Experiments on Stock trading Via Feedback Control

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***Abstract*—This paper analyzes the predictivity and return of the Barmish-Iwarere trading algorithm described in [1]. In the first part of the paper, we study the trade triggering algorithm using either an Ito process model, or real data from indexes and ETFs. It is shown through hypothesis testing that the trigger provides mixed results in predicting the sign of the single trade, for both the Ito process and real indexes. However, we show empirically the trigger is sufficiently good in identifying a trend, while it fails in detecting side movements. In the second part of the paper, the effect of parameters of the feedback controller will be analyzed under various market circumstances, the efficiency of a pre-optimization on the last data will appear controversal. Some changes will be tried with the objective of improving the returns. In particular, the trigger is modified to detect anomalous falls during a rising trend using the estimated volatility.**

***Keywords*-Trading system; trigger; controller;**

I. Introduction

A mathematical model consistent with the behavior of real markets is the Ito process [2], a Brownian motion with drift, Brownian motion (also known as Wiener process) has three properties:
- is a Markov process, that is the probability distribution for all future values of the process depends only on its current value;
- has independent increments, the probability distribution for the change in the process over any time interval is independent of any other (non overlapping) time interval;
- changes over any finite interval of time are normally distributed.

An Ito process is described by the equation:

$$dS = a(S,t)dt + b(S,t)dz \tag{1}$$

where, dz is the increment of a Wiener process, $a(S,t)$ is the drift parameter, dz can be represented as $dz = \epsilon_t\sqrt{dt}$, where ϵ_t is a normal random variable with zero mean and unit standard deviation. A special case of (1) is the geometric Brownian motion with drift, here $a(S,t) = \mu S$, $b(S,t) = \sigma S$, where μ and σ are constant, and the equation becomes:

$$dS/S = \mu dt + \sigma\epsilon_t\sqrt{dt} \tag{2}$$

So the instantaneous rates of return dS/S are normally distributed, as confirmed roughly by data analysis for real returns of stocks, and, for the Ito's lemma, the increment $d(lnS)$, will be [2]: $d(lnS) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma\epsilon_t\sqrt{dt}$

However in practice often equation (2) is used to simulate stock prices, the differences are negligible for high frequency data. Mean reversion is one of the observed deviation from the model [3], it is the tendency of stock prices to be attracted towards their long term mean, interest rates and raw commodities exhibit a mean reverting behavior. Some studies have shown the tendency of stock prices to overreact, investors are subject to waves of optimism and pessimism that cause prices to deviate systematically from their fundamental values and later to exhibit mean reversion, in such cases the "contrarian" strategy could be profitable, this strategy has been used in the experiments.

II. The Trading System

The trading system is composed by a trigger and a controller [1], the trigger gives the signal for entering or exiting a trade, the controller, modulates the amount invested trying to improve the return. The system was tested at first with simulations of an Ito process, the model used for generating the sequences is equation (2), hence the price $S(k+1)$ is given by: $S(k+1) = (1 + \mu\Delta t + \sigma\epsilon(k)\sqrt{\Delta t})S(k)$, where Δt is the time interval between potential trades measured in years, for a one day interval is set to $\Delta t = 1/252$, being a trading year composed by around 252 days, μ is the annualized drift of the stock, σ the annualized volatility of the stock, and $\epsilon(k)$ a normal random variable with zero mean and unit standard deviation. An estimation $\hat{\mu}$ of μ and $\hat{\sigma}$ of σ is computed from n simulated or real market data, then $\hat{\sigma}$ is used to build the corresponding confidence interval for $\hat{\mu}$, $[L, U]$.

The one period return used to obtain the estimation is: $\rho(k) = S(k+1)/S(k) - 1$ for real market data $S(k)$ is the closure price of the stock. Finally the estimates were:

$$\hat{\mu}(k) = \frac{1}{n\Delta t}\sum_{i=1}^{n}\rho(k-i)$$

$$\hat{\sigma^2}(k) = \frac{1}{n-1}\sum_{i=1}^{n}(\rho(k-i)/\Delta t - \hat{\mu})^2$$

$$[L(k), U(k)] = [\hat{\mu}(k) - \frac{t_{\frac{\alpha}{2},n-1}\hat{\sigma}(k)}{\sqrt(n)}, \hat{\mu}(k) + \frac{t_{\frac{\alpha}{2},n-1}\hat{\sigma}(k)}{\sqrt(n)}]$$

Where $1-\alpha$ is the chosen confidence level, $t_{\alpha/2,n-1}$, the critical value from the Student t-distribution with $n-1$ degrees of freedom. With daily data there is no need to use the exact formulas from the Black-Scholes model to compute $\hat{\mu}$ and $\hat{\sigma}$ [4]. Computed the interval $[L, U]$, the rule to trigger a trade is:

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- if the lower extreme of the confidence interval L satisfies $L \geq 0$, a long trade is triggered;
- if the upper upper extreme U satisfies $U \leq 0$, a short trade is triggered;
- for the case when $L < 0 < U$, no trigger results.

If the trigger returns a signal of trade, the controller determines the amount to invest, for instance, if at time k^* after a period of no trade there is a long signal, then a long trade begins, assuming no commissions and an account value $V(k^*)$, the initial investment is: $I(k^*) = \gamma_0 V(k^*)\ \ 0 < \gamma_0 < 1$.

Optimization of γ_0 is an open problem [1], we have used a training sequence of previous data and the Kelly criterion, as exposed in the following sections. For a short trade γ_0 is negative, in this paper it is introduced a γ_0 different also in absolute value, γ_{0L} for a long trade, and γ_{0S} for a short one. At the following step the stock price evolves, and the amount invested is tuned with the rule:

$$I(k^* + 1) = [1 + K\rho(k^*)]I(k^*) \tag{3}$$

where K is the feedback gain, $K = 1$ is a "buy and hold" strategy until the position is open, i. e. the trade is begun with the initial amount and no action is set out until the trigger changes the signal; $K > 0$ for a long trade, $K < 0$ for a short one. Also the value of K need to be optimized, like for γ_0, K was also set different in absolute value for a short trade. As the trade evolves the amount invested is updated according to the (3), however there is a limit, the saturation condition: $I(k^* + j) = \gamma_{max} V(k^* + j) = I_{max};\ \ j > 0$.

Taking into account the two conditions:
$I(k^* + j + 1) = \min\{[1 + K\rho(k^* + j)]I(k^* + j), I_{max}\}$.
Finally, considering also the possibility of short trading:
$I(\bullet + 1) = \max\{\min\{[1 + K\rho(\bullet)]I(\bullet), I_{maxL}\}, I_{maxS}\}$

III. The Kelly Criterion

As seen in the previous section, one of the open issue of the trading system is the value to assign to γ_0, i. e. how much capital to allocate to the risky investment and how much to keep in cash, a possible choice can be the *optimal Kelly fraction* [5], (also known as Latané strategy [6]), the goal of the strategy is to maximize the growth of the capital over the long term. Supposing to invest in m trials and that the amount invested is $I(k) = \gamma_0 V(k)$, then the capital after m trials is: $V_m = V_0(1 + \gamma_0 g)^S (1 - \gamma_0 l)^F$ where S and F are the number of successes and failures, $S + F = m$, g is the gain and l is the loss during a single trial, if $0 < l < 1$, it is not possible to lose more than the amount invested. If $0 < \gamma_0 < 1$, $Pr(V_m = 0) = 0$ also if $l = 1$. Since $e^{m\ln(\frac{V_m}{V_0})^{1/m}} = \frac{V_m}{V_0}$, $\ln(\frac{V_m}{V_0})^{1/m} = \frac{S}{m}\ln(1 + \gamma_0 g) + \frac{F}{m}\ln(1 - \gamma_0 l)$, the last quantity measures the exponential rate of increase per trial, for growth it has to be greater than zero, the criterion maximizes the expected value: $E\{\ln(\frac{V_m}{V_0})^{1/m}\} = E\{\frac{S}{m}\ln(1+\gamma_0 g) + \frac{F}{m}\ln(1-\gamma_0 l)\}$, i. e. $p\ln(1+\gamma_0 g)+q\ln(1-\gamma_0 l)$, where p is the probability of gain, $q = 1-p$ the probability of loss, after some calculations the unique optimal fraction is:

$$\gamma_0 = \frac{pg - ql}{gl} = \frac{p(l + g) - l}{gl}; \quad pg - ql > 0 \tag{4}$$

for that value the expected growth factor for trial is: $p\ln p + q\ln q + p\ln(1 + g/l) + q\ln(1 + l/g)$.

However the strategy is very aggressive, even if all the parameters were exact, i. e. in absence of estimation and chance errors, there would be a very high volatility of wealth levels, in fact the expected growth factor times m, gives the natural logarithm of the median wealth, but the distribution is dispersed [6]. Moreover, there is a high sensitivity to parameter values, either in the fraction, or in the return, wrong estimates impact heavily the median return, investing a fraction greater than the optimal one may have dramatic consequences, so it is common to use a lower fraction, typically a half, but lower values are common in derivatives trading. On the other hand, it can be shown that the mean first passage time to arbitrary large wealth targets is minimized, and the probability of reaching those targets is maximized. Such formula (4) is useful in gambling situations and in bond markets, $1 - l$ may be the recovery value of a high yield bond, $1 + g$ the total amount in excess of a risk-free bond of similar maturity, q the probability of default, however for stock trading, where there is a continuum of outcomes it can be found that the optimal Kelly fraction is: $\gamma_0 = \frac{\mu - r}{\sigma^2}$, where r is the risk-free interest rate.

IV. The Predictivity of the Trigger

In order to apply the Kelly criterion to the parameter γ_0, it was investigated at first if $\hat{\mu}$ is "near" the real return, even if the algorithm was not designed for that, but only to establish the sign of the return with a given level of confidence. Unfortunately either for simulated or real stock prices, the correlation coefficient between $\hat{\mu}$ and ρ is low, from about 7% to 13%, the coefficient is computed only when there is a signal of trade.

After it was studied if the sign of the predicted return is meaningfully predicted by the trigger: the following random variable is defined: $X = \text{Sign}(\rho)f$, where f is the flag of trading assuming value 1 for a long trade, -1 of a short trade, and 0 for no trade. Assuming as null hypothesis that the trigger is unable to predict the sign of the return, and it behaves like a source emitting symbols $\{-1, 0, 1\}$ with probabilities $\{q, r, p\}$, given N_+ the number of positive or zero returns, N_- the number of negative returns, $N = N_+ + N_-$, if the output of the trigger is independent from the sign of the return, then:

$$E\{X\} = E\{\frac{p(N_+(1) + N_-(-1)) + r(N_+ + N_-)(0) +}{N} \frac{+q(N_+(-1) + N_-(1))}{N}\}$$

$$E\{X\} = pp' - qp' - pq' + qq' = (p - q)(p' - q')$$

where $E\{N_+/N\} = p'$, and $E\{N_-/N\} = q' = 1 - p'$.
In the same manner for $E\{X^2\}$:

$$E\{X^2\} = pp' + qp' + pq' + qq' = (p + q)(p' + q') = p + q$$

finally an upper bound for the variance is:

$$\sigma^2 = E\{X^2\} - E^2\{X\} = p + q - (p - q)^2(p' - q')^2 \leq 1$$

because $p+q \leq p+q+r=1$ and $(p-q)^2(p'-q')^2=0$ if $p=q$ or $p'=q'$. Fixing the significance level to 99%, m^*, the margin of error to 1%, N, the number of simulated trades, has to be at least 66.349: $N=\lceil(\sigma z/m^*)^2\rceil$. For every set of parameters 5 simulations of different seed were run involving 67.000 trials, the high number of trials justifies the assumption of normality for $E\{X\}$ [7]. The results show a direct dependence on the drift to volatility ratio, and inverse with n, for $\mu/\sigma=0.5$ per year the null hypothesis is already falsified, in table I are reported the number of predicted sign minus the unpredicted ones for some values of the parameters. The prediction power decreases with n, but this relation is stronger for low values of μ/σ.

Table I
NUMBER OF PREDICTED MINUS UNPREDICTED RETURN SIGNS

n	Pred-Unpr	%	μ/σ y.
60	5,585	9.1%	0.5
110	4,084	6.6%	0.5
160	3,415	5.5%	0.5
210	3,341	5.4%	0.5
60	6,399	10.3%	1.0
110	4,782	7.7%	1.0
160	4,267	6.7%	1.0
210	3,822	6.0%	1.0
60	7,918	12.4%	2.0
110	7,272	11.2%	2.0
160	7,467	11.4%	2.0
210	6,889	10.4%	2.0

Observing all the data available it can be seen that α is not an important factor, the number of positive trade varies slowly with it, at least in the range $[0.01, 0.2]$, moreover the dependence appears a bit erratic, however the better choice seems to be from 0.8 to 0.9, nevertheless, as shown in the following paragraph, for volatile equities also a little number of good trades may lead to important differences.

Using real indexes predictivity seems to improve, as it can be seen from table II for the S&P500 Index from 1950 to February 2010 and the EEM ETF replicating the MSCI emerging markets index, from April 2003 to February 2010.

However the predictive power of the trigger does not appear sufficiently strong to use the Kelly criterion for the choice of γ_0.

Table II
PERCENTAGE OF PREDICTED MINUS UNPREDICTED RETURN SIGNS

Index	60	110	160	210	μ/σ y.
S&P 500 1950-2010	10.9%	9.8%	8.4%	7.8%	0.53
S&P 500 2003-2010	10.3%	11.3%	8.9%	7.7%	0.25
EEM 2003-2010	13.4%	10.1%	9.1%	8.7%	0.70

After investigating the predictive power of the trigger for a single day of trading it was studied its capacity to identify a trend through an empirical study, in figure 1 it can be seen the behavior of the trigger in detecting the trends for the EEM ETF, with $n=60$, the trigger identifies reasonably well the trends, however it lacks for precision in marking periods of side movements, where it should be desirable a signal of "no trade".

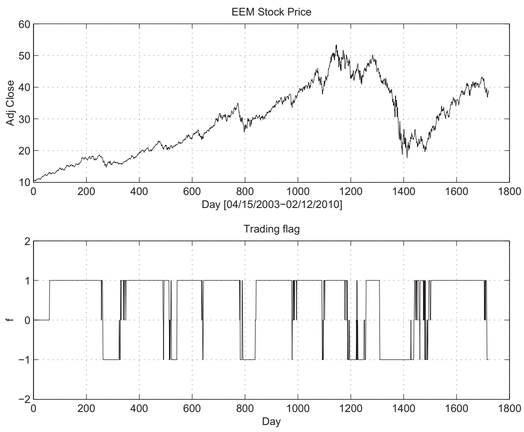

Figure 1. EEM and Trading flag, f=1 long, f=-1 short, f=0 no trade; n=60

Finally a return comparison using markets data was conducted between the Barmish system with gain $K=1$, $\gamma_0=1$ and $\gamma_{max}=1$, and a system based on a classical technical analysis indicator [8], a moving average of equal width, the first one outperforms the moving average strategy almost always, the results for EEM are in table III.

Table III
BARMISH SYSTEM VS MOVING AVERAGE, FINAL TRADED VALUES

n	Barmish	Mov. Avg.
60	23.3	17.6
90	31.8	22.6
120	18.6	27.9
150	38.1	21.7
180	44.8	22.8

V. OPTIMIZING THE PARAMETERS AND THE TRIGGER

Simulations on the Ito process and real markets data were run at first to study the sensitivity of the final traded value on the parameters, and then to try an optimization of them, moreover some modifications were introduced in order to take into account the results about the trigger described in the preceding paragraph. Transaction costs and bid-ask spreads were not considered.

A. *The window width n and the confidence level*

As previously seen the window width is very important for the process of triggering, n in the range of 50 - 80 captures well the trends, over all for volatile equities, like a single stock or an exotic index, but side movements usually are not detected, so the system may incur in important losses, particularly if the controller gain K is greater than one. Increasing the window width n reduces the jitter and increases the results, even if sometimes the results are inferior to a "buy and hold" strategy.

In pictures 2 - 5 the first subplot shows the value of S&P500 and EEM from April 2003 to February 2010, and the subplots below the values $V(k)$ of the trading system for some n, $K=0.2$ and $\alpha=0.1$. In order to compare the

Table IV
EEM FINAL TRADED VALUE FOR CONFIDENCE LEVEL AND WINDOW WIDTH (EEM FINAL VALUE 38.4; K=0.2)

n	80%	90%	95%	99%
60	28.0	26.4	26.7	26.6
70	32.9	36.4	40.0	40.4
80	47.4	48.9	50.4	44.0
90	35.2	37.3	43.2	40.8
100	32.4	36.8	37.5	38.8
110	33.2	34.8	34.7	38.4
120	20.7	21.9	26.1	33.2
130	25.8	25.9	29.4	32.7
140	56.3	51.2	37.5	35.8
150	46.4	45.1	45.6	45.5
160	47.3	45.1	40.8	47.5
170	43.5	45.9	48.1	42.4
180	49.7	49.3	47.8	44.9

series it is supposed that the entry value $V(0)$ is equal to the value of the index in the first day of trading, it can be seen very well that 180 days is a good choice for n, but not the best one, in table IV is shown how the final traded value varies abruptly with n (e. g. n from 120 to 140), moreover even if $1-\alpha$ rarely is an important factor, i. e. the number of triggered trades is almost insensitive to it, the outcomes during high volatility periods dramatically change. Finally another fact emerges from the observation of the subplots, the optimal n changes during the time of trading, it is as if the financial signal varies his time constant: for EEM the last rising trend is well caught by the 90 days system, while the collapse of fall 2008 is matched for $n = 180$ and almost ignored for $n = 120$, while for S&P500 the 90 days system is unable to exploit both the trends, so the usefulness of a pre-optimization on the last n data [1] seems doubtful.

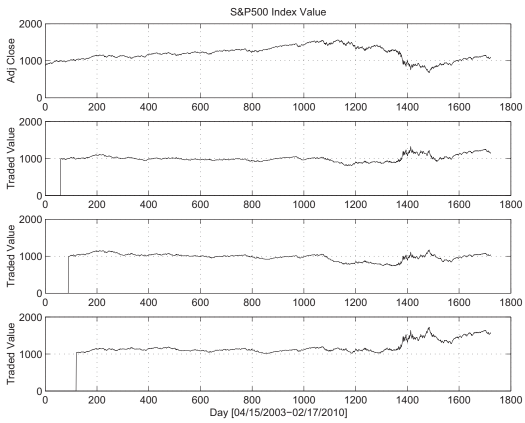

Figure 2. S&P500 - Index value and traded values, n=60,90,120

B. The gain K and γ_0

The preceding results were found with $K = 0.2$, great values of K often decreases the return, but this is true only for the last years, for S&P500 a K near to one was found optimal in previous times. A K greater than one can cause huge losses during financial turbulences, over

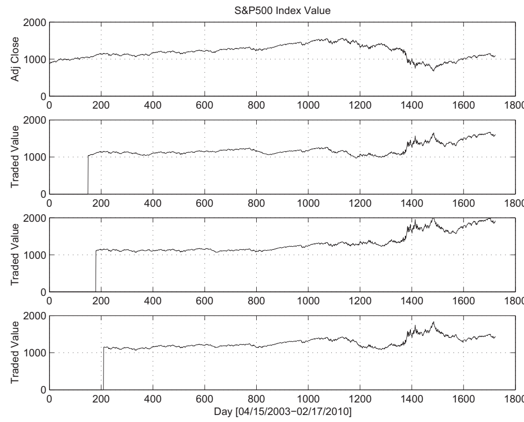

Figure 3. S&P500 - Index value and traded values, n=150,180,210

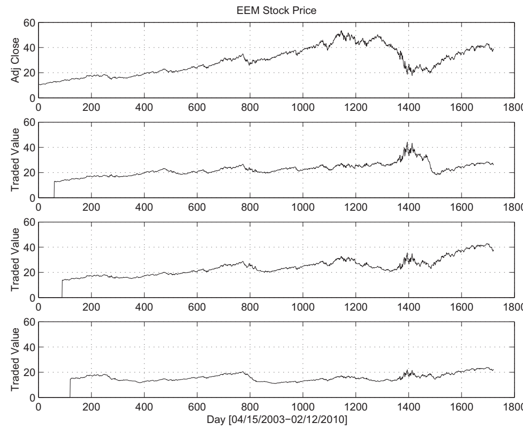

Figure 4. EEM - ETF quote and traded values, n=60,90,120

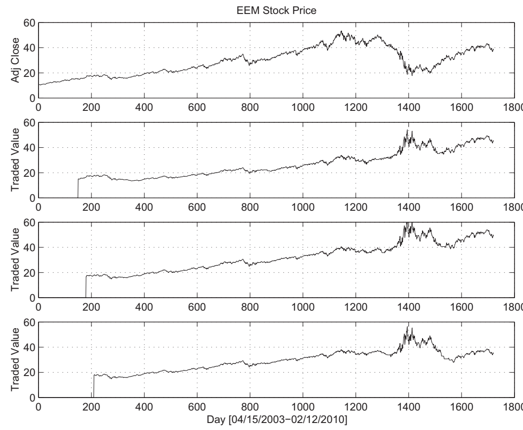

Figure 5. EEM - ETF quote and traded values, n=150,180,210

all for firm shares, like investment banks during the 2008 crisis, owing to the low predictivity of the trigger over a single day it is very likely to encounter a sequence where a high portion of the capital is invested in "bad days" and almost nothing in "good days" leading to early bankruptcy. It was investigated also if there is a convenience to use two gains, K_L for long positions, K_S for short positions, the motivation lies in the different behavior of financial markets during rising and falling cycles summarized by the expressions *"Up a staircase, down an elevator"* and *"The bull walks up the stairs and the bear jumps out the window"*, however no evidence was found to introduce two gains, but improvements were obtained modifying the trigger only with falling markets. Finally, the best results were obtained with a value of $\gamma_0 = \pm 1$, in index trading it seems the optimal choice, being relatively small the risk of a huge loss, in practice the purchase of options out of the money will hedge the position partially.

C. The integrative controller and the reverting to the mean process

The trigger is partially able to predict a single positive trade, but detects pretty good the trends, so it was introduced in the controller an integrative part, the amount invested for a trade beginning at time k^* is modulated by the algebraic sum of the preceding l gains, however there is no advantage to use this technique. After this simulation it was taken a positive sign for K during a short trade, i. e. a rising amount was invested proportionally to losses, surprisingly the final traded value increased for some stocks, most of the positive trades happened near the minimum following the 2008 crisis confirming the mean reversion of stock prices [3] and the profitability of the "contrarian" strategy.

D. Improving the trigger

We have attempted to use the estimated volatility to detect anomalous changes in the quotations, within a frame of long trading, the relative deviation of the current price versus the maximum in the frame is compared with a multiple of the volatility, if the deviation exceeds the bound then a short trade is triggered:

- if during a long trade beginning at $k = k^*$

$$\frac{S(k)}{S_{max}} - 1 < -z_L \hat{\sigma} \sqrt{\Delta t}; \quad S_{max} = \max_{k \geq k^*}[S(k)]$$

  then a short trade is triggered.

As usual z_L is set out in order to maximize the final value. We have tried also to detect anomalous deviations within a short trading, but the rule is not effective. In table V can be seen the final traded values for EEM and S&P500 with the Barmish trading system and with the modified trigger. In the simulation $K = 1$ (the shares are bought and sold only at triggering times, reducing dramatically the penalty for transactions costs and bid-ask spreads), $\alpha = 0.1$, $z_L = 4.5$, in most cases results improve, over all for the volatile EEM, the period of trading is April 2003 - February 2010.

Table V
FINAL TRADED VALUES (FINAL VALUES: EEM 38.4, S&P500 1,100)

	EEM		S&P500	
n	Barm.	Mod. Barm.	Barm.	Mod. Barm.
60	23.3	23.2	1,074	1,135
70	32.5	32.7	1,019	1,121
80	43.0	47.6	1,037	1,138
90	31.8	39.5	997	1,025
100	32.2	37.1	1,163	1,147
110	31.2	36.5	1,472	1,511
120	18.6	22.0	1,505	1,523
130	21.2	27.0	1,337	1,337
140	41.8	51.5	1,284	1,299
150	38.1	37.1	1,514	1,582
160	38.0	32.0	1,543	1,551
170	37.1	31.8	1,492	1,485
180	44.8	45.3	1,783	1,710
190	41.5	44.1	1,632	1,648
200	34.8	37.3	1,372	1,333
210	34.6	37.1	1,318	1,275

VI. CONCLUSION

We have studied the trading system proposed by Barmish and Iwarere, the system is composed by a trigger and a controller, it has been found that the trigger shows some predictivity and the outcomes of the whole system are usually good with moderate volatility indexes, like S&P500, sometimes there are troubles with more volatile indexes, and over all with firm shares.

Many simulations were run to optimize parameters, the window width n is the most important, however an optimization with a short learning sequence is not opportune, in fact the optimal n changes and it cannot be estimated preemptively, so it is better to use a suboptimal value but such to guarantee robustness.

The confidence level $1 - \alpha$ is not determinant for non volatile stocks and over all difficult to optimize, moreover the controller gain K has to be kept low or moderate $K \leq 1$. Eventually, it was introduced a change in the trigger in order to detect the inversion of a rising trend, the attempt was successful, over all for volatile indexes.

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