CSCD439/539 Homework1, Winter 2012, Eastern Washington University. Cheney, Washington.

Name: **EWU ID:** **Due:** 11:59pm, Feb. 5, 2012 (Sunday)

Please follow these rules strictly:

1. Write your name and EWUID on **EVERY** page of your submission.
2. Verbal discussions with classmates are encouraged, but each student must independently write his/her own solutions, without referring to anybody else's solution.
3. The deadline is sharp. Late submissions will **NOT** be accepted (it is set on the Blackboard system). Send in whatever you have by the deadline.
4. Submission must be computer typeset in the **PDF** format and sent to the Blackboard system. I encourage you all to use the LaTeX system for the typesetting, as what I am doing for this homework as well as the class slides. LaTeX is a free software used by publishers for professional typesetting and are also used by nearly all the computer science and math professionals for paper writing.
5. Your submission PDF file must be named as: **firstname_lastname_EWUID_cscd439_539_hw1.pdf**
   (1) We use the underline '_' not the dash '-'.
   (2) All letters are in the lower case including your name and the filename's extend.
   (3) If you have middle name(s), you don't have to put them into the submission's filename.
6. Sharing any content of this homework and its keys in any way with anyone who is not in this class of this quarter is NOT permitted.

Note: All the following problems are open for you all and myself. I do not have keys for them. Use your creativity and the knowledge that you learnt from this course to solve them. I will do my side. I have no guarantee that I can solve all the problems correctly, so let us do our best together! If anything is wrong in my solutions which will be published after your submissions, please feel free to let me know, and I will appreciate for your contributions to this course.

Problem 1 (15 points)**.** *I have a fair coin and a two-headed coin. I choose one of the two coins randomly with equal probability and flip it. Given that the flip was head, what is the probability that the coin I flipped is the two-headed coin ?*

Problem 2 (15 points)**.** *A medical company touts its new test for a certain genetic disorder. The false negative rate is small: if you have the disorder, the probability that the test returns a positive result is* 0.999*. The false positive rate is also small: if you do not have the disorder, the probability that the test returns a positive result is only* 0.005*. Assume that* 2% *of the population has the disorder. If a person chosen uniformly from the population is tested and the result comes back positive, what is probability that the person has the disorder ?*

Problem 3 (10 points)**.** *There may be several different min-cuts in a graph. Using the analysis of the randomized min-cut algorithm, argue that there can be at most* $n(n-1)/2$ *distinct min-cuts.*

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Problem 4 (10 points)**.** *A monkey types on a 26-letter keyboard that has lowercase letters only. Each letter is chosen independently and uniformly at random from the alphabet. If the monkey types* $1,000,000$ *letters, what is the expected number of times the sequence "proof" appears ?*

Problem 5 (10 points)**.** *If* X *is a* $B(n, 1/2)$ *random variable with* $n \geq 1$*, show that the probability that* X *is even is* $1/2$*.*

Problem 6 (15 points)**.** *We draw cards uniformly at random with replacement from a deck of* n *cards. What is the expected number of cards we must draw until we have seen all* n *cards in the deck ? If we draw* $2n$ *cards, what is the expected number of cards in the deck that are not chosen at all ?*

Problem 7 (10 points)**.** *For a coin that comes up head independently with probability* p *on each flip, what is the expected number of flips until the* k*th heads ?*

Problem 8 (15 points)**.** *Let* $a_1, a_2, \ldots, a_n$ *be a random permutation of* $\{1, 2, \ldots, n\}$*, equally likely to be any of the* $n!$ *possible permutations. When sorting the list* $a_1, a_2, \ldots, a_n$*, the element* a_i *must move a distance of* $|a_i - i|$ *places from its current position to reach its position in the sorted order. Find*

$$\mathrm{E}\left[\sum_{i=1}^{n} |a_i - i|\right]$$

the expected total distance that elements will have to be moved.