Physics 5CL Prelab 0

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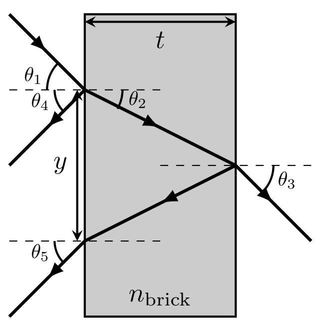
[NOTE:] Although we have separate prelab submissions, I worked extensively with **Andrew Binder** to complete this prelab, so that is the reason why our work shares relatively similar styles.

Problem 1: Brick Geometry

Consider the geometry of a ray of light incident on a rectangular brick of thickness t as shown. You may assume that the index of refraction of the surrounding air is 1.

Refer to the following diagram for solutions, thanks to Andrew Binder for the TikZ:



a) *Determine z in terms of the lengths t and y (you just need geometry for this part - no Snell's law is allowed)*

Purely from geometry, we get

$$z = \sin\theta_2 = \frac{\frac{y}{2}}{\sqrt{\left(\frac{y}{2}\right)^2 + t^2}}$$

b) *Use propagation of errors to determine α_x as a function of θ_1 and α_{θ_1}. Then determine α_z as a function of t, y, α_t and α_y.*

We have:

$$\alpha_Z = \sqrt{\left(\frac{\partial z}{\partial y}\alpha_y\right)^2 + \left(\frac{\partial z}{\partial t}\alpha_t\right)^2}$$

Thus, computing the partial derivative separately:

$$\frac{\partial z}{\partial y} = \frac{1}{2}\left(\frac{\sqrt{\frac{y^2}{4}+t^2} + \frac{y}{2\left(\frac{y^2}{4}+t^2\right)^{1/2}}}{\left(\frac{y}{2}\right)^2 + t^2}\right)$$

$$\frac{\partial z}{\partial t} = \frac{y}{2} \cdot \frac{1}{\left[\left(\frac{y}{2}\right)^2 + t^2\right]^{3/2}} \cdot -\frac{1}{2}(2t)$$

$$\therefore \alpha_z = \left[\left(\frac{\alpha_y}{2}\left(\frac{\sqrt{\frac{y^2}{4}+t^2} + \frac{y}{2\left(\frac{y^2}{4}+t^2\right)^{1/2}}}{\frac{y^2}{4}+t^2}\right)\right)^2 + \left(\frac{yt}{2\left[\frac{y^2}{4}+t^2\right]^{3/2}}\alpha_t\right)^2\right]^{1/2}$$

c) *Show that the geometry of reflection and refraction implies that $\theta_1 = \theta_3 = \theta_4 = \theta_5$*

By the law of reflection, the angle of incidence equals the angle of reflection, and thus $\theta_1 = \theta_4$. From Snell's law, we have $\sin\theta 1 = n\sin\theta_2$ and $n\sin\theta_2 = \sin\theta_3$, so $\sin\theta_1 = \sin\theta_3 \implies \theta_1 = \theta_3$. This is also true because all $\theta < 90$ due to the way we interpret the angles, so there is only one unique soluiton to $\sin\theta_1 = \sin\theta_3$.

Further, by parallel lines we get $n\sin\theta_2 = \sin\theta_5 = \sin\theta_1$, so this leads to $\theta_1 = \theta_5$, and thus $\theta_1 = \theta_3 = \theta_4 = \theta_5$.

d) *Use Snell's law and your result from (a) to determine the index of refraction of the brick n_{brick} first in terms of x and z and then in terms of t, y and θ_1.*

From Snell's law earlier, we had $n\sin\theta_2 = \sin\theta_1$, so we have:

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{x}{z}$$

To express this in terms of t, y and θ_1:

$$n = \frac{\sin\theta_1}{\frac{y/2}{\sqrt{\frac{y^2}{4}+t^2}}} = \frac{2\sin\theta_1\sqrt{\frac{y^2}{4}+t^2}}{y}$$

e) *Perform a propagation of errors to determine the uncertainty in n_{brick} based on the uncertainties of α_x and α_z.*

Since $n = \frac{x}{z}$, the error propagation is as follows:

$$\alpha_n = \sqrt{\left(\frac{-x}{z^2}\alpha_z\right)^2 + \left(\frac{1}{z}\alpha_x\right)^2}$$

$$= \frac{1}{z}\sqrt{\frac{x^2}{z^2}\alpha_z^2 + \alpha_x^2}$$

Problem 2: Least-Squares with Hypothesis $y = -x + b$

*Consider the hypothesis $y = -x + b$. The function χ^2 for a **weighted** least squares analysis is*

$$\chi^2 = \sum w_i(y_i + x_i - b)^2 = \sum \left(\frac{y_i + x_i - b^2}{\alpha_{y,equiv,i}}\right)^2$$

where $\alpha_{y,equiv,i}$ is given by $\sqrt{\alpha_{yi}^2 + \alpha_{xi}^2}$

1. *Find the best fit value b which minimizes χ^2. You may use the weights w_i in your answer as we did in Eqs. 5.3.1 and 5.3.3-5.3.5 in the Statistics Review Sheet*

   From equation 5.4.5b on the statistics review sheet, we have

   $$b = \frac{\sum w_i y_i - m \sum w_i x_i}{\sum w_i}$$

   And since our model is $y = -x + b$, we have $m = -1$ so:

   $$b = \frac{\sum w_i y_i + \sum w_i x_i}{\sum w_i}$$

2. *Use propagation of errors on your answer from (a) to determine the error in your best fit value α_b. Hint: Use the trick of shifting all the errors from x over to y, so $\delta x_i = 0$ and $\delta y_i = \delta y_{equiv,i}$(You may use the weights w_i in your answer.)*

   Following the hint, we assume that $\delta x_i = 0$, and so all errors are in δy_i. For this specific formula, the error propagation is:

   $$\alpha_b = \sqrt{\sum \left(\frac{\partial b}{\partial y_i}\alpha y_i\right)^2} = \sqrt{\sum \left(\frac{w_i}{\sum w_i}\alpha_{yi}\right)^2} = \sqrt{\frac{(\sum w_i \alpha_{yi})^2}{(\sum w_i)^2}}$$

Part 3: Laser Safety

Here's the screenshot for laser safety: