ASE 376 K HOMEWORK #7

1. A turbine stage with a mass flow rate 25 kg/s, has equal inlet and outlet velocities at the mean radius and the following design data

Inlet temperature T_{o1}	850 K	Axial velocity (const through stage) c_z	300 m/s
Inlet pressure p_{o1}	400 kPa	Blade speed U_m	420 m/s
Nozzle efflux angle α_2	60°	Stage exit swirl α_3	-15°

(a) Determine the rotor blade gas angles ($\beta_{2,3}$) degree of reaction, blade loading coefficient ψ, and power output.

(b) Assuming a nozzle loss coefficient λ_n of 0.05, calculate the nozzle throat area required ignoring the effect of boundary layers.

Methodology Steps for part (b):

1. First determine if the Mach number at station 2 (at the exit to the stator/nozzle) is supersonic. If it is, the nozzle is shaped like a CD nozzle and you have to find the nozzle throat area. The throat area is determined from $A_t = \dot{m}/(\rho_t c_t)$, where c_t is the throat velocity. In this step, T2 is calculated using total enthalpy conservation from station 1 to 2.
2. From the nozzle loss coefficient, calculate T2s (the temperature that would be reached isentropically). Given, P01, T01 and T2s, calculate, P2.
3. State 1 properties can be calculated from using enthalpy equation and pressure temperature isentropic relation.
4. Since State 1 properties and State 2 properties are known, determine the polytropic efficiency from the pressure temperature relationship involving polytropic efficiency.
5. Once polytropic efficiency is known the same calculation in step 4 can be done from State 1 to the throat of the nozzle.
6. Nozzle loss coefficient is given by

$$T_2 - T_{2s} = \frac{\lambda_n c_2^2}{2c_{ph}}$$

7. Also, assume $\gamma = 1.33, C_{ph} = 1148 \frac{J}{(K.kg)}$ (However, if you have already solved the problem using standard $\gamma = 1.4, C_{ph} = 1005 \frac{J}{(K.kg)}$, the solution will be graded without any loss in credit.

2. An axial flow turbine stage has a free-vortex design and the following data:

Inlet stagnation temperature, T_{o1}	1500 K	Inlet stagnation pressure, p_{o1}	850 kPa
No swirl at exit, exit velocity, c_3	180 m/s	Degree of reaction at mean radius, R'_m	0.5
Nozzle loss coefficient, λ_n	0.04	Turbine adiabatic efficiency	0.93
Hub-to-tip ratio, $\zeta = r_h/r_t$	0.6	Flow coefficient, c_z/U_m	0.6

Calculate:

(a) the stagnation pressure ratio across the stage;

(b) the static pressures p_1, p_2 at the mean radius;

(c) the angles β_{2t}, β_{3t} and degree of reaction R_t' at the tip. Sketch the velocity triangles at the blade tip.

(d) the angles β_{2h}, β_{3h} and degree of reaction R_h' at the root (hub). Sketch the velocity triangles at the root of the blades.