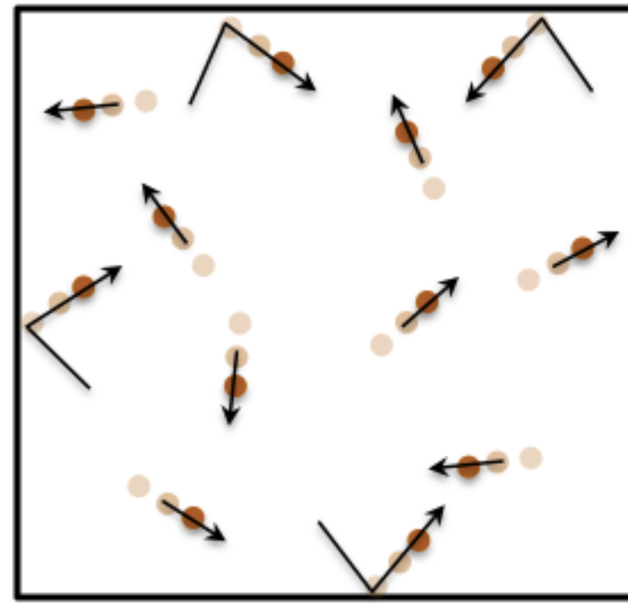
Ideal Gas Mixtures

Lecture 4

Ideal Gases

- An ideal gas consists of a large number of molecules, which are in random motion and obey Newton's laws of motion
- The volume of the molecules is negligibly small compared to the volume occupied by the gas;
- No forces act on the molecules except during elastic collisions of negligible duration.
- These assumptions were used to derive the ideal gas equation from Kinetic theory of gases in the 1800s.
- However, elements of the assumption were seen well before that in the 1600s and 1700s by Boyle, Bernoulli etc.



http://clearscience.tumblr.com/post/729199399/to-consider-absolute-zero-lets-mention-an-ideal

T ↑: they go faster

P: is caused by them hitting the walls

Equation of State for an Ideal Gas

Properties

Density = ρ Pressure = p Temperature = T Volume = V Mass = M

Observations

Boyle: For a given mass, at constant temperature, the pressure times the volume is a constant. $pV = C_1$

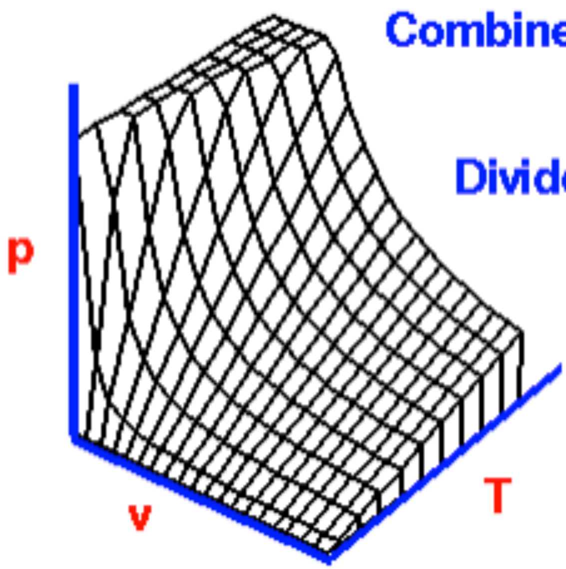
Charles and Gay–Lussac: For a given mass, at constant pressure, the volume is directly proportional to the temperature. $V = C_2 T$

Combine: $pV/T = n\bar{R}$ $\bar{R}$ = 8.31 J / mole / K (Universal)

$pV = n\bar{R}T$ n = number of moles

Divide by mass: $pv = \dfrac{n\bar{R}T}{M}$ Specific Volume = v

$v = \dfrac{\text{volume}}{\text{mass}} = \dfrac{1}{\rho}$



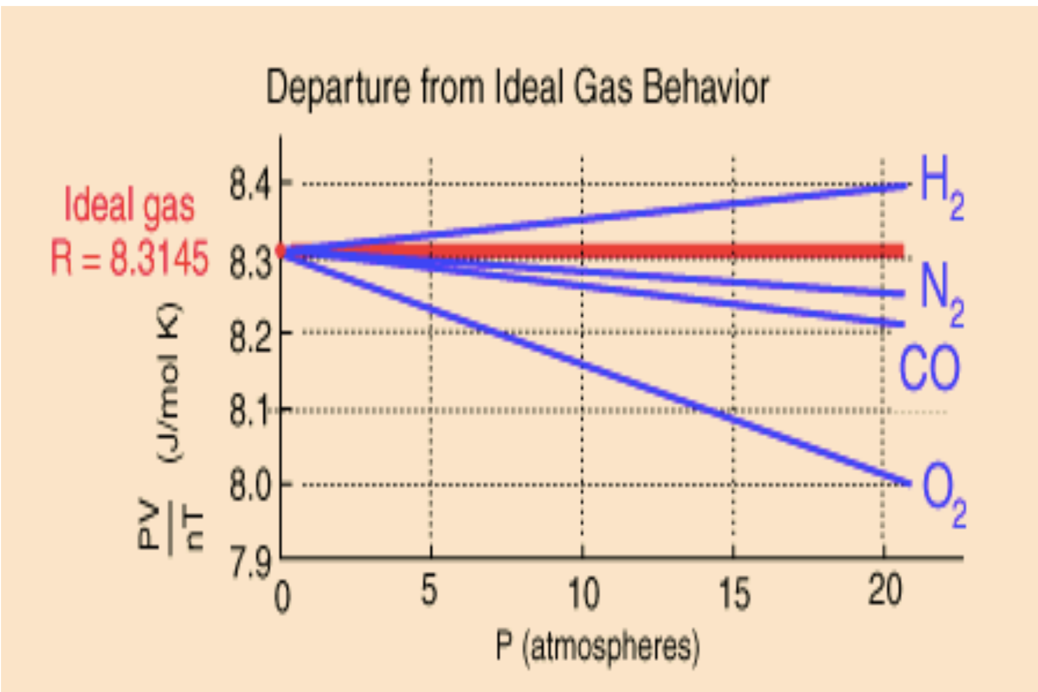
$$\boxed{pv = RT \quad \text{or} \quad p = R\rho T}$$

R = Constant value for each gas

= .286 kJ/ kg / K (for air)

https://www.grc.nasa.gov/www/k-12/airplane/shortp.html

Effect of pressure on ideal gas behavior



Engine	Overall pressure ratio	Major applications
Rolls-Royce Trent XWB	52:1	A350 XWB
General Electric GE90	42:1	777
General Electric CF6	30.5:1	747, 767, A300, MD-11, C-5
General Electric F110	30:1	F-14, F-15, F-16
Pratt & Whitney TF30	20:1	F-14, F-111

Pressures in engines are high enough that departure from ideal gas may need to be considered

Non-ideal gases

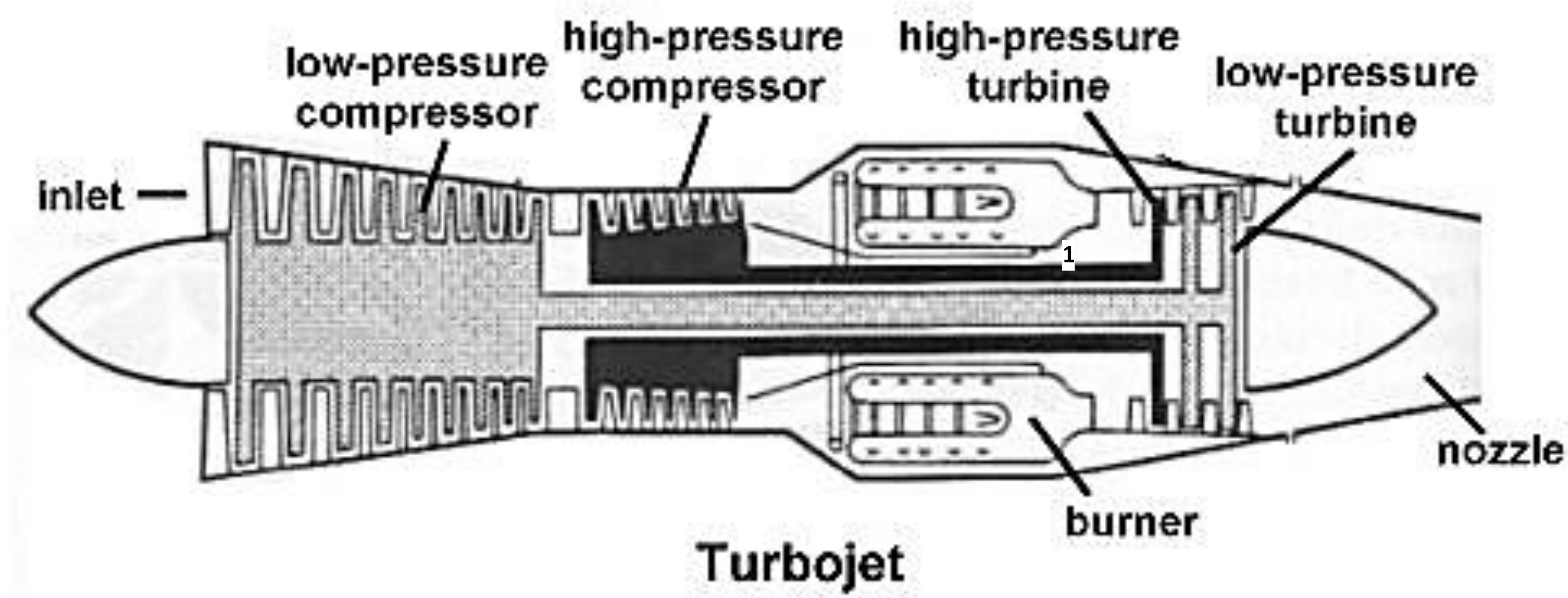
- The ***van der Waals equation*** is useful for gases that do not behave ideally.
- The constants “a” and “b” are dependent on the gas.
- Constant “a” corrects for intermolecular forces and “b” corrects for finite molecular size

Experimentally measured pressure

Container volume

$$\underbrace{\left(P + \frac{an^2}{V^2}\right)}_{\text{corrected pressure term}}\underbrace{\left(V - nb\right)}_{\text{corrected volume term}} = nRT$$

Mixtures of Gases: Why are they important



gas mixture
(*O_2, N_2, CO_2, H_20, Ar, Nox, SOx*)

- Air is primarily a mixture of gases; engines breathe air, not oxygen
- Post combustion products are also a mixture of gases

Mixtures of Ideal Gases: Behavior

- Temperature: From the Zeroth law of thermodynamics, all gases in a mixture will be at the same temperature when in equilibrium
- Volume:
  - Any gas present in the mixture will attempt to occupy the entire volume. The large mean free paths ensure this.
  - When two or more gases are placed in a container, each gas behaves as though it occupies the container alone.
- Pressure: Each gas behaves independently of the other when creating pressure on the walls of the container.

$PV = nRT$ $\Rightarrow$ Only thing that matters is the number of moles of a gas

Basic laws for mixtures of ideal gases

- The pressure of a mixture of gases is equal to the sum of the partial pressures of each constituent gas as if it occupies the entire container itself
- The internal energy of the mixture is equal to the sum of the internal energies of the constituents
- The entropy of the mixture is equal to the sum of the entropies of the constituents

Basic laws for mixtures of ideal gases

Thus for a mixture of n constituents:

Temperature	$T_m = T_1 = T_2 = \cdots = T_n\,,$	(2.23a)
Pressure	$p_m = p_1 + p_2 + p_3 + \cdots + p_n\,,$	(2.23b)
Volume	$\mathcal{V}_m = \mathfrak{M}_m v_m = \mathfrak{M}_1 v_1 = \mathfrak{M}_2 v_2 = \cdots = \mathfrak{M}_n v_n\,,$	(2.23c)
Energy	$E_m = \mathfrak{M}_m e_m = \mathfrak{M}_1 e_1 + \mathfrak{M}_2 e_2 + \cdots + \mathfrak{M}_n e_n\,,$	(2.23d)
Entropy	$S_m = \mathfrak{M}_m s_m = \mathfrak{M}_1 s_1 + \mathfrak{M}_2 s_2 + \cdots + \mathfrak{M}_n s_n\,,$	(2.23e)
Enthalpy	$H_m = \mathfrak{M}_m h_m = \mathfrak{M}_1 h_1 + \mathfrak{M}_2 h_2 + \cdots + \mathfrak{M}_n h_n\,,$	(2.23f)

in which $\mathfrak{M}$ signifies mass, subscript m refers to the mixture, and subscripts 1, 2, . . . , n refer to a series of constituents.

Example

A 1.00-L vessel contains 0.215 mole of N_2 gas and 0.0118 mole of H_2 gas at 25.5° C. Determine the partial pressure of each component and the total pressure in the vessel.

Method Use the ideal gas equation to find the partial pressure of each component of the mixture, and sum the two partial pressures to find the total pressure.

Solution $T = 298.65\ \text{K}$

$$P_{N_2} = \frac{(0.215\ \text{mol})(0.08206\ \text{L·atm/K·mol})(298.65\ \text{K})}{1.00\ \text{L}} = 5.27\ \text{atm}$$

$$P_{H_2} = \frac{(0.0118\ \text{mol})(0.08206\ \text{L·atm/K·mol})(298.65\ \text{K})}{1.00\ \text{L}} = 0.289\ \text{atm}$$

$$P_{\text{total}} = P_{N_2} + P_{H_2} = 5.27\ \text{atm} + 0.289\ \text{atm} = 5.56\ \text{atm}$$

Mole Fractions

The relative amounts of the components in a gas mixture can be specified using *mole fractions*.

$$\chi_i = \frac{n_i}{n_{total}}$$

X_i is the mole fraction.

n_i is the moles of a certain component

n_{total} is the total number of moles.

There are three things to remember about mole fractions:

1) The mole fraction of a mixture component is always less than 1.

2) The sum of mole fractions for all components of a mixture is always 1.

3) Mole fractions are dimensionless.

Mole Fractions: Example

Nitric oxide (NO) is used to treat and prevent lung disease, which occurs commonly in premature infants. The nitric oxide used in this therapy is supplied to hospitals in the form of a N_2/NO mixture. Calculate the mole fraction of NO in a 10.00-L gas cylinder at room temperature (25° C) that contains 6.022 mol N_2 and in which the total pressure is 14.75 atm.

Method Use the ideal gas equation to calculate the total number of moles in the cylinder. Subtract moles of N_2 from the total to determine moles of NO. Divide moles NO by total moles to get mole fraction.

Solution The temperature is 298.15 K.

$$\text{total moles} = \frac{PV}{RT} = \frac{(14.75\ \text{atm})(10.00\ \text{L})}{(0.08206\ \text{L·atm/K·mol})(298.15\ \text{K})} = 6.029\ \text{mol}$$

$$\text{mol NO} = \text{total moles} - N_2 = 6.029 - 6.022 = 0.007\ \text{mol NO}$$

$$\chi_{\text{NO}} = \frac{n_{\text{NO}}}{n_{\text{total}}} = \frac{0.007\ \text{mol NO}}{6.029\ \text{mol}} = 0.001$$

Stoichiometric ratio

- For engines, we are usually more interested in mass fraction than mole fraction, since we want to know the fuel/air mass ratio.
- Stoichiometric ratio: The fuel/air ratio that results in complete combustion.
  - If fuel/air ratio is higher than stoichiometric, the mixture is called “rich”
  - If fuel/air ratio is lower than stoichiometric the mixture is called “lean”

Computing Mass Fractions/ Stoichiometric Fuel Air Ratio

- $2H_2 + O_2 + 79/21N_2 = 2H_2O + 79/21N_2$
  - 2 Moles of Hydrogen has a molecular weight of 4gms.
  - 1 Mole of Oxygen has a molecular weight of 32gms.
  - 79/21 Moles of Nitrogen has a weight of 105 gms

- This means to completely burn 4gms of Hydrogen, you need to supply 137 gms of Air.
- So Stoichiometric Fuel/Air ratio is 1/34.
- For 1 kg of Hydrogen, if you supply more than 34 kg of air, the fuel/air mixture is called lean
- For 1 kg of Hydrogen if you supply less that 34 kg of air, the fuel/air mixture is called rich