ASE 376K Homework # 2

2-1 A jet engine is designed for maximum thrust on a test stand (static thrust) of 53kN. The tests are conducted at 25C and 1 atm ambient conditions. The engine has a inlet mass flow rate of 77 kg/s at designed static thrust. Assume that the nozzle is perfectly expanded to begin with and check if that assumption may have a problem. The fuel used in the engine is Kerosene, which produces 46 MJ/kg of fuel combusted. Assume the engine design is such that the maximum allowable turbine temperature of 1500K is reached.

a) Assuming stoichiometric fuel-air ratio, compute the nozzle exit velocity.

Answer:
Stoichiometric fuel air ratio for Kerosene is about 0.07. (See HW1). For static conditions,

$$\mathfrak{I} = \dot{m}_a u_e$$

We are assuming the nozzle is perfectly expanded, i.e., pe=pa, but remember we don't know that for sure, at this point it is just an assumption.

Since inlet mass flow rate is 77kg/s and stoichiometric fuel-air ratio is 0.07, exit mass flow rate is 1.07*77 = 82.1 kg/s.
Thrust is 53kN, Nozzle exit velocity, u_e = 53000/82.1 = 645 m/s.

This could be supersonic or subsonic depending on the speed of sound of the exhaust gases. If it is subsonic, our assumption of perfectly expanded would be fine. If supersonic, it depends on nozzle shape and length.

b) The gases enter the turbine at relatively low velocity, and assume this is zero for this calculation. Assuming isentropic expansion across the turbine and nozzle, what is the exit temperature of the exhaust gases. The exhaust gases contain the mass fractions of CO_2, H_2O and N_2 in the stoichiometric ratio (See class notes). Assume that the R for the exhaust gases is close to that of air and gamma is a constant.

Answer:

Assuming isentropic expansion in the nozzle, we know that the stagnation temperature stays constant. Therefore the turbine stagnation temperature is 1500K.

Assuming isentropic expansion through the turbine and nozzle,

$T_0/T_{exit} = 1 + ((\gamma-1)* M_{exit}$^2)/ 2
But we know, $M_{exit} = U_{exit}$ /sqrt($\gamma*R_{exit}*T_{exit}$)

$T0/T_{exit} = 1 + (\gamma-1)*U_{exit}$^2/($\gamma$*2*$R_{exit}*T_{exit}$)

Or, $1500/T_{exit}$ = 1 + 0.4*645^2/(1.4*2*287*T_{exit})

Or, (1500-208)/T_{exit}=1, or T_{exit} = 1292K.

c) What is the Mach No of the exhaust gases? What does this tell you about the assumption of pe=pa?

Answer:

Mach number of exhaust gases = U_{exit}/sqrt(γ*R*T_{exit}) = 645/sqrt(1.4*287*1292) = 645/720 = 0.895. So, the assumption of perfectly expanded, i.e., p_e=p_a is ok.

d) What is the temperature rise of the air due to the stoichiometric combustion of Kerosene? Is the stoichiometric burning of Kerosene viable? Why or why not?

Answer:

The amount of fuel burnt for stoichiometric combustion is 0.07*77 = 5.4 kg/s
The power produced is mfuel*specific heat of combustion = 5.4 kg/s*46 MJ = 248.4 MW.

The temperature rise of the air-fuel mixture is mdot * Cp * ($T_{turbine}$-$T_{combustor}$) = 248.4 *1e6
Therefore $T_{turbine}$-$T_{combustor}$ = 248.4*1e6/((77+5.4)*1005) = 2994K

Since $T_{combustor}$ > 0, $T_{turbine}$ > 2994K
This is higher than the allowable turbine inlet temperature! So stoichiometric burning is not viable.

e) Usually in real operation, engines are operated in a lean fuel-air mixture. Assuming a lean fuel-air mixture, with a fuel-air ratio of 0.02, what is the compressor pressure ratio and efficiency of the Brayton cycle for the engine?

Answer:

The amount of fuel burnt for lean combustion is 0.02*77 = 1.54 kg/s
The power produced is mfuel*specific heat of combustion = 5.4 kg/s*46 MJ = 70.8 MW.

The temperature rise of the air-fuel mixture is m_{dot} * C_p * ($T_{turbine}$-$T_{combustor}$) = 70.8 *1e6
Therefore $T_{turbine}$-$T_{combustor}$ = 70.8*1e6/((77+1.54)*1005) = 897K

Now, $T_{combustor}$ = 1500-897 = 603K

Therefore, Temperature ratio = 603/298 = 2.02
And compressor pressure ratio = (2.02)^1.4/0.4 = 11.8

The efficiency of the Brayton cycle is: 1-1/2.02 = 0.505

2-2 To estimate the effect of combustion on the gas constant of the combustion products in a jet engine, consider the following typical gas compositions:

		Mole Fraction		
	Molar Mass (kg/kmol)	Pure Air	Mixture 1 ($f = 3.32\times10^{-2}$)	Mixture 2 ($f = 1.66\times10^{-2}$)

N_2	28.0	0.783	0.761	0.775
O_2	32.0	0.208	0.101	0.155
Ar	40.0	9.30×10^{-3}	7.23×10^{-4}	7.37×10^{-4}
CO_2	44.0		6.47×10^{-2}	3.30×10^{-2}
H_2O	18.0		7.28×10^{-2}	3.71×10^{-2}

Calculate the gas constant of pure air, and of mixtures 1 and 2 assuming an ideal gas mixture in each case. For each case, what is the percentage error in R if one assumes R = 287 J/kg-K?

Need: Gas constant for different mixtures, % error in assuming $R_{nominal}$ = 287 J/kg-K.
Given:

		Mole Fraction (χ)		
	Molar Mass (*kg/kmol*)	Pure Air	Mixture 1 ($f = 3.32\times10^{-2}$)	Mixture 2 ($f = 1.66\times10^{-2}$)
N_2	28.0	0.783	0.761	0.775
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Solution:

$$R = \frac{\hat{R}}{M_{mix}};\ M_{mix} = \sum_s \chi_s M_s$$

Assumptions:
(1) Ideal gas behavior

Using a simple spreadsheet to perform the calculations one gets:

	Pure Air	Mixture 1 ($f = 3.32\times10^{-2}$)	Mixture 2 ($f = 1.66\times10^{-2}$)
M_{mix}	28.94	28.72	28.78
R_{mix}(*J/kg-K*)	287	289	289
*Error**	0.1%	0.9%	0.6%
Error	0	1%	0.7%

***In calculating this percentage error I have carried a few extra digits to prevent loss of significant digits when computing R_{mix}–$R_{nominal}$. If one carried 3 digits throughout one gets the values shown on the last line.**

2-3. A rocket engine uses a H_2-O_2 mixture as a propellant. The mean molar mass of the propellant is 11.58 kg/kmol with ratio of specific heats γ = 1.20. The *stagnation* pressure at the throat of the nozzle is 8.26 MPa and the *static* temperature at the throat is 3300 K. The throat area A_t = 750.4 cm^2. The ratio of the exit plane area to the throat area is 39.8 and the static pressure at the exit plane of the nozzle is 18.1 kPa.

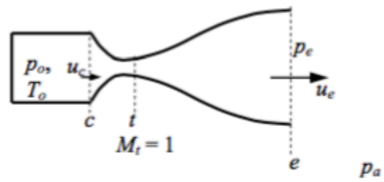
(a) Calculate the mass flow rate of propellant through the nozzle.

(b) Calculate the vacuum thrust of the rocket motor.

(Picture from http://www.boeing.com/defense-space/space/propul/SSME.html)

Need: (a) $\dot{m}$ (b) $\Im_{vac}$

Given: H_2-O_2 propellant, $\hat{M}_{prop}$ = 11.58 kg/kmol,
γ = 1.20, p_{ot} = 8.26 MPa, T_t = 3300 K,
A_t = 750.4 cm^2, p_e = 18.1 kPa, A_e/A_t = 39.8



Solution:

Assumptions:
(1) Steady, quasi-1D flow
(2) Supersonic flow in nozzle
(3) Ideal gas behavior, with constant c_p
(4) Adiabatic nozzle

$\dot{m} \overset{(1)}{=} \rho_t A_t u_t = \rho_e A_e u_e$

$u_t = \cancel{M_t}_{1}\sqrt{\gamma R T_t}$;

$R_{prop} \overset{(4)}{=} \dfrac{\hat{R}}{\hat{M}_{prop}} = \dfrac{8314 \text{ J/kmol-K}}{11.58 \text{ kg/kmol}} = 718$ J/kg-K $\Rightarrow u_t$ = 1.69 km/s

$p_t = \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} p_{ot} = 4.66$ MPa; $\rho_t \overset{(3)}{=} \dfrac{p_t}{R_{prop}T_t} = 1.97$ kg/m^3 ; $\dot{m} = \rho_t A_t u_t \Rightarrow \boxed{\dot{m} = 249 \text{ kg/s}}$

$\Im_{vac} \overset{(1)}{=} \dot{m}u_e + \left(p_e - p_a\right)A_e = \dot{m}u_e + p_e A_e$ ($p_a = 0$ in vacuum)

Know $\dot{m}$, p_e, A_e = 39.8 A_t; need u_e.

$\dot{m} = \rho_e A_e u_e = \dfrac{p_e}{R_{prop}T_e} A_e u_e$; need T_e; But $T_{oe} = T_e + \dfrac{u_e^2}{2c_p} \overset{(4)}{=} T_{ot}$; $c_p \overset{(3)}{=} \dfrac{\gamma}{\gamma-1} R_{prop} = 4308$ J/kg-K

$T_{ot} = \dfrac{\gamma+1}{2} T_t = 3630$ K (equivalently $T_{ot} = T_t + \dfrac{u_t^2}{2c_p} = 3630$ *K*)

We have 2 equations, mass and energy (i.e. T_o), in 2 unknowns u_e, T_e. Can solve in principle. One possible strategy: From mass conservation we have $T_e = \left(p_e / R_{prop}\dot{m}\right) A_e u_e$. Substituting in equation for T_{ot} we get a quadratic equation in u_e: $\dfrac{u_e^2}{2c_p} + \left(p_e A_e / \dot{m} R_{prop}\right) u_e - T_{ot} = 0$

$\Rightarrow u_e^2 + b u_e + c = 0$, where $b \equiv \dfrac{2\gamma}{\gamma-1}\dfrac{p_e A_e}{\dot{m}}$; $c \equiv -2c_p T_{ot}$

Substituting numerical values: $b = 2.605\times10^3$ m/s; $c = -3.127\times10^7$ m^2/s^2 and picking positive root of the quadratic equation $u_e = \dfrac{-b+\sqrt{b^2-4c}}{2}$ = 4.44 km/s.

Finally $\Im_{vac} = \dot{m}u_e + p_e A_e = 1.16\times10^6$ N or $\boxed{\Im_{vac} = 1.16 \text{ MN}}$

(Even in a vacuum the pressure term only contributes ~5% because of the large expansion ratio in the diverging section ~40, and hence relatively low pressure in the exit plane.)

Note that the expansion process in the nozzle from throat to exit is ***not*** isentropic. We can calculate the exit stagnation pressure from $p_{oe} = p_t\left(T_{ot}/T_e\right)^{\frac{\gamma}{\gamma-1}}$ where $T_e \overset{(4)}{=} T_{ot} - u_e^2/2c_p$ =1342 K.

This gives p_{oe} = 7.08 MPa < p_{ot} = 8.26 MPa.