Eric Du
Lecture Notes

Special and General Relativity

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1 Introduction

- Let's go back to 1974, e
- No classical theory permits this gradual energy loss, except in General Relativity!
- Speaking of General relativity, one thing it predicted was the present of gravitational waves (GW), and this was where the lost energy was going. Specifically, we can calculate its power:

$$P = -\frac{2}{5}\frac{G^4M^5}{R^5c^5}$$

- We commonly think of $G = 6.67 \times 10^{-11}$, but later in the course we're going to work in units where $G = 1$, to simplify things.
- In electrodynamics, an charge q that experiences an acceleration also emits electromagnetic waves. This is called synchrotron radiation:

$$P = -\frac{2}{3}\left(\frac{q^2}{4\pi\epsilon_0}\right)\frac{a^2}{c^3}$$

  To get a sense of the scale of this power, the amount that our solar system is losing due to the sun and Jupiter is around 200 W. But Hudson and Taylor found a power of $P = -7 \cdot 10^{34}$ W!

- In 1983, they measured this, in 1993 they won the Nobel prize for their indirect detection of gravitational waves. In 2015, LIGO detected these waves directly, and found a power $P = 3.6 \times 10^{49}$ W. In 2017, they won the Nobel prize for this discovery.

1.1 Why General Relativity?

- In Newton's gravity, we have the equation $\vec{F}_i = \frac{\mathrm{d}^2\vec{r}_i}{\mathrm{d}t^2}$, and for universal gravitation, we had:

$$\vec{F}_i = \sum_{j\neq i}\frac{Gm_im_j}{|\vec{r}_i - \vec{r}_j|^3}(\vec{r}_j - \vec{r}_i)$$

  Note that it's only formatted like this so that we can talk about vectors.

- One problem with this interpretation is that things are instantaneous: this is an issue because objects don't react instantaneously to changes (information can't travel faster than the speed of light), which Newton's equations seem to imply.

  We can say the same about Coulomb's law: and the solution there was to replace the notion of a force with *fields*. Now, the force can be written as:

$$\vec{F}_i = q(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B})$$

We'll use Gaussian units, mainly because $\vec{E}$ and $\vec{B}$ now have the same units. With this,

$$\begin{aligned}
\vec{\nabla} \cdot E &= 4\pi\rho \\
\vec{\nabla} \cdot B &= 0 \\
\vec{\nabla} \times E &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \\
\vec{\nabla} \times B &= \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}
\end{aligned}$$

So the question then becomes: why didn't we do this for Gravitation? Well, this is a thing, but it's only an approximation.

- Let's talk about energy conservation: in E&M, the energy is written as:

$$\mathcal{E} = \frac{1}{8\pi}\int (\vec{E}^2 + \vec{B}^2) dv + \sum_i k_i$$

What happens when we change the sign on everything to accomodate for gravitation? Then, we introduce instability into the system!