Header styling inspired by CS 70: https://www.eecs70.org/

Collaborators

I worked with the following people on this assignment:

- Teja Nivarthi: 3036508567
- Nikhil Maserang: 3036978230

Problem 1

A Vandermonde matrix is a $(m+1)\times(m+1)$ matrix of the form

$$V = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^m \\ 1 & x_1 & x_1^2 & \dots & x_1^m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \dots & x_m^m \end{bmatrix}$$

for some complex numbers $x_0, x_1, \dots, x_m$. A Vandermonde matrix always has determinant:

$$\det V = \prod_{j=1}^{m}\prod_{i=0}^{j-1}(x_j - x_i)$$

a) Show that V is invertible if and only if no two x_i are the same.

Solution: We know from linear algebra that a matrix is invertible if and only if the determinant is nonzero. Consequently, this is also only true if no two x_i's are the same, since the above formula for the determinant shows us that the determinant would be zero if that were the case. □

We can use V to perform **polynomial interpolation**. In particular, consider a degree at most m polynomial $p(x) = a_0 + a_1 x + \cdots + a_{m-1}x^{m-1} + a_m x^m$. We do not know the a_j, but we have pairs (x_j, y_j) for $j = 0$ to m such that $y_j = p(x_j)$.

b) Write a matrix-vector equation which allows one to recover

$$a := \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix} \text{ from } y := \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_m \end{bmatrix}$$

as long as the x_j 's are not equal.

Solution: We know that based on the vector equation, that $y = Va$, so in order to determine a, then we multiply both sides by V^{-1} on the left:

$$a = V^{-1}y$$

Again, this requres that V is invertible, which is guaranteed as long as the x_j's are not equal. □

For the next subpart, assume we use the DFT matrix formulation:

$$\vec{x} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \quad \vec{X} = \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} e^{-j\frac{2\pi}{N}0\cdot 0} & e^{-j\frac{2\pi}{N}1\cdot 0} & \cdots & e^{-j\frac{2\pi}{N}N\cdot 0} \\ e^{-j\frac{2\pi}{N}0\cdot 1} & e^{-j\frac{2\pi}{N}1\cdot 1} & \cdots & e^{-j\frac{2\pi}{N}N\cdot 1} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\frac{2\pi}{N}0\cdot N} & e^{-j\frac{2\pi}{N}1\cdot N} & \cdots & e^{-j\frac{2\pi}{N}N\cdot N} \end{bmatrix}$$

where $\vec{X} = \mathbf{F}\vec{x}$

c) The DFT matrix for an $(m+1)$-length signal $z[n]$ where $z[n] = 0$ outside the interval $0 \leq n \leq m$ is actually a Vandermonde matrix. Show this by picking suitable values for $x_0, x_1, \ldots, x_m$. This gives us another interpretation of DFT: it transforms polynomials to their evaluations on this set of points.

Solution: Looking at the matrix $\mathbf{F}$, we can see that in each row, we have one of the N roots of unity, then along the row we exponentiate it from 1 to N. Therefore, one way we can pick $x_0, \ldots, x_m$ is:

$$x_k = e^{-j\frac{2\pi}{N}k} = \omega_1^k$$

where ω_1 is the first nontrivial N-th root of unity. □

Problem 2

Find the CTFT $X(\omega)$ of $x(t)$, where

$$\forall t \in \mathbb{R}, x(t) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\sigma^2}\right)$$

and $\sigma > 0$. You may or may not find it useful to know that

i) $\int_{-\infty}^{\infty} x(t)\ dt = 1$

ii) $\int_a^b u\ dv = uv\Big|_a^b - \int_a^b v\ du$

iii) $tg(t) \leftrightarrow i\frac{\mathrm{d}G(\omega)}{\mathrm{d}\omega}$

iv) $\int_0^\omega \frac{1}{G(\lambda)}\frac{\mathrm{d}G(\lambda)}{\mathrm{d}\lambda} = \ln G(\omega)\Big|_0^\omega$

Hint: Take the derivative of the given equation for $x(t)$ and use CTFT properties and the above hints to take the Fourier transform of both sides.

Solution: First, we can write out the Fourier transform:

$$X(\omega) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(t^2/2\sigma^2 + i\omega t)}\ dt$$

To continue doing this integral, I prove the following relation first:

$$\int_{-\infty}^{\infty} e^{-\frac{a}{2}x^2+bx} = e^{\frac{b^2}{2a}}\sqrt{\frac{2\pi}{a}}$$

First, we factor out $-\frac{a}{2}$ from the exponent:

$$I = \int_{-\infty}^{\infty} e^{-\frac{a}{2}(x^2+\frac{2b}{a}x)}dx$$

And now we complete the square:

$$I = \int_{-\infty}^{\infty} e^{-\frac{a}{2}\left(\left(x-\frac{b}{a}\right)^2-\frac{b^2}{a^2}\right)}dx = \int_{-\infty}^{\infty} e^{-\frac{a}{2}\left(x-\frac{b}{a}\right)^2+\frac{b^2}{2a}}dx = e^{\frac{b^2}{2a}}\int_{-\infty}^{\infty} e^{-\frac{a}{2}\left(x-\frac{b}{a}\right)^2}dx$$

Now we perform a u-substitution $u = x + \frac{b}{a}$ so $du = dx$, therefore:

$$I = e^{\frac{b^2}{2a}}\int_{-\infty}^{\infty} e^{-\frac{a}{2}u^2}du$$

Now perform a second substitution $r = \sqrt{\frac{a}{2}}u$, so therefore $r^2 = \frac{a}{2}u^2$:

$$I = \sqrt{\frac{2}{a}}e^{\frac{b^2}{2a}}\int_{-\infty}^{\infty} e^{-r^2}\ dr \tag{1}$$

It's a well known result that the remaining integral simplifies to $\sqrt{\pi}$, but I'll prove it as well. I'll use x instead of r, since we're going to use that later. First, we calculate the square of the integral:

$$I'^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\ dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\ dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\ dx\ dy$$

We know that $x^2+y^2 = r^2$, and switching this to polar coordinates:

$$I'^2 = \int_0^\infty\int_0^{2\pi} e^{-r^2}r\ d\theta\ dr = \int_0^{2\pi}\ d\theta\int_0^\infty e^{-r^2}r\ dr$$

The second integral can be solved via a u-substitution $u = r^2$ so $du = 2r\ dr$, so:

$$I'^2 = 2\pi \frac{1}{2} \int_0^\infty e^{-u}\ du$$

The integral evaluates to 1 (I'm just too lazy to show it explicitly at this point), meaning that $I' = \sqrt{\pi}$ after taking the square root. Returning to the original integral (equation 1), we get:

$$I = \sqrt{\frac{2\pi}{a}} e^{\frac{b^2}{2a}}$$

as desired. Now we can finally proceed with our Fourier transform. In the equation, we identify that $a = \frac{1}{\sigma}$, and $b = -i\omega$. Therefore, the integral simplifies to:

$$X(\omega) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\omega^2\sigma^2/2} \sqrt{2\pi\sigma^2} = e^{-\omega^2\sigma^2/2}$$

This is also an expected result: we know in literature that the Fourier transform of a Gaussian is a Gaussian; in fact, it is this property that makes the Gaussian the "minimum uncertainty" wavepacket between position and momentum in quantum mechanics. I imagine it to work the exact the same between temporal support and the bandwidth in frequency space since both things (position/momentum vs. frequency/time) are related by a simple Fourier transform. □

Problem 3

Consider a discrete-time signal $x : \mathbb{Z} \to \mathbb{C}$ that has a Fourier transform (DTFT) $X : \mathbb{R} \to \mathbb{C}$.

a) Let $\hat{X}$ be such that

$$\forall \omega \in \mathbb{R}, \quad \hat{X}(e^{j\omega}) = j\frac{\mathrm{d}X}{\mathrm{d}\omega}(e^{j\omega})$$

Determine an expression for $\hat{x}(n)$.

Solution: Using the definition for the derivative property:

$$nx[n] \quad \leftrightarrow j\frac{\mathrm{d}X(e^{j\omega})}{\mathrm{d}\omega}$$

since the right hand side of this is what we have for $\hat{X}e^{j\omega})$, then it means that $\hat{x}[n]$ must correspond to the left hand side. This means that:

$$\hat{x}[n] = nx[n]$$

□

---

b) Let $\hat{x}$ be such that

$$\forall n \in \mathbb{Z}, \quad \hat{x}[n] = \begin{cases} x\left(\frac{n}{N}\right) & \text{if } n \bmod N = 0 \\ 0 & \text{otherwise} \end{cases}$$

for some integer N.

i) Show that $\hat{X}$, the DTFT of $\hat{x}$, is

$$\forall \omega \in \mathbb{R}, \quad \hat{X}(e^{j\omega}) = X(e^{jN\omega})$$

Solution: The proof of this was covered in lecture: let's take the Fourier transform of $\hat{x}[n]$:

$$\begin{aligned}\sum_{n=-\infty}^{\infty} \hat{x}[n]e^{-j\omega n} &= \sum_{k=-\infty}^{\infty} x[kN]e^{-j\omega kN} \\ &= \sum_{k=-\infty}^{\infty} x[k]e^{-j(N\omega)k} \\ &= X(e^{jN\omega})\end{aligned}$$

as desired. □

---

ii) Consider the causal echo system characterized by the linear, constant-coefficient difference equation

$$\hat{y}[n] = \hat{x}[n] + \alpha\hat{y}[n-N]$$

where $|\alpha| < 1$ and $\hat{x}$ and $\hat{y}$ denote the input, and output, respectively. Show that the frequency response $\hat{H}$ of this system is

$$\forall \omega \in \mathbb{R}, \hat{H}(e^{j\omega}) = \frac{1}{1 - \alpha e^{-i\omega N}}$$

Solution: Recall that to compute frequency response, we let $\hat{x}[n] = Ae^{i\omega n}$, and $\hat{y}[n] = H(\omega)Ae^{j\omega n}$. Therefore:

$$\hat{H}(\omega)Ae^{i\omega n} = Ae^{i\omega n} + \alpha\hat{H}(\omega)e^{i\omega(n-N)}$$

dividing both sides by $Ae^{i\omega n}$ and collecting $\hat{H}(\omega)$, we get:

$$\hat{H}(\omega) = \frac{1}{1 - \alpha e^{-i\omega N}}$$

as desired. □

iii) How is $\hat{H}$ related to the frequency response H of the causal system described by the linear, constant-coefficient difference equation

$$y[n] = x[n] + \alpha y[n-1]$$

where $|\alpha| < 1$, and x and y denote the input and output, respectively?

Solution: This system is the special case where $N = 1$, so therefore we have:

$$H(\omega) = \frac{1}{1 - \alpha e^{-i\omega}}$$

So in terms of input, we have $H(\omega N) = \hat{H}(\omega)$. □

---

iv) Determine the impulse response values $h[n], \forall n \in \mathbb{Z}$. Then use the up-sampling property to determine the sample values of the impulse response $\hat{h}[n]$.

Solution: Solving the LCCDE from the previous part, we have:

$$y[n] = \sum_{k=0}^{n} \alpha^{n-k} x[k]$$

Therefore, the impulse response $h[n]$ is given by feeding in $x[k] = \delta[k]$:

$$h[n] = \sum_{k=0}^{n} \alpha^{n-k} \delta[k] = \alpha^n u[n + N]$$

where the last equality comes from noticing that for every n, there is only one term in this sum that is nonzero, which is the delta function with α^k attached to it. We then have a step function to enforce that values above n are zeroed out. Then, using the up-sampling property, we know that $\hat{h}[n] = h[\frac{n}{N}]$:

$$\hat{h}[n] = \sum_{k=0}^{n/N} \alpha^{n/N} u[n/N + N]$$

□

Problem 4

Consider a ***real-valued***, *causal* discrete-time signal x and its Fourier transform X. The real part of the signal's Fourier transform is given by

$$\mathrm{Re}\{X(e^{j\omega})\} = 1 + \cos(\omega) - \cos(2\omega)$$

On midterm 1, we derived $X(e^{j\omega})$ by splitting it up into its real and imaginary parts. Now, we use DTFT properties! :)

a) Using DTFT properties, evaluate the integral $\int_{\langle 2\pi\rangle} X(e^{j\omega})\, d\omega$.

Solution: This is basically an exercise in rewriting an integral:

$$\begin{aligned}\int_{\langle 2\pi\rangle} \mathrm{Re}\{X(e^{j\omega}\}\, d\omega &= \int_{\langle 2\pi\rangle} \frac{1}{2}(X(e^{j\omega}) + X^*(e^{j\omega}))\, d\omega \\ &= \frac{1}{2}\int_{\langle 2\pi\rangle} X(e^{j\omega})\, d\omega + \frac{1}{2}\int_{\langle 2\pi\rangle} X^*(e^{j\omega})\, d\omega\end{aligned}$$

Since $x[n]$ is real-valued, then we know that $X^*(e^{j\omega}) = X(e^{-j\omega})$, which we can now write as:

$$\int_{\langle 2\pi\rangle} \mathrm{Re}\{X(e^{j\omega})\}\, d\omega = \frac{1}{2}\int_{\langle 2\pi\rangle} X(e^{j\omega})\, d\omega + \frac{1}{2}\int_{\langle 2\pi\rangle} X(e^{j(-\omega)})\, d\omega$$

So this means that if we pick a suitable (odd) interval $\omega \in [-\pi, \pi]$, then the two integrals are both identical, therefore, we have:

$$\int_{-\pi}^{\pi} \mathrm{Re}\{X(e^{j\omega})\}\, d\omega = \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$$

Computing the left hand side since we're given $\mathrm{Re}\{X(e^{j\omega})\}$, we get:

$$\int_{-\pi}^{\pi} X(e^{j\omega}) = 2\pi$$

It then follows from the fact that the exponentials along any interval of width 2π is periodic that:

$$\int_{\langle 2\pi\rangle} X(e^{j\omega})\, d\omega = 2\pi$$

□

---

b) Determine, and provide a well-labeled plot of the signal x.

Solution: From the earlier part, because we've shown that computing the integral for $\mathrm{Re}\{X(e^{j\omega})\}$ is the same as computing the integral for the total $X(e^{j\omega})$, this implies that

$$\mathrm{Re}\{X(e^{j\omega})\} = X(e^{j\omega})$$

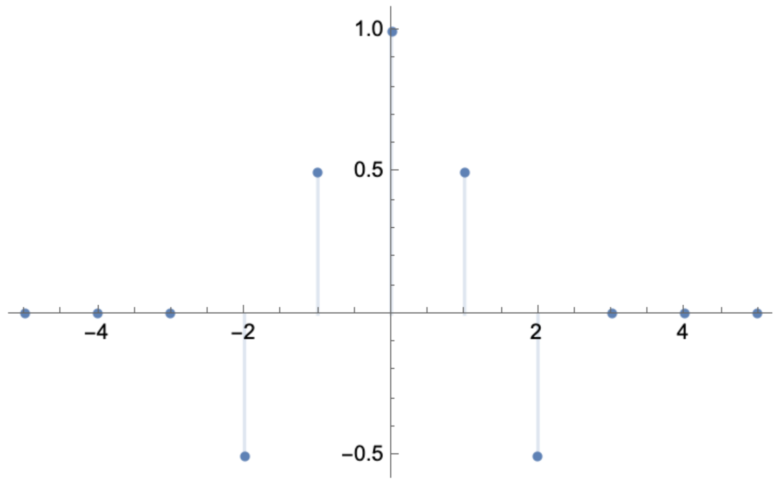
Therefore, now we can just find $x[n]$ via the DTFT synthesis equation:

$$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} (1 + \cos\omega - \cos(2\omega))e^{j\omega n}\, d\omega$$

This evaluates to:

$$x[n] = \frac{1}{2}(-\delta[n-2] + \delta[n-1] + 2\delta[n] + \delta[n+1] - \delta[n+2])$$

As for the plot, we've done this a million times but here's how I understand it: at $n = \pm 2$, we have peaks of $-\frac{1}{2}$, at $n = \pm 1$ we have peaks of $\frac{1}{2}$, and at $n = 0$ we have a peak of 1 because we have $2\delta[n]$ which cancels out the prefactor of $\frac{1}{2}$. For all other n, we have $x[n] = 0$. I'm explaining it like this to show that I do understand how to plot functions like these; I am simply using Mathematica becuase it takes less time:



□

c) Evaluate the integral $\int_{\langle 2\pi\rangle} X(e^{j\omega})\cos(\omega)\, d\omega$.

Solution: Written out, we're basically asked to compute:

$$\int_{\langle 2\pi\rangle} (1+\cos\omega - \cos(2\omega))\cos\omega\, d\omega = \int_0^{2\pi} \cos\omega\, d\omega + \int_0^{2\pi} \cos^2\omega\, d\omega - \int_0^{2\pi} \cos\omega\cos(2\omega)\, d\omega$$

We can notice that the first and third integrals evaluate to zero (since we're integrating a cosine wave over its entire period), so we're left with the second integral:

$$\begin{aligned}\int_0^{2\pi} \cos^2\omega\, d\omega &= \int_0^{2\pi} \frac{1+\cos(2\omega)}{2}\, d\omega \\ &= \left[\frac{\omega}{2} + \frac{1}{4}\sin(2\omega)\right]_0^{2\pi} \\ &= \pi\end{aligned}$$

Therefore, we conclude:

$$\int_{\langle 2\pi\rangle} X(e^{j\omega})\, d\omega = \pi$$

□

d) Evaluate the integral

$$\int_{\langle 2\pi\rangle} |X(e^{j\omega})|^2\, d\omega$$

Solution: Well, we know that $X(e^{j\omega}) = \mathrm{Re}\{S(e^{j\omega}\}$, so basically we just have to compute:

$$\int_{\langle 2\pi\rangle} |1+\cos(\omega) - \cos(2\omega)|^2\, d\omega$$

We can use Parseval's theorem to massively simplify our life, which states:

$$\frac{1}{2\pi}\int_{\langle 2\pi\rangle} |X(e^{j\omega})|^2\, d\omega = \sum_n |x[n]|^2$$

Therefore, we have:

$$\int_{\langle 2\pi\rangle} |X(e^{j\omega})|^2\, d\omega = \frac{1}{2\pi}\left(1 + 2*\left(-\frac{1}{2}\right)^2 + 2*\left(\frac{1}{2}\right)^2\right) = 4\pi$$

□

e) Determine a reasonably simple expression for $\mathrm{Im}\{X(e^{j\omega})\}$, the imaginary part of the signal's Fourier transform, where, as you know,

$$X(\omega) = \mathrm{Re}\{X(e^{j\omega})\} + i\mathrm{Im}\{X(e^{j\omega})\}$$

Solution: We've already concluded earlier in this problem that $X(e^{j\omega} = \mathrm{Re}\{X(e^{j\omega})\}$, so this would directly imply that $\mathrm{Im}\{X(e^{j\omega})\} = 0$.

I'm going to preface by saying that I probably did something wrong in this problem, but I don't have enough time to figure out what that is. $\square$

Problem 5

Consider a finite impulse response (FIR) filter $H : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ having impulse response h and frequency response H.

Part (a) below discloses several pieces of information about the filter H. Your task in that part is to determine the impulse response h completely.

In the subsequent parts you explore the filter h of the part (a) further.

Suppose you're given the following pieces of information of the filter H:

I) H is a causal filter

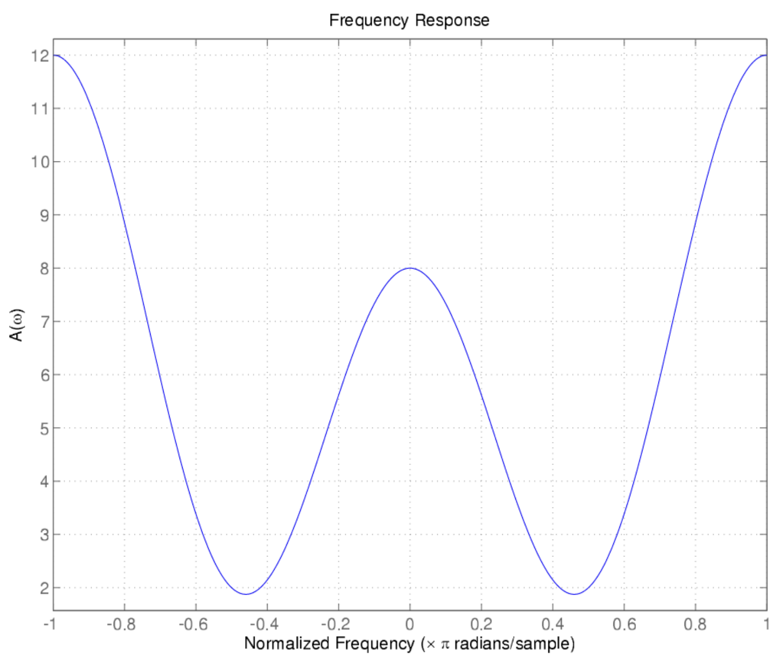
II) There exists a filter A : $[\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ having impulse response a and frequency response A, about which we know the following:

  i) $\forall \omega \in \mathbb{R}, A(e^{j\omega}) = H(e^{j\omega})e^{j2\omega}$.

  ii) $\int_{\langle 2\pi \rangle} A(e^{j\omega})\, d\omega = 12\pi$, where $\langle 2\pi \rangle = [0, 2\pi], [-\pi, \pi]$, or another continuous interval of length 2π.

  iii) $\forall \omega \in \mathbb{R}, A(e^{j\omega}) \in \mathbb{R}$

  iv) The figure below depicts $A(e^{j\omega}), \forall \omega \in [-\pi, +\pi]$.



a) Determine, and provide a well-labeled plot of, the impulse response h.

Solution: From the frequency shift property:

$$x(t - t_0) = e^{j\omega t_0} X(\omega)$$

This means that $a[n] = h[n + 2]$, since we've picked up a phase of $e^{2i\omega}$. Then, condition (iii) says that A is real-valued, and the plot for condition (iv) shows us that A is also even, meaning that the filter a is also real-valued and even.

We also know that H is a causal filter, so this means that since $a[n] = h[n+2]$ this implies that $a[n] = 0$ for all $n < -2$. Combining this with the fact that $a[n]$ is even, this implies that $a[n] = 0$ for all $n > 2$ as well.

Now, we look at the plot of $A(e^{j\omega})$ in order to figure out the values of a. Recall the synthesis equation:

$$A(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a[n]e^{-j\omega n}$$

since $a[n] = 0$ outside of $[-2, 2]$, we can restrict our interval:

$$A(e^{j\omega}) = \sum_{n=-2}^{2} a[n] e^{-j\omega n}$$

Now, since $a[n]$ is even, we only have to calculate three equations. To do this, let's first pick $\omega = 0$:

$$A(\omega = 0) = a[0] + 2a[1] + 2a[2] = 8$$

At $\omega = \pi$, we have:

$$A(\omega = \pi) = a[-2]e^{2\pi j} + a[-1]e^{\pi j} + a[0] + a[1]e^{-\pi j} + a[2]e^{-2\pi j} = a[0] - 2a[1] + 2a[2] = 12$$

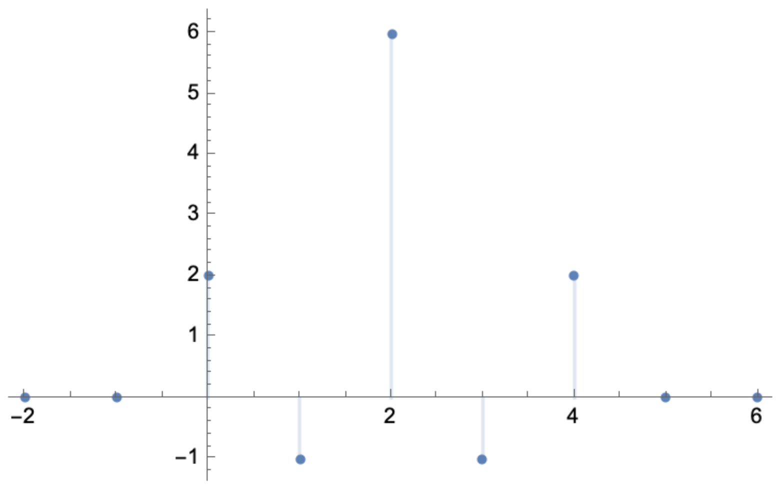
Finally, to get our third equation, we use the fact that

$$\frac{1}{2\pi} \int_{\langle 2\pi \rangle} A(\omega)\, d\omega = a[0] = \frac{12\pi}{2\pi} = 6$$

Therefore, solving for $a[1]$ and $a[2]$, we get:

$$a[1] = -1 \quad a[2] = 2$$

Therefore, with a determined, we can determine h using the fact that $a[n] = h[n+2]$. We know that since $a[-2] = a[2]$, this implies that $h[0] = h[4] = a[2]$, and $a[-1] = a[1]$ implies that $h[1] = h[3] = -1$. Finally, $h[2] = a[0] = 6$. Therefore, the plot of $h[n]$ is as follows (done in mathematica because it looks nicer than my iPad drawings):



□

---

b) Let x be the input and y the corresponding output of the FIR filter H. Determine the linear, constant-coefficient difference equation governing the input-output behavior of the filter.

Solution: Here, we just have to write $h[n]$ in terms of delta functions. With the ceofficients we have from earlier, we can conclude that:

$$h[n] = 2\delta[n] - \delta[n-1] + 6\delta[n-2] - \delta[n-3] + 2\delta[n-4]$$

To find $y[n]$, we just have to turn all the deltas into x:

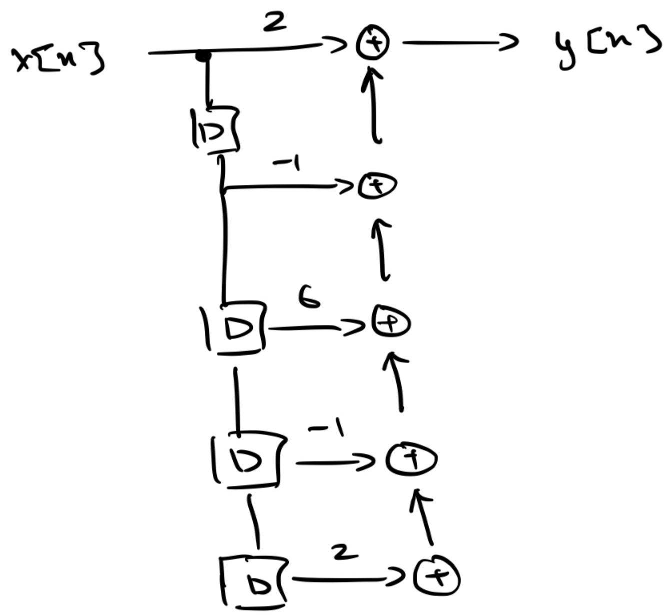
$$y[n] = 6x[n] - x[n-1] + 6x[n-2] - x[n-3] + 2x[n-4]$$

□

---

c) Show – by providing a delay-adder-gain (DAG) block diagram – how you would implement the filter using a minimal number of delay elements and scalar multiplications.

Solution: Image below:



□

d) Determine an expression each for the magnitude response $|H(e^{j\omega})|$ and phase response $\angle H(e^{j\omega})$ of the FIR filter.

Solution: First we find $H(\omega)$, using the fact that

$$H(\omega) = \sum_n h[n]e^{-j\omega n} = 2 - e^{j\omega} + 6e^{-2j\omega} - e^{-3j\omega} + 2e^{-4j\omega}$$

Factoring out $e^{-2j\omega}$, we get:

$$\begin{aligned} H(\omega) &= e^{-2j\omega}(2e^{2j\omega} - e^{j\omega} + 6 - e^{-j\omega} + 2e^{-2j\omega}) \\ &= e^{-2j\omega}(4\cos(2\omega) - 2\cos\omega + 6) \end{aligned}$$

Therefore, the magnitude of this equation is everything but the prefactor:

$$|H(e^{j\omega})| = 4\cos(2\omega) - 2\cos\omega + 6$$

The phase is just the prefactor in front:

$$\angle H(\omega) = -2\omega$$

□

e) For each of the following output signals x, determine the corresponding output signal y:

a) $\forall n, \quad x[n] = 1.$

Solution: Here we'd have $y[n] = x[n] * h[n]$, so we'd get:

$$y[n] = \sum_k x[k]h[n-k] = \sum_k h[k] = 8$$

□

b) $\forall n, \quad x[n] = (-1)^n$

Solution: Again, we'd have the same thing:

$$y[n] = \sum_k x[k]h[n-k] = \sum_k (-1)^k h[n-k] = (-1)^n \cdot 12$$

□

c) $\forall n \quad x[n] = \cos\left(\frac{\pi}{4}n\right)$

Solution: Same thing:

$$y[n] = \sum_k x[k]h[n-k] = \sum_k \cos\left(\frac{\pi}{4}n\right)h[n+k]$$

Unfortunately, I didn't have enough time to completely finish the evaluation. □