$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

$\nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$

$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ F_x & F_y & F_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$

$\nabla \cdot (\nabla f) = \nabla^2 f = \frac{\partial^2 f}{\partial x} + \frac{\partial^2 f}{\partial y} + \frac{\partial^2 z}{\partial z}$

Midterm 2 Cheat Sheet

$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$

B-S Law:

$B_{wire} = \frac{\mu_0 I}{2\pi r}$, $\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$ $\quad d\vec{B} = \frac{\mu_0 I\, d\vec{\ell}}{4\pi r^2}$ ← only useful to calculate net $\vec{B}$ at one point.

$\int B \cdot d\ell = \mu_0 I_{enc}$, $I_{enc} = \frac{dq_{enc}}{dt}$

↳ choose smart Amperian loops!

$\mathcal{E} = -\frac{d}{dt}\Phi_B$ — Faraday + Lenz's law → Flux does not change due to current loop $\Phi = B(t) \cdot \pi a^2$ for circular solenoid

↳ Always Kirchhoff loop + junction rule to solve sys. of equations.

Inductors: $\mathcal{E} = -L\dot{I}$ (opposes current) Capacitors: $\mathcal{E} = \frac{Q}{V}$ Resistors: $V = IR$

Switch behavior: $t=0 \Rightarrow$ break in circuit; $t=\infty \Rightarrow$ normal wire | $t=0 \Rightarrow$ normal wire; $t=\infty \Rightarrow$ break in circuit. | same at all t

Impedances: $Z = i\omega L$ $\quad Z = \frac{i}{\omega C}$ $\quad Z = R$

series: $Z_{eq} = Z_1 + Z_2$ parallel: $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

w/ mutual inductance: $L_{eq} = L_1 + L_2 + 2M$ (series) $\quad L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$ (parallel)

$U_B = \int dV \frac{|B(t)|^2}{2\mu_0}$ — magnetic energy density $\quad$ Sim.: $U_E = \int dV\, \varepsilon_0 \frac{|E(t)|^2}{2}$

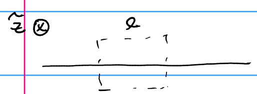
complex current: $I(t) = I_0 \cos(\omega t + \phi) \rightarrow I_0 = |\tilde{I}(t)| = \sqrt{Re(I)^2 + Im(I)^2}$, find current via $\tilde{V} = \tilde{I}Z$

Kirchhoff for AC: $\mathcal{E} - IR - \sum_n I Z_i = 0$ (w/o inductor) $\quad \mathcal{E} - IR - \sum_n I Z_i = \mathcal{E}_{faraday}$ (w/ inductor)

Transformers:  $\frac{V_2}{V_1} = \frac{N_2}{N_1} \rightarrow \frac{I_2}{I_1} = \frac{N_2}{N_1}$ In power line: $\frac{\partial P}{\partial V} < 0$ to minimize losses.

other specifics: $\mathcal{E} = \oint \vec{E} \cdot d\ell = -\frac{d\Phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{a} \Rightarrow \overset{\text{curl}}{\nabla \times \vec{E}} = -\frac{\partial B}{\partial t}$ → Induced $\vec{E}$ and $\vec{B}$ fields.

$\nabla \times \vec{B} = \mu_0 J$, $\nabla \cdot \vec{B} = 0$ (no free endings)

Infinite sheet:  $\vec{K} = K\hat{z} \Rightarrow \oint_C B\, d\vec{\ell} = B \cdot 2\alpha = \mu_0 K \alpha$

$\boxed{B = \frac{\mu_0 K}{2}}$

Self-inductance: $\Phi = LI$ $\quad \frac{d\Phi}{dt} = L\frac{dI}{dt}$ $\quad -\frac{d\Phi}{dt} = \mathcal{E} \Rightarrow \mathcal{E} = -L\dot{I}$

Relativity: $E_{\parallel} = E_{\parallel}$, $E_{\perp} = \gamma E_{\perp}$, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, Note final result should be a rotation.