Prove the limit statements
$$\lim_{x\to\sqrt{3}} \frac{1}{x^2} = \frac{1}{3}.$$

First, we see that $f(x) = \dfrac{1}{x^2}$, $x_0 = \sqrt{3}$, $L = \dfrac{1}{3}$ Let $\epsilon > 0$ be given then

$$
\begin{gathered}
|f(x) - L| < \epsilon \\
\left|\frac{1}{x^2} - \frac{1}{3}\right| < \epsilon \\
-\epsilon < \frac{1}{x^2} - \frac{1}{3} < \epsilon \\
\frac{1}{3} - \epsilon < \frac{1}{x^2} < \frac{1}{3} + \epsilon \\
\frac{1}{\frac{1}{3} - \epsilon} > x^2 > \frac{1}{\frac{1}{3} + \epsilon} \\
\text{Assuming } \epsilon < \frac{1}{3} \\
\sqrt{\frac{1}{\frac{1}{3} - \epsilon}} > x > \sqrt{\frac{1}{\dfrac{1}{3} + \epsilon}} \\
\sqrt{\frac{1}{\frac{1}{3} + \epsilon}} < x < \sqrt{\frac{1}{\dfrac{1}{3} - \epsilon}}
\end{gathered}
$$

And we want

$$
\begin{gathered}
|x - x_0| < \delta \\
|x - \sqrt{3}| < \delta \\
-\delta < x - \sqrt{3} < \delta \\
\sqrt{3} - \delta < x < \sqrt{3} + \delta
\end{gathered}
$$

Therefore,

$$
\begin{aligned}
\sqrt{3} - \delta_1 &= \sqrt{\frac{1}{\frac{1}{3} + \epsilon}} \\
\delta_1 &= \sqrt{3} - \sqrt{\frac{1}{\frac{1}{3} + \epsilon}} \\
\sqrt{3} + \delta_2 &= \sqrt{\frac{1}{\frac{1}{3} - \epsilon}} \\
\delta_2 &= \sqrt{\frac{1}{\frac{1}{3} - \epsilon}} - \sqrt{3} \\
\Rightarrow \delta = \min(\delta_1, \delta_2).
\end{aligned}
$$

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