Physics 5A Homework

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1 Question 1

Label the velocity of Huck relative to the raft to be $v_{H,R}$ and the velocity of the current to be $v_{R,G}$, it follows that:

$$\vec{v}_{H,R} + \vec{V}_{R,G} = \vec{v}_{H,G}$$

Since Huck's velocity and the river current is perpendicular to each other, we can just use pythagorean theorem:

$$|\vec{v}_{H,G} = \sqrt{(0.70)^2 + (1.5)^2} = 1.66m/s, \tan^{-1}\left(\frac{1.5}{0.7}\right) = 64.98^\circ$$

So therefore his velocity is 1.66 m/s [N 64.98° E], if we can assume that upwards points in the north direction.

2 Question 2

Given that she angles the boat at an angle θ to the vertical, we have her velocity in the $\hat{\jmath}$ direction to be $v_b \cos\theta$. That means that the time for her to cross the river is $t_{cross} = \dfrac{d}{v_b \cos\theta}$.

We also have that her velocity in the $\hat{\imath}$ direction is going to be equal to $v_c - v_b \sin\theta$ the time that it takes her to run to her desintation after crossing the river is going to be: $t_r = v_r \cdot t_{cross}(v_c - v_b \sin\theta)$

If we sum both of these up, we get:

$$T = t_r + t_{cross} = \frac{d}{v_b \cos\theta} + \frac{d}{v_r v_b \cos\theta}(v_c - v_b \sin\theta)$$

We now take $\frac{dT}{d\theta}$:

$$\frac{dT}{d\theta} = \frac{d}{v_b}\left[\sec\theta\tan\theta(1 + v_r(v_c - v_b\sin\theta)) + \sec\theta \cdot -v_r v_b \cos\theta\right]$$

After much algebra, we get the equation:

$$\sin\theta = \left(\frac{\frac{v_b}{v_r}}{1 + \frac{v_c}{v_r}}\right)$$

Giving a value of $\theta = 24.9^\circ$.

3 Question 3

3.1 Part a

Let $\vec{r}_a$ be the displacement vector from the origin of frame A to a paticle in space, and let $\vec{r}_b$ denote the same thing except with frame B. We can also connect the origins of the two frames (going from frame A to B) together with a vector $\vec{r}_c$.

From the diagram, we an see that:

$$\vec{r}_c + \vec{r}_b = \vec{r}_a$$
$$\therefore \dot{\vec{r}}_c + \dot{\vec{r}}_b = \dot{\vec{r}}_a$$

3.2 Part b

We can write $\vec{r}_{ab} = \vec{r}_a - \vec{r}_b$, so $\dot{\vec{r}}_{ab} = \dot{\vec{r}}_a - \dot{\vec{r}}_b$. Substituting in the cartesian coordinates:

$$\dot{\vec{r}}_{ab} = r\omega(\cos\omega t\hat{\jmath} - \sin\omega t\hat{\imath}) - r\omega(-\sin\omega t\hat{\imath} + \cos\omega t\hat{\jmath})$$

Note that we need to factor the second equation by a factor of -1 because they have opposite values for ω. Looking at the equation, we also see that the $r\omega\cos\omega t\hat{\jmath}$ terms cancel each other. This should make sense since they are falling down at the same rate. Therefore, we get:

$$\dot{\vec{r}}_{ab} = 2r\omega\sin(\omega t)\hat{\imath}$$

4 Question 4

For the whole system: $a = \dfrac{F}{m_1 + m_2}$. If we consider the two red normal vectors labelled N in the diagram, and we consider the free body diagram on m_2 only:

$$N = m_2 a_2$$

We also have the constraint (given the axes in the diagram):

$$x_2 - x_1 = \text{const.} \implies \ddot{x}_2 = \ddot{x}_1$$

So from this we can conclude that $a_2 = a$, so we can simply substitue a_2 for that to get our contact force:

$$N = \frac{m_2 F}{m_1 + m_2} = 1 \text{ N}$$

5 Question 5

Variables and their directions are defined as shown in the diagram.

For m_2, we have $m_2 g - T = m_2\ddot{x}_2$, and we also have $T = m_1\ddot{x}_1$ for m_1. To find the constraint, use the following:

$$x_1 - x_2 + \frac{\pi R}{2} = l_{rope} \implies \ddot{x}_1 = \ddot{x}_2$$

So we now combine the two equations and solve, by cancelling T. From here, I will be removing the indices on $\ddot{x}$ terms since we've already proven that they are the same.

$$m_2 g - m_1\ddot{x} = m_2\ddot{x}$$
$$\therefore \ddot{x} = \frac{m_2 g}{m_1 + m_2}$$

6 Question 6

For m_A and m_B, we have the following two equations:

$$T - m_A g \sin\theta_A = m_A \ddot{x}_A$$
$$m_B g \sin\theta_B - T = m_B \ddot{x}_B$$

To find the constraint, we again find a way to express the length of the rope to be constant. Namely, we can use the following:

$$x_A - x_B + K = l_{rope} \implies \ddot{x}_A = \ddot{x}_B$$

Here I use K to denote the amount of spring that is being passed thorugh the rope (analogous to πR in some of the previous questions). Since it's a constant, I really don't care enough about K to calculate an exact value.

So now we can put the two equations together again, by eliminating T. Again, I remove indices on $\ddot{x}$ since we've proven that accelerations are the same for both blocks.

$$T = m_A\ddot{x} + m_A g \sin\theta_A$$
$$m_B\ddot{x} =\therefore m_B g \sin\theta_B - m_A\ddot{x} - m_A g \sin\theta_A$$
$$\therefore \ddot{x} = \boxed{\frac{g(m_B \sin\theta_B - m_A \sin\theta_A)}{m_A + m_B}}$$

7 Question 7

We have the general equation for $\ddot{\vec{r}}$ in polar coordinates:

$$\ddot{\vec{r}}(t) = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

Since we're talking about uniform circular motion, we can eliminate a lot of terms. So $\ddot{r} = 0$, $\dot{r} = 0$, $\ddot{\theta} = 0$. So we're left with:

$$\ddot{\vec{r}}(t) = (-r\dot{\theta}^2)\hat{r}$$

Applying Newton's second law:

$$F = -mr\dot{\theta}^2$$

If we want the concrete to not stick, then we equivalently want that at the highest point, the centripetal force F must only be equal to the gravitational force at that time, or mg. So if we set those two equal:

$$mg = -mr\dot{\theta}^2 \implies \dot{\theta} = \sqrt{\frac{g}{r}}$$

This evaluates to roughly 1.4 rad/s.

8 Problem 8

We have the equations for Newtons' second law on both the masses:

$$\begin{aligned} m_1 g - T_1 &= m_1 \ddot{x}_1 \\ T_2 - m_2 g &= m_2 \ddot{x}_2 \end{aligned}$$

If we look at the free body diagram of the moving pulley, we can deduce that:

$$T_1 - 2T_2 = m_p \ddot{x}_p$$

And since $m_p = 0$, this directly implies that $T_1 = 2T_2$.

The difficult part of the problem is coming up with the constraint. Again, refer to the diagram for the variables that I will be using here. For the first mass, we have:

$$h_2 - h_1 - x_1 + \pi R = l_{rope1} \implies |\ddot{x}_1| = |\ddot{h}_2|$$

For the second mass:

$$\pi R + (H - h_2) + h_3 - h_2 = l_{rope2} \implies |\ddot{h}_3| = 2|\ddot{h}_2| = 2|\ddot{x}_1|$$

> **Remark 8.1.** Due to the fact that I have two axes, I treat everything going in one direction (counterclockwise) as positive, which allows me to omit the negative signs that might come as a result of using one axis and treating $\ddot{x}_1$ as a vector. This is mmmwhy I'm able to write $\ddot{x}_2 = 2\ddot{x}_1$ instead of $\ddot{x}_2 = -2\ddot{x}_1$.

From the diagram, we can also see that $\ddot{h}_3 = \ddot{x}_2$, since the movement of x_2 is accounted for in h_3. So finally we conclude that $\ddot{x}_1 = 2\ddot{x}_2$. We can now move to solving the equation, which from here is just a lot of algebra:

$$\begin{aligned} m_1 g - 2T_2 &= m_1 \ddot{x}_1 \\ T_2 - m_2 g &= m_2(2\ddot{x}_1) \longrightarrow 2T_2 - 2m_2 g = 4m_2\ddot{x}_1 \\ \therefore m_1 g - (4m_2\ddot{x}_1 + 2m_2 g) &= m_1\ddot{x}_1 \\ \therefore \ddot{x}_1 &= \frac{g(m_1 - 2m_2)}{m_1 + 4m_2} \end{aligned}$$

9 Problem 9

We have the equations:

$$m_A\ddot{\vec{r}}_A = -T = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$
$$m_B\ddot{\vec{r}}_B = -T = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

We also have that $\vec{r}_A - \vec{r}_B = l_{rope}$, meaning that $\ddot{\vec{r}}_A = \ddot{\vec{r}}_B$. Notice also that the accelerations here are all in the radial direction, so we can dot both equations by $\hat{r}$:

$$m_A\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r}$$
$$m_B\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r}$$

10 Problem 10

Since we want m_3 to be stationary, then we must also have that $\ddot{x}_3 = 0$. Looking at the free body diagram for m_3:

$$m_3 g - T = m_3\ddot{x}_3 \implies T = m_3 g$$

We also have $m_2\ddot{x}_2 = T$ so $\ddot{x}_2 = \frac{m_3 g}{m_2}$ once we substitute the previous conclusion. If we want m_3 to not move, then it also means that the acceleration of the whole system should be equal to the acceleration of m_2. What this means is that $\ddot{x}_1 = \ddot{x}_2$. If we consider the whole thing as a single free body diagram:

$$F = (m_1 + m_2 + m_3)\ddot{x}_1 \implies \boxed{F = (m_1 + m_2 + m_3) \cdot \frac{m_3 g}{m_2}}$$