Eric Du Physics 5CL
Prelab 3 Introduction to Experimental Physics II October 26, 2022

Collaborators

I worked with **Andrew Binder** to complete this prelab.

Problem 1

The ***small angle approximation*** is the first-order Taylor approximation and states that, when θ is given in radians and $\theta \ll 1$, $\sin\theta \approx \tan\theta \approx \theta$.

a) At what angle does the small-approximation reach a 1% relative error for $\sin\theta$? For $\tan\theta$? At what angle does approximating $\sin\theta$ by $\tan\theta$ yield a 1% error?

Solution: Relative error is measured as $\delta = \left|\frac{\theta - \sin\theta}{\sin\theta}\right|$, so we essentially want to solve

$$\left|\frac{\theta - \sin\theta}{\sin\theta}\right| = 0.01$$

Solving this (we used WolframAlpha to compute this numerically) we get $\theta = 13.71°$. Mathematica also solved the equation for $\theta \approx \tan(\theta)$, and gave us $\theta = 9.91°$. Approximating by $\sin\theta \approx \tan\theta$ gives us $\theta = 8.06°$.

□

Consider a single-slit setup with light of wavelength λ and slit width α.

b) At what minimum ratio a/λ does approximating the angle of the fifth-order diffraction minimum with the small-angle approximation $\sin\theta \approx \tan\theta \approx y/L$ give a 1% relative error? [*Hint: Think about your answer from a*]

Solution: For diffraction minima, we have $a\sin\theta = \pm m\lambda$, so therefore:

$$a\sin\theta = 5\lambda$$
$$\left(\frac{a}{\lambda}\right)_{\text{crit}} = \frac{5}{\sin\theta_c}$$

Since we know the critical angle for the approximation $\sin\theta \approx \tan\theta$ is at around $6.28°$, then

$$\left(\frac{a}{\lambda}\right)_{\text{crit}} \approx 35.66$$

Although this value appears to be high, we have to note that λ is the wavelength of light, which is very small, so this value actually might not be as insane as it might seem. □

Now consider a screen at a distance of $L = 1.00$ m from a single thick slit of width a and let light of wavelength $\lambda = a/125$ be incident on the slit. You may assume the small-angle approximation.

c) In terms of the variables L, λ and a, if you measure the central maximum to have a width (with uncertainty) of $\Delta y_{\text{cent}} = w \pm \Delta w$, what is the slit-width (with uncertainty), $a \pm \Delta a$?

Solution: Here we do error propagation - we have errors in Δy_{cent} and we are asked to find errors in a. Given the small angle approximation, the following formula relates a and y:

$$a = \frac{2L\lambda}{y}$$

So this means

$$a = \frac{2L\lambda}{w}$$

and the error:

$$\begin{aligned}\Delta a &= \sqrt{\left(\frac{\partial a}{\partial w}\Delta w\right)^2} \\ &= \left|\frac{\partial a}{\partial w}\Delta w\right| \\ &= \left|-\frac{2L\lambda}{w^2}\Delta w\right| \\ &= \frac{2L\lambda}{w^2}\Delta w\end{aligned}$$

So this means that our final expression is:

$$a = \frac{2L\lambda}{w} \pm \frac{2L\lambda}{w^2}\Delta w$$

$\square$

Problem 2

Consier a double thick-slit setup consisting of two thick slits each of width a, with a center-to-center slit separation of d. Recall that the intensity function for this setup is the product of the double-thin slit intensity function and the single-thick slit intensity function. Thus the observed pattern will be the double-thin slit interference pattern inside a single-thick slit diffraction pattern envelope. When an interference maximum coincides with a diffraction minimum, the interference fringe has a very low intensity. We call this a ***missing order***.

a) Suppose the m^{th} interference order is "missing" because it coincides with the n^{th} diffraction minimum. Determine the ratio of center-to-center slit separation d to the slit width a.

Solution: The diffraction minima occur at $a\sin\theta_1 = \pm n\lambda$, and $d\sin\theta_2 = \pm m\lambda$. For simplicity, let $m, n > 0$. Furthermore, we assume that $d \ll L$, so $\sin\theta_1 = \sin\theta_2$. As a result, it suffices to look at the diffraction minima for one slit.

Equating these two equations, we get:

$$\frac{m\lambda}{d} = \frac{n\lambda}{a} \implies \frac{d}{a} = \frac{m}{n}$$

□

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b) Suppose the ratio of the center-to-center slit separation d to the slit width a were f (that is, $d = fa$). What would be the order m of the lowest missing interference fringe and what would be the corresponding order n of the diffraction minimum?

Solution: We use the equation from the previous part, while substituting $d = fa$:

$$\frac{fa}{a} = \frac{m}{n} \implies m = nf$$

For the minimum missing order, we can't have $n = 0$ since diffraction minima don't occur at $n = 0$, so we must choose $n = 1$, and therefore $m = f$. □

Consider a screen at a distance of $L = 1.00\text{m}$ from a *pair* of thick slits of width a and with center-to-center slit separation $d = 4a$. Again let light of wavelength $\lambda = a/125$ be incident on the setup. You may assume the small-angle approximation in this part.

c) Plot a graph of intensity I vs. position y for this diffraction pattern for the range $-2\text{ cm} \leq y \leq 2\text{cm}$. Add in a graph of the corresponding single-slit pattern, which serves as an envelope. Label where the interference maxima and diffraction minima occur on the graph and identify the missing order.

Solution: We use the function given in the lab manual:

$$I(\theta) = I_{\max}\text{sinc}^2(\alpha)\cos^2\beta$$

Here, using $I_{\max} = 1$ is fine since we're simply looking for the shape of the curve, and a scale factor really doesn't mean anything to us here. To write this explicitly as a function of y, we write out α and β:

$$\alpha \equiv \frac{\pi a \sin\theta}{\lambda} \qquad \beta \equiv \frac{\pi d \sin\theta}{\lambda}$$

The first simplification we can make is that $d = 4a$ as given in the problem. Further, since the small angle approximation applies here, we write $\sin\theta = y/L$. Doing so gives:

$$\alpha \equiv \frac{\pi a y}{\lambda} \qquad \beta \equiv \frac{4\pi a y}{\lambda}$$

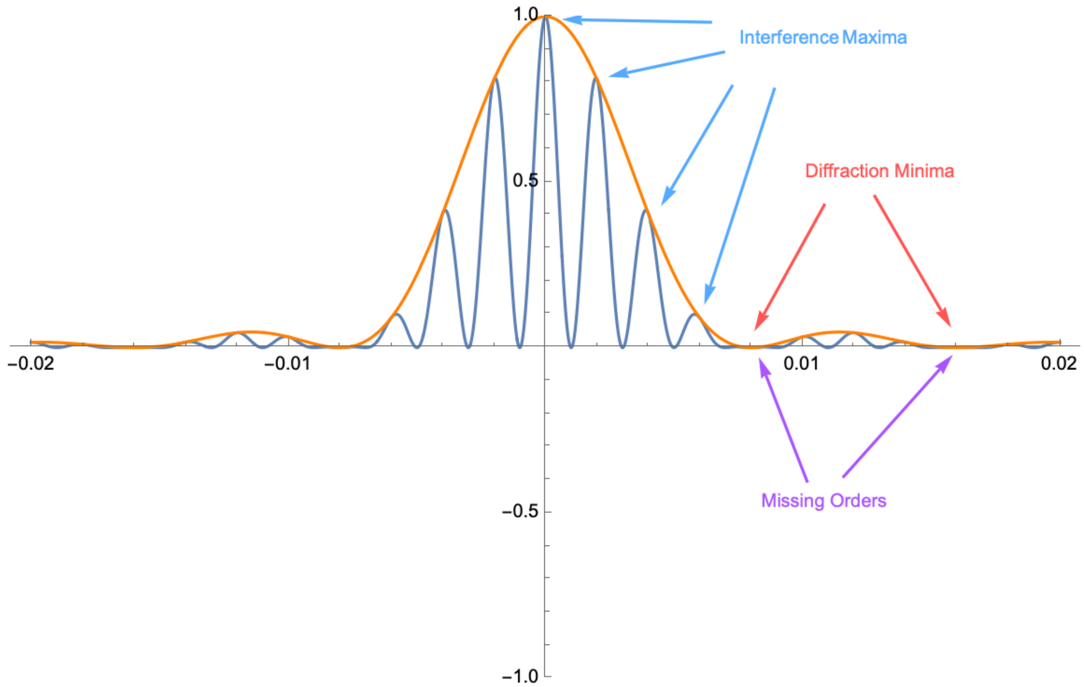
As a final simplification, we can write $\lambda = a/125$, so now we get

$$\alpha \equiv 125\pi y \qquad \beta \equiv 500\pi y$$

Overall, this changes our function $I(\theta)$ into a function strictly in terms of y:

$$I(y) = \text{sinc}^2(125\pi y)\cos(500\pi y)$$

Plugging this into Mathematica from $-0.02 \leq x \leq 0.02$ (since we're going from 2cm to 2cm), we get the following plot:



Here the envelope is in orange, and the interference pattern is in blue. We can see that there are interference maxima occur at $y = 0, \pm 0.002, \pm 0.006$ and so on. Similarly, the diffraction minima is the minimum of the envelope, which is at $y = \pm 0.008, \pm 0.016$ and so on. The missing orders are where the diffraction minima and interference maxima coincide, so at $\pm 0.08, \pm 0.016$, etc.

$\square$