

Chapters 3: Statistical Mechanics

- Stirling's approximation: $\ln n! = n \ln n - n$
- Discrete probability distribution $\langle x \rangle = \sum_i x_i P_i$, where P_i denotes the probability of finding x_i .
- Continuous Probability Distribution $\langle x \rangle = \int x P(x) dx$
- For independent variables, $P(u, v) \Delta u \Delta v = P(u) du \cdot P(v) dv$, and $\langle uv \rangle = \langle u \rangle \langle v \rangle$
- Random Walk: defined as a situation where it is equally likely for a particle to walk a distance $+a$ and $-a$. The resulting mean is $\langle x \rangle = 0$, but $\sigma_x^2 = na^2$.

Linear Transforms

- $y = mx + b$ gives us $\langle y \rangle = m \langle x \rangle + b$, and $\sigma_y^2 = m^2 \sigma_x^2$

Ideal Gas Law

States that $PV = nRT$, or that $PV = nk_B T$, where $R = N_A k_B$ and $n = \frac{N}{N_A}$.

Chapter 5: Maxwell Boltzmann Distribution

An incredibly general statement about the distribution of variables which follow from a statistical distribution. That is, for a function like $\Omega(E)$ which is defined as the number of microstates, and the number of microstates is defined via a combinatorial argument, then we get this distribution shape.

Mathematically, we have the equation

$$\frac{d \ln \Omega(E)}{dT} = \frac{1}{k_B T}$$

Chapter 6: Pressure

Ideal Gas Law

States that $PV = nRT$, or that $PV = nk_B T$, where $R = N_A k_B$ and $n = \frac{N}{N_A}$. It is a combination of multiple laws coming together.

Chapter 11: Energy

Functions of State

They are defined when a state is at equilibrium. The idea is that a variable is a function of state if we can *generate* a system which, at equilibrium, has a value as that function of state.

Heat Capacity

$c = \frac{dU}{dT}$ always when temperature is changing. Otherwise, $c = \frac{dQ}{dT}$ if temperature is constant.

- If there's no work, then $dU = dQ$, so therefore $c = \frac{dQ}{dT}$ is also a valid formula.

Note that heat capacity is actually a very poor term for this, we should really be thinking of this as energy capacity instead.

Laws of Thermodynamics

- **Zeroth Law:** If two systems A and B are in thermal equilibrium with another system C , then A and B are also in thermal equilibrium with each other.
- **First Law:** The total energy in a system is given by the total heat transferred to the system and the work done on the system. Conversely, the total change in energy of a system is given by the sum total of the change in heat as well as the work done (or done by) the system. Essentially, think of it as a reframing of the conservation of energy

$$dU = dQ + dW$$

For a thermodynamic process, we often have $dW = -pdV$ as the system does work on the outside, so we often write

$$dU = dQ - pdV$$

- **Second Law:** The total change in entropy of a reversible system is zero, whereas the change in entropy of an irreversible system is always positive. The change in entropy cannot be a net negative number. In other words,

$$\Delta S \geq 0$$

for any thermodynamic system.

Office Hours Questions

- Since dQ is labelled as an *inexact differential*, what form does it take on that makes it an inexact one?
- Why is the Boltzmann Distribution the way it is?
- Why is it that a system can be described exactly using three variables, and not more? What gives?
- How did we derive the relation $S = k_B \ln \Omega$?
- Particle mixing problem: why is it that having two identical particles in both chambers give us no changing entropy?