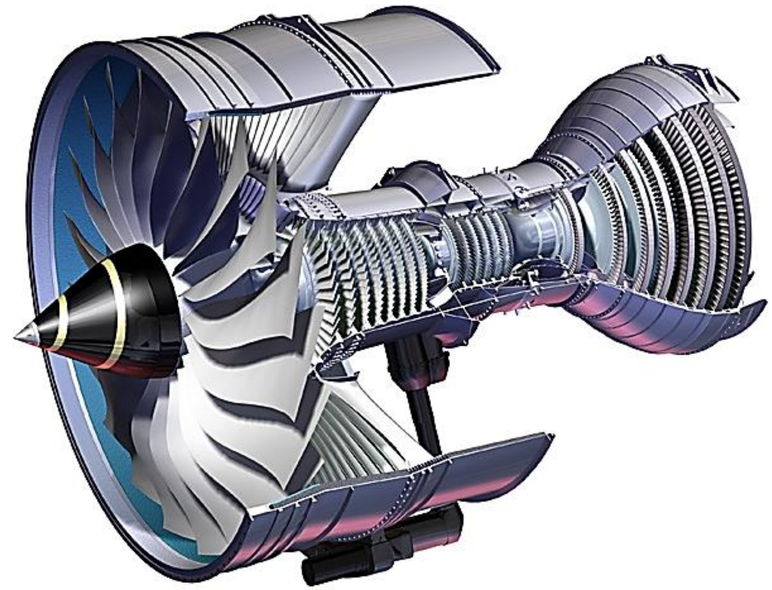
Lecture 16-17

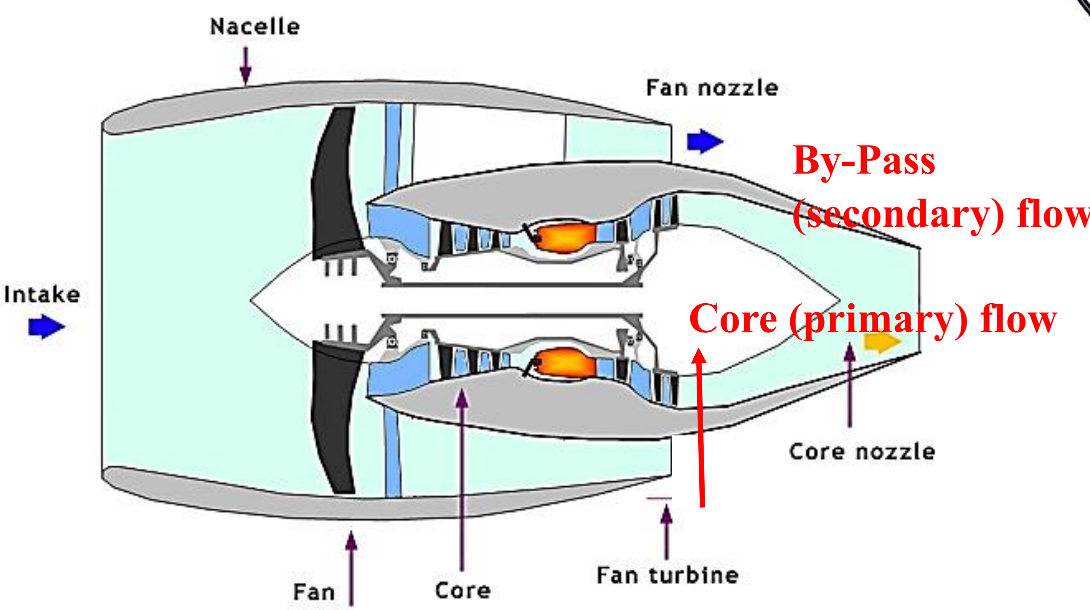
Turbofan Performance Analysis and Design

The Turbofan Engine

Trent 1000 on a Boeing 787 prototype aircraft



Trent 1000 Turbofan Engine (Courtesy of Rolls-Royce, plc)



An aside about Propfans: https://www.airspacemag.com/history-of-flight/the-short-happy-life-of-the-prop-fan-7856180/

Efficiency comparison of propeller thrust to jet thrust

PROPELLER

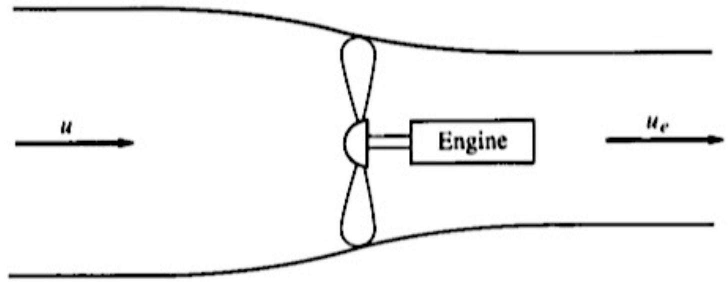


FIGURE 1.7 Acceleration of a stream tube of air through a propeller.

Thrust Produced $$\Im_{prop} = \dot{m}_a(u_e - u)$$

Minimum fuel-energy consumption rate, assuming thermal efficiency of the engine is 1 $$\dot{E} = \frac{\dot{m}_a({u_e}^2 - u^2)}{2}$$

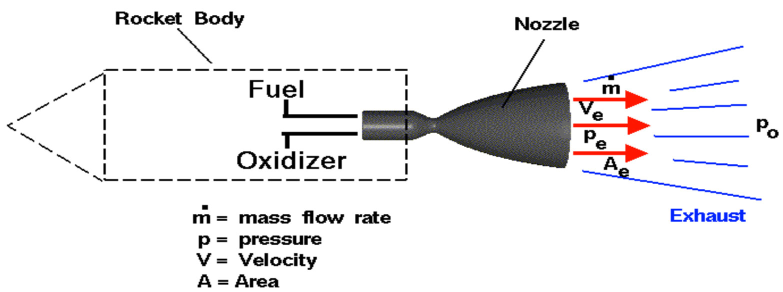
Therefore, ratio of thrust to minimum fuel consumption rate is $$\frac{\Im_{prop}}{\dot{E}} = \frac{2}{(u_e + u)}$$

If ue < u, no thrust is produced, so maximum possible value of thrust per fuel consumption rate is at ue=u, for which

$$\frac{\Im_{prop}}{\dot{E}} = \frac{1}{u_e}$$

Efficiency comparison of propeller thrust to jet thrust

ROCKET



Thrust Produced (propellant flow rate times exhaust velocity)

$$\Im_{jet} = \dot{m}_p u_{jet}$$

Minimum fuel-energy consumption rate, since there is no inflow

$$\dot{E} = \frac{\dot{m}_p\left(u_{jet}{}^2\right)}{2}$$

Therefore, ratio of thrust to minimum fuel consumption rate is

$$\frac{\Im_{jet}}{\dot{E}} = \frac{2}{(u_{jet})}$$

For the same minimum energy conversion rate, $\dot{E}$

$$\frac{\Im_{prop}}{\Im_{jet}} = \frac{u_{jet}}{(2u_e)}$$

Since typically $u_{jet} >> u_e$, propeller aircraft can deliver much higher thrust for same efficiency than purely jet driven devices

Propeller driven aircraft limitation

Parameter	Symbol	Units
propeller diameter	D	m
propeller speed	n	rev/s
torque	Q	Nm
thrust	T	N
fluid density	ρ	kg/m^3
fluid viscosity	μ	m^2/s
fluid bulk elasticity modulus	K	N/m2
flight velocity	u_0	m/s

With dimensional analysis we can show that

$$T = k_T \rho n^2 D^4,$$

Therefore for high thrust, one needs high density (which limits altitude), or high RPM (which increases losses due to tip effects) or large propeller diameter (which also increases losses due to tip effects)

- The need is to go
  - Higher to increase range (due to lower drag) without losing thrust
  - Faster (need for military applications/lower time of flight)
  - Stronger (better Thrust to Weight) so more payload

- This led to the development of jet engines, where more thrust was obtained by increasing Ue instead of mass flow rate.

- However, jet engines are less efficient, so operational costs went up – thus, Turbofan and Turboprop engines

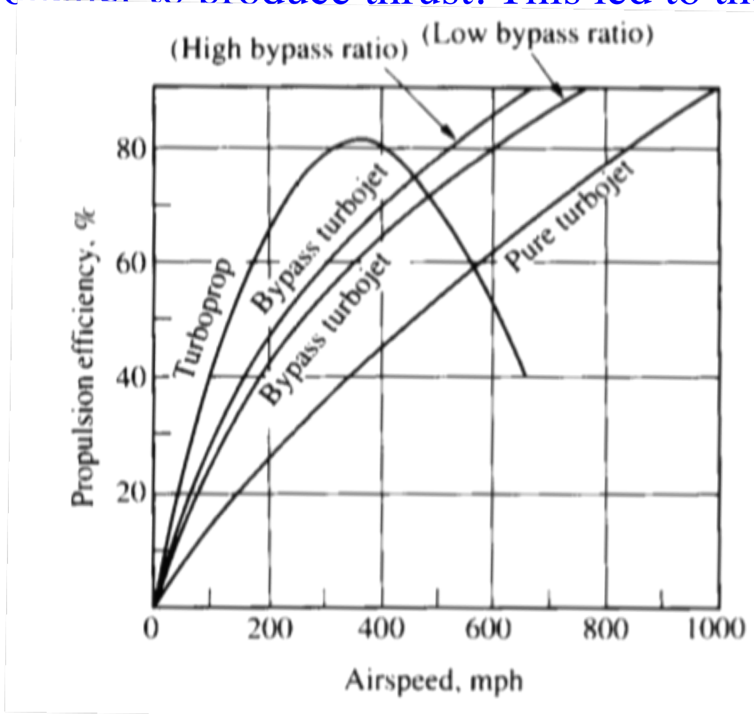
The argument for Turbofans

For f << 1 $\mathcal{T} = \dot{m}_a(u_e - u)$.

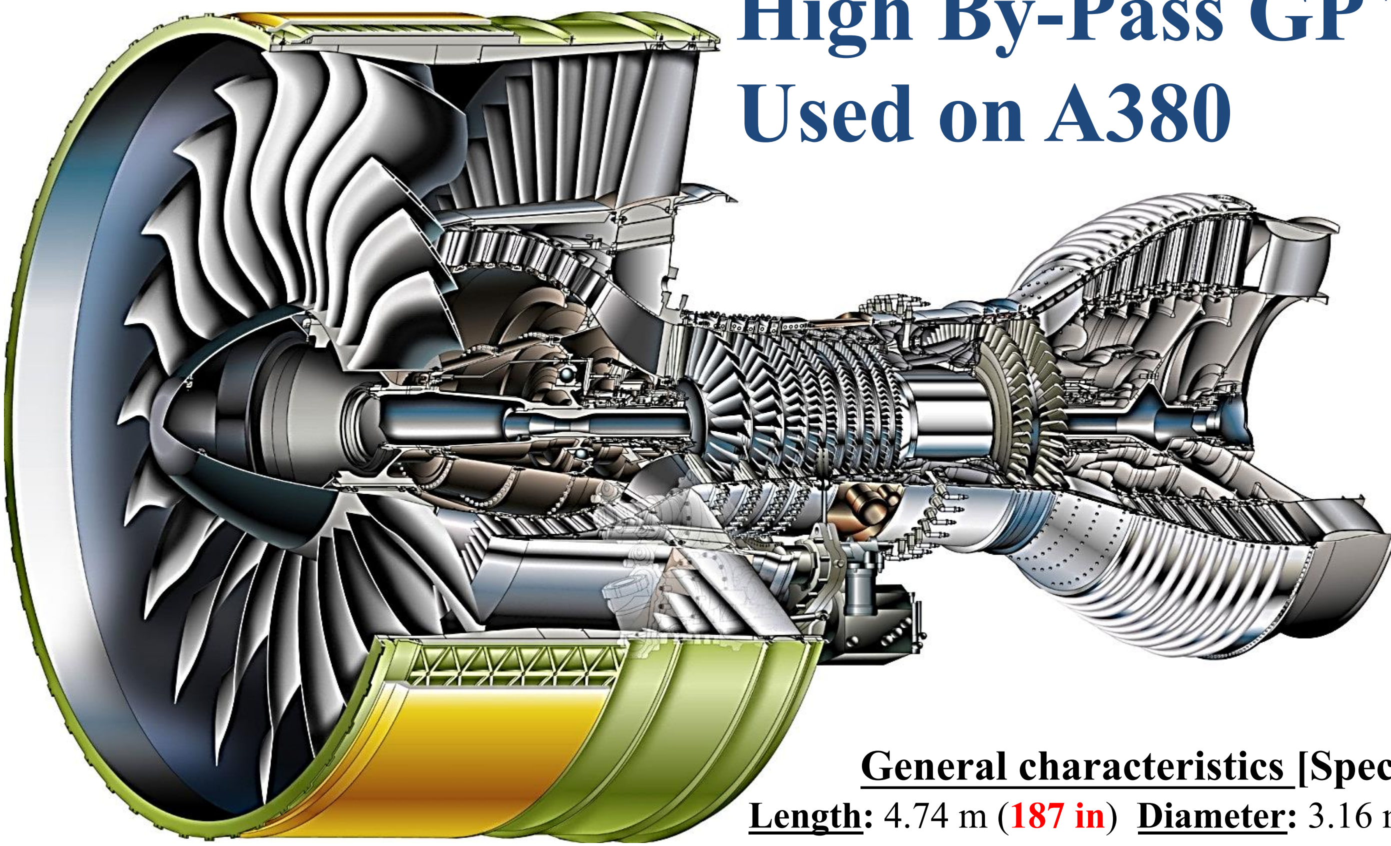
$$\eta_p = \frac{2u}{u_e + u}$$

Eliminating u_e, $\eta_p = \dfrac{1}{1 + \dfrac{\mathcal{T}}{2\dot{m}_a u}}$.

- Therefore, decreasing the specific thrust will increase efficiency for a given flight speed, but that implies driving more mass flow through the engine and therefore a heavier engine.
- The alternative to that is to drive part of the mass flow through the engine and the rest through a propeller to produce thrust. This led to the development of Turbofans



High By-Pass GP 7000 Cutaway Used on A380



General characteristics [Specifications (model GP7270)]:

Length: 4.74 m (**187 in**) **Diameter:** 3.16 m (**124 in**), fan tip 2.95 m (116 in)

Dry weight: 6,712 kg (**14,800 lb**)

Compressor: 24 swept, wide-chord hollow titanium fan blades, **8.7:1 by-pass ratio**; 5-stage low-pressure axial compressor; 9-stage high-pressure axial compressor

Combustors: low-emissions single **annular combustor**

Turbine: 2-stage high pressure axial flow, **single crystal blades**; **split blade cooling w/ thermal barrier coatings**; 6-stage low-pressure axial flow

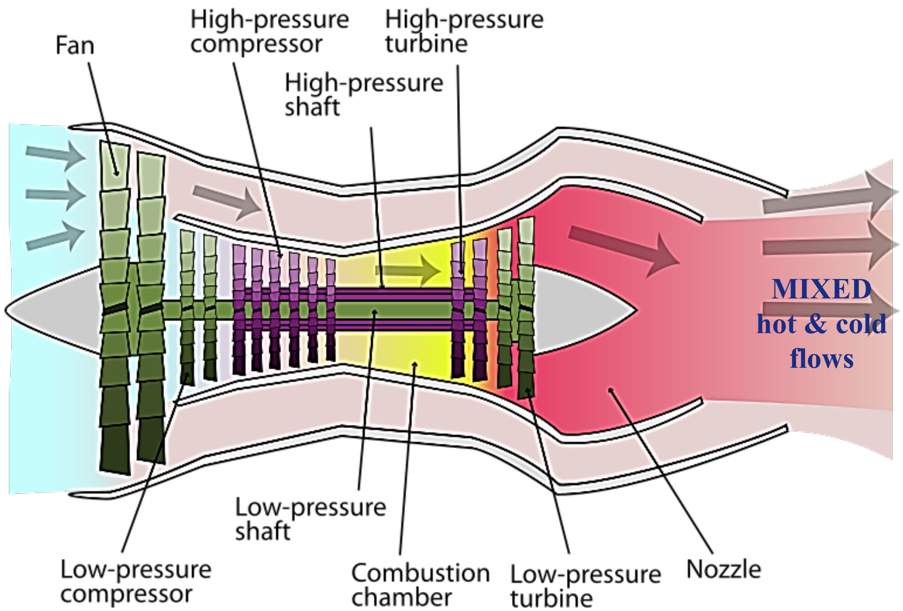
Maximum Thrust: 363 kN or **81,500 lbf** **Overall pressure ratio: 43.9 (compounded)**

Thrust-to-weight ratio: 4.73 (including fuel weight)

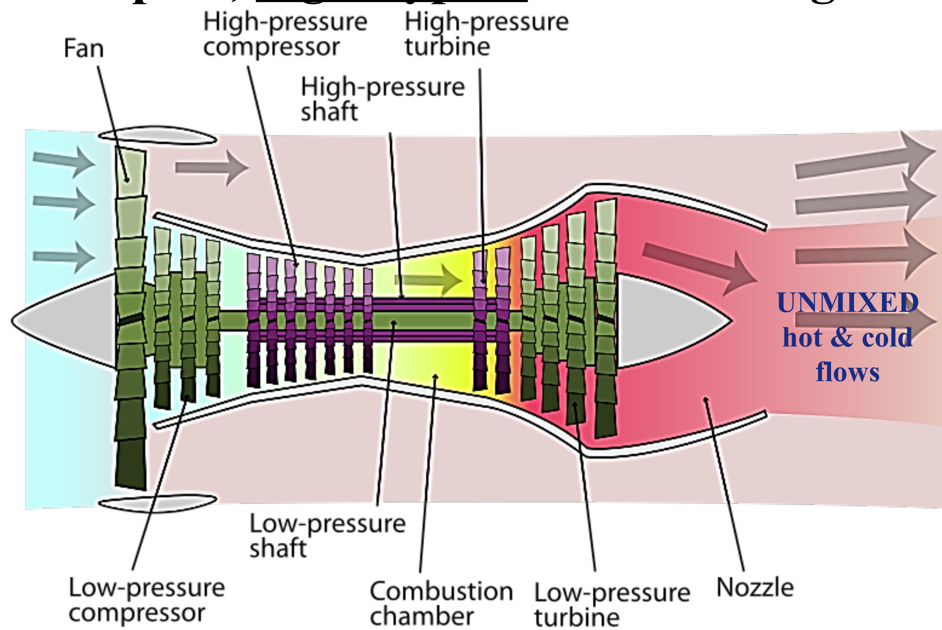
TURBOFAN ENGINES

- Thrust ratio, bypass ratio, and fan pressure ratio are terms you should become familiar with.
- Thrust ratio is the comparison of the thrust produced by the fan to the thrust produced by the engine core exhaust.
- Bypass ratio is the ratio of incoming air that bypasses the core to the amount of air that passes through the engine core.
- Fan pressure ratio is the ratio of air pressure leaving the fan to the air pressure entering the fan.

2-spool, <u>low-bypass</u> turbofan engine



2-spool, <u>high-bypass</u> turbofan engine



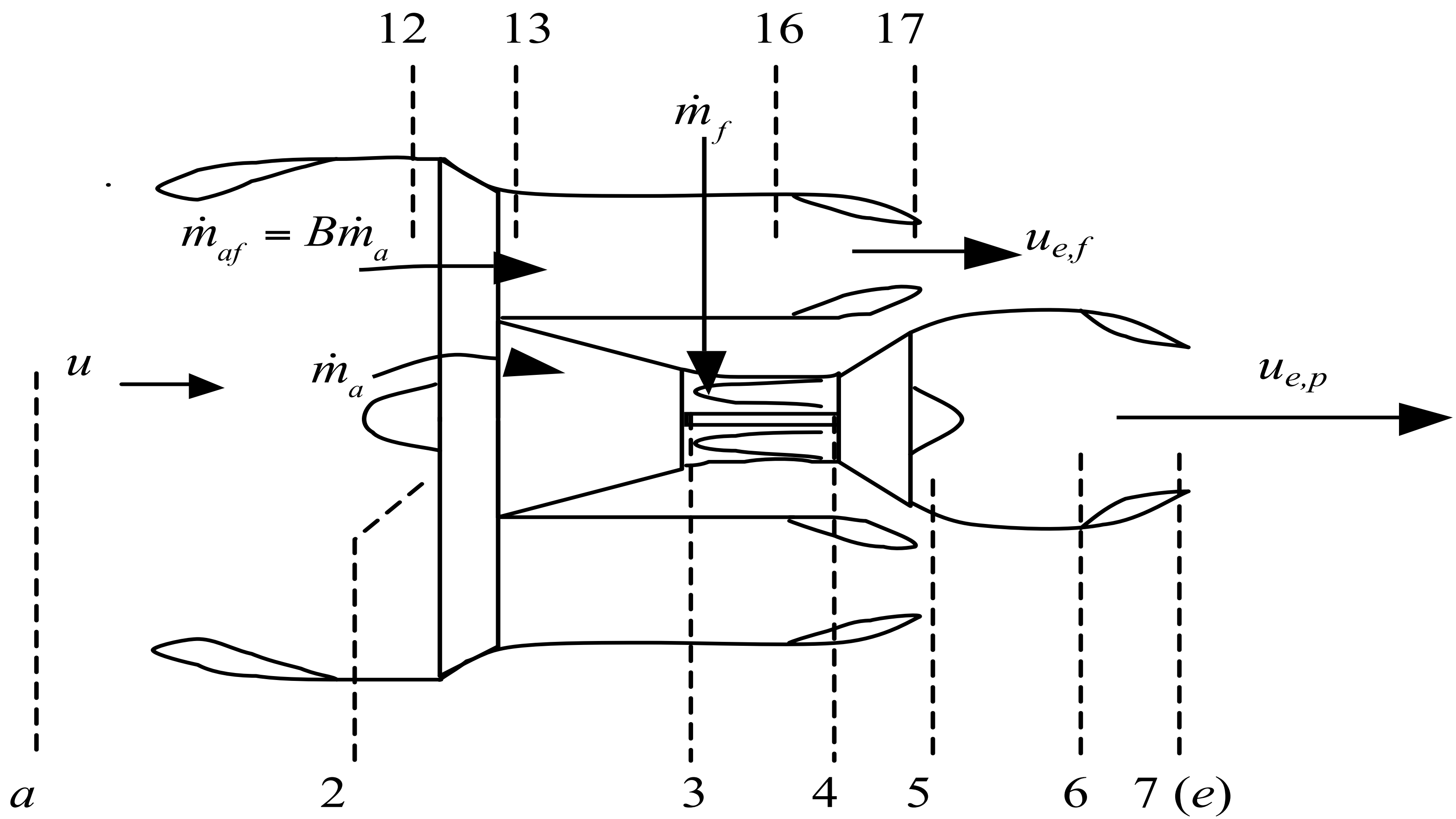
Low bypass (BPR < about 5) :

- **small portion of thrust** derived from ducted fan
- favored for **military combat aircraft**
- **compromise** between improved fuel economy and combat requirements
- **high power-to-weight** ratios
- **supersonic** performance
- ability to use **afterburners**

High-bypass: (BPR > about 5)

- **majority of thrust** is derived from a ducted fan
- **lower TSFC** at cruise speed for most commercial jet aircraft
- Dominant type for **all commercial passenger aircraft and transports**
- **lower exhaust velocities** result in **lower noise**

Turbofan Analysis



Compute

a) fuel-air ratio fb, b) specific thrust c) TSFC, and d) efficiencies ηp, ηth, and ηov.

Flight and Engine Data

Flight Mach number, M_a	0.84	Enthalpy (heat) of reaction (combustion) of fuel, Q_R or $\lvert\Delta H_R\rvert$	45 MJ/kg
Ambient temperature, T_a	220 K	Burner efficiency, η_b	0.99
Inlet adiabatic efficiency, η_d	0.92	Burner stagnation pressure ratio, r_b	0.95
Compressor pressure ratio, π_c (PR)	30	Turbine inlet temperature, T_{max}	1600 K
Compressor adiabatic efficiency, η_c	0.86	Hot gas specific heat, γ_h	1.33
Fan pressure ratio, π_f (PR_f)	2	Turbine adiabatic efficiency, η_t	0.93
Bypass ratio, B	6	Primary stream nozzle efficiency, η_n	0.95
Fan adiabatic efficiency, η_f	0.90	Fan stream nozzle efficiency, η_n	0.97

Assumptions:

1. Steady, quasi-1D, uniform flow
2. $p_e = p_a$, $p_{ef} = p_a$
3. Ideal gas behavior, $R_{air} = 0.287$ kJ/kg-K, Constant specific heats, $\gamma_c = \gamma = 1.4$, $\gamma_h = 1.33$ (Note: we are using a different value for the hot gases)
4. Turbine exit = nozzle inlet, i.e. 5 = 6, with no stagnation pressure drop
5. Adiabatic components

Solution: Marching Methodology

$$\Im = \dot{m}_a \left\{ \left[(1+f_b) u_{e,p} - u \right] + B \left[u_{e,f} - u \right] \right\} + \cancel{(p_{e,p} - p_a)}_{0(2)} A_{e,p} + \cancel{(p_{e,f} - p_a)}_{0(2)} A_{e,f}$$

Core thrust Bypass Thrust

$$u = M_a \sqrt{\gamma R T_a} = 0.84 \times 297.3 = 250\ m/s$$

From the ambient to the compressor inlet

$$T_{02} = T_a \left(1 + \frac{\eta_d (\gamma - 1)}{2} M^2 \right) = \sim 251K$$

Across the Compressor

$$\Delta T_{oc} = T_{o3} - T_{o2} = \frac{T_{o3s} - T_{o2}}{\eta_c} = \frac{T_{o2}\left(\frac{T_{o3s}}{T_{o2}} - 1 \right)}{\eta_c} = T_{o2} \left(\frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right) = 480\ K$$

$$T_{o3} = T_{o2} + \Delta T_{oc} = 731\ K$$

Solution : Calculation of f (or f_b)

the subscript b here is for the burner fuel input, if there is an afterburner we would term it f_{ab}

- $T_{03} = 731K$
- $T_{04} = 1600K$
- $\eta_b = 0.99$
- Q_r or $|\Delta H_R| = 45$ MJ/kg
- $C_{ph} = \gamma_h/(\gamma_h-1) * R = 1160$ J/kg.K

$$f_b = \frac{T_{o4} - T_{o3}}{\left[\dfrac{\eta_b Qr}{c_{ph}} - T_{o4}\right]} = 2.36\times10^{-2}$$

From energy balance across combustor

Note that for typical aviation fuels, the first term in the denominator is usually much larger than the second one, so we can approximate further

$$f_b = \frac{c_{ph}(T_{o4} - T_{o3})}{\eta_b Qr}$$

Which yields, $f_b = 2.26 \times 10^{-2}$

Solution: Turbine Work

$$\left|\dot{W}_t\right| = \dot{W}_c + \dot{W}_f \Rightarrow (\dot{m}_a + \dot{m}_f)\left|h_{o5} - h_{o4}\right| = \dot{m}_a(h_{o3} - h_{o2}) + B\dot{m}_a(h_{o13} - h_{o12})$$

Dividing by air-flow rate through primary and using assumption (3)

$$(1+f_b)c_{ph}\left|T_{o5} - T_{o4}\right| = c_{pc}\left[(T_{o3} - T_{o2}) + B(T_{o13} - T_{o12})\right]$$

$$\Rightarrow \left|\Delta T_{ot}\right| = \frac{c_{pc}}{c_{ph}(1+f_b)}\left[\Delta T_{oc} + B\Delta T_{of}\right]$$

- $T_{02} = 251K$, $T_{03} = 731K$, $T_{04} = 1600K$
- $f_b = 0.0236$
- $C_{pc} = 1005$ K/kg.K
- $C_{ph} = (\gamma_h/\gamma_h\text{-}1) * R = 1160$ J/kg.K
- $B = 6$

- We are looking to calculate T_{05}
- First, we still need to find what ΔT_{0f} is

Calculation of T_{012} and T_{05}

Across the fan

$$\Delta T_{of} = T_{o13} - T_{o12} = \frac{T_{o13s} - T_{o12}}{\eta_f} = \frac{T_{o12}\left(\frac{T_{o13s}}{T_{o12}} - 1\right)}{\eta_f} = T_{o12}\left(\frac{\pi_f^{\frac{\gamma-1}{\gamma}} - 1}{\eta_f}\right) = 61.1\ K$$

- $T_{012} = T_{02} = 251K$
- $\eta_f = 0.9$
- $\pi_f = PR_f = 2$

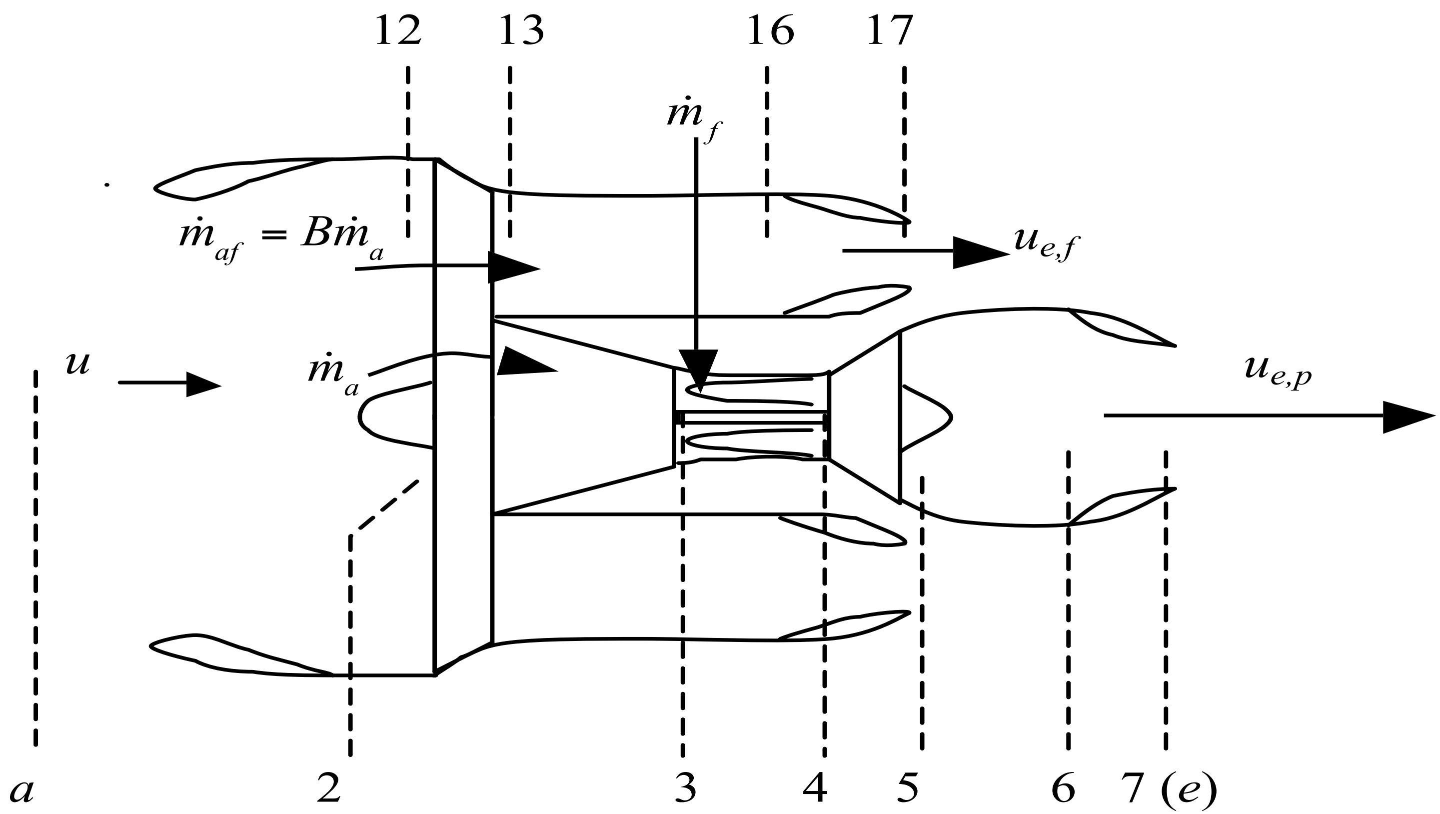
- Therefore, $T_{013} = 251 + 61.1 = 312.1K$

$$(1+f_b)c_{ph}\left|T_{o5} - T_{o4}\right| = c_{pc}\left[(T_{o3} - T_{o2}) + B(T_{o13} - T_{o12})\right]$$

$$\Rightarrow \left|\Delta T_{ot}\right| = \frac{c_{pc}}{c_{ph}(1+f_b)}\left[\Delta T_{oc} + B\Delta T_{of}\right]$$

$$T_{o5} = T_{o4} - \left|\Delta T_{ot}\right| = 1600 - 718 = 882\ K$$

Calculation across primary nozzle



$$\frac{p_{o6}}{p_{e,p}} = \frac{p_{o6}}{p_{o5}}\frac{p_{o5}}{p_{o4}}\frac{p_{o4}}{p_{o3}}\frac{p_{o3}}{p_{o2}}\frac{p_{o2}}{p_a}\cancel{\frac{p_a}{p_{e,p}}} = \cancel{r_{ab}}\pi_t r_b \pi_c \left(1+\eta_d \frac{\gamma-1}{2} M_a^2\right)^{\frac{\gamma}{\gamma-1}}$$

Primary Nozzle Velocity

$$\frac{p_{o6}}{p_{e,p}} = \frac{p_{o6}}{p_{o5}}\frac{p_{o5}}{p_{o4}}\frac{p_{o4}}{p_{o3}}\frac{p_{o3}}{p_{o2}}\frac{p_{o2}}{p_a}\cancel{\frac{p_a}{p_{e,p}}} = \cancel{r_{ab}}\,\pi_t r_b \pi_c \left(1+\eta_d \frac{\gamma-1}{2} M_a^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\pi_t = \left(\frac{T_{o5}}{T_{o4}}\right)^{\frac{\gamma_h}{\gamma_h-1}} = (882/1600)^{\wedge(1.33/0.33)} = 0.091$$

- $T_{012} = T_{02} = 251K$
- $\eta_d = 0.92$
- $\pi_c = PR = 30$
- $r_b = 0.95$
- $Ma = 0.84$
- $\eta_n = 0.95$

Using these values,
$P_{06}/P_e = 3.976$, and $T_{06} = T_{05} = 882$ K

$$T_{es} = T_{o6}\left(\frac{p_e}{p_{o6}}\right)^{\frac{\gamma_h-1}{\gamma_h}}$$

$$T_{es} = 882 * (0.251)^{\wedge(0.33/1.33)} = 625K$$

$$u_e = \sqrt{2c_{ph}\eta_n\left(T_{o6} - T_{es}\right)} = 752\ m/s$$

Fan Nozzle Velocity

$$\frac{p_{o16}}{p_{e,f}} = \frac{p_{o13}}{p_{e,f}} = \frac{p_{o13}}{p_{o12}} \frac{p_{o12}}{p_a} \cancel{\frac{p_a}{p_{ef}}}_{1(2)} = \pi_f \left(1 + \eta_d \frac{\gamma - 1}{2} M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 3.07$$

- $T_{013} = T_{016} = 312.1K$
- $\eta_d = 0.92$
- $\pi_f = PR_f = 2$
- $Ma = 0.84$
- $\eta_{nf} = 0.97$

$$T_{es,f} = T_{o16} \left(\frac{p_{e,f}}{p_{o16}}\right)^{\frac{\gamma-1}{\gamma}} = 226\ K$$

$$u_{e,f} = \sqrt{2\eta_{nf} c_{pc} \left(T_{o13} - T_{es,f}\right)} = 408\ m/s$$

Specific Thrust

$$\Im = \dot{m}_a \left\{ \left[\left(1 + f_b \right) u_{e,p} - u \right] + B \left[u_{e,f} - u \right] \right\}$$

The total mass flow is the sum of the mass flow into the core and mass flow into the fan

$$\frac{\Im}{\dot{m}_{tot}} = \frac{1}{1+B} \left\{ \left[\left(1 + f_b \right) u_{e,p} - u \right] + B \left[u_{e,f} - u \right] \right\} = 2.09 \times 10^2 \ m/s$$

$$TSFC = \frac{\dot{m}_f}{\Im} = \frac{f_b}{\Im / \dot{m}_a} = \frac{f_b}{(1+B)\left(\Im / \dot{m}_{tot} \right)} = 1.61 \times 10^{-5} \ kg/s\text{-}N$$

Efficiencies

Propulsive efficiency = Thrust Power/Kinetic Energy change of airflow
Thermal Efficiency = Kinetic Energy Change of airflow/Heat input
Overall efficiency = Thrust Power/Heat Input

Thrust Power = Total Thrust x Flight Speed

$$\Im . u = \dot{m}_a u \left\{ \left[\left(1 + f_b\right) u_{e,p} - u \right] + B \left[u_{e,f} - u \right] \right\}$$

Heat Input = fuel flow rate x heat of reaction

$$Heat = f_b \dot{m}_a Qr$$

Kinetic Energy Change of Airflow

$$\Delta KE = 0.5 * \dot{m}_a \left\{ \left[\left(1 + f_b\right) u_{e,p} * u_{e,p} \right] + B \left[u_{e,f} * u_{e,f} \right] - (1 + B) \left[u * u \right] \right\}$$

Efficiencies

- Caluculate these as Homework
- Thermal Efficiency = 0.537
- Propulsive Efficiency = 0.643
- Overall Efficiency = 0.345

Bypass Ratio Effect