1. (1) d (2) a (3) b (4) b
(5) d (6) a

2. (a) $PV = nRT$ n, T are constant.

$\therefore P_0 V_0 = PV$

$$P = \frac{P_0 V_0}{V} = \frac{P_0 \cdot A \cdot L}{A\,(L+\Delta x)} = P_0 \frac{L}{L+\Delta x}$$

$$P_0 = \frac{k x_0}{A} \qquad \therefore P = \frac{k x_0}{A} \cdot \frac{L}{L+\Delta x}$$

(b) $F = -k(x_0 + \Delta x) + P \cdot A$

$$= -k(x_0 + \Delta x) + k x_0 \cdot \frac{L}{L+\Delta x}$$

for small Δx : $\frac{L}{L+\Delta x} = \frac{1}{1+(\Delta x/L)} \approx 1 - \frac{\Delta x}{L}$

$$\therefore F = -k(x_0 + \Delta x) + k x_0 \left(1 - \frac{\Delta x}{L}\right)$$

$$= -\Delta x \cdot k\left(1 + \frac{x_0}{L}\right)$$

(c) $\omega = \sqrt{\frac{k_{total}}{M}}$ $k_{total} = k\left(1 + \frac{x_0}{L}\right)$

$$\therefore \omega = \sqrt{\frac{k}{M} \cdot \left(1 + \frac{x_0}{L}\right)}$$

3. (a) $S = k_B \ln g$

$$= k_B \cdot \left[\ln C + N\ln(V-bN) + \frac{3N}{2}\ln\left(E+\frac{N^2a}{V}\right)\right]$$

(b) $$\frac{1}{k_BT} = \frac{d\ln g}{dE}$$

$$= \frac{d}{dE}\left[\ln C + N\ln(V-bN) + \frac{3N}{2}\ln\left(E+\frac{N^2a}{V}\right)\right]$$

$$= \frac{3N}{2}\ \frac{1}{E+\frac{N^2a}{V}}$$

$$\therefore\ T = \frac{1}{k_B}\cdot\frac{2}{3N}\cdot\left(E+\frac{N^2a}{V}\right)$$

(c) $$E+\frac{N^2a}{V} = \frac{3N}{2}k_BT$$

$$\therefore E = \frac{3N}{2}k_BT - \frac{N^2a}{V}$$

Energy per particle $\varepsilon = \frac{E}{N} = \frac{3}{2}k_BT - \frac{N}{V}\cdot a$

$$= \frac{3}{2}k_BT - \rho\cdot a$$

(d) $$TdS = dE + pdV$$

$$\therefore\ dS = \frac{1}{T}dE + \frac{p}{T}dV$$

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_E$$

$$= \frac{\partial}{\partial V}\left\{k_B\left[\ln C + N\ln(V-bN) + \frac{3N}{2}\ln\left(E+\frac{N^2a}{V}\right)\right]\right\}$$

$$= \frac{\partial}{\partial V}\left\{k_B\left[N\ln(V-bN) + \frac{3N}{2}\ln\left(E+\frac{N^2a}{V}\right)\right]\right\}$$

$$= k_BN\cdot\frac{1}{V-bN} + k_B\frac{3N}{2}\cdot\frac{1}{E+\frac{N^2a}{V}}\cdot\left(-\frac{N^2a}{V^2}\right)$$

$$= k_B N \left[\frac{1}{V-bN} - \frac{3}{2} \cdot \frac{1}{V} \cdot \frac{N^2 a/V}{E + \frac{N^2 a}{V}} \right]$$

$$\therefore P = Nk_B T \cdot \left[\frac{1}{V-bN} - \frac{3}{2} \frac{1}{V} \frac{N^2 a/V}{E + \frac{N^2 a}{V}} \right]$$

$$= \frac{N}{V} k_B T \left[\frac{V}{V-bN} - \frac{3}{2} \cdot \frac{\frac{N}{V} \cdot Na}{E + \frac{N}{V} \cdot Na} \right]$$

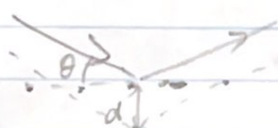
$$= \rho k_B T \cdot \left[\frac{1}{1-b\rho} - \frac{3}{2} \frac{\rho \cdot a}{\left(\frac{E}{N}\right) + \rho \cdot a} \right]$$

(e) $\frac{N^2 a}{V} \sim E$ in unit

$\therefore a \sim \frac{EV}{N^2} = (J \cdot m^3)$

$bN \sim V$ in unit

$\therefore b \sim \frac{V}{N} = m^3$ in unit

4. 

(a) Bragg condition:

$$2kd \cos\theta = n \cdot 2\pi \quad ; \; k = \frac{2\pi}{\lambda}$$

$$\therefore \cos\theta = \frac{n\lambda}{2d}$$

(b) $E = \frac{p^2}{2m} \quad \therefore \quad p = \sqrt{2mE}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2 \cdot mc^2 \cdot E}}$$

$$= \frac{1973 \text{ eV} \cdot \text{Å} \cdot 2\pi}{\sqrt{2 * 0.02 \text{ eV} * 939 \text{ MeV}}} \approx 2 \text{ Å}$$

This is comparable to typical lattice spacing.

5. (a) $\psi = C\left(\sin\frac{\pi x}{2L}\cos\frac{3\pi x}{2L} - 2\sin\frac{\pi x}{L}\right)$

$= C\left\{\left[\frac{\sin\left(\frac{\pi x}{2L}+\frac{3\pi x}{2L}\right)+\sin\left(\frac{\pi x}{2L}-\frac{3\pi x}{2L}\right)}{2}\right] - 2\sin\frac{\pi x}{L}\right\}$

$= C\left\{\frac{\sin\frac{2\pi x}{L} - \sin\frac{\pi x}{L}}{2} - 2\sin\frac{\pi x}{L}\right\}$

$= C\left\{\frac{1}{2}\cdot\sin\frac{2\pi x}{L} - \frac{5}{2}\sin\frac{\pi x}{L}\right\}$

Normalization $\int_0^L |\psi|^2 dx = 1$

$\therefore C^2\cdot\left[\frac{1}{4}\int\sin^2\frac{2\pi x}{L}dx + \frac{25}{4}\int\sin^2\frac{\pi x}{L}dx\right] = 1$

$\therefore C^2\cdot\left[\frac{1}{4}\cdot\frac{L}{2} + \frac{25}{4}\cdot\frac{L}{2}\right] = 1$

$\therefore C = \sqrt{\frac{8}{26L}} = \sqrt{\frac{4}{13L}}$

(b) $E_1 = \frac{\hbar^2}{2m}\cdot\left(\frac{\pi}{L}\right)^2$ $\quad E_2 = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2$

$\psi(x,t) = \frac{1}{\sqrt{13L}}\cdot\sin\frac{2\pi x}{L}\, e^{-i\frac{E_2}{\hbar}t} + \frac{5}{\sqrt{13L}}\sin\frac{\pi x}{L}e^{-i\frac{E_1}{\hbar}t}$

(c) $\langle x\rangle = \int_0^L \psi(x,t)^*\cdot x\,\psi(x,t)\,dx$

$= \int_0^L \frac{1}{13L}\cdot\sin^2\frac{2\pi x}{L}\cdot x\,dx + \int_0^L \frac{25}{13L}\sin^2\frac{\pi x}{L}\,x\,dx$

$+ \int_0^L \frac{5}{13L}\sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}\,x\left(2\cos\frac{E_2-E_1}{\hbar}t\right)dx$

$= \frac{L}{2} + \frac{10}{13L}\left(\cos\frac{E_2-E_1}{\hbar}t\right)\cdot\int_0^L \sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}\,x\,dx$

⑤

$$\int_0^L \sin\frac{2\pi x}{L} \sin\frac{\pi x}{L}\, x\, dx$$

$$= \int_0^L \frac{\cos\frac{3\pi x}{L} - \cos\frac{\pi x}{L}}{2}\, x\, dx$$

$$\int_0^L \cos\alpha x * x\, dx = \frac{1}{\alpha}\int_0^L x\, d\sin\alpha x$$

$$= \frac{1}{\alpha}\left[x\sin x\Big|_0^L - \int_0^L \sin\alpha x\, dx\right]$$

$$= \frac{1}{\alpha}\left[x\sin x\Big|_0^L + \frac{1}{\alpha}\cos\alpha x\Big|_0^L\right]$$

$$\therefore \int_0^L \cos\frac{3\pi x}{L} x\, dx = \frac{1}{(\frac{3\pi}{L})^2}\cdot[-1-1] = -\frac{2L^2}{9\pi^2}$$

$$\int_0^L \cos\frac{\pi x}{L} dx = \frac{1}{(\frac{\pi}{L})^2}[-1-1] = -\frac{2L^2}{\pi^2}$$

$$\therefore \int_0^L \sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}\, x\, dx = \frac{L^2}{\pi^2} - \frac{L^2}{9\pi^2} = \frac{8}{9}\frac{L^2}{\pi^2}$$

$$\therefore \langle x\rangle = \frac{L}{2} + \frac{80}{117}\frac{L}{\pi^2}\cos\frac{E_2-E_1}{\hbar}t$$

(d) $$\langle p\rangle = \int_0^L \psi^* \left(i\hbar\frac{\partial}{\partial x}\right)\psi\, dx$$

$$= \int_0^L \frac{1}{13L}\left[\sin\frac{2\pi x}{L} e^{i\frac{E_2}{\hbar}t} + 5\sin\frac{\pi x}{L} e^{i\frac{E_1}{\hbar}t}\right]$$

$$\cdot i\hbar\frac{\partial}{\partial x}\left[\sin\frac{2\pi x}{L} e^{-i\frac{E_2}{\hbar}t} + 5\sin\frac{\pi x}{L} e^{-i\frac{E_1}{\hbar}t}\right] dx$$

only cross term will be non-zero

$$\therefore \langle p\rangle = \int_0^L \frac{1}{13L}\left[i\hbar\frac{2\pi}{L}\cos\frac{2\pi x}{L}\sin\frac{\pi x}{L} e^{-i\frac{E_2-E_1}{\hbar}t}\right.$$

$$\left. + i\hbar\frac{\pi}{L}\cos\frac{\pi x}{L}\sin\frac{2\pi x}{L} e^{i\frac{E_2-E_1}{\hbar}t}\right] dx$$

$$= \frac{i\hbar}{13L}\int_0^L\left[\frac{\sin\frac{3\pi x}{L} - \sin\frac{\pi x}{L}}{2}\frac{2\pi}{L} e^{-i\frac{E_2-E_1}{\hbar}t}\right.$$

$$\left. + \frac{\sin\frac{3\pi x}{L} + \sin\frac{\pi x}{L}}{2}\cdot\frac{\pi}{L} e^{i\frac{E_2-E_1}{\hbar}t}\right] dx$$

$$= \frac{i\hbar}{13L}\cdot\left[\frac{2\pi}{L}\cdot\left(\frac{L}{3\pi} - \frac{L}{\pi}\right)e^{-i\frac{E_2-E_1}{\hbar}t} + \frac{\pi}{L}\cdot\left(\frac{L}{3\pi} + \frac{L}{\pi}\right)e^{i\frac{E_2-E_1}{\hbar}t}\right]$$

$$= \frac{i\hbar}{13L}\cdot\frac{4}{3}\cdot 2i\sin\frac{E_2-E_1}{\hbar}t = \frac{-8}{39}\frac{\hbar}{L}\sin\frac{E_2-E_1}{\hbar}t$$

(e) $\langle F \rangle = \frac{d\langle p \rangle}{dt}$

$$= -\frac{8}{39}\frac{\hbar}{L}\frac{E_2-E_1}{\hbar}\cos\frac{E_2-E_1}{\hbar}t$$

$$= -\frac{8}{39}\left(\frac{E_2-E_1}{L}\right)\cos\frac{E_2-E_1}{\hbar}t$$

6. Resonance transmition takes place when

$$2kL = n \cdot 2\pi \qquad \text{where} \quad k = \frac{\sqrt{2m(E-V)}}{\hbar}$$

$$\therefore \cancel{2} * \frac{\sqrt{2m(E-V)}}{\hbar} L = n \cdot \cancel{2}\pi$$

$$\sqrt{2m(E-V)} = \frac{n\pi\hbar}{L}$$

$$\therefore E = \frac{\left(\frac{n\pi\hbar}{L}\right)^2}{2m} + V$$

$$= \frac{n^2\pi^2\hbar^2}{2mL^2} + V$$