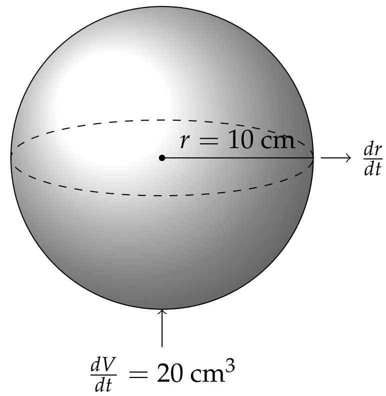
1. Air is pumped into a spherical balloon at a rate of 20 cm^3 per minute. How fast is the radius of the balloon increasing when the diameter is 10 cm?



The volume of a sphere is

$$\begin{aligned}
V\frac{4}{3}\pi r^3 \\
\frac{d}{dt}\left[V\right] &= \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right] \\
\frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \\
\frac{dr}{dt} &= \frac{1}{4\pi r^2}\frac{dV}{dt} \\
\frac{dr}{dt} &= \frac{1}{4\pi 5^2}20 \\
\frac{dr}{dt} &= \boxed{\frac{1}{5\pi}\ {}^{\text{cm}}\!/_{\text{min}}}.
\end{aligned}$$

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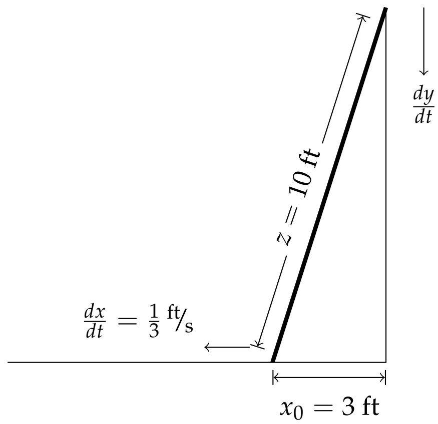
2. A 10-foot ladder is resting against a wall. The bottom of the ladder is initially 3 feet away from the wall, and is being pulled away from the wall at a rate of $^1\!/_3$ feet per second. How fast is the top of the ladder moving after 4 seconds?



The base of the ladder is initially 3 ft from the wall, however we want to know the rate of change in the top of the ladder after $t = 4$ s. Therefore, we must find the distance the base of the ladder is when $t = 4$ s. So,

$$\begin{aligned} x &= x_0 + t \cdot \frac{dx}{dt} \\ &= 3 + 4\frac{1}{3} \\ &= \frac{13}{3} \text{ ft} \end{aligned}$$

and

$$\begin{aligned} y &= \sqrt{z^2 - x^2} \\ &= \sqrt{10^2 - \left(\frac{13}{3}\right)^2} \\ &\approx 9.01 \text{ ft} \end{aligned}$$

$$\begin{aligned} x^2 + y^2 &= z^2 \\ \frac{d}{dt}\left[x^2 + y^2\right] &= \frac{d}{dt}\left[z^2\right] \\ 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} &= 2z \cdot \cancelto{0}{\frac{dz}{dt}} \\ \frac{dy}{dt} &= -\frac{x\frac{dx}{dt}}{y} \\ \frac{d3}{dt} &= -\frac{4/3 \cdot 1/3}{9.01} \\ \frac{d3}{dt} &\approx \boxed{-0.160 \text{ ft/s}}. \end{aligned}$$

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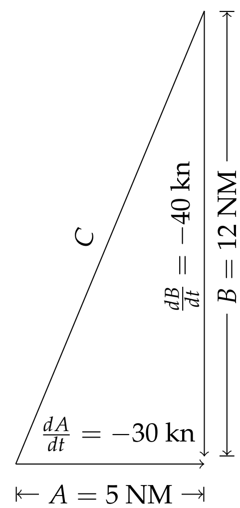
3. Boat A is traveling east at 30 knots. Boat B is traveling south at 40 knots. Suppose the boats are both headed to the same point. How fast is their distance from each other decreasing when boat A is 5 nautical miles from its destination and boat B is 12 nautical miles from its destination?



$$\begin{aligned}
A^2 + B^2 &= C^2 \\
2A\frac{dA}{dt} + 2B\frac{dB}{dt} &= 2C\frac{dC}{dt} \\
\frac{dC}{dt} &= \frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{C} \\
\frac{dC}{dt} &= \frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{\sqrt{A^2+B^2}} \\
\frac{dC}{dt} &= \frac{-30\cdot 5 + -40\cdot 12}{\sqrt{25+144}} \\
\frac{dC}{dt} &= -\frac{630}{13} \\
\frac{dC}{dt} &\approx \boxed{-48.5 \text{ kn}}.
\end{aligned}$$

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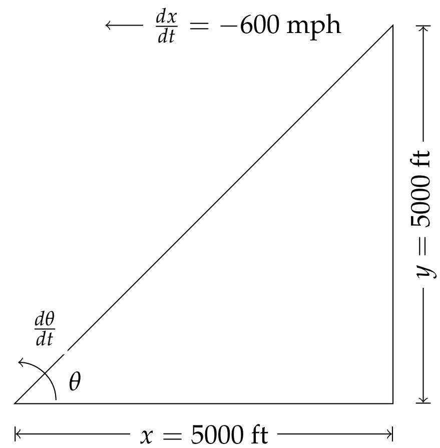
4. Suppose you are sitting on the ground watching a plane flying towards you. The plane is flying level at 5000 ft and is traveling 600 mph. If θ is the angle between the ground and your line of sight to the plane, at what rate is this angle changing the instant the plane is 5000 ft from you along the ground.



First we must convert $\frac{dL}{dt}$ to ft/sec to have a more reasonable answer. Since a change of an angle over an hour really isn't that useful.

$$\begin{aligned}\frac{dL}{dt} &= -600\,{}^{\text{miles}}/_{\text{hr}} \\ &= -600\,{}^{\text{miles}}/_{\text{hr}} \cdot 5280\,{}^{\text{ft}}/_{\text{mile}} \cdot \frac{1}{3600}\,{}^{\text{hr}}/_{\text{s}} \\ &= -880\,{}^{\text{ft}}/_{\text{s}}.\end{aligned}$$

Now notice that the angle of inclination is equal to the angle of declination and so the equation relating the sides and the angle is

$$\begin{aligned}\tan\theta &= \frac{5000ft}{x} \\ \theta &= \tan^{-1}\frac{5000ft}{x} \\ \frac{d}{dt}[\theta] &= \frac{d}{dt}\left[\tan^{-1}\frac{5000}{x}\right] \\ \frac{d\theta}{dt} &= -\frac{1}{1+\left(\frac{5000}{x}\right)^2}\cdot\frac{5000}{x^2}\cdot\frac{dx}{dt} \\ \frac{d\theta}{dt} &= \frac{1}{2}\cdot\frac{1}{5000}\cdot 880 \\ &= \boxed{0.088\,{}^{\text{rad}}/_{\text{s}}}.\end{aligned}$$

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