LEARNING HOW TO WORK WITH OPTICAL INSTRUMENTS

Physics 5CL Lab 1

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Introduction

This lab involved working with optical instruments and measuring different quantities using tools from geometric optics.

Contributions

The contribution breakdown was fairly similar to that of the last lab.

Eric Du

Eric completed most of the data analysis for this lab via Python (Jupyter Notebooks). He also helped with the data collection. Eric completed the analysis and completed the writeup for experiment 1, taking approximately **10 hours** to complete. This is mostly due to the poor quality of our lab notebook.

Andrew Binder

Andrew completed all of the Ti*k*Z diagrams and most of the writeups for the lab report. He also helped with the data collection. Andrew spent about **seven hours** on the lab writeup and Ti*k*Z diagrams.

Experiment 1: Multiple lens systems

Theory

In order to calculate the focal length of lenses, we use the thin lens equation:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

So rearranging for f:

$$f = \frac{d_i d_o}{d_i + d_o}$$

The error in the focal length is given by the following formula, following the standard method of error propagation:

$$\alpha_f = \left[\left(\frac{d_o(d_i + d_o) - d_i d_o}{(d_i + d_o)^2} \cdot \alpha_{d_i} \right)^2 + \left(\frac{d_i(d_i + d_o) - d_i d_o}{(d_i + d_o)^2} \cdot \alpha_{d_o} \right)^2 \right]^{\frac{1}{2}}$$

Calculating focal lengths of lenses

In order to calculate the focal length of the lenses, we used the F-object as the object, then placed each respective lens in front of the F-object and a screen after so that an image could be produced. When a sharp image was produced, we recorded the location of the lens and the screen. The location of the object remained constant throughout the entire experiment, so only one measurement was made for the position of the object. The data we collected is shown below:

	A		**B**		**C**	
Object Pos	**Lens Pos**	**Image Pos**	**Lens Pos**	**Image Pos**	**Lens Pos**	**Image Pos**
23.5	58.4	93.2	45.4	94.8	33	44.5
	63.3	95	49.6	86.1	37.4	45.7
	61.5	94.5	60.4	86.1	50.6	57.7
	54.6	95	46.6	92.5	62.2	68.5
	60.2	93.4	68.6	91.8	77	83.1
	56.1	94.4	60.7	86.7	87.8	93.1

Note that the column "object pos" was simply copied over for all three lenses, since the position of the object did not move throughout the entire experiment. Further, all positions are measuredin centimeters. To calculate the focal lengths of these lenses, we used the thin lens equation mentioned above, giving us the following mean focal lengths:

$$f_A = 17.57 \text{ cm}$$
$$f_B = 15.25 \text{ cm}$$
$$f_C = 5.32 \text{ cm}$$

And computing the error in the focal lengths via error propagation:

$$\alpha_{f_A} = 0.18 \text{ cm}$$
$$\alpha_{f_B} = 0.21 \text{ cm}$$
$$\alpha_{f_C} = 0.35 \text{ cm}$$

Calculating Magnification

In this experiment we were asked to measure the object and image distances, as well as measure the object and image heights. To calculate the magnification, we can use the relation:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Then, the error in the magnification is given by the following two formulas, depending on whether we are using the heights or the distances:

$$\alpha_m = \left[\left(\frac{\partial m}{\partial h_i}\alpha_{h_i}\right)^2 + \left(\frac{\partial m}{\partial h_o}\alpha_{h_o}\right)^2\right]^{\frac{1}{2}} = \left[\left(\frac{1}{h_o}\alpha_{h_i}\right)^2 + \left(-\frac{h_i}{h_o^2}\alpha_{h_o}\right)^2\right]^{\frac{1}{2}}$$
$$\alpha_m = \left[\left(\frac{\partial m}{\partial d_i}\alpha_{d_i}\right)^2 + \left(\frac{\partial m}{\partial d_o}\alpha_{d_o}\right)^2\right]^{\frac{1}{2}} = \left[\left(\frac{1}{d_o}\alpha_{d_i}\right)^2 + \left(-\frac{d_i}{d_o^2}\alpha_{d_o}\right)^2\right]^{\frac{1}{2}}$$

Here is the raw data below:

Object Position	**Lens Position**	**Image Position**
33.4	44	54.2

Object Height (cm)	**Image Height (cm)**
1.50	1.56

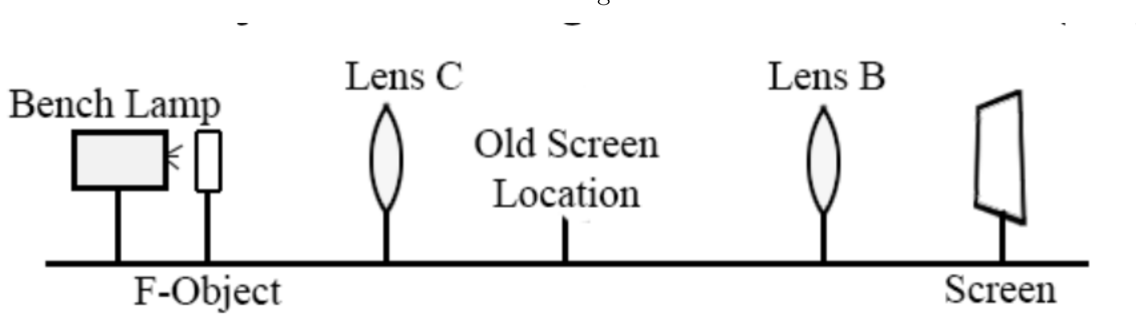
We split the data into two tables so that they fit properly onto the page. We can compute the magnification using the formula describe above, along with its error. Doing so, we get

$$|m_h| = 1.03 \pm 0.05$$
$$|m_d| = 1.04 \pm 0.06$$

As we can see, the values match quite well, so our formula for linear magnification checks out.

Using two Lenses

In this setup, we are asked to take the image from lens C, and add another lens (lens B) in between the setup and move the screen behind lens B. Here's the diagram from the lab document:



From here, we were asked to measure the object and image distances, which were 15.8 and 102 cm respectively. And as per the instructions, the object distance calculated here refers to the distance between the new screen location and the old screen location.

Lens at old screen location

When we performed this experiment, we noticed that the size of our object (the F) remained more or less the same. This is expected, since this reinforces the idea that the thin lens equation can be used multiple times in compound lens systems, which is indeed true.

A virtual object

Here, we place lenses C and B such that B is located before the location of the image had the light only passed through lens C. Our measured object and image distances and magnification are as follows:

Object Distance	Image Distance	Magnification
13.8	33.2	-1.33

To find the focal length of B, we can just use the lens equation here:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \implies f = \frac{d_o d_i}{d_o + d_i}$$

And since the object is virtual so the object distance is negative, this gives us:

$$f = -23 \pm 1.479$$

This is far from our expected value for the focal length of B, which should be around 15 cm. This could very likely be due to the fact that the way we measured our distances incorrectly, as the lab notebook we are using does not mention how the distances were taken. As a result, there is the possibility that we cannot simply use the thin lens equation, since the distances that we measure are not the same distances as they are in the thin lens equation.

Virtual Image acting as a Real Object for Converging Lens

Theory

Here we have a two-lens system where the focal length of lens A is known, but we assume the focal length of lens B is unknown. Using the distance from the image to lens A, we know that :

$$d_{oA} = \frac{f d_i}{f - d_i}$$

The error attached to this value is $\alpha_{d_{oA}}$:

$$\alpha_{d_{oA}} = \left[\left(\frac{f(f-d_i) + d_i f}{(f-d_i)^2} \cdot \alpha_{d_i} \right)^2 + \left(\frac{d_i(f-d_i) - d_i f}{(f-d_i)^2} \cdot \alpha_f \right)^2 \right]^{\frac{1}{2}}$$

Then, since we know that the position of this object is where the image of B should exist, we can now compute the image distance:

$$d_{iB} = s - d_{oA}$$

where s denotes the separation between the two lenses. The uncertianty in this this is

$$\alpha_s = \sqrt{\alpha_s^2 + \alpha_{d_{oA}}^2}$$

Since we've computed the distance from the object to lens B, we can now find the focal length of B using:

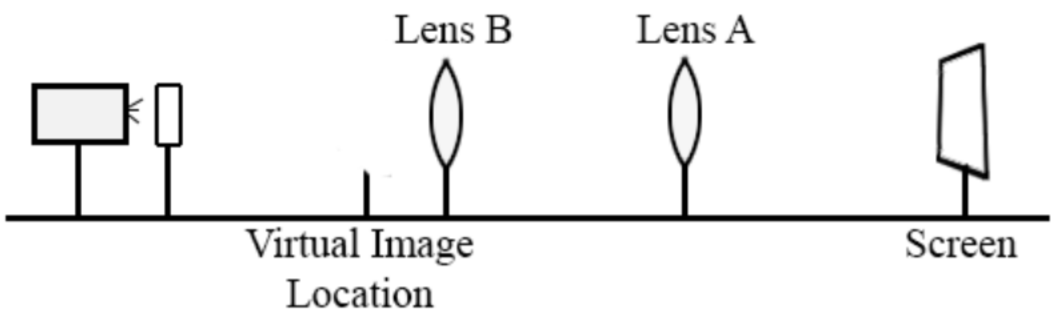
$$f = \frac{d_o d_{iB}}{d_o + d_{iB}}$$

and the uncertainty in f was given earlier:

$$\alpha_f = \left[\left(\frac{d_o(d_{iB} + d_o) - d_{iB} d_o}{(d_{iB} + d_o)^2} \cdot \alpha_{d_{iB}} \right)^2 + \left(\frac{d_i(d_{iB} + d_o) - d_{iB} d_o}{(d_{iB} + d_o)^2} \cdot \alpha_{d_o} \right)^2 \right]^{\frac{1}{2}}$$

Experimental Design

Here, our setup is exactly as described in the lab manual:



Here we used lens B with a 15cm focal length as lens B in the diagram, and the 17.5 cm lens A as lens A. We chose this setup because the lenses with the longest focal lengths also made it easier to measure distances, since had we used a lens with a focal length of 5cm the lenses would be too close to one another.

Data Collection

Our raw data is as follows:

Object Position	**Lens B Position**	**Lens A Position**	**Image Position**	**Magnific.**
17.2	27.9	58.9	83	-1.07E+00
		63.2	86.6	-1.00E+00
		51	76.5	-1.33E+00
		35.5	65	-2.07E+00
		34.1	62.7	-2.13E+00
		62.1	85.7	-1.13E+00

The object position and the image position did not change throughout our measurements.

Analysis

Following our theory from above, we calculate that the focal length of lens B is:

$$f_B = 12.55 \pm 2.83$$

This computed value then falls just within our previous calculated focal length for f_B, which serves as further confirmation that the focal length of lens B is approximately 15 cm. However, we can see that this value is further from the true accepted value (15 cm) than our previous method, which was a simple setup with a single lens. Thus, adding a virtual object in this case weakened our measurement, as it increased the error and also gave us a result which is farther away from the accepted value.

This is somewhat to be expected, since adding more complications to an experimental setup will naturally increase the error as it propagates through each equation, and the complexity of the setup also introduces more subtleties in the system that must be accounted for, which can easily go unnoticed.

Overall, however, we can see a relatively good agreement between our experimental results and our theoretical, accepted results, so overall this experiment was successful in determining the focal length of lens B.

Experiment 2: Simple Magnifier

Measuring Near Points

Eric and Andrew both measured the near points of their right eyes. Eric's near point was at $\boxed{8.7 \text{ cm}}$, while Andrew's was at $\boxed{6.7 \text{ cm}}$.

Observing Angular Magnifications

Now, we set up exactly how the diagram showed:

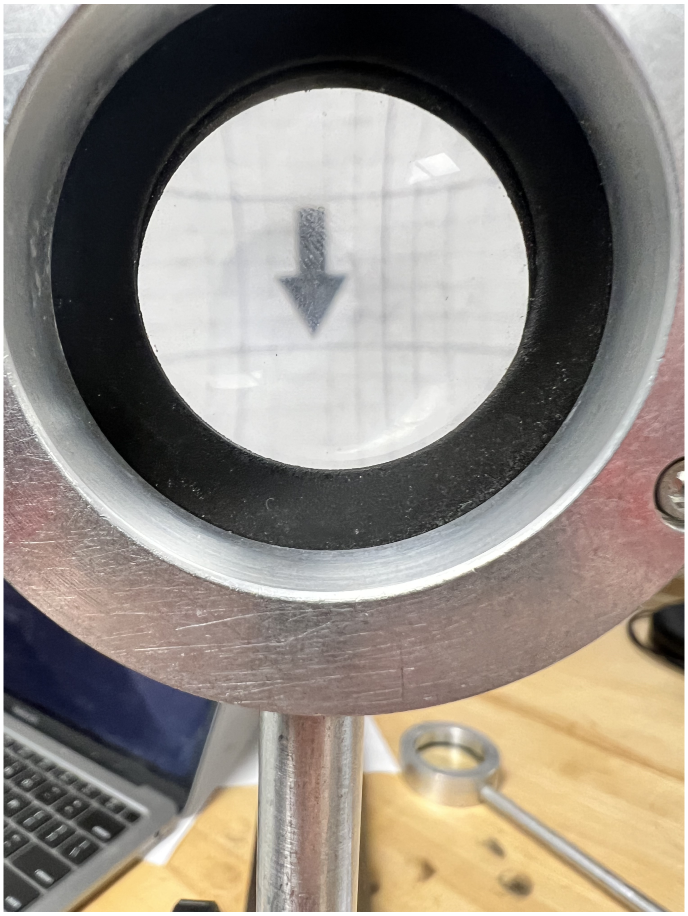
From this, we estimated the angular magnification to be around $\boxed{1.5}$ (Andrew looked through the eyepiece and estimated). As for the actual magnification, we got that through our formula:

$$m_{\text{eyepiece}} = \frac{d_{\text{NP}}}{f_{\text{eyepiece}}} = \frac{6.7}{5.4} \approx \boxed{1.24}$$

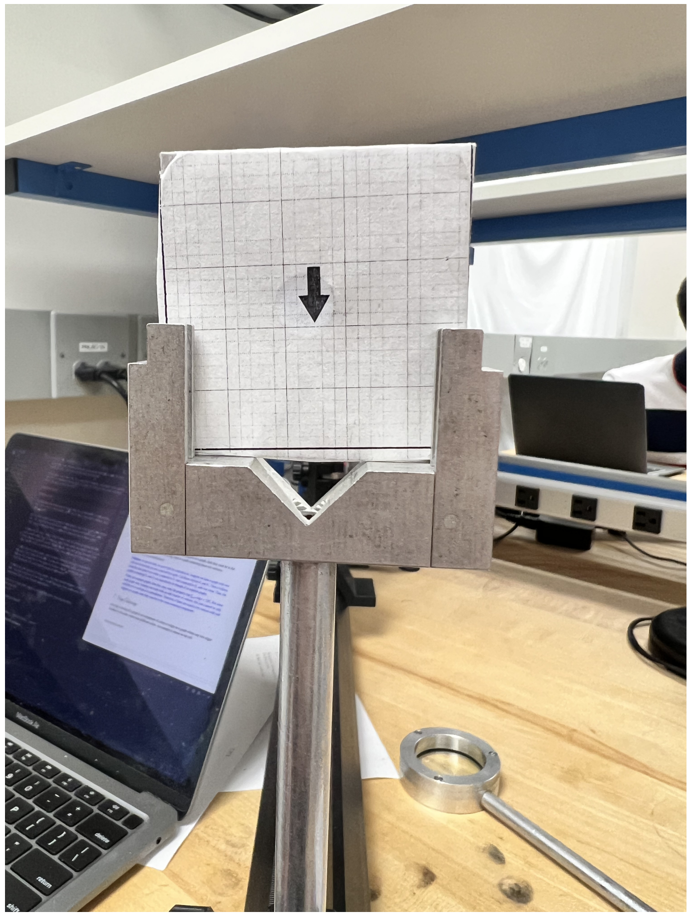
As a note, we used Andrew's near point for this. So, the actual magnification wasn't exactly what we estimated it to be, but our eyes aren't perfect and we don't have a great gauge for ratios, so this discrepancy is excused (since it was still within the ballpark).

Quantitative Magnification Data

We now proceed to the more quantitative data collection, which involved taking a picture of the magnified arrow with a phone camera. We used Andrew's iPhone camera for this:



Similarly, we took a picture of the unmagnified arrow at exactly the same distance from the arrow with the exact same configurations of the actual camera, this time without our magnification lens present:



The distance between the camera lens and the printed arrow was noted to be $\boxed{12 \text{ cm}}$.

Analysis

In order to quantitatively assess the magnification, we measured the size of an individual grid square in both images. The reason we did not use the arrow was that in the magnified image, achieving maximum zoom for that image would cause the arrow to become too large (off the screen), and hence not measurable. Thus, we chose the next best thing - to use the grid square instead. We did this in the same fashion as we would have if we measured the arrow, by maximizing the zoom on the phone then taking the measurement via a tape measure. Then we use the relation that

$$m = \frac{\theta_i}{\theta_o} = \frac{\frac{h_i}{d}}{\frac{h_o}{d}} = \frac{h_i}{h_o}$$

to compute the magnification. Doing so with our values (1.905 cm for unmagnified and 5.715 cm for magnified) we get:

$$m = \frac{5.715}{1.905} = 3$$

Propagating the error in this:

$$\alpha_m = \left[\left(\frac{\partial m}{\partial h_i} \alpha_{h_i} \right)^2 + \left(\frac{\partial m}{\partial h_o} \alpha_{h_o} \right)^2 \right]^{\frac{1}{2}}$$

Our errors in the heights are $\alpha_{h_i} = \alpha_{h_o} = 0.05$ cm, so we get

$$\alpha_m = 0.082$$

Thus, our final reported measurement for the magnification is 3 ± 0.082. While this is rather off from our value that we got as the angular magnification obtained earlier, this is not incredibly surprising because what we measured previously was based upon an estimation of our near point distance, which was more qualitatively than quantitatively measured since there was no way to precisely determine whether our eyes could resolve a clear image or not.

Additional Notes

For this experiment, it's important to keep in mind that we had different near points, so we could measure the angular magnification to be slightly different. Similarly, we are not sure if all of the auto-focus features on Andrew's phone were completely disabled, so if we were to run this experiment again, we would either bring a different camera where we could manually disable all of the auto-focus or we would be very careful with all of the phone camera application presets.

Experiment 3: Divergence

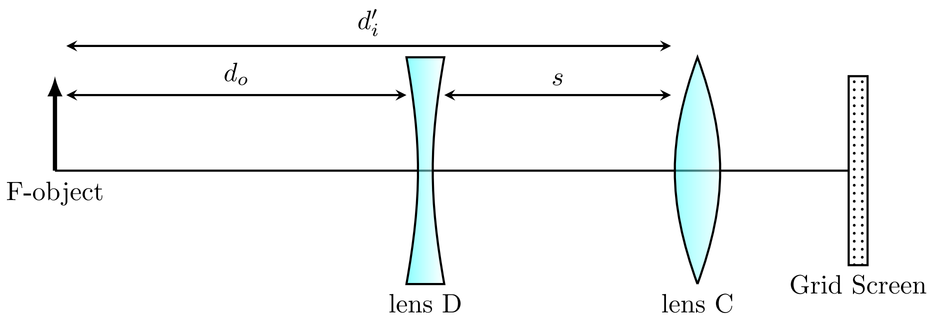
Determining the Focal Length of the Double-Concave Lens

Firstly, we want to determine the focal length of the double-concave lens.

Theory Recap

To determine the focal length of the diverging lens, we use a compound lens system. Here, we will make use of the fact that in a compound lens system, the image formed by the first lens, real or virtual, can be used as the object for the next lens in the system, so on and so forth. In other words, we can use the thin lens equation repeatedly in order to obtain a mathematical relation between the image distance and the focal lengths of the lenses in a compound lens system.

A schematic is shown below:



To calculate the focal length of the diverging lens, we use the thin lens equation to calculate the image location for the diverging lens:

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Solving for d_i:

$$d_i = \frac{d_o f}{d_o - f} \tag{1}$$

Now we use this image and use the thin lens equation again, except this time with the converging lens:

$$\frac{1}{f} = \frac{1}{d_o'} + \frac{1}{d_i'}$$

$$\therefore d_o' = \frac{d_i' f'}{d_i' - f'}$$

Note now that $d_o' = -(d_i + s)$. We add s because the virtual image from the diverging lens is formed *before* the lens, and the negative sign is to indicate that the image is virtual. As a result, we can substitute this in ans solve for d_i:

$$-(d_i + s) = \frac{d_i' f'}{d_i' - f'}$$

$$d_i + s = -\frac{d_i' f'}{d_i' - f'}$$

$$\therefore d_i = \frac{-d_i' f'}{d_i' - f'} - s$$

Which is the formula for the image distance. Then, once teh image distance is found, we can now solve for f using equation 1:

$$f = \frac{d_o d_i}{d_o + d_i}$$

Which would give us the focal length of the diverging lens.

A quick check that our formula is correct is by noticing that since $d_i < 0$, then this means that $f < 0$, since d_o is necessarily positive. As a result, we would get a negative focal length, which is exactly what we expect since the focal length of a diverging lens should be negative.

Experimental Design

We set up our experiment in exactly the same fashion as described in our theory. The diverging lens D was used, and the converging lens we chose was lens C. We chose this because we wanted to guarantee that the converging lens we used was stronger than that of the diverging lens, and not knowing the strength of the diverging lens, it made sense to choose the most powerful converging lens we had. This is because if the diverging lens is stronger than the converging lens, the converging lens would not be able to converge the initially diverging rays enough to generate a clear image (we can think of this as a push and pull situation).

A screen was also used at the end of the table in order to see if we obtained a clear image or not. Once a clear image as obtained, we noted down the positions of all lenses, objects and the screen.

Data Collection

Here is our necessary data. As usual, we measure their position along the optical bench, so the distances still come out to what we expect them to be if we were to actually measure the distances rather than objective position:

Object Position	Div Lens Position	Con Lens Position	Distance (Object-Div)	Distance (Lens)	Distance (Con-Image)	Distance (Div-Image)	Image Position
16.6	30.7	38	14.1	7.3	8.4	15.7	46.4
	25.1	33.1	25.1	8	8.8	16.8	41.9
	43.3	51	43.3	7.7	7.6	15.3	58.6
	43.2	66.1	43.2	22.9	6.7	29.6	72.8
	43.2	79.8	43.2	36.6	6.4	43	86.2
	43.2	88.4	43.2	45.2	6.3	51.5	94.7

All of the above measurements are in centimeters. As a note, we used the shorthand "div" for diverging lens and "con" for converging lens, to save space when making the table. Similarly note that we did not change the position of the object on the optical bench, though the other distances will change because of the new positions.

Analysis

Having calculated the focal length with our formula, we will check against what the value was listed as. Note that there was some discrepancy within the lab as to what the actual focal length of this lens was (the lab document said it was 10 centimeters, but others stated it was 15), so we will use the final answer as our accepted value: 15 centimeters. With this in mind, we can compare that against our computed values:

Experiment #	Focal Length
1	-14.832
2	-15.340
3	-16.147
4	-3.368
5	4.197
6	7.728

And so, we see that our earlier measurements were definitely pretty close to the accepted value (or at least the value that we deemed to be the accepted one). As previously stated, the first three measurements were pretty good, whilst the other ones were pretty far off. We believe this is because in the last three measurements, as we can see from our raw data, the distance between the F object and the diverging lens did not change, while the distance between the two lenses did change. This is problematic because while a clear image did form, the size of that image was not the same as that for our first three measurements, and so we were unknowingly performing a completely different measurement. In other words, our experimental process was not consistent throughout the entire experiment. As a result of this, we can effectively ignore the last three data points and continue working with the first three instead. Doing so, we compute the following mean and standard error:

$$f_{\text{experiment}} = -15.44 \text{ cm} \pm 0.54$$

Here, we argue that standard error can be used in place of propagating the error, since when we made our measurements, we implicitly introduced a random error as we determined what constituted a "clear image" by eye. Furthermore, since the precision in our measurements for the positions could be precise down to half a millimeter but any range within approximately 0.5 cm of the "clearest" satisfied our definitions for a clear image, using standard error here instead of propagating the error is justified.

Measuring the Radius of Curvature of the Double-Concave Lens

Now we want to measure. the radius of curvature.

Theory

The most reliable way to solve for the radius of curvature is using the lensmaker's equation:

$$\phi_{\text{lens}} = \frac{n_{\text{lens}} - n_0}{n_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

ϕ here denotes the optical power of our lens. Notice that this is the thin-lens approximation of this, which is valid in our case. Now, since $R_1 = R_2$ in our case (we have a symmetric lens), we can simply call this radius R:

$$\phi_{\text{lens}} = \frac{n_{\text{lens}} - n_0}{n_0}\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{n_{\text{lens}} - n_0}{n_0}\left(\frac{2}{R}\right)$$

We negate the second one, because the radius of curvature will be negative (given the Cartesian sign convention). Now, notice that n_0 is simply $n_{\text{air}} = 1$ in our case, so now we can solve for R:

$$\frac{2}{R} = \phi_{\text{lens}}\frac{1}{n_{\text{lens}} - 1}$$
$$\therefore R = \frac{2(n_{\text{lens}} - 1)}{\phi_{\text{lens}}}$$

So, to measure the radius of curvature, we need to know the optical power of our lens ϕ_{lens} and the index of refraction of the lens n_{lens}.

Experimental Design

We ran into one major issue upon trying to start measuring the radius of curvature: measuring it is impossible when not given the exact index of refraction or the optical power. So, we need to measure the optical power and the index of refraction. So, we need to take a few liberties.

For this experiment, we simply assumed that our double-concave was made of regular glass, giving it a refractive index $\boxed{n_{\text{lens}} = 1.53}$. Now, with this, we used a lens clock (calibrated to the index of refraction of standard glass) to measure the optical power. We took two measurements (one for each side) to confirm that both sides were roughly the same curvature. Then, given those values, we can plug them into our formula to calculate the radius of curvature.

Measurements and Calculations

Firstly, as before, as assumed that our double-concave lens was made of standard glass with an index of refraction of 1.53. Thus, we have an index of refraction that we can plug into our formula for the radius of curvature, as well as one we can use to calculate the optical power. Now, since we can't calculate the optical power using the lensmaker's equation, we will use a lens clock calibrated to standard glass to measure. Upon measuring both sides, we got that the optical power was roughly $\boxed{-2.75 \text{ Diopters}}$ (remember that the measurement will be negative because our lens is concave). Now, with this, we can plug in these values into our formula to get the radius of curvature:

$$R = \frac{2(n_{\text{lens}} - 1)}{\phi_{\text{lens}}} = \frac{2(1.53 - 1)}{-2.75} = -\frac{2 * 0.53}{2.75} = \boxed{-0.38\text{m}}$$

These measurements are in meters, so this should really be $\boxed{-38 \text{ centimeters}}$. The radius is negative, because we have a diverging lens.

Analysis

There wasn't really much in the way of analysis that could be performed here, since there wasn't really anything we could use to check our result against. One option we had considered as attempting to measure the radius of curvature directly by using calipers, but after communicating with another group that attempted this, the methodology contained too many errors and also produced incredibly inaccurate results, so we concluded that this was not a reliable way to check our answer.

However, we did try to reason our result qualitatively. Firstly, the result being negative makes sense, since by convention a converging lens has a positive radius of curvature, so by consequence a diverging lens should have a negative radius of curvature. Further, we also reasoned that 38 cm did not seem too outside of the realm of possibility when considering the radius of curvature of the diverging lens.

Experiment 4: Compound Microscope

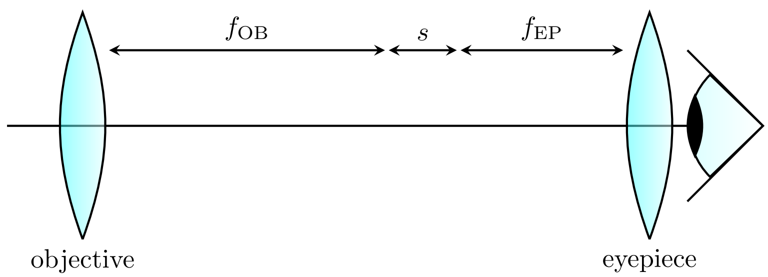
Here, we created a compound microscope using two of our double-convex lenses.

Theory Recap

As a recap, in the case of a compound microscope, the total angular magnification of the microscope is given by the following expression:

$$m_{\text{microscope}} = M_{\text{OB}} m_{\text{EP}} = -\frac{s d_{\text{NP}}}{f_{\text{OB}} f_{\text{EP}}}$$

Here, f_{OB} refers to the focal length of the objective, and f_{EP} is the focal length of the eyepiece. The two lenses are separated by a distance $f_{\text{OB}} + f_{\text{EP}} + s$, where s is some arbitrary distance needed to create the crisp image. A recreation of the diagram is shown below:

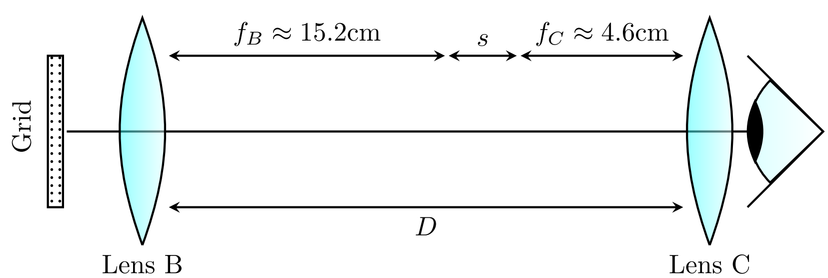


Notice that we choose the objective to have a longer focal length than the eyepiece.

Experimental Design

Firstly, we picked two of our double-convex lenses as the parts of our compound microscope. We picked lens B and lens C, since lens A would have too long of a focal length that wouldn't allow for a good measurement on our limited optical bench (although in principle any two should work). Since lens B had a longer focal length than C, we used lens B as the objective and then lens C as the eyepiece. We then positioned them accordingly on the optical bench, and used the grid as our object to be magnified (we didn't use the F-object, since it was difficult to analyze the magnification for an object with holes in it). Next, we adjusted the positions so that we achieved a clear magnification (no blurry squares). When we were satisfied, we had our s and could proceed with the rest of the experiment.

Our final setup then looked like so:



Now, as a second measurement method, we also used a ruler to measure one square on the grid when nothing is magnified to get our standard object height. Then, we put in the lenses for our microscope, and then measured the height of one of the magnified squares by pressing the ruler against the eyepiece lens. Then, we took the ratio to get our magnification. These two methods can help us compare.

Data Collection and Calculation

Now that we have the setup, we can proceed with data collection and the calculation of our magnification. As with previous measurements, we found the position of the different objects along the optical bench, so the distances then were calculated by subtracting positions on the optical bench:

Lens B Position	**Lens C Position**
45	81.8

As always, all of our measurements here are in centimeters. Then, our calculations come together:

$$\begin{aligned} D &= 81.8 - 45 = 36.8 \\ s &= D - (f_B + f_C) = 36.8 - 15.2 - 4.6 = 17 \end{aligned}$$

This time, Eric observed the microscope, so everything was calibrated for his near point, which is 8.7 cm. So, keeping that in mind, we get our final calculation for the total angular magnification of our compound microscope setup:

$$m_{\text{microscope}} = -\frac{sd_{\text{NP}}}{f_{\text{OB}}f_{\text{EP}}} = -\frac{17 * 8.7}{15.2 * 4.6} \approx \boxed{-2.115}$$

Calculating the error in this value using the uncertainty of the lenses from experiment 1 and the uncertainty in the distances measured:

$$\begin{aligned} \alpha_m &= \left[\left(\frac{d_{NP}}{f_{OB}f_{EP}} \alpha_s \right)^2 + \left(\frac{s}{f_{OB}f_{EP}} \alpha_{f_{OB}} \right)^2 + \left(\frac{sd_{NP}}{f_{OB}^2 f_{EP}} \alpha_{d_{OB}} \right)^2 + \left(\frac{sd_{NP}}{f_{OB}f_{EP}^2} \alpha_{f_{EP}} \right)^2 \right]^{\frac{1}{2}} \\ &= 0.17 \end{aligned}$$

Thus this gives us a magnification reading of

$$m = -2.1 \pm 0.2$$

Then, we also observed by eye (using a camera) as a secondary measurement. To do this, we measured the height of a square in the grid when unmagnified, and then measured the magnified square in the lens. For unmagnified, we got the height to be $\boxed{0.2 \pm 0.05\text{cm}}$, and the magnified one was $\boxed{0.4 \pm 0.05 \text{ cm}}$, which means the angular magnification turns out to be $\boxed{-2 \pm 0.7 \text{ cm}}$.

Analysis

Here, we can see that the two methods of measurement agree via the agreement test, which confirms that the theoretical equation for the angular magnification is correct and applies to our experimental setup.

One major source of error in our measurements was the fact that determining the near point distance of our eyes was a difficult task. Firstly, the way we measured this near point distance was simply by holding up the arrow to our eye and moving it closer and closer until we could no longer resolve it clearly. The issue in this is that although we believe that we are measuring the distance between our eye to the arrow, we do not know if our eye is completely level with the arrow, and thus we do not know whether we are measuring the exact distance between our eye to the arrow. To mitigate this in the future, the railing could be used instead, and the arrow could be mounted to the railing and then slowly slid closer. This way, a more accurate measurement can be made since now we can guarantee that the eye and the arrow are level with each other.

Conclusion

Overall, the sources of error throughout these experiments were fairly similar to the error sources found in the previous lab report. The biggest sources of error were systematic errors that came during the measurement process. The most significant errors came in the tier 2 experiments (no real surprise there), especially because there wasn't really a great way to measure the radius of curvature. The best way to improve for future repetitions of such experiments would be to eliminate systematic errors by taking more repeated measurements and getting better tools for measurement. The problem is that some of these measurement errors cannot be rectified through repeat measurements (like our radius of curvature measurements), so better equipment or more information about the objects we are measuring would allow us to not have to eyeball or guess.