Need: (a) π_{st} (b) π_c'; (c) N_{st}'

Given: GE 90-85B engine, $\pi_c = 39.3$, $N_{st} = 14$, π_{st} = constant; 4% improvement in stage pressure ratio.

Solution:

If π_{st} = constant, then $\pi_{st}^{N_{st}} = \pi_c \Rightarrow \pi_{st} = \pi_c^{1/N_{st}} \Rightarrow \pi_{st} = 1.30$

If compression is increased 5%, then $\pi_{st}' = 1.352$ and $\pi_c' = \left(\pi_{st}'\right)^{N_{st}} = 1.352^{14} = 68.1$ (73% increase)

If overall compression ratio is maintained, $\left(\pi_{st}'\right)^{N_{st}'} = \pi_c \Rightarrow N_{st}' = \dfrac{\ln \pi_c}{\ln \pi_{st}'} = 12.2$

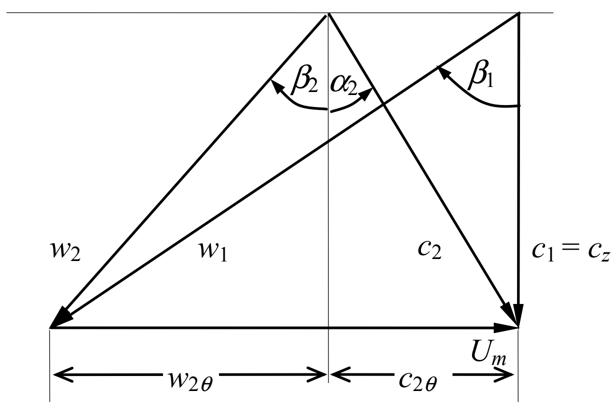
One cannot have fractional stages, so one would need 13 stages. Thus one could save the cost and weight associated with 1 stage if one could improve the stage compression ratio by 4% while maintaining constant efficiency. A further small improvement (4.5% improvement over original) could result in a drop to just 12 stages.

Note this example is meant to illustrate the dramatic effect of improvements in stage compression ratio. In real engines the stage compression ratio is usually NOT constant. The stage stagnation temperature rise ($\Delta T_{o,st}$) is approximately constant in the low pressure compressor and a different constant in the high pressure compressor. Constant stagnation temperature rise per stage gives a decreasing stage pressure ratio in successive stages.

Need: (a) Velocity triangles, β_1, α_2, and β_2 at r_m; (b) $\Delta T_{o,st}$; (c) π_{st}

Given: $T_{o1} = 260\ K$, $M_1 = 0.42$; Axial inlet flow; $r_m = 0.52\ m$; $N = 6000\ rpm$, $\lambda = 0.97$, $w_2 = 0.74\ w_1$, $\eta_{st} = 0.93$

Solution:



Assumptions:
1. Steady flow
2. Mean radius analysis with negligible radial displacement of streamlines
3. c_z = constant at mean radius
4. Fluid angles same as blade angles
5. Radial equilibrium, $c_r \approx 0$
6. Ideal gas behavior

$\Omega = 2\pi N = 628\ rad/s$; $U_m = \Omega r_m = 327\ m/s$;

$$T_1 = \frac{T_{o1}}{1+\frac{\gamma-1}{2}M_1^2} = 251\ K. \quad a_1 = \sqrt{\gamma R T_1} = 318\ m/s.$$

Axial inlet flow $\Rightarrow \alpha_1 = 0$; $c_z = c_1 = M_1 a_1 = 0.42\times 318\ m/s = 133\ m/s$.

By inspection of the velocity triangles $w_1 = \sqrt{c_{1z}^2 + U_m^2} = 353\ m/s \Rightarrow M_{rel} = \frac{w_1}{a_1} = \boxed{1.11}$

Again, by inspection of the sketch $\beta_1 = \tan^{-1}\frac{U_m}{c_z} = \boxed{67.8°}$

$w_2 = 0.74\ w_1 = 261\ m/s$; $\cos\beta_2 = \frac{c_z}{w_2} \Rightarrow \boxed{\beta_2 = 59.3°}$

$w_{2\theta} = c_z \tan\beta_2 = w_2 \sin\beta_2 = 225\ m/s$; $c_{2\theta} = U_m - w_{2\theta} = 102\ m/s$; $\tan\alpha_2 = \frac{c_{2\theta}}{c_z} = 0.766$

$\Rightarrow \boxed{\alpha_2 = 37.5°}$

$$\Delta T_{o,st} = \frac{\lambda U_m c_z}{c_p}\left(\tan\alpha_{2m} - \cancel{\tan\alpha_{1m}}_{(0)}\right);\ c_p = \frac{\gamma}{\gamma-1}R_{air} = 1005\ J/kg\text{-}K;\ \Rightarrow \Delta T_{o,st} = \boxed{32.3\ K}$$

$$\pi_{st} \equiv \left[1+\eta_{st}\frac{\Delta T_o}{T_{o1}}\right]^{\frac{\gamma}{\gamma-1}} = \boxed{1.47}$$

Need: (a) N_{stage} (b) Ω (c) b for last stage of compressor

Given: $\pi_{overall} = 10$, axial compressor; $p_{oi} = 220\ kPa$, $T_{oi} = 372\ K$. $\Delta T_{o,stage,\max} = 30\ K$, $\lambda = 0.85$, $\eta_{pc} = 0.90$, $c_1 = 150\ m/s$, $\alpha_1 = 30°$, $d_m = 0.50\ m$, $\dot{m} = 56\ kg/s$

Solution:

$$\pi_{overall} = \frac{p_{oe}}{p_{oi}} = \left(\frac{T_{oe}}{T_{oi}}\right)^{\eta_{pc}\frac{\gamma}{\gamma-1}} \Rightarrow \frac{T_{oe}}{T_{oi}} = \left(\pi_{overall}\right)^{\frac{\gamma-1}{\eta_{pc}\gamma}} = 2.08;$$

$T_{oi} = 372\ K \Rightarrow T_{oe} = 773\ K$

So $\Delta T_{oc} = 401\ K$; $N_{stage} = \dfrac{\Delta T_{oc}}{\Delta T_{o,stage,\max}} = \dfrac{401K}{30K} = 13.4 \Rightarrow$

Because we cannot have a fractional number of stages $\boxed{N_{stage} = 14}$ and corresponding $\Delta T_{o,stage} = 28.6\ K$

Assumptions:

1 Steady flow
2 Mean radius analysis with negligible radial displacement of streamlines, $c_r \approx 0$
3 c_z = constant at mean radius
4 Ideal gas behavior