Eric Ortega
Rodriguez

Unigram Assignment Proof

Maximize $\prod_k P_k^{n_k}$ with Constraint $\sum_k P_k = 1$

rewritten to be: $\sum_k n_k \ln(P_k)$

Use Lagrauge Multiplier: $\mathcal{L}(x,\lambda) = f(x) - \lambda g(x)$

$$\sum_k n_k \ln(P_k) - \lambda\left(\sum_k P_k - 1\right) = 0$$

Partial Derivatives: $\frac{\partial}{\partial P_k}$

$$\frac{\partial}{\partial P_k}\left(\sum_k n_k \ln(P_k)\right) = \left(\lambda \sum_k P_k - 1\right)\frac{\partial}{\partial P_k}$$

$$n_k \cdot \frac{1}{P_k} = \lambda \cdot 1$$

$$n_k = \lambda P_k$$

Bring Sum to both sides:

$$\sum_k n_k = \lambda \sum_k P_k$$

↳ the constraint

Apply Constraint (equal to one):

$$\sum_k n_k = \lambda \cdot 1$$

Then:

$$\frac{n_k}{p_k} = \lambda = \sum_k n_k$$

Solving for P_k

$$n_k = P_k \sum_k n_k$$

$$\frac{n_k}{\sum_k n_k} = P_k$$

Therefore:

$\prod_k P_k^{n_k}$ is maximized

when $P_k = \dfrac{n_k}{\sum_k n_k}$