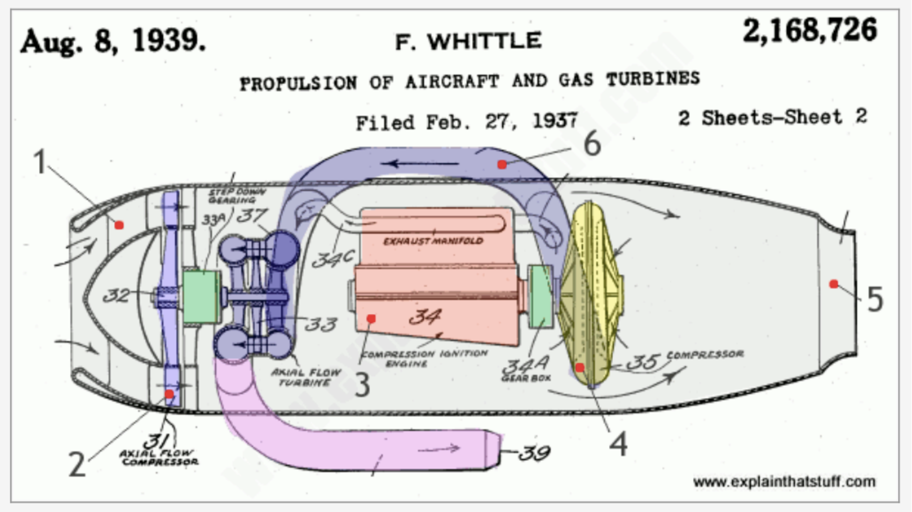
# Lecture 14-15

## Turbojet Performance Analysis and Cycle Design

# History of Turbojets

Sir Frank Whittle and Dr. Hans Von Hain in 1938

First Patent on Turbojet AeroEngines

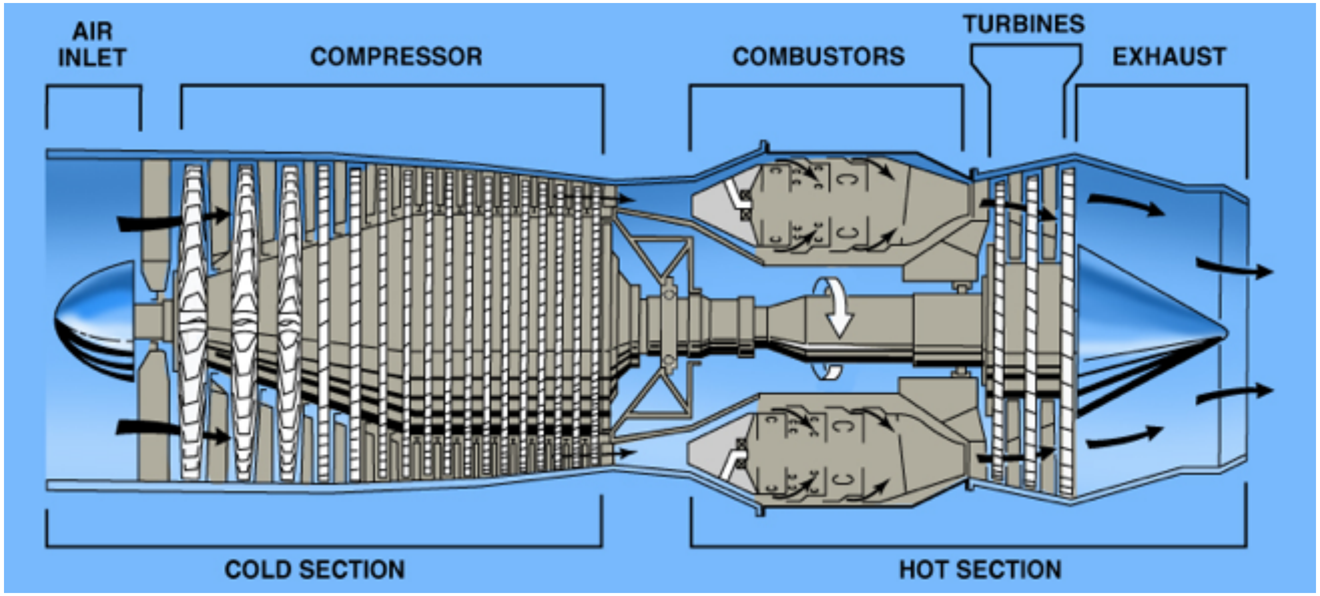


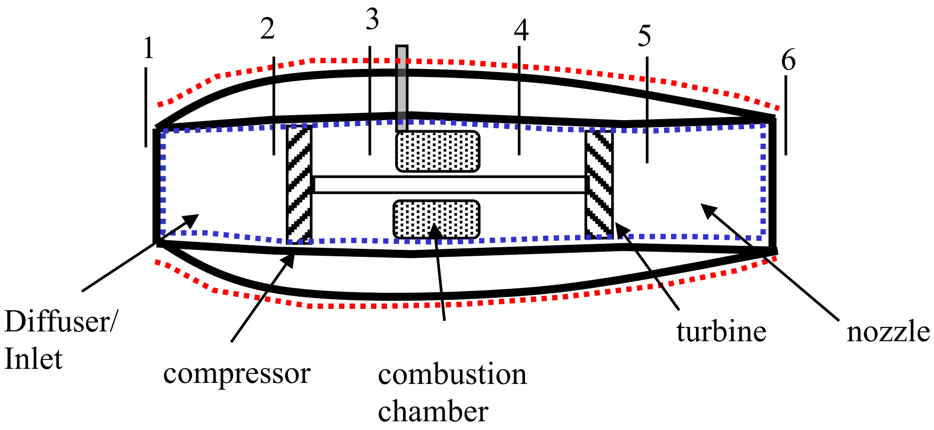

- The invention is credited to Sir Frank Whittle and Dr. Hans Von Hain (with Heinkel)
- Fascinating pre-WWII story on the race to building the first jet engine
- http://www.historynet.com/frank-whittle-and-the-race-for-the-jet.htm

Gloster E.28/39 (W4041), Farnborough, Sqn Ldr J Moloney, c. 1941

Heinkel He 176, Peenemünde in 1938.

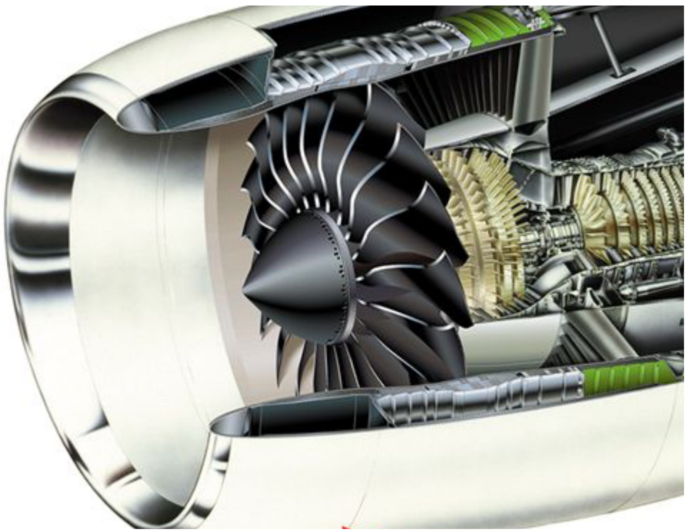
# Idealized Turbojet Engine




- 1-2 Isentropic compression in diffuser/inlet
- 2-3 Isentropic compression through compressor
- 3-4 Constant pressure heat addition in combustion chamber
- 4-5 Isentropic expansion through turbine
- 5-6 Isentropic expansion in nozzle
- 6-1 Constant pressure heat rejection

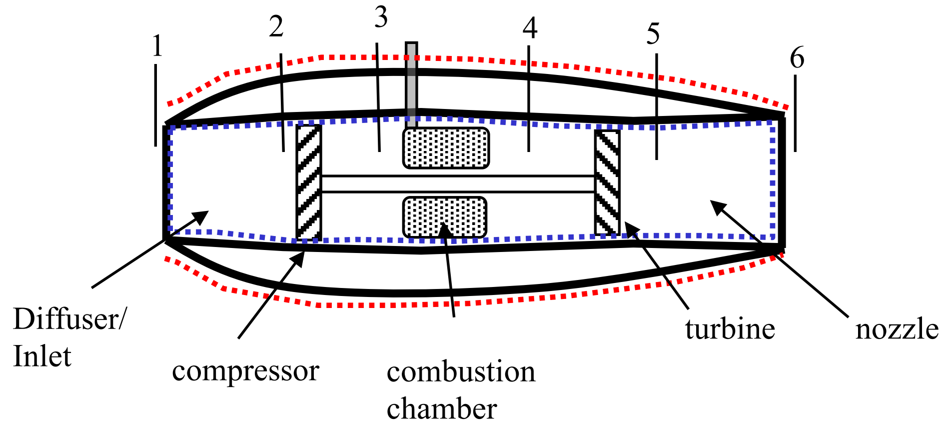
# INLETS



- Functions of air inlet duct:
  - To recover as much of the total pressure of the free airstream as possible and deliver it to the compressor, while slowing it down.
  - Provide a uniform supply of air to the compressor so it can operate efficiently.
  - Must cause as little drag as possible.
- Typical subsonic engines require compressor inlet Mach number ~ 0.5-0.6

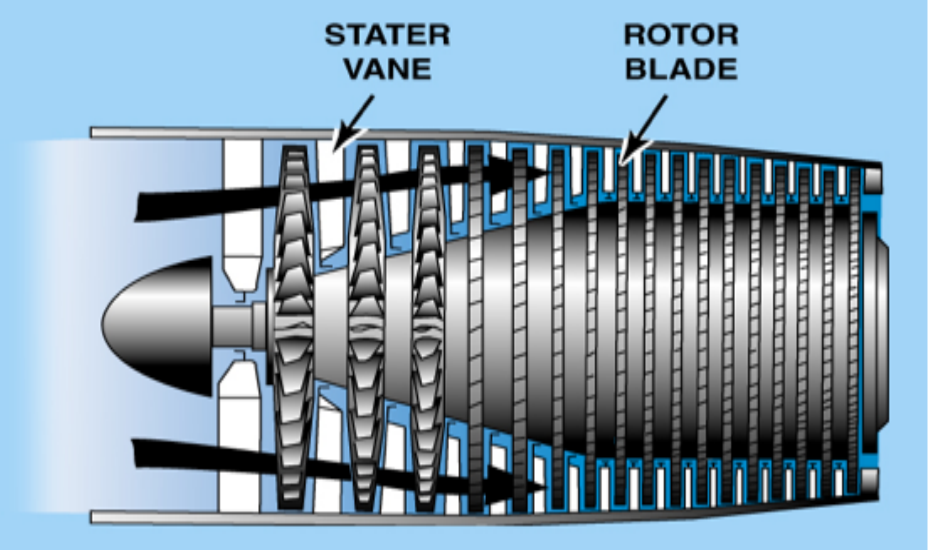
# Ideal Inlet Analysis

**An engine flying at 280m/s and an ambient temperature of 270K (M=0.85) and pressure of 72kPa, what is the compressor inlet temperature if the inlet to compressor must be at M = 0.5 ?**




- $Cp \times T_2 + u_2^2/2 = Cp \times T_a + u^2/2$
- $T_2 + M_2^2\,(\gamma * R * T_2)/2Cp = T_a + u^2/2Cp$
- Or, $T_2\,(1+M_2^2 * \gamma * R/2Cp) = 270 + 280*280/2/1005 = 309$ K
- Or, $T_2 = 309/(1+0.5\text{^}2*1.4*287/2/1005) = 309/1.05 = 294$K
- Also, $P_2/P_a = (T_2/T_a)^{(\gamma/\gamma-1)} = (294/270)\text{^}3.5 = 1.35$, so P2 = 97 kPa
- P02 = P2 * (1+0.5*0.4*0.5^2)^3.5 = 115 kPa

# AXIAL FLOW COMPRESSOR

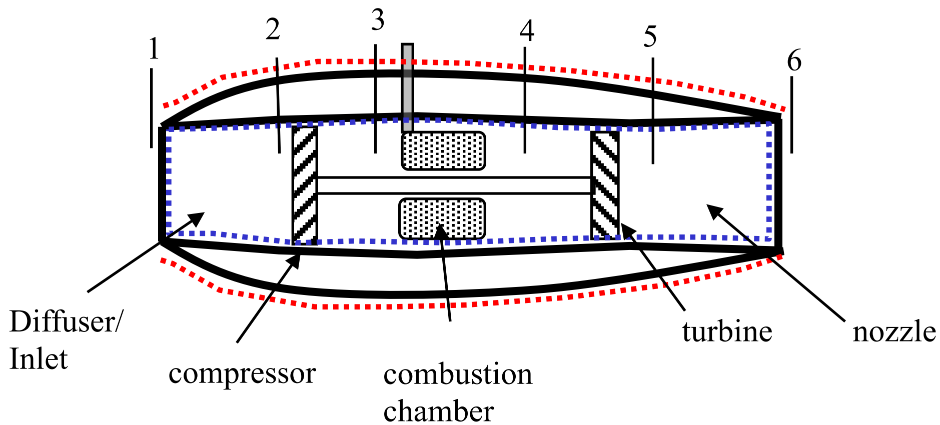



- Compressors of two types
  - Radial or Centrifugal
  - Axial (most modern engines use this)
- Consist of a ROTOR and STATOR.
- The rotor consist of rows of blades fixed on a rotating spindle.
- The stator vanes are arranged infixed rows between the rows of rotor blades and act as a diffuser at each stage.
- Each pressure stage consist of one row of blades and one row of vanes.

- Modern compressors increase the air pressure 15 to 40 times above the ambient air pressure.
- Compressor pressure ratio (PR) is the ratio of the exit stagnation pressure to the inlet air stagnation pressure across the compressor.
- Compressors also supply bleed air:
  – to cool the hot section
  – heated air for anti-icing
  – cabin pressurization and air conditioning.
  – fuel system deicing..

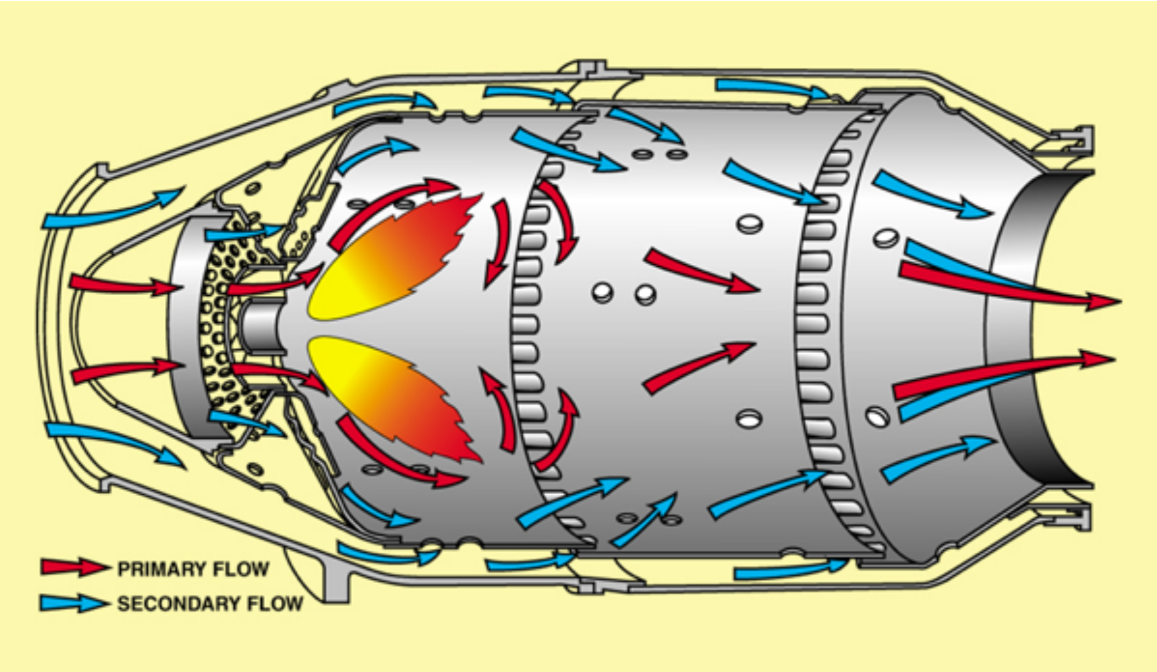
# Ideal Compressor Analysis

**What is the compressor exit stagnation and static temperature, given a compression ratio of 25? Assume M=0.3 at the exit of the compressor**




- Compressor inlet stagnation pressure is 1.59 * 72kPa = 115 kPa
- Total compression ratio = 25
- Therefore, $P_{03} = P_{02}$ * PR = 25 * 115 = 2875 kPa
- And TR = $PR^{(\gamma-1/\gamma)} = 25^{(0.4/1.4)} = 2.5$
- $T_{03}$= 309*2.5 = 772K
- T3 = 758 K (using isentropic static to stagnation relationship and M = 0.3)

# COMBUSTION CHAMBER

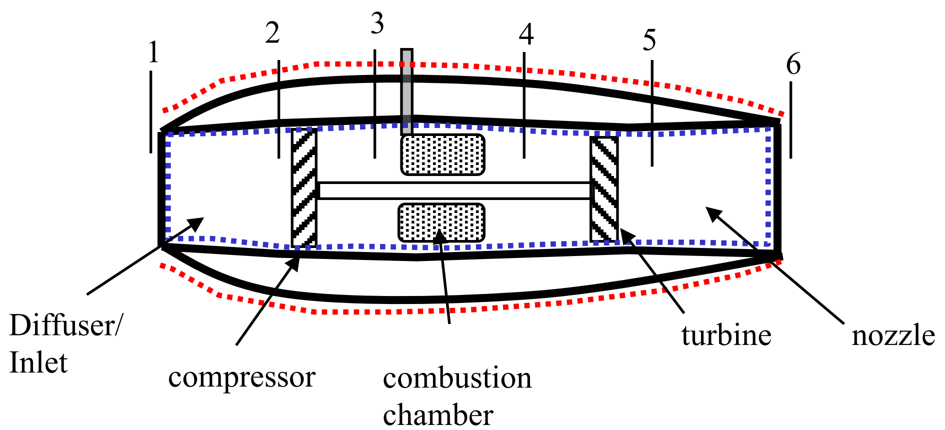



- Combustion chamber or combustors is where the fuel and air are mixed and burned.
  - Located directly between the compressor and turbine section.
- Basic components of combustion section:
  - one or more combustion chambers (combustors).
  - fuel injection system.
  - ignition source.
  - fuel drainage system.

- To accomplish the task of burning the fuel air mixture efficiently the C.C must
  – Mix fuel and air effectively in the best ratio for good combustion.
  – Burn the mixture as efficiently as possible.
  – Cool the hot combustion gases to a temperature the turbine blades can tolerate.
  – Distribute hot gases evenly to the turbine.

# Combustion Chamber Analysis

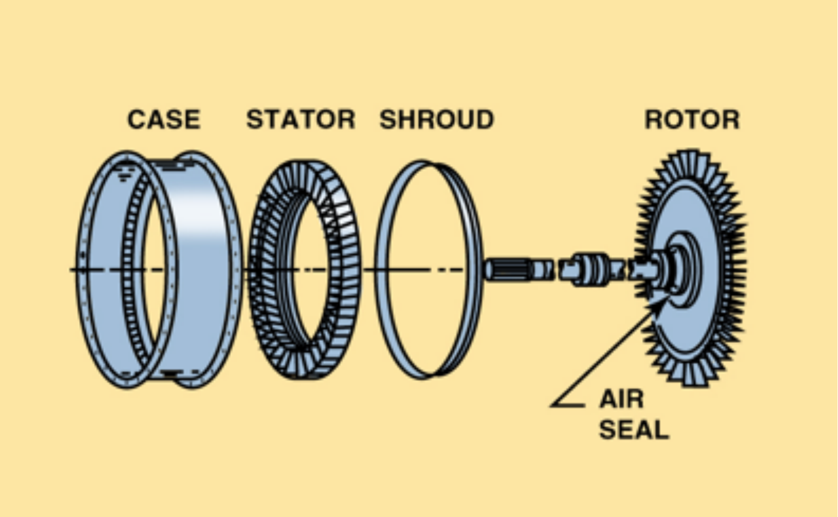
**Given combustion inlet temperature of 772K, and a combustion energy input rate of 46 MJ/kg of fuel, what is the combustion exit temperature? Assume 180 kg/s of air and f = 0.02**




(Since f << 1)

$$\text{mdot}_f * Q_R = \text{mdot}_a\, Cp\, (T_{04}-T_{03})$$
$$f * Q_R = Cp\, (T_{04}-T_{03})$$
$$(T_{04}-T_{03}) = 0.02*46000000/1005 = 915\text{K}$$
$$T_{04} = 915 + 772 = 1687\text{ K},$$
$$P04 = P03 = 2862\text{ kPa}$$

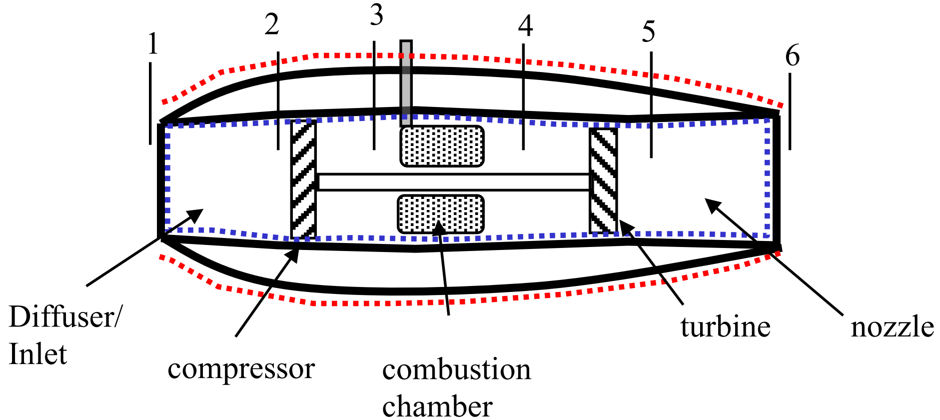
# TURBINE




- Transforms a portion of the kinetic energy in the hot exhaust gases into mechanical energy to drive the compressor and accessories.
- In a turbojet engine the turbine absorbs approximately 60 to 80 % of the total pressure energy from exhaust gases.
- Consist of:
  - 1- case.
  - 2- stator.
  - 3- shroud.
  - 4- rotor.

# Turbine Analysis

**Given turbine inlet temperature of 1687K, and 90MW is required to power the compressor, what is the turbine exit temperature? (Homework: Also, compute the transmission efficiency, assuming the compressor is 100% efficient)**




(Since f << 1)

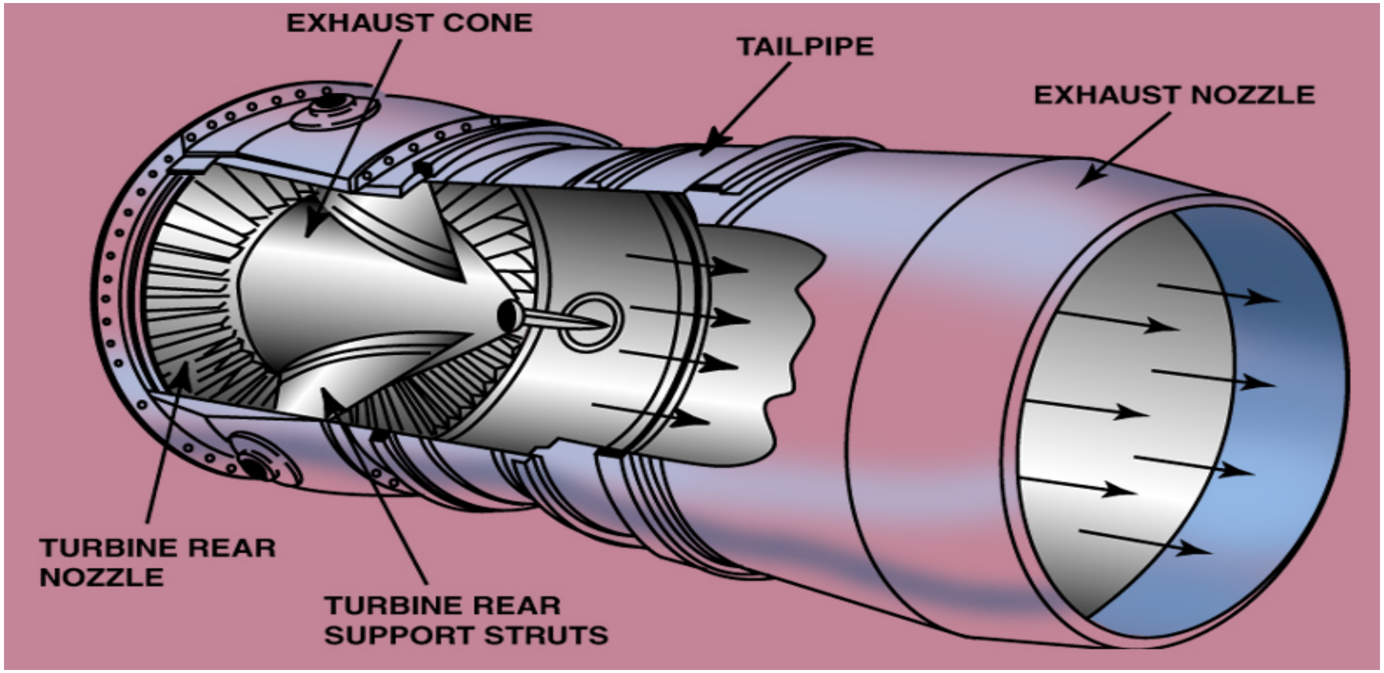
$$-P_{shaft} = mdot_a \, Cp \, (T_{05}-T_{04})$$

$$(T_{05}-T_{04}) = - 90 * 1000000/180/1005$$

$$T_{05} = 1687 - 497 \text{ K} = 1190 \text{ K}$$

$$P05 = P04 * (T05/T04)^{(\gamma/\gamma-1)} = 2867 * (1190/1687)\text{^}3.5 = 845\text{kPa}$$

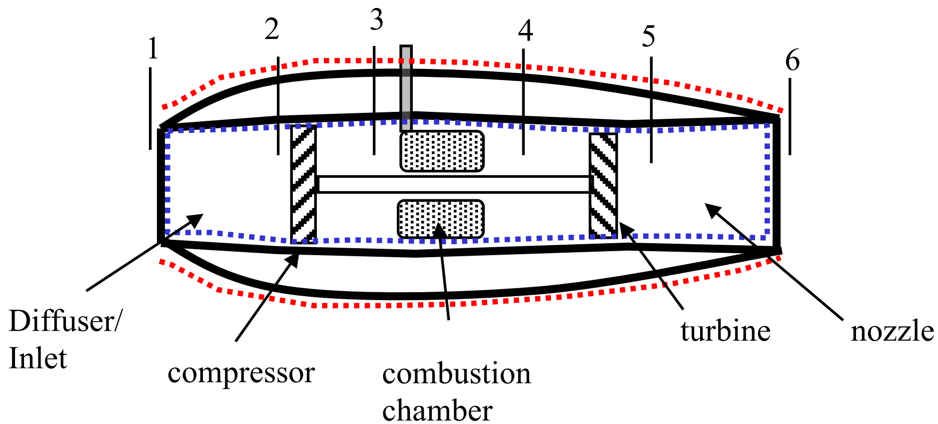
# Nozzle




- Exhaust section extends from the rear of the turbine section to the point where the exhaust gases leave the engine.
- The nozzle exhaust velocity determines amount of thrust developed.
- Nozzle may be converging, converging-diverging or variable
- Nozzles may be equipped with reverse-thrust mechanisms

# Nozzle Analysis

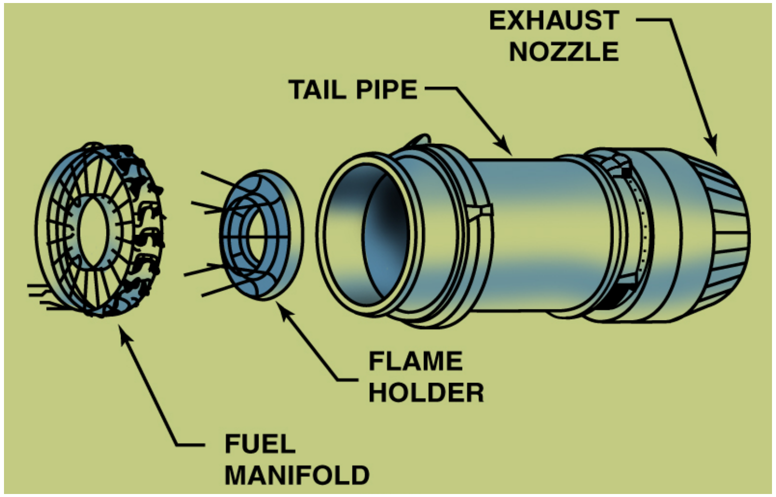
**Given nozzle inlet temperature of 1190K, and perfect expansion, i.e., pe=pa, what is the thrust of the engine?**

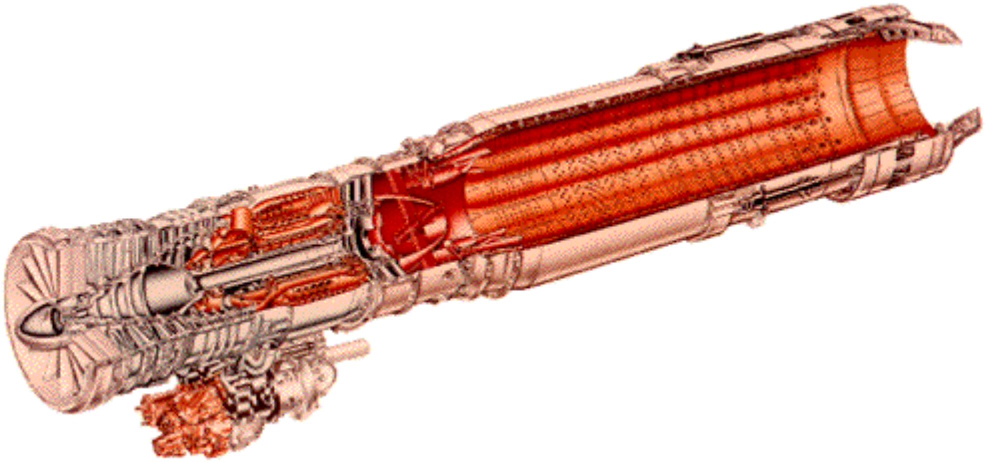



(Since f << 1)

- P6 = Pe = Pa = 72kPa, P05 = 1334 kPa, P06 = 1334kPa, T06=T05 = 1190K
- Me = 2.55 (from the P06/P6 isentropic relationship)
- T6 = 517K (from the T06/T6 isentropic relationship, knowing Mach No)
- a6 = sqrt (1.4*287*589) = 455 m/s, Ve = Me * a6 = 2.55*455 = 1160 m/s
- Thrust = 183.6 * 1160-180*280 = 163kN

# Turbojet (w/ afterburner)

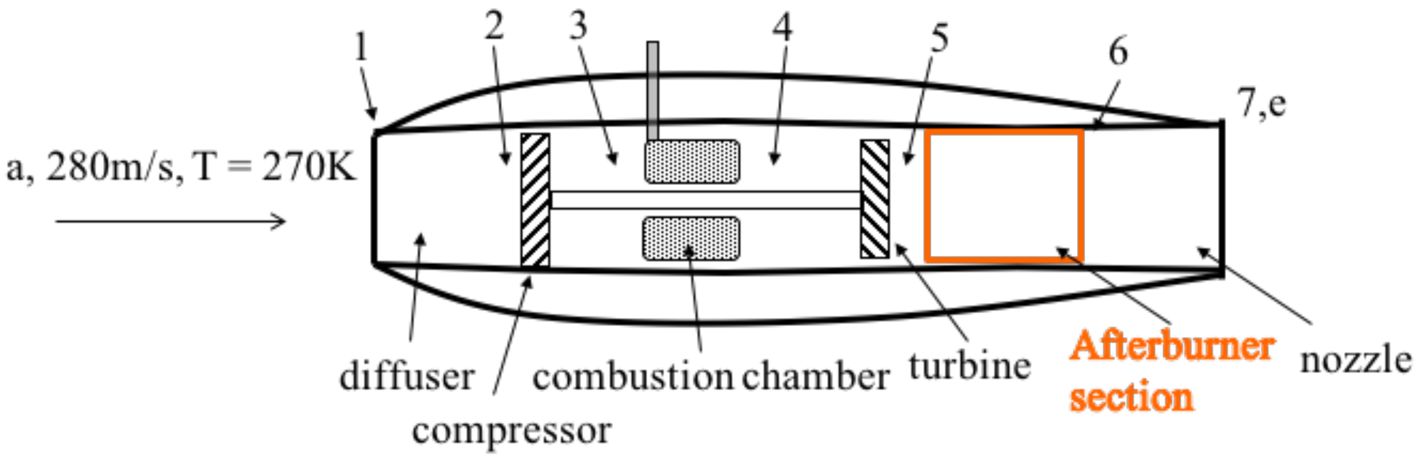


- The gases in the tailpipe sill contain a large quantity of oxygen.
- Used to accelerate the exhaust gases to increase thrust.
- Installed after the turbine and in front of exhaust nozzle.
- Consist of fuel manifold, ignition source and flame holder

# Afterburner Analysis

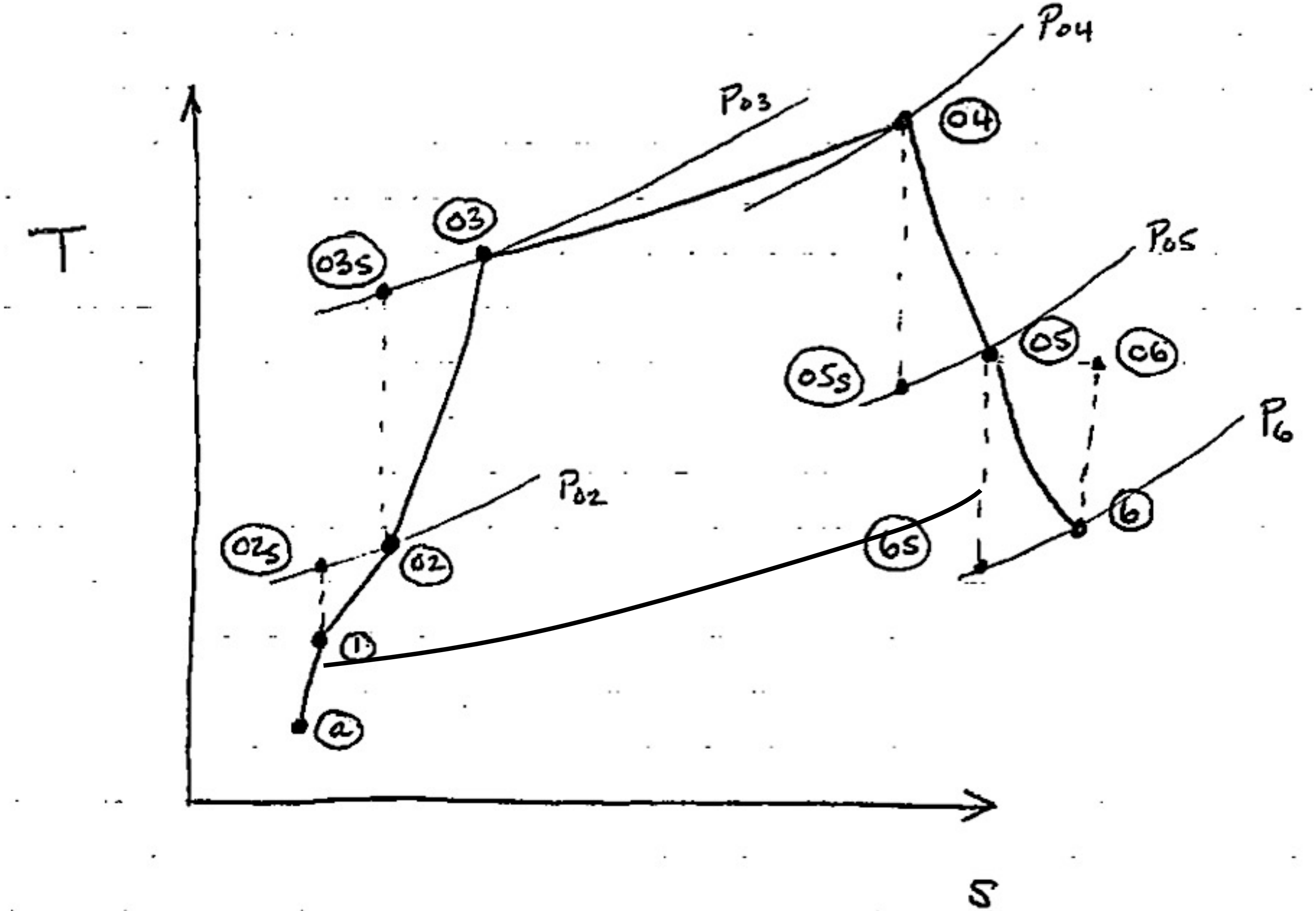
**Given turbine exit temperature of 1190K and pressure of 1334kPa, and perfect expansion, i.e., pe=pa, what is the maximum additional thrust of the engine if the afterburner walls can handle 2500K?**




- P07=1334kPa, T05 = 1190K
- T06 = T07 = Tmax = 2500K
- Me = 2.55
- Te/T07 = 1/2.3 (from Me)
- Te = 1086K

- a7 = sqrt (1.4*287*1086) = 660 m/s
- Ve = Me * a6 = 2.55*660 = 1683 m/s
- Thrust = 183.6 * 1683-180*280 = 259kN
- Additional thrust = 96 kN (59% higher than without afterburner)

# Turbojet non-ideal analysis



# Component Efficiencies

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a}$$

Ratio of ideal to actual enthalpy change in diffuser/inlet

$0.7 < \eta_d < 0.9$

$$\eta_c = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}}$$

Ratio of ideal to actual work required in compressor

$0.85 < \eta_c < 0.9$

$$\eta_t = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}}$$

Ratio of actual to ideal turbine work

$0.9 < \eta_t < 0.95$

$$\eta_n = \frac{h_{06} - h_7}{h_{06} - h_{7s}}$$

Ratio of actual to ideal enthalpy change in nozzle

$0.95 < \eta_n < 0.98$

<u>Combustor</u>

$$\eta_b = \frac{\text{actual heat released / time}}{\dot{m}_f Q_R}$$

$0.97 < \eta_b < 0.99$

# Remember Goal: Calculate Thrust and TSFC

$$\frac{\mathcal{T}}{\dot{m}_a} = [(1 + f)u_e - u] \qquad \text{TSFC} = \frac{\dot{m}_f}{\mathcal{T}} = \frac{f}{(1 + f)u_e - u}$$

Given flight speed u, ambient conditions Pa, Ta, compressor pressure ratio P03/P02, and turbine inlet temperature T04, determine the fuel-air ratio, f, and the exhaust velocity $u_e$. Also given are all the efficiencies

$$\frac{u_e^2}{2} = h_{05} - h_6 \Rightarrow u_e^2 = 2\eta_n (h_{05} - h_{6s})$$

$$h_{05} - h_{6s} = c_p (T_{05} - T_{6s})$$

$$h_{05} - h_{6s} = C_p T_{05}\left[1 - \left(\frac{P_6}{P_{05}}\right)^{\gamma-1/\gamma}\right]$$

$$u_e = \sqrt{2C_p T_{05}\, \eta_n \left[1 - \left(\frac{P_6}{P_{05}}\right)^{\gamma-1/\gamma}\right]}$$

# Goal: Calculate Thrust

Need to calculate these two

$$\frac{P_{o5}}{P_b} = \frac{P_{o5}}{P_{o4}} \cdot \frac{P_{o4}}{P_{o3}} \cdot \frac{P_{o3}}{P_{o2}} \cdot \frac{P_{o2}}{P_a}$$

Assume $P_e = P_6 = P_a$

Stagnation Pressure Ratio for Combustor, $r_c$ (defined as in Ramjet Lecture)

Pressure Ratio for Compressor, PR (or $p_{rc}$)

$$T_{o2} = T_{o1} \rightarrow \frac{T_{o1}}{T_a} = 1 + \frac{\gamma - 1}{2} M^2$$

$$T_{o2} - T_a = T_a \frac{\gamma - 1}{2} M^2$$

# Goal: Calculate Thrust

(1) $$h_{o2} - h_a = C_p T_a \frac{\gamma - 1}{2} M^2 = \frac{h_{o2s} - h_a}{\eta_d}$$

(2) $$h_{o2s} - h_a = C_p (T_{o2s} - T_a) = C_p T_a \left[ \left( \frac{P_{o2}}{P_a} \right)^{\gamma-1/\gamma} - 1 \right]$$

Substituting (2) into (1)

$$C_p T_a \eta_d \frac{\gamma - 1}{2} M^2 = C_p T_a \left[ \left( \frac{P_{o2}}{P_a} \right)^{\gamma-1/\gamma} - 1 \right]$$

$$\frac{P_{o2}}{P_a} = \left[ 1 + \frac{\gamma - 1}{2} \eta_d M^2 \right]^{\gamma/\gamma-1}$$

# Goal: Calculate Thrust

Compressor work input per unit mass $= h_{03} - h_{02}$

(3) $$h_{03} - h_{02} = \frac{h_{03s} - h_{02}}{\eta_c} = \frac{C_p}{\eta_c}\left(T_{03s} - T_{02}\right) =$$

$$= \frac{C_p T_{02}}{\eta_c}\left(\frac{T_{03s}}{T_{02}} - 1\right) = \frac{C_p T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\gamma-1/\gamma} - 1\right]$$

Turbine work output per unit mass $= h_{04} - h_{05}$

(4) $$h_{04} - h_{05} = \eta_t\left(h_{04} - h_{05s}\right) = \eta_t C_p\left(T_{04} - T_{05s}\right)$$

$$= \eta_t C_p T_{04}\left(1 - \frac{T_{05s}}{T_{04}}\right) = \eta_t C_p T_{04}\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}\right]$$

# Goal: Calculate Thrust

Assuming that the mass flow rate through the turbine and compressor are about equal ($\dot{m}_a \sim \dot{m}_a + \dot{m}_f$) and all transmission efficiencies are unity

$$h_{04} - h_{05} = h_{03} - h_{02}$$

From (3) and (4)

$$\frac{c_p T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\gamma-1/\gamma} - 1\right] = \eta_t c_p T_{04}\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}\right]$$

$$\frac{P_{05}}{P_{04}} = \left\{1 - \frac{T_a}{T_{04}\,\eta_t\,\eta_c}\left[1 + \frac{\gamma-1}{2}M^2\right]\left[\left(\frac{P_{03}}{P_{02}}\right)^{\gamma-1/\gamma} - 1\right]\right\}^{\gamma/\gamma-1}$$

# Goal: Calculate TSFC

Next parameter to be calculated : f

From energy equation :

$$\eta_b \dot{m}_f Q_R = \dot{m}_e h_{04} - \dot{m}_a h_{03} = \dot{m}_a \left[ (1+f) h_{04} - h_{03} \right]$$

$$f \eta_b Q_R = \left[ (1+f) h_{04} - h_{03} \right]$$

$$\boxed{f = \frac{\dfrac{T_{04}}{T_{03}} - 1}{\dfrac{\eta_b Q_R}{C_p T_{03}} - \dfrac{T_{04}}{T_{03}}}}$$

# Goal: Calculate TSFC

Finally, need to calculate, T03 and T05

$$T_{02} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$T_{03} - T_{02} = \frac{T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\gamma-1/\gamma} - 1\right]$$

$$\boxed{T_{03} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right)\left[1 + \frac{(P_{03}/P_{02})^{\gamma-1/\gamma} - 1}{\eta_c}\right]}$$

$$T_{05} = T_{04}\left\{1 - \eta_t\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}\right]\right\}$$

# Thrust Calculation Steps

- Calculate
  - P02/P0a (needs $\eta_d$, M)
  - P05/P04 (needs Ta, Tmax, $\eta_t$, M, PR)
  - T03 (needs Ta, M, PR, $\eta_c$)
  - T05 (needs $T_{max}$, $\eta_t$, $P0_5/P0_4$)
  - f ( needs $T_{max}$, $T0_3$, $\eta_b$, Qr)
  - P05/P6 (needs, P02/P0a, P05/P04, rc, PR)
  - ue (needs $T0_5$, $P0_5/P_6$, $\eta_n$)
  - Thrust (needs f, $u_e$)