Workspace

Start with a weighted directed graph $\mathbf{G}$ containing n vertices $v \in \mathbf{V}$ with and m edges $e \in \mathbf{E}$.

We start by defining a weight function for the vertices $F : \mathbf{V} \to \mathcal{R}$ with a corresponding weight value $f_i = F(v_i)$. Similarly we add a weight for each edge given by the weight function $W : \mathbf{E} \to \mathcal{R}$ with corresponding weights $w_i = W(e_i)$. Finally, define an activation function on each vertex taking an affine parameter λ as $\mu[v] : \mathcal{R}x\mathcal{R} \to \mathcal{R}$. For simplicity let $\mu_i(\lambda) := \mu[v_i](\lambda)$.

Now we can define the potential energy of the system by:

$$\mathcal{E} = \sum_i \left[f_i - \mu_i(\sum_j w_j f_j, \lambda) \right]^2 \tag{1}$$

I am going to switch to Einstein notation, since I think it will really help make things easier. Values of weights will be represented with superscript. We can then write the potential energy of the system using:

$$\mathcal{E} = \sum_i \left[f_i - \mu_i(w^j f_j, \lambda) \right]^2 \tag{2}$$

One idea is to direct-sum the spaces of nodes and edges. Each edge would also be a node, and the connection matrix becomes a matrix of 1's and 0's. This allows writing the potential energy using:

$$\mathcal{E} = \sum_i \left[f_i - \mu_i(\Delta^{jk} w_j f_k, \lambda) \right]^2 \tag{3}$$

Now we define a new vector $\mathbf{g}$ by combining $\mathbf{f}$ and $\mathbf{w}$:

$$\begin{aligned} g_i &= f_i | i \in \{1..., n\} & (4) \\ g_i &= w_{i-n} | i \in \{n+1..., n+m\} & (5) \\ \Delta^{ij} &\in \{0, 1\} & (6) \end{aligned}$$