Eric Du | Physics 112
Office Hours | Introduction to Statistical and Thermal Physics | December 12, 2023

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- Study the occupancy stuff (energies larger than n_F go to zero).
- Bose-Einstein Condensation: solve the number integral, then say that the number of excited particles is restricted the max value given by the number integral. Then, the remainder of the particles must be in the ground state (we get BEC from that.)
- The issue arises in the fact that there are a large number of particles in the ground state, hence $g(\epsilon)$ isn't smooth, so we can't replace this with an integral. Therefore, only the number of particles in the excited states can be transferred to an integral equation:

$$N = \sum_j n_B(\epsilon_j - \mu) = \underbrace{n_B(\epsilon_0 - \mu)}_{N_0} + \underbrace{\sum_{j>0} n_B(\epsilon_j - \mu)}_{N_{\text{ex}}} = N_0 + \int_0^\infty g(\epsilon) n_B(\epsilon - \mu) d\epsilon$$

  (The N_0 here is the source of the Bose-Einstein condensation. The proportion of particles between 0 and ϵ_1 goes to 0 in the thermodynamic limit, so this approximation as an integral on $[0, \infty)$ is good enough.

- Stefan-Boltzmann Law: sphere of temperature T and radius R, how much power does it radiate?

$$P = (\text{surface area}) \cdot T^4 \cdot \sigma_B = 4\pi R^2 T^4 \sigma_B$$

  σ_B is the Stefan-Boltzmann constant.

- Given $g(\epsilon)$ (single particle density of states) and a temperature T, how do we calculate the occupancy?
- Relation between $g(\epsilon)$ and momentum space:

  Given $\epsilon(k) = \frac{\hbar^2 k^2}{2m}$, we can find the number of particles:

$$N = \sum_{\vec{k}} n_{F/B}(\epsilon_k - \mu)$$

  We converted this then to an integral and calculated N. There's another way to calculate this:

$$N = \int d\epsilon g(\epsilon) n_{F/B}(\epsilon - \mu)$$

  where $g(\epsilon)$ is given by the number of single-particle with energy within the interval $[\epsilon, \epsilon + d\epsilon]$. For k^2 dispersion, read book to figure out $g(\epsilon)$. In lecture we did:

$$N = \sum_{\vec{k}} n_{F/B}(\epsilon_k - \mu) = \left(\frac{L}{2\pi}\right)^3 \int d^3k \cdot n_{F/B}(\epsilon_k - \mu)$$

  We can replace this sum with an integral when the function is "smooth" around ϵ_F.

- Learn how to do this: given a density of states, calculate the total number of particles in the system. Is the solution to literally do the integral?
- Dispersion relation relates ϵ to k.