Midterm Assignment: Building a Star

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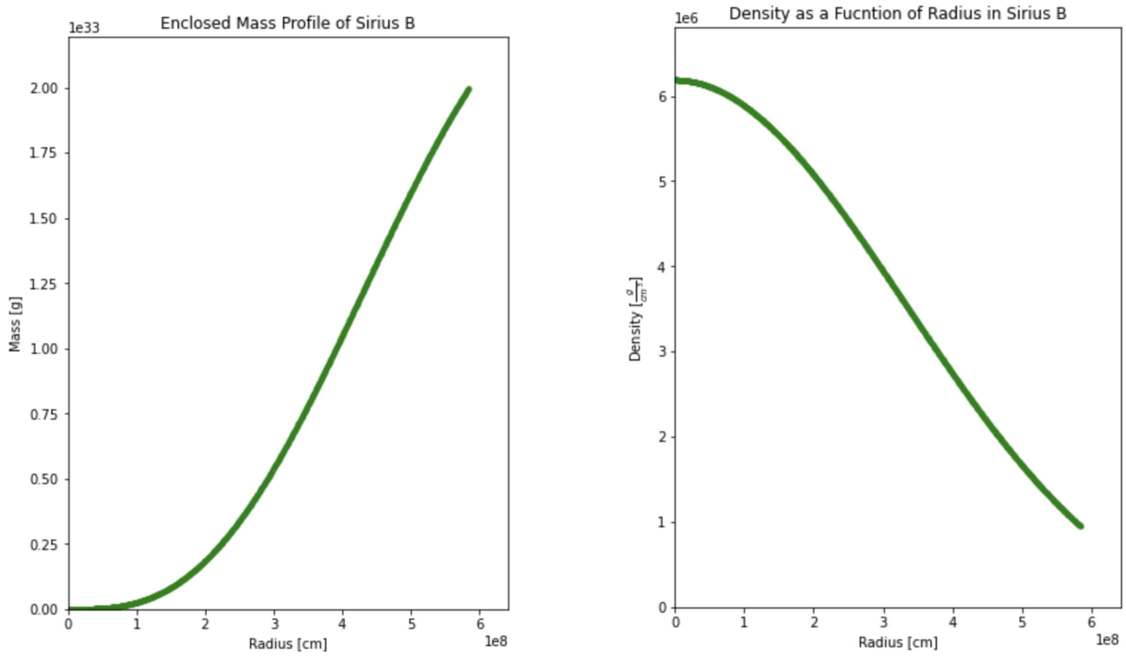
1. The enclosed mass of the white dwarf Sirius B is shown in **Figure 1 (a)** and the density as a function of radius is shown in **Figure 1 (b)**. The code that produced these plots used the given density at r = 0, $\rho(0) = 6.19 \cdot 10^6 \frac{g}{cm^3}$, which was set as the first value in the density array. This value was then used to calculate the pressure at r = 0 using the non-relativistic equation of state for a star dominated by electron degeneracy pressure. This calculation was made instead of making an initial guess for pressure. Additionally, each differential equation was converted to a difference equation (e.g. $\delta r \rightarrow \Delta r$), allowing for numerical evaluation of the system. The Δr in my system was constant, simple as the radius of Sirius B divided by the total number of times my code would iterate. My code first, solved for the change in mass per spherical shell, then added this change to the previous iteration, such that $m_i = \Delta m + m_{i-1}$. This allowed for the change in pressure to be calculated for a given spherical shell, then the total pressure at that shell, and subsequently the density at that shell.

2. The enclosed mass, density profile and temperature profile of the sun are shown in **Figure 2**. The same process of iteration for finding the mass, density and pressure throughout the Sun was the same as in Sirius B, with the exception of using the equation of state for a star dominated by an ideal gas pressure, thus the sun is dependent on temperature changes. Energy transport is dominated by radiative transport in which the opacity can be described by Thomson scattering, requiring a hydrogen fraction, and in the sun the hydrogen fraction is $X = 0.7$. In order for energy transport to be calculated, the change in luminosity at each spherical shell is solved for using the energy generation equation, which is dominated by p-p chain energy production. Additionally the value for μ is found by using $X = 0.7$, $Y = 0.28$ and $Z = 0.02$ for the chemical abundances in the Sun.

3. As indicated by question 3, the temperature gradient produced from the assumptions in question 2 is not representative of the actual gradient in the sun. Using the inequality that says if convection will happen at a given shell, the convective regions from the model of the Sun in problem 2 can be seen in **Figure 3**. It can be seen that for both a constant adiabatic exponent and an adiabatic exponent that is numerically found using difference equations produces convective regions only at the center of the Sun. This does not match reality as convection in the Sun happens at $r > 2 \cdot 10^{10} cm$, which is nearly the opposite of what is seen in **Figure 3**.

4. In problem 2 we assume that the entire star is dominated by an ideal gas equation of state, energy transport dominated by radiative diffusion via electron scattering, energy production dominated by the p-p chain, that the inner (r = 0cm) temperature is $T = 1.47 \cdot 10^7 K$ and that the inner (r = 0cm) density is $\rho = 149 \frac{g}{cm^3}$. Since the inner density and temperatures are free parameters in this model, these assumptions can be assumed to be correct as the final mass produced is close to being correct and these values can varied to better fit our model. Additionally, we know from **Homework 3** that our Sun is dominated by p-p chain (~ 0.916 of energy produced by p-p chain), so our energy production assumption is also quite close to reality as well.

   This leaves our pressure support and energy transport assumptions as the possible sources of error. Our Sun should contain not just pressure support from ions, but also from radiation. The assumption of using the ideal gas equation of state works for the denser, hotter regions of the sun, however in the cooler, less dense regions of the Sun it would be a better assumption to

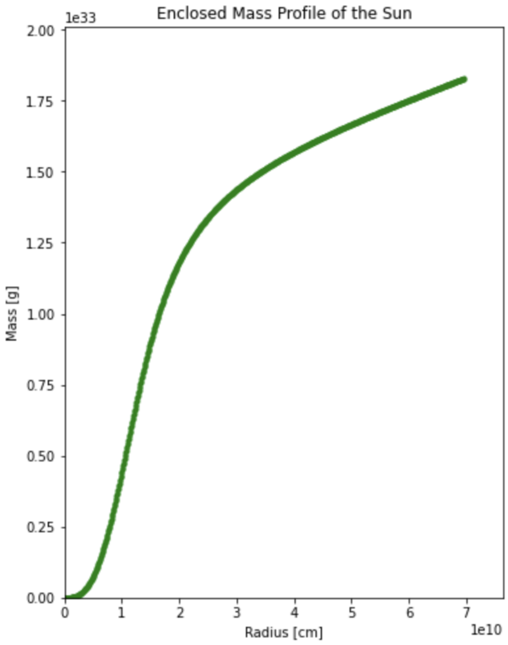


(a) Enclosed Mass

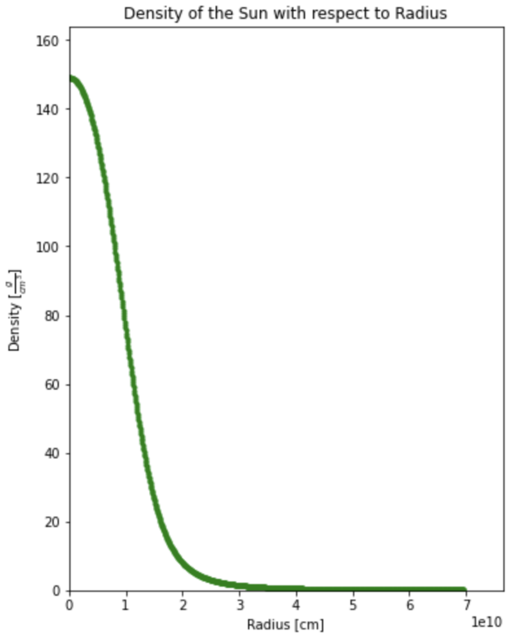
(b) Density Profile

Figure 1: The enclosed mass **(a)** and density profile **(b)** of the white dwarf Sirius B.

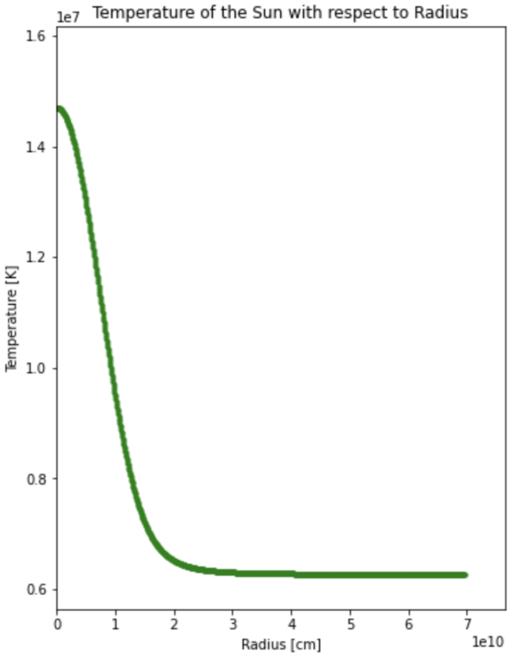
describe the pressure support with the radiation equation of state. The relationship between temperature and density can be seen in the plot shown in **Figure 4**. As for the energy transport, the model in problem 2 uses the opacity described by Thomson scattering for the entire Sun. However, it would be a more realistic model if the opacity changed from Thomson scattering to both bound-bound and bound-free absorption as this is the opacity that best describes the surface of stars.



(a) Enclosed Mass



(b) Density Profile



(c) Temperature Profile

Figure 2: The enclosed mass **(a)**, density profile **(b)** and temperature profile **(c)** of the Sun.

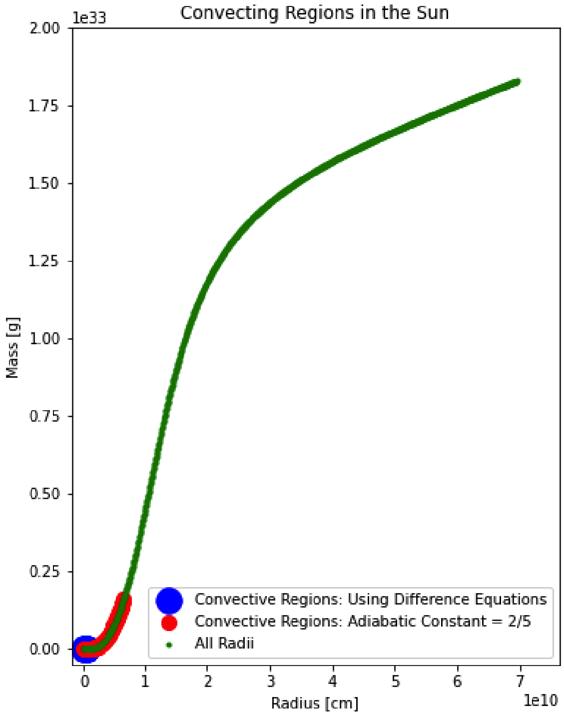


Figure 3: The enclosed mass of the Sun versus radius. The red being regions of the Sun unstable for convection found using $\frac{dlog(T)}{dlog(P)} = \frac{2}{5}$, the blue being the regions of the Sun unstable to convection for the $\frac{dlog(T)}{dlog(P)}$ values derived using the model from problem 2 and the green shows all regions of the Sun.

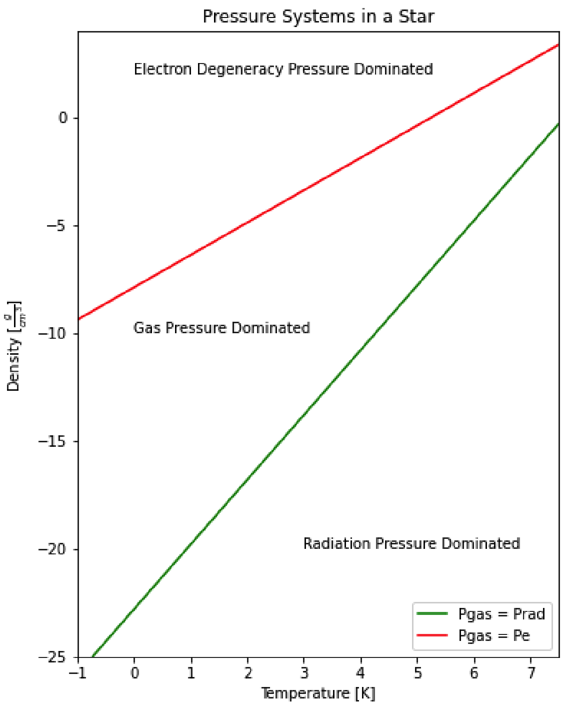


Figure 4: The intersections of the density and temperature for where electron degeneracy pressure is equal to ideal gas pressure and where ideal gas pressure is equal to radiation pressure.