1. Suppose f is defined on $[-1,3]$ and satisfies:

$$f(x) = \begin{cases} x, & -1 \leq x < 0 \\ -\frac{1}{2}x^2, & 0 \leq x < 1 \\ \sqrt{1-(x-2)^2}, & 1 \leq x \leq 3, x \neq 2 \\ 0, & x = 2 \end{cases}$$

(a) Sketch the graph of the function given above.

(b) Does $\lim_{x\to 2} f(x)$ exist? Justify your answer.

(c) Does $\lim_{x\to 1} f(x)$ exist? Justify your answer.

(d) Does $\lim_{x\to 4} f(x)$ exist? Justify your answer.

2.

(a) Suppose that a function $f(x)$ is defined for all $x \in [-2,2]$. Can anything be said about the existence of $\lim_{x\to 0} f(x)$? Give reasons for your answer.

(b) Suppose that g is a function defined for all x. If $g(1) = 5$, must $\lim_{x\to 1} g(x)$ exist? If it does, then must $\lim_{x\to 1} g(x) = 5$? Can we conclude anything about $\lim_{x\to 1} g(x)$? Explain!

3. Find the following limits.

(a) $\lim_{h\to 0} \dfrac{\sqrt{5h+4}-2}{h}$

(b) $\lim_{x\to -2} \dfrac{x+2}{\sqrt{x^2+5}-3}$

(c) $\lim_{t\to 0} \dfrac{1+t+\sin(t)}{3\cos(t)}$

(d) $\lim_{u\to 1} \dfrac{u^6-1}{u^4-1}$

4. Use Sandwich Theorem and limit laws to show that

(a) $\lim_{t\to 0} t^2 \cos(20\pi t) = 0$

(b) $\lim_{x\to 0} \sqrt{x^3+x^2}\sin(\frac{\pi}{x}) = 0$

(c) $\lim_{h\to 0} \left(h^2\cos(\frac{2}{h})+1\right)\left(h^2\cos(\frac{2}{h})-1\right) = -1$

(d) $\lim_{u\to 0} u^2 4^{\sin(\frac{\pi}{u})} = 0$