Collaborators

I worked with **Andrew Binder, Christine Zhang, Teja Nivarthi** on this assignment.

Problem 1

The electric field of a solid sphere with radius R and uniform charge density ρ is given by

$$E = \begin{cases} \dfrac{\rho r}{3\epsilon_0} & (r < R) \\ \\ \dfrac{kQ}{r^2}\hat{r} & (r > R) \end{cases} \tag{1}$$

where Q is the total charge of the sphere. The magnetic field of an infinitely long thick table with radius a is given by

$$B = \begin{cases} \dfrac{\mu_0 J s}{2}\hat{\phi} & (s < a) \\ \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & (s > a) \end{cases}$$

where the net current I flows in the $+z$-direction. Note that the E-field and B-field are expressed in spherical and cylindrical coordinates respectively.

(a) Calculate the divergence and curl of E wiht spherical coordinates

(b) Calculate the divergence and curl of B in cylindrical coordinates.

Independent of the previous part, consider a vector field $V = s(2 + \cos^2\phi)\hat{s} + s\sin\phi\cos\phi\hat{\phi} + 3z\hat{z}$.

(c) Calculate the divergence and curl of the vector V.

(d) Verify the divergence theorem holds true using the quarter-cylinder of radius 1 and height 2. shown in the figure below.

(e) Verify that Stoke's theorem holds true using the surface S shown in the figure below.

Problem 2

The vector field

$$E = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right) - \frac{p}{3\epsilon_0}\delta^3(r)$$

where p is a constant in the z-direction, can be written as a gradient of some scalar function $V(r)$. Find the scalar functi $V(r)$ for $r \neq 0$. *Note:* The second term including the delta function is added for completeness, but you do not need to worry aboutit here. I do NOT recommend you using the Helmholtz theorem where

$$V(r) = \frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\nabla \cdot E}{\mathscr{r}} d\tau'$$

because the divergence at the origin is very tricky to deal with as it's not mathematically well-defined. Instead, thikn of this problme as solving the differential equation $E = -\nabla V$ for $r \neq 0$

Problem 3

Show the following integral theorems:

(a) $\int_{\mathcal{V}} (\nabla T) d\tau = \oint_S T da$

(b) $\int_{\mathcal{V}} (\nabla \times V) d\tau = -\oint_{\mathcal{S}} V \times da$

(c) $\int_{\mathcal{V}} (T\nabla^2 U - U\nabla^2 T) d\tau = \oint_S (T\nabla U - U\nabla T) \cdot da$

Here $\mathcal{V}$ is a three-dimensional region in 3D flat space and $\mathcal{S}$ is its boundary. T, U are scalar fields, while V is a vector field. For (a), you can use the divergence theorem but with the vector field to be cT where c is a constant vector field. For (b), you can again consider divergence theorem but with the vector field to be $V \times c$ where again c is a constant vector field.