Lecture 11

Review for Test 1

Key things to focus on

- Speed ranges for different types of propulsion
- Laws of Thermodynamics
- Behavior of Mixtures of Ideal Gases
- Basics of compressible flow, Mach No, Normal shock behavior
- Brayton cycle P-V and T-S diagrams, efficiency, temperature and pressure ratios for maximum power
- Stoichiometric fuel air ratio, fuel equivalency ratio
- Meaning and Calculation of Thrust, Specific Impulse, Thrust Specific Fuel Consumption
- Air-breathing engine efficiencies: propulsive, thermal, overall – understand meanings
- Basics of Ramjet Propulsion, Ideal Ramjet

Key equations

- Mach Number u/a, where u is flight speed and a is speed of sound.
- Speed of sound $a = \sqrt{\gamma RT}$
- Efficiency of Brayton Cycle = 1 – 1/TR, where TR is compressor temperature ratio
- Pressure ratio relation to temperature ratio, $PR = TR^{\left(\frac{\gamma}{\gamma-1}\right)}$
- Fuel equivalency ratio = actual fuel flow rate/stoichiometric fuel flow rate
- $Thrust = \dot{m}e * Ue - \dot{m}a * Ua + (Pe - Pa) * Ae$
- TSFC = mdot(fuel)/Thrust
- Isp = Thrust/(mdot(fuel)*g) (Note g = 9.8 m/s^2)
- Optimal compressor exit temperature for maximum power
  - $Tb = \sqrt{Ta\ x\ Tc}$, where Ta is Tambient and Tc = Turbine Inlet Temperature

Example Problem 1

- A liquid rocket consumes 200 kg/s of oxidizer and 50 kg/s of fuel. After combustion the gas is accelerated in a C-D nozzle, where Vexit = 4000m/s, Pexit = 200kPa. If the exit diameter of the nozzle is 2m, calculate
  - (a) Pressure Thrust for an ambient pressure of 100kPa
  - (b) Net Thrust

Solution

- Pressure thrust = $(P_e - P_a) * A_e$
  - (200000-100000) * (3.1417 * D_e^2/4)= 314.16 kN

  Where, D_e is exit diameter
- Momentum thrust = $\dot{m}e$ * Ve – $\dot{m}a$*Vin

  = 250*4000 = 1000kN (There is no incoming flow, or ram drag, so Vin=0)

- Net thrust = 1000 + 314.16 = 1314.16 kN

Example Problem 2

- An aircraft engine in takeoff condition has the following parameters
- $\dot{m}a$ = 100kg/s
- Vin= 200m/s
- $\dot{m}f$ = 2 kg/s
- Ve = 900 m/s
- Calculate the thrust of the engine, assuming pressure thrust is zero.
- Compute thrust specific fuel consumption and specific impulse

Solution

- Thrust = $(\dot{m}a+\dot{m}f)$*V_e - $\dot{m}a$ * V_{in}

  = 102 * 900 – 100 * 200 = 71.8kN
- Fuel consumption = 2 kg/s
  - Thrust = 71.8kN
  - TSFC = $\dot{m}f$/Thrust = 2/71.8*1e3 = 27.8e-6 kg/Ns = 27.8 gm/kN-s
- Specific Impulse
  - Thrust/(Weight of fuel flow rate)

  = 71.8*1e3/(2 *9.8) = 3663 s

Example problem 4

- An aircraft has an TO weight of 450000 kg. It cruises at 900 km/hr at a L/D ratio of 17.
- It has a fuel capacity of 250000 liters and Jet fuel has a density of 800 gms/Liter.
- It has an SFC of 17.1 g/(kN-s)
- The engine has an inlet diameter of 3m and cruises at an altitude of 13km, where the air density is 0.24 kg/cu.m
- Assume fuel is uniformly burnt throughout operation and completely consumed during flight
  - What is the range of the aircraft?
  - Estimate the thrust of each engine during cruise conditions (there are four engines), assume pe=pa
  - Calculate fuel/air ratio
  - Calculate the exit velocity of the exhaust gases, propulsive efficiency, thermal efficiency and overall efficiency of each engine – assume jet fuel has a heat of reaction of 43 MJ/kg

Solution - Range

- We know range is
  - R = Vin * (L/D) * Isp * ln (Wi/Wf)
  - Vin = 900 km/hr = 250 m/s
  - L/D = 17
  - Wi = 450000 kg * 9.8
  - Wf = Wi – Wfuel = 450000 * 9.8 – ρ_{fuel} * $Vol_{fueltank}$ * g

    = 450000 * 9.8 – 800 * 250000 * 9.8/1000

    = 4320000-1960000 = 2450000
  - Therefore, Wi/Wf = 450000*9.8/2450000 = 1.8
  - Ln(Wi/Wf) = 0.588

  - SFC = 17.1 g/KN-s = 17.1e-6kg/N-s = $\dot{m}f$/Thrust
  - This means Thrust/$\dot{m}f$ = 56818, or Isp = 56818/9.8 = 5798 s

  - Therefore Range = 250 * 17 * 5798 * 0.588 = 14489 km

Solution – Engine Thrust

- Given TSFC = 17.1e-6 kg/N-s
- First, we need to calculate fuel burn rate.
  - Given flight speed of 900km/hr and a range of 14489km, time of flight = 16.1 hrs.
  - The mass of fuel = volume x density = 250000 * 0.8 = 200000 kg.
  - Therefore, $\dot{m}f$ = 200000/(16.1*3600)=3.45kg/s
- Thrust = $\dot{m}f$ /TSFC = 3.45/17.1e-6 = 201754N.

Solution – fuel/air ratio and Exit Velocity

- Fuel/Air ratio, f = $\dot{m}f/\dot{m}a$
- We need to calculate $\dot{m}a$ = ρ X Ain x Vin
  - $\dot{m}a$ = 0.24 * pi * (3*3)/4 * 250 = 424 kg/s
  - $\dot{m}f$ = 3.45 kg/s
  - Fuel/Air ratio, f = 3.45/(424) = 0.008
- Given Thrust = $\dot{m}e$ * Ve – $\dot{m}a$*Vin
    - 201754 = (424+3.45) * Ve – 424 * 250
    - Ve = 307754/427.45 = 720 m/s

Calculate Efficiencies

$$\eta_{th} = \frac{[(1+f)(u_e^2/2) - u^2/2]}{fQ_R}.$$

= (720^2-250^2) /(2*0.008*43e6)
= 0.66

$$\eta_p \approx \frac{(u_e - u)u}{(u_e^2/2) - u^2/2} = \frac{2u/u_e}{1 + u/u_e}$$

= 2*250/(250+720)
= 0.515

$$\eta_o = \eta_p \eta_{th} = \frac{\mathcal{T} u}{\dot{m}_f Q_R}.$$

= 0.66*0.515 = 0.34