

1. Use the Intermediate Value Theorem to show that there exist a root of the given equation in the specified interval.

(a) $\cos(x) = x$ for $x \in (0, 1)$

(b) $\sqrt[3]{x} = 1 - x$ for $x \in (0, 1)$

2. Is there a real number that is exactly 1 more than its cube? Why?

3. Find the slope of the tangent line to

$$f(x) = \sqrt{x}$$

at the point $x_0 = 1$ and then determine the equation of the tangent line.

4. Using the definition of the derivative find the derivative of

$$f(x) = \sin x$$

Hint: Use the relation $\sin x - \sin y = 2 \cos \left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right)$

5. Suppose $f(x)$ is given by

$$f(x) = \begin{cases} x^3 + x - 1 & x < 0 \\ x^2 - 1 & 0 \leq x \leq 1 \\ x^3 - 3x^2 + 3x & x > 1 \end{cases}.$$

(a) Does $f'(0)$ exist? If so, what is it? If not, why not? Explain!

(b) Does $f'(1)$ exist? If so, what is it? If not, why not? Explain!