Exercise 1

A discrete-time signal: $x(n) = \begin{cases} 1 + n/3, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & elsewhere \end{cases}$

1. Determine its values and sketch the signal x(n)

Case 1.

$$x(n) = 1 + n/3$$

$$n(-3) = 1 + -3/3 = 0$$

$$n(-2) = 1 + -2/3 = 1/3$$

$$n(-1) = 1 + -1/3 = 2/3$$

Case 2.

$$n(0) = 1$$

$$n(1) = 1$$

$$n(2) = 1$$

$$n(3) = 1$$

$$x = [-5:5]$$

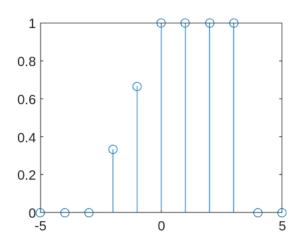
$$x = 1 \times 11$$
 -5 -4 -3 -2 -1 0 1 2 3

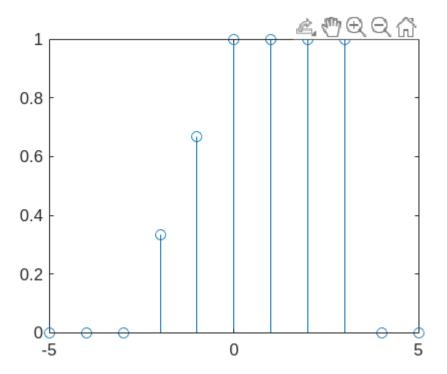
$$y = [0, 0, 0, 1/3, 2/3, 1, 1, 1, 0, 0]$$

5

$$y = 1 \times 11$$
0 0 0 0.3333 0.6667 1.0000 1.0000 1.0000 ...

stem(x, y)





2. Sketch the signals that result if we:

• First fold x(n) and then delay the resulting signal by four samples

Fold signal

y = [0, 0, 0, 1/3, 2/3, 1, 1, 1, 1, 0, 0, 0, 0]

x = [-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7]

y = [0, 0, 0, 0, 1, 1, 1, 1, 2/3, 1/3, 0, 0, 0]

x = [-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7]

Delay four samples

y = [1, 1, 1, 1, 2/3, 1/3, 0, 0, 0, 0, 0, 0, 0]

x = [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

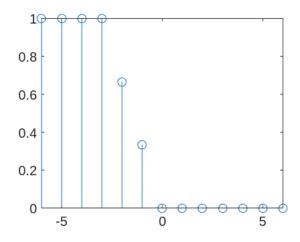
x = [-6:6]

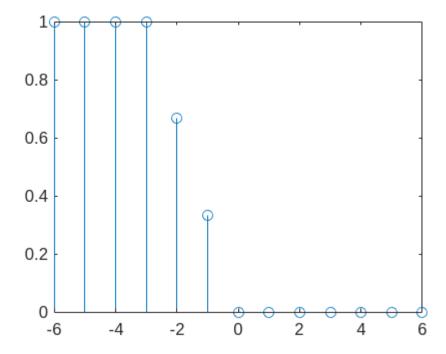
 $x = 1 \times 13$ -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

y = [1, 1, 1, 1, 2/3, 1/3, 0, 0, 0, 0, 0, 0]

 $y = 1 \times 13$ 1.0000 1.0000 1.0000 0.6667 0.3333 0 0 · ·

stem(x, y)





• First delay x(n) four samples and then fold.

Delay four samples

$$y = [0, 0, 0, 0, 1/3, 2/3, 1, 1, 1, 1, 0, 0, 0]$$

$$x = [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]$$

$$y = [1/3, 2/3, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0]$$

$$x = [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]$$

Fold the signal

$$y = [1/3, 2/3, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0]$$

$$x = [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]$$

y = [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2/3, 1/3]

x = [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

x = [-6:6]

 $x = 1 \times 13$

-5 -4 -3 -2 -1 0 1 2 3 4 5 6

0

y = [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2/3, 1/3]

 $y = 1 \times 13$

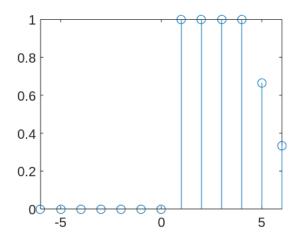
0 0

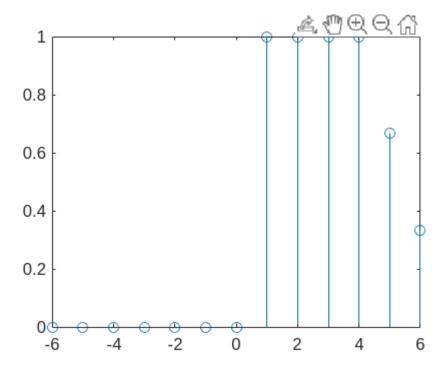
0

0

0 1.0000 ...

stem(x, y)





3. Sketch the signal x(-n+4)

$$n=0; x(4); y = 0$$

$$n=1; x(3); y = 1$$

$$n=2; x(2); y = 1$$

$$n=3; x(1); y = 1$$

$$n=4$$
; $x(0)$; $y=1$

$$n=5$$
; $x(-1)$; $y=2/3$

$$n=6$$
; $x(-2)$; $y = 1/3$

$$n=7$$
; $x(-3)$; $y=0$

$$x = [-2:7]$$

$$x = 1 \times 10$$

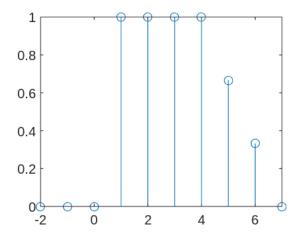
0

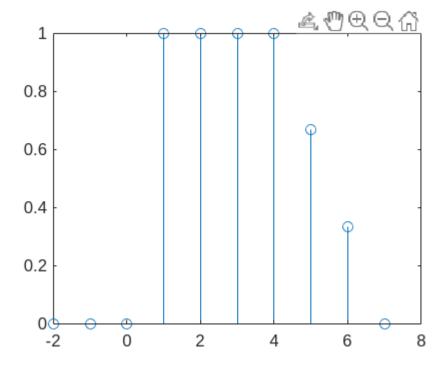
$$y = [0, 0, 0, 1, 1, 1, 1, 2/3, 1/3, 0]$$

$$y = 1 \times 10$$

7

stem(x, y)





4. Compare the results in parts b) and c) and derive a rule for obtaining the signal x(-n+k) from x(n)

The rule would be:

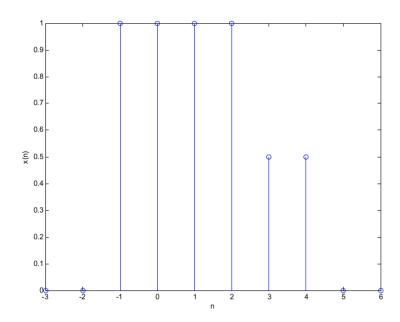
$$x(-n+k) = \begin{cases} 1 + (-n+k)/3, & -3 \le -n+k \le -1 \\ 1, & 0 \le -n+k \le 3 \\ 0, & elsewhere \end{cases}$$

5. Can you express the signal in terms of x(n) in terms of signals $\delta(n)$ and u(n)?

No. For $\delta(n)$ or even $\delta(n-k)$, the signal will result in a shifted impulse, instead of the signal described by x(n) $\mu(n)$ and $\mu(n-k)$ also cannot describe x(n) as will just shift a train of impulses.

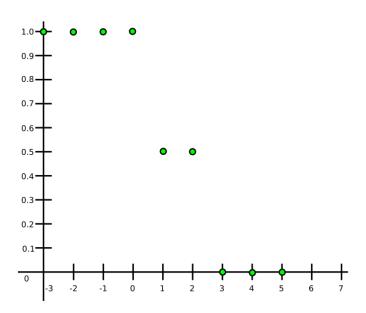
Exercise 2

A discrete-time signal x(n) is shown in the next figure.

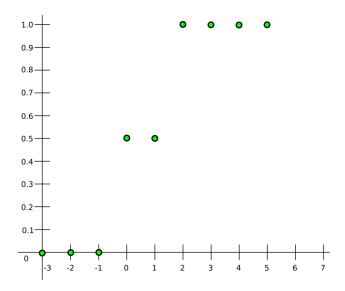


Sketch and label carefully each of the following signals:

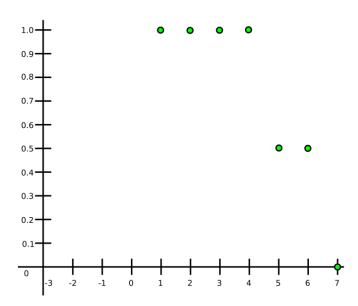
a.
$$x(n-2)$$



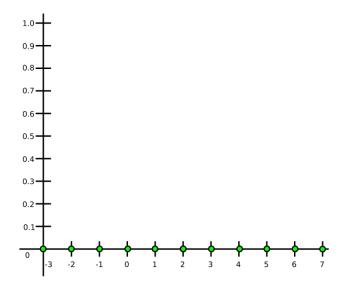
b.
$$x(4 - n)$$



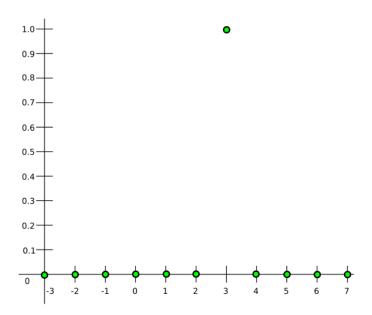
c. x(n + 2)



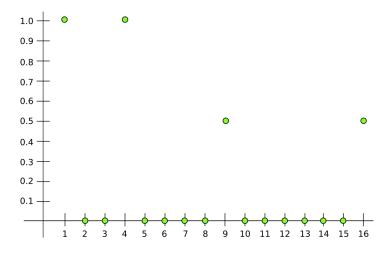
d.
$$x(n)\mu(2-n)$$



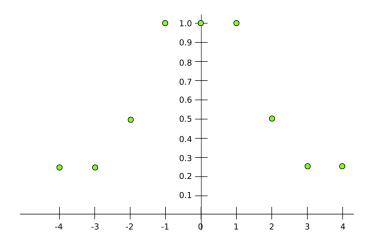
e. $x(n+1)\delta(n-3)$



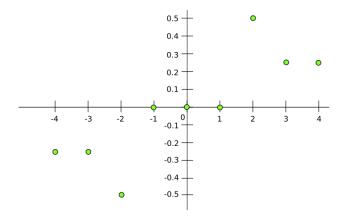
f. $x(n^2)$



g. Even part of x(n)



h. Odd part of x(n)



Exercise 3

Plot in matlab the Exercise 2

x(*n*)

```
x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0, 0]
```

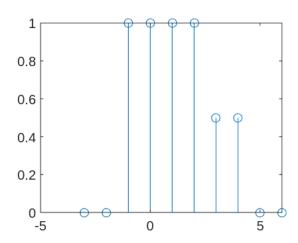
 $x = 1 \times 10$

0 0 1.0000 1.0000 1.0000 1.0000 0.5000 0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$ -3 -2 -1 0 1 2 3 4 5 6

stem(n, x)



a. x(n-2)

x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0]

 $x = 1 \times 10$

0 1.0000 1.0000 1.0000 1.0000 0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

-3 -2 -1 0 1 2 3 4 5 6

-5 -4 -3 -2 -1 0 1 2 3 4

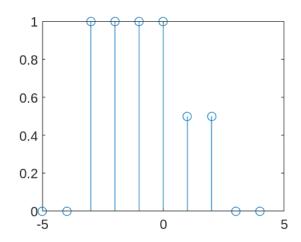
[xf, nf] = sigshift(x, n, -2)

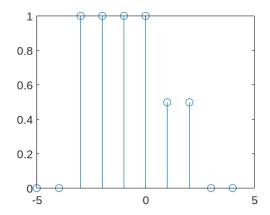
 $xf = 1 \times 10$

 $nf = 1 \times 10$

0 1.0000 1.0000 1.0000 1.0000 0.5000 ...

stem(nf, xf)





b. x(4 - n)

```
x = [0, 0, 1.0, 1.0, 1.0, 0.5, 0.5, 0.5, 0]
```

 $x = 1 \times 10$

0 1.0000 1.0000 1.0000 1.0000

2 3 4 5 6

0.5000 0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

[xF, nF] = sigfold(x, n)

-3 -2 -1 0 1

 $xF = 1 \times 10$

0 0.5000 0.5000 1.0000 1.0000 1.0000 1.0000 ...

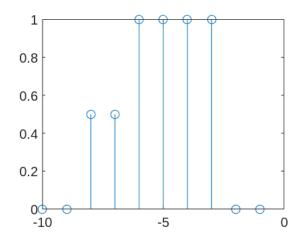
 $nF = 1 \times 10$ -6 -5 -4 -3 -2 -1 0 1

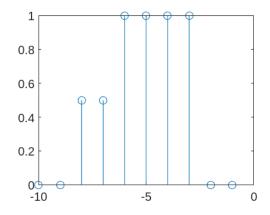
[xf, nf] = sigshift(xF, nF, -4)

 $xf = 1 \times 10$ 0 0.5000 0.5000 1.0000 1.0000 1.0000 1.0000 ...

 $nf = 1 \times 10$ -10 -9 -8 -7 -6 -5 -4 -3 -2 -1

stem(nf, xf)





c. x(n + 2)

x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0, 0]

 $x = 1 \times 10$

0 1.0000

1.0000

1.0000

1.0000

0.5000

0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

-3 -2 -1 0 1

0 1.0000

2

3

5

[xf, nf] = sigshift(x, n, 2)

 $xf = 1 \times 10$

 $nf = 1 \times 10$ -1 0

1

2 3

4 5

1.0000 1.0000

6

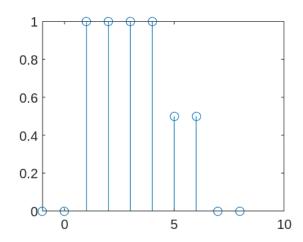
1.0000

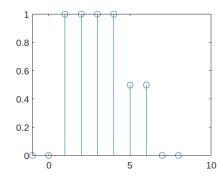
7 8

0.5000

0.5000 ...

stem(nf, xf)





d. $x(n)\mu(2-n)$

x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0, 0]

 $x = 1 \times 10$

0 1.0000

1.0000

1.0000

1.0000

0.5000 ...

0.5000

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

-2 -3

-1

0

1

2 3 4

5 6

0

u = [1, 1, 0, 0, 0, 0, 0, 0, 0, 0]

 $u = 1 \times 10$

1 1

0

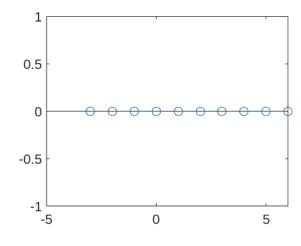
0

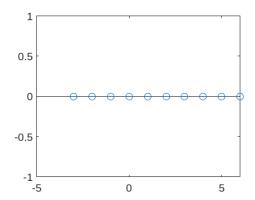
0

0 0 0

0

stem(n, x.*u)





e. $x(n + 1)\delta(n - 3)$

x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0]

 $x = 1 \times 10$

0 1.0000

1.0000

1.0000

1.0000

0.5000

0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

-2 -1 0 1 2 3 -3

5 6

s = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0]

 $s = 1 \times 10$

0

0 0 0

0 0

1

0

[xf, nf] = sigshift(x, n, 3)

 $xf = 1 \times 10$

0 1.0000

1.0000 1.0000

0

1.0000

0

0.5000 ... 0.5000

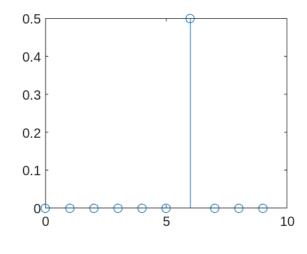
 $nf = 1 \times 10$

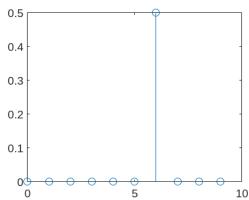
2 3 4

5

6 7 8 9

stem(nf, xf.*s)





f. $x(n^2)$

x = [0, 0, 1.0, 1.0, 1.0, 0.5, 0.5, 0.5, 0]

 $x = 1 \times 10$

0 1.0000 1.0000 1.0000 1.0000 0.5000 0.5000 ...

n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

 $n = 1 \times 10$

-3 -2 -1 0 1 2 3 4 5 6

xexp = [0, 1.0, 0, 0, 1.0, 0, 0, 0, 0.5, 0, 0, 0, 0, 0, 0.5]

xexp = 1x17

0 1.0000

0 0 1.0000

0

0 0 • • •

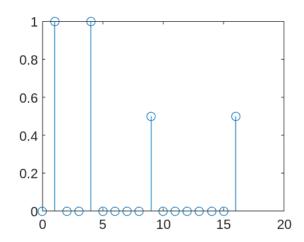
nexp = [0:16]

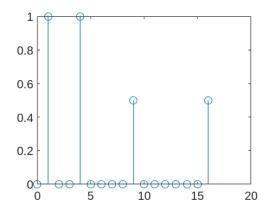
 $nexp = 1 \times 17$

0 1 2

3 4 5 6 7 8 9 10 11 12 ...

stem(nexp, xexp)





g. Even part of x(n)

x = [0, 0, 1.0, 1.0, 1.0, 1.0, 0.5, 0.5, 0]

 $x = 1 \times 10$

0 0 1.0000 1.0000 1.0000 0.5000 0.5000 ...

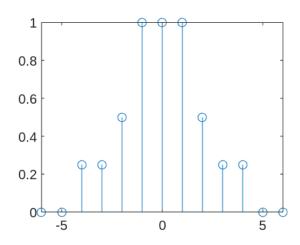
n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

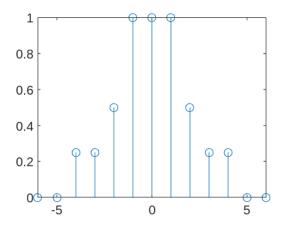
 $n = 1 \times 10 \\
 -3 -2 -1 0 1 2 3 4 5 6$

[e, o, m] = evenodd(x, n)

 $e = 1 \times 13$ $0 \quad 0 \quad 0.2500 \quad 0.2500 \quad 0.5000 \quad 1.0000 \quad 1.0000 \quad \cdots$

stem(m, e)





h. Odd part of x(n)

x = [0, 0, 1.0, 1.0, 1.0, 0.5, 0.5, 0, 0]

 $x = 1 \times 10$ 0 0 1.0000 1.0000 1.0000 0.5000 0.5000 ...

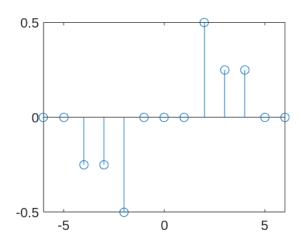
n = [-3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

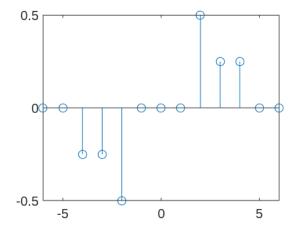
 $n = 1 \times 10 \\
 -3 -2 -1 0 1 2 3 4 5 6$

[e, o, m] = evenodd(x, n)

 $e = 1 \times 13$ $0 \quad 0 \quad 0.2500 \quad 0.2500 \quad 0.5000 \quad 1.0000 \quad 1.0000 \quad \cdots$ $e = 1 \times 13$ $e = 1 \times 13$

stem(m, o)





Exercise 4

Mitra exercises 2.1, 2.3, 2.4, 2.5, 2.8 and 2.23

Exercise 2.1

Find the norm L_1, L_2 and L_inf of the next finite length sequences.

x1 = [4.5, -2.68, -0.14, 3.91, 2.62, -0.43, -4.81, 3.21, -0.55]

```
x1 = 1x9

4.5000 -2.6800 -0.1400 3.9100 2.6200 -0.4300 -4.8100 3.2100 ···

% L_1 norm

L_1 = sum(abs(x1))
```

```
L_1 = 22.8500
```

```
% L_2 norm
L_2 = sqrt(sum(x1.^2))
```

```
L_2 = 9.1396
```

```
% L_inf norm
L_inf = max(abs(x1))
```

```
L_{inf} = 4.8100
```

```
% Matlab verification.
L1m = norm(x1, 1)
L1m = 22.8500
L2m = norm(x1, 2)
L2m = 9.1396
Linf = norm(x1, "inf")
Linf = 4.8100
x2 = [0.92, 2.34, 3.37, 1.9, -2.59, -0.75, 3.48, 3.33]
x2 = 1x8
                            1.9000 -2.5900 -0.7500
   0.9200
            2.3400 3.3700
                                                         3.4800
                                                                  3.3300
% L_1 norm
L_1 = sum(abs(x2))
L_1 = 18.6800
% L_2 norm
L_2 = sqrt(sum(x2.^2))
L_2 = 7.1944
% L_inf norm
L_{inf} = max(abs(x2))
L_{inf} = 3.4800
% Matlab verification.
L1m = norm(x2, 1)
L1m = 18.6800
L2m = norm(x2, 2)
L2m = 7.1944
Linf = norm(x2, "inf")
Linf = 3.4800
```

Exercise 2.4

Write the sequence x[n] = [1, 3, -2, -4] in terms of the step unit sequence $\mu[n]$

Assuming the values in x[n] are n, then

$$\mu[n] = [1, 1, 0, 0]$$

But, if the signal is correct and the n values are not specified, then $\mu[n]x[n] = [1, 3, -2, -4]$, if the signal starts at n=0

Exercise 2.5

Consider the next sequences (values not specified are zero), get the sequences:

$$x(n) = \{-4, 5, 1, -2, -3, 0, 2\} \quad -3 \le n \le 3$$

$$y(n) = \{6, -3, -1, 0, 8, 7, -2\} \quad -1 \le n \le 5$$

$$w(n) = \{3, 2, 2, -1, 0, -2, 5\} \quad 2 \le n \le 8$$

a.
$$c(n) = x(-n + 2)$$

$$x = [-4, 5, 1, -2, -3, 0, 2]$$

$$x = 1 \times 7$$
 -4 5 1 -2 -3 0 2

$$n = [-3, -2, -1, 0, 1, 2, 3]$$

$$[xf, xn] = sigfold(x, n)$$

$$xf = 1 \times 7$$
2 0 -3 -2 1 5 -4
 $xn = 1 \times 7$
-3 -2 -1 0 1 2 3

$$[cf, cn] = sigshift(xf, xn, -2)$$

cf =
$$1 \times 7$$

2 0 -3 -2 1 5 -4
cn = 1×7
-5 -4 -3 -2 -1 0 1

cf

cf =
$$1 \times 7$$

2 0 -3 -2 1 5 -4

b.
$$d(n) = y(-n - 3)$$

$$y = [6, -3, -1, 0, 8, 7, -2]$$

$$y = 1 \times 7$$
 $6 -3 -1 0 8 7 -2$

$$n = [-1, 0, 1, 2, 3, 4, 5]$$

$$n = 1 \times 7$$
 $-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

```
[yf, nf] = sigfold(y, n)
 yf = 1 \times 7
         7 8 0 -1 -3
    -2
 nf = 1 \times 7
   -5
       -4 -3 -2
                     -1
                           0
                                 1
 [df, dn] = sigshift(yf, nf, 3)
 df = 1 \times 7
         7 8
                 0 -1
   -2
                           -3
 dn = 1 \times 7
   -2
       -1
             0
                  1
                      2
                           3
                                 4
 df
 df = 1 \times 7
    -2 7 8 0 -1 -3
c. e(n) = w(-n)
 w = [3, 2, 2, -1, 0, -2, 5]
 w = 1 \times 7
                         -2
 3
        2 2 -1 0
                                 5
 n = [2, 3, 4, 5, 6, 7, 8]
 n = 1 \times 7
        3
             4
                 5 6
                           7
                                 8
 [wf, nf] = sigfold(w, n)
 wf = 1 \times 7
     5
        -2
             0
                  -1
                       2
                            2
                                 3
 nf = 1 \times 7
        -7
  -8
             -6
                  -5
                       -4
                           -3
                                -2
 wf
 wf = 1 \times 7
    5 -2 0 -1 2 2
                                3
d. u(n) = x(n) + y(n-2)
 x = [-4, 5, 1, -2, -3, 0, 2]
 x = 1 \times 7
        5 1 -2 -3 0
                                 2
 nx = [-3, -2, -1, 0, 1, 2, 3]
 nx = 1x7
 -3 -2 -1 0 1 2
                                 3
 y = [6, -3, -1, 0, 8, 7, -2]
 y = 1 \times 7
   6 -3 -1 0 8 7 -2
```

```
ny = [-1, 0, 1, 2, 3, 4, 5]
 ny = 1 \times 7
  -1 0 1 2 3 4 5
 [yf, nyf] = sigfold(y, ny)
 yf = 1 \times 7
        7 8 0 -1 -3 6
  -2
 nyf = 1 \times 7
  -5 -4 -3 -2 -1 0 1
 [sa, san] = sigadd(x, nx, yf, nyf)
 sa = 1 \times 9
  -2
        7
            4 5 0 -5
                               3
                                    0
                                        2
 san = 1 \times 9
   -5 -4 -3 -2 -1
                          0
                               1
                                    2
                                        3
 sa
 sa = 1 \times 9
  -2 7 4 5
                     0 -5
                              3
                                        2
e. v(n) = x(n)w(n + 4)
 x = [-4, 5, 1, -2, -3, 0, 2]
 x = 1 \times 7
  -4 5 1 -2 -3 0
                               2
 nx = [-3, -2, -1, 0, 1, 2, 3]
 nx = 1x7
  -3 -2 -1 0 1 2
                               3
 w = [3, 2, 2, -1, 0, -2, 5]
 w = 1 \times 7
        2 2 -1 0 -2
                               5
 nw = [2, 3, 4, 5, 6, 7, 8]
 nw = 1 \times 7
 2 3 4 5 6 7
                               8
 [wf, nf] = sigshift(w, nw, -4)
 wf = 1 \times 7
        2 2 -1 0 -2
                               5
    3
 nf = 1 \times 7
  -2 -1 0
                 1
                     2
                          3
 [sm, snm] = sigmult(x, nx, wf, nf)
 sm = 1 \times 8
  0 15 2 -4 3 0 -4
                                    0
 snm = 1 \times 8
```

-3 -2 -1 0 1 2 3 4 sm $sm = 1 \times 8$ 0 15 2 -4 3 0 -4 0 f. s(n) = y(n) - w(n-4)y = [6, -3, -1, 0, 8, 7, -2] $y = 1 \times 7$ 6 -3 -1 0 8 7 -2 ny = [-1, 0, 1, 2, 3, 4, 5] $ny = 1 \times 7$ -1 0 1 2 3 4 5 W = [3, 2, 2, -1, 0, -2, 5] $w = 1 \times 7$ 3 2 2 -1 0 -2 nw = [2, 3, 4, 5, 6, 7, 8] $nw = 1 \times 7$ 3 4 5 [wf, nf] = sigshift(w, nw, 4) $wf = 1 \times 7$ 2 2 -1 0 -2 5 3 $nf = 1 \times 7$ 7 8 9 10 11 12 [sa, san] = sigadd(y, ny, wf.*-1, nf) $sa = 1 \times 14$ -1 8 7 -2 -3 1 0 2 • • • 6 -3 0 -2 -2 $san = 1 \times 14$ 5 7 11 • • • 1 2 3 4 6 8 9 10 -1 0 sa $sa = 1 \times 14$ -1 0 8 7 -2 -3 -2 -2 1 0 2 • • • g. r(n) = 3.5y(n)

y = [6, -3, -1, 0, 8, 7, -2]

 $y = 1 \times 7$ 6 -3 -1 0 8 7 -2

ny = [-1, 0, 1, 2, 3, 4, 5]

$$r = y.^3.5$$

Exercise 2.8

Find the symmetric and asymmetric conjugate of the next sequences.

Use the following formulas:

$$X_{1 cs}(n) = \frac{1}{2} \left[X(n) + X_1^*(-n) \right] \qquad X_{1 ca}(n) = \frac{1}{2} \left[X(n) - X_1^*(-n) \right]$$

$$x_1(n) = \{1 + j4, -2 + j5, 3 - j2, -7 + j3, -1 + j1\}, -2 \le n \le 2$$

$${x_1^*[n]} = {1 - j4, -2 - j5, 3 + j2, -7 - j3, -1 - j1}$$

$${x_1^*[-n]} = {-1 - j1, -7 - j3, 3 + j2, -2 - j5, 1 - j4}$$

$$x_{1cs}(n) = \{j1.5, -4.5 + j1, 3, -4.5 - j1, -j1.5\}$$

$$x_{1ca}(n) = \{1 + j2.5, 2.5 + j4, -j2, -2.5 + j4, -1 + j2.5\}$$

$$x_2(n) = e^{j\pi \frac{n}{3}}$$

 $x_3(n) = je^{-j\pi\frac{n}{5}}$ C.