TIER 1

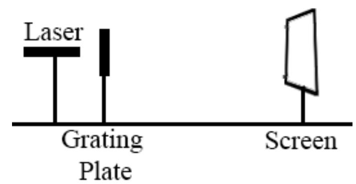
Work Deliberation:
Aren - Experiment 1, Experiment 4, Experiment 5 up to Data
Eric - Experiment 2, Experiment 3, Experiment 5 Data Analysis and Conclusion

Experiment 1 - Observing the Single-Slit Diffraction Pattern

We will start by observing the pattern formed by a single slit and determine a way to measure slit-widths from an observed diffraction pattern.

1.1 - A Single-Slit Diffraction Pattern

Our light source will be a red diode laser, which produces a coherent beam of light of wavelength $\lambda = 635$ nm.



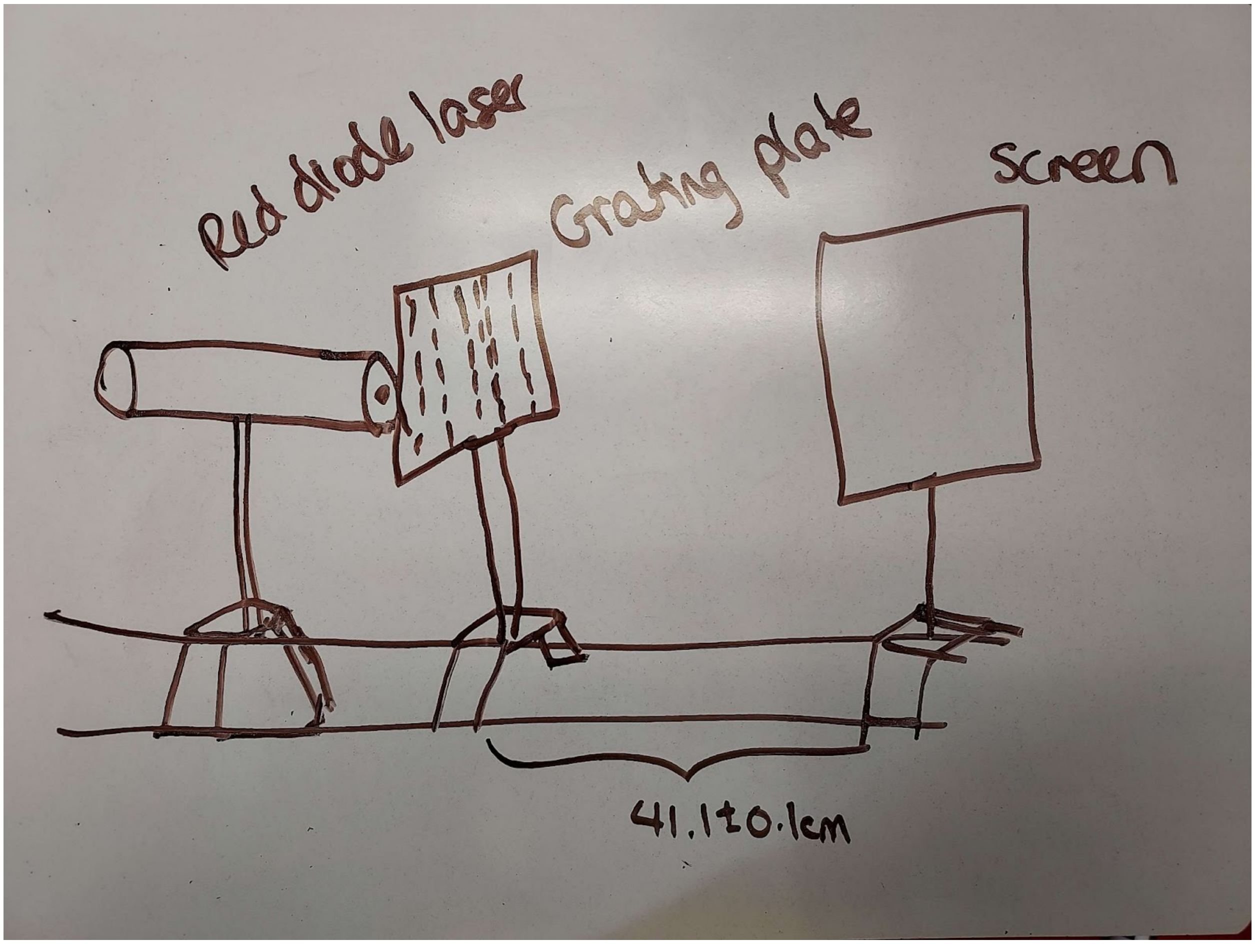
Setup: Mount the red diode laser, Cornell grating plate, and screen on the optical bench.

Setup: Adjust the grating plate so the laser can strike the ***single slit of width 4pts***. This is the "1/4/–" slit, second from the bottom of the left-most row of the grating plate.

Note: To get a good pattern visible on the screen you may need to move the screen farther away from the plate than the optical bench allows. You can simply remove the bench stand from the optical bench. You will need to determine and describe an accurate way of measuring the distance between the grating plate and the screen when you do this.

Reminder: Always make sure the grating plate is normal to the path of the beam. Do not rotate the laser to strike these openings. Rather, slide the grating plate farther in or farther out of the grating plate holder. You may need to make minor adjustments to the horizontal and vertical positioning of the grating plate to get the best pattern formed on the screen.

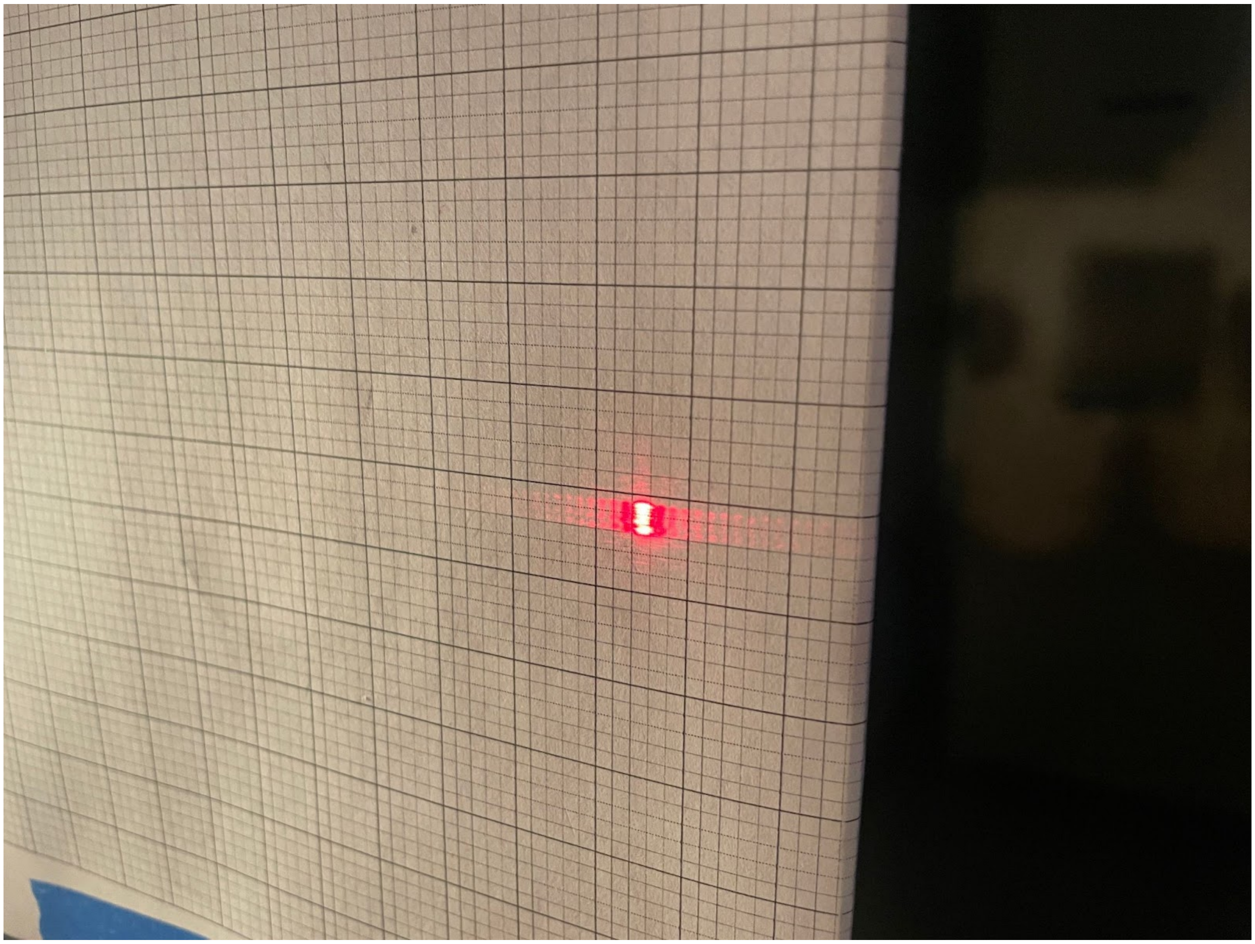
Measure: Sketch your setup and label and measure the distance between the grating plate and the screen.



41.1 ± 0.1 cm.

Record: Trace the diffraction pattern, labeling the very center of the pattern (where the laser struck the screen before the grating plate was introduced). Attach this tracing into your lab notebook.

Rather than tracing, you can also just try taking a picture! If you do this you need to take care to adjust the settings so the intensity of the light doesn't drown out the fine details of the pattern. You also need a length scale (such as the graph paper grid on the screen) visible and need to make sure you are minimizing distortions of scale.



Observation: Qualitatively describe the defining features of this pattern. In particular, describe how the widths of each fringe compare to each other.

The fringes look like thick sets of horizontal line segments (larger on the top and bottom than in the middle) arranged vertically. The fringes mimic this behavior, except they appear horizontally to the left and right, but the intensity of each is less and decays the further out we are. The length of each segment also decreases, which means that the widths of the fringes also decrease.

In Question Pre1(b) we determined the ratio a/λ at which the small-angle approximation would yield a 1% error (roughly 36). Now we will revisit that question with the observed pattern.

Analysis: Look at the length scales you are dealing with in this setup (particularly the grating-to-screen distance L and the widths you are measuring on the screen) and determine the order of magnitude of the relative error that would be incurred by using the small-angle approximation.

Length between slits and screen: L = 41.1 ± 0.1 cm

Distance between centers of fringes: 0.2cm, hence y/L ~ 0.2 / 41, and using small angle approximation, 41/0.2 >> 35.6 so we can use the small angle approximation.

Hence the relative error is roughly ((0.2/41) - sin(0.2/41))/sin(0.2/41), which is roughly 4 * 10^-6, which is much smaller than a 1% relative error.

Measure: Measure the width of the central maximum.

Since our pattern is small the reading uncertainty will be significant.

y = 0.20 cm ± 0.05 cm

Analysis: Using the known wavelength λ = 635 nm, determine (with uncertainty) the width of the slit.

Using the equation found in the prelab: a = 2L*lambda/w ± 2L*lambda/w^2 δw = 0.026 ± 0.006 cm

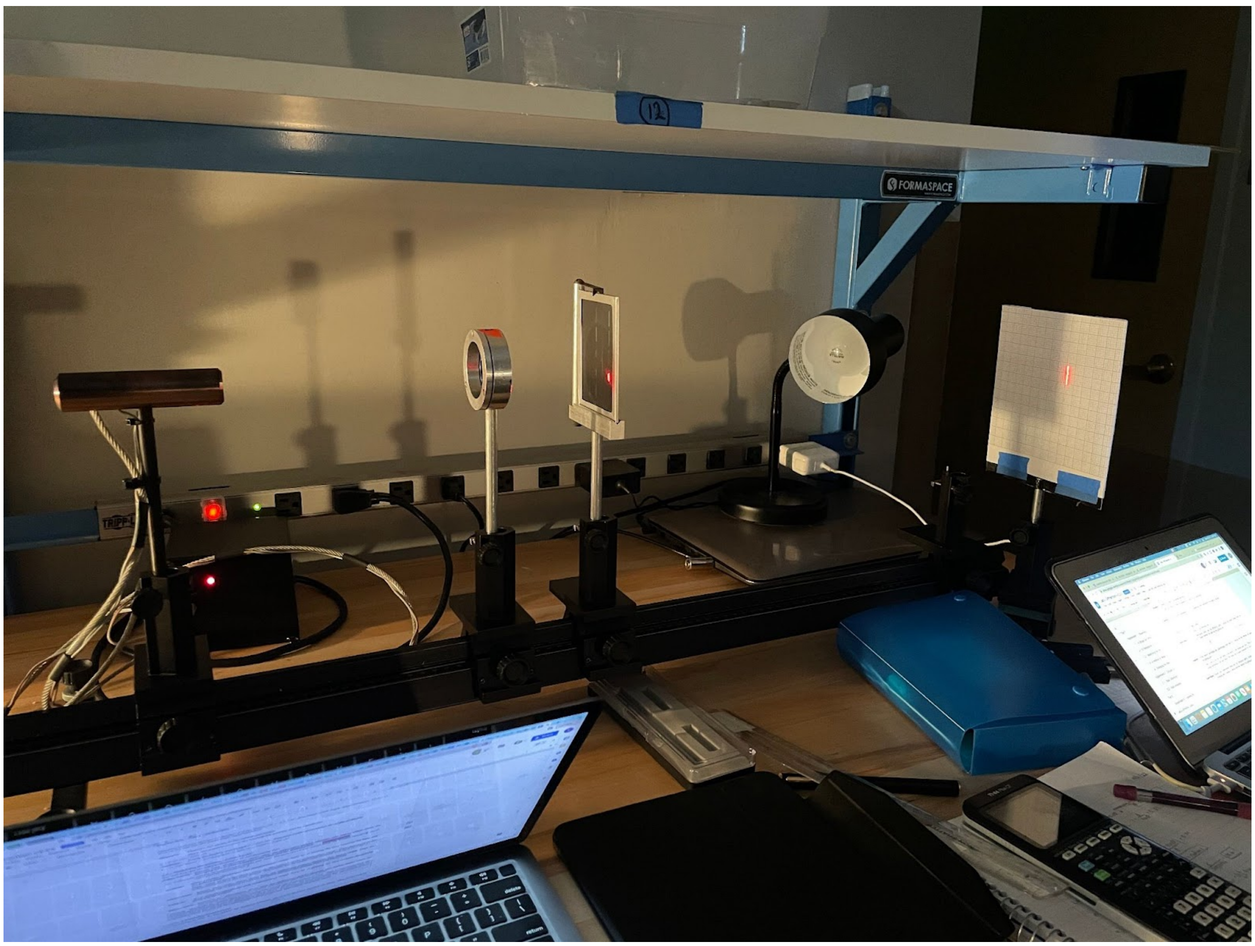
Note that the error in length is negligible here (2*lambda / w * δL) << 2L*lambda/w^2 δw, so we can effectively ignore its contribution.

1.2 – Modifying the Pattern - Using a Lens

We will now see the effects of using a 15-cm diverging lens.

NOTE: in both these experiments we were unable to get the laser light to pass through the center of the lens.

Observation: Record your observations about how the diffraction pattern changes size and/or shape when the diverging lens is placed between the laser and the grating plate.



The diffraction pattern is significantly taller, there are sections where the horizontal line segments disappear, and the fringes disappear much more quickly horizontally. Also, the resulting image has a lower intensity than the light without the diverging lens.

Analysis: What possible advantages and disadvantages might there be in placing the lens between the laser and the grating plate?

Hint: Look at how the laser shines on the surface of the grating plate itself. What happened to the actual size/spread of the laser beam? Why might this be useful (especially for slits such as 1/32/–).

Adding a diverging lens allows the diffraction pattern to be more clear (i.e. it's magnified, so we can see key features much easier), but there is the downside that we get less fringes. This may actually be a positive for narrow slits, which result in a very large number of fringes, and hence reducing this number via a diverging lens would be useful.

As we bring the diverging lens close to the laser, there is more distance for the rays to diverge, and so the laser effectively acts as a lamp. Conversely, if we bring it closer to the slits we are able to see more interference since the laser light is being diverged less. It definitely changes in height, but the width also seems to change.

Observation: Record your observations about how the diffraction pattern changes size and/or shape when the diverging lens is placed at various positions between the grating plate and the screen.

The number of fringes increases as we bring the diverging lens closer to the screen, and the height also decreases, but the intensity seems to remain largely the same.

Analysis: What possible advantages and disadvantages might there be in placing the lens between the grating plate and the screen?

Adding a diverging lens increases the size of the image, making it easier to view the laser light and the fringes, making it easier to measure the central maximum and distance to the first fringe, but it comes at the disadvantage of not being able to see all of the fringes.

It would be interesting to see at what point does this become disadvantageous due to the extra error of introducing the lens equation.

Observation: Record your observations about how the diffraction pattern changes size and/or shape when the converging lens (f = +15 cm) is placed immediately after the grating plate.

This section should have been removed, and we were advised to ignore it.

Note: if we are careful enough, we can use a converging lens for our capstone project.

Warning! You might be curious how a converging lens would affect the pattern as well. With a converging lens, the far-field diffraction pattern can be generated over a shorter distance. We are ***not*** *providing a converging lens or including that in this lab, however, due to laser safety issues. If you want to study something like this as part of your capstone project, we will ask that you consult with us to go over laser safety standards again.*

1.3 - A Method of Measurement

According to Eqs. 8, we expect the distance between the two m-th order minima on either side of the central maximum to be $2m$ times the location of the first minimum y_1. In this part we will first verify this statement and then *use* this fact to come up with a reliable method of determining y_1.

Parts of this analysis can be done outside of lab but we want enough information to devise an efficient and precise measurement method for use later in the lab.

Measure: Measure the distance between the m-th order minimum on one side of the central maximum to the m-th order minimum on the other side for at least four different orders m.

Measuring the distance between the two m = 2 minima

Note, your diffraction pattern will look red instead of green, of course.

Value of m	Observed Distance
m=1	0.4cm ± 0.1cm

m=2	0.65 cm ± 0.1cm
m=3	1 cm ± 0.1cm
m=4	1.4 cm ± 0.1cm
m = 8	2.7 cm ± 0.2cm

Our error in the observed distance is mainly because the width of the minima is challenging to record accurately due to the laser light covering the lines of the ruler.

Analysis: Divide each of your results (with relevant uncertainties) by the order 2*m* to get a "calculated first minimum location". How does the relative uncertainty vary with *m*?

Value of m	ratio of distance/2m
m = 1	0.20 ± 0.05 cm
m = 2	0.16 ± 0.03
m = 3	0.17 ± 0.02
m = 4	0.18 ± 0.01
m = 8	0.17 ± 0.01

The relative uncertainty decreases or stays constant as m increases, which makes measuring very large values of m incredibly challenging.

Analysis: Based on the results from this section, what are the advantages and disadvantages of directly measuring the location of the first minimum? Of using a low order minimum-to-minimum distance measurement (say, $m = 1$ or 2)? Of using a high order minimum-to-minimum distance measurement (say, $m = 10$)?

Using a low order distance makes it slightly easier to measure, since the distances between minima is so small that sometimes we lose count. Low order distances also have the benefit of being brighter, making them easy to see even under the presence of a lamp. Directly measuring the location of the first minimum should theoretically give a better result. However, due to the large uncertainty of measuring such a small distance with calipers, it is not as useful as low order distance. To support this hypothesis, the y itself we calculated from m=1 was somewhat larger than the y calculated for higher orders, which suggests that either our measurements were off (i.e. we were inconsistent) or the fringes are not evenly distributed on the screen, which is possible since our screen is slightly angled in our experimental setup.

Experimental Design: Determine a consistent measurement procedure that you will use to determine the first minima locations. Be sure to balance accuracy and precision with time, computational cost, and observability.

Note: This mainly consists of determining which order m you are measuring and how many independent measurements of a given length you are making.

First, fix the screen, diode laser, and grating such that the laser is orthogonal to both, so that the fringes are evenly distributed on the screen. From our previous results we realize that m = 1 gave us a slightly larger ratio than the others, so we decided to start from m = 2 up to m = 6, for a total of 5 data points. Measure the distance from the center of one minima to the corresponding minima on the other side for each m. These low order measurements balance the accuracy of measuring a large number of fringes and dividing with the challenge of losing count of the number of fringes between each minima.

1.4 - Changing the Wavelength λ

There is a green diode laser at your lab station. Recall that green is a *shorter* wavelength than red.

Observation: Record your observations about how the diffraction pattern changes as we replace the red diode laser with the green diode laser. Include in your discussion things like the distance between fringes, width of the fringes and the total number of fringes visible.

Note: Our eyes are very sensitive to green light so even at the same power and intensity you may be able to identify many more fringes than with the red laser.

More fringes are visible, but they are significantly closer to each other, and it is challenging to see an individual fringe.
More of the light seems to be scattered vertically or off in angles. This seems to be a byproduct of the laser itself, not of the difference in wavelength. (the red laser had a rectangular pattern whereas the green one is a circular pattern)

At the end of Experiment 1, you came up with a way of measuring the distance to the first minimum of a diffraction pattern. We will use this method to determine the wavelength of the green laser.

Measure: Using the same single-slit that you used in Experiment 1, collect data to determine y_1, the location of the first minimum, for the pattern formed by the green laser.

We used a lamp to illuminate the fringes, making them easier to measure. This does not affect the fringe pattern.

Value of m	Distance (cm) measured by Eric	Distance (cm) measured by Aren
m=2	0.6 ± 0.1	0.5 ± 0.1
m=3	0.8 ± 0.1	0.7 ± 0.1
m=4	1.0 ± 0.1	1.0 ± 0.1
m=5	1.2 ± 0.1	1.4 ± 0.1
m=6	1.5 ± 0.1	1.5 ± 0.1

Taking both measurements and averaging

Value of m	Distance (cm)/2m, i.e. distance to first minima
m=2	0.14 ± 0.02
m=3	0.12 ± 0.01
m=4	0.125 ± 0.009

m=5	0.128 ± 0.007
m=6	0.123 ± 0.006

distance from grating to screen: 47.4 ± 0.1cm

Analysis: Use your data and Eqs. 7 and 8 to determine the wavelength of the green laser (with uncertainty). How does this compare to the accepted value of $\lambda = 532$ nm?

Hint: Even though you determined the slit-width in 1.1, you do not necessarily need that value. What is the ratio of the widths of the central maxima for two different wavelengths of light assuming L and a are held fixed?

We originally did a quick check on values: m = 2 analysis: picking 0.125 we get about 463 nm. However, we were unable to replicate this value with any method, including the one suggested by the hint. Other methods give us values which are even more unrepresentative, such as 6.85 * 10^-5 cm.

Applying the hint, observe that equation 7 tells us that, denoting the green laser data as L1, y1, and the red laser as L2, y2, we get:

$$\frac{a}{\lambda} = \frac{mL_1}{y_1},$$

and $$\frac{a}{6.35*10^{-5}cm} = \frac{mL_2}{y_2},$$

so $$\lambda = \frac{L_2 y_1}{L_1 y_2},$$

i.e. $$\alpha_\lambda = (6.35 * 10^{-5} cm)\sqrt{(\frac{\alpha_{L_2} y_1}{L_1 y_2})^2 + (\frac{L_2 \alpha_{y_1}}{L_1 y_2})^2 + (\frac{L_2 y_1 \alpha_{L_1}}{L_1^2 y_2})^2 + (\frac{L_2 y_1 \alpha_{y_2}}{L_1 y_2^2})^2}$$

The calculated values of lambda we obtain using this method are:

Lambda	alpha_lambda
3.85421E-05	1.10996E-05
4.12951E-05	8.47595E-06
4.04854E-05	5.5883E-06
3.91539E-05	3.05934E-06
3.98376E-05	3.05164E-06

We do not need to do further analysis to see the existence of a very large systematic error! Notice that these are much smaller than the suggested wavelengths, and we would expect these wavelengths for violet and even ultraviolet light! This suggests significant sources of error, and indeed our errors are very large. Part of this may have been due to the choice to measure from the edge of the fringe rather than the center, which made the distances y_1 smaller and consequently made the our wavelengths smaller as well.

Experiment 1 Checkpoint

Call the instructor over to discuss Experiment 1 at this point before moving on.

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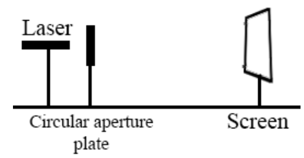
Experiment 2 - Circular Diffraction

In this next experiment, we will observe the diffraction pattern from a circular aperture.

2.1 - Data Collection



The circular aperture plate contains three small circular apertures of different diameters.

Setup: Place the red diode laser, circular aperture plate, and screen on the optical bench.

Observation: Play around and experiment with shining the red diode laser through the three different apertures. Describe the overall/common features of the diffraction patterns and how the pattern varies between the different aperture sizes. Include in your discussion things like the size of the fringes and the total number of fringes visible.

Diffraction pattern observed, for the one on the bottom the rings are fairly far apart, but for the middle one and the top one the rings are much closer together, to the point where we couldn't discern the diffraction pattern out in the top aperture. There are a few rings that are more explicit, but it is nearly impossible to see the dark space between rings. If we move things farther apart, the rings and the dim spaces between them are clearer.

Observation: For one of the diffraction patterns, make a detailed tracing of the diffraction pattern.

Note: It is obviously advantageous to choose the aperture that allows the clearest pattern to form.

Data Collection: Collect data for r_m, the radius of the m-th minimum, for the first four minima of the pattern formed by the smallest aperture.

We measured diameters instead of radii because measuring minimum to minimum is easier than measuring center to minimum. We will then divide by 2 to get our radii measurements.

Mth- minimum	Diameter (cm)
1	0.23 ± 0.01
2	0.38 ± 0.01
3	0.64 ± 0.01
4	0.79± 0.01

Analysis: Use r_1 and Eq. 12b to determine the diameter of the smallest aperture and compare to the given value.

Note: Be sure to describe how you are determining the radius and include a discussion of the various sources of uncertainty involved.

L = 78.0 ± 0.1 cm

lambda = 6.35 * 10^-5 cm

y/L ~= 1.22lambda / D => D ~= 1.22lambda * L / r_1

D ~= 0.052 cm

Experimentally measured aperture distance ~0.08

Experiment 2 Checkpoint Call the instructor over to discuss Experiment 2 at this point before moving on.

2.2 - Data Analysis

We took data for the radii of various minima for a circular diffraction pattern. For the data analysis you may assume the listed aperture diameters are accurate to the number of significant figures listed.

a) How well do your r_m values match what is predicted by theory?

Note: Give quantitative values for "how well", don't just say "meh, it fit pretty okay." The information in the Theory and Background section on Circular Diffraction will be of use.

The equation which gives us the diffraction distances is $D = \beta_{1m}\lambda L/(y\pi)$. The values of β_{1m} are as follows: $\beta_{11} \approx 3.8317$, $\beta_{12} \approx 7.0156$, $\beta_{13} \approx 10.1735$, and $\beta_{14} \approx 13.3237$. Therefore, substituting these values and our experimentally determined values of y we get:

m	aperture distance
1	0.052
2	0.058
3	0.050
4	0.053

To propagate our errors, we use the relation:

$$\alpha_D = \frac{\beta_{1m}\lambda}{y\pi}\sqrt{[\frac{L^2}{y^2}\alpha_y^{\ 2} + \alpha_L^{\ 2}]}$$

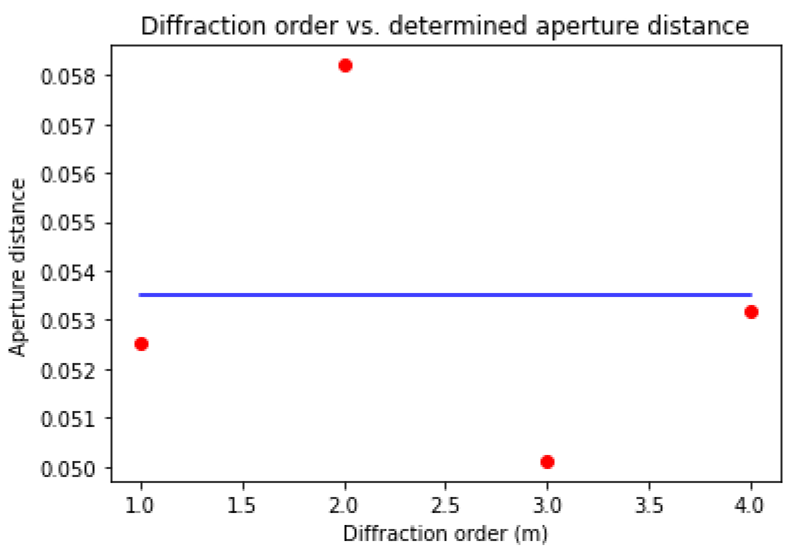
Our errors are $\alpha_y = 0.01$ cm for all values, and $\alpha_L = 0.1$ cm, so we get:

m	uncertainty
1	0.004
2	0.003
3	0.001
4	0.001

So therefore:

m	aperture size (cm)
1	0.052 ± 0.004
2	0.058 ± 0.003
3	0.050 ± 0.001
4	0.053 ± 0.001

As the next section suggests, we cannot simply use standard error to find a mean as well as an error. In order to find the error here, we fit this curve to a constant (as shown below), which gives us 0.053 ± 0.0001.



While this error may seem quite horrible at first, it's important to remember that we are on the order of 0.01 cm here (less than 1mm), so it makes sense that the graph looks like this.

One simple way of determining the radius would be to measure the *diameter* of one of the dark rings in the Airy disk and divide by two. This method has an uncertainty (as do all measurement methods) but there is a new twist - the uncertainty isn't necessarily symmetric and repeated measurements won't necessarily form a Gaussian around the true diameter! Rather, this method will tend to give either an under-estimation of the diameter.

b) What am I referring to in the above paragraph? That is, why would such a length measurement give an under-estimate of a length quantity when (presumably) other length measurements would form a symmetric bell curve around the true value? Does this produce an overestimate or an underestimate of the diameter? Propagating forward, does this give an overestimate or an underestimate of the diameter of the aperture?

Hint: This is tied to the geometry of a circle and is not some esoteric statistical subtlety. Hopefully this strikes a chord with you and you don't wind up overthinking.

∇

The reason we will tend to underestimate the diameter of the airy disk is because when we try to measure the diameter, it is difficult to determine which point around the circle is exactly diametrically opposite to the initial point. As a result, we generally tend to choose a point *around* the true diameter, which will always be an underestimate since the diameter is the largest distance between two given points on a circle. This is not an issue if we measure the radius instead, since it's rather easy to find the center of the circle, then finding when the bright spot ends in a diffraction minimum.

Since the diameter is underestimated, this means that in our equation for the aperture: $D = \beta_{1m}\lambda L/(y\pi)$ the value of y is lower than the true value. As a result, we will tend to measure a larger value for the aperture than the true value, and thus an overestimate.

One of the reasons we believe this did not occur (or at least prominently) during our experiment is because we also used the grid lines on the screen to make our measurement process easier, meaning that we could more reliably find the true diameter of the airy disk.

EXPERIMENT 3

Objective:

The goal of this experiment was to verify the theoretical relationship between the slit width and its observed diffraction pattern.

Theory and Background:

For a single slit diffraction pattern, we know that the diffraction minima are given by the equation

$$a \sin \theta = \pm\, m\lambda$$

If we use use the small angle approximation, then we get

$$\frac{ay}{L} = \pm\, m\,\lambda$$

Therefore, the slit width is given by

$$a = \frac{\pm m\lambda L}{y}$$

Where y represents the distance between the central maximum and the m^{th} minimum. In our experiment, we measured the distance from minimum to minimum, so to fit our experiment, we use instead:

$$a = \frac{\pm 2m\lambda L}{y_{min}}$$

Where y_{min} now represents the distance from minimum to minimum. The uncertainty associated with the slit width is then

$$\alpha_a = \frac{2mL\lambda}{y_{min}^{2}} \alpha_{y_{min}}^{2}$$

The wavelength of the laser is known (635nm), and the distance between the screen and the cornell grand L remains constant throughout the measurement process, so it too is effectively a constant. Therefore, we will be seeing how the diffraction pattern as measured measured by y, the location of the diffraction minima, varies with slit width. Then, we will use these measured values of y to experimentally measure the slit width a.

Experimental Procedure:

The red laser was placed on one end of the lab bench, and it was aimed at one of three single slits on the Cornell grating. The diffracted light was then collected onto a screen, which was placed on the other end of the lab bench. The distance between the Cornell grating and screen were measured, then data regarding the diffraction minima were gathered.

In order to simplify our measurements, we measured the distance between the minima of the same order on either side of the central maximum. We used calipers to measure the fringe distances, and used a meter stick to measure the distance between the Cornell grating and the screen. To measure the fringe distances, we placed one end of the caliper next to the diffraction minima on one side, then extended the caliper so that it measured the minimum to minimum distance on both sides of the central maximum.

Experimental Setup:

An image of the experimental setup is shown below:

In this image, the laser was shining through the second slit from the bottom. The bottom three slits were used in this experiment - the other two larger single slits were not used, because they could not produce a reliable diffraction pattern we could measure.

Data:

The length between the Cornell grating and the screen was 79.9 ± 0.1 cm. The raw data collected is shown in the tables below:

Slit 3

Value of m	Distance (cm)
m=2	0.44 ± 0.05

m=3	0.70 ± 0.1
m=4	1.09 ± 0.1
m=5	1.44 ± 0.1
m=6	1.76 ± 0.1

Slit 4

Value of m	Distance (cm)
m=2	0.95 ± 0.05
m=3	1.40 ± 0.05
m=4	1.80 ± 0.05
m=5	2.32 ± 0.05
m=6	2.76 ± 0.05

Slit 5

Value of m	Distance (cm)
m=2	1.60 ± 0.05
m=3	2.4 ± 0.1
m=4	3.0 ± 0.2
m=5	4.0 ± 0.2
m=6	stop here

Note that for slit 5, a distance between minima was not recorded because the interference pattern became too faint to discern properly.

Data Analysis:

Using the equation given in the theory section, we can calculate the slit width as well as the propagated errors. After having computed these values, we take their mean in order to get a singular value. Doing so, we obtain the following mean values for the slits and their propagated errors:

Slit	Experimentally Determined Width (cm)

Slit 3 (1/8/-)	0.039 ± 0.004
Slit 4 (1/4/-)	0.022 ± 0.0007
Slit 5 (1/2/-)	0.012 ± 0.0006

Comparing these values with the theoretical slit widths, as given by the lab manual:

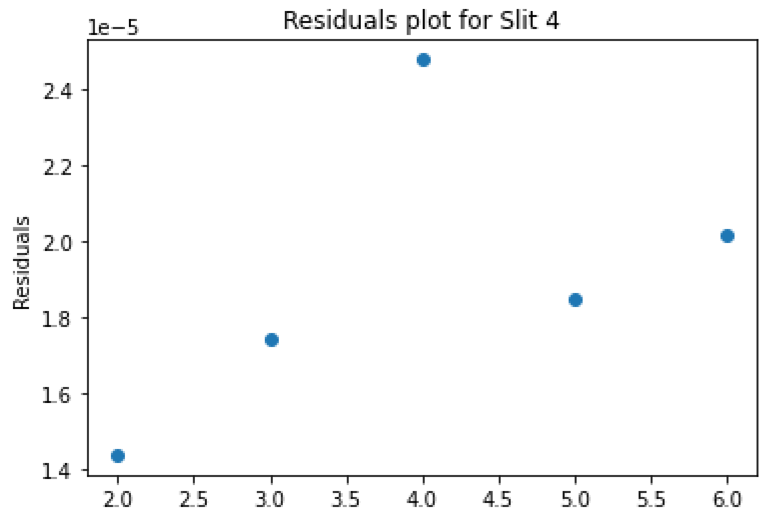
Slit	Actual Width (cm)
Slit 3 (1/8/-)	0.035144
Slit 4 (1/4/-)	0.017572
Slit 5 (1/2/-)	0.008786

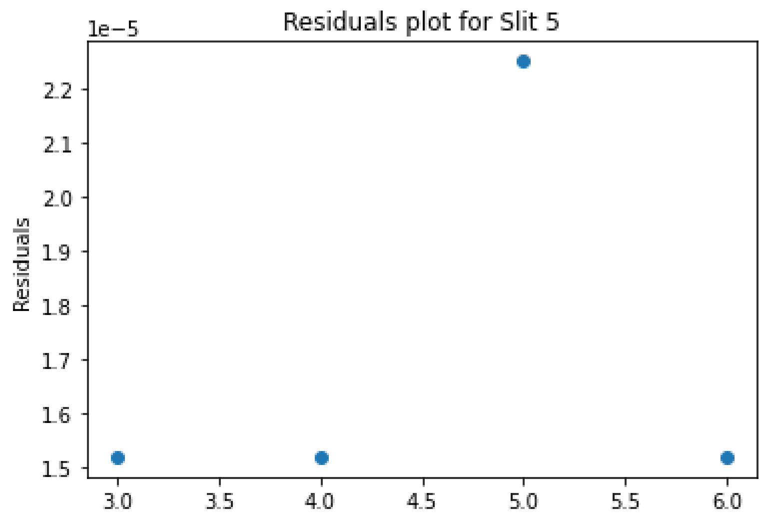
As we have the theoretical values as well as experimental values, we can first perform an agreement test using the following relation:

$$|y - z| < 2\Delta y$$

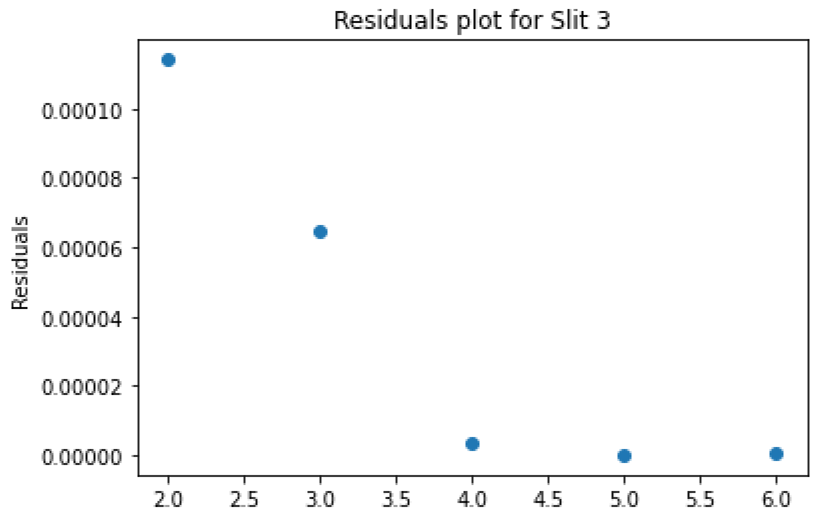
This equation is determined using the standard formula for an agreement test, then applying the conditions that our theoretical values have zero uncertainty. Therefore, we can see that only the widest slit (Slit 3) passes the agreement test. All things considered, this is not incredibly surprising, since as the slit width became smaller, we observed that the diffraction pattern also grew narrower (as mentioned previously), and thus the diffraction pattern became harder and harder to measure as distances became smaller and smaller. As a result, it is entirely possible that our measurements were poorly done due to the sheer difficulty in measuring the distances between diffraction minima - specifically the fact that in order to be conservative with our measurements, we inadvertently introduced a systematic error into our data collection process.

To reinforce this conclusion, we also computed the squared residuals for slits 4 and 5, from which we obtained the following plots:





As is evident from the plots, there appears to be some sort of trend with the residuals, hovering around 1.8×10^{-5} for slit 4 and 1.5×10^{-5}. Due to this apparent trend, although it is not possible to conclude, there appears to be reasonable evidence to suggest that some systematic error (albeit incredibly small) did exist during our measurement process. Comparing this with Slit 3, which was much easier to measure:



We can see that although residuals were high in the first two measurements, they eventually reach near zero, to a minimum of 4.25×10^{-8}, suggesting that our measurements had high accuracy and that there was no apparent systematic error. Again, this intuitively makes sense, because slit 3 was also significantly easier to measure than the other two slits, so we were also less prone to error here.

Conclusion:

The goal of this experiment was to verify the relationship between the diffraction pattern and the observed slit width, using the equation we derived in the prelab.

Despite the failed agreement tests with slits 4 and 5, this experiment was overall a success since we were able to determine the likely source of error for slits 4 and 5, and the passing of the agreement test with slit 3 indicates that our experimental setup isolated for as many variables as possible, and the lack of agreement with slits 4 and 5 are simply due to systematic measurement errors.

To eliminate these measurement errors, a number of modifications could be made. One way we could have spread out the diffraction pattern was to further increase the distance between the Cornell grating and the screen. While our experimental setup had a fairly large distance between the grating and the screen, it was certainly not maximal, so extending this distance could have helped with the measurement process. Another way the pattern could have been made to spread out more was to place a diverging lens between the grating and the screen, which would naturally spread out the diffraction pattern. However, this method is not entirely recommended, since introducing a diverging lens would mean that more error would be propagated, and it is unknown if the benefits of being able to measure the diffraction pattern more clearly outweighs the error that placing the diverging lens would introduce due to error propagation.

EXPERIMENT 4

Objective:

The goal of this experiment is to verify Babinet's principle for laser light blocked by 26 Gauge and 39 Gauge wires.

Theory and Background:

Babinet's principle states that the diffraction pattern produced via an obstruction is identical to the pattern produced by an aperture of the same shape, and thus can be used to determine the thickness of objects. In effect, due to the wire being nearly cylindrical, we can think of the laser light obstructed by the wire as passing through a linear rectangular aperture.

Our equations used will be identical to the ones used in question 3: we know that for the mth diffraction minimum, we have the relation: $\frac{a}{\lambda} = \frac{m}{sin\theta}$. Since our distance from the wire to the screen will be much larger than the distance between diffraction minima, we can apply the small angle approximation that $\theta \approx y_{min}/L$ to get $a = \frac{mL\lambda}{y_{min}}$. However, we must note additionally that in this experiment we will be measuring the distance from one minima to another, so we are missing an extra factor of two; indeed, the final equation we will obtain is $a = \frac{2mL\lambda}{y_{min}}$.

The only two uncertain values in this expression are L and y_{min}, but as seen in experiment 1 the contribution from the error in L is completely negligible. Therefore, we can write

$\alpha_a = \frac{2mL\lambda}{y_{min}^2}\alpha_{y_{min}}^{\;\;2}$.

Experimental Procedure:

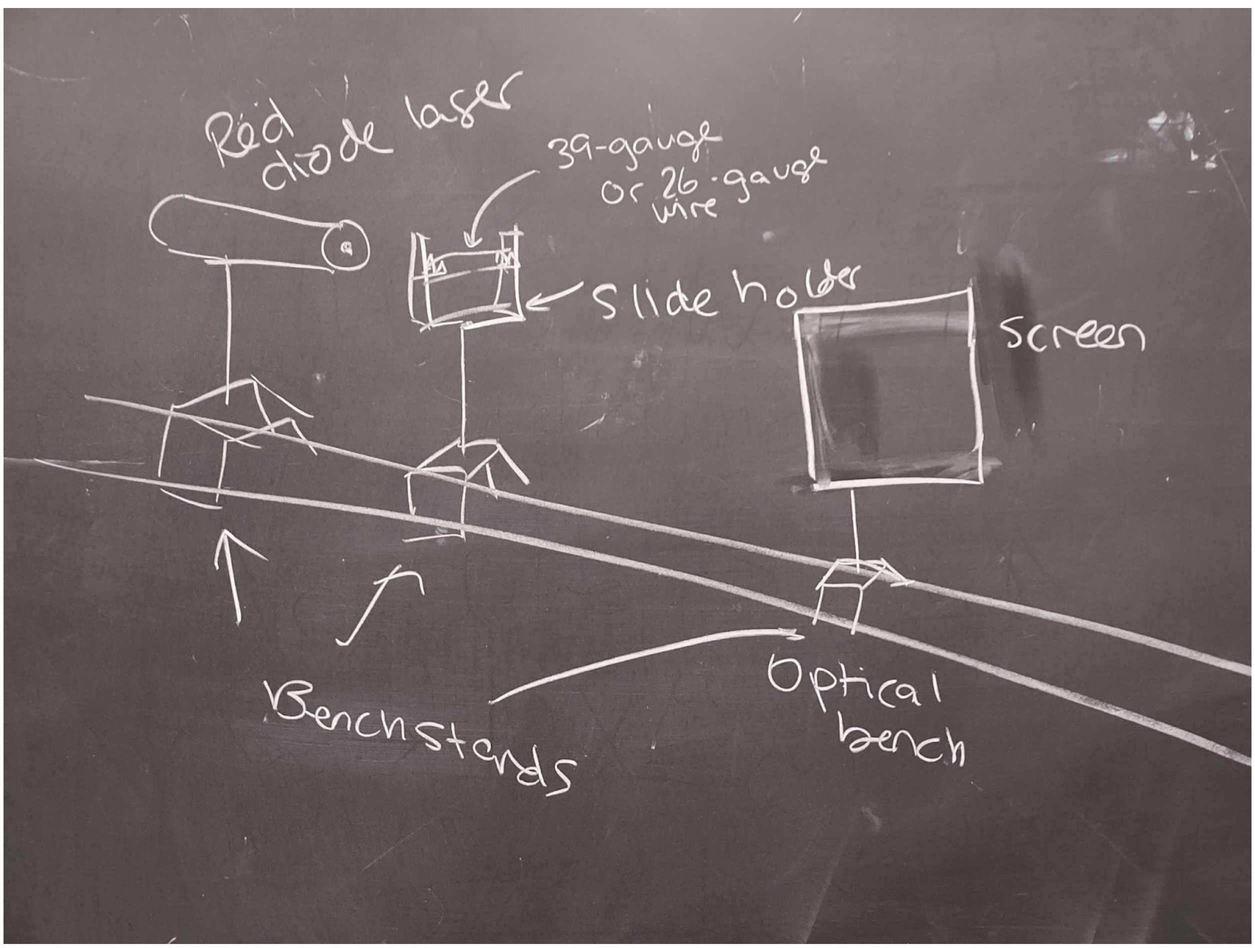
To determine the effective aperture and consequently the width of the wire, we use a nearly identical procedure to the previous experiment. First, place the red diode laser, the slide holder, and the screen such that the laser and slide holder are relatively close, and the screen is substantially far back. Next, place the

26-gauge wire in the slide holder so that it is horizontally aligned. Turn the laser on, making sure to vertically adjust so that the laser beam exactly hits the wire and also such that the diffraction pattern is visible on the screen. If the fringes are too close together, push the screen farther back along the optical bench so that they are farther apart.

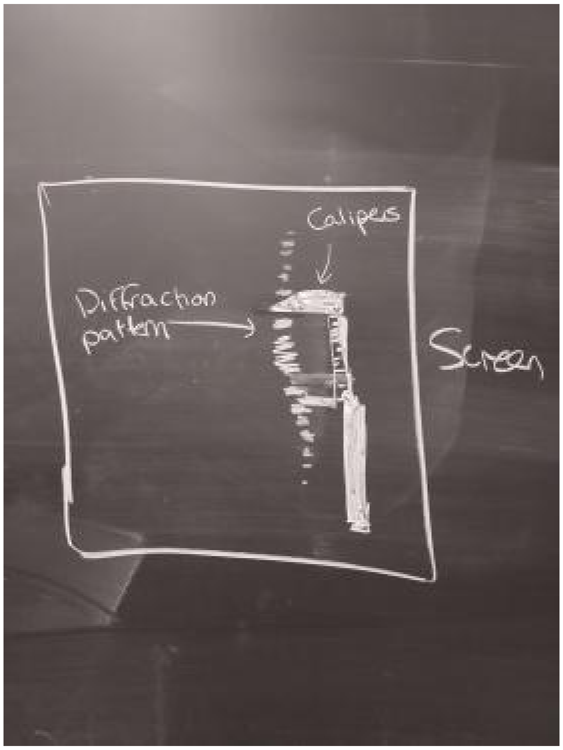
Measure and record the distance between the screen and the wire. Then, using calipers, measure and record the distance between the centers of the two second minima, one on either side. Repeat this for the third through the sixth minimas.

Then, replace the 26-gauge wire with the 39-gauge wire and repeat the procedure.

Experimental Setup:



The figure above details the experimental setup. The red diode laser is mounted so that the laser beam is obstructed by the wire, and the screen is placed to view the resulting diffraction pattern. Below, calipers are used to determine the diffraction minima.



Data:

Distance from wire to screen: 88.1 ± 0.2 cm

26-gauge

Value of m	Distance between minima (cm)
m=2	0.57 ± 0.05
m=3	0.78 ± 0.05
m=4	1.09 ± 0.05
m=5	1.39 ± 0.05
m=6	1.67 ± 0.05

39-gauge

Value of m	Distance between minima (cm)
m=2	2.0 ± 0.1
m=3	3.2 ± 0.2
m=4	4.5 ± 0.2
m=5	5.2 ± 0.3
m=6	not measured

Data Analysis:

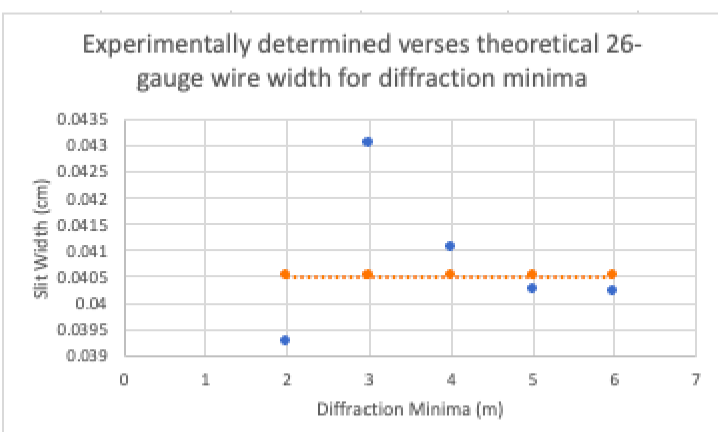
Since we have several data points and are confirming an expected value, we can perform a χ^2 test against the constant function, since we expect the calculated width of the aperture, and consequently the thickness of the wire, to be a constant as we vary the minima measurements.

First, we analyze the data for the 26-gauge:

Applying the equations to find a with its appropriate error we get:

a	alpha_a
0.0392586	0.00344374
0.04303346	0.00275856
0.04105945	0.00188346
0.04024712	0.00144774
0.04019892	0.00120356

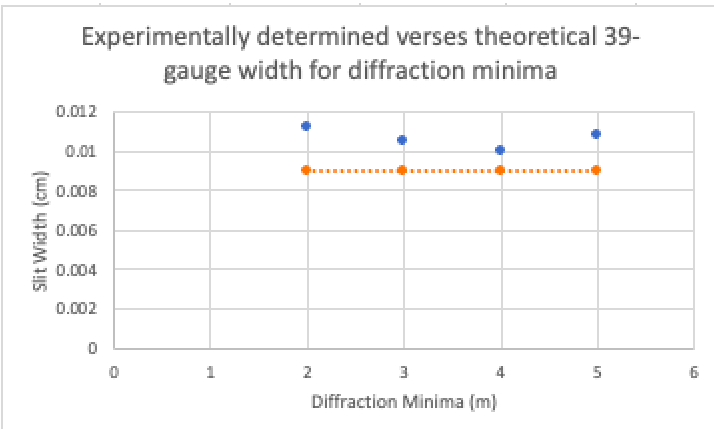
Then, plotting against the expected value, we get the following graph.



This suggests the data is relatively normally distributed, which we expect since the data in this case are effectively residuals. We can very easily calculate the expected χ^2 and $\overline{\chi}^2$, and we obtain:

Chi^2	Reduced chi^2
1.15472481	0.2886812

For the 39-gauge, similar calculations yield:



a	alpha_a
0.0111887	0.00055944
0.01048941	0.00065559
0.00994551	0.00044202
0.01075837	0.00062067

Chi^2	Reduced chi^2
34.2728104	8.56820259

In this case, we observe that the actual data points are consistently above the theoretical, which suggests some systematic error. Additionally, the reduced chi^2 is large but not substantially large to the point where we would begin to question our model.

Conclusion:

The goal of this experiment was to verify Babinet's principle for two wires, a 26-gauge wire and a 39-gauge wire. Explicitly, this meant demonstrating that the diffraction pattern of a laser beam obstructed by an opaque body such as the wire is exactly the same as what would be produced by an aperture of the same width. By applying the procedure we had developed in experiment 1 and measuring the distances between low-order minima, we could obtain data sufficient to calculate the experimentally determined slit width and compare it to the theoretical.

Overall, our experiment was largely successful. For the 26-gauge wire, we obtained a reduced chi^2 of about 0.29, which is reasonable when considering that we were only able to gather five total data points; normally we would check whether our reduced chi^2 value falls within the interval 0.5 < $\bar{\chi}^2$ <2.0, but due to the very small sample size it is acceptable to be more lenient. A smaller reduced chi^2 in general suggests that we may have underestimated our errors, which is reasonable since it is somewhat challenging to measure the distance between successive minima with calipers and be precise.

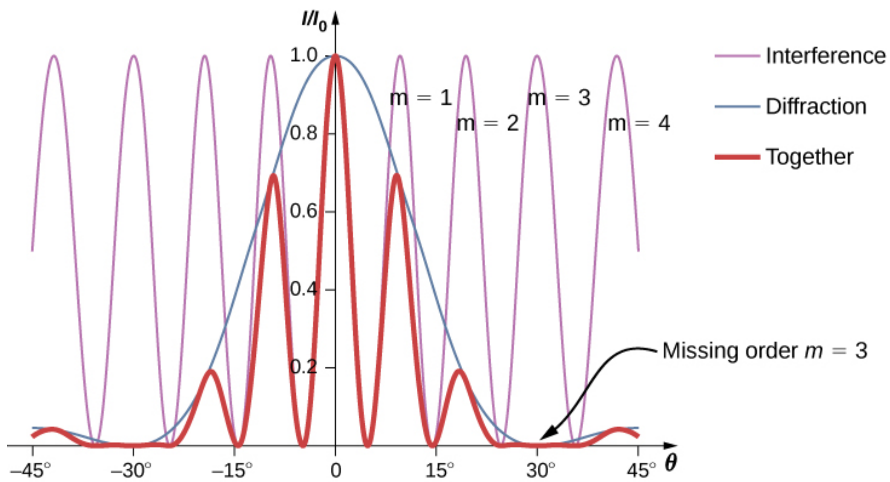
On the other hand, the 39-gauge data produced a reduced chi^2 of 8.57. This result is consistent with the observed distribution, which indicates that the data points are not normally distributed above and below the line: in fact, they all fall above the line, which suggests some systematic error in measurement is present. This may have been due to the immense difficulty in measuring the diffraction pattern for the 39-gauge, and consequently in our attempts to measure from the center of one minima to another it was sometimes easier to measure from the end of one maximum to the end of another. This would cause a smaller y_{min}, and therefore a larger a, which is consistent with the results we found. The reduced chi^2 is not large enough to suggest the constant model is not an appropriate fit; rather, it seems slightly off due to some systematic error. Additionally, it would have been beneficial to gather more data points for higher orders as well to be more confident in our assertion of systematic error.

<u>EXPERIMENT 5</u>

Objective:

The goal of this experiment is to determine the slit width a and slit separation d for double thick slits.

Theory and Background:



The double thick slit behavior effectively models the superposition of the single thick slit and the double thin slit patterns; we observe an overall "envelope" from the single thick slit which follows the equation $a = n\lambda/sin\theta$, which by small angle approximations gives us $a = 2n\lambda L/y_{min}$, with associated error

$\alpha_a = \frac{2nL\lambda}{y_{min}^2}\alpha_{y_{min}}^2$.

Similarly, we have the equation for the maxima from the double thin slit pattern, following the equation $d = m\lambda/sin\theta$, and again by the small angle approximation $d = 2m\lambda L/y_{max}$, and our associated error is exactly $\alpha_d = \frac{2mL\lambda}{y_{max}^2}\alpha_{y_{max}}^2$.

Using these relations, we can also experimentally determine missing orders: when an interference maxima is completely obstructed by a diffraction minima. Experimentally, this will appear as a completely dark spot between interference maxima. Note that there will be locations where the interference maxima is slightly dimmer, and this is *not* a missing order, since it will be partially but not completely obstructed.

From there, we can determine m and n, and then use the relation $a/d = n/m$ to compare whether the missing order agrees with our calculations of a and d.

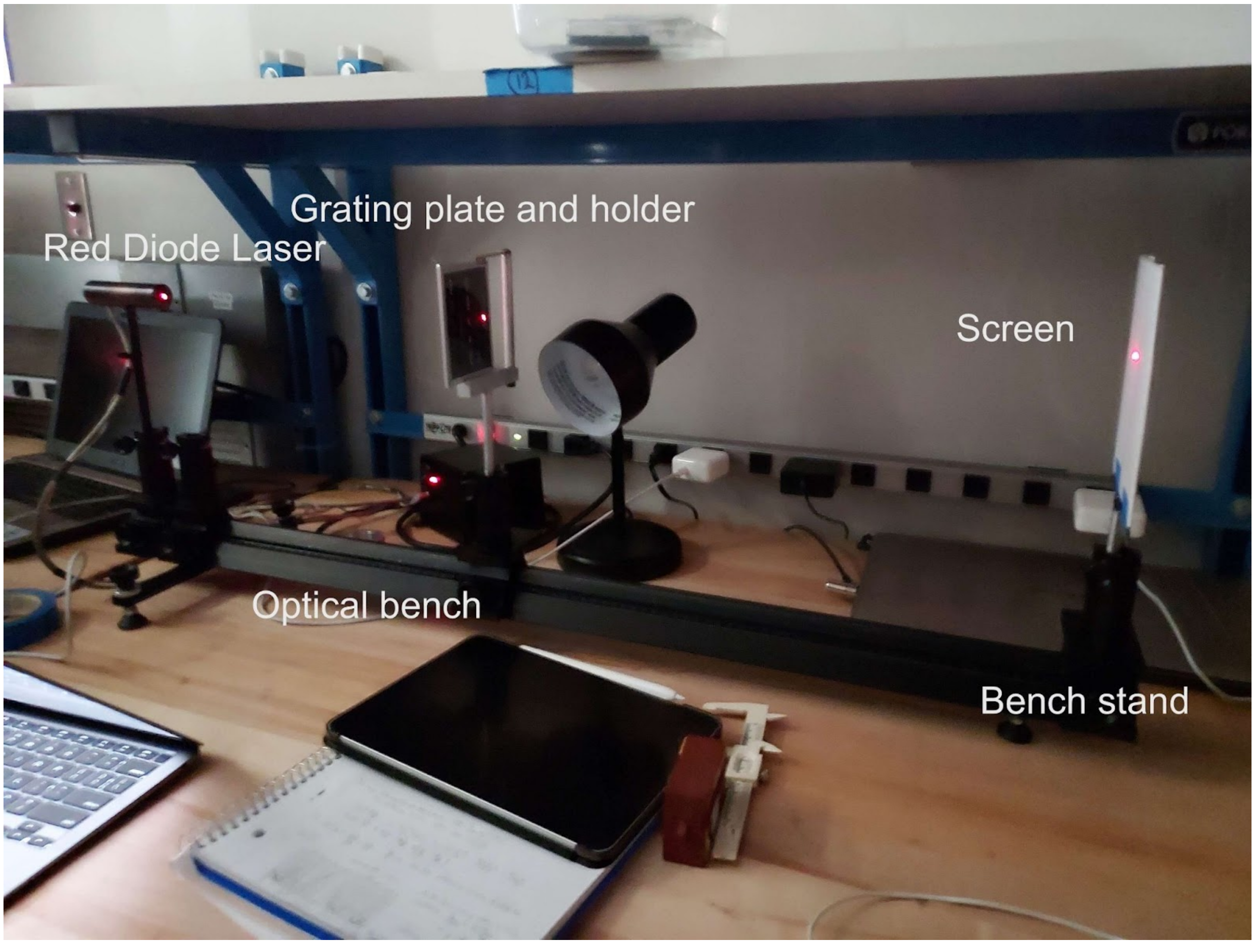
Experimental Procedure:

Line up the red diode laser, the grating plate and holder, and the screen along the optical bench. Make sure that the grating plate is sufficiently far from the red diode laser, but also sufficiently far from the screen. Angle the red diode laser and pull the grating plate out such that the resulting beam shines through the center of the middle double slit section of the plate, and position the screen so that the full diffraction pattern and the interference pattern can be both seen and accurately measured. Measure and record the distance between the grating plate and the screen.

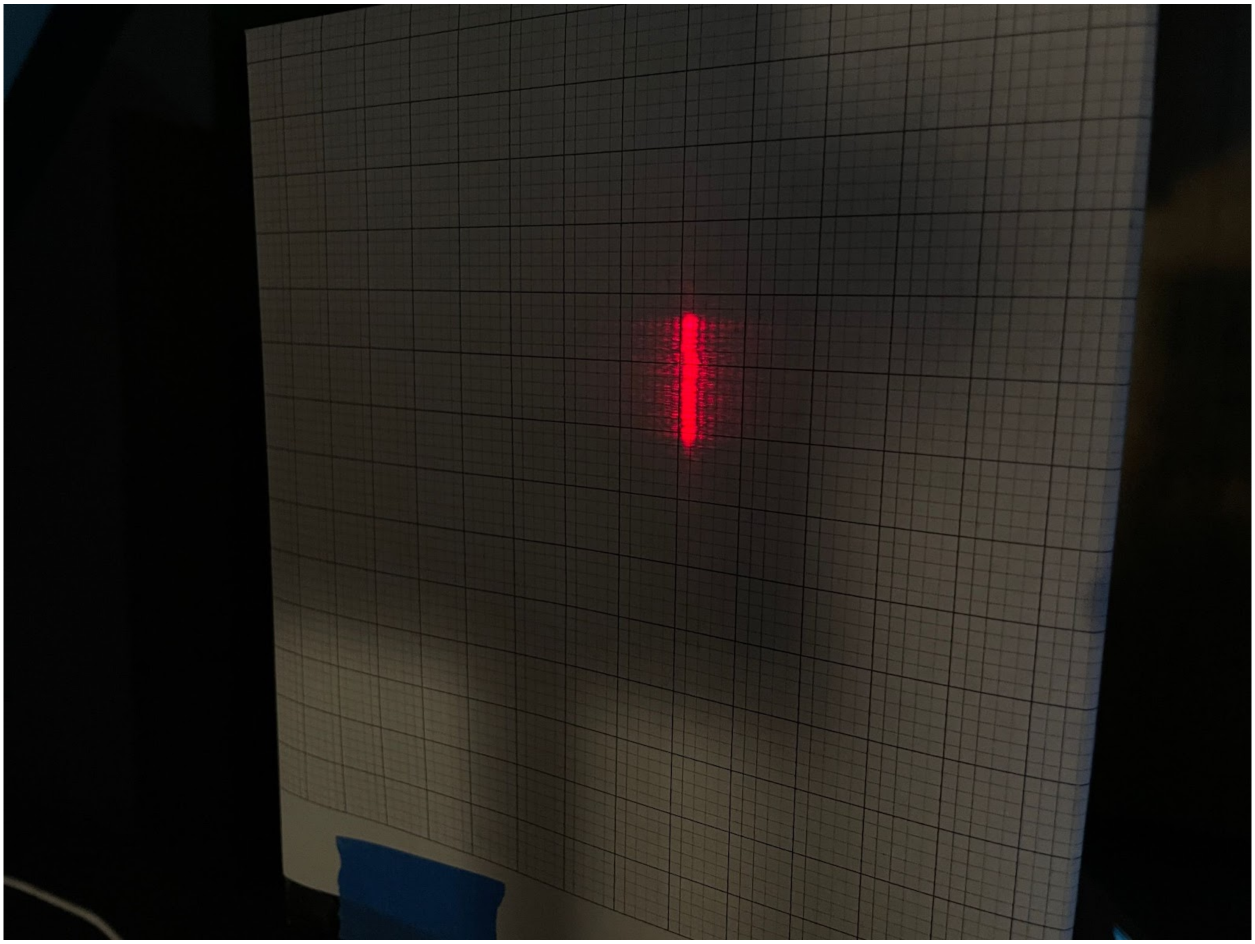
Next, proceed in two steps. The first is to determine the relative locations of the diffraction minima y_{min}. To do this, use the calipers to measure and record the distance between the low order diffraction minima, i.e. where the envelope itself dims. Since it is challenging to measure envelope minima due to the spread of the diffraction pattern, it is reasonable to measure the distance first four or five diffraction minima. After that, determine the distance y_{max} between each interference maxima for the first five or six low order maxima, and record.

Finally, adjust the laser vertically so that it now aligns with two slits that are closer together, and repeat the outlined procedure.

Experimental Setup:



The experimental setup is depicted above. The red diode laser is shined through the center and off-center thick double slits to produce a diffraction pattern on the screen. The diffraction pattern itself is shown in the image below:



The first one was simply to verify the double thick slit diffraction pattern, but we ended up pulling the diffraction grating farther back to get a thinner and wider diffraction pattern as seen in the second image.

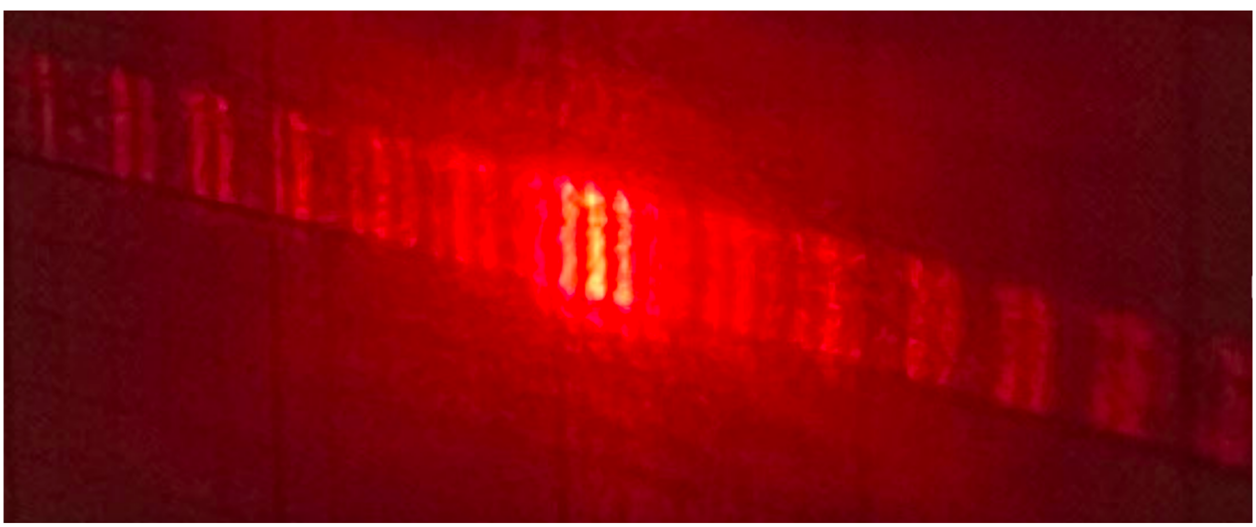


Figure 1: Close up view of the diffraction pattern given by the diffraction grating

Data

First, the distance between the grating plate and the screen was 83.7 + 0.2 cm

For the middle slit:

Trial 1

Minima for a:

Value of m	Distance (cm)
n=1	0.88 ± 0.10
n=2	1.70 ± 0.10
n=3	2.75 ± 0.10
n=4	3.68 ± 0.10
n=5	4.64 ± 0.10

Maxima for d:

Value of m	Distance (cm)
m=2	0.20 ± 0.03
m=3	0.38 ± 0.03
m=4	0.54 ± 0.05
m=5	0.70 ± 0.05
m=6	0.90 ± 0.05

For the one directly above it (smaller separation)

Trial 2

Minima for a:

Value of m	Distance (cm)
n=1	0.90 ± 0.05

n=2	1.8 ± 0.1
n=3	2.9 ± 0.1
n=4	3.7 ± 0.2

Maxima for d:

Note: this was unmeasurable unless our distance was fairly large; therefore, we extended this setup all the way to the cabinet, using meter sticks to measure the distance. For these measurements, our distance L = 236 ± 10 cm

Value of m	Distance (cm)
m=1	0.25 ± 0.03
m=2	0.40 ± 0.05
m=3	0.7 + 0.1
m=4	1.0 ± 0.1
m=5	1.3 ± 0.1
m=6	1.6 + 0.1

Data Analysis:

We use the equation derived in the theory section to obtain the following mean values for the slit widths and slit separations and their associated errors :

	Slit Width (cm)	Slit Separation (cm)
Slit 1 (Trial 1)	$0.01180 \pm 6 \times 10^{-5}$	0.0831 ± 0.0003
Slit 2 (Trial 2)	$0.01150 \pm 7 \times 10^{-5}$	0.1240 ± 0.0009

From the lab manual, we also know the slit distances:

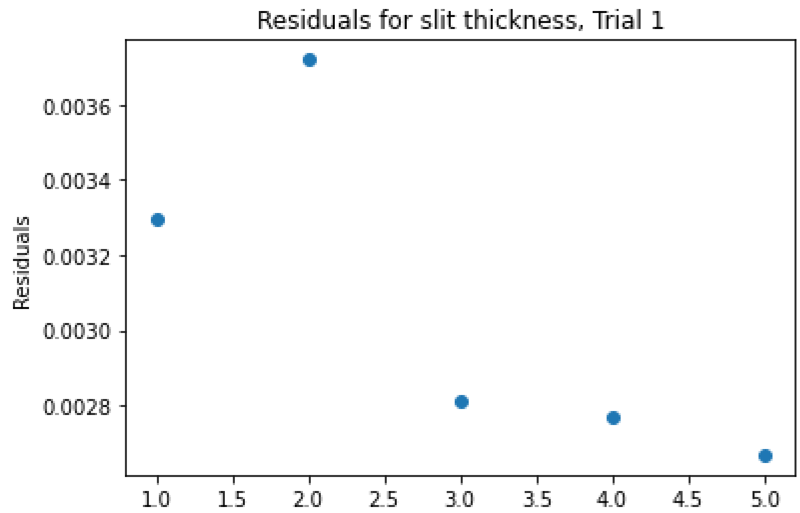
	Slit Width (cm)	Slit Separation (cm)
Slit 1 (Trial 1)	0.008786	0.026358

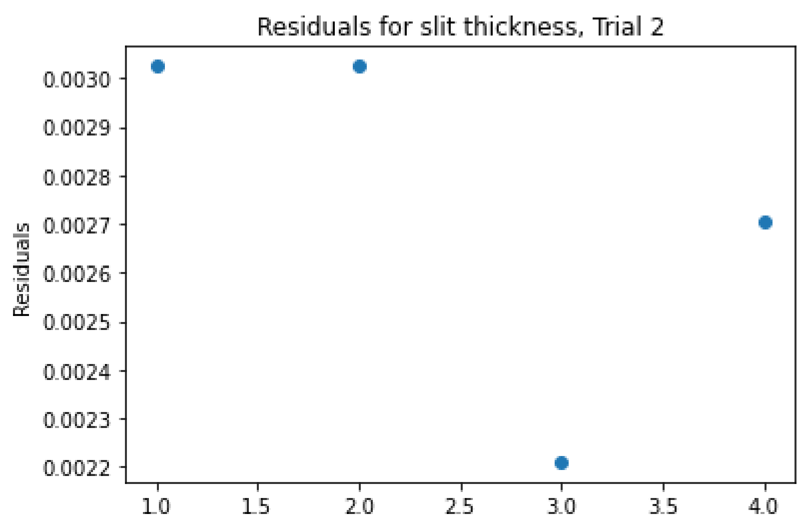
Slit 2 (Trial 2)	0.008786	0.008786

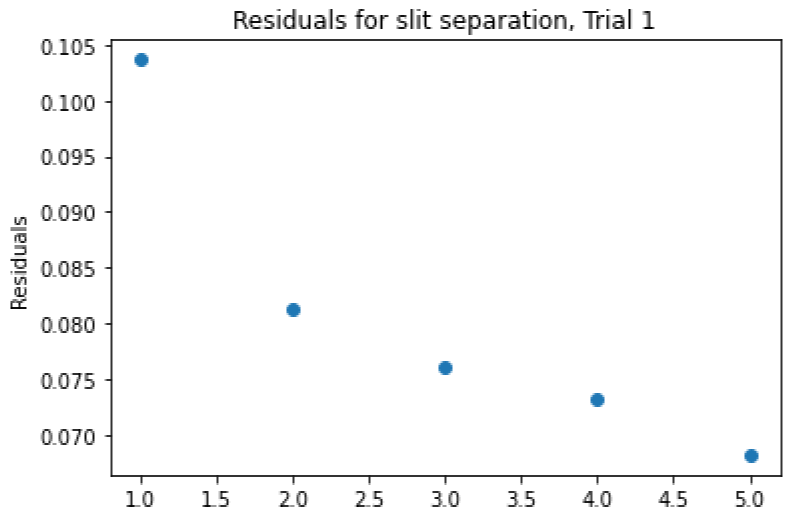
Firstly, notice that the errors that we obtained from the error propagation are incredibly small, and do not pass an agreement test (using the same equation as we did in Experiment 3). However, because we are dealing with measurements on the order of hundredths to thousandths of a centimeter, we argue that the uncertainty introduced by the limitations in our ability to accurately take measurements by hand is far greater than the uncertainties we propagated. As a result, we argue that these error propagated values should not be trusted as the true error.

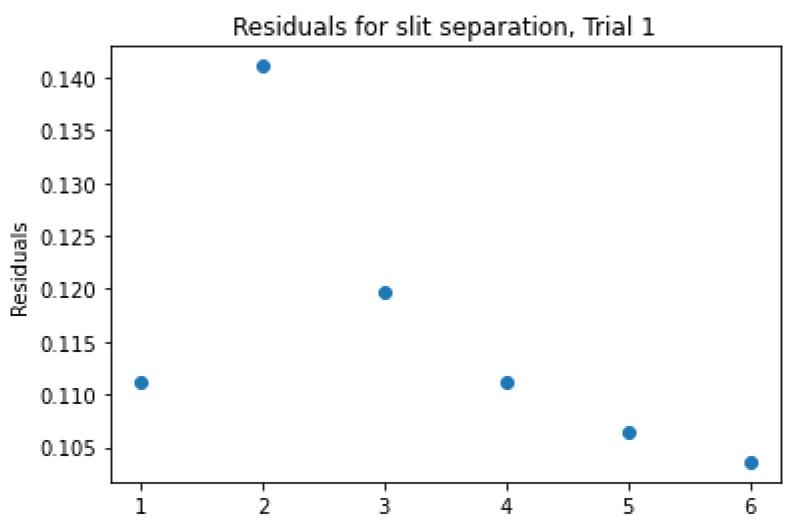
Roughly speaking, it's clear that the difference between our experimental and theoretical value in the slit thickness, which is on the order of 3 thousandths of a centimeter for both 1 and trial 2, is quite good considering the fact that we were measuring these distances by hand. Therefore, it is safe to conclude that there is good agreement between our theoretical equations and the experimental model. Furthermore, the distance between the cornell grating and the "screen" for Trial 2 was approximately 236cm but this was measured using three separate rulers concatenated together, and with no real way of maintaining the fact that all three rulers were level, this was a highly inaccurate measurement, and likely could have contributed significantly to the differences between our theoretical and experimental values, especially for quantities such as the slit separation.

To reinforce this conclusion, we can compute the residuals (i.e. raw distance) between our experimental values and the theoretical. Doing so, we generate the following plots:


Residuals for slit thickness, Trial 1
Residuals
0.0036
0.0034
0.0032
0.0030
0.0028
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Residuals for slit thickness, Trial 2
Residuals
0.0030
0.0029
0.0028
0.0027
0.0026
0.0025
0.0024
0.0023
0.0022
1.0
1.5
2.0
2.5
3.0
3.5
4.0





Perhaps the most evident feature about all these plots is that our residuals appear to be relatively high, reinforcing the claim that there are multiple sources of error in our experimental procedure that are significantly higher than what we theoretically were obtaining with our error propagations. However, as mentioned before, because we are dealing with such small scales of measurement, any slight perturbation of our system due to human error will greatly outweigh the systematic errors calculated through error propagation.

However, despite this, the fact that our experimental procedure was able to get within a thousandth of a centimeter in the best case and a tenth in the worst case is nevertheless a testament to the accuracy of our experimental model, barring our large errors.

In regards to the missing orders, we see that in the image for Trial 1 (Figure 1) we need to count to the fifth diffraction minimum to get an exact missing order, which corresponds to a value of $n = 5$ and $m = 17$, giving an experimental ratio of

$$\frac{a}{d} = \frac{n}{m} = \frac{5}{17} = 0.29$$

Comparing this to the theoretical ratio of 2/6 = 0.33, we can see that this value is incredibly close to the theoretical value, so despite our experimentally measured values not agreeing with our theoretical model, the fact that this measurement agrees demonstrates further that our experimental setup was correct, but the human errors associated with our measurements were simply to great to generate a good agreement between theoretical and experimental values.

Conclusion:

Overall, we still consider this experiment to be a success. Despite our experimental values not agreeing with our theoretical ones, the fact that our experimental procedure was able to get within a thousandth of a centimeter in the best case and a tenth in the worst case is nevertheless a testament to the accuracy of our experimental model. Furthermore, our experimentally determined missing order corresponded quite well with the theoretical model, further demonstrating the accuracy of our experimental setup.

However, as evident by the fact that our results do not agree with theoretical ones, our experimental setup is still clearly lacking in some respects. One of our major sources of error throughout this entire experiment was the fact that measuring the distances of the diffraction pattern on the screen was incredibly challenging since the interference lines were extremely close together. To remedy this, one potential solution is to increase the distance between the grating and the screen, but in a way where its distance could be accurately determined. In other words, a longer table would be preferable and a measuring laser could be used to determine the grating to screen distance. Just like Experiment 3, one other potential option is to use a diverging lens to help spread out the diffraction pattern and make it easier to measure, though introducing a diverging lens introduces error associated with the lens, and it is unknown whether the benefits outweigh the increases in error.