EXPLORING THE WACKY WORLD OF WATER OPTICS

Physics 5CL Capstone Project Report

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Introduction

Before anything, let's talk about what we want to accomplish with these experiments.

Objectives

The goal of these three experiments was to exhibit some interesting optics behavior initially discussed in **Physics 5B (Introductory Electromagnetism, Waves, & Optics)**.

For our first experiment, the primary objective was to experimentally exhibit and confirm the behavior of air-filled lenses underwater; more specifically, we expect diverging air-filled lenses to behave as converging lenses, and vice versa.

For our second and third experiments, we wanted to verify a peculiar feature of prisms and refraction. The fact that prisms are able to produce a wavelength indicates that the index of refraction of any material is dependent on its wavelength. For that reason, it should be possible to determine the differences in index of refraction for red and violet light in the visible spectrum, and confirm them to previously experimentally determined results. These two experiments carried different approaches: in the second experiment we found Brewster's angle, whereas in the third experiment we used Snell's Law.

Contributions

The following are the contributions from each of the group members for the overall capstone project (report, presentation slides, and actual experiment).

Eric Du

Eric was responsible for the Data Collection/Analysis for Experiment 1, as well as a good chunk of the overall data analysis for these experiments (done in Python and various other tools). He also helped with data collection and setting up the experiments. He also helped with formatting the Presentation slides.

Andrew Binder

Andrew was responsible for the diagrams across the report, as well as Experimental Design/Procedure/Setup/Results & Conclusions for Experiment 1; and the Conclusion. He also helped with data collection throughout the experiment, and was responsible for the additional (scrapped) experimental designs. Finally, he was responsible for the majority of the write-up for the Presentation.

Aren Martinian

Aren was responsible for the Introduction; Objectives; Theory and Background; Experimental Setup/Procedure/Data/Analysis/Results for Experiments 2 and 3; and overall Results and Conclusions for the three experiments, as well as helping Eric with some Data Analysis in Experiment 1. He was also responsible for coming up with the final design of the experiment as well as a lot of the data collection.

Theory and Background

This section will contain all of the theory required to conduct all three experiments, but they will be sectioned off accordingly.

Experiment 1 Theory

To quantitatively determine the behavior of diverging and converging lenses underwater, we first need the standard **thin lens equation** which is approximately true for all media, as long as lenses are thin:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Here, d_o is the object distance (i.e. the distance from the object to the lens) and d_i is the image distance.

The corresponding error, propagated in the relevant term (d_i), is then:

$$\alpha_{d_i} = \sqrt{\left(\frac{-\alpha_{d_o} f^2}{(d_o - f)^2}\right)^2 + \left(\frac{\alpha_f d_o^2}{(d_o - f)^2}\right)^2}$$

Note that when multiple lenses are present, the image of one lens becomes the object for the other, which we will need when computing distances for our first experiment.

However, unlike experiments done purely in air, we also need to keep track of the effect of different media with refractive indices that aren't equal to 1. For this, we make use of the **Lensmaker's equation**:

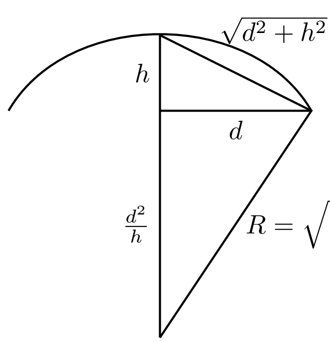
$$\frac{1}{f} = \left(\frac{n_{\text{lens}} - n_0}{n_0}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now, the corresponding error formula is the rather messy, mainly because every term in this expression carries with it an associated error:

$$\alpha_f = \sqrt{\left(\frac{R_2^2 \alpha_{R_1}}{\left(\frac{n_{\text{lens}} - n_0}{n_0}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^2}\right)^2 + \left(\frac{-R_1^2 \alpha_{R_2}}{\left(\frac{n_{\text{lens}} - n_0}{n_0}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^2}\right)^2 + \left(\frac{-n_0 \alpha_{n_{\text{lens}}}}{(n_{\text{lens}} - n_0)^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}\right)^2 + \left(\frac{R_2^2 \alpha_{R_1}}{(n_{\text{lens}} - n_0)^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}\right)^2}$$

When we use the image of one lens as the object for another, we need to account for the difference between the lenses, so we use $d_{ob} = d_{ic} - (d_{fb} - d_{fc})$ and associated error $\alpha_{d_{ob}} = \sqrt{\alpha_{d_{ic}}^2 + \alpha_{d_{fb}}^2 + \alpha_{d_{fc}}^2}$.

In order to calculate the **radius of curvature** (which is the R term in the Lensmaker's equation), we used calipers to measure the difference in height between the center and edge of the lens h and the radial distance d. To get an explicit expression for the radius, we used the approximation that the triangles shown in the image below are similar to get:



$$R = \sqrt{\frac{d^4}{h^2} + d^2} = d\sqrt{\frac{d^2}{h^2} + 1} = \boxed{\frac{d\sqrt{d^2+h^2}}{h}}$$

$$R = d\frac{\sqrt{d^2 + h^2}}{h}$$

Similarly, we get the associated error:

$$\alpha_R = \sqrt{\left(\frac{\alpha_d(2d^2+h^2)}{h\sqrt{d^2+h^2}}\right)^2 + \left(\frac{-\alpha_h d^3}{h^2\sqrt{(d^2+h^2)}}\right)^2}$$

Then, finally, for the **magnification**, we use the expression

$$M = \frac{d_{ib}d_{ic}}{d_{ob}d_{oc}}$$

Since in our experiment we are using two lenses, the magnification is simply the product of the two magnification expressions for each lens, which turns the negative sign positive.

The associated error is

$$\alpha_M = \sqrt{\left(\frac{d_{ic}\alpha_{d_{ib}}}{d_{ob}d_{oc}}\right)^2 + \left(\frac{d_{ib}\alpha_{d_{ic}}}{d_{ob}d_{oc}}\right)^2 + \left(\frac{d_{ib}d_{ic}\alpha_{d_{ob}}}{d_{ob}^2 d_{oc}}\right)^2 + \left(\frac{d_{ib}d_{ic}\alpha_{d_{oc}}}{d_{ob}d_{oc}^2}\right)^2}$$

Now we can finally move on to the second and third experiments.

Experiment 2 and 3 Theory

The **Sellmeier Equation** is an experimentally determined equation that gives the index of refraction of some media as a function of the wavelength of incident light to the media. The most general form is:

$$n^2(\lambda) = 1 + \sum_i \frac{B_i\lambda^2}{\lambda^2 - C_i}$$

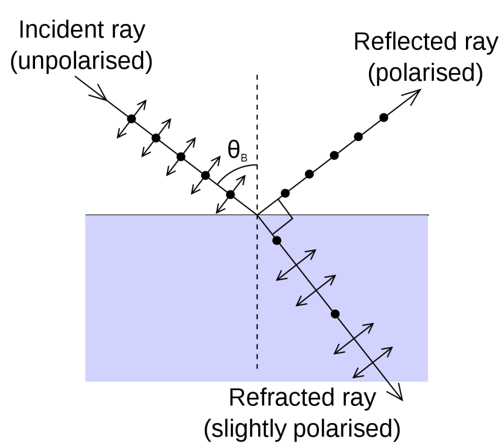
Where B_i and C_i are experimentally determined coefficients.

To compare results explicitly, we will use the experimentally determined coefficients up to second order from a paper from 1936, which deduced that for water,

$$n^2(\lambda) \approx 1.7726479 - \frac{0.0315734}{2.2535795 - \lambda^2} + \frac{0.00701841}{\lambda^2 - 0.0092513}$$

[2] Here, the coefficients are calibrated for λ when expressed in micrometers. [1]

For the second experiment, we will need the expression for **Brewster's Angle**, which is the angle at which an incident unpolarized ray will have a completely polarized reflected ray. Therefore if we begin with an incident ray polarized within the plane as shown in the diagram, we should expect to see no reflected ray.



[1] It should be noted that there are many slight differences in these coefficients across literature (see [3]), but they all return an equation which is good enough for our purposes.

The equation for Brewster's angle is then

$$\theta_B = \tan^{-1}\left(\frac{n_w}{n_a}\right),$$

[1, 4] where n_w and n_a are the refractive indices of water and air, respectively. Thus, our associated error becomes:

$$\alpha_{n_w} = \sqrt{(\alpha_{n_a}\tan(\theta_B))^2 + (n_a\sec^2(\theta_B)\alpha_{\theta_B})^2} \approx \sqrt{(n_a\sec^2(\theta_B)\alpha_{\theta_B})^2}$$

The last approximation is made because the error in the index of refraction in air is typically on the order of 10^{-5}, which is substantially smaller than the error we obtain using the second term.

Finally, for our last experiment we'll need **Snell's Law**, which states that for light incident at some angle traveling through two different media with indices of refraction n_1 and n_2,

$$n_1 sin(\theta_1) = n_2 sin(\theta_2)$$
$$\frac{n_2}{n_1} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

For our purposes we take n_1 to be air and n_2 to be water, and as before we argue that the error in n_1 is negligible, so we have roughly:

$$\alpha_{n_w} = \sqrt{\left(\frac{\cos(\theta_a)\alpha_{\theta_a}}{\sin(\theta_w)}\right)^2 + \left(\frac{\sin(\theta_a)\alpha_{\theta_w}}{\sin(\theta_w)\tan(\theta_w)}\right)^2}$$

With all of the theory in place, we can move on now to discussing the actual procedures for the three experiments.

Experiment 1: Observing How Lenses Change Behavior in Water

This was our primary experiment of the project, and here is how we did it.

Experimental Design

This is the main experiment we are interested in, which will check if lenses with smaller refractive indices than the medium they are immersed in will exhibit inverted behavior (ie, will converging lenses diverge light and will diverging lenses converge light). The way we wish to demonstrate this is actually quite simple: we submerge a diverging lens underwater and produce an image. If we produce a crisp image, that already means that, qualitatively, our diverging lens actually became a converging lens underwater, as desired. In order to actually accomplish this, we are going to need a fairly specific set of materials, however.

Most importantly, we'll need a diverging lens with a smaller refractive index than water. The way we accomplished this was with a glass lens that had an air pocket inside of it, though in principle it is equally as viable to try to find a lens made of a material with a smaller refractive index (though this may be slightly more difficult). Next, we need an object which we will use to produce an image (we went with the F object commonly used in our optics labs). It needed to be somehow secured to our setup, so we used a lab stand with some clamps. With this, we also need a light source that can be used to help produce the image. We went with a collimated projector for ours. Then, we need a tank with water (it's important that this tank not be too thick or made of an opaque material, since we want the light to actually pass through it and not worry about the glass walls interfering with the light by diffracting it or refracting it). Finally, we need a screen at the back on which our image will appear.

As it turns out, this unfortunately wasn't enough for our setup, because the focal length of the lens we chose was so great that the image was forming way further behind the tank than expected, so we would need either a significantly larger tank or a lens with a smaller focal length (both of which were unavailable). To rectify this, we added one last thing to our setup: a standard glass converging lens (because it was solid glass, it would have a higher refractive index than the water, meaning it would function like a normal converging lens). This lens would help further converge the light and actually have the image form on the screen behind the tank.

This was not the first design that we went with, and we'll discuss possible alternatives in a later section.

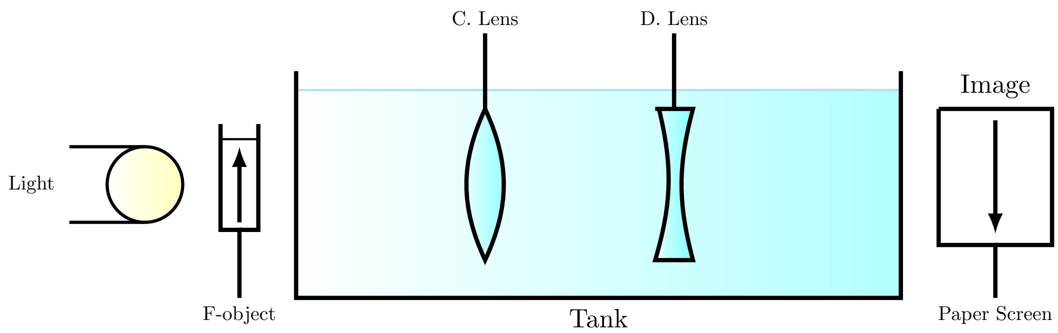
Experimental Procedure

Unlike the design that just preceded it, the procedure itself is fairly straightforward once everything is in place:

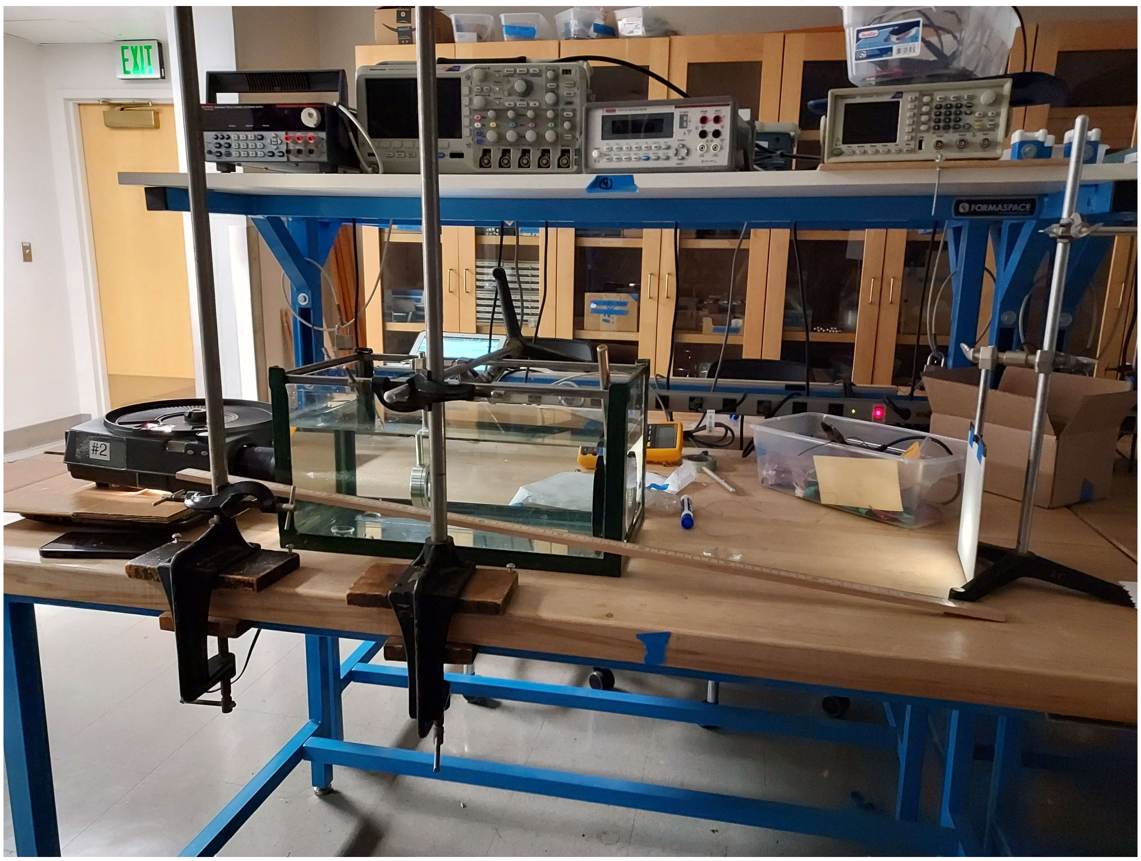
1. First, we placed the collimated projector on one side of the tank filled with water, and the screen some distance away on the other side such that the light hit the screen.
2. Next, attach the object used to create the image to the holder such that it is between the light source and the tank with no gaps (since we used an F-object in a circular grip, we simply anchored the grip to a stand).
3. For simplicity in data collection, fix the distance from the light source to the converging lens by using a cross bar to hold the converging lens in place, and from the light source to the screen.
4. With the setup in place, move the diverging lens to several different positions between the converging lens and the edge of the tank, and record both the size of the F object and all other relevant distances. Make sure that the image produced is crisp.
5. With the image produced, record the size of the F object image and the final distances from the F object to both lenses and to the screen.
6. Repeat this process for different distances.

Experimental Setup

Below is a sketch of the setup for the main experiment (confirming the inverse behavior of lenses when the refractive index of the medium is greater than that of the lens):



Our actual setup ended up looking like this:



With everything in place, it's time to move on to collecting some data and analyzing it.

The Data

Setting up the experiment is quite important, but it is equally as important to actually collect some data from the setup and attempt to confirm our theories, both qualitatively and quantitatively. We start by collecting raw data from our setup, and then we will move on to analyzing it. The results and conclusions of the experiment will be discussed in a later section.

Data Collection

As described in the procedure, the distance from the F-object to the converging lens was fixed at 17.8 cm. Furthermore, the size of the F-object was measured to be 1.55 ± 0.5 cm.[2] The following table shows the data that we collected (with corresponding errors)[3]:

Trial	**F to D Lens ($\pm$0.1 cm)**	**F to Screen ($\pm$0.1 cm)**	**Size of F ($\pm$0.1 cm)**
1	23.7	85.3	6.3
2	28.0	85.3	6.0
3	31.8	85.3	5.5
4	34.8	85.3	5.1
5	38.2	85.3	5.0
6	42.9	85.3	4.5

Then, to calculate the radius of curvature using the method we outlined in the theory section, we collected the data in the following table:

Lens	**Center**	**Side**	**Height Difference ($\pm$0.02)**	**Length ($\pm$0.01)**
Diverging lens	0.69	0.23	0.46	5.7
Double convex	0.88	0.34	0.54	1.6

All values in the above tables are in centimeters, and the associated errors are listed side-by-side.

Data Analysis

In order to make sense of the data, we first compute the experimentally calculated distances using equations from our theory. To do so, we take the following steps:

1. Compute the radius of curvature of both lenses using our approximation; then compute their focal lengths using the Lensmaker's equation.
2. Compute the distance of the "hypothetical" image formed by the converging lens underwater using the thin lens equation.
3. Use this new image distance as the object distance to the diverging lens, then apply the thin lens equation once more to calculate the final distance from the diverging lens to the image.

Of course, the error is also propagated in the same vein. Ultimately, we obtain the following set of results (see the Python notebook attached at the end of this report for a detailed look into the code that got us here):

Calculated Image Distance	**Error**
484.5	69.8
461.5	60.8
443.7	54.1
430.7	49.6
417.7	45.2
401.3	39.9

Magnification	**Error**
26.4	4.4
24.6	3.9
23.3	3.5
22.3	3.2
21.3	2.9
20.0	2.6

[2]Here, by "size" we mean the longer of the two horizontal lines on the 'F', since we expect the rest to scale proportionally.
[3]Here, "F" refers to the position of the F-object, and "D Lens" refers to the diverging lens

The image distance values were calculated from our raw data tables, then the values for the magnification were obtained using the calculated data. However, as we also took data on the size of the F-object, we can also compute the magnification by comparing the image and object sizes directly:

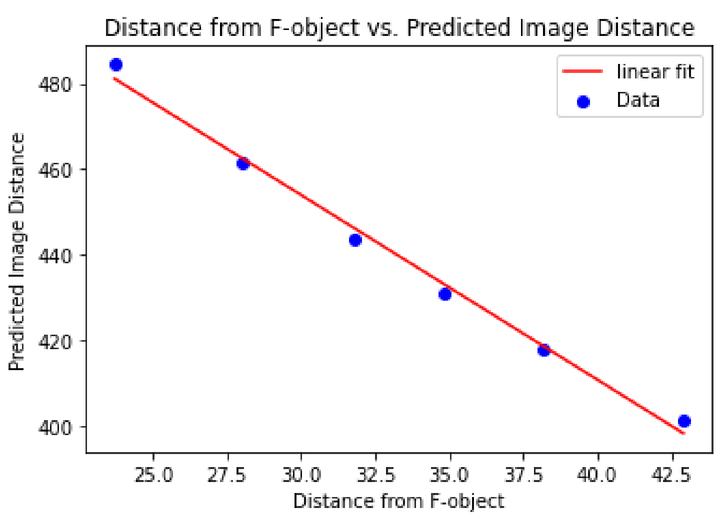
Magnification (from image size)	**Error**
4.1	0.1
3.9	0.1
3.5	0.1
3.3	0.1
3.2	0.1
2.9	0.1

Confirming Our Theory: Image Distances

Our first step in confirming the theory is to analyze the image distances. Here, it makes little sense to compute an average distance using an arithmetic mean, since the variance in these values was intentional. Clearly, our experimentally determined values for the image distance are significantly larger then the actual measured distance, which is approximately 69.8 cm. This is certainly alarming, but we do have a couple of explanations for this. Firstly, it is entirely possible that our calculations for the radius of curvature and hence the focal length of the lens to have been inaccurate, since the way we performed our measurements were quite crude.[4] A slight change in the focal length would cause a significant change in the theoretical results, since the size of images (and therefore distances) is dependent heavily on the object position relative to the focal length.

Secondly, this discrepancy could be explained by the fact that the light spent a very substantial portion in air after passing through the diverging lens, whereas the calculations made here assumed that water was the only medium at play. This could indeed explain the discrepancy, as the changes in optical properties are quite significant - the focal length of the strong converging lens changed from 5 cm to approximately 19 cm, turning the strong lens into a very weakly converging lens. Considering this, it is entirely possible to suspect that it is due to light spending a substantial time in air which caused our observed discrepancy. This is also substantiated by the fact that the focal length of the converging lens was changed by a factor of approximately 4, which is also almost precisely the factor of discrepancy that our measurements had.

Despite this discrepancy, what is arguably more important here is the overall trend that we observe when the distance between the F-object and the diverging lens is varied. Putting these experimental distances on a scatter plot, we see the following trend:



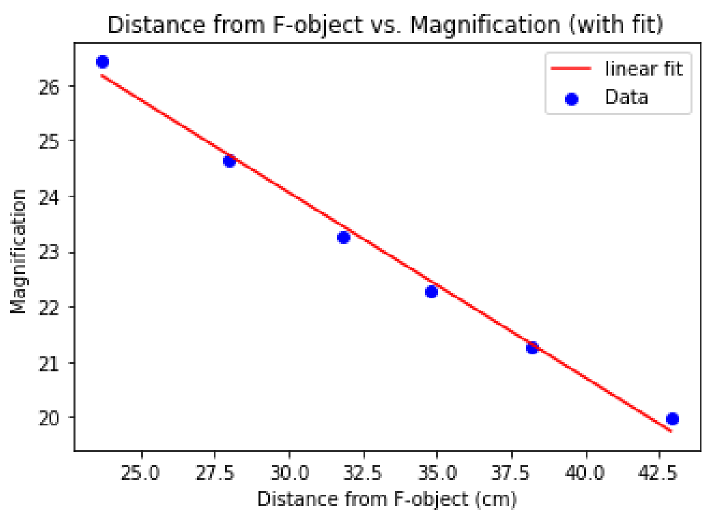
[4] We could've tried to use a lens clock here, but the one we had wasn't calibrated for this particular lens.

The linear fit here shows that our trend is roughly linear, with a slope of -4.3 ± 0.2 and a y-intercept of 583.5 ± 6.4. Again, while the fit parameters here carry little to no significance, what is important is the confirmation that this plot is roughly linear, as a linear plot fits quite perfectly with the experimentally obtained values. The calculated image distances also appear to decrease, which is precisely the trend that we expect, since as the distance between the the F-object and the diverging lens increases, the light between the two lenses now converges more, and thus produces a smaller image distance. The reduced χ^2 (i.e. $\tilde{\chi}^2$) value for this fit is $\boxed{0.019}$, indicating that our residuals are also small, and thus we can confirm that a linear trend is an appropriate fit for the data we have. However, it is worth noting that with such a small reduced χ^2, we have very likely overestimated the errors, which makes sense since a small disturbance in these measurements causes a drastic shift in the location of the final image.

Confirming our Theory: Magnification

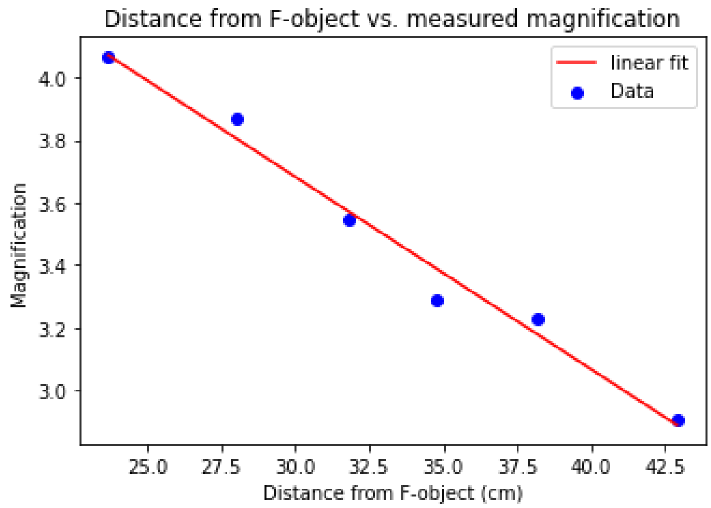
To further confirm that we are observing the correct phenomenon, we can take a look at our values for magnification. Firstly, it is important to note that the calculated magnification is much higher than the experimentally measured magnification. While this may appear to be alarming at first, note that our experimentally calculated values does not account for the fact that the light was refracted once more when exiting the fish tank, and since light converges faster when it exits into a faster medium, it makes sense that our measured magnifications will be smaller than the ones derived using our calculations as rays converging faster will inherently produce a smaller image.

Just like the image distances, what is arguably more important than the values themselves is the fact that the calculated trend matches the observed trend - the magnifications decrease as the distance between the F-object and the lens is increased. Again, this also makes sense theoretically since the magnification can also be calculated as a function of image distance, and since our calculated image distances are decreasing, it then makes sense that the magnification decreases as well. Furthermore, we also observe that the trend is roughly linear:



This further confirms our theory, as a linear trend between the object and image distances (and thus the magnification also) is exactly what we expect based upon our theoretical model, showing that our theoretical setup was a good rough model of our experimental system. The reduced χ^2 of this plot is $\boxed{0.023}$, so just like the image distance, this confirms that the residuals are small and that the linear fit is a good trend for the data we have.

These trends observed in the calculated magnification also align with those observed in the magnification measured using the image sizes, as shown below:



Here, we see that the decreasing trend is observed, so the magnification calculated using image distances is accurate in the sense that it produces the correct trend that we expect. The slopes of these two trends are quite different however: the measured magnification has a slope of -0.061 ± 0.004, whereas the calculated magnification has a slope of -0.33 ± 0.01. This can be explained again due to the fact that the light rays spent a substantial time in air, which converges the rays much faster than water. The reduced χ^2 of this plot is $\boxed{0.21}$, meaning that our errors were relatively well estimated, and is further confirmation that our theoretical model and experimental model roughly match each other.

Results and Conclusions

Finally, with everything collected, let's look back on what we obtained and see if our experiment worked as desired to give us the results we were looking for.

We expected diverging lenses to exhibit converging lens behavior when their refractive index is smaller than the surrounding medium. In this case, we chose a glass lens with an air pocket inside to function as our lens and water as our medium. So, if we were to pass light through the lens, we expected an real, magnified, inverted image to form on the screen at the back of the tank. Firstly, qualitatively, this is what we saw (albeit with a little assistance from a regular converging lens). We saw a real, enlarged, inverted image show up on the screen at the back, demonstrating that the lens was exhibiting the properly expected behavior. Then, when we actually took the raw data and analyzed it, we saw that moving around our lens generated the exact trend we were expecting: the further away the F object was, the smaller the magnification on the back. Similarly, the predicted image distances matched up linearly to the actual distance from the diverging lens, as expected.

While our values were technically pretty off, they still followed the trend we would expect, and all of the errors that we have can be explained pretty simply by issues with our setup, our lenses, and our actual data collection procedures, since we lost precision in a lot of these places.

In other words, our experiment confirmed our predictions in more way than one (since it was possible that the lens was still exhibiting diverging behavior, but the converging lens simply was more powerful and properly focused the light). In the end, this made this experiment a success.

There were many issues that we encountered when conducting the experiment, and the most notable is probably the actual setup. Lenses with smaller refractive indices than water with the proper focal length to give us good results in a small glass tank aren't exactly the easiest things to get our hands on, so it was important to work with what we had. We did run through multiple alternatives to this setup, each of which could've produced better results, and these will be discussed at the very end.

Experiments 2 and 3 (Brewster's Angle and Snell's Law)

These were secondary experiments that we came up with after finishing the first experiment, given that we had some spare time. While the first experiment was generally a success with confirming the inverted behavior, we were also curious about and wanted to experimentally confirm whether or not the refractive index actually depends on more than just the medium, which brings us here.

Introduction and Objectives

The goal of these two experiments were both the same: to experimentally measure and determine the effect of the wavelength of incident light on the index of refraction of water. This was done through two separate means: first, via measuring the Brewster angle of water for lasers with fixed frequencies, and second using Snell's law for different observed angles resulting from the refraction of light. The relevant theory was established previously in the theory section.

Experiment 2 Procedure

Our second experiment utilized a plastic semicircle filled with water, as shown in the image below. A laser light was shone at some angle incident to the flat stretch of the semicircle, a screen was placed in the direction of the light reflection, and a polarizer was placed in between the laser and the semicircle such that the beam of laser light passed through it.

First, we needed to calibrate the polarizer such that we knew the incident light was polarized purely in the horizontal plane. To do this, we first used the marks on the polarizer to determine which orientations resulted in vertical and horizontal light.

We then estimated a reasonable angle where Brewster's angle should occur (roughly 50 degrees) and adjusted the polarizer such that the light passing through on the reflection was minimized (but not due to blocking the laser's own polarization[5]).

From there, we then fine tuned our angle with small precise movements back and forth, making sure the laser beam's trajectory passed through the origin of the protractor throughout to minimize error. This was done for three lasers: red, green, and violet with wavelengths 635, 532, and 405 nanometers respectively.

Due to difficulty seeing the beam through the water, for red and green lasers we introduced chalk dust, and similar for the near ultra-violet we used UV fluorescent dye to achieve the same effect.

Once an angle was obtained, we recorded the angle measured with associated error.

Experiment 3 Procedure

The final experiment subsisted of shining an iPhone camera light into a similar water-filled plastic rectangular prism. The side of the prism was taped except for a small vertical aperture through which the light could pass. At a specific angle, prism-like behavior could be observed, and the light would scatter into colors of various wavelengths.

First, we confirmed that the water was indeed responsible for this effect by shining the light through the rectangular box without any water and making sure no effect was observed. Water was then added.

The second thing to confirm was that the light passing through the aperture was responsible for the rainbow. This was simply done by blocking the aperture with a finger or some other object and noticing that the refraction no longer occurred.

[5] This was particularly annoying, as it's important to move the polarizer in tandem with the laser to avoid issues.

Then, the light was shined at a specific angle such that the rainbow was visible. A screen was placed in the direction of the rainbow, and a photo was taking directly above the experimental setup.

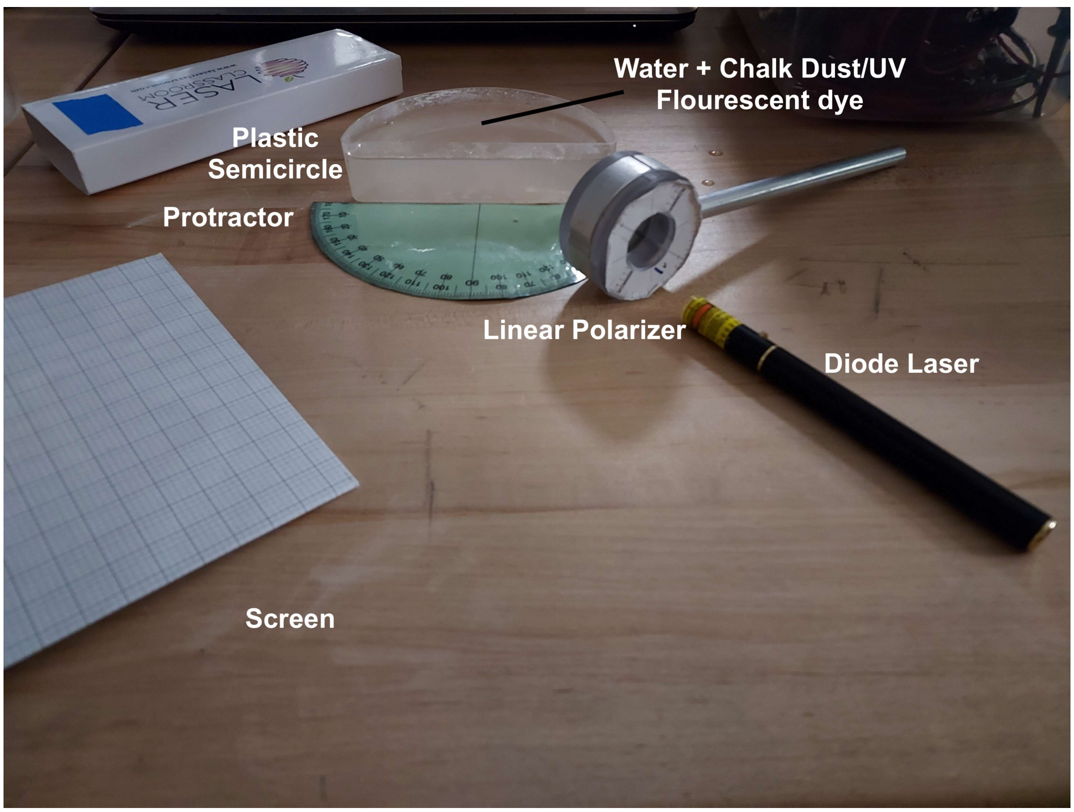
From this image, angles were measured explicitly via an imaging software and compared, which could then be used to deduce the index of refraction through Snell's Law.

In order to collect decent data, to make sure some consistency was applied, the incorrect assumption was made that the angle was roughly fixed for all light in the water, and most of the refraction occurred in air. This was purely for ease of calculations, since it was impossible to see the light in the water or the point where the light exited the water.

Additionally, all lines comprising the angles were fixed except when wavelengths were varied.

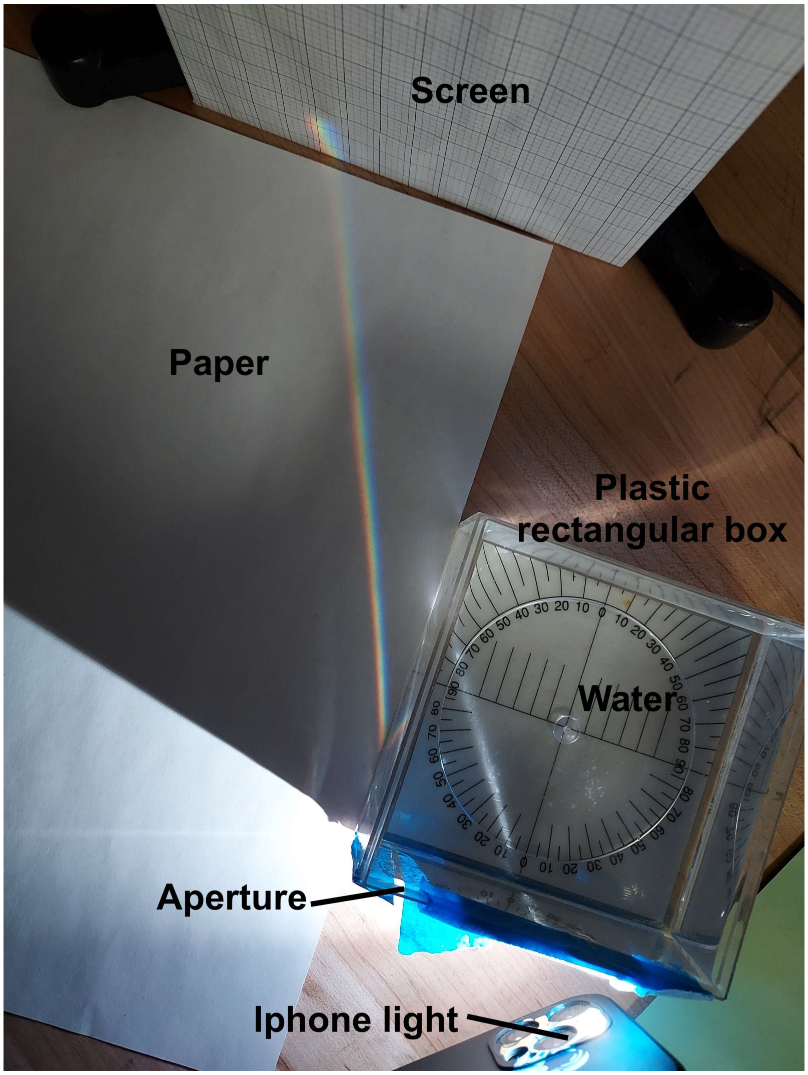
Experimental Setups

First, we have the experimental setup for Experiment 2, which involved confirming Brewster's angle:



The laser was held by hand and oriented such that the light shining through traveled through the polarizer and met the origin of the protractor, which is aligned with the plastic semicircle filled with water. The screen was held vertically, but is oriented horizontally here so that it is visible in the image.

Next, we have the setup for Experiment 3:



This was one of the images used to collect data using an image processing software. The full rainbow spectrum is visible, and a paper was used to identify where, to a reasonable amount of certainty, the light exited the rectangular box and began to travel through air. Note that on the side of the box some reflection is visible; this is *not* the point where the light exits.

The Data Collection and Analysis

Again, with the experimental design and setup in place, it's time to collect some data. We did this in order of experiment number, first collecting/analyzing data for experiment 2, and then for experiment 3 (since it was crucial that we got good results for experiment 2 in order for experiment 3 to run properly).

Experiment 2 Data Collection

The measured Brewster's angles (with error) for each laser are:

Laser Color	**Wavelength (nm)**	**Brewster Angle (°)**	**Error**
Red	634.6	53.5	0.5
Green	532	53	0.5
Violet	405	52	0.5

Note that an error of 0.5 degrees was chosen for each color, since our ability to fine-tune the angle, as well as to clearly see the beam in the water, allowed us to very precisely determine the point at which the light dimmed the most. Therefore, we were able to measure with enough certainty to be within one marking of the protractor.

Experiment 2 Data Analysis

Using the equations for Brewster's Angle and Associated Errors, we find that the experimentally determined indices of refraction for the Red, Green, and Violet lasers were $\boxed{1.35 \pm 0.02}$, $\boxed{1.33 \pm 0.02}$, and $\boxed{1.28 \pm 0.02}$ respectively.

When the given wavelengths of the lasers ($0.6346 \mu m$, $0.532 \mu m$, and $0.405 \mu m$) were plugged into the experimentally determined second-order Sellmeier equation, the corresponding indices of refraction obtained for red, green, and violet were 1.331, 1.334, and 1.342.

Unfortunately, we do not have errors for these coefficients so we are unable to determine the errors associated with the indices of refraction, but due to the precision to which these coefficients are known it is safe to assume the error is substantially smaller than our errors calculated via Brewster's angle.

Performing a one-sided agreement test gives that the Red and Green lasers pass, but unfortunately the Violet laser fails.

More importantly, we exhibited the opposite trend, and in fact observed a trend much more significant in the opposite direction than what was expected. There are several reasons why this is possible. First, we notice that the procedure we used was only precise up to roughly half of a degree, since the protractor was in degrees, but this may not be entirely accurate. The width of the laser beam is roughly about the same size as one of the degree markings on the protractor, making it difficult to measure with immense precision. Additionally, the laser beam would need to be perfectly lined up with the origin of the protractor through the polarizer, which was not always a simple task and likely lead to some substantial error.

One of the largest reasons, however, was that there were only three data points, making it difficult to exhibit an actual pattern, and whatever pattern we did measure may have subconsciously been influenced by our desire to see a pattern exhibited in the first place. For this reason, we shifted to an experiment where all visible light could be seen at the same time, so that we could more accurately verify the expected trend of index of refraction as a function of wavelength.

Now, we move on to Experiment 3.

Experiment 3 Data Collection

Our determined angle in water was as follows:

Trial 1	**Trial 2**	**Average**
53.10 ± 0.10	52.18 ± 0.10	52.68 ± 0.07

Similarly, our determined angles in air were the following[6]:

Purple	**Red**
67.43 ± 0.10	65.54 ± 0.10

Recall that we made the assumption that the angle in water was fixed. This was incorrect since most of the refraction actually occurred in water. However, attempting to determine the exact angle for red and purple in the water proved to be very challenging and gave even worse results.

Experiment 3 Data Analysis

The equations for Snell's Law ($n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$) and its associated error together give us the corresponding indices of refraction of 1.145 ± 0.001 and 1.161 ± 0.001

Immediately we notice that the indices of refraction are *significantly* off from what we would expect. This is likely due to the fact that it was very challenging to find precise angle measurements using the imaging software; for instance, it was challenging to use perspective from the image to determine where the normal to the surface of the plastic box should be to measure the incident angle via Snell's law, and even using the lines in the image did not produce an entirely accurate result. A small shift in the point at which we decided the lines caused the angles to shift dramatically.

Additionally, since the indices of refraction fall roughly halfway between those of air and those of water, we can guess that the decision to not factor in the angle of refraction occurring in water likely contributed a substantial amount to this index of refraction. However, as noted in the data section, attempting to factor this in produced indices of refraction less than one, which we could not use as viable data.

This was not entirely a failure, however. We observed that if we kept all lines fixed and only vary the one parallel to the specific wavelength of light, we could control the differences of the indices of refraction, even if we could not get accurate measurements of the index of refraction itself.

The difference we obtained was about 0.016 ± 0.002. Plugging in the visible wavelength for an average person, roughly 380 to 750 nanometers, we get that the indices of refraction we expect are 1.329 and 1.345, giving us an expected difference of 0.016, so we pass the agreement test! Of course, this is not completely accurate because the differences may not be translationally invariant, since we are taking sin and cos of angles, but it is a good indication that we get the expected behavior.

[6]We used 'Purple', 'Ultraviolet', and 'Violet' interchangeably here: they all refer to the same color laser

Results and Conclusions

The goal of both of these experiments was to quantitatively demonstrate and isolate the effect of the wavelength of incident light on the refractive index of water.

While our first experiment was fairly substantial at determining the actual index of refraction of water, since both the red and the green lasers passed the one-sided agreement test (namely $|1.35 - 1.333| \leq 0.02$ and $|1.33 - 1.333| \leq 0.02$), unfortunately the value of the violet laser, 1.28 ± 0.02, did not fall in the range. Furthermore, we got the <u>opposite trend from what we would expect</u>, which largely came from problems with precision, such as the thickness of the beam and ensuring the laser passed through the origin of the protractor, as well as not being able to collect enough data points.

In our second experiment, we observed a very different set of indices of refraction than expected, 1.145 ± 0.001 for red to 1.161 ± 0.001 for purple, but when comparing to the theoretical values of 1.329 and 1.345 we see the same exact difference between the two indices of refraction in the correct direction, which suggests we can confirm the effect of wavelength of light on the index of refraction. The reason why these values are so off are likely due to incorrect assumptions we had to make, such as keeping the angle fixed within the water, and difficulty determining where the normals to the box were through the image. It was only when we held everything but the line parallel to the rainbow fixed that we actually observed the correct trend.

In the future, it makes sense to replicate the third experiment rather than the second, since it would be substantially easier to collect more data points and be more precise using imaging software. Pictures should be taken directly above the setup and from multiple points, and more data points should be collected and averaged to get a more accurate result.

For problems with visibility of the light, we attempted to use a piece of paper to determine exactly where the light left the water and entered air, but we could also introduce chalk dust or UV fluorescent dye into the water to see if that would assist in making the light beam more visible in the water.

Conclusion

Overall, our first experiment gave us the exact behavior we anticipated: our theories predicted that a lens with a smaller refractive index than the medium it is submerged in would exhibit inverted behavior, and that's exactly what we saw with our diverging lens converging light rays when submerged in water. We had a bit more trouble when it came to quantitatively establishing this, since we had all sorts of minor issues with our setup (our lens had too large a focal length, the tank had a thickness that might not necessarily be negligible, etc). Despite this, we still considered this experiment a success, since it confirmed our hypothesis.

As for the next two experiments, they unfortunately did not go as well, giving us opposite trends from what our theories predict. This means that either our theories need to be refined, or our experiment was faulty; we are inclined to believe that the latter is the case.

Future Directions

We mentioned before that we had several different approaches to an experimental setup for the first experiment that all fell through. This was either due to monetary constraints or simple timing. Should we ever wish to repeat this experiment in the future, it may be worth trying one of these setups, since they could yield more exact and precise results.

Alternative 1: Glycerin

Our first alternative was using Glycerin as our medium instead of water. Because Glycerin has a higher index of refraction than water (closer to glass), we would simply need to find a material with a slightly lower refractive index than glycerin in order to conduct our experiment. However, this didn't end up working out, because we would (a) need a lot of glycerin (enough to fill a tank), which we couldn't get our hands on, and (b) a lens with a smaller refractive index than glass, which are prohibitively expensive to get. So, because of those constraints, we ended up scrapping this idea. Despite this, if we had more resources, we could try this direction.

Alternative 2: Hollow Plastic Hemispheres

Our second idea had to do with attempting to create a better lens with a refractive index smaller than water, since the big glass lens we had access to had a focal length that was a bit big for our experiment. This involved using two plastic hemispherical shells that we could then glue together to form a spherical lens. However, this didn't work out, because the hemispheres didn't glue together well enough to create an airtight seal, and we couldn't get a variant shaped like a diverging lens.

Alternative 3: Hollow 3D Printed Transparent Plastic Lenses

Our third idea had to do with refining our second idea with better lenses. This involved 3D printing two halves of a shell for a lens with transparent plastic filament, and then gluing the pieces together to form a hollowed-out lens. This would produce a lens with a smaller refractive index than water, and more importantly, it would be a lens that was (a) thinner, and (b) one whose focal length we could control. However, this idea fell through, because the translucent material we had access to wasn't actually as translucent as we had initially hoped, and our lenses didn't end up letting any light through. If we were to try this again, we would try to use even more translucent material and polish the printed lenses more so that they could actually let light through.

Alternative 4: Epoxy Resin

Our final alternative had to do with trying to cast our entire setup out of resin. We found that epoxy resin also has a higher refractive index than water, and it would be perfect for actually replicating what we saw in Physics 5B: an air bubble in water. We wanted to cast lens shapes (converging/diverging) that we would then fill with air, while the rest of the tank was filled with resin. This eliminates the need for a tank, since resin will solidify, and it allows us to actually cast air bubbles (which definitely will have a smaller refractive index than the surrounding resin). However, we scrapped this idea, because it would require too much preparation and too many materials for us to be able to realistically get our hands on in the week or two that we had to complete these experiments from start to finish. However, if we were to ever replicate this experiment, and we had all of the necessary materials and skills, this would definitely be the best and most reliable approach to get the most accurate data possible.

Concluding Thoughts

In the end, all of these different setups are viable, but all can have a lot of different problems associated with them (whether it be the accuracy of the setup in collecting good results or the feasibility of actually creating said setup), but if anyone had sufficient time and resources, they could make a better setup and get better results that more definitively confirm our theories. However, the main takeaway will be the same: **Physics 5B did not lie to us about the optics.**

References

[1] Brewster's angle, November 2022.

[2] L. W. Tilton. Accurate representation of refractive index of distilled water as a function of wave length. *Journal of Research of the National Bureau of Standards*, volume 17, November 1936.

[3] L. Weiss, A. Tazibt, and A. Tidu. Water density and polarizability deduced from the refractive index determined by interferometric measurements up to 250 mpa. *Journal of Chemical Physics*, 136, March 2012.

[4] A. C. William Emery. Basic electromagnetic concepts and applications to optical sensors. *Introduction to Satellite Remote Sensing*, 2017.

Click here to see the Python notebook for Experiment 1 analysis (Berkeley account needed to view).