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# Urban Growth Modeling with Road Network Expansion and Land Use Development

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## Abstract

Land use and transportation systems are considered as two most important subsystems determining urban form and structure, and are assumed to mutually influence each other overtime. To better understand the relationship between them, we build a simple dynamic model to simulate long-term urban growth instead of a static one. Our urban simulation combines vector road network growth with grid land use dynamics. Vector model has advantages in topological analysis and we use space syntax metrics to control road network growth by estimating road traffic flow, which is also adopted to calculated accessibility for land use simulation. The land use model includes five land use categories and two behavior sub-models: mobility model and location choice model. Our preliminary simulation results show similar land use patterns of how real cities grow.

## 1- Introduction

Urban is a complex system, which consists of various interactive subsystems and is influenced by a variety of variables, such as governmental land policies, population growth, transportation infrastructure, market behavior, etc. Modeling and simulation is regarded as an important tool to study urban issues with numerous applications including mapping and visualization, urban planning, emergency response, entertainment and etc. (Vanegas et al. 2009 a).

Among nine types of major urban subsystems (Wegener 2004), perhaps land use changes and transportation system are the two most important

ones determining urban form and structure in the long term. Transportation networks and development of land use are assumed to mutually influence each other over time. Construction of a new road link or enlargement of an existing network would influence firm/household location choice, real estate development, land development and building density. At the same time land use changes in turn influence the demand for travel or the accessibility (Kelly 1994, Iacono et al. 2008).

Three main traditional methods to study the reciprocal relationship between land use and transportation are surveys, empirical studies and mathematical models (Wegener, 2004). In recent years some microsimulation models are introduced, such as activity-based travel model, cell-automata model, and multi-agent model, using “bottom up” modeling frameworks. Many models have been developed, for exmaples, Ramblas (Veldhuisen et al. 2000), ILUMASS (Moeckel et al. 2003, Wagner and Wegener 2007), UrbanSim (Waddell et al. 2003, 2007), ILUTE (Miller et al. 2004), PUMA (Ettema et al. 2005). They use transportation networks to calculate travel times and costs, traffic assignment, congestion levels and etc. when simulating residents’ activity and travel patterns, firms’ location, flow of goods and services in the market, etc. While for some other models, such as ABLOoM (Otter et al. 2001), CityDev (Semboloni 2005), and Kou’s artificial urban (Kou et al. 2008), transportation is simply used as a road network to calculate the accessibility for different agents. When we intend to simulate long-term urban growth, one obvious disadvantage in these models is that transportation networks are assumed to be static.

Currently a variety of methods have been developed to generate road networks. In geometric urban simulation, generating an underlying road network is critical to urban reconstruction. Parish and Muller (2001) used a procedural approach based on L-systems and road network templates to create a street map. Similar works were modified by Kelly and McCabe (2007) and Weber et al. (2009). Besides, Vanegas et al. (2009 b) generated a set of seeds, which are converted to intersections of the arterial road network, and then constructs arterial road segments. Sun et al. (2002) used a template-based model to generate a virtual traffic network. Esch et al. (2007) and Chen et al. (2008) adopted user-guided tensor fields as the input to generate street. Yamins (2002) used FindMax procedure to get a pair of points which there is the greatest need for new transport structure, and then builds a road between the points with least-cost function. Jiang (2007) used a transition potential map to select road nodes. Route is generated with minimum elevation difference or highest transition potential. A very different approach to road generation is the use of an agent based model.

Lechner et al. (2004, 2006) and Watson (2007) applied extender and connector agents for the generation of tertiary and primary road networks.

Most of land use models use static transportation networks, while most road generation methods use existing population density maps or current land use maps as input data. Because we intend to build a long term urban growth model, our idea is to combine two dynamics together to observe and study the relationship between land use change and transportation.

Our urban growth model mainly consists of two parts: transportation network growth and land use changes. The major contributions of our works are:

1. We generate a dynamic urban model, instead of creating a static one.
2. We propose an urban simulation combining vector road network and grid land use. Vector model has advantages in topological analysis, especially for road network, while grid model is easy to visualize land cover changes and to hatch agents which could make individual choices and build behavioral sub-models.
3. Space syntax metrics are introduced for road network generation by selecting important roads according to predicted traffic flow, although these metrics have been widely used in network analysis and generalization.

The paper is organized as follows. Section 2 introduces road growth method. Section 3 describes traffic flow computation, which is used to select road segments and calculate accessibility. Section 4 is the land use simulation. The implications of the experiments are discussed in section 5 before the paper draws to a conclusion in section 6.

## **2- Road growth**

Our Road network mainly consists of two types of roads: major roads and minor roads. Minor roads provide access to the major road network and service the local areas. Roads can be regarded as multiple agents at micro-scale, in which every road interacts with each other to form a connected network. We simply use two types of agents to generate road network. One is node; the other is road segment, which links two nodes. Road segments form individual roads according rules, which will be discussed later. Both major and minor road networks have the same data structure. Each node

has an attribute to distinguish major node from minor node. And road segments have attributes to tell which individual roads they belong to.

Our road growth strategy is based on several previous works (e.g., Parish and Muller 2001, Kelly and McCabe 2007), with two differences.

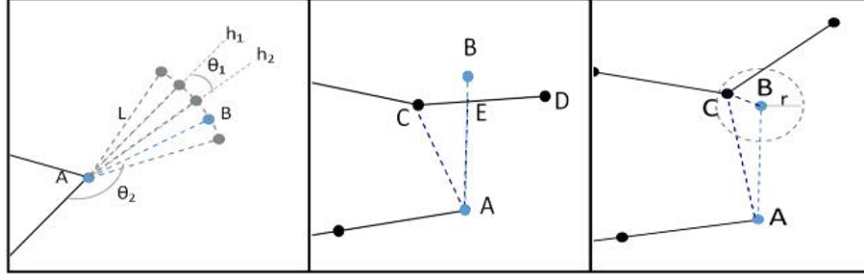
1. We don't use static population density map as input data, which they use and thus road networks in their paper are not generated in the same order as real city.
2. We add a traffic model to control road growth by selecting necessary road segments.

Major roads are generated by starting from user defined city center or existing nodes. The heading  $h_1$  for a start node  $A$  is created firstly. Angle of deviation  $\theta_1$  is decided by road patterns. The node  $A$  shoots a number of rays radically with a preset move step length  $L$ . Along each ray, elevation data is extracted. The direction with minimum elevation difference is chosen for continuing the growth (for example: segment  $AB$ ). Calculate the angle between proposed segment (blue dash line in Figure 1, left) and existing segments (black solid lines) started from or ended to the node  $A$ . Choose the minimal angle  $\theta_2$  and make sure it meets the threshold.

Parameters could be used to control road pattern. Road patterns mainly consist of strokes, grids (raster), stars (radial and optional rings) and irregulars. Strokes are basic elements in grids and stars patterns (Marshall 2005, Heinze et al. 2007). We implemented three different road patterns (i.e. grid, radial, and organic). Major road network applies radial and organic patterns, while minor road network applies a grid-like pattern, which simply means angle  $\theta_2$  closes to 90 and no more than 4 segments are allowed to own the same node. Other parameters include move length  $L$  and density of intersection nodes.

Similar to local constraints of Parish's paper (2001) and legality tests of Weber's paper (2009), proposed road segments should meet the following constraints: 1) Make sure proposed segment ends inside a legal area. If not, rotate the segment up to a maximal angle. Some areas, such as river, could be cross up to a specified length. 2) Check the relationship between proposed segment and existing nodes and segments. If this proposed segment intersects with an existing segment, generate a new intersection node  $E$  (Figure 1 Middle). A new segment  $AE$  or  $AC$  (only when segment  $EC$  is too short) is chosen. If the end node of the proposed segment closes to an existing node or segment, extend it to the existing node or segment (Figure 1

Right). If angles between segments are too small or segment  $BC$  is too short, we choose segment  $AC$  instead of segments  $AB + BC$ .



**Figure 1:** Left: Road growth from node A. Middle: Intersection test. Right: Snapping test

After constraints are executed for proposed road segments, we need to compute traffic flow for new segments, especially for major roads which apply organic pattern. The new road segments with high traffic flow would be confirmed as major roads.

### 3- Traffic computation

Traffic model is built to select major roads and prove travel demands for land use simulation. Weber et al. (2009) pointed out that existing traffic demand models have some disadvantages. These models are too complex, need many external variables and data, and work with static cities. In order to compute traffic flow over time to show the evolution of major road network, they presented a simplified stochastic model which uses sampling to distribute a discrete number of trips in the road network.

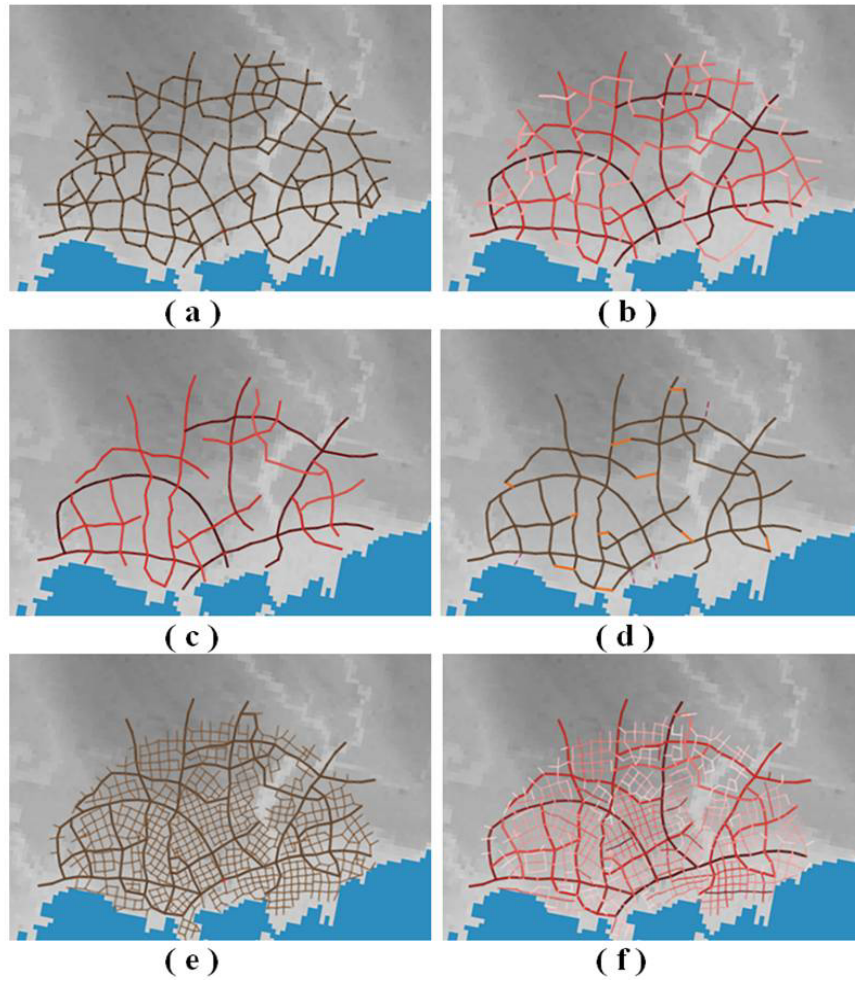
In this paper we propose a new method by using space syntax metrics to select important roads according to predicted traffic flow. In space syntax community, integration is a key measure, and can be measured at both local and global levels. For a node  $A$  in a space syntax graph, firstly we calculate the shortest distance from others to node  $A$ . Here the distance is the length of the graph geodesic between two nodes and the maximum shortest distance is denoted by  $md$ . Then we count the number of nodes  $N_d$  with the distance  $d$ . Finally we get the following expression,  $\sum_{d=1}^m d \times N_d$ . If  $m = 1$ , we get the connectivity (degree) of node  $A$ . if  $m = md$ , it represents global integration, and if  $1 < m < md$ , it is a local integration expression.

Local integration is the default measure of space syntax for traffic flow prediction. Usually we adopt two steps for the calculation, i.e.  $m$  is equal to 2.

According to Jiang et al. (2007, 2008), point-based metrics assigned by summing into individual roads tend to have a much better correlation with traffic flow than line-based metric. Thus, we use node's local integration to estimate the traffic flows. Although this assumption might not hold for precise predictions, they allow for displaying the general spatial distribution of traffic flows in a road network.

First of all, we join some road segments into individual roads following a self-organized process based on three different join principles: every-best-fit, self-best-fit and self-fit. Jiang et al. (2008) pointed out self-best-fit principle is the best option for metric-flow correlation, and we use self-best-fit principle which means each segment only considers itself to find a best fit of its neighboring segments. Then we derive characteristic points from existing nodes to draw visibility graph, which shows how each characteristic point is visible to others (Jiang and Claramunt 2002). Angle deviation of two adjacent road segments and maximum distance a node could see should be considered. Thus connectivity values of characteristic points are derived. After computing each node's local integration, we sum them into individual roads to describe the road traffic flow (Figure 2, b).

We choose roads with high local integrations as major roads (Figure 2, c). A threshold is defined as the percentage of the maximum local integration. Confirmed major roads need to be modified to connect each other well as a network. We create a new link if one road end is near to another road (orange line in Figure 2, d). If an end part of one individual road, which is divided by an intersection node, is too short, we erase it or degrade it to a minor road (magenta dash line in Fig. 2, d). In the area serviced or surround by major roads, minor roads are created with a grid-like pattern (Figure 2, e). We also calculate the local integration of the whole road network (Figure 2, f), and find that proposed major roads generally maintain the framework of the whole road network according to the traffic flow, and some minor roads near city center also show heavy traffic flow. This phenomenon is quite obvious and common in a real road network.



**Figure 2:** (a) is initial high density major road network. Background is water area and elevation map. (b) shows local integration of each individual road, dark red means high value while light is low one. (c) is major roads after selection. Modification is adopted in (d). (e) displays the whole road network. Light brown roads are minor roads. (f) shows local integration of the whole road network

#### 4- Land use simulation

Land use type determines demography distribution and influence travel demand, besides different land use types have different road patterns.



There are various types of land use that could be found in a city. We currently define five land development categories: residential (high-density and low-density), commercial (community-oriented and highway-oriented), industrial (heavy and light), transportation (road development) and parks (parks and forests), since they make up the majority of land use in most cities. Residential, commercial and industrial areas are together result in roughly 60% of urban land use (8: 1: 2). Percentage of area for transportation is 25% - 30%. And it is well know that the percentage of each category changes a bit at different stages of urban development.

#### 4.1 Mobility model

Our mobility algorithm moves a subset of land developer agents to new places in a period. Size of the subset is  $S_t = \alpha D_t$  at time  $t$ , where  $\alpha$  is a small fraction and  $D_t$  is a set of developed land area, and it could be residential, commercial or industrial land use category.  $S_t$  is chosen at random, placed in a waiting list  $L_t$  and removed from  $D_t$ .  $S_t \cup L_t \rightarrow L_{t+1}$ . It is rather similar to employment mobility model and household mobility model of UrbanSim (Waddell et al. 2003, 2007), but we mainly focus on land use types. As the urban growth, there are more people and new lands are developed as urban area. We simply add these new demands as developer agents into the waiting list  $L_t$ .

#### 4.2 location choice model

For each developer agent  $a \in S$ , we choose a small random subset  $T$  of vacant places to be located.  $T = V - D$ , where  $V$  is the set of areas served by roads. The probability of location  $l(i, j) \in T$  to be selected is equal to the probability that the utility value at this location is greater than any other candidate locations (Li and Liu 2008).

$P(a, l) = \exp(U(a, l)) * \text{Con}(l = \text{suitable}) / \sum \exp(U(a, l'))$ ,  $l, l' \in T$ .  $\text{Con}()$  is a constraint function, for example, some locations are forbidden to be transferred.

Utility calculation depends on the various attributes. Transportation area could be easily derived from road network. Road width depends on road type and its traffic flow. Big parks often appear in rough terrain or flood

plains. As far as the location of residential, commercial, and industrial development, many attributes would influence the decision. 1) Distance to center. Commercial developers prefer to city center, while industrial developers are opposite. 2) Terrain, including proximity to water and slope. Commercial and some industrial developers like to near water. Commercial developers prefer flat land. 3) Accessibility. Factories need convenient transportation so they are located near major road. Commercial area always has a higher accessibility than residential area. 4) Clustering. For example, residential developers intend to choose land close to existing residential area, because it reduces investment and development risk. 5) Neighborhood. Industrial development should far away from residential area, while commercial development should near residential area and market.

For residential agents,

$$U_r = \beta_{Dw}x_{Dw} + \beta_Ax_A + \beta_{Nr}x_{Nr} + \beta_{Ni}(1 - x_{Ni})$$

For commercial agents,

$$U_c = \beta_{Dc}x_{Dc} + \beta_{Dw}x_{Dw} + \beta_Ax_A + \beta_{Nc}x_{Nc} + \beta_{Nr}x_{Nr}$$

For industrial agents,

$$U_i = \beta_Tx_T + \beta_{Dw}x_{Dw} + \beta_{Apr}x_{Apr} + \beta_{Ni}x_{Ni} + \beta_{Nr}(1 - x_{Nr}) + \beta_{Nc}(1 - x_{Nc})$$

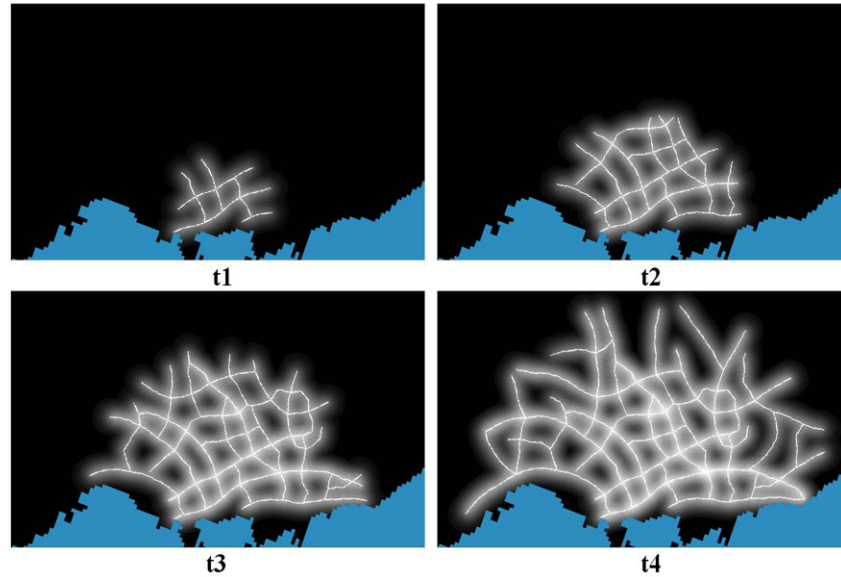
where  $\beta$  represents relative importance weight for each attribute.  $x_{Dw}$  and  $x_{Dc}$  represent proximity to water and city center.  $x_D = \exp(-\beta d)$ ,  $d$  is Euclidean distance.  $x_T$  represents the terrain slope, i.e. flatness.  $x_N$  represents the clustering and neighborhood influence of each land use type, which is simply calculated as the percentage of developed grids which have the same land use type in the neighborhood.  $x_{Na} = \sum N(landuse = a) / (\sum N - 1)$ , where  $a$  could be  $r$ ,  $c$  or  $i$ .

$x_A$  represents the accessibility.  $x_A = \gamma_{Apr}x_{Apr} + \gamma_{Asr}x_{Asr}$ .  $x_{Asr}$  is the accessibility to minor road network,  $x_{Asr} = \exp(-\gamma d)$ , and  $x_{Apr}$  is the accessibility to major road network. The novelty here is that we not only calculate the nearest road, but other roads in a threshold distance. Besides we use the estimated traffic flow as a weight parameter. It is clear that a place has a better accessibility if it could easily access to more important roads.

$x_{Apr} = \sum (1 - \exp(-\gamma' LI)) * \exp(-\gamma'' d)$ , LI is the value of local integration of the road being calculated. Fig. 3 shows the dynamics of accessibility maps.

## 5- Experiments and discussion

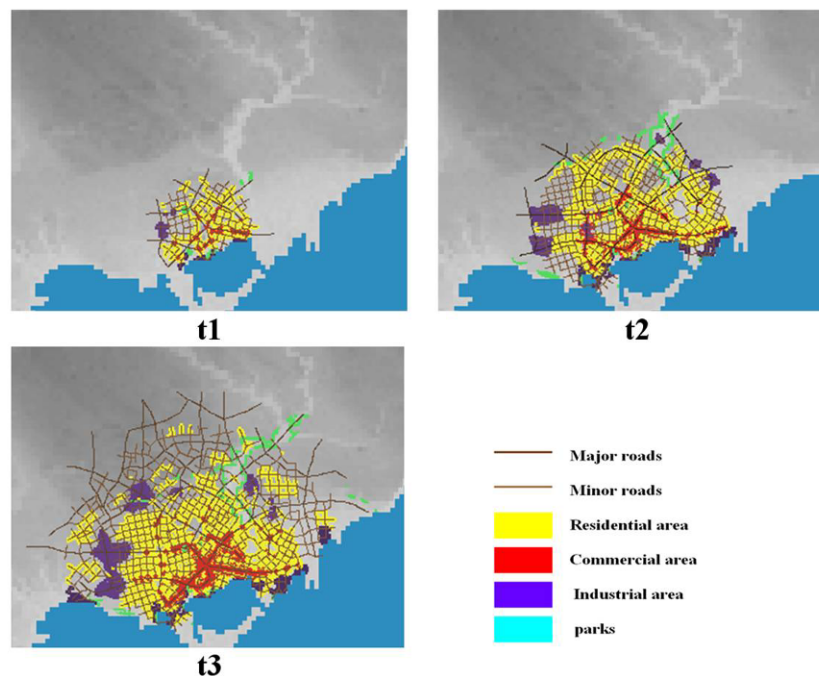
In our experiments, the background data, i.e. basic land use and elevation data, is from the Greater Toronto Area. As described above, accessibility is not only related to road network, but also the traffic flow. Considering road network growth and traffic flow changes in each road, we have different accessibility maps for major roads in four periods. See Figure 3.



**Figure 3:** Major roads (white line) growth and accessibility (lighter means higher accessibility value) changes in four periods

Figure 4 shows one example of the long term development of a growing city. Initial parameters include water and elevation. Higher elevation is represented by increased luminance. Simulation starts with a given center. Residential areas are denoted by yellow, commercial are denoted by red, and industrial are represented by blue. Roads are transformed from vector data into grid data. Major roads are distinguished from minor roads in that they are wider and a deeper shade of gray.

One challenge for urban growth modeling is the validation of the modeling result. Similar to other social or economic fields, the reality of urban development is hard to be reproduced, mainly because our observation usually comes from statistical surveys and entities in social systems have cognitive capabilities in contrast with natural science (Semboloni 2005). Utilized parameters in many large scale social models are calibrated through a lot of statistical data and not from direct observation. However in our behavior model, we use agents to simulate different types of entities, which interact with other entities. We could formalize the entities the same we observe in the reality. And parameters from observation, which describe entities' choices, have a meaning not only to the model but also to the reality.



**Figure 4:** Simulation results of a growing city in three periods

Actually we do not intend to reproduce any existing city, but the typical urban patterns. We have some methods to judge our simulation results. One is to visually compare it with land use map of a real city. The other is to use measure metrics to describe land cover, such as spatial configuration, which refers to the distribution and clustering of land use (for example Houston, TX in Fig.16 in Lechner et al. 2006). Our simulation results show similar patterns of land use as that of real cities. Commercial areas

are drawn to city center and areas of high road density and close to the roads. Industry attempts to develop in urban periphery. With city growth, some are left, while some move to new city fringe. Residential areas tend to develop with the accessibility to road network and segregate themselves from industry. Major road network shows a skeleton of the growing city and demonstrate connectivity among the denser parts of the city. Note that some roads may be major roads in an early period, but are minor roads in the next period, depending on the traffic flow. It reveals road hierarchy changes well in a real world city.

## 6- Conclusion and future work

We have presented a dynamic model to study long-term urban growth based on land use changes and transportation growth. Therefore our model mainly consists of two sub-models: vector road growth model and grid land use model. Our simulation spans a long time, so building a dynamic road network is necessary. We use space syntax to analysis road network, and local integration is calculated to predict traffic flow, which are used to select and confirm major roads, and calculate accessibility for land use simulation. Our accessibility maps are dynamic because both road network and traffic flow in the same individual roads change over time. Simulation results show some similar patterns of a real city, although our preliminary model is quite rough.

Our model has several limitations and needs to be improved in the future.

- (1) Behavioral model used here is rather simple. We intend to add more elements, such as location choice model for household and employment.
- (2) A dynamic population distribution and density map should be calculated, which is very important for road growth.

## References

- Chen G, Esch G, Wonka P, Müller P, Zhang E (2008) Interactive procedural street modeling. *ACM Trans. Graph.*, 27(3)
- Esch G, Wonka P, Müller P, Zhang E (2007) Interactive procedural street modeling. In *SIGGRAPH '07: ACM SIGGRAPH 2007 sketches*, New York, NY, USA. ACM

- Ettema D, de Jong K, Timmermans H, Bakema A (2005) PUMA multi agent modelling of urban systems. In: 45th Congress of the European Regional Science Association, Vrije Universiteit Amsterdam
- Glass KR, Morkel C, Bangay SD (2006) Duplicating road patterns in south african informal settlements using procedural techniques. In Afrigraph '06: Proceedings of the 4th international conference on Computer graphics, virtual reality, visualization and interaction in Africa, pages 161-169, New York, NY, USA. ACM Press
- Heinzle F, Anders K, Sester M (2007) Automatic Detection of Patterns in Road Networks: Methods and Evaluation. In: Proc. of Joint Workshop Visualization and Exploration of Geospatial Data, Stuttgart, vol. XXXVI -4/W45
- Iacono M, Levinson D, El-Geneidy A (2008) Models of transportation and land use change: a guide to the territory. *Journal of Planning Literature*, 22(4), pp. 323-340
- Jiang B, Claramunt C (2002) Integration of space syntax into GIS: new perspectives for urban morphology. *Transactions in GIS*, 6(3), 295-309
- Jiang B (2007) A topological pattern of urban street networks: universality and peculiarity. *Physica A: Statistical Mechanics and its Applications*, 384, 647-655
- Jiang B, Zhao S, Yin J (2008) Self-organized natural roads for predicting traffic flow: a sensitivity study. *Journal of Statistical Mechanics: Theory and Experiment* July, P07008
- Jiang Z (2007) The Road Extension Model in the Land Change Modeler for Ecological Sustainability of IDRISI. ACMGIS'07, Seattle, WA
- Kelly ED (1994) The transportation land-use link. *Journal of Planning Literature* 9(2): 1, 28-45
- Kelly G, McCabe H (2007) Citygen: An Interactive System for Procedural City Generation. In Proceedings of GDTW 2007: The Fifth Annual International Conference in Computer Game Design and Technology, pages 8-16, Liverpool, UK
- Kou X, Yang L, Cai L (2008) Artificial Urban Planning: Application of MAS in Urban Planning Education. 2008 International Symposium on Computational Intelligence and Design, 349-353
- Lechner T, Watson B, Ren P, Wilensky U, Tisue S, Felsen M (2004) Procedural modeling of land use in cities. Technical report, Northwestern University
- Lechner T, Ren P, Watson B, Brozefski C, Wilenski U (2006) Procedural modeling of urban land use. In SIGGRAPH '06: ACM SIGGRAPH 2006 Research posters, New York, NY, USA. ACM
- Lechner T, Moeckel R, Schwarze B, Spiekermann K, Wegener M (2007) Simulating Interactions Between Land Use, Transport and Environment. 11th WCTR, 24-28 June 2007, UC Berkeley
- Li X, Liu X (2008) Embedding sustainable development strategies in agent-based models for use as a planning tool. *International Journal of Geographical Information Science*, Vol. 22, No. 1, January 2008, 21-45
- Marshall S (2005) *Streets & Patterns*. Spon Press, Taylor & Francis Group, New York
- Miller JE, Hunt DJ, Abraham JE, Salvini PA (2004) Microsimulating urban systems. *Computers, Environment and Urban Systems* 28:9-44
- Moeckel R, Spiekermann K, Schurmann C, Wegener M (2003) Microsimulation of land use, transport and environment. Paper presented at the 8th International Conference on Computers in Urban Planning and Urban Management, May 27-29, Sendai, Japan
- Parish Y, Müller P (2001) Procedural Modeling of Cities. In Proceedings of ACM SIGGRAPH 2001, ACM Press, E. Fiume, Ed., 301-308
- Otter HS, van der Veen A, Vriend HJ (2001) ABLOoM: Location behavior, spatial patterns, and agent based modelling. *Journal of Artificial Societies and Social Simulation*, 4 (4), published online
- Semboloni F (2005) Multi-agents simulation of urban dynamic. Proceedings of XII CUPUM. Ed. London
- Sun J, Baciú G, Yu X, Green M (2002) Template-Based Generation of Road Networks for Virtual City Modeling. VRST '02, Hong Kong

- Vanegas CA, Aliaga DG, Wonka P, Muller P, Waddell P, Watson B (2009 a) Modeling the appearance and Behavior of Urban Spaces. *COMPUTER GRAPHICS forum*. Volume 28, Number 2, pp.1-18
- Vanegas CA, Aliaga DG, Benes B, Waddell PA (2009 b) Interactive design of urban spaces using geometrical and behavioral modeling. *ACM SIGGRAPH Asia 2009. SESSION: Urban modeling*. Article No.: 111
- Veldhuisen J, Timmermans H, Kapoen L (2000) RAMBLAS: a regional planning model based on the microsimulation of daily activity travel patterns. *Environment and Planning A* 32 (3), 427-443
- Waddell P, Borning A, Noth M, Freier N, Becke M, Ulfarsson G (2003) Microsimulation of Urban Development and Location Choices: Design and Implementation of UrbanSim, *Networks and Spatial Economics*, vol. 3, no. 1, pp. 43-67
- Waddell P, Ulfarsson G, Franklin J, Lobb J (2007) Incorporating Land Use in Metropolitan Transportation Planning. *Transp. Res. Part A: Policy and Practice*, 41, 382/410
- Wagner P, Wegener M (2007) Urban Land Use, Transport and Environment Models. *DISP*, 2007, 170, 3, 45-56
- Watson B (2007) Real and virtual urban design. In *SIGGRAPH '07: ACM SIG-GRAPH 2007 courses*, pages 167-228, New York, NY, USA. ACM
- Weber B, Müller P, Wonka P, Gross M (2009) Interactive Geometric Simulation of 4D Cities. *EUROGRAPHICS 2009*. Volume 28, Number 2
- Wegener M (2004) Overview of Land-use Transport Models. Chapter 9 in David A. Hensher and Kenneth Button (Eds.): *Transport Geography and Spatial Systems. Handbook 5 of the Handbook in Transport*. Pergamon/Elsevier Science, Kidlington, UK, 2004, 127-146
- Xie F, Levinson D (2009) Modeling the Growth of Transportation Networks: A Comprehensive Review. *Netw Spat Econ* (2009) 9:291-307
- Yamins D, Rasmussen S, Fogel D (2003) Growing Urban Roads. *Networks and Spatial Economics*, 3: 69-85