Eric Du Physics 111B
Prelab 1 Advanced Experimentation Laboratory September 17, 2024

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Prelab Questions

1. What is Non-linear Dynamics (NLD)? Specifically what is it that's non-linear?

   Solution: Nonlinear dynamics deals with the dynamics of nonlinear systems. Namely, this means that the differential equation that governs the motion of the system has a nonlinear term, so a power of 2 or above in one of the $x, \dot{x}, \ddot{x}, \ldots$ terms.

   For instance, the equation $\ddot{x} + \frac{g}{L}\sin x = 0$ is a nonlinear dynamical equation. □

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2. What is chaos? What are the defining characteristics of chaos?

   Solution: The book defines it as follows:

   *Chaos is **aperiodic long-term behavior** in a **deterministic** system that exhibits **sensitive dependence on initial conditions.*** Explained:

   - Aperiodic just means that over the long term the system doesn't settle into a periodic state.
   - Deterministic means that the system doesn't have any noisy inputs – the irregular behavior arises as a result of the equations rather than noise
   - Sensitive dependence on initial conditions means that trajectories which start off very similar diverge exponentially quickly.

   □

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3. Present and solve on the board the problems 9.3.1 and 11.4.2.

   1) (Quasiperiodicity ≠ chaos) The trajectories of the quasiperiodic system $\dot{\theta}_1 = \omega_1, \dot{\theta}_2 = \omega_2$ (ω_1/ω_2 is irrational) are not periodic.

      (a) Why isn't this system considered chaotic?

      Solution: This system is not considered chaotic because the distance $\|\delta(t)\|$ does not scale exponentially. □

      (b) Without using a computer, find the largest Liapunov exponent for the system.

      Solution: I imagine if we consider two trajectories separated by some $\|\delta\|$, that they won't diverge at all – they'll just be parallel to each other. There's no reason for them to diverge if ω_1, ω_2 are identical for both systems. Thus, $\|\delta(t)\| = \|\delta_0\|$, so max $\lambda = 0$. □

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   2) Find the box dimension of the Sierpinski carpet.

      Solution: The book gives the following definition for the box dimension:

$$d = \lim_{\epsilon \to 0} \frac{\ln N(\epsilon)}{\ln(1/\epsilon)}$$

      Here $N(\epsilon)$ is the number of ϵ-sized squares are required to cover the entire carpet.

Let the carpet have side length 1. Then, S_1 is covered by 8 squares with side length $\frac{1}{3}$. Then, S_2 is covered by 8^2 squares with side length $\epsilon = \left(\frac{1}{3}\right)^2$. (this follows the same process as Example 11.4.2 in the text) Then, we have:

$$d = \lim_{\epsilon \to 0} \frac{\ln 8^n}{\ln 3^n} = \frac{\ln 8}{\ln 3}$$

□

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4. What is the Feigenbaum ratio? What does it mean? See the Wolfram Math World subjects for help.

Solution: The Feigenbaum constant is a universal constant for systems which approach chaos via the process known as **period doubling**. Basically, for some chaotic systems where the period appears to double, we can let μ_n denote the point where the period doubles to 2^n, then we can express the Feigenbaum ratio as:

$$\delta = \lim_{n \to \infty} \frac{\mu_{n+1} - \mu_n}{\mu_{n+2} - \mu_{n+1}}$$

It is known that $\delta > 1$, and for a particular structure called the logistic map, we know $\delta \approx 4.669$.

Really, it just tells us that we increase our parameter, the time between subsequent period doublings gets faster and faster, roughly 4.5x faster every time.

What's also interesting about this is that it's considered **universal**, meaning that all period doubling systems, when we perform the calculation above, yields the same constant. □

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5. What is a return map? What is a Poincare map?

Solution: I honestly am not really sure. As for a Poincare section (which I imagine to be very closely related), it basically a traced version of the state space as time evolution is applied to the system.

After reading the textbook a bit more, basically the Poincare map is a way for us to denote when a system comes back on itself. The textbook has the following example: let $\dot{\mathbf{x}} = f(\mathbf{x})$ be an n-dimensional system, and consider the $n-1$ dimensional surface S. Then, the evolution of the dynamical system passes perpendicular to S always. Now, let $\mathbf{x}_k$ be the k-th intersection, and $\mathbf{x}_{k+1}$ be the $(k+1)$-th intersection through S. Then, the Poincare map $P : S \to S$ is definde by $P(\mathbf{x}_k) = \mathbf{x}_{k+1}$

The Poincare map is also called the **first return map**, which I think is what the first part of this problem is referring to. □

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6. What is a Fourier Transform? What does the power spectrum of a square wave look like? What is aliasing?

Solution: A Fourier transform gives us a way to view a signal in frequency space instead of temporal space. It's occasionally useful :)

The Fourier transform is a sinc function, so the power is a sinc^2 function.

Aliasing is the phenomenon when we undersample our points, and as a result of our undersampling we get losses in the data. Bascially, the sampling frequency must be at least half the bandwidth (in frequency space) in order to prevent aliasing. (This is the Nyquist criterion) □
