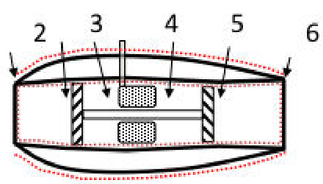
ASE 376K Homework # 4

1. A jet aircraft moves with a velocity of 200 m/s where the air temperature is 20°C and the pressure is 101 kPa. The inlet and exit areas of the turbojet engine of the aircraft are 1 m^2 and 0.6 m^2, respectively. It is known that the exit jet nozzle velocity is 1522 m/s (from lab calculation) if the exhaust gases expand to 101 kPa at a temperature of 1,000°C. The mass flow rates of the inlet and exhaust flow are 240 kg/s and 252 kg/s, respectively. As a thermal engineer, your task is to (a) determine if the temperature of the exhaust gases is too high for the turbine blades as they exit from the combustion chamber. (b) Determine the amount of combustion energy necessary to provide the thrust. The maximum tolerable temperature of the blades is 3,000 K. It is known that the pressure ratio of the multi-stage compressor is 8 to 1.
Assumptions and simplifications:

- neglect all losses and irreversibilities/inefficiencies. All processes are isentropic.
- Neglect all kinetic energy components except at the inlet and the nozzle.
- Air and fuel mixture behaves as an ideal gas and has the same thermal properties as the air.
- All shaft work produced by the turbine are used to drive the compressor.
- Air (& mixture) has a constant C_P=1 kJ/kg.K, and k=1.4

2. For problem 1, calculate the overall efficiency of the engine and compare with the Brayton Cycle efficiency.

- Between sections 1 and 2, the incoming flow slows down to increase both the pressure and the temperature before it enters the compressor section. It is an isentropic process.



$$h_1 + \frac{V_1^2}{2} = h_{02},\ C_P T_1 + \frac{V_1^2}{2} = C_P T_{02}$$

$$T_{02} = T_1 + \frac{V_1^2}{2C_P} = 293 + \frac{(200)^2/1000}{2(1)} = 313[\mathrm{K}]$$

$$\text{From isentropic relation: } \frac{T_{02}}{T_1} = \left(\frac{P_{02}}{P_1}\right)^{\frac{k-1}{k}}$$

$$P_{02} = P_1 \left(\frac{T_{02}}{T_1}\right)^{\frac{k}{k-1}} = 101 \left(\frac{313}{293}\right)^{3.5} = 127.3[\mathrm{kPa}]$$

- Across the compressor, the pressure is further increase to eight times of P_2 and this is accompanied by an increase of temperature also. $P_3=8P_2=1018.4$(kPa)

$$\text{Isentropic compression: } \frac{T_{03}}{T_{02}} = \left(\frac{P_{03}}{P_{02}}\right)^{\frac{k-1}{k}},\ T_{03} = T_{02}(8)^{0.286} = 567.3[\mathrm{K}]$$

• The shaft work of the compressor is equal to the difference of enthalpy before and after the compressor.

$$W_{compressor} = \dot{m}_1(h_{03} - h_{02}) = \dot{m}_1 C_P(T_{03} - T_{02}) = (240)(1)(567.3 - 313) = 61037[\text{kJ}]$$

• The same amount of shaft work is produced across the turbine section as assumed.

$$W_{compressor} = W_{turbine} = \dot{m}_{mixture}(h_{04} - h_{05}) = \dot{m}_m C_P(T_{04} - T_{05})$$
$$= (240 + 12)(1)(T_{04} - T_{05}) = 61037[\text{kJ}]$$

$T_{04} - T_{05} = 242.2[\text{K}]$. That is, there is a 242.2[K] temperature drop as the mixture expands through the turbine.

• In order to determine the temperature entering the turbine (T_4), we need to find the temperature exiting the turbine (T_5) and it is related to the temperature exiting the nozzle (T_6) as it is expanding in the nozzle through an isentropic process.

$$h_{05} = h_6 \frac{v_6^2}{2}, \; T_{05} = T_6 + \frac{V_6^2}{2C_P} = 1273 + \frac{(1522)^2/1000}{2(1)} = 2431[\text{K}]$$

• Therefore, the temperature of the hot gas entering the turbine section will be at a temperature of T_4=T_5+242.2=2673.2(K) and it is below the maximum tolerable temperature of 3,000 K.

• The total thermal energy supplied into the engine can be determined as the difference of the energy in and out of the combustion chamber:

$$\dot{Q} = \dot{m}_m h_{04} - \dot{m}_a h_{03} = C_P[(252)T_{04} - (240)T_{03}] =$$
$$(1)[(252)(2673.2) - (240)(567.3)] = 537494[\text{kJ}]$$

$$\text{The overall efficiency of the engine} = \frac{\text{Power output}}{\text{Heat input}} = \frac{TV}{\dot{Q}}$$
$$= (335.5)(200)/537494 = 12.5\%$$

This is significantly lower than the Brayton cycle's efficiency

$\eta = 1 - r_P^{(1-k)/k} = 48.2\%$, where $r_P =$ pressure ratio across the compressor and inlet

• Note: I neglect the energy of the fuel by assuming that is small compared to the combustion energy.

3. An engine is flying at M=0.85 at an altitude of 27000ft. The maximum temperature that the turbine can handle is 2800K. The compressor pressure ratio, PR = 35. Make assumptions for any other variables you need. Using the equations derived in Lecture 16, compute the thrust of the engine for efficiency values of 1, 0.9 and 0.8 (i.e., set all the η values and rc to the same for each calculation)
What is the loss in thrust because of the inefficiencies? Report the answer in the form of this table.

$\eta_n = \eta_t = \eta_c = \eta_b = \eta_d = rc$	Specific Thrust (k-Ns/kg)
1	1523
0.9	1369
0.8	1190

Writing a short computer code for this problem might be the best way to solve this problem, since you have to repeat the calculation three times.