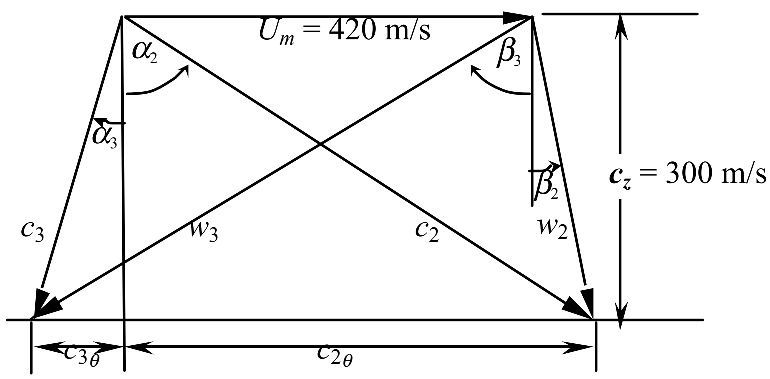
**Need**: (a) $\beta_2$, $\beta_3$ $R', \psi, \dot{W}_{out}$ ;(b) $A_t$

**Given:** Axial flow turbine, $c_1 = c_3$ at $r_m$, $T_{o1}$ = 850 K, $p_{o1}$ = 400 kPa, $c_z$ = 300 m/s, $U_m$ = 420 m/s, $\alpha_2$ = 60°, $\alpha_3$ =-15°, $\dot{m}$=25 kg/s, $\lambda_n$ = 0.05.

**Solution:**




**Assumptions:**
1 Steady, adiabatic flow
2 Mean radius analysis with negligible radial displacement of streamlines, $c_r \approx 0$
3 $c_z$ = constant at mean radius
4 Fluid angles same as blade angles
5 Ideal gas behavior, $R = R_{air}$ = 287 J/kg-K
6 Constant $\eta_{pe}$ in nozzle row

$$\tan\beta_2 = \frac{-c_z\tan\alpha_2 + U}{c_z} = -\tan\alpha_2 + \frac{U}{c_z} = -0.332 \Rightarrow \boxed{\beta_2 = -18.4°}$$

$$\tan\beta_3 = \frac{-c_z\tan\alpha_3 + U}{c_z} = -\tan\alpha_3 + \frac{U}{c_z} = 1.668 \Rightarrow \boxed{\beta_3 = 59.1°}$$

$$R'_m = \frac{c_z}{2U_m}\left(\tan\beta_3 + \tan\beta_2\right) \Rightarrow \boxed{R' = 0.477}$$

$$\psi \equiv \frac{|w|}{U^2} = \frac{|\Delta h_o|}{U^2} = \left[(\tan\alpha_2 + \tan\beta_3)\frac{c_z}{U} - 1\right] \Rightarrow \boxed{\psi = 1.43},$$

$$\dot{W}_{out} = \dot{m}|\Delta h_o| = \dot{m}U^2\psi = 6.30\times10^6\ W \Rightarrow \boxed{\dot{W}_{out} = 6.3\ MW}$$

Before calculating nozzle throat area, check if nozzle exit flow is supersonic: $M_2 = c_2 \big/ \sqrt{\gamma R T_2}$

$$T_2 = T_{o2} - \frac{c_2^2}{2c_{ph}};\ T_{o2} \overset{(1)}{=} T_{o1} = 850\ \text{K},\ c_{ph} = \frac{\gamma_h}{\gamma_h - 1}R = 1148\ J/kg\text{-}K,\ c_2 = \frac{c_z}{\cos\alpha_2} = 600\ \text{m/s}$$

$\Rightarrow T_2$ = 693 K, $M_2$ = 1.16 > 1, hence there must be a throat in nozzle row.

At the throat $M_t = 1 \Rightarrow T_t = \dfrac{T_{o1}}{1 + \frac{\gamma_h - 1}{2}\cancel{M_t^2}_{1}} = \dfrac{2T_{o1}}{\gamma_h + 1}$ = 729 K.

From nozzle loss: $T_2 - T_{2s} = \dfrac{\lambda_n c_2^2}{2c_{ph}} \Rightarrow T_{2s} = T_2 - \dfrac{\lambda_n c_2^2}{2c_{ph}}$ = 685 K

Because states o1 and 2$s$ are on a constant entropy line, $p_2 = p_{o1}\left(\frac{T_{2s}}{T_{o1}}\right)^{\frac{\gamma_h}{\gamma_h-1}} = 169$ kPa

To calculate polytropic process $1 \rightarrow t \rightarrow 2$, need to calculate state 1 properties also.

$$T_1 = T_{o1} - \frac{c_1^2}{2c_{ph}};\ c_1 = c_3;\ c_3 = \frac{c_z}{\cos\alpha_3} = 311 \text{ m/s}$$

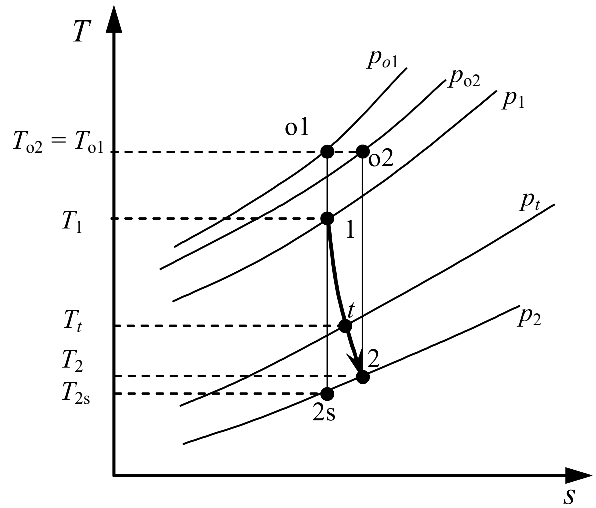
$$\Rightarrow T_1 = 808 \text{ K},\ p_1 = p_{o1}\left(\frac{T_1}{T_{o1}}\right)^{\frac{\gamma_h}{\gamma_h-1}} = 327 \text{ kPa}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma_h}{\eta_{pe}(\gamma_h-1)}} = \left(\frac{T_2}{T_1}\right)^{\kappa} \Rightarrow \kappa = \frac{\ln(p_2/p_1)}{\ln(T_2/T_1)} = 4.297$$

(correspondingly $\eta_{pe} = 0.931$)

$$p_t = p_1\left(\frac{T_t}{T_1}\right)^{\kappa} = 209 \text{ kPa} \Rightarrow \rho_t = \frac{p_t}{RT_t} = 1.00 \text{ kg/m}^3;$$

$$c_t = \sqrt{\gamma R T_t} = 528 \text{ m/s}\quad A_t = \frac{\dot m}{\rho_t c_t} \Rightarrow \boxed{A_t = 4.73\times10^{-2} \text{ m}^2}$$



**Need**: (a) $\pi_{st}$; (b) $p_1, p_2$ at $r_m$; (c) $\beta_{2t}, \beta_{3t}, R'_t$; velocity triangles at blade tip (d) $\beta_{2h}, \beta_{3h}, R'_h$; velocity triangles at blade root.

**Given:** Axial flow turbine stage, free-vortex design, $T_{o1}$ = 1500 K, $p_{o1}$ = 850 kPa,
No swirl at exit, $c_3$ = 180 m/s, $R'_m$ = 0.5; $\lambda_n$ = 0.04, $\eta_{st}$ = 0.93, $r_h/r_t$ = 0.6
$c_z/U_m$ = 0.6, $\gamma_h = {}^4/_3$

**Solution:**

$$\pi_{st} \equiv \frac{p_{o3}}{p_{o1}} = \left[1 - \frac{\Delta T_o}{\eta_{st} T_{o1}}\right]^{\frac{\gamma_h}{\gamma_h - 1}} = \left[1 - \frac{\Delta T_o}{\eta_{st} T_{o1}}\right]^4 \text{ for } \gamma_h = \tfrac{4}{3}$$

No swirl at exit $\Rightarrow \alpha_3 = 0^\circ$ and $c_3 = c_z$ = 180 *m/s*, $c_{3\theta m} = 0$

$$U_m = \frac{c_z}{\phi} = 300 \text{ m/s}$$

$R'_m = 0.5 \Rightarrow$ symmetric velocity triangles, $\beta_{2m} = \alpha_{3m}$, $\alpha_{2m} = \beta_{3m}$

$$\Delta T_o = \frac{|w|}{c_{ph}}; \quad |w| = U_m(c_{2\theta m} + \cancel{c_{3\theta m}}_{0}) = U_m^2$$ by inspection of symmetric triangles below

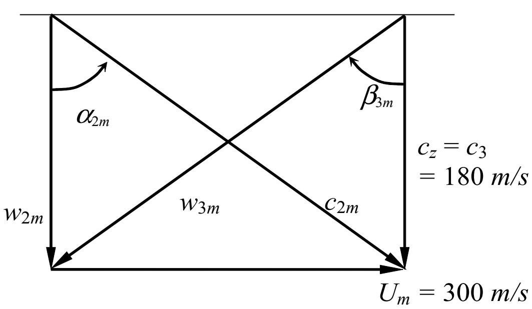
**Assumptions:**

1 Steady, adiabatic flow
2 Mean radius analysis with negligible radial displacement of streamlines, $c_r \approx 0$
3 $c_z$ = constant at mean radius
4 Fluid angles same as blade angles
5 Ideal gas behavior, $R = R_{air}$ = 287 *J/kg-K*
6 $c_1 = c_3$

So $w = 9\times10^4$ m²/s² $c_{ph} = \dfrac{\gamma_h}{\gamma_h - 1} R_{air}$ = 1148 J/kg-K

$$\Rightarrow \Delta T_{o,st} = \frac{U_m^2}{c_{ph}} = 78.4\ K$$

$\boxed{\pi_{st} = 0.793}$




$$T_{2m} = T_{o2} - \frac{c_{2m}^2}{2c_{ph}}; \quad c_{2m} = w_{3m} = \sqrt{U_m^2 + c_z^2} = 350 \text{ m/s}$$ ; $T_{o2} = T_{o1}$ (adiabatic nozzle) $\Rightarrow T_{2m}$ = 1447 K

$$T_2 - T_{2s} = \lambda_n \frac{c_{2m}^2}{2c_{ph}} \Rightarrow T_{2s} = T_{o2} - (1 + \lambda_n)\frac{c_{2m}^2}{2c_{ph}} = 1445 \text{ K};$$

$$p_2 = p_{o1}\left(\frac{T_{2s}}{T_{o1}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} \Rightarrow \boxed{p_2 = 731 \text{ kPa}}$$

Assuming repeating stages so $c_1 = c_3$

$$T_1 = T_{o1} - \frac{c_1^2}{2c_{ph}} \overset{(6)}{=} T_{o1} - \frac{c_1^2}{2c_{ph}} = 1486 \text{ K}; \quad p_1 = p_{o1}\left(\frac{T_1}{T_{o1}}\right)^{\frac{\gamma_h}{\gamma_h - 1}} \Rightarrow \boxed{p_1 = 818 \text{ kPa}}$$

$$\zeta \equiv \frac{r_h}{r_t} = 0.6; \quad r_m = \frac{r_h + r_t}{2} = \frac{\zeta + 1}{2} r_t = \frac{\zeta + 1}{2\zeta} r_h \Rightarrow \frac{r_h}{r_m} = \frac{2\zeta}{1 + \zeta} = 0.75, \quad \frac{r_t}{r_m} = \frac{2}{\zeta + 1} = 1.25.$$

$$U_t = \frac{r_t}{r_m} U_m = 375 \text{ m/s} \qquad U_h = \frac{r_h}{r_m} U_m = 225 \text{ m/s}$$

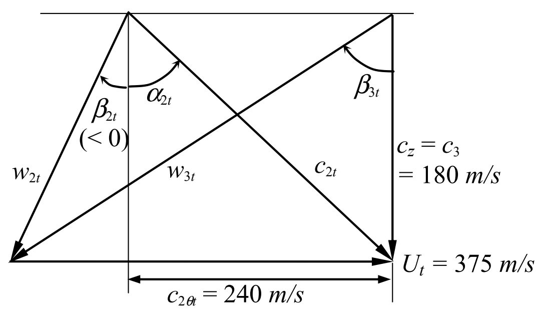
Free-vortex design $\Rightarrow rc_\theta = \text{const.}$

So $r_t\, c_{2\theta t} = r_m\, c_{2\theta m} \Rightarrow c_{2\theta t} = \frac{r_m}{r_t} c_{2\theta m} = 240\ m/s;\ c_{3\theta t} = \frac{r_m}{r_t} \cancel{c_{3\theta m}}_{0} = 0$.

$c_z \neq f(r)$ in a free-vortex design $\Rightarrow c_{3t} = c_z = 180\ m/s$

$$\tan\beta_{2t} = \frac{-c_{2\theta t} + U_t}{c_z} = +0.75 \Rightarrow \boxed{\beta_{2t} = +36.9°}\ ;\ \tan\beta_{3t} = \frac{U_t}{c_z} = 2.08 \Rightarrow \boxed{\beta_{3t} = 64.4°}$$

$$R_t' = \frac{c_z}{2U_t}\left(\tan\beta_{3t} + \tan\beta_{2t}\right) \Rightarrow \boxed{R_t' = 0.680}$$




Similarly, $r_h\, c_{2\theta h} = r_m\, c_{2\theta m} \Rightarrow c_{2\theta h} = \frac{r_m}{r_h} c_{2\theta m} = 400\ m/s;\ c_{3\theta h} = \frac{r_m}{r_h} \cancel{c_{3\theta m}}_{0} = 0$.

$c_z \neq f(r)$ in a free-vortex design $\Rightarrow c_{3h} = c_z = 180\ m/s$

$$\tan\beta_{2h} = \frac{-c_{2\theta h} + U_h}{c_z} = \mathbf{-0.972} \Rightarrow \boxed{\beta_{2h} = -44.2}\ ;\ \tan\beta_{3h} = \frac{U_h}{c_z} = 1.25 \Rightarrow \boxed{\beta_{3h} = 51.3°}$$

$$R_h' = \frac{c_z}{2U_h}\left(\tan\beta_{3h} + \tan\beta_{2h}\right) \Rightarrow \boxed{R_h' = 0.111}$$