1. If $g(x)$ is a differentiable function and assuming that $g(x) \neq 0$ show that

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$$

(a) By using the definition of the derivative.

(b) By using the quotient rule.

(a)

$$\begin{aligned}
\frac{d}{dx}\left[\frac{1}{g(x)}\right] &= \lim_{h\to 0}\frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h} \\
&= \lim_{h\to 0}\frac{g(x) - g(x+h)}{g(x)g(x+h)h} \\
&= \lim_{h\to 0}\frac{g(x) - g(x+h)}{h}\lim_{h\to 0}\frac{1}{g(x)g(x+h)} \\
&= -\lim_{h\to 0}\frac{g(x+h) - g(x)}{h}\frac{1}{[g(x)]^2} \\
&= -\frac{g'(x)}{[g(x)]^2}
\end{aligned}$$

■

(b)

$$\begin{aligned}
\frac{d}{dx}\left[\frac{1}{g(x)}\right] &= \frac{\frac{d}{dx}[1]g(x) - \frac{d}{dx}[g(x)]}{[g(x)]^2} \\
&= \frac{0\cdot g(x) - g'(x)}{[g(x)]^2} \\
&= -\frac{g'(x)}{[g(x)]^2}
\end{aligned}$$

■

2. Find the derivative of

$$f(x) = \frac{x}{1+x^2}$$

and the equation of the tangent line to $f(x)$ at the point $(3, 0.3)$.

$$\begin{aligned}
f'(x) &= \frac{1\cdot(1+x^2) - 2x\cdot x}{(1+x^2)^2} \\
&= \boxed{\frac{1-x^2}{(1+x^2)^2}.}
\end{aligned}$$

The equation of the tangent line is given by the point slope form

$$\begin{aligned} y - y_0 &= f'(x_0) \cdot (x - x_0) \\ y - 0.3 &= \frac{1-9}{(1+9)^2} \cdot (x-3) \\ y &= -\frac{8}{100} \cdot (x-3) + 0.3 \\ y &= -0.08x + 0.24 + 0.3 \\ &\boxed{y = -0.08x + 0.54.} \end{aligned}$$

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3. Find f' and f'' for

$$f(x) = e^x(x^3 + \sqrt{x} + 1)$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}[e^x](x^3 + \sqrt{x} + 1) + \frac{d}{dx}[x^3 + x^{1/2} + 1]e^x \\ &= \boxed{e^x(x^3 + x^{1/2} + 1) + (3x^2 + \frac{1}{2}x^{-1/2})e^x} \\ f''(x) &= \frac{d}{dx}[e^x](x^3 + x^{1/2} + 1) + \frac{d}{dx}[x^3 + x^{1/2} + 1]e^x + \frac{d}{dx}[3x^2 + \frac{1}{2}x^{-1/2}]e^x + \frac{d}{dx}[e^x](3x^2 + \frac{1}{2}x^{-1/2}) \\ &= \boxed{e^x(x^3 + x^{1/2} + 1) + (3x^2 + \frac{1}{2}x^{-1/2})e^x + (6x - \frac{1}{4}x^{-3/2})e^x + e^x(3x^2 + \frac{1}{2}x^{-1/2})} \end{aligned}$$

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4. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the following numbers

(a) $(fg)'(3)$

(b) $\left(\frac{f}{f-g}\right)'(3)$

(a)

$$\begin{aligned} (fg)'(3) &= f'(3)g(3) + g'(3)f(3) \\ &= -6 \cdot 2 + 5 \cdot 4 = \boxed{8.} \end{aligned}$$

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(b)

$$\begin{aligned} \left(\frac{f}{f-g}\right)'(3) &= \frac{f'(3)(f(3) - g(3)) - (f(3) - g(3))'f(3)}{[f(3) - g(3)]^2} \\ &= \frac{f'(3)(f(3) - g(3)) - (f'(3) - g'(3))f(3)}{[f(3) - g(3)]^2} \\ &= \frac{-6 \cdot (4-2) - (-6-5) \cdot 4}{[4-2]^2} \\ &= \frac{-12 + 44}{4} = \boxed{8.} \end{aligned}$$

■

5. Find f' and f'' for

$$f(x) = x - 3x^{1/3}$$

$$\boxed{f'(x) = 1 - x^{-2/3}}$$

$$\boxed{f''(x) = \frac{2}{3}x^{-5/3}.}$$

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