1. Find the derivatives of the following functions at $x = 3$ given
$f(3) = 6$, $f'(3) = 0$, $f(1) = 1$, $f'(1) = 5$, $g(3) = 1$, $g'(3) = 2$, $g(4) = 2$, $g'(4) = 1$, $h(3) = 4$, and $h'(3) = 2$

(a) $f(g(x)) \cdot h(x)$

(b) $\dfrac{f(x)}{g(h(x))}$

(a)

$$\begin{aligned}
\frac{d}{dx}\left[f(g(x)) \cdot h(x)\right] &= \frac{df(g(x))}{dg(x)}\frac{dg(x)}{dx}h(x) + f(g(x))h'(x) \\
&= f'(g(x)) \cdot g'(x) \cdot h(x) + f(g(x))h'(x) \\
\Rightarrow \frac{d}{dx}\left[f(g(x)) \cdot h(x)\right]_{x=3} &= f'(g(3)) \cdot g'(3)h(3) + f(g(3))h'(3) \\
&= f'(1) \cdot 2 \cdot 4 + f(1) \cdot 2 \\
&= 5 \cdot 8 + 1 \cdot 2 \\
&= \boxed{42}.
\end{aligned}$$

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(b)

$$\begin{aligned}
\frac{d}{dx}\left[\frac{f(x)}{g(h(x))}\right] &= \frac{f'(x) \cdot g(h(x)) - f(x)\frac{dg(h(x))}{dh(x)}\frac{dh(x)}{dx}}{g^2(h(x))} \\
&= \frac{f'(x) \cdot g(h(x)) - f(x) \cdot g'(h(x)) \cdot h'(x)}{g^2(h(x))} \\
\Rightarrow \frac{d}{dx}\left[\frac{f(x)}{g(h(x))}\right]_{x=3} &= \frac{f'(3) \cdot g(h(3)) - f(3) \cdot g'(h(3)) \cdot h'(3)}{g^2(h(3))} \\
&= \frac{0 \cdot g(4) - 6 \cdot g'(4) \cdot 2}{g^2(4)} \\
&= \frac{-12 \cdot 1}{2^2} \\
&= \boxed{-3}.
\end{aligned}$$

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2.

(a) Find y' if

$$y = \tan(\cos(\cos(x^2)))$$

(b) Find $y^{(1123)}$ if

$$y = \sin^2 x$$

(a)

$$\begin{aligned}
y' &= \frac{d\tan(\cos(\cos(x^2)))}{d\cos(\cos(x^2))}\frac{d\cos(\cos(x^2))}{d\cos(x^2)}\frac{d\cos(x^2)}{dx^2}\frac{dx^2}{dx} \\
&= \sec^2(\cos(\cos(x^2)))\cdot\Big(-\sin(\cos(x^2))\Big)\cdot\Big(-\sin(x^2)\Big)\cdot 2x \\
&= \boxed{2x\cdot\sec^2(\cos(\cos(x^2)))\cdot\sin(\cos(x^2))\cdot\sin(x^2)}.
\end{aligned}$$

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(b) For this problem we will need to determine a pattern.

$$\begin{aligned}
y' &= \frac{d\sin^2 x}{d\sin x}\frac{d\sin x}{dx} \\
&= 2\sin x\cdot cosx \\
&= \sin 2x \\
\Rightarrow y'' &= \frac{d\sin 2x}{d[2x]}\frac{d[2x]}{dx} \\
&= 2\cos 2x \\
\Rightarrow y''' &= -2\frac{d\cos 2x}{d[2x]}\frac{d[2x]}{dx} \\
&= -4\sin 2x \\
&= -(2)^2\sin 2x \\
\Rightarrow y^{(4)} &= -(2)^2\frac{d\sin 2x}{d[2x]}\frac{d[2x]}{dx} \\
&= -(2)^3\cos 2x \\
\Rightarrow y^{(5)} &= -(2)^3\frac{d\cos 2x}{d[2x]}\frac{d[2x]}{dx} \\
&= (2)^4\sin 2x
\end{aligned}$$

From this we see that the derivative follows a pattern similar to $\sin 2x \rightarrow \cos 2x \rightarrow -\sin 2x \rightarrow -\cos 2x$ and then it repeats. We must however account for the number in front. As can be seen from the pattern I have demonstrated the number in from should 2^{i-1} where i is the order of the derivative. To use the pattern for the trigonometric function we determine the remainder after dividing 1123 by 4. In this case we see that the remainder is 3. Therefore, we want the trigfunction in the third position of the established pattern, i.e. $-\sin 2x$. Finally we see that

$$\boxed{y^{(1123)} = -(2)^{1122}\sin 2x}$$

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3. Using implicit differentiation show

(a) $\frac{d}{dx}\left[\sin^{-1} x\right] = \dfrac{1}{\sqrt{1-x^2}}$ Hint: Set $y = \sin^{-1} x$ solve for x and then differentiate.

(b) $\frac{d}{dx}\left[\ln x\right] = \dfrac{1}{x}$ Hint: Set $y = \ln x$ solve for x and then differentiate.

(a) Let $y = \sin^{-1} x$ then

$$\begin{aligned}
&\sin y = x \\
&\Rightarrow \frac{d}{dx}[\sin y] = \frac{d}{dx}[x] \\
&\frac{d \sin y}{dy}\frac{dy}{dx} = 1 \\
&\cos y \cdot y' = 1 \\
&y' = \frac{1}{\cos y} \\
&y' = \frac{1}{\cos(\sin^{-1} x)} \\
&y' = \frac{1}{\sqrt{1 - \sin^2(\sin^{-1} x)}} \\
&\boxed{y' = \frac{1}{\sqrt{1 - x^2}}}.
\end{aligned}$$

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(b) Let $y = \ln x$ then

$$\begin{aligned}
&e^y = x \\
&\Rightarrow \frac{d}{dx}[e^y] = \frac{d}{dx}[x] \\
&\frac{de^y}{dy}\frac{dy}{dx} = 1 \\
&e^y \cdot y' = 1 \\
&y' = \frac{1}{e^y} \\
&y' = \frac{1}{e^{ln x}} \\
&\boxed{y' = \frac{1}{x}}.
\end{aligned}$$

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4. Given the following functions find y'

(a) $\cos y + \tan y^2 = \dfrac{x+1}{e^{x^2}}$

(b) $y^2 + y = \cot(3x + 2)$

(a)

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\begin{aligned}
\frac{d}{dx}\left[\cos y + \tan y^2\right] &= \frac{d}{dx}\left[\frac{x+1}{e^{x^2}}\right] \\
\frac{d\cos y}{dy}\frac{dy}{dx} + \frac{d\tan y^2}{dy^2}\frac{dy}{dx} &= \frac{e^{x^2} - (x+1)\frac{de^{x^2}}{dx^2}\frac{dx^2}{dx}}{\left[e^{x^2}\right]^2} \\
-\sin y \cdot y' + \sec^2 y^2 \cdot 2y \cdot y' &= \frac{e^{x^2} - (x+1)e^{x^2} \cdot 2x}{\left[e^{x^2}\right]^2} \\
\left[-\sin y + 2y \cdot \sec^2 y^2\right] \cdot y' &= \frac{e^{x^2} - (x+1)e^{x^2} \cdot 2x}{\left[e^{x^2}\right]^2} \\
y' &= \frac{e^{x^2} \cdot [1 - (x+1) \cdot 2x]}{[-\sin y + 2y \cdot \sec^2 y^2] \cdot \left[e^{x^2}\right]^2} \\
&\boxed{y' = \frac{1 - 2x^2 - 2x}{[-\sin y + 2y \cdot \sec^2 y^2] \cdot e^{x^2}}}.
\end{aligned}
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(b)

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\begin{aligned}
\frac{d}{dx}\left[y^2 + y\right] &= \frac{d}{dx}\left[\cot(3x+2)\right] \\
\frac{dy^2}{dy}\frac{dy}{dx} + \frac{dy}{dx} &= \frac{d\cot(3x+2)}{d[3x+2]}\frac{d[3x+2]}{dx} \\
2y \cdot y' + y' &= -3\csc^2(3x+2) \\
[2y \cdot +1] \cdot y' &= -3\csc^2(3x+2) \\
&\boxed{y' = \frac{-3\csc^2(3x+2)}{2y \cdot +1}}.
\end{aligned}
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