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Abstract

In this lab, we investigate the Hall effect, and how it can be used to calculate fundamental constants such as the carrier mobilities and densities of a semiconductor. We use a hole-doped germanium crystal (which we will confirm to be so in the lab) as our semiconductor, and use the Van der Pauw technique to experimentally determine the resistivity. From these measurements, we determine the Hall coefficient, the hall mobility, the individual carrier mobilities and densities and their relationship to temperature in this lab. We then discuss the nature and accuracy of our results to verify our experiment.

1 Introduction

This report concerns the SHE experiment in the Physics 111B Experimentation Laboratory. In this report, we will begin by detailing the theoretical background for the phenomena, then analyze the data we collect to verify our theoretical results.

2 Theory

In this section, we will cover all the theory relevant to our experiment and our analysis. Unlike other labs, this lab is particularly theory heavy, so it is imperative that the underlying theory be understood so that the data and results may be effectively interpreted. As such, this lab report will follow a slightly different structure than the others: we will go over *all* the requisite theory in this section, instead of saving some theory for the analysis. As it turns out, the analysis and results will be relatively straightforward,

2.1 Conductors, Insulators and Semiconductors

Before discussing the hall effect, it is useful to outline the underlying physics of conductors, semiconductors and insulators, as the theoretical background will be important for our analysis performed later in the lab.¹ To begin, the most important question to ask is: what actually causes current to flow through a material? Or, in other words, what causes a material to become electrically conductive?

Before tackling this question, we need to first examine the energy levels of electrons in a solid. In contrast to singular atoms, where the energy levels are discrete and separated by a known energy gap, the energy levels in a solid exist in *energy bands*, separated by some energy gap E_g . The reason this occurs is because unlike a singular atom, an electron not only feels electrostatic attraction to its host atom, but also to all the positively charged neighboring nuclei in the solid. Thus, these extra electrostatic forces cause the electron to become "delocalized" in the material, and thus the electrons now exist in an energy bands instead of levels.

To study current, we need to become intimately familiar with two particular energy bands, called the *valence band* and the *conduction band*. At absolute zero, due to the Pauli exclusion principle, electrons cannot occupy the same state, so electrons do the next best thing – occupy all the lowest energy levels up to a certain energy, which we call the Fermi energy E_F . All energy levels below this energy are occupied, and are what we define to be the *valence band*. The valence band is (mostly) filled at all times, at least for the temperature regimes we will be working with.

By contrast, the conduction band is a band of allowed energy levels at which an electron is able to freely roam through the material. Because of this freedom, these electrons will move when a voltage difference is applied through the material, which manifests itself as measurable current. This energy band is generally unfilled, unlike the valence band. As such, this band exists at a higher energy than the valence band, and the difference between the highest energy in the valence band and the lowest energy in the conduction band is called the *band gap*, denoted as E_g . When an electron has energy greater than $E_F + E_g$, then it has the

¹This is actually especially true for this lab, more so than the other ones.

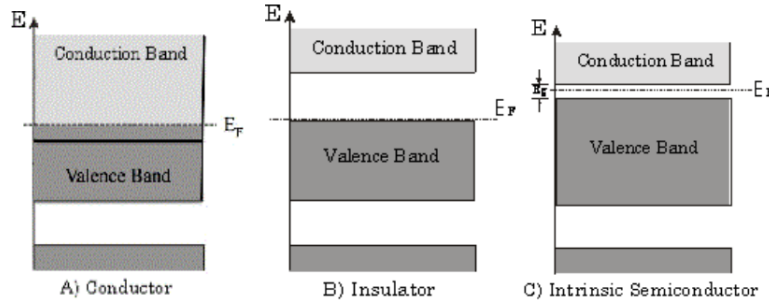


Figure 1: Diagram showing the different valence and conduction band configurations for (A) a conductor, (B) an insulator, and (C) a semiconductor. The semiconductor here is labeled *intrinsic*, meaning that there are zero impurities in the semiconductor crystal.

ability to be promoted to the conduction band, and in doing so it also leaves behind a *hole*, which can be thought of as a positron, or a positively charged electron. Because of this effect, holes and electrons in the conduction band come in pairs, a fact which will become useful later. So, the study of current is effectively dictated by the transition of electrons between the valence and conduction bands. Below, we study the structure of the valence and conduction bands inside a conductor, an insulator, and finally a semiconductor.

First, we consider a conductor. By definition, conductors are materials that allow current to flow when even a small $V > 0$ is applied to both ends of the material. In order for this to happen, this would mean that there must exist electrons in the conduction band at all times; the only way for this to happen is if the valence and conduction band overlap, so that at any given temperature there are electrons in the conduction band able to move through the material. Thus, for a conductor, there is no band gap between the valence and conduction bands – they completely overlap.

An insulator exhibits the exact opposite of this phenomenon. In an insulator, we know that we require a large voltage difference in order for a current to be produced. Thus, not only does this mean that there aren't any electrons in the conduction band, but the band gap between the valence and conduction band must be large. In particular, we know from statistical mechanics that the variance (or the spread) in electron energies around E_F is on the order of $k_B T^2$, so for there to be few electrons in the conduction band we must require that $E_g \gg k_B T$ in an insulator.

A semiconductor is a material that lives within these two regimes. In a semiconductor, the valence band and conduction band don't overlap, but they are on the order of $k_B T$. Because the band gap is on this order, it means that a semiconductor's properties are highly dependent on temperature, and is one of the main effects we will leverage in this lab. Again, leveraging the spread of the energies to be on the order of $k_B T$, then when $k_B T \gtrsim E_g$, then there are electrons in the conduction band, so the material behaves more like a conductor. When $k_B T \lesssim E_g$, the material behaves more like an insulator.

These conclusions are summarized in figure 1. Notice the large energy gap in an insulator, the lack thereof in a conductor, and a small one ($E_g \sim k_B T$) in the case of a semiconductor.

2.2 Doping

The small band gap of semiconductors lends itself very well to a process called *doping*, which allows us to alter the band gap energy. We do this by introducing some selected impurities into the semiconductor, which has the effect of either increasing the energy of the valence band or decreasing the energy of the conduction band. The type of effect we receive is determined by the kind of dopant atom we use: if we use an atom with an ionization energy lower than that of the intrinsic material, then we lower the conduction band, since the electron in this atom excites to the conduction band at a lower energy. By contrast, if we use an atom with a high ionization energy, then electrons will bind to this atom at a higher energy, thus increasing the energy of the

²To be a bit more specific, about this, the distribution of electron energies in a semiconductor follow the Fermi-Dirac distribution, $f(E) = \frac{1}{1+e^{(E-E_F)/k_B T}}$, which gives a nonzero probability of electrons existing at above E_F by a width of $k_B T$.

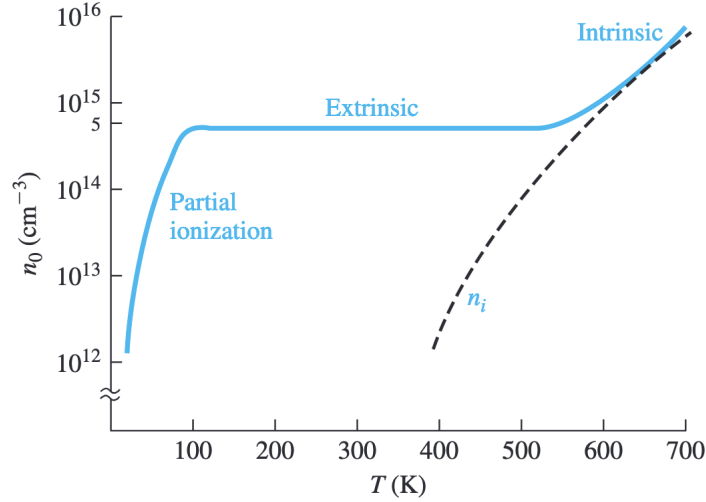


Figure 2: Diagram showing the electron concentration of a semiconductor against temperature, with the different semiconductor regimes labeled. The objective of this figure is just to illustrate the different regimes we're working in, so if there's anything to take away it's that the semiconductor behaves very differently depending on which regime it's in. This diagram is taken from [1].

valence band. The former case is also known as "electron doping" and is called an n-type semiconductor, and the latter is called "hole doping" and called a p-type semiconductor.

One characteristic effect of this doping is that it splits the semiconductor behavior into two regimes based on temperature. To see why, consider a semiconductor doped with an electron with lower ionization energy.³ At temperatures where $k_B T \sim E_g$ of the intrinsic material, both the doped electrons and the intrinsic electrons can excite to the conduction band. Since the electrons from the intrinsic material participate in conduction, this regime is often called the *intrinsic regime*. However, due to the lower ionization energy of the doped atoms, there exists a temperature range in which only the doped electrons can promote to the conduction band, which is called the *extrinsic regime*. The "phase transition" between these two regimes is completely determined by the properties of the doping atom we use, and the physics of the phase transition is complex, so we will make simplifications of the physics here in our analysis. Figure 2 shows the transition between what the extrinsic and intrinsic regime looks like in terms of electron concentration (in essence, the number of electrons participating in conduction). Note that there is also the regime of partial ionization on the graph, but this regime will not be relevant for us in this lab.

2.3 Hall Effect

2.3.1 One Carrier

Now, we turn to the Hall effect, the derivation of which we follow [1] closely. To explain this phenomenon, consider a semiconductor placed in a uniform magnetic field \mathbf{B} . Then, we pass a current \mathbf{I} perpendicular to \mathbf{B} . Then, because current is modeled as moving charge carriers, they will be deflected by the magnetic field, due to the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. Due to this deflection, we will end up with an accumulation of charge on one side of the semiconductor, which generates an electric field \mathbf{E} . This strength of the \mathbf{E} field is exactly enough such that the net force on the electrons is zero. That is,

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{0} \implies \mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0} \quad (1)$$

From this formula, we can deduce that the electric field strength \mathbf{E} is dependent on \mathbf{v} and \mathbf{B} . Here, \mathbf{v} is controlled by the current, and \mathbf{B} is the external magnetic field we apply. This \mathbf{E} field can also be measured via the relation that $\mathbf{E} = -\nabla V$, and due to the uniformity of the \mathbf{E} field we get $V_H = E_H W$, where W is the length of the semiconductor along the direction of the \mathbf{E} field.

³the exact reverse of this argument is the case where you hole dope instead of electron dope.

The polarity (or sign) of V_H is determined by whether the charge carriers are positively or negatively charged – in a p-type semiconductor the charge carriers are positive, so $V_H > 0$, and the opposite is true for n-type semiconductors.

Other quantities can also be determined from the Hall effect. To illustrate this, first we notice that from equation 1, we have the equation:

$$q\mathbf{E} = q(\mathbf{v}_x \times \mathbf{B}_z)$$

combining this with $V_H = E_H W$, we obtain, in terms of magnitude:

$$V_H = v_x W B_z$$

here, v_x represents the (average) velocity of the electrons through the semiconductor; the book calls this the *drift velocity*. Finally, the last equation we will use is the expression for the drift velocity in terms of the current and hole density p :

$$\mathbf{v}_x = \frac{\mathbf{J}_x}{ep} = \frac{\mathbf{I}_x}{(ep)(Wd)}$$

To explain this formula, it is perhaps more useful to write it as $I_x = v_x(ep)(Wd)$. v_x represents the drift velocity, ep is just a representation of how much charge is moving as it is a charge multiplied by a density, and Wd is just the dimensions of the semiconductor. Put this way, it should make sense why the formula for I_x is written like this:⁴ the current is a measure of how much moving charge there is, and the right hand side represents that by multiplying together a drift velocity and the "amount" of charge, which is a density times a volume. Finally, putting this together with the previous result, we end up with the first of two equations:

$$V_H = \frac{I_x B_z}{epd} \quad (2)$$

An analogous approach can be followed for an n-type semiconductor can also be done, where we arrive at the equation:

$$V_H = -\frac{I_x B_z}{end} \quad (3)$$

Notice the change in sign here, so V_H is negative when we have an n-type semiconductor, and positive when we have a p-type. This difference in V_H is what will allow us to determine the type of semiconductor we deal with in the experiment. With these equations, we can also calculate a quantity called the *Hall coefficient* R_H :

$$R_H = \frac{E_H}{J_x B_z} = \begin{cases} \frac{1}{en} & \text{n-type} \\ -\frac{1}{ep} & \text{p-type} \end{cases} \quad (4)$$

Intuitively, this Hall coefficient is effectively a measure of the strength of the generated electric field \mathbf{E} relative to the strength of the magnetic field that we introduce into the system. The Hall coefficient is something that we will be able to measure, and its sign will tell us which type of semiconductor we are working with.

2.3.2 Multiple Carriers

It should also be mentioned here that this above equation for R_H only works in the case of a single charge carrier, and a more complete equation is required when both positive and negative charge carriers are present. For this, we turn to [2] for a complete treatment. The first thing to notice is that despite there being different charges present, there is still no current flowing in the y direction, so therefore there should be no current flowing along this axis. Assuming that the drift velocities of both positive and negative charge carriers to be the same, this relation can be expressed as:

$$e(\mathbf{v}_y^+ n - \mathbf{v}_y^- n) = 0 \quad (5)$$

⁴I also want to acknowledge that here, I'm not writing I_x as a vector quantity as I did in the line above; this is because there is only one direction the current could flow in, so the direction is obvious.

Here, v_y^+ represents the velocity of the positive charges in the y direction, and v_y^- is the same but for negative charges. [2] also provides a formula for the drift velocity which we will make use of:

$$\mathbf{v}_y = \left(\frac{1}{2} \frac{\mathbf{F}}{m} d^2 \right) \frac{1}{d}$$

Now, \mathbf{F} acts on the carriers differently based on their charge. In particular, we have:

$$\begin{aligned}\mathbf{F}^+ &= e[(\mathbf{v}_x^+ \times \mathbf{B}) + \mathbf{E}_y] \\ \mathbf{F}^- &= -e[(\mathbf{v}_x^- \times \mathbf{B}) + \mathbf{E}_y]\end{aligned}$$

Here, we introduce a new term, called the *carrier mobility*. Intuitively, the carrier mobility represents how fast a charge carrier can make its way through a material when under the presence of an electric field, and it is defined as:

$$\mu = \frac{v}{E}$$

This carrier mobility is also a fundamental property of the semiconductor, and is one of the quantities we are interested in calculating for our sample. Here, because we have two different charges, we will consider the positive and negative carriers separately, and using this new relation between \mathbf{v}_x and μ , we can combine this with equation 5 to arrive at the following relationships for \mathbf{v}_x^+ and \mathbf{v}_x^- :

$$\begin{aligned}\mathbf{v}_x^+ &= \mu_p(\mu_p \mathbf{E}_x \mathbf{B} + \mathbf{E}_y) \\ \mathbf{v}_x^- &= \mu_n(\mu_n \mathbf{E}_x \mathbf{B} - \mathbf{E}_y)\end{aligned}$$

and finally, plugging this back into equation 5, we finally get the following relationship between \mathbf{E}_y and \mathbf{E}_x :

$$\mathbf{E}_y = \mathbf{E}_x B \frac{\mu_p^2 p - \mu_n^2 n}{\mu_p p + \mu_n n}$$

Finally, since we have a formula for R_H from equation 4, we may write:

$$R_H = \frac{\mathbf{E}_y}{\sigma \mathbf{E}_x \mathbf{B}} = \frac{\mu_p^2 p - \mu_n^2 n}{e(\mu_p p + \mu_n n)^2}$$

which is the "complete" form for R_H given two carriers present. Note that this equation does actually become 4 if we set either p or n to zero, as expected. The σ term represents the *conductivity*, which is related to the resistivity by $\sigma = 1/\rho$; this will be one of the values we are interested in.

2.4 The Van Der Pauw Technique

In our experiment, the main technique we will employ to determine the semiconductor characteristics is called the *Van Der Pauw Technique*. This technique, popularized by Leo van der Pauw in 1958, allows us to measure characteristics such as the resistivity and the carrier mobilities in a semiconductor. This technique is also very powerful, as it can be used on a semiconductor of arbitrary shape, provided that the following criteria are satisfied:

1. The contacts are at the circumference of the sample.
2. The contacts are sufficiently small.
3. The sample is an even thickness.
4. The sample does not have holes (i.e. the sample is a slab with no holes in it; this isn't referring to electron holes).

[3]. In our experiment, the semiconductor is a square shape of even thickness, with small leads placed on the corners of the shape (see figure 5), so in this case all four of these conditions are actually satisfied.

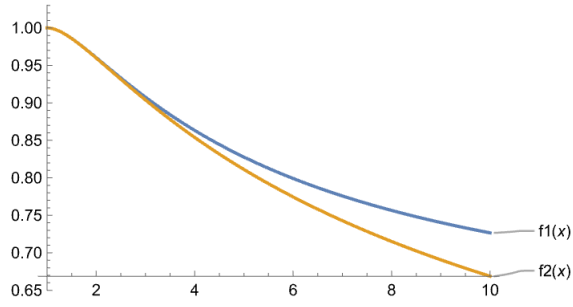


Figure 3: Comparison of the function f provided by Van der Pauw and the function f we will use in our approximation. While the approximation does get significantly worse past $x = 10$, our ratios (x -values) do not exist within that regime so this is a valid approximation.

When these conditions are satisfied, then the Van der Pauw theorem says that if we measure the voltage and current at these points A, B, C, D , then we can find resistances $R_{AB,DC}$ and $R_{AD,BC}$ (more on how to do this later), and we have the following relation between these two quantities:

$$\exp\left(-\frac{\pi d}{\rho} R_{AB,DC}\right) + \exp\left(-\frac{\pi d}{\rho} R_{AD,BC}\right) = 1$$

This equation can then be solved for ρ , which yields the following form:

$$\rho = \frac{\pi d}{\ln 2} \frac{(R_{AB,DC} + R_{AD,BC})}{2} \cdot f\left(\frac{R_{AB,DC}}{R_{AD,BC}}\right)$$

The function $f(x)$ is a function which satisfies the relation:

$$\frac{R_{AB,DC} - R_{AD,BC}}{R_{AB,DC} + R_{AD,BC}} = f \cosh^{-1}\left(\frac{\exp(\ln 2/f)}{2}\right)$$

This f is called the *correlation factor*, and it is a function only of the ratio of resistances $R_{AB,DC}$ and $R_{AD,BC}$. There are many ways we can approximate f ; van der Pauw himself suggested the following approximation that applies when $R_{AB,DC} \approx R_{AD,BC}$:

$$f_1 \approx 1 - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}}\right)^2 \frac{\ln 2}{2} - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}}\right)^4 \left(\frac{(\ln 2)^2}{4} - \frac{(\ln 2)^3}{12}\right) \quad (6)$$

While $R_{AB,DC} \approx R_{AD,BC}$ in our experiment, we will instead use a simpler approximation for f , given to us by [4]:

$$f_2(x) = \frac{1}{\cosh(\ln x/2.403)} \quad (7)$$

which is good enough. To illustrate how good of an approximation this is, figure 3 shows the plots of $f_1(x)$ and $f_2(x)$, where $f_1(x)$ is equation 6 and $f_2(x)$ is 7. As shown in the figure, the approximations are very good for x -values up to about $x \approx 4$, which is exactly the regime that we will be working in. The approximation is poor past $x = 10$, but this isn't a regime we should care about at all because neither f_1 nor f_2 are actually meant to approximate the analytically solved f in these regimes.

With the resistivity determined, we can then move on to determining the carrier mobilities, using the relation:

$$\frac{1}{\rho} = e (n\mu_n + p\mu_p) \quad (8)$$

where μ_n and μ_p are the electron and hole densities, respectively. With this determined, we now have full information about the semiconductor in the sample, which is what we ultimately set out to determine.

3 Experimental Procedure

Now, with the theory out of the way, we move on to the experimental procedure. We will split this section into two portions: first, we will explain the setup containing the semiconductor chip, and then we will discuss the surrounding apparatus separately.

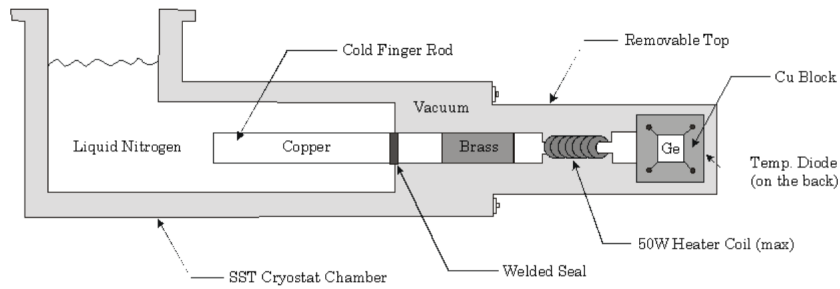


Figure 4: Block diagram of the central apparatus.

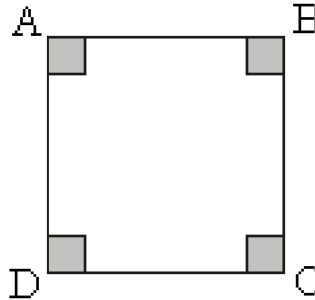


Figure 5: Schematic of the semiconductor chip, with leads placed at points *A*, *B*, *C*, *D* to perform voltage and current measurements.

To begin, figure 4 shows the semiconductor chip and the surrounding apparatus. On the left, there is an opening where we will pour in liquid nitrogen to cool the entire apparatus down. This cooling is transferred to the semiconductor via a copper rod, as shown in the figure. We place a brass intermediary here to control how quickly the sample cools down – we want to take measurements at varying temperatures, so it makes sense to not want the temperature to vary too fast. On the right side, we have the germanium crystal (our semiconductor), surrounded by four leads from which we will make voltage and current measurements. We also have a temperature diode placed next to the crystal, from which we will take temperature measurements. Finally, there is a 50W heater, which we will use to slowly heat the sample and take measurements over a wide range of temperatures.

Figure 12 shows a block diagram setup of the surrounding apparatus. The primary elements that we care about in this diagram are the electromagnet (a Helmholtz coil), which generates the uniform **B** field in the experiment, and the various meters in the bottom left. The meters record valuable information: the strength of **B** field, the voltage and current measurements, as well as the temperature. All of this information is fed into the DAQ box, which is then fed into the computer and displayed on the control_program.vi LabVIEW program. From here, we then export the data to a .csv file, from which it is ready to be analyzed.

Our experimental procedure for this experiment is relatively straightforward. First, we start the water cooling pumps, and turn on the vacuum pump⁵ and wait for the pressure in the chamber to below 90 millitorr. At this point, we begin pouring LN2 to the left side, and wait until the temperature drops to around 240 K. At this point, we pour the LN2 a second time, which will cool the sample to a temperature of around 90 K. While the apparatus is cooling, we turn on the other requisite equipment: the power supply to the heater, the Helmholtz coils, and also turn on the computer to record the measured data. This data is automatically collected for us via the "gold box", so all we have to do at this point is to just wait for all the data to be collected and upload it as a .csv file.

While pouring LN2 is the only manual thing we do in terms of data collection, we should still become familiar with what the apparatus is doing to perform its measurements. From what we could gather, the following steps are carried out for a given temperature reading:

⁵The reason for a pump is to prevent the germanium crystal from oxidizing; it has no measurable effect on the measurements themselves.

1. Record the temperature of the semiconductor.
2. With the magnetic field switched on (current flowing through the Helmholtz coils), record the strength of the magnetic field using the 450 Gaussmeter.
3. Using the probes attached to the semiconductor, send a current along AB , AD , AC , BD and also along the reverse directions.
4. For each current measurement, measure the voltage along the two probes that are not used to send current (i.e. if current is sent along AB , then we measure voltage along CD , and so on).
5. Reverse the polarity of the magnetic field (amounts to switching the direction of current) and repeat steps 2-4.
6. Once the positive and negative polarity data is taken, we then take a "zero field" measurement, by effectively running a vanishingly small current through the Helmholtz coils so that we approximate 0 G.

This process is done repeatedly for different temperature values, so we have data over a range of 95 K to approximately 350 K. Finally, we repeat this entire process different values of current, six of them to be exact. For some of our datasets, unfortunately the lab period ended just before the data could finish collecting, so some of our datasets stopped at around 330 K, but as we will see this won't affect the results that much. Now with the theory and procedure complete, we are ready to move on to the analysis.

4 Analysis

Now we move on to the analysis of the report. Above, we detailed how we took measurements for different current values, so this analysis section will detail the calculations performed for a single current dataset (30uA), then we will present our findings over all current values at the end. We will also go in order of the lab manual, detailing the things that are required in this report.

4.1 Resistivity and Hall Coefficient at Room Temperature

This section is relatively simple: directly from our measurements, we can use the Van der Pauw method and equation 6 to determine the resistivity for all temperatures. Then, we can then take its reciprocal and compute the conductivity as a function of temperature as well. This is then plotted against inverse temperature, and shown in figure 6. One thing to notice about the plot is that the resistivity is just ever so slightly different for positive and negative field; this is a phenomenon we observe but we don't have a concrete explanation for. One leading theory we have is that this has to do with Earth's ambient magnetic field, but we do not know how to confirm or reject this hypothesis.

The plot for the Hall coefficient is more relevant for the next section, so we will discuss it further there. The plot is given in figure 7b.

4.2 Semiconductor Type

Now, with the theory established, we are ready to move on to the central focus of this report. To begin, it is beneficial to go over all the data we collect, so that the steps taken in the analysis are clear. The quantities we recorded throughout the experiment were the temperature, the surrounding B field in Gauss, and the voltage and current measurements from each pair of leads A , B , C , D . To compute the resistivity, we will use the measurements that come from sending current through adjacent points, and for the Hall voltage we will compute from the current being driven across non-adjacent points.

To begin, the first quantity we should calculate is the Hall coefficient R_H , because calculating it will allow us to determine crucial information about what kind of semiconductor we are dealing with. To see why this is, consider what happens when we plugin equation 4 into equation 2 or 3. In either case, working out the signs, we the following relation between R_H and V_H :

$$V_H = -\frac{R_H I_x B_z}{d}$$

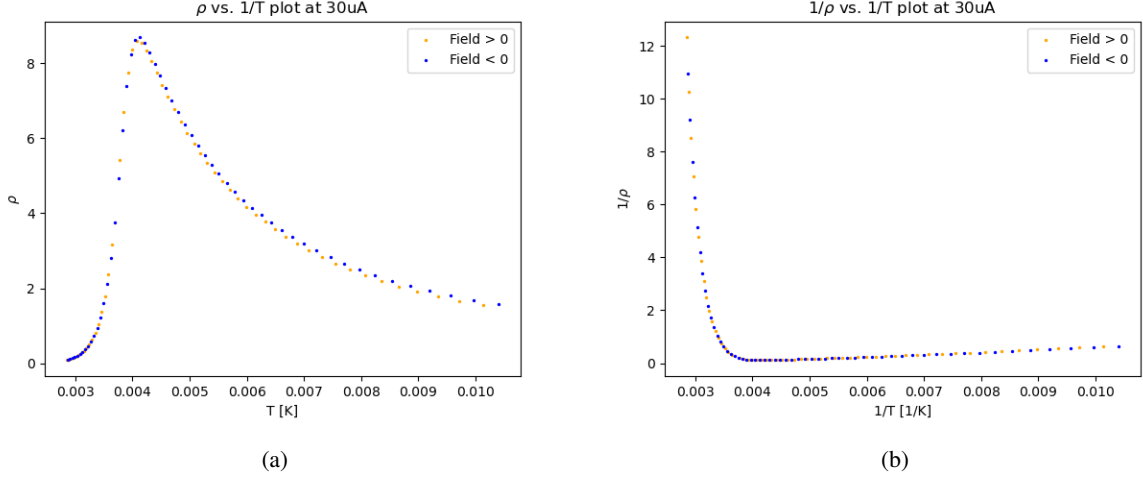


Figure 6: Plot of (a) resistivity and (b) conductivity versus inverse temperature. Note that these relationships do make sense intuitively: the conductivity increases as temperature decreases (or equivalently, as $1/T$ decreases, so this is a good indication that our plots are accurate.

Since we measure V_H , I_x and B_z , then plotting V_H against R_H will give us the sign of R_H . Specifically, from this equation we can deduce that the slope is given by:

$$m = -\frac{I_x B_z}{d} \text{sgn}(R_H)$$

where $\text{sgn}(\cdot)$ is the sign function. So, for B_z of a given sign, the sign of the slope will allow us to determine the sign of R_H , and hence whether R_H is given by $1/en$ or $-1/ep$. In the latter case, we know we have a p-type, and in the former we conclude it's an n-type. Figure 7a shows the resulting plot, where we color-code the points based on the sign of the surrounding field. As shown in the figure, points with a positive B_z exhibit a positive slope, so hence R_H is negative, and thus we have a p-type semiconductor. In addition, as explained in appendix B, another way to confirm this is to plot R_H as a function of temperature: if the sign of R_H flips, then this is an indication of a p-type semiconductor. This is precisely what we observe (figure 7b), so this further confirms the fact that we are indeed working with a p-type semiconductor. Note that while we do plot R_H against inverse temperature in figure 7b, the change in sign should still exist; plotting against inverse temperature does not change this fact.

4.3 Carrier Mobilities

4.3.1 Calculating the hole mobility

The next thing we calculate is the carrier mobilities of the semiconductor. We do this via equation 8, except in this equation we have both μ_n and μ_p to determine. In order to simplify this, we first consider the semiconductor in the extrinsic regime. In this regime, the intrinsic electrons from the germanium crystal don't have enough energy to be promoted to the conduction band; only the doped atoms can because of their lower ionization energy. Therefore, in the extrinsic regime, we have $n = 0$, and thus our equation simplifies to:

$$\frac{1}{\rho} = ep\mu_p = -\frac{\mu_p}{R_H} \quad (9)$$

which is a much simpler equation to calculate. Since we know R_H from earlier, we can use it and equation 4 to solve for p , and we can also use the Van der Pauw method to determine ρ , using equation 6 as our choice for f . With these two quantities determined, we now have μ_p in the extrinsic regime. According to [1], the carrier mobility has a nice scaling with temperature, it scales as $\mu_p \propto T^{-\alpha}$, so we can plot μ_p versus T and perform a least-squares fit to experimentally determine n . This least squares fit is shown in figure 8, and we get an n -value of $n = -1.883 \pm 0.0012$ for a current $I_x = 15 \mu\text{A}$, with a table showing the α fit values for all currents displayed in table 1. Taking the average of all α values, we get a mean value of $\alpha = -1.88 \pm 0.0009$. According to [1], first-order scattering theory predicts that $\alpha = -\frac{3}{2}$, but in most experimental circumstances this $T^{-3/2}$ relationship is never

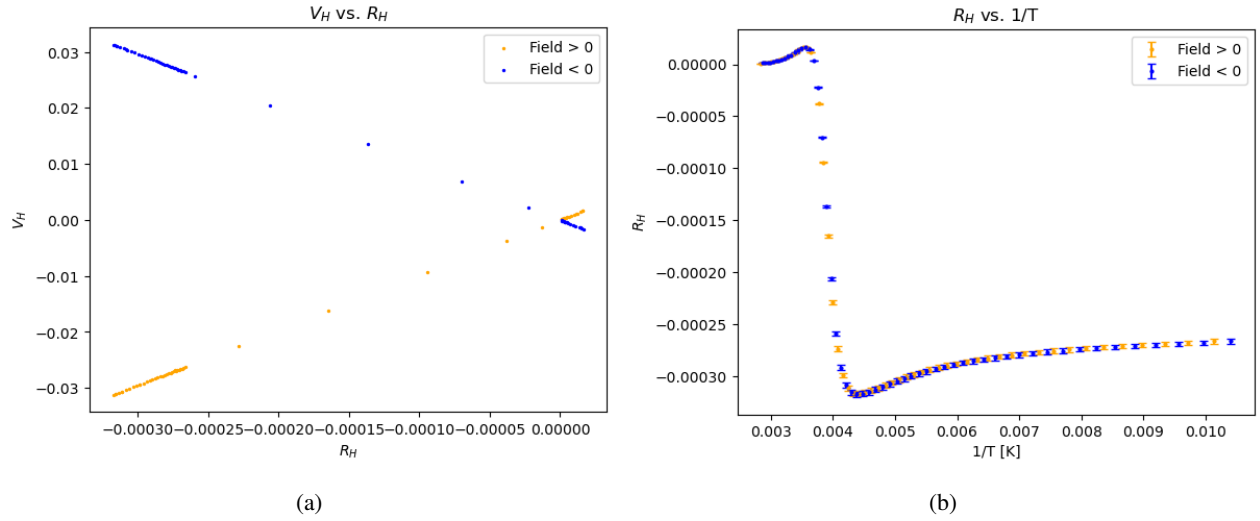


Figure 7: Two different ways determining the identity of the semiconductor. (a) A plot of the Hall voltage against the Hall coefficient, R_H . The positive slope of the line for positive B_z allows us to conclude that the semiconductor is a p-type. (b) A plot of the Hall coefficient R_H against inverse temperature, notice the flip in sign at around $1/T \approx 0.0035$, and the presence of a sign flip is indicative of a p-type semiconductor.

observed [2]. Thus, it's really hard to say whether our values are accurate, but at the very minimum, our values are consistent (see table 1), and this alone is strong evidence for the fact that our fitted dependence is likely accurate.

Looking at figure 8 more closely, we see that the decaying exponential relationship becomes inaccurate at around 240 K, and this signals the transition between the extrinsic and intrinsic regimes of the semiconductor, as n no longer becomes negligible and thus equation 9 no longer holds. However, the from this fit we determined μ_p for all values of T , which will be useful in the next section.

4.3.2 Calculating the electron mobility

Now with the hole mobility determined, it is now possible to determine the electron mobility. For this section, we will follow the analysis method provided in [2]; here we will only highlight the relevant equations we need for the analysis, a more detailed explanation can be found in the appendix.

The key to determining the electron mobility is to look closely at the temperature where $R_H = 0$, we will denote this as T_0 . This temperature is special because it provides some special relationships between quantities we want to calculate. In particular, [2] mentions the following approach: using our fitted value of μ_p , we can use equation 9 to *extrapolate* the conductivity to the point $T = T_0$ to get $\sigma_e(T = T_0)$, and from the Van der Pauw method we can experimentally determine the conductivity at T_0 using $\sigma_0 = 1/\rho(T = T_0)$. Then, comparing the two values, we then have the following relationship:

$$\frac{\sigma_0}{\sigma_e} = \frac{b}{b-1} \quad (10)$$

where $b = \mu_n/\mu_p$, the ratio of carrier mobilities. We can experimentally calculate this ratio σ_0/σ_e , which means that for this equation we can solve for b , and thus we can now solve for μ_n since we already have μ_p . The combined plot of μ_p and μ_n along with their exponential fits are shown in figure 9. As demonstrated in the plot, the exponential fit becomes significantly worse past $T \approx 240$ K, but this is to be expected, since our formula we used to compute μ experimentally only holds when there's one carrier present. So, it is natural that the data does not match the plot, but in this case the plot is to be trusted instead.

Current (μA)	α	T_0
5	-1.8790 ± 0.0009	269.813
15	-1.8918 ± 0.0017	270.259
30	-1.8832 ± 0.0011	270.525
45	-1.8744 ± 0.0006	270.408
60	-1.8792 ± 0.0009	269.925
75	-1.8802 ± 0.0010	269.785
90	-1.8805 ± 0.0009	270.642

Table 1: Fit values for α for all current values.

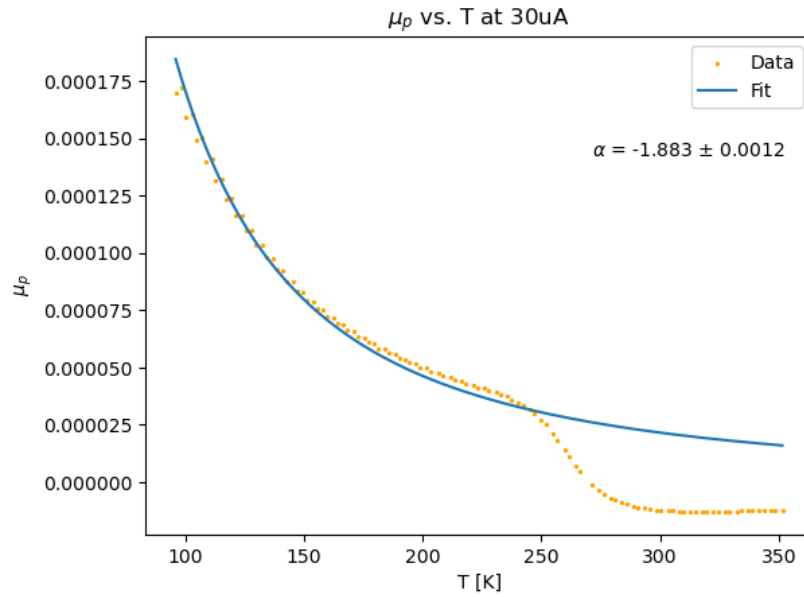


Figure 8: Plot of the hole mobility against temperature, with a fitted constant of $\alpha = -1.883 \pm 0.0012$. Notice that the exponential fit clearly fails to account for the transition between the extrinsic and intrinsic regimes of the semiconductor, as expected. However, in the extrinsic regime, the decaying exponential plot fits the data relatively well.

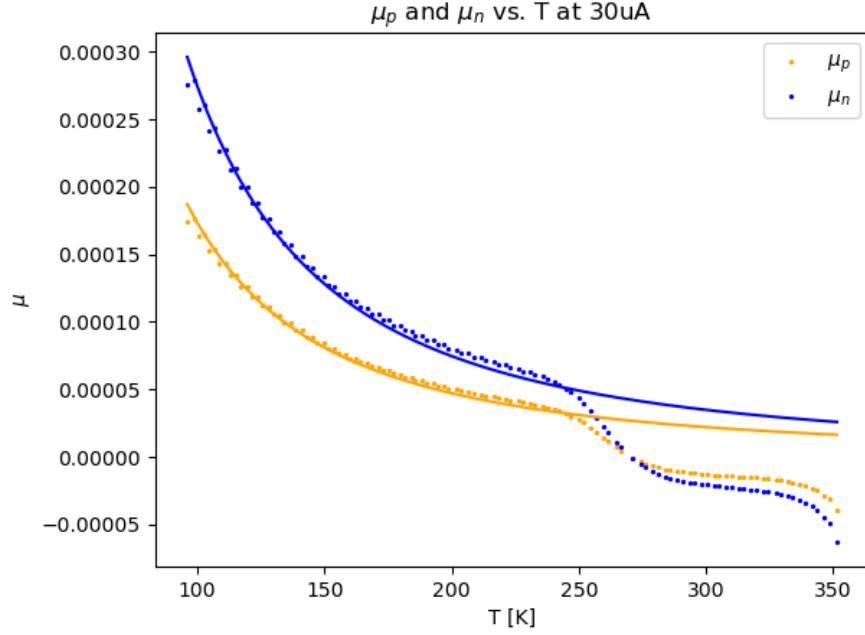


Figure 9: Plot of μ_p and μ_n as a function of temperature, along with their fitted decaying exponential curves. Again, notice that the fit does not work in the extrinsic regime, which makes sense since this exponential is only true in the case where there is one carrier present.

4.4 Calculating the Electron and Hole Densities

Now we move on to calculating the electron and hole densities. To do this, we will continue following the analysis given in [2]. First, we will make the following assumption about the behavior of our semiconductor: within the intrinsic and extrinsic regimes, the semiconductor behaves ideally, so in the extrinsic regime there are *zero* intrinsic carriers participating, and *all* dopant carriers participate. Given this, then we may write that p is equal to the total number of dopant atoms, or $p = N_a$. Likewise, the number of negative carriers n we will set to zero in this regime. In the intrinsic regime, the intrinsic carriers do participate, so let $n = N$. Because the electrons and holes now come in pairs, then there is a matching hole for every electron, so $p = N_a + N$. Using these two relations, [2] shows that we have:⁶

$$n = \frac{N_a}{b^2 - 1} \quad (11)$$

where b is the same ratio of mobilities as in 10. So, in summary, the following is what we need to do to determine the densities in the extrinsic regime: we first calculate p in the extrinsic regime to get N_a , and combine this with our value of b and equation 11 to determine n . With this relation, we now have the electron and hole densities in both the intrinsic and extrinsic regimes, and we've determined all the relevant quantities in the semiconductor. The n and p values in both regimes, for all current values are given in table 2.

4.5 Hall Mobilities and Zero Field

The last two quantities that we want to calculate is the Hall mobility and also the resistance at zero field. The Hall mobility is really just another measure of the carrier mobility, and is computed as $\mu_H = R_H \sigma$. We have both these values on the right from earlier already, so we can just calculate it and plot it against temperature. The plot is shown in figure 10.

⁶It should also be obvious that this is indeed a simplification on the true physics of the system – from this analysis, the result we obtain is that n and p are constants in both the extrinsic and intrinsic regime, but this is at least somewhat untrue since from statistical mechanics, there should still be some atoms that don't have the appropriate amount of energy to promote to the conduction band. We theorize that the reason we can still make this assumption is because we are using extremely small currents, on the order of microamps. Therefore, the number of carriers required to facilitate such a current immediately saturates after the phase transition.

Current (μA)	Extrinsic		Intrinsic	
	p	n	p	n
5	2.2082e+22	0	3.5670e+22	1.3587e+22
15	2.2084e+22	0	3.8055e+22	1.5970e+22
30	2.2149e+22	0	3.7278e+22	1.5129e+22
45	2.1752e+22	0	3.5300e+22	1.3547e+22
60	2.2030e+22	0	3.5687e+22	1.3657e+22
75	2.2006e+22	0	3.5536e+22	1.3530e+22
90	2.2183e+22	0	3.6632e+22	1.4449e+22

Table 2: Table of the positive and negative charge carrier densities in the extrinsic and intrinsic regimes. In the extrinsic regime, we have $n = 0$, and in the intrinsic regime we have $p = N_a + n$, where N_a is the value for p in the extrinsic regime, and n is the electron carrier density.

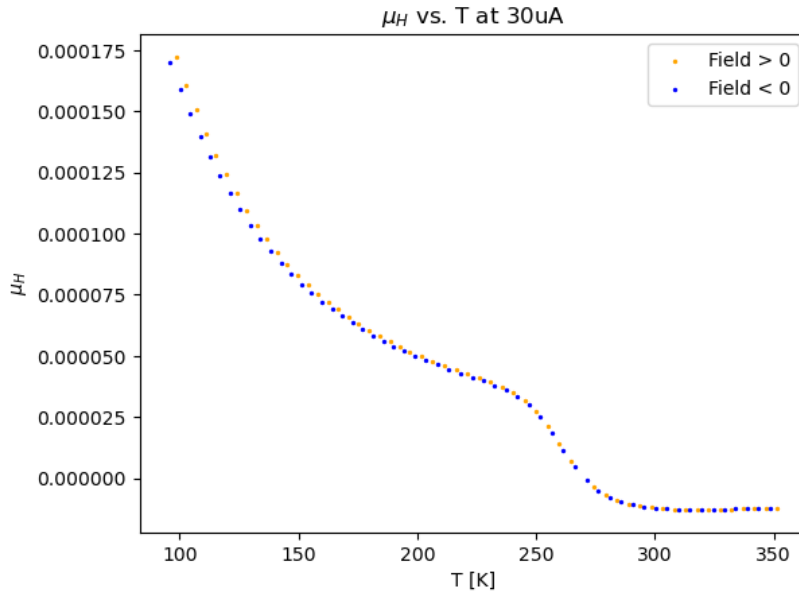


Figure 10: Plot of the Hall mobility $\mu_H = R_H \cdot \sigma$ as a function of T . Note the minus sign is because we have a p-type semiconductor. This plot makes sense intuitively; the Hall mobility is just a measure of the carrier mobility, so we should expect the shape of the curve to look identical to that of the carrier mobilities we calculated earlier.

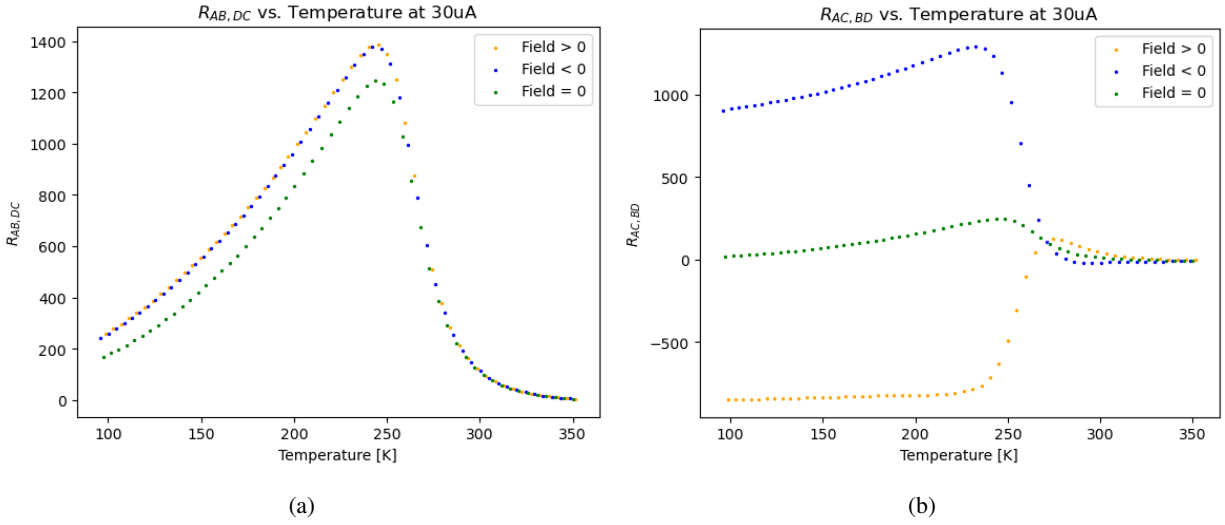


Figure 11: Plot of the (a) resistance and (b) trans-resistance of the semiconductor as a function of temperature. Notice that $R_{AB,DC}$ scales and equals approximately the same values as that in a nonzero field, but $R_{AC,BD}$ does not.

So far, all the plots we've been working with have involved using nonzero field measurements. The final quantity that we look at is the resistance and trans-resistance at zero field compared to nonzero field, and the plot is shown in figure 11. The resistance as measured by $R_{AB,DC}$ at zero field scales in the same fashion as the resistance with nonzero field – this is expected, since this quantity is used to measure the resistivity, an intrinsic property of the material that should not have a heavy dependence on the Hall effect. This is what we see in the plot, as we see a minor variation in the resistivity, but overall the scaling is unaffected. The trans-resistance, however, is affected by the presence of the \mathbf{B} field, as it is the quantity we use to measure the Hall coefficient. Put this way, without the Hall effect in action, we expect the Hall coefficient R_H to equal zero, and this is reflected in our plots as the trans-resistance is nearly zero by comparison. It's not exactly zero however, because even in the case of "zero field" we are applying a very small current to the Helmholtz coils, which explains why the trans-resistance is not exactly zero.

5 Reflection

Overall, the data we collected in this lab was a success. To be completely transparent, there wasn't really much that *could* go wrong in our experiment anyway, because all we had to do to get the experiment running was pour the LN2 into the cryostat chamber and the gold box did the rest for us. As a result, there aren't really many sources of error for this experiment either, or rather, there isn't one that we can necessarily "do better" on, because the machine already does all the data collection for us. All we had to do was run Python code to analyze the data.

That said, there were a couple things out of our control which happened that affected our data, although not in an extremely impactful way. Just to name a few, there were instances where despite making it clear that we wanted the vacuum pump to be left on overnight it was not done, so we had to spend lab time reinitializing the vacuum. There was also an instance where we ran out of LN2, and that also took a while to fix. Overall, I'd estimate that we lost about 1.5 hours of data collection time as a result of these delays. Luckily enough, this didn't affect our experiment quality very much, but it very well could have. For instance, in the 45uA trial, the first 20 data points were unreliable because we forgot to turn the power supply to the coils on; luckily this didn't really affect our data as the solution was just to remove these points in our analysis, but in the ideal case we would have re-taken this dataset if we had the time.

That said, this experiment was still a lot of fun to do, especially when all the data analysis worked out. To summarize, we successfully used the Hall effect in semiconductors to learn about the properties of the material indirectly, using the Van der Pauw technique and some clever math. First, we were able to determine the type of semiconductor from the Hall coefficient, then, through some crucial assumptions, we were able to determine quantities such as the electron and hole densities and mobilities for

both the extrinsic and intrinsic regime. Figuring out *what* to do for the analysis was definitely the hard part of this lab, but I did enjoy reading through different papers and gathering together what they were all saying so that I could formulate the logic myself – after all, this is what doing research kind of boils down to. What’s more interesting is the fact that even in such a simple system there are still things we don’t know: for instance, what is the true scaling for the mobility in terms of temperature? What is the true model that characterizes the transition region between extrinsic and intrinsic? The fact that such a simple system can exhibit such complex behavior is incredibly interesting to me, and it’s what made this lab so fun to investigate for me.

References

- [1] Donald A. Neamen. *Semiconductor Physics and Devices: Basic Principles*. 4. ed. New York, NY: McGraw-Hill, 2012. 758 pp. ISBN: 978-0-07-352958-5.
- [2] Adrian Melissinos. *Experiments in Modern Physics*.
- [3] Leo van der Pauw. “A Method of Measuring Specific Resistivity and Hall Effect of Discs of Arbitrary Shape”. In: *Philips Research Report* 13 (1958), pp. 1–9. URL: https://experimentationlab.berkeley.edu/sites/default/files/SHE/References/01-Measuring_Discs.pdf.
- [4] Physics 111B: Advanced Experimentation Laboratory. *Semiconductor Hall Effect*. URL: http://experimentationlab.berkeley.edu/sites/default/files/writeups/SHE_Manual_2024-05-03.pdf.

A Figures

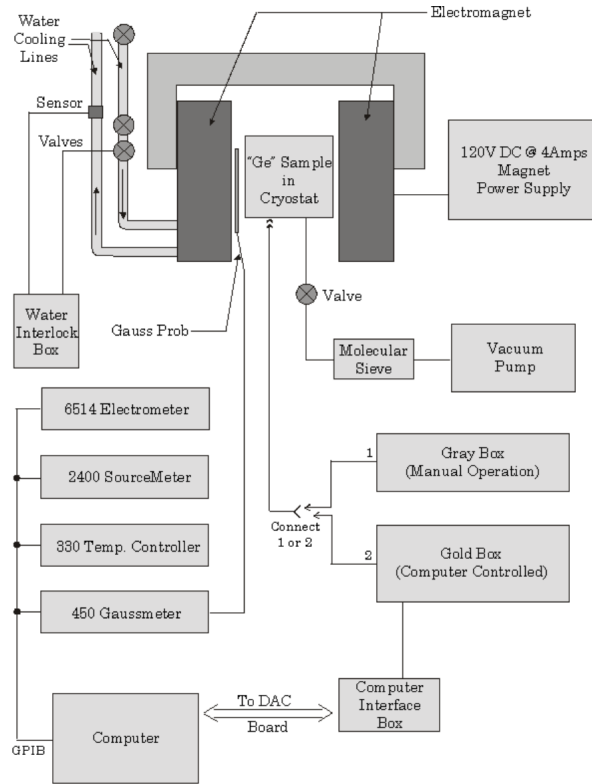


Figure 12: Macroscopic setup of the experiment apparatus. The germanium sample here is shown in the middle, with the vacuum and other relevant electronic equipment surrounding it. The measurements are made by all the meters in the bottom left, which are sent to the computer and compiled together in a .csv file. The reason I put this figure here is because it was too big, so it made more sense to include it here.

B Calculating Carrier Mobilities

As mentioned in the analysis section, a large part of the logic in the analysis section is driven by the work of [2], and in this section we will present their logic in full. To begin, [2] (and also the equation we derive in section 2.3.2) provides the following equation for the Hall coefficient R_H in the case where there are two carriers present:

$$R_H = \frac{\mu_p^2 p - \mu_e^2 n}{e(\mu_p p + \mu_e n)^2}$$

This is a more complex formula than the one we introduce in the theory section; this is because when there is more than one charge carrier present, the Hall effect acts differently on the positive and negative carriers, so as a result the equation for E_H and J_x , giving us a more complicated equation. It is important to note that aside from a minus sign, this equation does collapse into equation 4 when we set either n or p to zero.

Now, given this relation, for p type semiconductors, we have p to be nonzero, so it now makes sense that R_H should equal zero at some point (as in, it's a physical point). At the point where R_H equals zero, then we have the following relation:

$$\mu_p^2 p - \mu_e^2 n = 0 \implies n \left(\frac{\mu_e}{\mu_p} \right)^2 - p = 0$$

From here on, we now denote $b = \mu_e/\mu_p$, so this equation reads $nb^2 - p = 0$. Now, we make the simplification that in the extrinsic regime, all the doped carriers participate in conduction and none of the intrinsic electrons do, so we have $p = N_a$ and $n = 0$ in the extrinsic regime. In the intrinsic regime, because the electrons and holes come in pairs, then if $n = N$ we have $p = N_a + N$. Plugging the values in the extrinsic regime into the above equation, we get:

$$Nb^2 - (N_a + N) = 0 \implies N = n = \frac{N_a}{b^2 - 1} \quad (12)$$

This result will serve as the central equation we use to solve for n in the intrinsic regime. There are also other quantities that exist at $T = T_0$, the temperature where $R_H = 0$. In particular, from equation 8 we can now write:

$$\sigma_0 = \frac{1}{\rho} = e(n\mu_n + (N_a + n)\mu_p)$$

This quantity we are directly measuring, since we compute $\sigma = \frac{1}{\rho}$ for all T . We also have another equation for the conductivity, which comes from the extrinsic regime. To get the value at $T = T_0$, we extrapolate our fit in the extrinsic regime to $T = T_0$, which is:

$$\sigma_e = \sigma(T = T_0) = eN_a\mu_p(T = T_0)$$

We get the μ_p relationship as a function of temperature by fitting it to T^{-n} , which allows us to extrapolate σ to the point $T = T_0$. Now, we can take the ratios of these two at $T = T_0$, which gives:

$$\frac{\sigma_0}{\sigma_e} = \frac{n\mu_n + (N_a + n)\mu_p}{\mu_p N_a} = \frac{N_a + n(1 + b)}{N_a} = \frac{b}{b - 1}$$

where the last equality comes from substituting equation 12 to get the ratio in terms of b only. From here, we may solve for b :

$$b = \frac{\sigma_0}{\sigma_0 - \sigma_e} = \frac{1}{1 - \frac{\sigma_e}{\sigma_0}}$$

Then, with b determined, we can now go back to equation 12 and solve for n , which completes the analysis. Here, hopefully it is clear to see why we need the assumption that $p = N_a$ in the extrinsic regime. Without this assumption, we would not be able to write a nice expression for n and we also wouldn't be able to write a nice expression for σ_0/σ_e either.

C Data

Because the raw dataset is rather large for this lab, so instead of pasting these all into this document, I will instead attach a link to the dataset instead. You can find the full dataset along with the relevant plots using: https://drive.google.com/drive/folders/1DcLZfVUPI55-ZmGKXyVG6E5kIr6W5yFh?usp=drive_link