1 Concentration Inequalities

- Markov's inequality is:

  $$P(X \geq k) \leq \frac{E(X)}{k}$$

  The proof comes from the expectation formula:

  $$E(X) = \sum_{\text{all } a} aP(X = a) \geq \sum_{a>=k} aP(X = a) \geq \sum_{a \geq k} kP(X = a) = k \sum_{a \geq k} P(X = a) = kP(X \geq k)$$

  From here, we can conclude that

  $$P(X \geq k) \leq \frac{E(X)}{k}$$

  this inequality is good, but its a fairly weak upper bound. Note also that Markov's has the inherent downside that it requires your random variable X to be nonnegative, which isn't always the case.

- Chebyshev's inequality is:

  $$P(|X - \mu| \geq c) \leq \frac{Var(X)}{c^2}$$

  this formula gives a much tighter bound for X, but it requires a bit more work to get there. Markov's is nice in that if you're given a value of k to work with, then you know to just plug k into the formula. However, with Chebyshev's, you need to do a bit of work in choosing μ and c in order to get the bound you want.

  Chebyshev's inequality has the downside that it gives you a two-sided bound, instead of the one-sided bound that is guaranteed by Markov's. This is because of the $|X - \mu| \geq c$ term.

- I also think that with Chebyshev's, the two-tailed sum will end up giving you some overhead (in the two-sided limit), but that's kind of the price you have to pay.

2 LLN

- LLN basically is an application of Chebyshev's, which basically says that as N grows large, then the sample mean approaches the true mean of the distribution. This is what it means when you flip a coin 10000 times and your sample mean (the flips) roughly matches the true mean of $\frac{1}{2}$.

  This is a result of the fact that the variance of the mean for a random variable scales with $\frac{1}{n}$, so as n increases, the variance in the mean grows smaller.

3 Continuous Probability

- We went over the continuous analogues last week, so I won't bother writing them here.

A Instrument plots

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