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1 2DFT

- 2DFT takes a signal $f(x, y)$ and converts it into a 2D signal $X(e^{j\omega_x}, e^{j\omega_y})$ that is defined as:

$$X(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j\omega_x t_1} e^{-j\omega_y t_2} \, dt_1 \, dt_2$$

  This is the definition of the Cartesian 2D Fourier transform. In frequency space, this is defined as:

$$X(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi j f_x t_1} e^{-j2\pi f_y t_2} \, dt_1 \, dt_2$$

  Then, to transform back:

$$f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(\omega_x, \omega_y) e^{j\omega_x x} e^{j\omega_y y} \, d\omega_x \, d\omega_y$$

  And in frequency space, this is written as:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f_x, f_y) e^{2\pi j f_x x} e^{2\pi j f_y y} \, df_x \, df_y$$

- A signal is separable if we can write a signal $f(x, y)$ into $f_x(x) f_y(y)$, in other words, into two functions that exhibit dependence in only one of the two variables. So, a 2-dimensional delta function $\delta[x_0, y_0]$ is separable into $\delta_x[x_0]\delta_y[y_0]$.
- In discrete time, the formulas are more or less the same:

$$X(e^{j\omega_1}, e^{j\omega_2}) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x[n_1, n_2] e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

2 Practice Problmes and Questions

2.1 Spring 2019

2.1.1 Problem 1

- $\forall t \in \mathbb{R}$, we have $f(t) = e^{-\alpha t^2}$.