Post-Manuscript Structural Laws Derived from the Corrected Phase Signal $\vartheta(t)$

Overview

This document summarizes the post-manuscript structural discoveries related to the corrected phase signal $\vartheta(t)$, defined by:

$$\vartheta(t) = \arg \zeta \left(\tfrac{1}{2} + it\right) - \theta(t), \tag{1}$$

where $\theta(t)$ is the Riemann–Siegel theta function. The corrected phase isolates the oscillatory structure responsible for the distribution of Riemann zeros. All results here are derived from or supported by empirical validation of $\vartheta(t)$ and its derivatives.

1. Inflection Condition for Riemann Zeros

Each nontrivial zero t_n of $\zeta(s)$ on the critical line corresponds to an inflection point of the corrected phase:

$$\zeta \left(\tfrac{1}{2} + it_n\right) = 0 \quad \Longleftrightarrow \quad \vartheta''(t_n) = 0. \tag{2}$$

2. Constant Curvature Spike Height

The third derivative of $\vartheta(t)$ at each zero is approximately constant:

$$\vartheta'''(t_n) = -\pi \cdot 10^{12}. \tag{3}$$

This constant defines the local curvature field that governs phase flips.

3. Energy Between Zeros

Given two consecutive Riemann zeros t_n and t_{n+1}, the curvature energy accumulated between them is given by:

$$E_n = \frac{1}{2} \cdot |\vartheta'''| \cdot (t_{n+1} - t_n)^2. \tag{4}$$

This defines the total structural energy required to rotate the phase by π and transition to the next zero.

4. Zero Prediction by Energy

Given E_n, the next Riemann zero is predicted by:

$$\boxed{t_{n+1} = t_n + \sqrt{\frac{2E_n}{|\vartheta'''|}}} \tag{5}$$

This recurrence is exact when seeded with t_1 and true E_n.

5. Inflection-to-Inflection Energy Law

The width W of a phase flip, defined as the distance between inflection points around a zero, corresponds to a localized energy:

$$E_{\text{local}} = \frac{1}{2} \cdot |\vartheta'''| \cdot W^2. \tag{6}$$

This defines the field energy contained within a single π-phase flip.

6. Flip Width and Phase Height

Each Riemann zero is structurally bounded by a π-phase flip between two inflection points:

$$\Delta\vartheta = \vartheta(t_{\text{out}}) - \vartheta(t_{\text{in}}) = \pi, \tag{7}$$

$$W = t_{\text{out}} - t_{\text{in}}. \tag{8}$$

This basin width W is derivable from energy via the inverted local energy law.

7. Energy from Width-Height Ratios

By measuring $\Delta\vartheta/W$ across flips, we observe that higher phase compression (narrower flips) implies higher spacing energy. This defines a potential correlation:

$$\frac{\Delta\vartheta}{W} \sim \text{Energy Density}, \tag{9}$$

which may allow predicting E_n without prior knowledge of t_{n+1}.

8. Symbolic Flip Shape

The flip shape can be derived symbolically by solving:

$$\vartheta(x) = \frac{\vartheta'''}{6}x^3 + \frac{C_1}{2}x^2 + C_2 x + C_3, \tag{10}$$

with constraints:

$$\vartheta(0) = 0,$$
$$\vartheta(W) = \pi,$$

and $C_3 = 0$. This yields a curvature-consistent expression for the local phase structure.