

## Collaborators

I worked with **Andrew Binder** to complete this assignment. The plots were mostly created by him (since he knows TikZ better than I do), but I'm slowly learning so the plot for the electric field in part c) and the last plot in part d) were created by me.

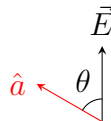
## Problem 1

Consider a linearly polarized beam of light propagating out of the plane of the page with a vertical polarization. Figure 1. shows how the electric field vector of the light wave changed over one full period at a particular point in space.

Consider the electric field vector  $\vec{E}$  of magnitude  $|\vec{E}| = E_0$  from Figure 1 at time  $t = 0$ . Let axis  $\hat{a}$  be oriented at an angle  $\theta$  counterclockwise of the  $+\hat{y}$ -axis. When the light passes through a **linear polarizing filter** with a transmission axis along the  $\hat{a}$ -direction, only the projection of the electric field onto the  $\hat{a}$ -axis gets transmitted.

- a) What is the magnitude of the electric field after passing through the polarizer? What is the intensity?

*Solution:* Refer to the following diagram:



As we can see, the intensity would be  $|E| = E_0 \cos \theta$ .

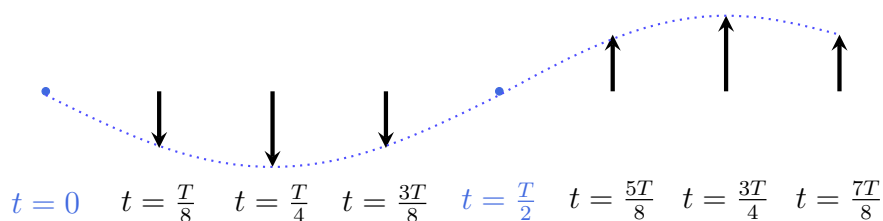
## Problem 2

Consider vertically polarized light incident on a **quarter-wave plate**. After passing through the quarter-wave plate, the phase of the component of the electric field parallel to the fast axis has advanced one quarter of a cycle more compared to the component of the electric field parallel to the slow axis.

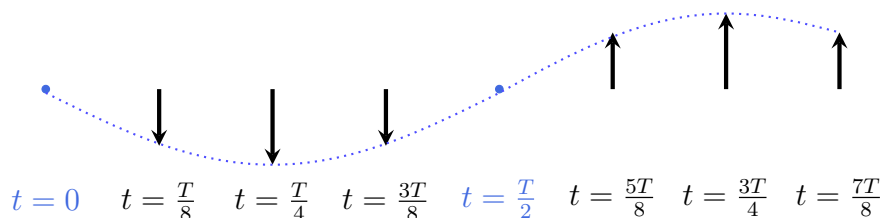
Consider a copy of the sketches from Figure 1.

- a) Suppose the light passes through a quarter-wave plate with a fast axis along the vertical. Draw sketches as in Figure 1 showing the  $\hat{x}$ - and  $\hat{y}$  components of the electric field vector *after* passing through the quarter-wave plate. Next to your sketches draw the shape traced out by the tip of the electric field vector, including an arrow indicating direction. [Note: The  $x$  component will be particularly boring in this case!]

*Solution:* Since the fast axis is along the vertical, we just take every single phase and shift it forward by a phase of  $T/4$ :



We can connect the tips of these vectors to produce a sine wave propagating in the  $\hat{y}$  direction:

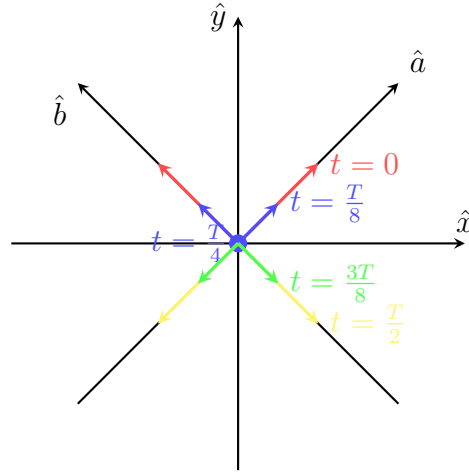


You should have found that your light was still vertically linearly polarized (even though it's starting at a different point in the cycle). Similarly, if the fast axis were oriented along the  $\hat{x}$ -axis, we would also find that the light remains a vertical linear polarization. But what happens if we use a different axis?

Consider axes  $\hat{a}$  and  $\hat{b}$ , oriented at angles of  $\pm 45^\circ$  relative to the  $+y$ -axis as shown to the right.

- b) Draw a copy of Figure 1 and add in the projections of the electric field vector onto the two axes. This is our “starting point” for light before it passes through the quarter-wave plate.

*Solution:* Our vector projection would look like this:



The idea is that as the wave is linearly polarized along the  $\hat{y}$  direction, its magnitude reduces and becomes negative, but it is even split among the  $\hat{a}$  and  $\hat{b}$  axes.

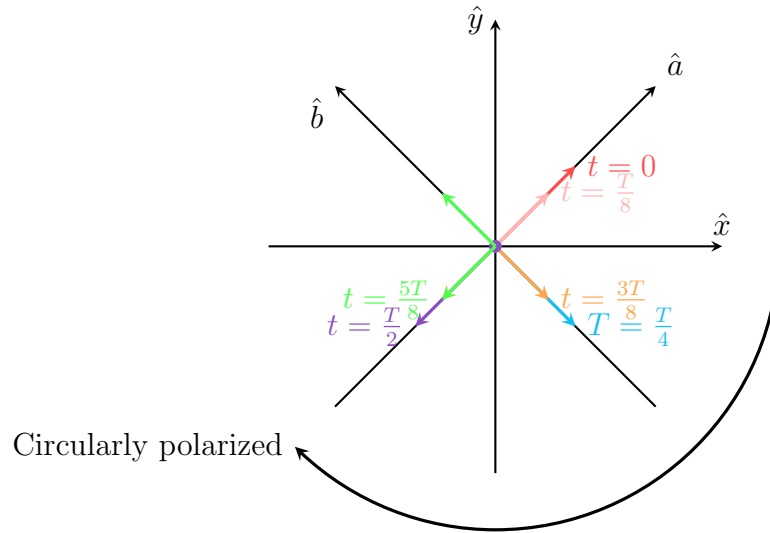
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- c) Suppose the fast axis of the quarter wave plate was oriented along the  $\hat{b}$  direction. using your sketches from part (b), draw sketches showing the  $\hat{a}$ - and  $\hat{b}$ -components of the electric field vector **after** passing thorough the quarter wave plate. In a different color, add the total electric field vector to your sketches. Is this left- or right-circularly polarized light?

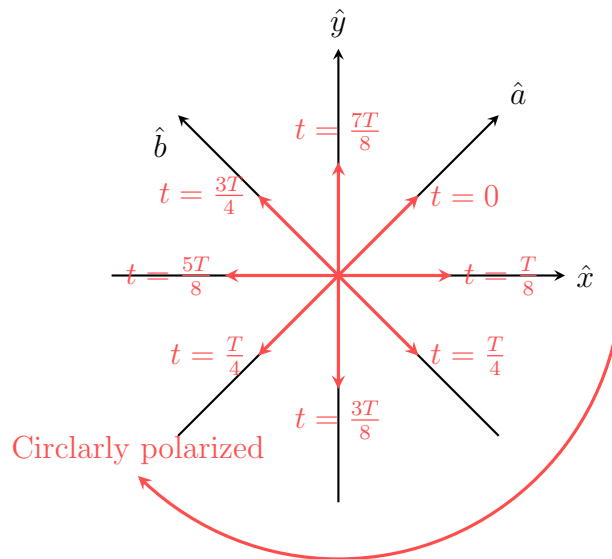
*Hint: It may help to draw the shape traced out by the tip of the electric field vector, including an arrow indicating direction.*

*Note: Aligning the optical axis with the  $\hat{a}$ -axis will produce the opposite circular polarization*

*Solution:* Refer to the following diagram:



Apologies for the bad diagram, the point is to show that at every point in time the vectors rotate along the  $\hat{a}$  and  $\hat{b}$  axes. If we sum up all these vectors, we get:

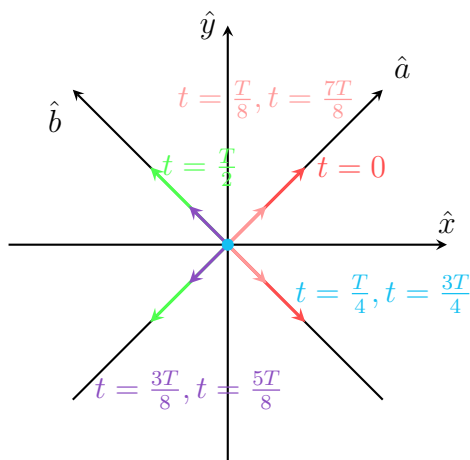


As we can see, the electric field vector appears to be moving clockwise, so this is **right circularly polarised light**.

Finally, suppose the light passes through a *half-wave plate* with the fast axis along the  $\hat{b}$  direction.

- d) Using your sketches from part (b), draw sketches showing the  $\hat{a}$ - and  $\hat{b}$ - components of the electric field vector *after* passing through the half-wave plate. In a different color, add the total electric field vector to your sketches. You should wind up with linearly polarized light. What is the polarization axis?

*Solution:* When the light passes through the half-wave plate, we see that the light is indeed linearly polarized, however we get that the light is polarized linearly along both the  $\hat{a}$  and  $\hat{b}$  axes.



Apologies for the diagram. The light is linearly polarized along the positive  $\hat{a}$  axis and the negative  $\hat{b}$  axis, then half a period later it flips to being a positive  $\hat{b}$  axis and a negative  $\hat{a}$  axis.

The net electric field sums to be linearly polarized light along the  $\hat{x}$  axis:

