Geometric Transformations

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12 – 14 July 2006

Homography

- The most general relationship between points on 2 planar images is the Homography:

$$\mathbf{u} \quad = \quad \mathbf{H} \quad \mathbf{x}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$u' = u / w, \quad v' = v / w,$$

$$x' = x / z, \quad y' = y / z,$$

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Common Transformations

- Translation, rotation, scaling, and shearing can all be expressed using homographies.

- To compute a new image:
  - First, determine the size of the output image.
  - Use the destination scan algorithm to calculate the pixel values of the output.

Destination Scan

1. for $u = 1$ to image width

2.   for $v = 1$ to image height

3.   Find (x,y), the point that (u,v) maps to in the input image, using the appropriate homography.

4.   Compute c, the color at (x,y). Interpolate where necessary. If (x,y) falls outside input image, let $c =$ black.

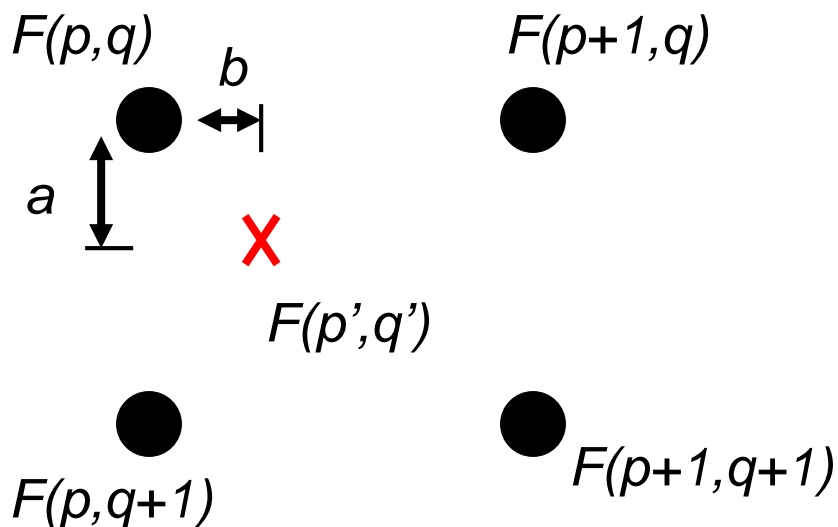5.   Set the color at (u,v) to c.

Nearest-neighbor Interpolation

- Use the value at the nearest corner

$F(p,q)$ $F(p+1,q)$

b

a

X

$F(p',q')$

$F(p,q+1)$ $F(p+1,q+1)$
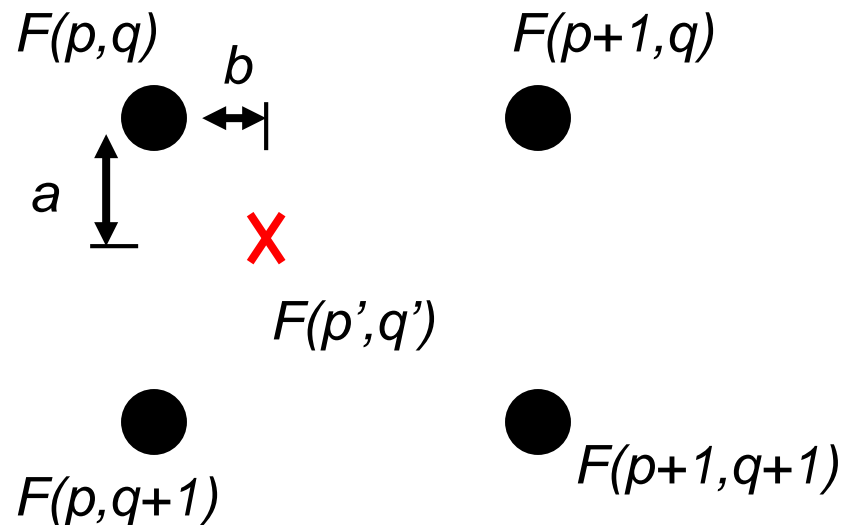
$F(p',q') = F(x,y)$ where (x,y) is the nearest corner

Bilinear Interpolation

- Weighted average of values at the 4 corners
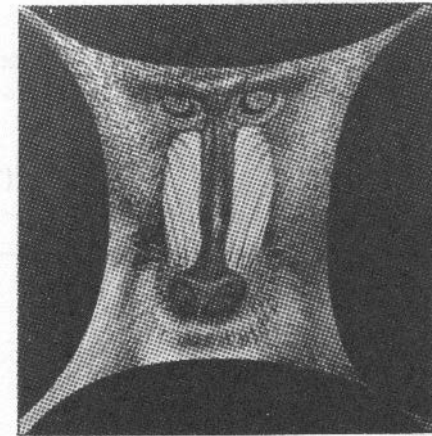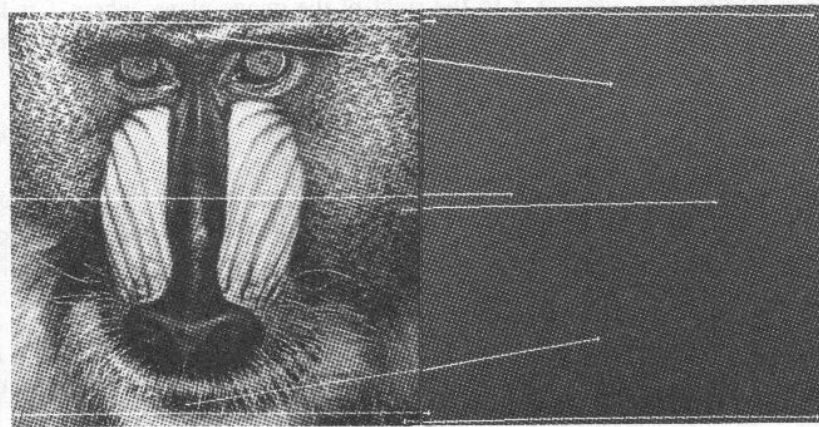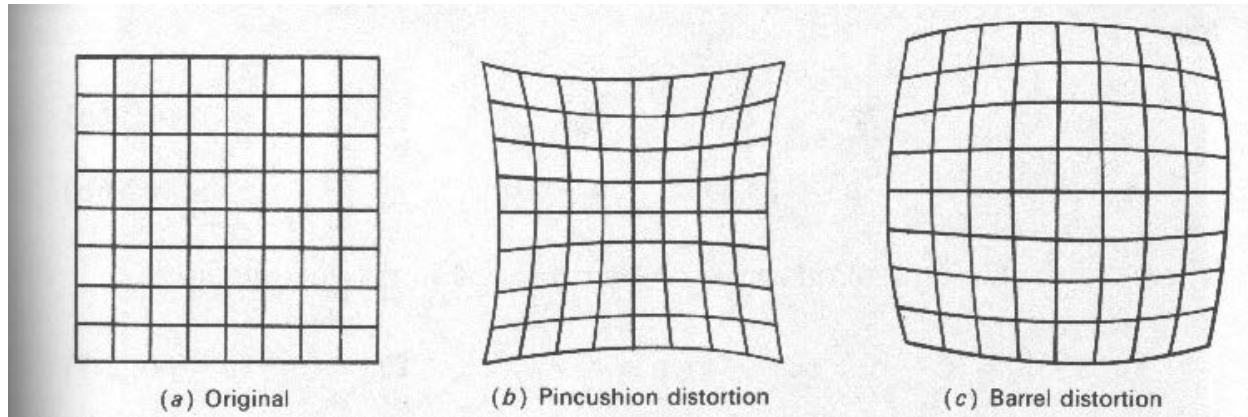 - Weights proportional to areas

F(p,q)   *F(p+1,q)*

b

a

X

F(p',q')

F(p,q+1)   *F(p+1,q+1)*

$$F(p',q') = (1-a)[(1-b)F(p,q) + bF(p+1,q)]$$
$$+ a[(1-b)F(p,q+1) + bF(p+1,q+1)]$$

Comments

- Bicubic Interpolation
  - Weighted average of 16 neighboring corners.
- If color image, process each channel independently.
- Exercise will explore this.

Non-planar Transformations

(a) Original   (b) Pincushion distortion   (c) Barrel distortion

(a) Source   (b) Destination   (c) Warped image

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Non-planar Transformations

- Polynomial warping

$$u = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2;$$

$$v = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2;$$

- In matrix form:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_5 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \\ x^2 \\ xy \\ y^2 \end{bmatrix}$$

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Summary

- Geometric transformations affect the shape of the image.

- 2D planar transformations can be expressed as a 3x3 matrix called a Homography.
  - and using homogeneous coordinates

- Non-planar transformations are possible too.

- These transformations relate the *coordinates* between the input and output pixels.
  - Pixel colors are determined using interpolation.

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