed by a total of 3 degrees, even after 10 minutes. This qualitatively confirms the fact that styrofoam is an extremely good insulator, as was taught to us by high school chemistry teachers.

Now, we quantitatively establish the thermal conductivity and measure its agreement with the theoretical value provided in the lab notebook. To do this, we compute the derivative dT/dt using a computerized numerical derivative procedure written in Python. Then, we substitute this value along with others into our expression for the thermal conductivity and heat flow, as discussed in our theory section. Doing so, we obtain the following values for H and K:

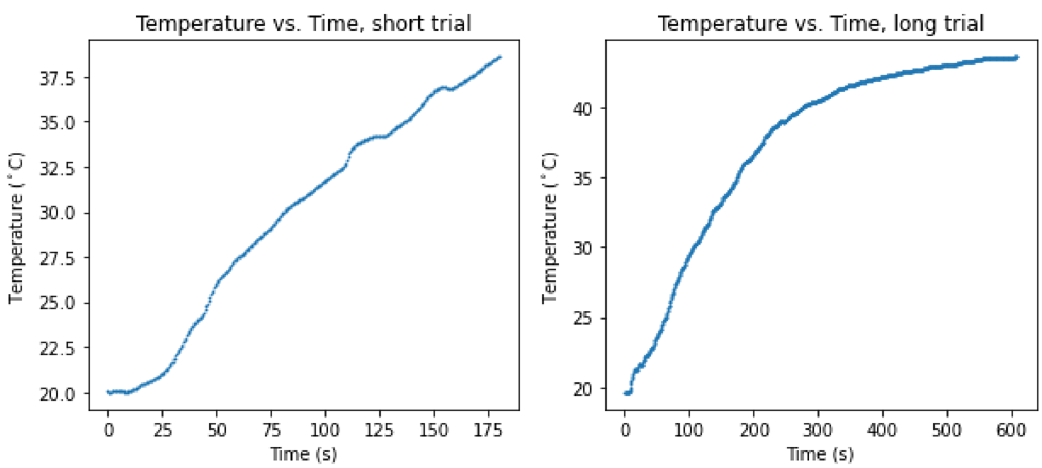
Trial	**Heat Flow**	**Thermal Conductivity**
Short (3 min)	4.08	0.033

Long (10 min)	9.81	0.031

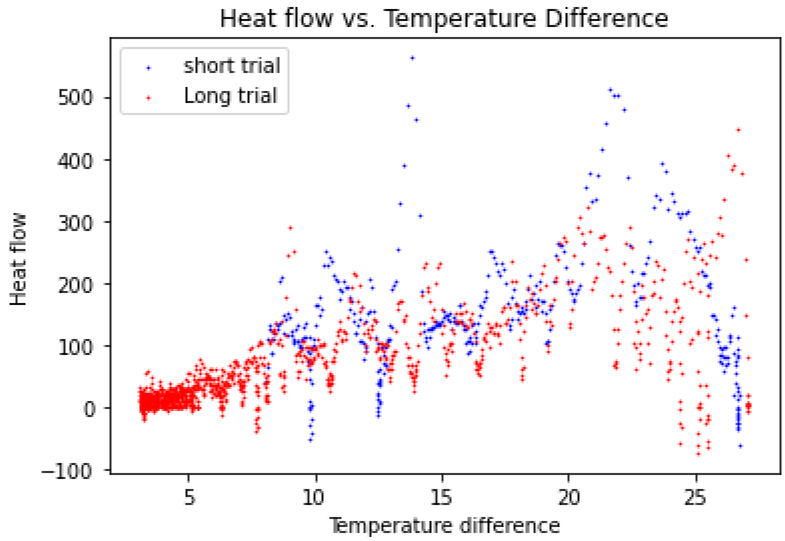
Note that these values we obtain are incredibly similar to the agreed value for the thermal conductivity, which is 0.033 given in the lab manual, which indicates an extremely good agreement between our theoretical and experimental values. Unfortunately, the error propagation equation is extremely massive, and we could not input the equation into Python and get any value for the error.

Nevertheless, these results demonstrate that styrofoam is an incredibly good material at inhibiting heat transfer, even in the presence of large temperature gradients. Therefore, these results indicate that our assumption of the styrofoam being a perfect insulator against the outside air (which already is a poor conductor of heat) is a perfectly reasonable and accurate assumption, therefore these results will not significantly alter the results we obtained in experiment 2.

Now we perform the same analysis with the metal can. The raw data plotted is shown below:



After calculating the slope over time, we can now plot this against the temperature gradient, which is calculated as the difference in temperature between the temperature in the water bath and that of the water inside the cup. Doing so, we obtain the following plot:



Visually, the first thing we notice is that at small temperature differences the proportionality is roughly linear, but at large temperature gradients, the linear correlation begins to break down relatively quickly. This makes sense with what we expect, since we only expect the proportionality to be *roughly* linear, where we're only truly allowed to use this approximation with small temperature gradients.

There are a couple ways that this discrepancy for large temperature gradients can be explained. Firstly, and perhaps most obviously, it might simply be due to the fact that a linear proportionality is not the right model for large temperature gradients, which would make sense, since the linear proportionality is an approximation. Alternative explanations could be the fact that at large temperature gradients, because the heat transfers quicker than that of small temperature gradients, that small fluctuations in our setup (such as the fact that we used our hands to hold the styrofoam cup in the heat bath) would have a much larger effect in these regimes when compared to small temperature differences. This would also make sense, since we can see that these small "oscillations" in the heat flow also exist at low temperature differences, but are much more localized than those at large temperature differences.

Conclusion

Overall, our experiment in verifying the thermal conductivity of styrofoam, along with the rough relation of the thermal conductivity was a success. Not only were we able to receive extremely good agreement (even without an error value, we believe that this is a reasonable conclusion to make) between the experimental and theoretical values, we were also able to verify that the heat flow is roughly proportional to the temperature difference, with this relation breaking down at large temperature differences, as expected.

Despite this success, we still believe that there are still some slight adjustments we could make to our experimental procedure to further improve our experiment. Firstly, it would be interesting to also verify these same relations if we put our styrofoam cup in an ice bath - in other words, see whether these relations hold when the temperature of the water inside the cup is higher than that of its surroundings. Secondly, in collecting the data for this experiment, we used our hands to hold the styrofoam cup into the heat bath, and held the cup in there by hand for the entirety of the data collection period. If this process could be more stable by fixing the styrofoam cup onto some type of stand, we would remove a large source of random human error, and likely obtain even better results.

References

Experiment 2

```
# @author Nikhil Maserang
# @date 2022/11/3

import numpy as np
import matplotlib.pyplot as plt

import sys
sys.path.append(r"../")

import analysis_utils as au
# get experimental data
data = au.read_from_docs_splitting("exp2data.txt", 1, (0, 0), float)

# chop last trial because the ice didn't melt
for i, (values, errors) in enumerate(data):
    data[i] = values[:-1], errors[:-1]

# assign names to data
(mc, mc_err), (mh, mh_err), (mf, mf_err), (Tc, Tc_err), (Th, Th_err), (Tf,
Tf_err) = data

# check that data is correct
au.print_datasets(["mc", "mh", "mf", "Tc", "Th", "Tf"], data)

# heat capacity of water
cw, cw_err = np.full(len(mc), 4.184), np.full(len(mc), 0.001)
```

```
# heat capacity of ice
ci, ci_err = np.full(len(mc), 2.093), np.full(len(mc), 0.001)
# create expression and symbols list
latent_heat_expr = "cw * ( mh * (Th - Tf) - mc * (Tf - Tc) ) / mc"
syms = ["mc", "mh", "Tc", "Th", "Tf", "cw"]

# get latex for error prop expression
print(f"Latex: {au.get_error_propagation_latex(latent_heat_expr, syms,
True)}")

# get latent heat values
L, L_err = au.calculate_derived_value(latent_heat_expr, syms, [], [], [mc,
mh, Tc, Th, Tf, cw], [mc_err, mh_err, Tc_err, Th_err, Tf_err, cw_err],
True)
au.print_dataset("Latent Heat (Ignore Ice)", L, L_err, 1)

# create expression and symbols list
latent_heat_expr = "( cw * ( mh * (Th - Tf) - mc * Tf ) - ci * mc * -1 *
Tc ) / mc"
syms = ["mc", "mh", "Tc", "Th", "Tf", "cw", "ci"]

# get latex for error prop expression
print(f"Latex: {au.get_error_propagation_latex(latent_heat_expr, syms,
True)}")

# get latent heat values
L, L_err = au.calculate_derived_value(latent_heat_expr, syms, [], [], [mc,
mh, Tc, Th, Tf, cw, ci], [mc_err, mh_err, Tc_err, Th_err, Tf_err, cw_err,
ci_err], True)
au.print_dataset("Latent Heat (Taking Ice Into Account)", L, L_err, 3)
```

exp2data.txt:
mc mh mf tc th tf
65 ± 1
71 ± 1
39 ± 1
124 ± 1

156 ± 1
175 ± 1

303 ± 1
148 ± 1

218 ± 1
243 ± 1
339 ± 1
263 ± 1

-1.3 ± 0.1
-1.3 ± 0.1
-1.0 ± 0.1
-1.4 ± 0.1

46.0 ± 0.1
45.7 ± 0.1
45.6 ± 0.1
45.3 ± 0.1

10.7 ± 0.1
11.9 ± 0.1
33.0 ± 0.1
-0.1 ± 0.1

Experiment 3