Prediction of Foreign Exchange Rate by Local Fuzzy Reconstruction Method

Tadashi Iokibe, Shoji Murata, Masaya Koyama
System Development Section
System Technology Division
Meidensha Corporation
36-2, Nihonbashi Hakozaki-cho, Chuo-ku, Tokyo, 103 Japan
TEL +81-3-5641-7507 FAX +81-3-5641-9303

Keywords: Deterministic chaos, Nonlinear short-term prediction, Timeseries, Embedding, Local fuzzy reconstruction method, Foreign exchange rate

1. INTRODUCTION

Several systems for the purpose of predicting trends in the foreign exchange market and stocks have been developed. They are knowledge based expert systems [2], or fuzzy expert systems [3]-[4]. Disadvantage of these system is mainly depend on knowledge base. It is not easy to obtain the knowhow of dealers completely. This paper presents the method of predicting timeseries data, which is considered as a deterministic chaos, based on deterministic dynamical system. The key technology is embedding and local reconstruction. At first, the authors explain about deterministic chaos. Next, we show the Takens' embedding theorem roughly, and local reconstruction especially the local fuzzy reconstruction method in detail. Finally, we applied the local fuzzy reconstruction method to prediction of foreign exchange rate, and show the results of prediction.

2. DETERMINISTIC CHAOS

An irregular phenomenon had been taken for an indeterministic phenomenon governed by contingency. But apparently irregular, unstable and complex behavior can often be generated from a differential or difference equation governed by determinism, for example the timeseries generated by equation (1) when $3.57 \le a \le 4$ which is called logistic map demonstrated by R. May. This is one of deterministic chaos of dynamical system.

$$x_{n+1} = f(x_n) = a\, x_n(1 - x_n) \qquad (1)$$

3. KEY TECHNIQUE OF PREDICTION BASED ON DETERMINISTIC DYNAMICAL SYSTEM

3.1 Embedding

A prediction from the viewpoint of deterministic dynamical system is based on the Takens' theory for "reconstructing the state space and the attractor of the original dynamical system from single observed timeseries data" [1]. The Takens' theory is summarized below.

Vector $\mathbf{x}(t)=(y(t), y(t-\tau), y(t-2\tau), \ldots, y(t-(n-1)\tau))$ is generated from the observed timeseries $y(t)$, where "τ" represents a time delay. This vector indicates one point of an n-dimensional reconstructed state space R^n. A trajectory can be drawn in the state space by changing "t" as shown in Fig.1. Let us assume that the target system is a deterministic dynamical system and that the observed timeseries is obtained through an observation system corresponding to C^1 continuous mapping from the state space of dynamical system to the 1-dimensional Euclidean space R.

Then, the reconstructed trajectory is an embedding of the original trajectory when "n (embedding dimension)" value is sufficiently large. Namely, if any attractor has appeared in the original dynamical system, another attractor, which retains the phase structure of the first attractor, will appear in the reconstructed state space.

In order that such reconstruction achieves "embedding", it has been proven that the dimension "n" should satisfy the condition of $n \ge 2m+1$, where "m" represents the state space dimension of the original dynamical system. However, this is a sufficient condition. Depending on data, embedding can be established even when "n" is less than $2m+1$.

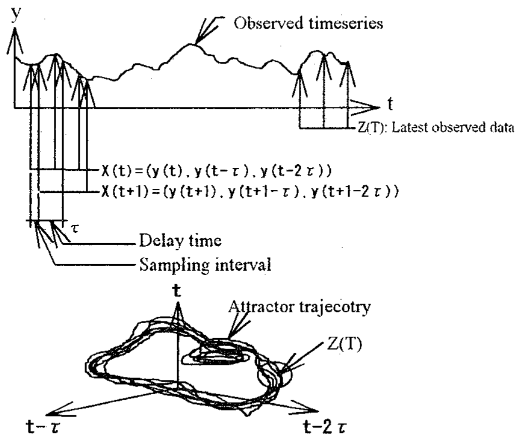

Fig.1 Embedding of Timeseries into n-Dimensional Reconstructed State Space

The data vector **Z**(T) resulting from the latest observation is plotted in the state space and the neighboring data vector is replaced with **X**(i). Since **X**(i) is the past data, the state **X**(i+s) at "s" steps ahead is already known as shown in Fig. 2. Utilizing this, the predicted value $\hat{z}$ (T+s) of **Z**(T+s) is obtained.

From the data vector $\hat{z}$ (T+s) at "s" steps ahead via the local reconstruction, the predicted value $\hat{y}$(T+s) of y(T+s) at "s" steps ahead of the original timeseries is obtained.

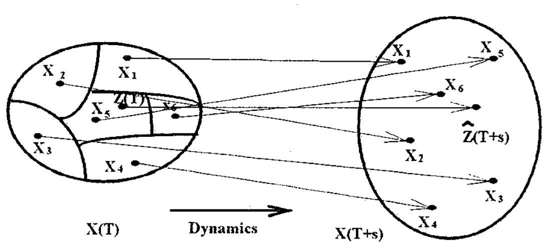

Fig.2 Dynamics of **X**(T) to **X**(T+s)

3.2 Local fuzzy reconstruction method

If the behavior of the observed timeseries corresponds to deterministic chaos, as shown in Fig.2 the transition from state **X**(i) to state **X**(i+s) after "s" steps can be assumed to be dependent on the dynamics subjected to determinism. This dynamics can be expressed by fuzzy function as follows.

$$\text{IF } \mathbf{X(T)} \text{ is } \tilde{\mathbf{X}}\mathbf{(i)} \text{ THEN } \mathbf{X(T+s)} \text{ is } \tilde{\mathbf{X}}\mathbf{(i+s)} \qquad (2)$$

Now let us remember the following relations.

$$\mathbf{X}(i)=(y(i),y(i-\tau),......,y(i-(n-1)\tau))$$
$$\mathbf{X}(i+s)=(y(i+s),y(i+s-\tau),...,y(i+s-(n-1)\tau)) \qquad (3)$$

Also, the trajectory from **Z**(T) to **Z**(T+s) is influenced by Euclidean distance from **Z**(T) to **X**(i). This formula can be rewritten as follows when focusing attention on the "j" axis in the n-dimensional reconstructed state space.

$$\text{IF aj(T) is } \tilde{y}\text{j(i) THEN aj(T+s) is } \tilde{y}\text{(i+s)} \qquad (4)$$
$$(j = 1 \sim n)$$

Where, aj(T) is J-axis component of **X**(i) value neighboring to **Z**(T), aj(T+s) is J-axis component of **X**(i+s) and "n" is dimension of embedding

Now let us remember the following .

$$\mathbf{Z}(T)=(y(T),y(T-\tau),......,y(T-(n-1)\tau))$$

Therefore, the j-axis component of **Z**(T) in the n-dimensional reconstructed state space becomes equal to yj(T).

Accordingly, the j-axis component of the predicted value $\hat{z}$ (T+s) of data vector **Z**(T+s) after "s" steps of **Z**(T) is obtainable as aj(T+s) by a fuzzy inference with yj(T) substituted into aj(T) of formula (4). We named this method "Local Fuzzy Reconstruction Method" [5]-[9].

4. PREDICTION OF FOREIGN EXCHANGE RATE

In this section, we try a short-term prediction by employing the local fuzzy reconstruction method proposed in this paper with regard to the timeseries of foreign exchange rates.

Concretely, 699 (1992.12.31-1994.11.2) timeseries data of the exchange rates (JP¥/US$, JP¥/CAN$) are tested. The first half of the observed timeseries data is embedded in an n-dimensional reconstructed state space. The embedded latest data vector is replaced with **Z**(T) and the timeseries data after "s" steps is predicted by the local fuzzy reconstruction method. Next, the timeseries data after 1 step is fetched, the data vector including this data is replaced with **Z**(T+1) and the value after "s" steps is predicted by the local fuzzy reconstruction method. This sequence is iterated up to the last data.

Figs.3 to 5 show the result of predicting the timeseries of the exchange rate of JP¥/US$ in case where the dimension of embedding (n), delay time (τ) and the number of neighboring data vector (N) are 7, 3 and 4, Figs. 6 to 8 show that of JP¥/CAN$ in case of n=5, τ=2 and N=5 respectively. And the correlation coefficient and RMSE of the JP¥/US$ were 0.980 and ¥0.645, that of the JP¥/CAN$ were 0.988 and ¥0.695 respectively.

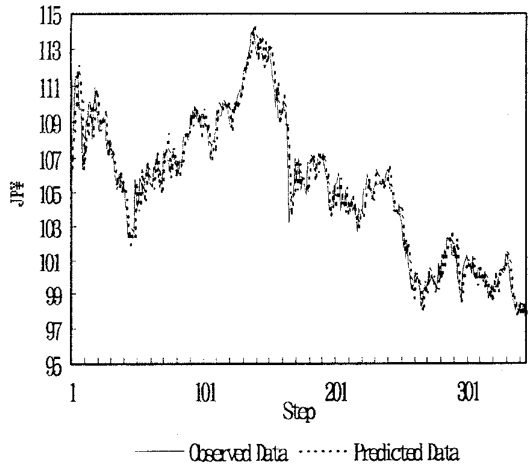

Fig. 3 Result of 1 Step Prediction of JP¥/US$ (Timeseries)

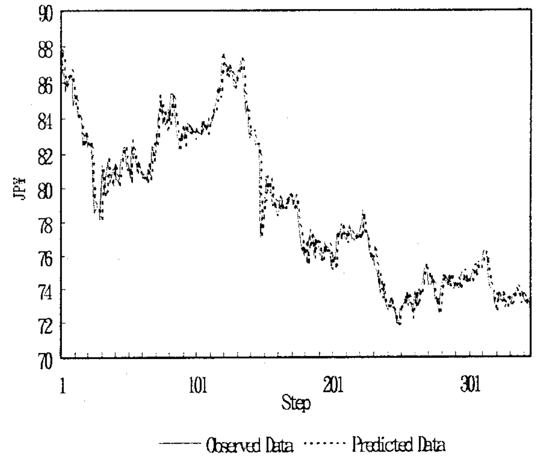

Fig. 6 Result of 1 Step Prediction of JP¥/CAN$ (Timeseries)

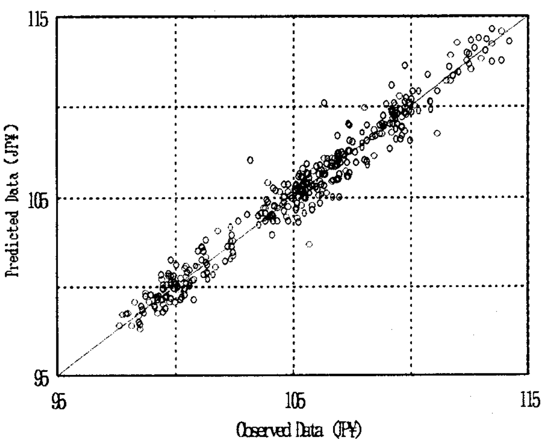

Fig. 4 Result of 1 Step Prediction of JP¥/US$ (Correlation)

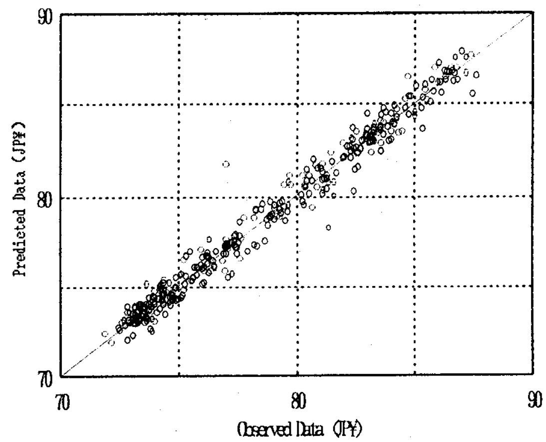

Fig. 7 Result of 1 Step Prediction of JP¥/CAN$ (Correlation)

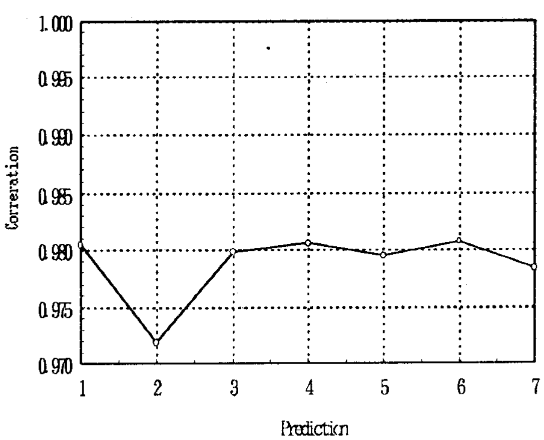

Fig.5 Change of Correlation Coefficient by Prediction Step of JP¥/US$

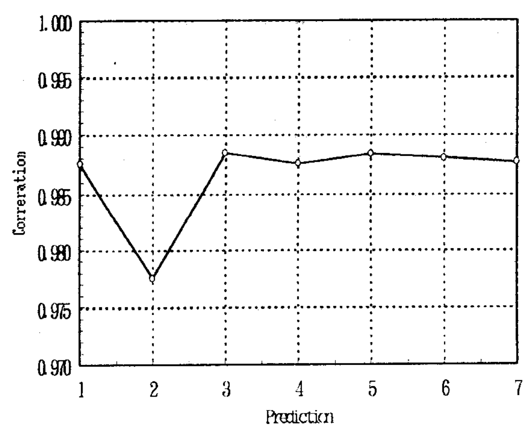

Fig.8 Change of Correlation Coefficient by Prediction Step of JP¥/CAN$

5. CONCLUSION

This paper explained about the prediction method based on deterministic dynamical system, especially explained the local fuzzy reconstruction method in detail, which linguistically expresses the dynamics of dynamical system according to fuzzy rules. And in order to verify the effectiveness of this method, it has been applied to the short-term prediction of each timeseries of foreign exchange rate, and it has been proven that a satisfactory result is obtainable as shown in Fig. 3 and 8.

6. REFERENCES

[1] F. Takens, "In Dynamical Systems and Turbulence", (eds. D. Rang and L. Young), Springer, Berlin,1981, pp. 366-381.

[2] D. Dubois and H. Prade, "Possibility Theory", Plenum Press, New York 1988.

[3] Y. Katoh, H. Yuize, M. Yoneda, K. Takahashi and S. Tano, "Gradual Rules in a Decision Support System for Foreign Exchange Trading", In Proc. of the 2nd International Conference on Fuzzy Logic and Neural Networks, 1992/7/17-7/22, Iizuka, Japan, 1992, pp.625-628.

[4] C. Yasunobu, M. Kosaka and K. Yokomura, "A Fuzzy Dealing Support System", Denshi Tokyo, No.32, IEEE Tokyo Section, 1993, pp.85-88 .

[5] T. Iokibe, M. Kanke, Y. Fujimoto and S. Suzuki, "Short-term Prediction on Chaotic Timeseries by Local Fuzzy Reconstruction Method", In Proc. of Brazil-Japan Joint Symposium on Fuzzy Systems, 1994/7/20-7/27, Campinas, Brazil, 1994, pp.136-139.

[6] T. Iokibe, M. Kanke, Y. Fujimoto and S. Suzuki, "Short-term Prediction on Chaotic Timeseries by Local Fuzzy Reconstruction Method", In Proc. of the 3rd International Conference on Fuzzy Logic, Neural Nets and Soft Computing, 1994/8/1-8/7, Iizuka, Japan, 1994, pp.491-492.

[7] T. Iokibe and N. Ohshima, "Industrial Application Based on Chaos and Fuzzy", In 1994 National Convention Record IEEJ, Industrial Applications Society, 1994/8/24-8/26, S.6-3, 1994, pp.S121-S126.

[8] T. Iokibe, "Application Based on Chaos and Fuzzy", In Proc. of the 3rd Fuzzy Workshop, SOFT 1994/9/27-9/28, 1994, pp.52-62.

[9] T. Iokibe and N. Mochizuki, "Short-term Prediction Based on Fuzzy and Chaos", In Proc. of Intelligent System Symposium, SICE, 1994/10/20-10/21, 1994, pp.77-82.