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## Improvement of the Mathematical Model for Determining the Length of The Runway at the Stage of Aircraft Landing

Evgeniya Ugненко<sup>a,\*</sup>, Elena Perova<sup>a</sup>, Yelizaveta Voronova<sup>a</sup>, Gintas Viselga<sup>b</sup>

<sup>a</sup>*Department of Researches and Designing of Highways and Airports, Kharkiv National Automobile and Highway University, Ukraine*

<sup>b</sup>*Vilnius Gediminas Technical University, Lithuania*

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### Abstract

The main methods for determining the length of the runway at the landing stage are shown in the given article. The shortcomings of these methods are proved. Statistical data that shows the need of landing distance recalculation for calculation of the required length of the runway are provided. The advanced mathematical model for calculating the required length of the runway at the landing stage is shown.

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**Keywords:** runway, required length of the runway, landing distance, reverse, aircraft, post-landing run, coefficient of friction

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### 1. Introduction

The pressing problem of European airfields and airfields of Ukraine consists in increasing the capacity of the airports that assumes reconstruction of airfields.

There are several ways of reconstruction:

- strengthening of the runway covering;

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\* Corresponding author.

E-mail address: [j.ugnenko@mail.ru](mailto:j.ugnenko@mail.ru)

- increasing the runway length;
- construction of new runways.

One of the factors that results in increasing the potential risk of aviation incidents are adverse weather conditions. Statistical data confirms that during the rainfall the number of aircraft crashes increases [1].

Landing in low visibility is perhaps one of the most “exciting” ways to operate an aircraft but is certainly the most demanding. Such progress in civil aviation was made possible by huge improvements in aircraft automatic control systems over the last 30 years coupled with stringent requirements for airfield equipment and crew qualification [2].

The complexity of the task is such that even in ideal conditions, a perfect landing is virtually impossible, while any deviation from the ideal adds to the actual landing distance.

**Runway Conditions.** The maximum landing mass and the landing speed depend on the runway braking conditions. If these have been inaccurately reported or if the runway is wet or contaminated when its condition was reported as being dry, the landing distance achieved will be increased. The presence of standing water, snow, slush or ice on the runway has a particularly serious effect on landing performance and if it cannot be cleared, it must be reported as accurately as possible. Special techniques must be used by pilots when landing on contaminated runways.

**Weather Conditions.** The maximum landing mass and landing speed is calculated based on the reported wind and temperature. Significant changes to the reported conditions will affect the landing distance achieved. Strong crosswinds, turbulence and wind shear make handling difficult and are likely to result in an increased landing distance [3].

## 2. Literature survey

The purpose of the given work was to research the influence of the value of coupling coefficient of aircraft chassis with the airfield covering on the value of the full path of braking under different weather conditions.

Aircraft operational performances, at landing or take-off, are strongly dependant on runway surface conditions. Bad weather conditions may severely degrade runway surface condition. For obvious safety reasons, when such events appear, methods and means must be implemented to characterize runway surface condition and to provide pilots with relevant information about how well the surface will perform [4].

In many cases, the landing distance of the aircraft defines the requirements to the runway length. The minimum landing distance is reached during landing at the minimum safety speed which would exclude stalling and provide the sufficient controllability and possibility of following the missed approach procedure. Usually, the landing speed makes up a certain share of stalling speed and the minimum evolutive speed for the aircraft landing configuration. Actually, the touchdown occurs in case of defined values of the thrust coefficient. The exact values of these dimensions depend only on the characteristics of the aircraft and don't depend on its weight, height above the sea level and wind.

To provide the minimum landing distance at a set landing speed, the forces acting on the aircraft are to provide the maximum negative speedup (slowdown) during the post-landing run. To achieve the maximum value of negative speedup, certain control actions may be required.

It is necessary to distinguish the steering procedures providing minimization of a landing distance, and usual steering of the post-landing run in the conditions of a considerable stock of the runway length. The minimum landing distance is reached due to providing the constant maximum of negative speedup, or, in other words, hard work of the braking system of the aircraft. On the other hand, the ordinary post-landing run in conditions of a considerable stock of the runway length allows intensive use of aerodynamic resistance for minimization of wear of brakes and tires. If the aerodynamic resistance is sufficient for reducing the speed of the aircraft, it can be used instead of brakes at an early stage of a post-landing run. Brakes and tires of the aircraft chassis wear out from constant intensive use while the aerodynamic resistance is available and has no wear. It is possible to use the aerodynamic resistance only for reducing the speed to 60–70% of landing. At a speed smaller than 60–70% from the landing one, the aerodynamic resistance decreases so that it becomes useless, and for further slowdown it is

necessary to use the brakes. As the main objective of the post-landing run lies in reduction of speed, the thrust of the power plant must have the minimum positive value (or the maximum negative — in case of reverse thrust) [5].

### 3. Justification of need of improvement of the method of definition of the landing distance of the aircraft

As a general principle, it is now accepted that runway safety encompasses runway incursions and runway excursions. A runway excursion is defined as when an aircraft departs the runway either by veering off the side or by overrunning the runway end. Meanwhile a runway incursion is when the protected area of a surface of designated for the takeoff or landing of aircraft (paved or unpaved) entered by an aircraft, vehicle or person in error. According to research carried out by three groups, The Flight Safety Foundation, the Netherlands Lab R and IATA, runway excursions are now the most common type of event leading to accidents in commercial operations. These excursions are generally as a result of a poor approach leading to an abnormal landing or a loss of control on the runway either during takeoff or landing [6].

According to the International Civil Aviation Organization (ICAO), the growth in volumes of air transportations is followed by growth of the number of aviation incidents in recent years. This tendency confirms the forecasts of specialists of the Boeing company who note that in case of the continuing decrease of the relative indicator of the number of aviation incidents by 1 million flights, an increase in the number of aviation incidents is possible in the future.

Every week, worldwide there occur at least two aircraft incidents while moving along the landing strip, generally this rolling out of limits of a landing strip and a course deviation. The statistics shows that rolling-out is a fixed problem and their number hasn't decreased for the last 20 years. Similar cases lead to the breakdown of the plane and influence the safety of passengers. In addition to rolling out of limits of a landing strip, cases of unauthorized departure on a landing strip, as well as violation of the safe distance between aircrafts or other objects (other means of transport or people), which became the reasons for concern around the world, are noted (Table 1) [7].

Table 1. Aviation incidents connected with rolling-out on the runway.

Year	Flight number	Causes of aircraft accidents	Number of victims
09.07.06	plane «Airbus A310» of Siberia airline	It rolled out of the runway limits then there occurred ignition of the plane, owing to collision with an obstacle near the airfield.	125 people died
27.08.06	Trip Comair 5191 from Lexington to Atlanta (USA)	It was wrecked right after the take-off. The pilots by mistake tried to fly up from a strip whose length was insufficient for take-off.	49 people died
18.09.07	plane McDonnell Douglas MD-82	It crashed the land because of bad weather conditions. The airplane rolled out of the runway and smashed into a hill.	Not available
20.08.08	plane MD-82 of Spanair	While taking-off it rolled out of borders of the runway, collapsed and has lit up.	154 people died
16.08.10	passenger plane Boeing 737-200 the liner performed flight Bogota—San Andres	There occurred an aircraft accident when landing on the Colombian island. San Andres	1 person died
04.12.10	plane Ty-154M Dagestan Airlines	After landing the plane was rolled out of borders of the runway and collapsed.	2 people died
14.05.12	passenger plane	An aircraft suffered accident in attempt to make landing.	14 people died
03.06.12	plane Boeing 727	When landing the aircraft rolled out of borders of the runway and crashed into a minibus	10 passengers of the bus died
29.12.12	plane Tu-204	The aircraft rolled out of limits of the runway at Vnukovo airport, collapsed and partially burned down.	5 people died
17.11.13	plane Boeing 737 airlines Tatarstan	At landing approach suffered accident.	50 people died

#### 4. Principle of definition the aircraft landing distance

Proceeding from table 1, it is visible that quite often aircrafts crashes are connected with an insufficiently long runway. Therefore, this question should be considered in more detail. As is well-known from the main sources, movement of the aircraft (A) on the take-off landing strip (TOLS) at the running start up stages before taking-off and running after landing can be described by the following system of equations:

$$R - X - F - mg \sin \theta = m \frac{dV}{dt}, \quad (1)$$

$$-mg + Y + N = 0.$$

The equation of the longitudinal movement can be presented in the following form [2]:

$$\frac{dV}{dt} = \frac{R}{m} - fg + \frac{(fC_y - C_x)\rho_0(K_v(V+u))^2 S}{2m} - g \sin \theta, \quad (2)$$

where:  $V$  – traveling speed of the movement VS;  $K_v$  – coefficient of air speed transfer to the indicator;  $u$  – longitudinal component of wind speed;  $R = R(\alpha_{rud}, P, t^\circ, V)$ , or  $R = R(n, P, t^\circ, V)$  – total thrust of engines;  $\alpha_{rud}$  – the operating mode of engines determined by the provision of the control levers of engines (CLE);  $n$  – average value of engine rotation;  $P$  – pressure of external air;  $t^\circ$  – temperature of external air; other designations according to [2].

Then the expression of run length, according to [2] is as follows:

$$L = \int_{V_n}^{V_k} \frac{V}{\frac{R}{m} - fg + \frac{(fC_y - C_x)\rho_0(K_v(V+u))^2 S}{2m} - g \sin \theta} dV_p. \quad (3)$$

The solution of equations (1) and (2) is necessary, for example, for definition of a required distance of the run (up to a full stop of LS) on landing or a required distance of the interrupted take-off, and in connection with high relevance of the problem of preventing the rolling-out of LS on landing or at the interrupted take-off, the necessity to find a solution with, perhaps, higher precision is obvious [8].

For creating an adequate mathematical model of aircraft movement during the run it is necessary to define the dependence  $R = R(n, P, t^\circ, V)$ , as well as the values of all included in (2) coefficients.

In each case of a run (running start) only the limits of integration of  $V_n$  and  $V_k$  set by the researcher, the well-known size of acceleration of gravity of  $g = 9.81 \text{ m/s}^2$  (for middle latitudes),  $\rho_0 = 1.225 \text{ kg/m}^3$  can be unambiguously determined by the table of the International Standard Atmosphere (ISA) and the area of a side panel of the plane  $S$  – from documentation of the aircraft. The coefficient of the transfer of air speed to indicator  $K_v$  can be defined on the basis of materials. As for a runway discharge angle  $\theta$ , the value of a bias given in help data of the airfield is average and as for the accuracy is not always sufficient to research the objectives, however, it is possible to use, for example, data where the values of relative height of points of a surface on the set geographical coordinates are given, and also the mode of measurement of distances between surface points is realized [9].

Coefficients of aerodynamic forces of  $C_x$  and  $C_y$  are set by the entity developer of the aircraft.

Concerning the provided coefficient of friction of swing  $f$  for running start and run of the plane (in the absence of use of brakes of wheels of the chassis) statistical values from 0.02 to 0.06 are given in various sources. Generally,  $f$  depends on the size of sinking of chassis pneumatics, speed of the movement and characteristics of a surface of the runway. In [2] it is noted that use for calculation of the constant coefficient of friction, averaged according to the speed for all types of aircrafts, when calculating the running start and the run gives good convergence with results of

flight tests. It allows simplifying a calculation procedure. However for the cases connected with threat of rolling-out of the aircraft out of borders of the runway, such approach can be insufficiently exact.

Thus, there is a problem of determination of the specified coefficient of friction of swing for specific conditions of LF run. This task can be solved, for example, with the use of records of flight information (FI) obtained from real flights.

After conducting pilot studies and calculations by the leading engineer of the Center of processing and analysis of flight information of JSC Volga-Dnieper V. V. Zavershinsky, it was defined that the coefficient of friction of swing at the movement of the aircraft on the runway, with other things being equal, significantly depends on the mass of the aircraft, and this dependence can be received on the basis of the analysis of data of flight information and used, for example, when calculating the specified distance of running start, the interrupted take-off, as well as the landing distance of an aircraft on the run.[10]

## 5. Method of determination of the length of the runway in settlement conditions for set aircraft norms of Ukraine

The runway length for set aircrafts in settlement conditions is considered proceeding from the landing length of a runway in standard conditions on application of correction coefficients on local conditions for the scheme “take-off” and “landing”.

Landing length of a runway according to the scheme “landing” is determined by a formula:

$$L_{noc} = L_{noc}^0 \cdot K_i \cdot K_{pt}, \quad (4)$$

where:  $L_{noc}$  – landing length of a runway for landing in settlement conditions, m;  $L_{noc}^0$  – the required landing length of a runway when landing the aircraft in standard conditions, m;  $K_i$  – the settlement coefficient considering a longitudinal bias of a runway;  $K_{pt}$  – the correction coefficient considering simultaneous influence of settlement air temperature and height of an arrangement of airfield.

These coefficients are determined by formulas:

$K_i$  – the coefficient considering an average longitudinal bias of a runway is determined by formulas:

$$K_i = 1 + 5i_{cp}, \text{ provided that } L_{vzl}^0 \leq 1000 \text{ m}, \quad (5)$$

$$K_i = 1 + 9i_{cp}, \text{ provided that } L_{vzl}^0 > 1000 \text{ m}, \quad (6)$$

$K_{pt}$  – the correction coefficient considering simultaneous influence of settlement air temperature and height of an arrangement of airfield:

$$K_{pt} = \frac{2,64(270 + 1,07t_{13})}{P_n}, \quad (7)$$

where:  $t_{13}$  – average monthly air temperature in airfield at 13 o'clock the hottest month in a year;  $P_n$  – air pressure on height, mm Hg.

Value of parameter air pressure by height of  $P_n$  is determined by interpolation between the values of height of  $H$  and is accepted depending on height of airfield arrangement according to the formula:

$$P_n = P_0 \cdot \left(1 - \frac{H_{aer}}{44300}\right)^{5,256}, \quad (8)$$

where:  $P_0$  – pressure at height  $H = 0$  m,  $P_0 = 1.01 \cdot 10^6$  Pa.

For height  $H \leq 1000$  m above the sea level a simplified dependence can be used:

$$P_n = 760 - 0.0865 \cdot H_{aer}, \quad (9)$$

where:  $H_{aer}$  – the highest point of a surface of airfield above the sea level, m. [11].

## 6. Method of determination of a required length of the runway by ICAO technique

In the absence of the corresponding flight manual the length of the runway shall be determined by means of application of general correction coefficients (the amendment to runway length on excess, temperature and a bias). At the first stage, the initial length of the runway should be chosen so that it should meet the operational requirements of planes for which this runway is intended. The initial length of the runway is called the runway length necessary for take-off or landing in case of standard atmospheric conditions, zero excess, lack of wind and a zero bias of the runway chosen for the purposes of planning an airfield.

1. The initial length of the runway should be increased by 7% for each 300 m of excess.
2. The runway length determined in item 2 should be increased by 1% for each 1 °C of that number of degrees by which settlement air temperature around the airfield exceeds the temperature of the standard atmosphere for this excess.
3. When the initial length of the runway constitutes 900 m or more, the length of the runway should be increased by 10% for each 1% of a bias of the runway.

The runway length used for landing adjusted for excess is determined by the formula:

$$L_{noc} = \left( L_{noc}^0 \cdot 0.07 \cdot \frac{H_{aer}}{300} \right) + L_{noc}^0, \quad (10)$$

where:  $L_{noc}^0$  – the runway length necessary for landing at sea level under standard atmospheric conditions;  $H_{aer}$  – excess of airfield [12].

## 7. Advanced method of determining the length of the run at aircraft landing

Landing is called the slowed-down movement of the plane from the moment of flight at the height of 15 m to the full stop or speed of taxiing.

Calculation of landing characteristics is made for standard atmospheric conditions and a normal landing configuration of the aircraft (Fig. 1).

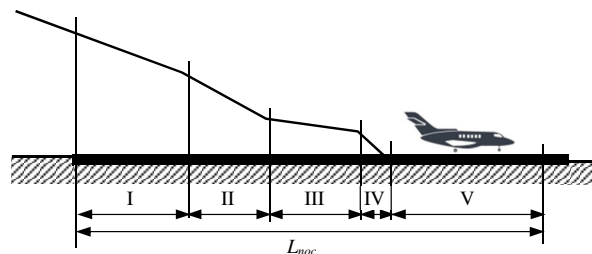


Fig. 1. Scheme of aircraft landing: I – prelanding drop, II – alignment, III – keeping, IV – landing (parachuting), V – run.

In the course of performance of work the following landing characteristics of the aircraft are defined:  $V_{noc}$  – landing speed, km/h;  $j_{T.cp.}$  – average speedup of braking when landing;  $L_{np}$  – run length.

Besides, the influence on the length of the run of the following operational factors is estimated:  $m$  – landing mass of the plane;  $f_{mp}$  – friction coefficient size.

The landing speed of the plane is a horizontal component of aircraft speed at the time of contact of wheels with the runway.

The run length – the distance passed by an aircraft from the moment of runway contact to the full stop (taxing speed) [1].

Plane run length when landing is expressed through the landing speed of  $V_{noc}$  and average speedup of braking

$$L_{np} = \frac{V_{noc}^2}{2j_{m.cp}}. \quad (11)$$

Speedup of braking depends generally on two forces operating aboard the plane at a run:

- front resistance  $X = C_x \cdot S \cdot \frac{V_{noc}^2}{2}$ ;
- friction of wheels  $F = f(mg - Y)$ ,

which depends on the coefficient of friction (clutch) of wheels with the surface of the runway and at most a safe pressure of the plane on the runway which in turn is defined by the landing mass and the upward force of the plane. Force  $F$  with reduction of speed continuously increases owing to the reduction of the upward force.

The average value of slow-down force  $R_{cp}$  in the course of the run can be defined as an arithmetic average of initial and final values of forces  $X$  and  $F$ :

$$R_{cp} = \frac{1}{2}(X + F). \quad (12)$$

Then the average value of acceleration of braking is the following:

$$j_{T.cp} = \frac{R_{cp}}{m}. \quad (13)$$

Besides, the length of run  $L_{np}$  is influenced by the speed and wind direction  $W_{eb}$ , as well as the bias exerted on the condition of the runway.

The landing speed is calculated by the formula:

$$V_{noc} = \sqrt{\frac{2mg}{C_{y.noc} \cdot \rho S}}, \quad (14)$$

where:  $m$  – the landing mass of the plane;  $g$  – acceleration of gravity;  $C_{y.noc}$  – coefficient of the carrying power of the wing when landing (is determined by the schedule of Fig. 2

for an angle of attack =  $6^\circ$ );

- $\rho$  – mass density of air;
- $S$  – covered area,  $m^2$ .

The average of acceleration of braking when landing –  $j_{T.cp}$  is calculated by the formula:

$$j_{T.cp.} = \frac{0,5 \left[ C_x \frac{\rho V_{noc}^2}{2} S + f_{mp} \left( mg - C \frac{\rho V_{noc}^2}{2} S \right) \right]}{m}, \quad (15)$$

where:  $\rho$  – mass firmness of air, kg/m<sup>3</sup>;  $V_{noc}$  – landing speed, m/s;  $S$  – area of a side panel, sq.m;  $f_{mp}$  – friction coefficient size;  $m$  – landing mass of the aircraft, kg;  $g$  – acceleration of gravity, m/s.

Aircraft run length when landing –  $L_{np}$ .

$$L_{np} = \frac{V_{noc}^2}{2j_{T.cp.}}, \quad (16)$$

where:  $V_{noc}$  – landing speed of the plane, m/s;  $j_{T.cp.}$  – average acceleration of braking when landing, m/s<sup>2</sup>.

As while determining the average speedup of braking of the plane when landing,  $J_{mp}$  is the size of coefficient of friction, which takes into account the condition of a dry covering. And for obtaining a specified aircraft run at landing, it is also necessary to consider the presence on the tarmac of water, snow, ice, as well as stratifications of rubber in zones of contact of tire pieces of aircraft chassis with the surface of the runway, this indicator should be specified by means of input of the coefficient of adhesion which as much as possible describes the condition of covering [5].

$$j_{T.cp.} = \frac{0,5 \left[ C_x \frac{\rho V_{noc}^2}{2} S + f_{mp} \cdot K_{c.VPP} \left( mg - C \frac{\rho V_{noc}^2}{2} S \right) \right]}{m}, \quad (17)$$

where:  $K_{c.VPP}$  – coefficient  $t$  of a condition of airfield covering.

We present  $L_{np}$  at landing into the formula of defining the length of the runway when landing according to the standards of Ukraine and obtain:

$$L_{noc} = \frac{V_{noc}^2}{2j_{T.cp.}} \cdot K_i \cdot K_{pt}. \quad (18)$$

Results of calculation of length of a runway are presented in Table 2.

Table 2. The run length.

Plane	The run length					
	dry	air standard conditions	wet	snow	slush	ice
Boeing 747	2180	2189	2286	2286	2408	2424
Airbus A380	2900	2925	3104	3104	3380	3418
AH - 225	1900	1928	1985	1985	2054	2068
AH-22	1040	1112	1160	1160	1235	1250
Airbus A340-600	1926	1928	1976	1976	2089	2097



## 8. Conclusions

1. To compensate the reduction of ability to braking under adverse conditions on the runway (such as moisture or slippery state), amendments to characteristics in the form of increase in length of the runway necessary for landing, or reduction of admissible take-off or landing weight are applied. For compensation of the lowered traveling controllability the admissible component of a side wind has to be lowered.
2. As all the known mathematical models consider coupling of the aircraft with the covering in ideal conditions, it was decided to improve the mathematical model of defining the required runway length, taking into account the coupling coefficient.
3. As a result of mathematical modeling the specified mathematical model of defining the defining the required landing distance.

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