USING THE RULES OF STATISTICS TO CONFIRM THE LAWS OF GEOMETRIC OPTICS

Physics 5CL Lab 0

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Introduction

This lab involved practicing and reviewing the statistics concepts covered in the sheet, as well as remembering all of the data and error analysis techniques covered in Physics 5BL.

Contributions

The following are the contributions from each of the group members.

Eric Du

Eric completed most of the data analysis for this lab via Python (Jupyter Notebooks). He also helped with the data collection. Eric spent about **five hours** on the data analysis.

Andrew Binder

Andrew completed most of the Ti*k*Z diagrams and the writeups for the lab report. He also helped with the data collection. Andrew spent about **five hours** on the lab writeup and Ti*k*Z diagrams.

Exercise: Parallax

Here are our observations and conclusions about the exercise on parallax.

Observation: Different Distances

When we held both pencils at different distances, we noticed that the pencil which was held **closer** to our head appeared to move more than the pencil held further away from us.

Observation: Same Distance

When we held both pencils to the same distance, they appeared to move by roughly the same amount.

Experimental Design

From these observations, we can use parallax to determine the distance from the observer to the object in question. Then, if we can measure the distance in this fashion for two different objects, we can then calculate the spatial separation between them. We can do this by drawing a triangle with vertices at the two objects and the observer, and use the Law of Cosines to determine the distance between them.

Experiment 1: The Law of Reflection

This experiment is aimed at utilizing and confirming the law of reflection.

Setup and Calibration

Firstly, it's important that we calibrate our experiment so that our data is accurate and correctly reflects what we are attempting to measure and confirm.

Calibration

We calibrated the laser by placing the plane mirror onto the center of the protractor. Then, we placed the protractor so that the laser came in at an angle of $90°$.

Experimental Design

The plane mirror was attached to the metal block and placed above the paper protractor. The mirror was placed in a way such that its reflective backing was aligned against the straight line running across the protractor, to the best of our ability. We then rotated the protractor about its center and recorded incidence and reflected angles.

One of the main difficulties we encountered was making sure that the laser light reflected off the back of the plane mirror, and that this reflective layer lined up with the protractor properly. Another difficulty we had was making sure that we were rotating the protractor about its center, where the mirror was located.

Verify/Refine

Our methods were fairly precise; doing repeated measurements with the same incident angle resulted in the same reading, down to the standard precision error.

Uncertainties

Our uncertainty in the measurement of the angles were half of the smallest tick, which happened to be $1°$. Thus, our uncertainty for this experiment was $0.5°$.

This is a reasonable uncertainty to choose not only because the tick markings for the protractor we used had precision up to $1°$, but also because this accounts for the width of the laser beam itself.

Data Collection

The raw data collected is as follows:

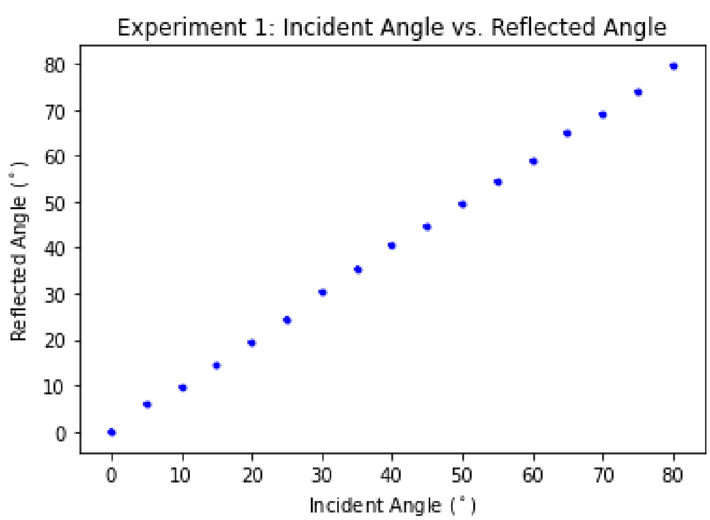
Incident Angle	Error	Reflected Angle
0		0.0
5		6.0
10		9.5
15		14.5
20		19.5
25		24.5
30		30.5
35		35.5
40		40.5
45	$\pm 0.5°$	44.5
50		49.5
55		54.5
60		59.0
65		65.0
70		69.0
75		74.0
80		79.5

Data Analysis

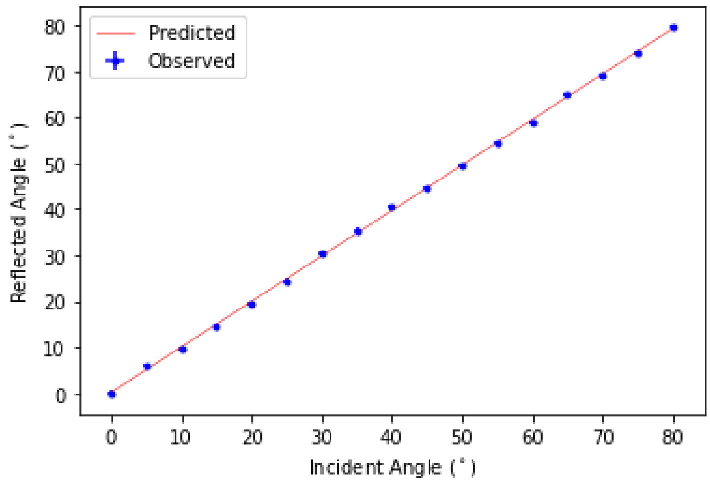
The following is the analysis of the data that we have just collected.

Scatter Plot

A scatter plot of our data is shown below:



We can see that there is a strong linear trend in the data, and applying a line of best fit:



We can see that our data fits the predicted data very well. We can also see that even given our extremely tiny error bars (barely visible on the plots), it is safe to say that at least $\frac{2}{3}$ of our observed data falls within the predicted data, and thus our predicted curve is a good model for the data.

Qualitative Assessment

Qualitatively, because our incident angles matched our reflected angles almost exactly, this suggests that the data tends to agree with the statement of the first law. Technically speaking, we had a few anachronisms with some of the angles being slightly off, this can be attributed to imperfections in the brick we used and our measurement methods, rather than the actual law being incorrect.

χ^2 Evaluation

Now comes the time to check just how good our data was in terms of estimating the errors. To perform a χ^2 evaluation, we calculate χ^2 using the following formula:

$$\chi^2 = \sum (mx_i - \theta_i)^2 = \boxed{4.205}$$

Where mx_i reflects the theoretical value, and θ_i reflects the measured value.

Now, for the reduced χ^2 (which is arguably the more useful result), we divide by the number of degrees of freedom:

$$\overline{\chi^2} = \frac{\chi^2}{2} = \boxed{2.103}$$

As we can clearly see, this value falls within our good range, which means that our experiment did a remarkably decent job of not overestimating or underestimating the errors and their impacts on our results. In other words, we did a good job, which is always something to be happy about.

Experiment 2: The Law of Refraction

This experiment essentially confirmed the law of refraction by calculating the indices of refraction and verifying that Snell's law indeed works.

Setup and Geometry

Firstly, we should understand the geometry behind what's going on and take a bit of a qualitative assessment so that the quantitative analysis makes more sense in context.

Experimental Design: Laser Alignment

To make sure that the laser is properly aligned, we moved the laser forward and backward on the optical bench and tested to see if the dot formed on the screen remained where it was. If it moved, that means that we did not properly calibrate the laser. Once it stopped moving, we had a properly-calibrated laser.

Experimental Design: Incident Angle

We started off with no angle between the laser and the bench and then gradually increased the angle by increments of $10°$ until around $80°$, at which point the angle was too steep to take accurate measurements. We measured these angles with respect to the optical bench, by aligning the protractor with the side of the pole holding up the laser. Immediately, we knew that there may be some problems, because the pole holding up the laser is circular, meaning that the protractor may rotate slightly during (and between) the measurements.

As for the distances between the beams, we simply measured the distances between the dots formed on the screen with a ruler. There were also potential issues here, because the angle may not accurately reflect the distance, and the dots themselves had a thickness that could affect the results that we got for the distances between them. Similarly, one of the dots got very faint at certain points in the angular increments, making it difficult to accurately measure the distance. Lastly, we needed to make sure that the screen at the back was perpendicular to the optical bench, which was also a bit challenging to maintain, as it got shifted while we were measuring the angles and distances.

One slight complication is that instead of measuring the distance of the reflected rays of light (due to partial reflection), we instead measured the distance of the two dots that were formed on the image screen. We will discuss how to compensate for this in our analysis.

However, overall, there was not too much difficulty with taking these measurements.

Verify/Refine

Once we actually started the measurements, by that point, our errors were not too big, and we were not able to decrease the uncertainty past the precision error. To minimize shifting during the measurement process, we secured the protractor to the optical bench, and when we took the angles, we brought it up so that it was better aligned with the laser. This gave us an even more precise angular measurement, though it was still not all that precise.

Data Collection: Thickness

The thickness of the block was 2.79 ± 0.05 cm. We ran into no issues while measuring the block's thickness. This means that our $\alpha_t = 0.05$.

Data Collection: Table Construction

Here is our table of values:

Incident Angle	Distance (cm)	Distance (reflected)
10	0.6	0.6
20	0.7	0.7
30	1	1
40	1.3	1.4
50	1.5	1.5
60	1.45	1.45
70	1.1	1.1
80	0.6	0.6
Additional Measurements		
Distance Between Laser & Block	29.9 cm	
Distance Between Block & Sheet	24.2 cm	
Width of Block	2.79 cm	

Analysis

As mentioned in our experimental design, we instead measured the distance between the two dots formed on the screen. In order to correct for this, we divide the distances we got by an overall $\cos\theta$ to compensate. This is due to the overall way we measured the distance of the points on screen, since we did not rotate the screen while rotating the block.

An Agreement Test

Now, we perform an agreement test to see if our data agrees with what we should expect in theory.

Determining n

Starting off, we want to determine our index of refraction using what we found in parts 1D and 1E from Prelab 0. Those were, respectively:

$$n = \frac{2\sin\theta\sqrt{\frac{y^2}{4}+t^2}}{y}$$
$$\alpha_n = \frac{1}{z}\sqrt{\frac{x^2}{z^2}\alpha_z^2+\alpha_x^2}$$

Now, we simply plug in the values we obtained from our experiment (using Python):

$$n =$$
$$\alpha_n =$$

Mean, Standard Deviation, and Standard Error

Now that we computed our index of refraction with error, we can proceed with computing the mean, standard deviation, and standard error:

$$\overline{n} = 2.05$$
$$\sigma_n = 0.3$$
$$\alpha_n = 0.1$$

Scatter Plot

With these values, here is the scatter plot we get when we plot n_i vs θ_i with error bars:

Agreement Test

With all tools in place, we can finally perform our agreement test:

As we can see, our data $\boxed{\text{fails the agreement test}}$. This most likely happened due to an error in our experimental procedure. In other words, we weren't entirely consistent with our measurements, and we may have underestimated the impact that those anachronisms may have had on our results and the corresponding errors of said measurements.

Experiment 3: The Focal Length of a Converging Lens

In this experiment, we measured and calculated the focal length of a converging lens.

Qualitatively Identifying Real and Virtual Images

Firstly, it's important that we qualitatively assess what is going on, so that our quantitative measurements make more sense in context.

Observation: Real Image

When we looked into the lens, and we observed that the image appeared to be in front of the lens as it was generally larger than the original F-object. Intuitively this also makes sense, since a real image is formed, and this image is formed at a distance closer to the eye, making it appear larger.

As we moved closer to the lens, we gradually observed the image get larger and larger until it completely disappeared, then appeared inverted. This is a result of us moving through the focal point of the lens, where objects are never focused.

Experimental Design

In order to make sure that we got the crispest image possible, we moved the screen both toward and away from the converging lens. Once movement in both directions gave us a fuzzy image, this means that we've found the optimal spot. Our image that formed was a flipped F, which is exactly what we expect from our standard ray-tracing diagrams. To accurately get a crisp image, we recommend to slowly adjust the object/lens until the lines that make up the F are as bright and thin as possible. As soon as the lines start to smudge, we know we've passed the focal point, and if the lines are still getting crisper the more we move, we know we haven't gotten to the focal point yet. And, as always, the best way to minimize our error is to take many measurements that will then begin to average out to a more exact and precise location.

Verify/Refine

Since we simply moved the screen and the lens along the optical bench, our precision was limited to the precision of the ruler markings on the optical bench, which corresponds to a precision error of 0.05 cm. In principle, if we wanted even better results, we could attempt to use some other measuring stick with more precise markings, but we didn't have such equipment available, so this was really the best we could do.

However, the other potential source of error is something we could probably fix, which is the consistency of the crispness of the image. In other words, the image we found may not have been the crispest possible image, meaning that we may not necessarily be measuring the actual focal length, but rather something just shy or just over the actual length. We could rectify this by taking multiple measurements so that it averages out to around what the actual focal length is. We took a few additional trials to make sure that we had the best possible data with our available tools and measurement instruments.

Observation: Virtual Image

As we moved the lens and the object around on the optical bench to try to get a clear image, we kept finding that the light just smeared on the screen and gave us a smudge rather than a nice crisp F. This is because the incoming rays diverged, since we were past the focal point and the light was no longer able to focus because there was simply not enough distance for it to do so.

Experimental Design

We used parallax. We know that the virtual image must be behind the actual lens, since we observe the virtual image to move less than the lens. Given the ratio, we can estimate roughly where the virtual image is relative to the lens.

Verify/Refine

Our method obviously contains a lot of room for error, especially with the creation of the virtual image. The biggest source of error, as always, comes from the precision error of the actual optical bench, as well as human error in predicting where it is, since we are essentially eyeballing. We can rectify this by taking a lot of measurements so that our data averages out to the expected solution.

Images

Measurements

The dimensions of our F-object were $\boxed{2.20 \pm 0.05\text{cm in height and } 1.20 \pm 0.05\text{cm in width}}$. We forgot to take the additional measurements, but we didn't really use them in our experiment data collection and analysis, so in the end, everything worked out just fine.

Data Collection

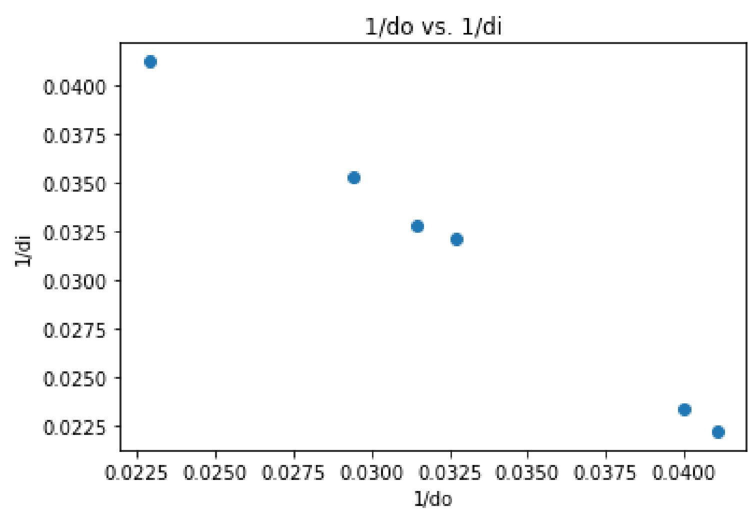
The data we obtained is as follows; all lengths here are measured in centimeters:

Object Position	**Image Position**	**Object Height**	**Image Height**	**Lens Position**
21.3	89.25	1.1	2.2	65
21.3	89.2		3.3	46.3
20.9	83.2		1.9	52.7
20.9	90.3		3.2	45.25
14.8	76.5		2.3	45.35
14.8	77.1		2	48.8

Performing a Linear Fit

Covariance and Coefficient of Linear Correlation

We perform a linear fit on the following scatter plot:



Doing so gives a slope of -1.06 ± 0.04. As our expected value for the slope is -1, this result indicates that our values support the linear hypothesis.

Calculating Error

To calculate error, we use the equation that was given in the prelab:

$$\alpha_{y,equiv,i} = \sqrt{\alpha_x^2 + \alpha_y^2}$$

In our case, we have

$$\alpha_x = \frac{\alpha_d}{d^2}$$

For both d_i and d_o. We computed this in python, which yielded an average value of 0.001 cm.

Weighted Least-Squares

We use the same formulas that we derived in the prelab. Doing so gives us $b = 0.06$ and $\alpha_b = 0.001$

Statistical Test

We use the following statistical test:

$$\frac{|b_1 - b_2|}{2\sqrt{\alpha_{b_1}^2 + \alpha_{b_2}^2}} < 1$$

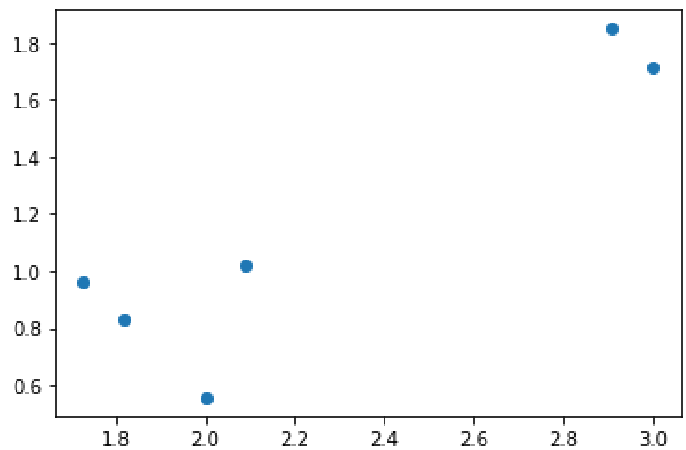
Performing this test for the value of b we calculated in the previous step with the value of b we obtained from the curve fit, we can see that they match.

Best-Fit Focal Length

Since our values passed the agreement test, it makes sense to take the mean of the two values of b and use that as our singular value. Since we know that $b = \frac{1}{f}$, we can take the reciprocal of b in order to get the focal length. Doing this gives us that the focal length is 15.34 cm.

More Statistical Tests

When we plot $\frac{h_i}{h_o}$ versus $\frac{d_i}{d_o}$ our magnification equation tells us that we expect a linear relationship between the two values. However, from the plot below, we can hardly say that the relationship we obtained is linear. Therefore, our data does not seem to support this relationship very strongly.



Experiment 4: Index of Refraction Redux

For this part, we designed our own experiment with accompanying theory to provide a different perspective on experiment 2.

Theory: Total Internal Reflection

In this version, we decided to utilize total internal reflection. This occurs at a critical angle θ_c when none of the incident light is refracted through the medium and all of it is reflected. This critical angle can then be used to calculate the index of refraction of the medium we are using via Snell's Law.

Experimental Design

To determine the critical angle for total internal reflection, we start with the configuration we had in experiment 2. Now, we slowly rotate the block until the refracted dot (the second brightest dot) disappears from the screen. At this moment, we know that total internal reflection has occurred. We then measure the angle we have rotated the block to obtain our critical angle.

Experimental Procedure

Once the critical angle was found where the refracted dot disappears, we took 5 independent measurements for the critical angle. To ensure that our measurements were independent of each other, we made sure to alternate turns for our measurement.

Data Collection

The data we collected for the critical angle is as follows:

Trial	**Critical Angle**
1	53
2	51.5
3	51
4	52
5	51.5
6	51.5

Analysis

Due to the way we measured the critical angle, we in fact need to take $90° - \theta_c$ to find the true critical angle using Snell's law. This is because we measured the angle we rotated the block, and not the angle the laser makes with the normal of the surface. Snell's law also states that for total internal reflection:

$$n_{glass} = \frac{n_2}{\sin\theta_c} = \frac{1}{\sin\theta_c}$$

Thus, computing these values, we obtain an average value of $n_{glass} = 1.615 \pm 0.08$. While this means that our experiment fails the agreement test assuming that $n_{block} = 1.53$, this isn't such a bad result, since nobody is certain about the real value for n_{glass}.

Conclusion

Overall, this was a nice jump back into experimental physics. It's definitely harder than 5BL, since we have to take more of an active role in designing and carrying out the experiments, but it is definitely an educational experience. As a suggestion, though, we hope that the lab documents are updated in the future with the proper values for some of the physical quantities, because it would be nice for us to easily be able to cross-reference our calculated values with the given expected values. Other than that, we're excited for future labs!