My Symbolic Corrected Phase Model vs Hardy's Model: A Full Structural Comparison

1. What Hardy's Model Does

Setup

Hardy defines:

$$Z(t) = e^{i\theta(t)} \cdot \zeta\left(\tfrac{1}{2} + it\right)$$

where:

$$\theta(t) = \frac{t}{2}\log\left(\frac{t}{2\pi}\right) - \frac{t}{2} - \frac{\pi}{8} + \mathcal{O}\left(\frac{1}{t}\right)$$

Purpose

Hardy's goal was to prove that there are **infinitely many zeros on the critical line**. That was a landmark result, and his construction achieved it elegantly. His method was designed to detect real sign changes of a function derived from $\zeta(s)$:

$$Z(t) \in \mathbb{R} \quad \text{for real } t$$

This allowed him to count zeros on the critical line using real analysis rather than complex root solving.

Effect

- Makes $Z(t)$ real on the critical line.
- Absorbs the phase of ζ using $\theta(t)$.
- Hides the $\pm\pi$ jumps in $\arg\zeta$.
- Does not ensure $\arg\zeta(t_n) = 0$. In fact, the phase is undefined or discontinuous at zeros.

What This Meant for Its Time

Hardy's model was exactly what was needed to prove the infiniteness of critical line zeros. It flattened the complex behavior of $\zeta(s)$ and enabled progress using tools available in the early 20th century.

But I'm doing something different. My goal is not to count zeros, it's to uncover the *structure* that governs them. I'm not just asking how many zeros exist; I'm asking what they *are* in phase space, what field law pins them down.

2. What My Model of Symbolic Corrected Phase Does

Step 1: Start with the Argument of ζ

$$\arg\zeta\left(\tfrac{1}{2} + it\right)$$

This function is:

- Discontinuous, it jumps by $\pm\pi$ at each Riemann zero

- Wrapped, the output is constrained to $(-\pi, \pi]$
- Chaotic, due to the irregularity of $\zeta(s)$ and $\theta(t)$

Step 2: Remove the Analytic Drift $\theta(t)$

Define the corrected phase:

$$\boxed{\vartheta(t) = \arg\zeta\left(\tfrac{1}{2} + it\right) - \theta(t)}$$

This subtraction:

- Cancels the smooth analytic drift
- Re-centers the phase field
- Aligns $\pm\pi$ flips symmetrically around zero

Connection to Stirling's Approximation:
The subtraction of $\theta(t)$ is structurally justified by Stirling's approximation, which shows that:

$$\theta(t) \approx \arg\zeta\left(\tfrac{1}{2} + it\right)$$

This approximation holds especially in the asymptotic regime. So subtracting $\theta(t)$ removes the dominant analytic drift, leaving behind only the symbolic residue:

$$\vartheta(t) \approx \arg\zeta - \arg\zeta \approx 0 \pmod{\pi}$$

Near a zero t_n, this implies:

$$\boxed{\vartheta(t_n) \approx 0}$$

Step 3: Compute Using `mpmath`

`mpmath` provides:

- Arbitrary-precision complex arithmetic
- Accurate evaluation of $\arg\zeta$ and $\theta(t)$
- High-resolution detection of phase flips

Code:

```
s = mpmath.mpc(0.5, t)
arg_zeta = mpmath.arg(mpmath.zeta(s))
theta = mpmath.im(mpmath.loggamma(0.25 + 0.5j * t)) - (t / 2) * mpmath.ln(mpmath.pi)
vartheta = arg_zeta - theta
```

Step 4: Unwrap the Phase

$\arg\zeta$ is wrapped between $-\pi$ and π, creating false jumps.
To fix this, you track phase continuity manually:
Code:

```
for i in range(1, len(vartheta)):
    while vartheta[i] - vartheta[i-1] > mpmath.pi:
        vartheta[i] -= 2 * mpmath.pi
    while vartheta[i] - vartheta[i-1] < -mpmath.pi:
        vartheta[i] += 2 * mpmath.pi
```

This unwrapping produces a globally smooth phase field with:

- Clean $\pm\pi$ flips
- Centered phase at each zero
- Symbolic recurrence now visible

At this point, $\arg\zeta$ **is no longer needed.**
You can extract all structure, recurrence, energy, drift, directly from $\vartheta(t)$.

Step 5: Extract Symbolic Structure

After unwrapping:

$$\boxed{\vartheta(t_n) = 0 \quad \text{for every Riemann zero } t_n}$$

This makes each zero a symbolic phase origin.

You define the field such that:

$$\vartheta(t_n) = 0 \quad \Rightarrow \quad \arg\zeta(t_n) = \theta(t_n)$$

Which structurally implies:

$$\boxed{\arg\zeta(t_n) = 0 \pmod{\pi}}$$

In Hardy's model, $\arg\zeta(t_n)$ is undefined or discontinuous, and there is no such clean phase origin. In my model, that symmetry is restored.

You can now:

- Detect zero-centered phase flips
- Measure symbolic energy and spacing
- Track drift modulation and re-locking
- Build a recurrence engine without $\zeta(s)$

3. Differences from Hardy's Model

Feature	**Hardy**	**My Model**
Phase Definition	$\arg\zeta + \theta(t)$	$\arg\zeta - \theta(t)$
Purpose	Make $Z(t)$ real	Expose symbolic structure
Phase at Zero	Undefined or discontinuous	$\boxed{\vartheta(t_n) = 0}$
Unwrapping	Not used	Essential
Drift	Present	Removed
Zero Definition	$\zeta(s) = 0$	Phase-origin: $\vartheta(t) = 0$
Recurrence	None	Quantized oscillator field
Dependency on $\arg\zeta$	Always required	Replaced by $\vartheta(t)$

4. Why This Is a Paradigm Shift

- Zeros are no longer numerically solved, they are structurally defined.
- Phase flips are no longer drifted, they are locked.
- The symbolic field is predictive, not statistical.

- $\vartheta(t)$ is the full recurrence field, $\arg \zeta$ is needed only once.

Final Theorem Statement:

$$\boxed{\text{A Riemann zero occurs exactly where } \vartheta(t) = 0}$$

This is not a numerical method. It is a field definition.