Mathematical Morphology

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Morphology Operations

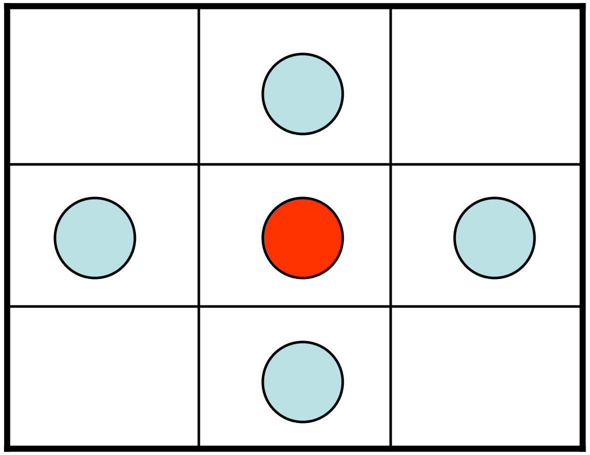
- Morphology
  – a branch of biology that deals with the form and structure of animals and plants.
- Mathematical morphology (MM)
  – a tool for extracting image components that are useful in the representation and description of region shape.
- The language of MM is set theory.
- Sets in MM represent the shapes of objects in an image.

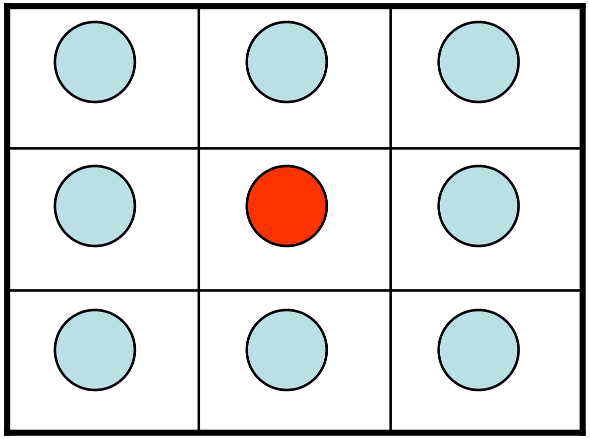
Binary Images

- Morphology works on binary images
  - But may be extended to grayscale images
- A binary image has pixel values = 0 or 1
- Typically 0 = background, 1 = foreground
- Binary images can be obtained from grayscale images by thresholding.
  - Gray values $< \theta$ are mapped to 0
  - Gray values $>= \theta$ are mapped to 1

Who is my neighbor?

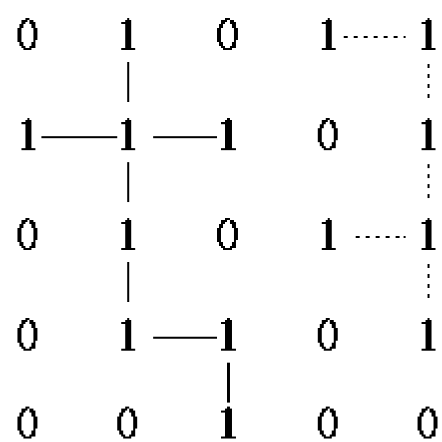
- 4-neighbor
- 8-neighbor





Connectivity

- Two pixels are *4-connected* if there is a *path* (unbroken sequence) of 4-neighbors between them.
  - Similar definition for 8-connectedness
- A *connected component* is a group of pixels that are connected
  - Can be 4-connected or 8-connected

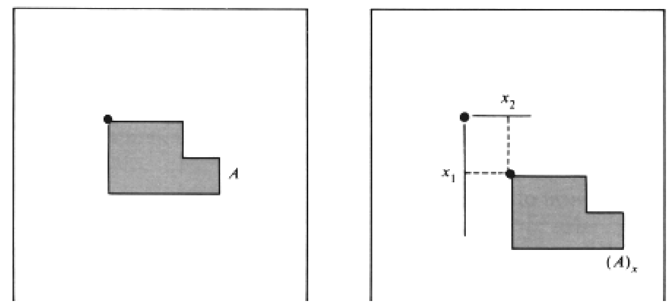


Basic Definitions

Let *A* and *B* be sets in the spatial plane with components $a=(a_1, a_2)$ and $b=(b_1, b_2)$

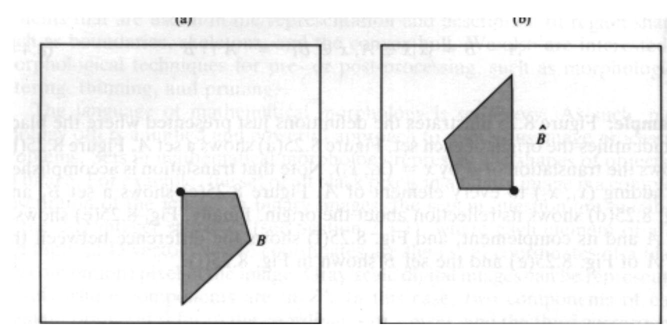
The translation of *A* by $x=(x_1, x_2)$, denoted by $(A)_x$ is defined as

$$(A)_x = \{c \mid c = a + x, \text{ for } a \in A\}$$



The reflection of B is defined as

$$\hat{B} = \{x \mid x = -b, \text{ for } b \in B\}$$



Basic Definitions

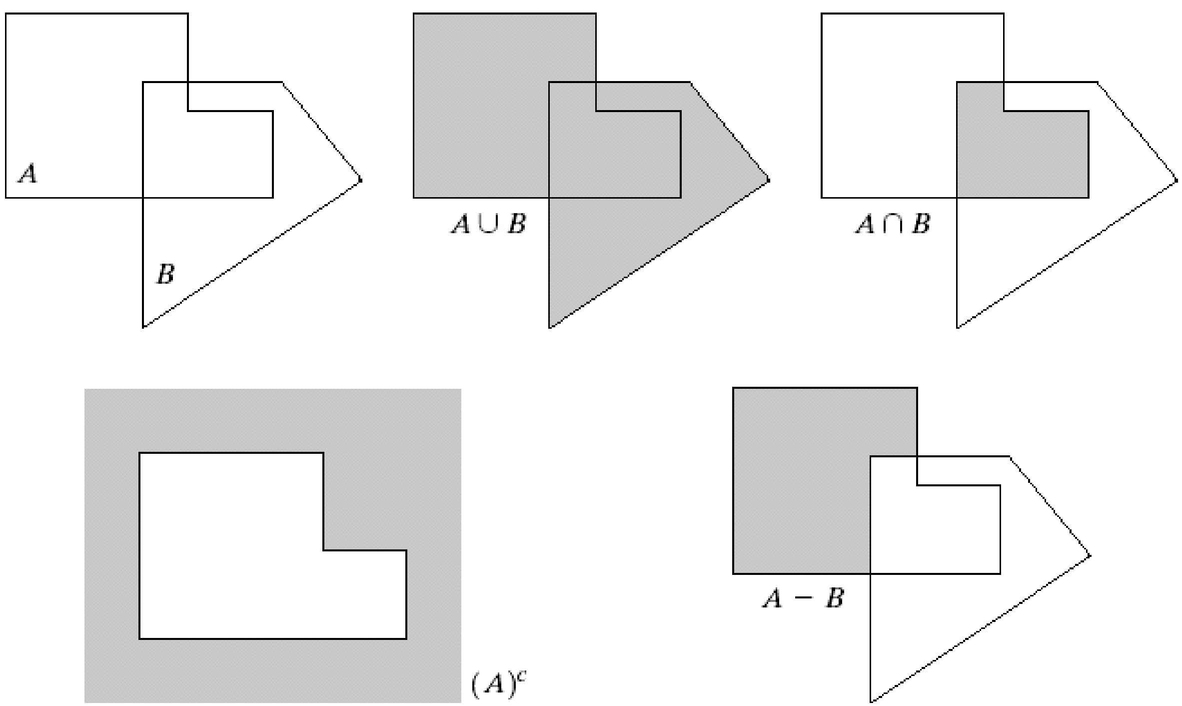


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Dilation

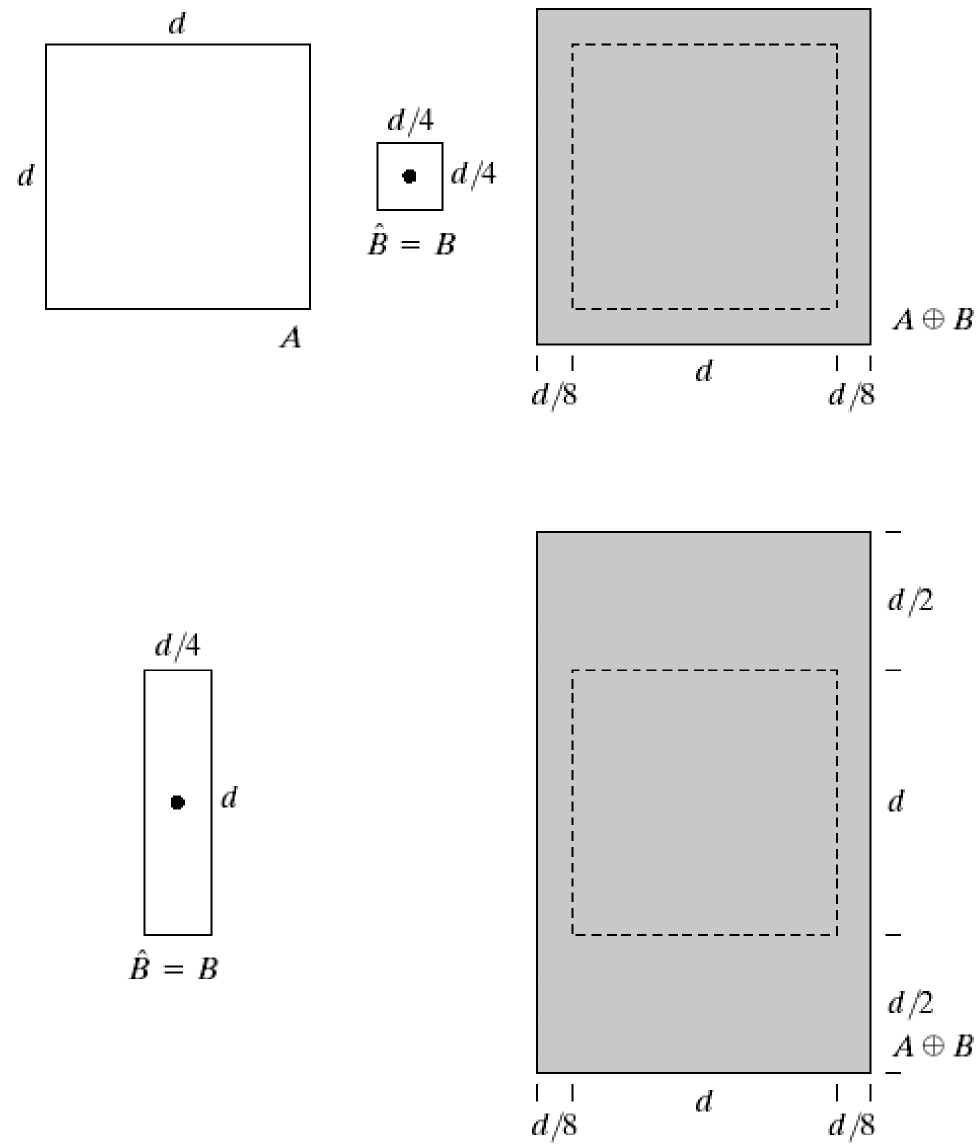
Let *A* and *B* be sets in the spatial plane and ϕ be the empty set, dilation of *A* by *B* is

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \phi\}$$
$$= \{x \mid \left[(\hat{B})_x \cap A\right] \subseteq A\}$$

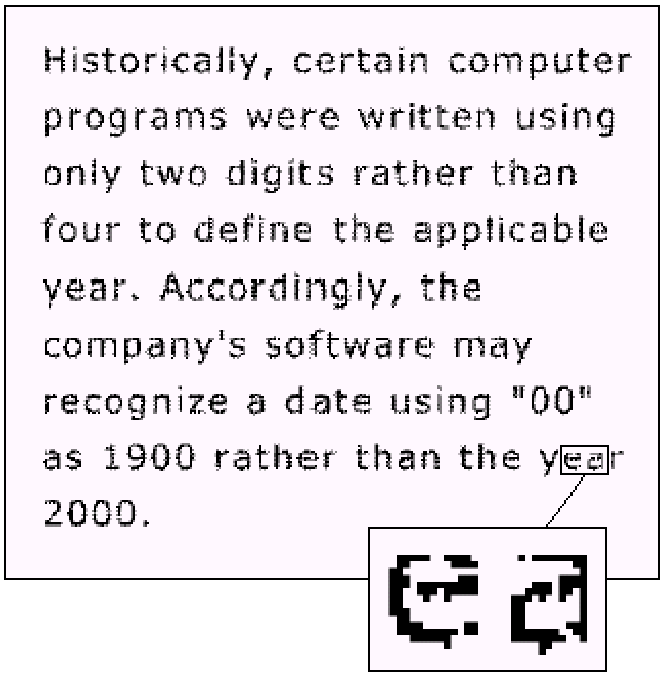
Hence $A \oplus B$ is the set of all x displacements such that *B* and *A* overlap by at least one non-zero element.
B is called the structuring element.

a	b	c
d		e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Dilation



0	1	0
1	1	1
0	1	0

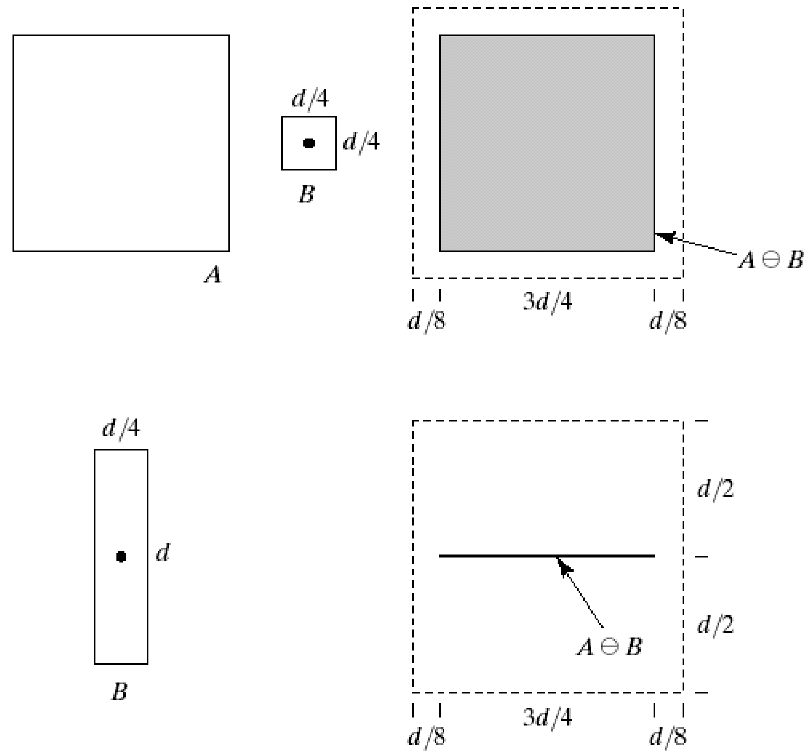
a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion

Erosion, *A¤B is the set of all points x such that B, translated by x, is contained in A.*

$$A¤\, B = \{x \mid (B)_x \subseteq A\}$$



| a | b | c |
| d | | e |

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Opening

Closing

Duals

$$A \circ B = \left(A \bullet B\right) \oplus B$$

$$\left(A \bullet B\right)^{C} = \left(A^{C} \circ \hat{B}\right)$$

$$A \bullet B = \left(A \oplus B\right) ¤ B$$

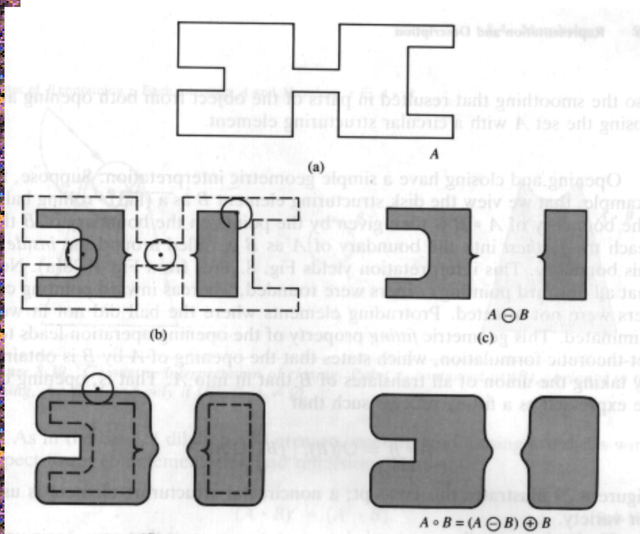


Illustration of opening

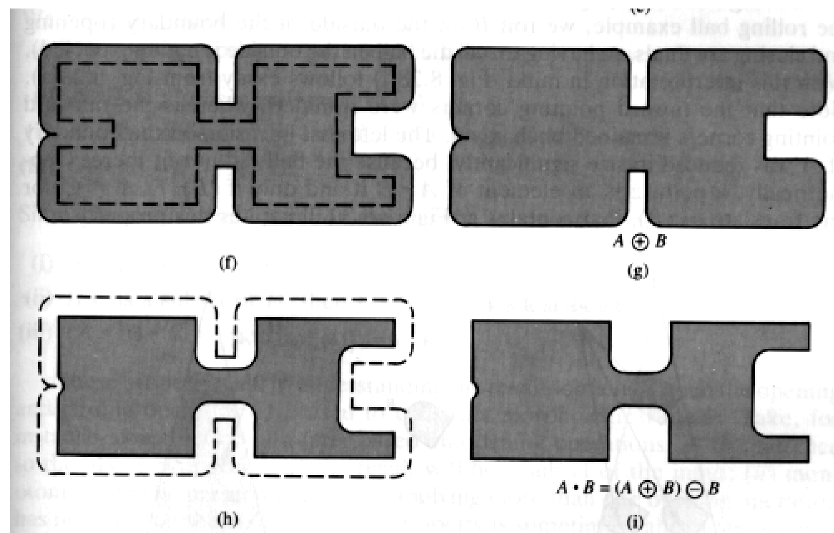
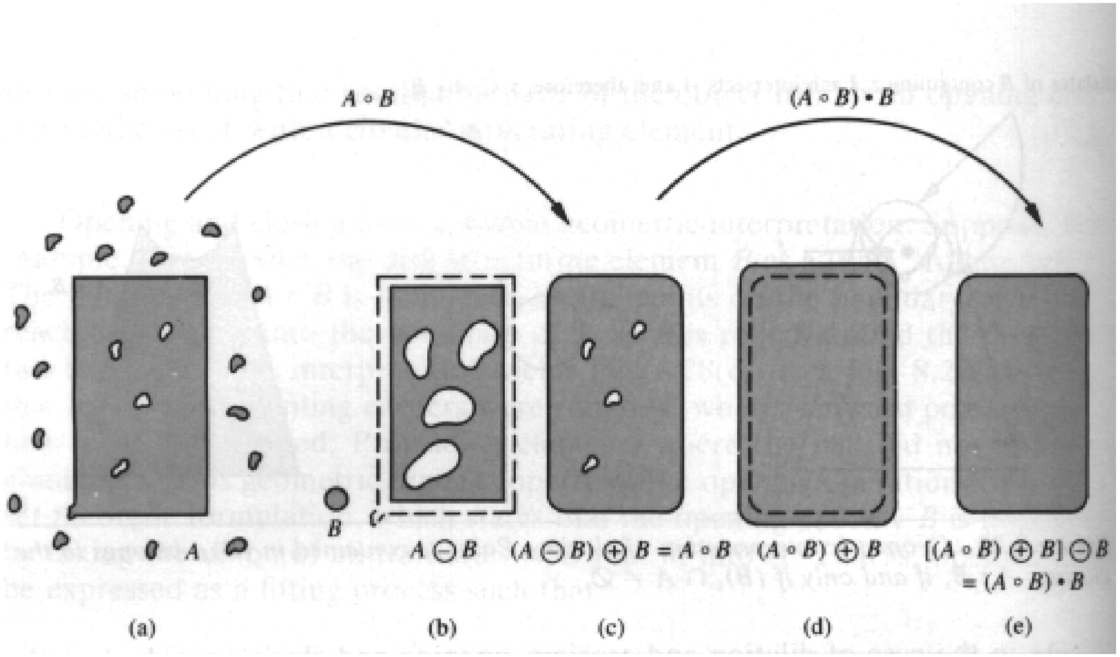


Illustration of closing

Morphological filtering

$(A \circ B) \bullet B$ can be used to eliminate the noise and its effect on the object.

Noise pixels outside object area are removed by opening with *B* and noise pixels inside object area are removed by closing with *B*



Example

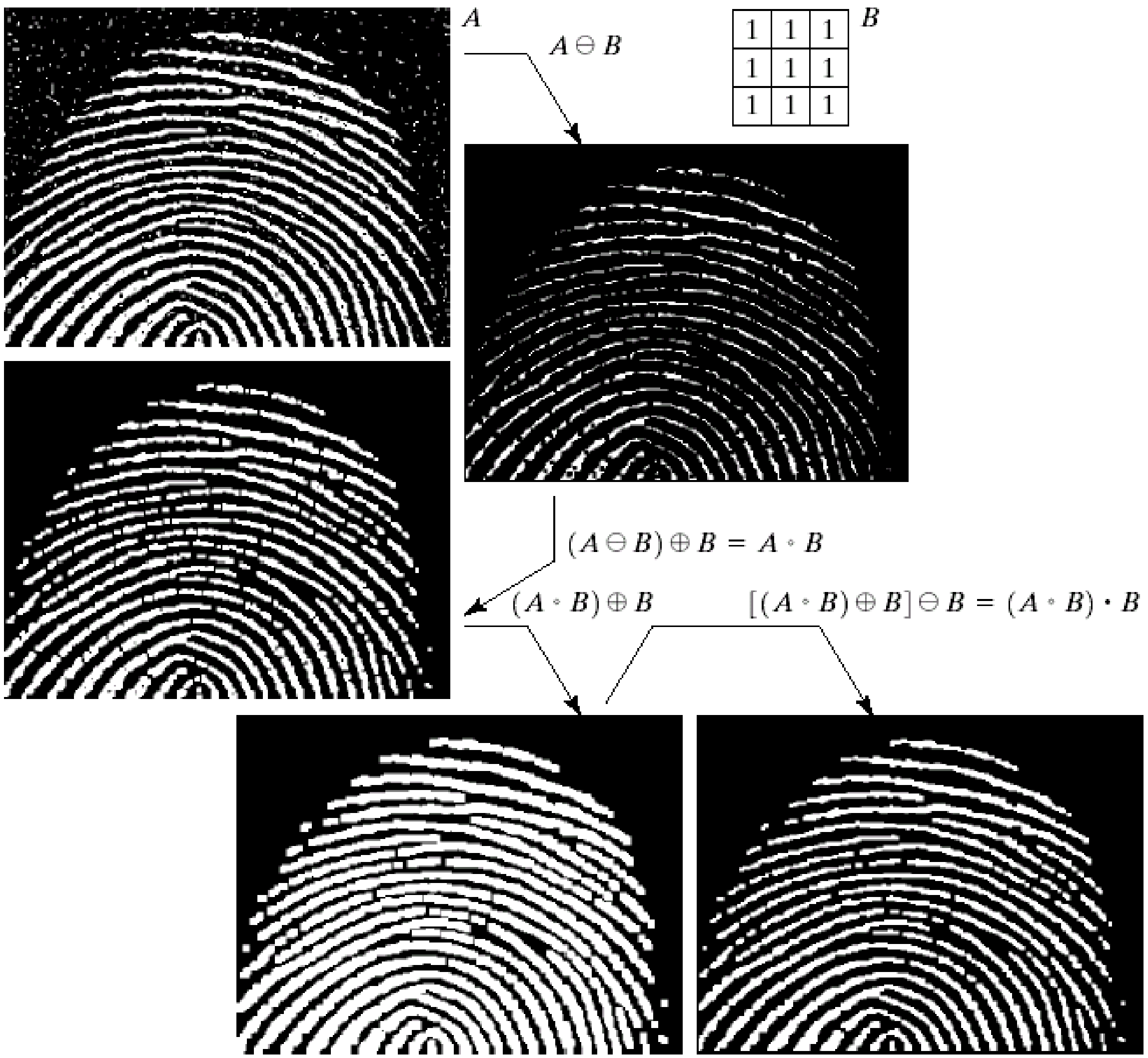


FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

<u>Hit-or-Miss Transform</u>

a basic tool for shape detection.

If the shape to be detected is found in the image, the output is a "hit".

Example

To find a square shape (represented by *X*) by Hit-or-Miss Transform, we can perform

$$A \S B = (A \text{¤} X) \cap [A^c \text{¤} (W - X)]$$

where *B* = (*X*, (*W*-*X*)), *X* is the set formed from elements of *B* associated with an object, *W* is the window enclosing *X*, and (*W*-*X*) is the set of elements of *B* associated with the corresponding background.

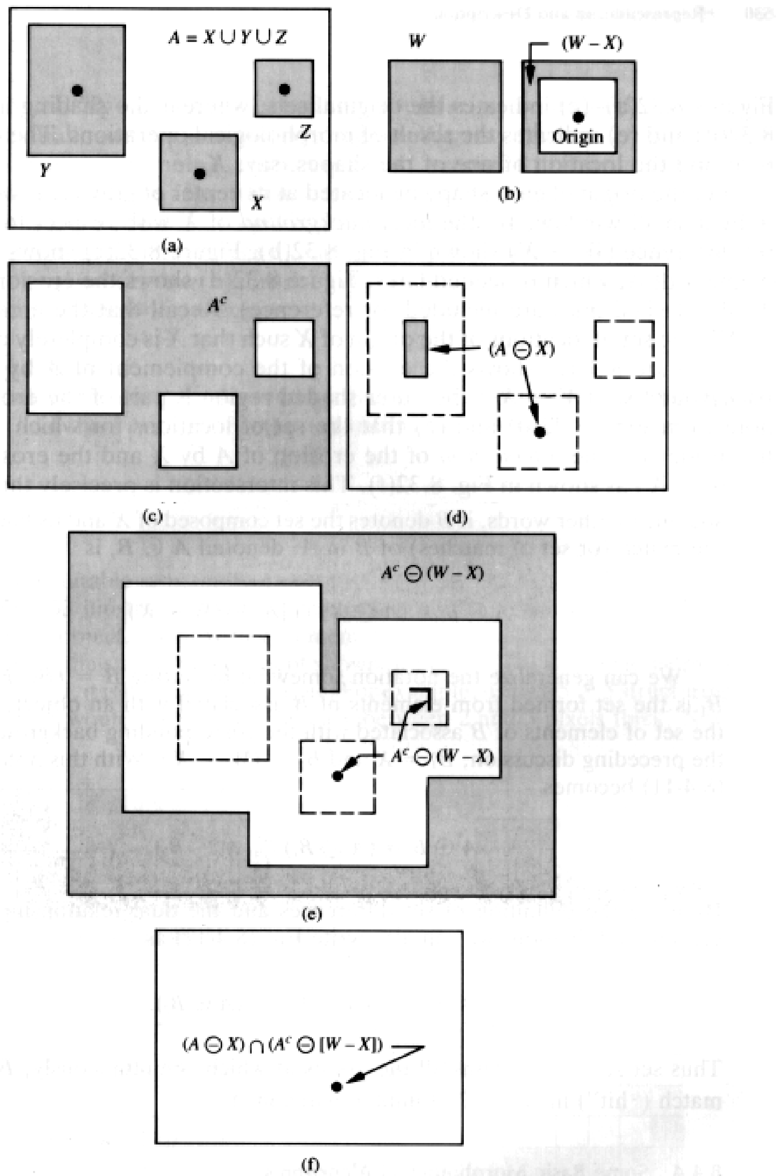
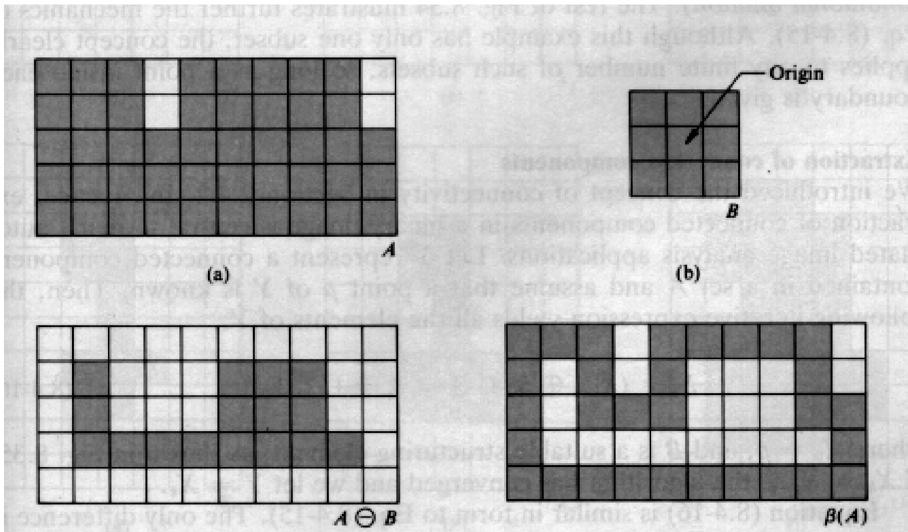


Illustration of Hit-or-Miss Transform

(a) Set A
(b) W and (W-X)
(c) complement of A
(d) A eroded with X
(e) complement of A eroded with (W-X)
(f) intersection of (d) & (e)

Boundary extraction using binary morphology

$$\beta(A) = A - \left(A \ominus B\right)$$



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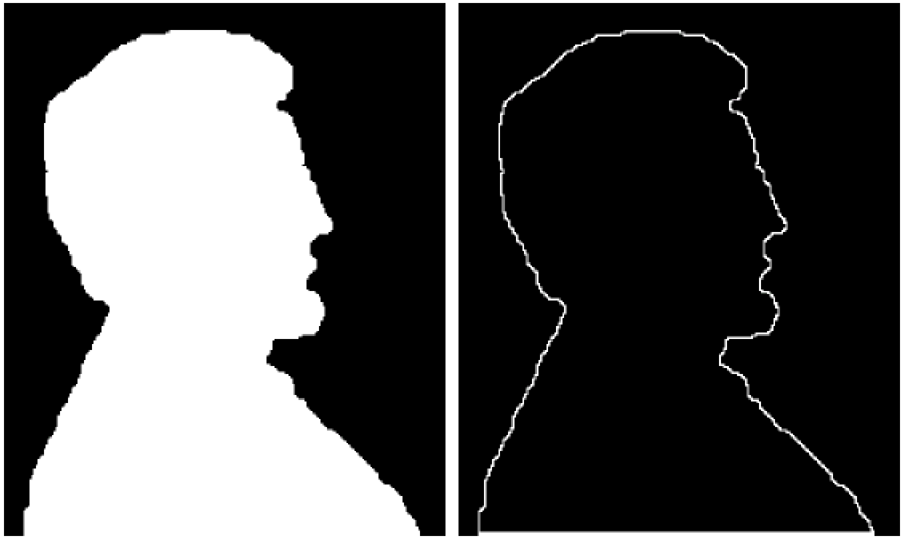
Boundary Extraction



Thinning using binary morphology

Thinning of set *A* by structuring element *B* can be defined in terms of Hit-or-Miss transform:

$$A \otimes B = A - (A \S B) = A \cap (A \S B)^c$$

B contains the pattern to be removed from *A*.

Thinning of set *A* is usually based on a sequence of structuring elements :

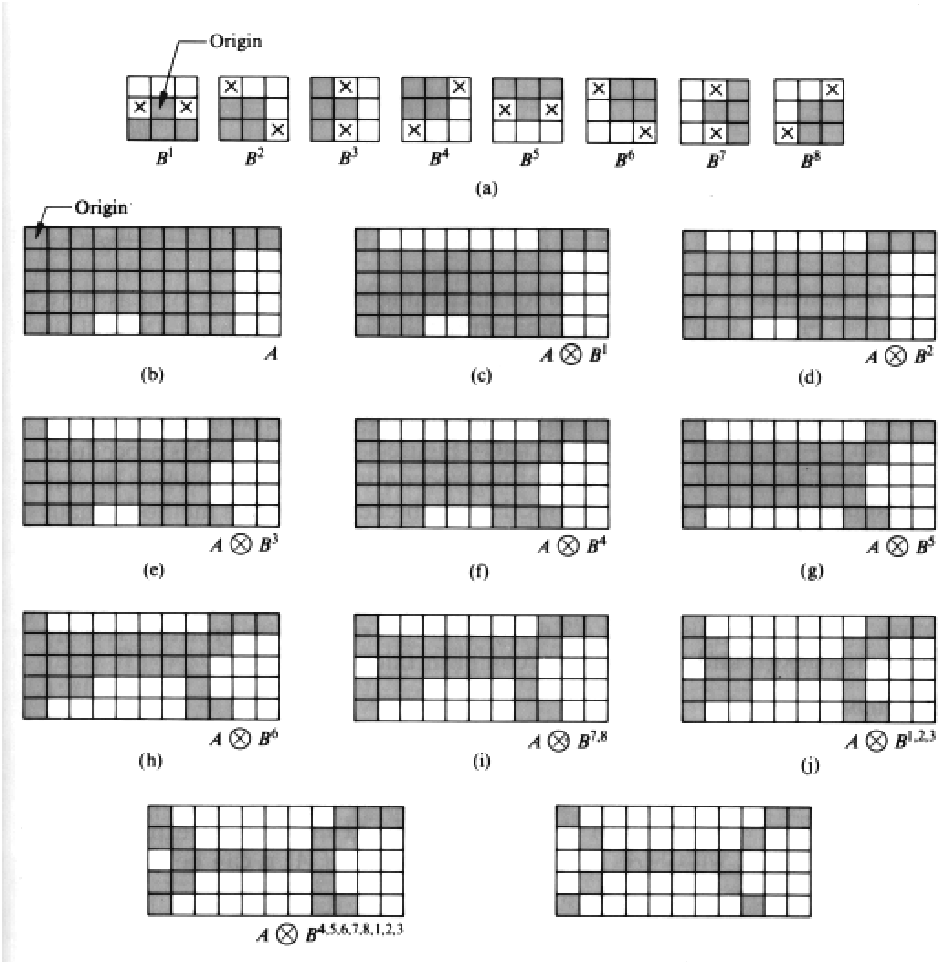
$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

where B^i is a rotated version of B^{i-1}

Hence, $A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2).....) \otimes B^n)$

The thinning operation is repeated until no more changes occur.

Example of thinning



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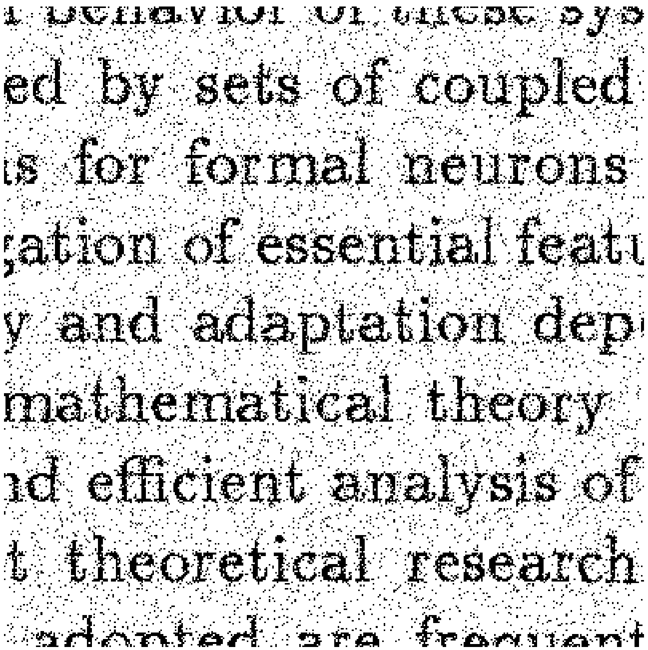
Application

- Morphological operations can help improve OCR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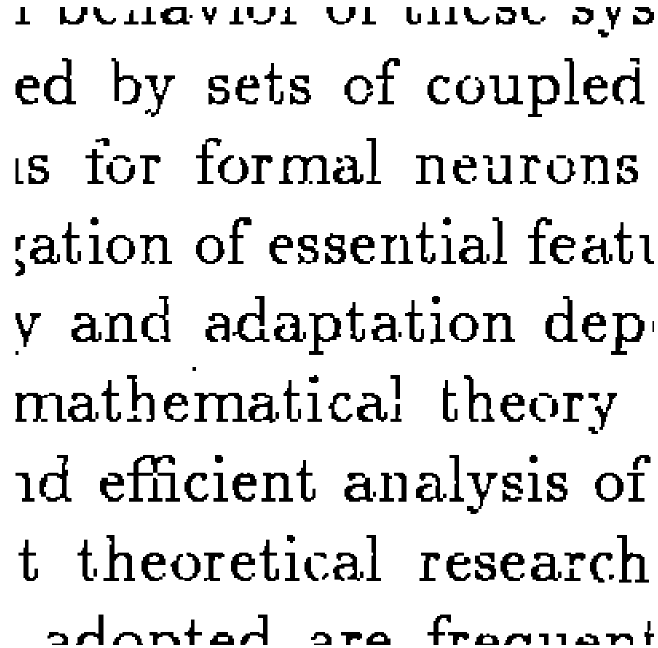
Noisy image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y and adaptation dep
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nd efficient analysis of
t theoretical research

After morphological operations

Summary

- Mathematical morphology is a set of image manipulation tools
- Good for binary/grayscale images
- Good for creating masks to extract regions of interest