Lecture 26-30

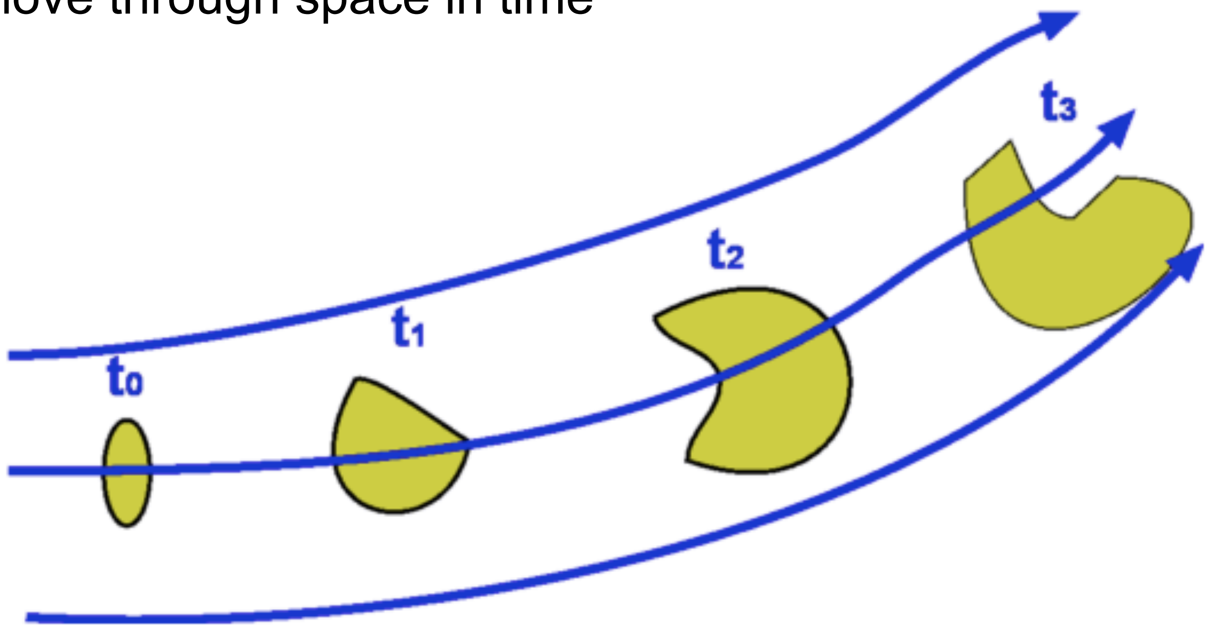
Compressor Stage Analysis (Velocity Diagrams, Pressure Ratio, Stage Parameters, Non-Dimensional Parameters)

Lagrangian Approach to Modeling

Lagrangian(Control Mass: *CM*)

- fixed mass; no mass flow
- system boundary
- surroundings

Laws describe what happens to group of fluid elements as they move through space in time



Fundamental Laws for Systems(*)

- Mass is conserved (i.e., mass can be neither created nor destroyed).

$$\boxed{\frac{D}{Dt}(mass) = \frac{D}{Dt}\int \rho \, d(volume) = 0}$$

- Newton's second law: Rate of change of momentum is equal to the unbalanced force on the system.

$$\boxed{\frac{D}{Dt}(momentum) = \frac{D}{Dt}\int \rho \, \bar{u} \, d(volume) = \sum \bar{F}_{on\ the\ fluid}}$$

- Energy is conserved; it can only change from one form to another.

$$\boxed{\frac{D}{Dt}(energy) = \frac{D}{Dt}\int \rho \, e \, d(volume) = \left(\dot{Q} - \dot{W}\right)_{on\ the\ fluid}}$$

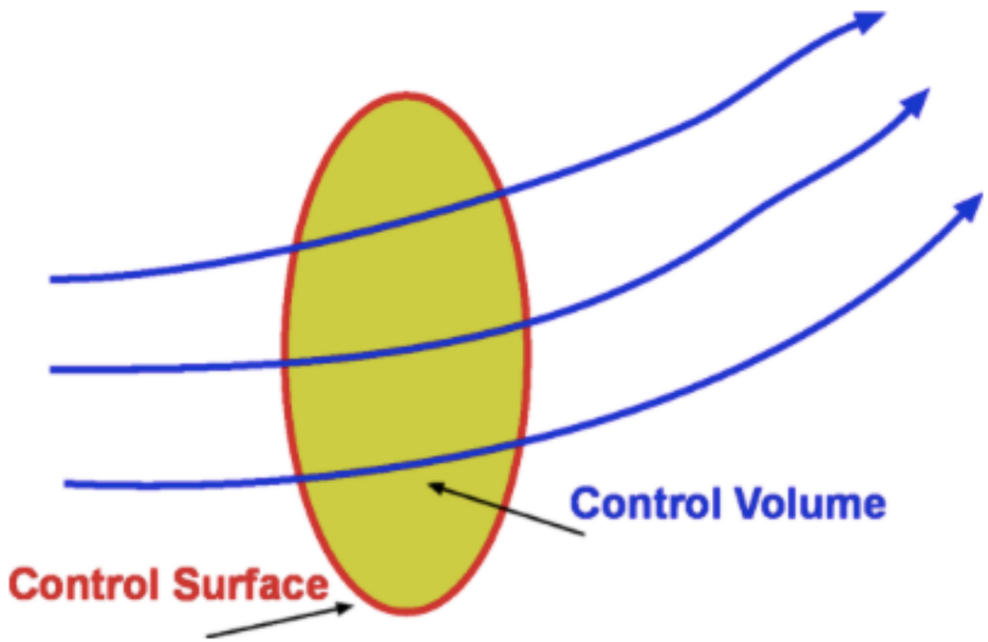
* Neglecting nuclear reactions and relativistic phenomenon

Eulerian Approach to Modeling

Eulerian(Control Volume, *CV*)

- mass flow; fixed region in space
- control surface
- surroundings

Laws describe the entire volume or "field"



Basic Laws for a steady flow in a CV

Conservation of Mass:

$$\boxed{\int_{CS} \rho(\bar{u} \cdot \hat{n})dA = 0 \quad \Rightarrow \quad \sum \dot{m}_{out} = \sum \dot{m}_{in}}$$

Conservation of Linear Momentum:

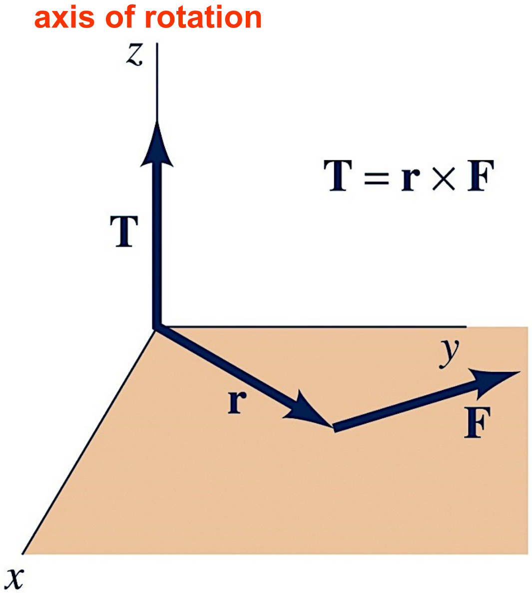
$$\boxed{\sum \bar{F} = \int_{CS} \rho \bar{u}(\bar{u} \cdot \hat{n})dA \quad \Rightarrow \quad \sum \bar{F} = \sum \dot{m}_{out}\bar{u}_{out} - \sum \dot{m}_{in}\bar{u}_{in}}$$

Conservation of Energy:

$$\sum\left(\dot{Q}_{net} - \dot{W}_{net}\right) = \int_{CS} \rho(e + \frac{1}{2}V^2 + gz)(\bar{u} \cdot \hat{n})dA \quad \Rightarrow$$

$$\sum\left(\dot{Q}_{net} - \dot{W}_{shaft}\right) = \sum \dot{m}_{out}(h + \frac{1}{2}u^2 + gz)_{out} - \sum \dot{m}_{in}(h + \frac{1}{2}u^2 + gz)_{in}$$

Angular Momentum (Moment of Momentum)



r **= position vector of fluid particle in an inertial coordinate system**

δF **= vector force acting on the fluid particle that creates a moment, and thus a torque, on the fluid particle**

In general ,

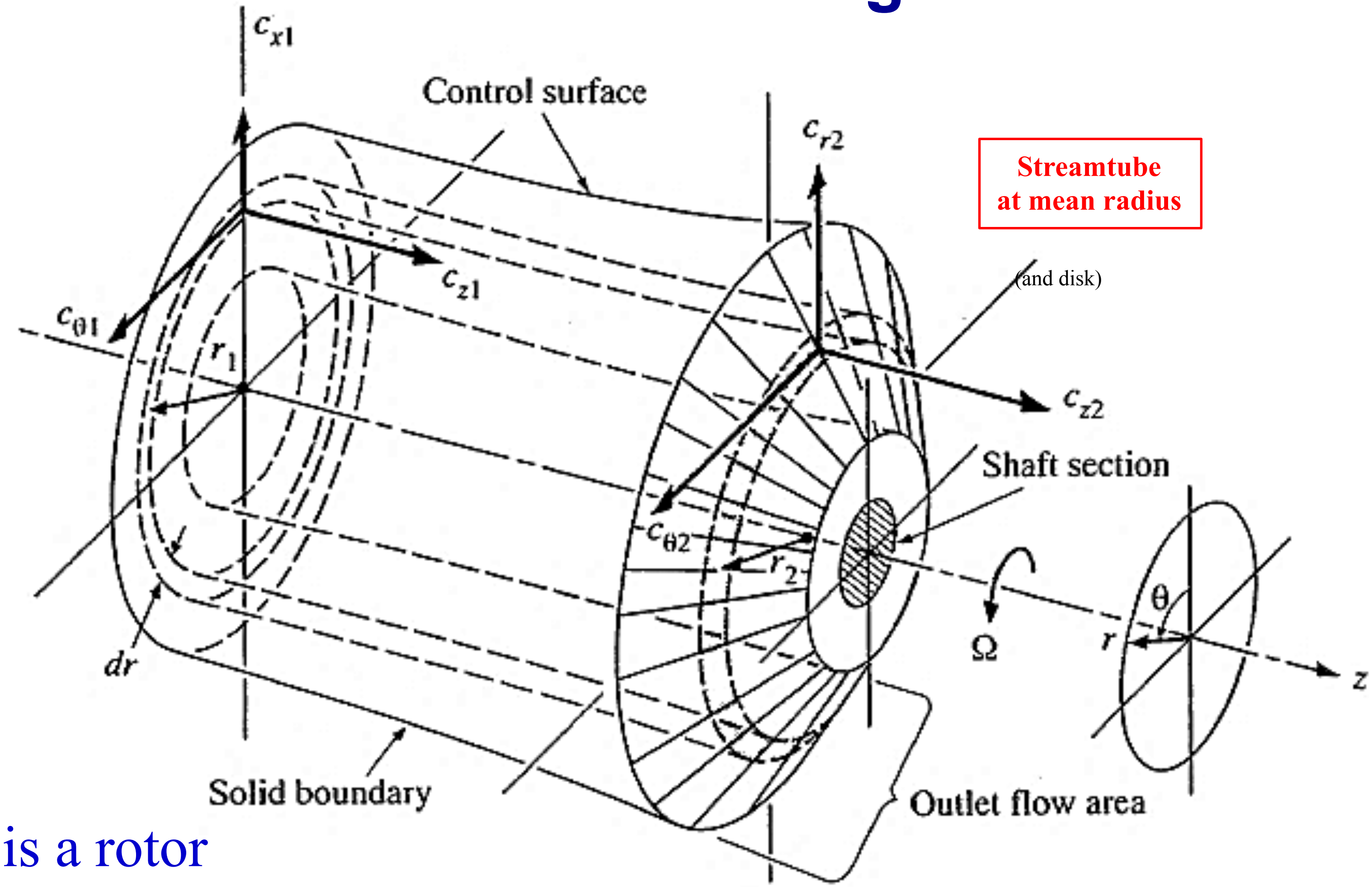
$$\mathbf{F} = \mathbf{r}F_r + \boldsymbol{\theta}F_\theta + \mathbf{z}F_z, \qquad \mathbf{c} = \mathbf{r}c_r + \boldsymbol{\theta}c_\theta + \mathbf{z}c_z,$$

- Jet engines are predominantly axial
- So we worry only about the torque about the z-axis, or the θ component
- We can show (refer, H&P pg. 277-280) that…

net torque = change in angular momentum

$$\sum \tau = \frac{d}{dt}\int_{cv} \rho r c_\theta \, d\mathcal{V} + \int_{cs} \rho r c_\theta (\mathbf{c}\cdot\mathbf{n})\, dA$$

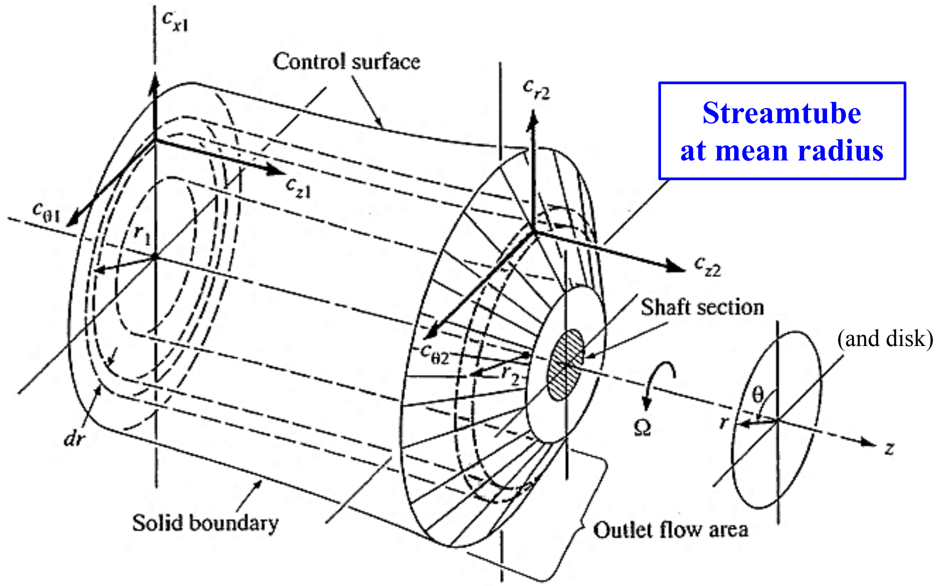
Conservation of Angular Momentum for an Axial Rotor Stage



There is a rotor between station 1 and 2

$$\sum \tau = \int_{A_2} (rc_\theta)\rho c_n \, dA - \int_{A_1} (rc_\theta)\rho c_n \, dA$$

Conservation of Angular Momentum



Assuming rc_θ is constant over the control surface (free vortex flow assumption)

Torque acting on the fluid $$\sum \tau = \dot{m}[(rc_\theta)_2 - (rc_\theta)_1]$$

Power required to drive the stage $$\mathcal{P}_s = \tau\Omega = -\dot{m}\Omega[(rc_\theta)_2 - (rc_\theta)_1]$$

If we can figure out the increment in tangential velocity, we can calculate the shaft power of each stage – <u>this is where velocity triangles come in</u>

Resultant Aerodynamic Force and Moment on an airfoil

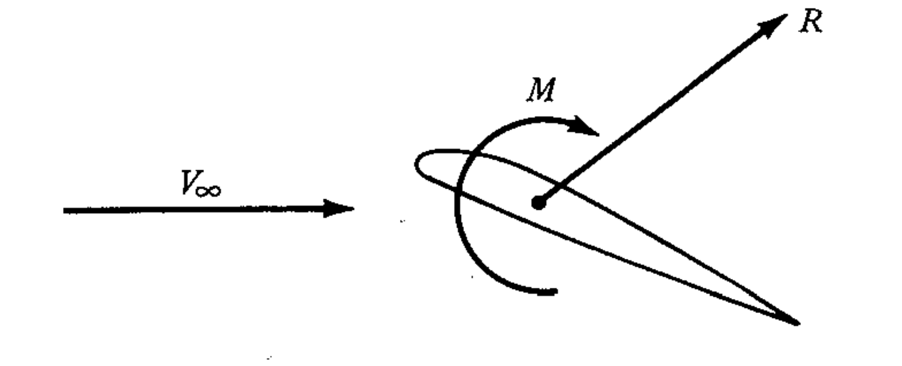


Figure 1.15 Resultant aerodynamic force and moment on the body.

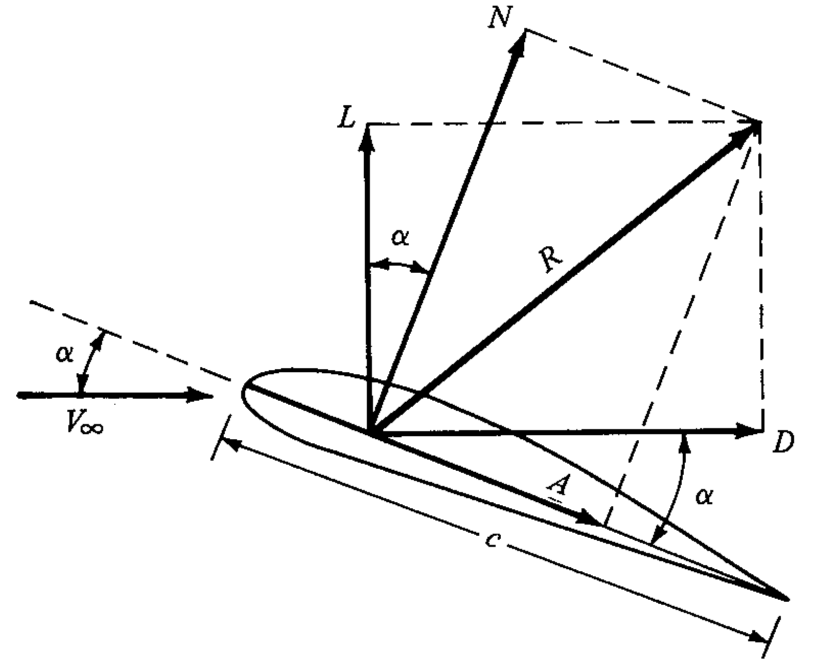


Figure 1.16 Resultant aerodynamic force and the components into which it splits.

L=lift=component perpendicular to freestream

D=drag=component parallel to freestream

Sometimes, it is easier to look at forces parallel and perpendicular to the chord line: (eg., if we measure forces in the wind tunnel using a support sting)

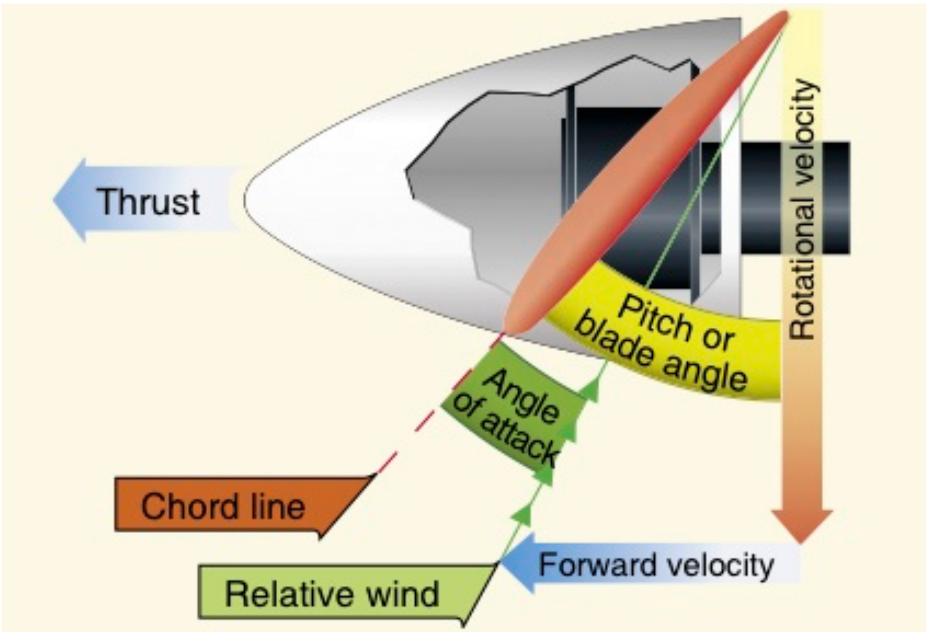
N=normal force=component of R perpendicular to chord, c

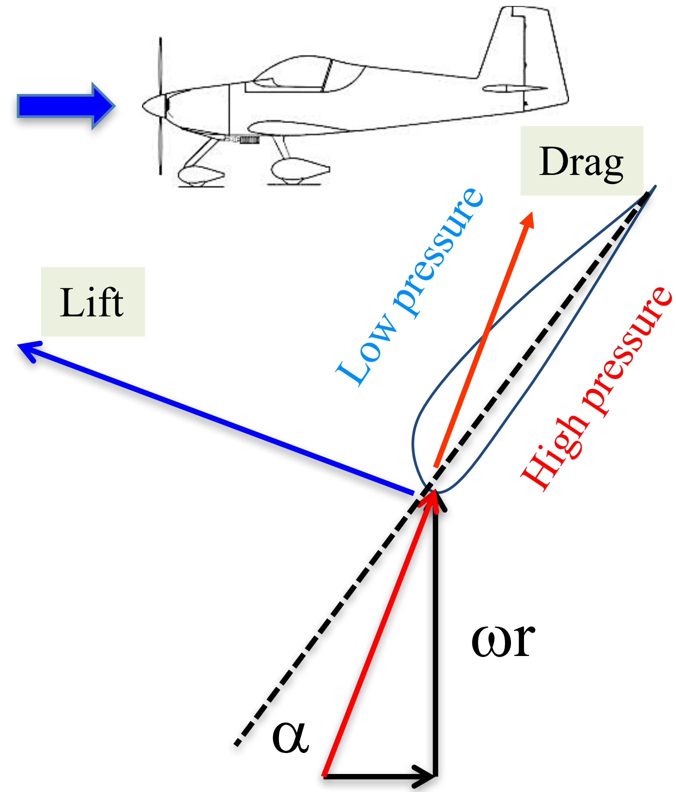
A=axial force=component of force R parallel to c

$L = N\cos\alpha - A\sin\alpha$

$D = N\sin\alpha + A\cos\alpha$

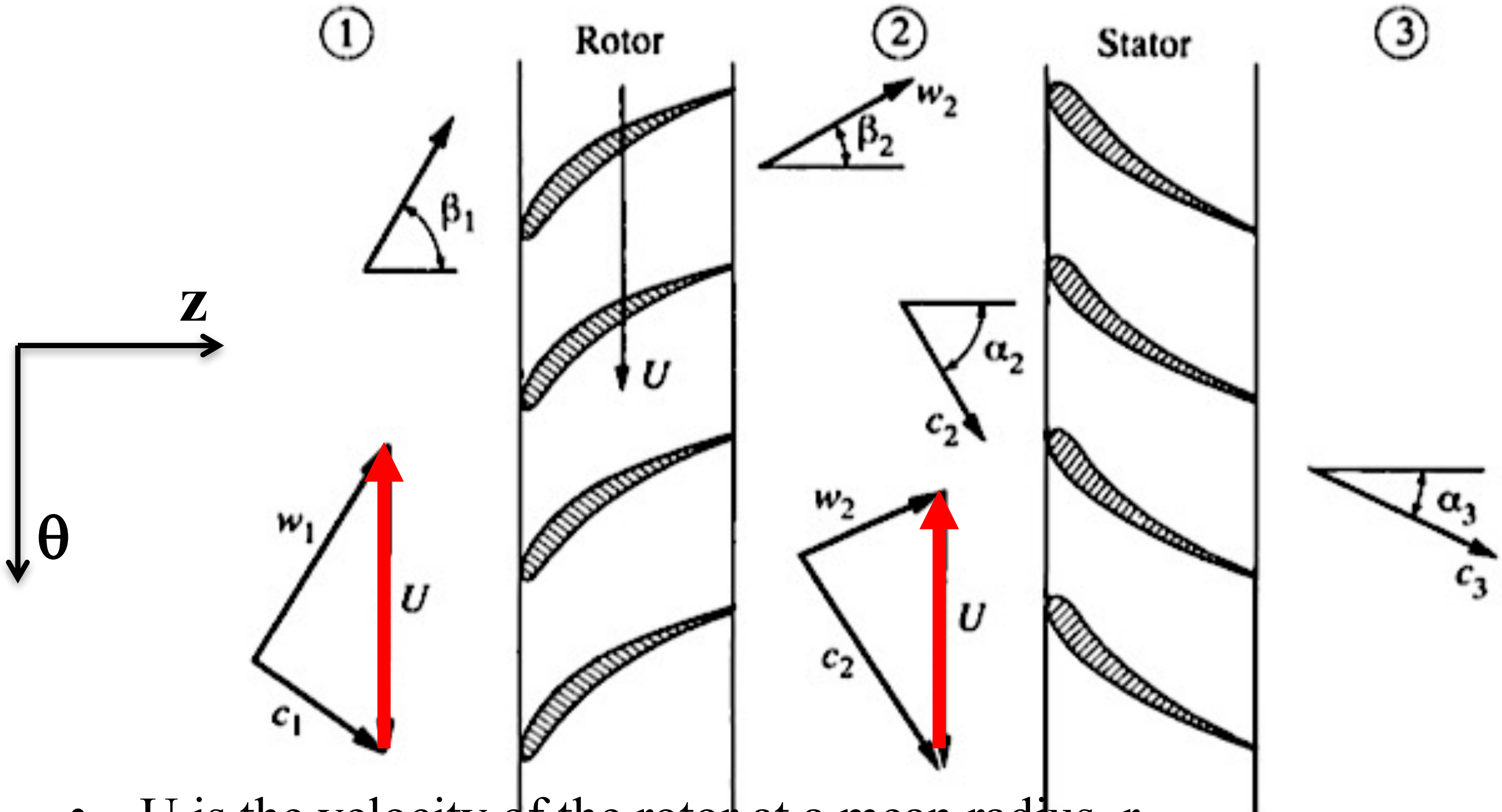
Concept of Velocity Triangles





- Consider a propeller aircraft flying at a speed of c
- In this case, the propeller creates thrust by increasing the pressure of the air coming into the propeller disk
- The vector diagram representing the effective incoming velocity is called a velocity triangle

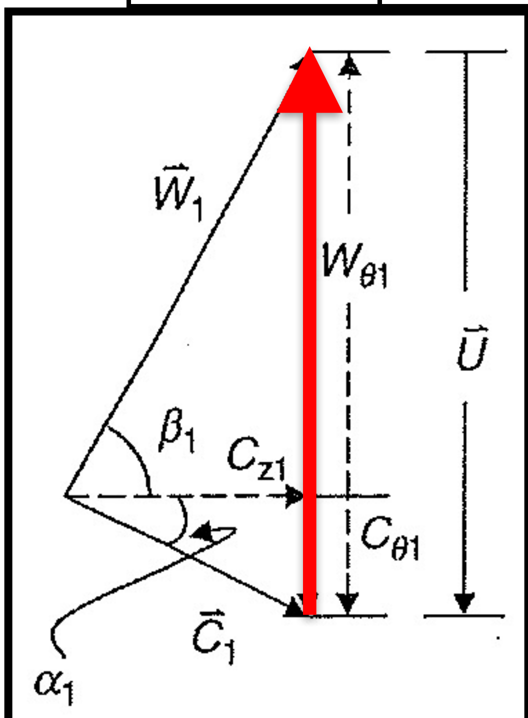
Velocity Triangles in a Compressor Stage



- U is the velocity of the rotor at a mean radius, r
- c is the velocity of the flow coming into the rotor,
  where $c = c_z.\,\mathbf{z} + c_\theta.\,\boldsymbol{\theta}$ **(z, θ are unit vectors)**
- w is the velocity relative to the rotor
- β is the angle of the resultant velocity, and turning angle $\Delta\beta = \beta_1 - \beta_2$
- α is the angle of the mean flow with respect to the axis of the engine

Frame of Reference Definitions

Variable	Stationary or Absolute	Relative	Velocity of moving frame
Velocity	C, (C_z, C_θ)	W, (W_z,W_θ)	U=ωr
Angle	α	β	
Direction	Positive clockwise below axis	Positive counterclockwise Above axis	Postive downward



$$\vec{C} = \vec{W} + \vec{U}$$

If stationary

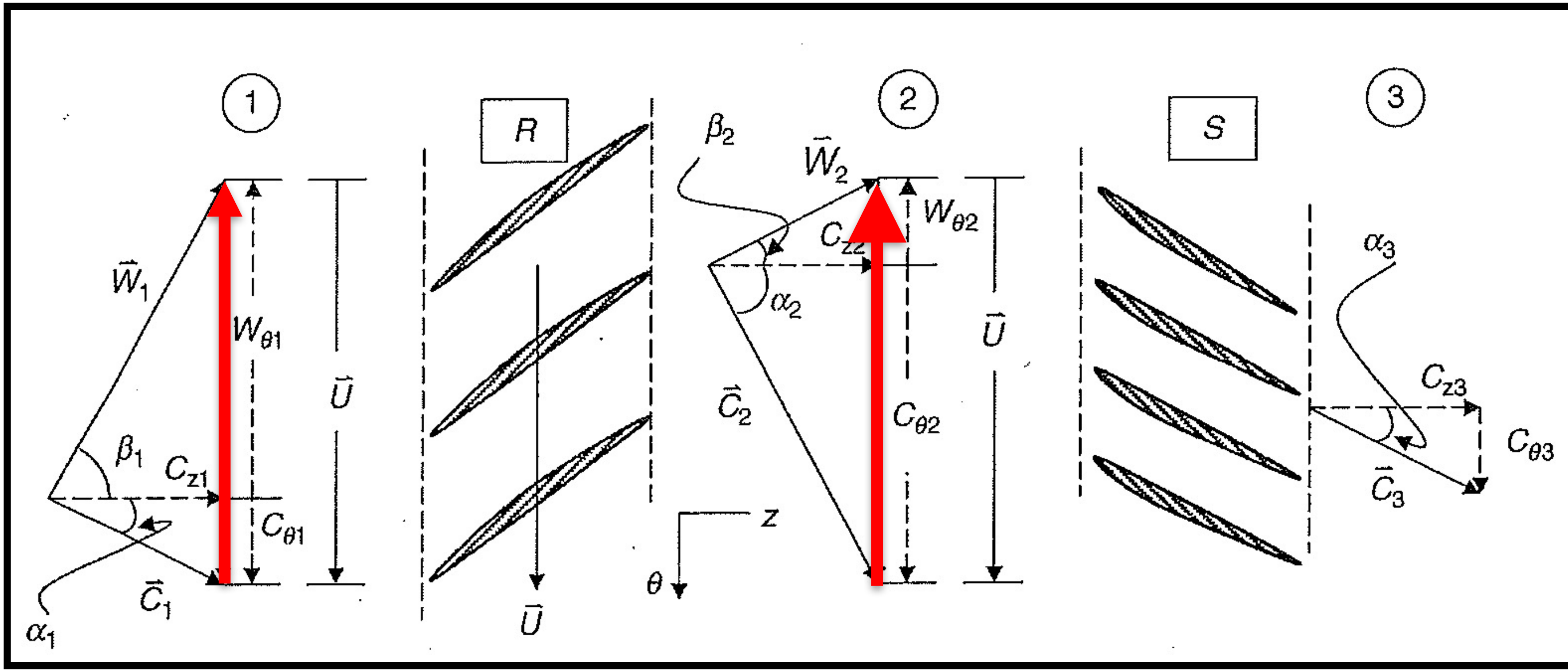
$$\vec{C} = \vec{W}, \alpha = \beta$$

Velocity Components

$$c_z = \textit{axial component}$$

$$c_\theta = \textit{circumferential component}$$

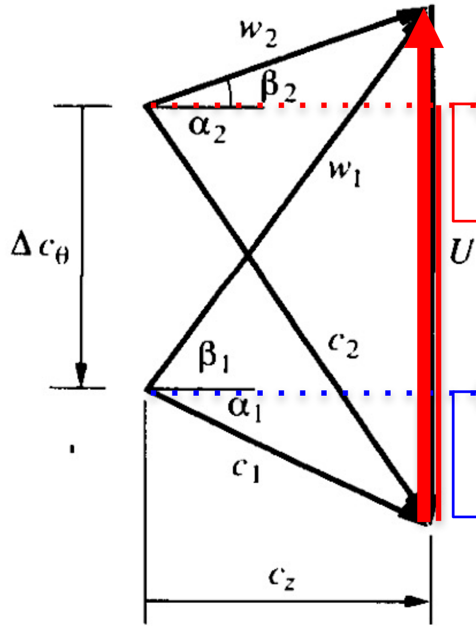
Axial Compressor Velocity Diagram



- The rotor imparts additional swirl to the flow
- Note the increase of the velocity vector from C1 to C2
- $C_{\theta 1} > C_{\theta 2}$

Analysis at the Mean Radius

$$c_{z1} = c_{z2} = c_z$$



$$\boxed{c_{\theta 2} = U - c_z \tan \beta_2}$$

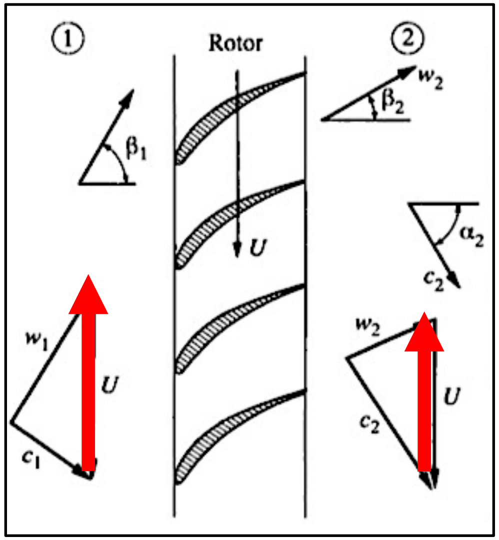
$$\boxed{c_{\theta 1} = c_z \tan \alpha_1}$$

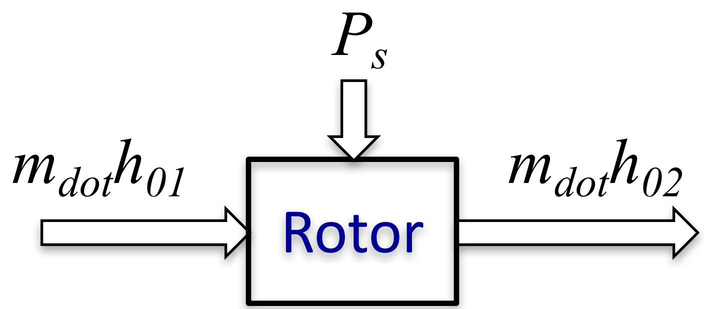
$$\mathcal{P}_s = \tau\Omega = -\dot{m}\Omega[(rc_\theta)_2 - (rc_\theta)_1]$$

$$\mathcal{P}_s = -\dot{m}U(c_{\theta 2} - c_{\theta 1})$$

- Key properties can be ascertained using analysis at the mean radius of the rotor
- Velocity triangles can be combined across rotor stages to create a composite diagram
- Assumptions
  - Axial component of c, i.e., c_z stays constant across the stage
  - Also, radial flow, i.e., $c_r = 0$

Energy Balance Across Rotor





$$m_{dot}h_{02} = m_{dot}h_{01} + P_s$$

$$\mathcal{P}_s = \dot{m}U(c_{\theta 2} - c_{\theta 1})$$

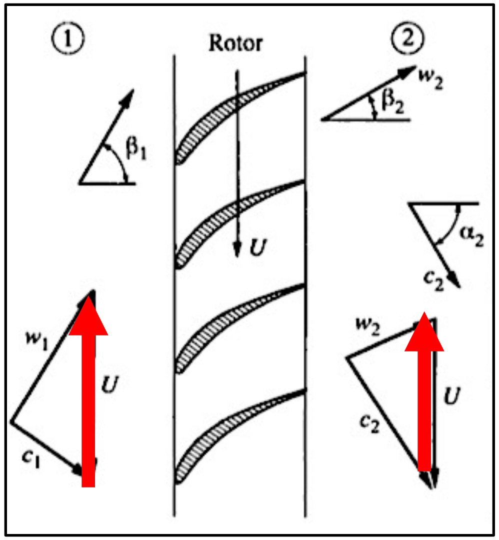
$$h_{02} - h_{01} = U(c_{\theta 2} - c_{\theta 1})$$

Temperature rise across rotor $\boxed{\dfrac{\Delta T_0}{T_{01}} = \dfrac{U\,\Delta c_\theta}{c_p\,T_{01}}}$

- Since no work is done across stator, this also represents temperature across the entire rotor-stator stage

Compressor Stage Analysis

- Across rotor, power input is $\boxed{P_s = \dot{m}c_p\left[T_{02} - T_{01}\right]}$
- Across stator, power input is $\boxed{\dot{W} = 0 \quad \Rightarrow \quad T_{03} = T_{02}}$
- Since, c_z = constant



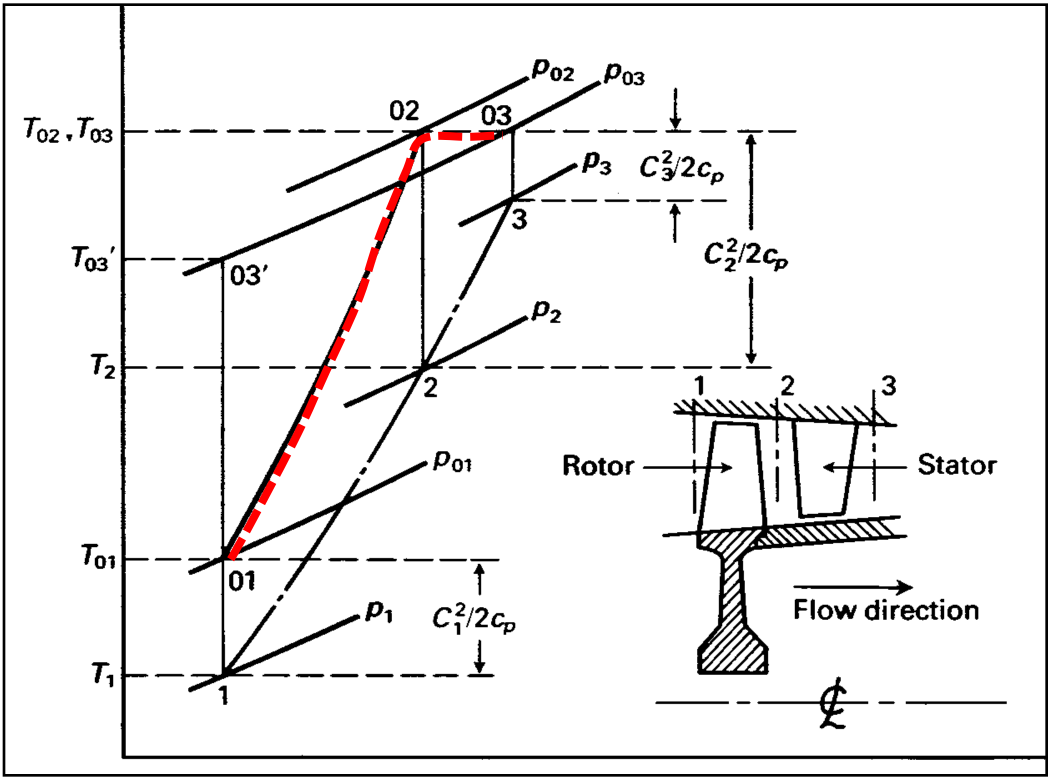
$$P_s = \dot{m}U\left[C_{\theta 2} - C_{\theta 1}\right] = \dot{m}UC_z\left[\tan\alpha_2 - \tan\alpha_1\right]$$

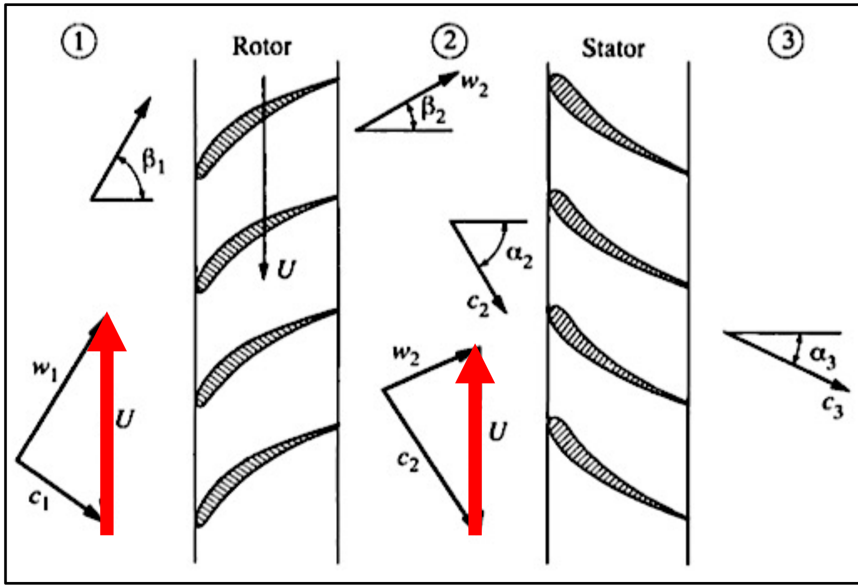
or in terms of rotor blade angles

$$P_s = \dot{m}UC_z\left[\tan\beta_1 - \tan\beta_2\right]$$

- Thermodynamics related to rotor kinematics, i.e., speed, U and flow turning angles

Compressor Stage Analysis





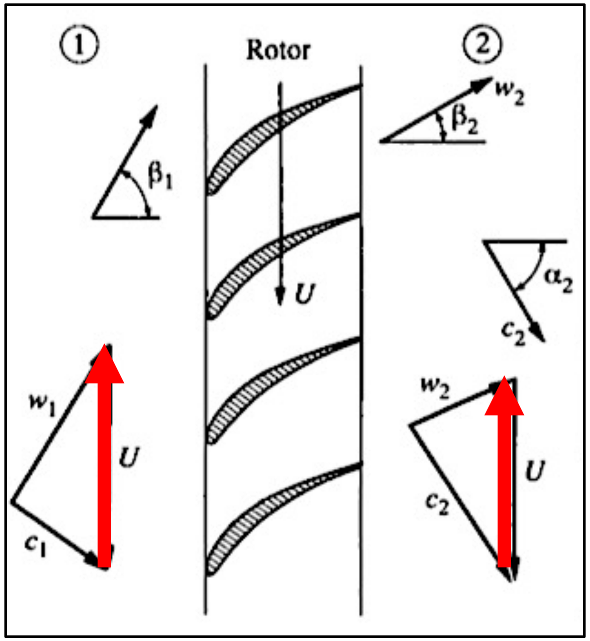
$$\mathrm{Pr}_s = \frac{p_{03}}{p_{01}}$$

$$\eta_{isen} = \eta_{st} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}}$$

$$\frac{p_{03}}{p_{01}} = \left[\frac{T_{03}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} = \left[1 + \eta_{st}\frac{\Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

Example Problem

Given a first single stage of an Axial Compressor with the following conditions: ambient pressure (p_{in}) 1 atmosphere, ambient temperature (T_{in}) 300K, Incoming speed (V_{in}) 170m/s, mean blade diameter (D) 0.5m, rotor rpm (ω_{rotor}) 8000rpm, turning angle ($\Delta\beta$) 15 degrees, specific heat ratio (γ) 1.4, air mass flow rate (m_{dot}) 35kg/s, and (C_p) 1005 J/kgK, calculate the first stage shaft power, and stage temperature and pressure ratio. Assume stage efficiency of 0.9.



Pin = 1 atm

Tin = 300K

Vin = C_{z1} = 170 m/s

$C_{\theta 1}$=0

Mean D = 0.5m (Radius = 0.25m)

(ω_{rotor}) = 8000rpm

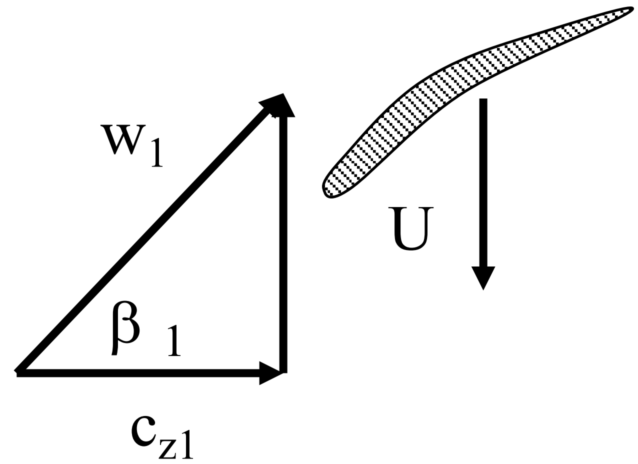
Turning angle ($\Delta\beta$) 15 degrees

m_{dot} = 35 kg/s

Solution

Step 1.

Create the velocity triangle and calculate the relative speed of the rotor blade from the rotational velocity. Note $c_{\theta 1}=0$.



$\omega = 2\pi f, where\ f\ is\ in\ rev/sec$

$$U \equiv r\omega \qquad U := \frac{D}{2} \cdot \frac{2 \cdot \pi}{60 \cdot s} \cdot 8000 \qquad U = 209.44 \frac{m}{s}$$

Step 2.

Calculate the air to blade relative velocity and the angle between the relative and actual air speed.

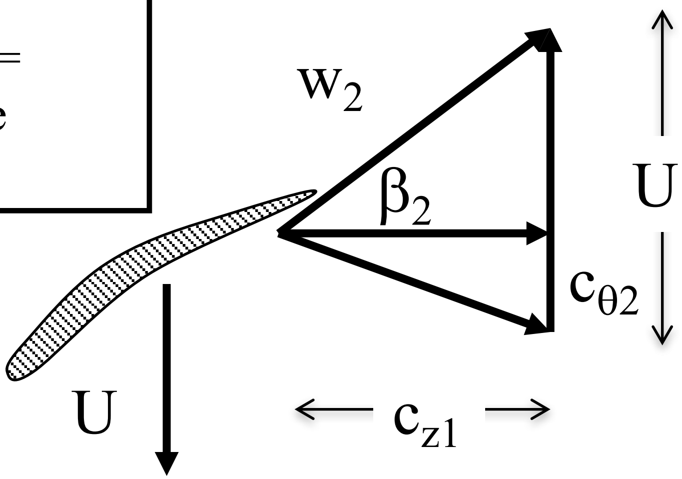
$$w_1 = \sqrt{(c_{z1}{}^2 + U^2)} = \sqrt{170^2 + 209^2} = 269.8\ m/s$$

$$\beta_1 = \tan^{-1}(U/c_{in}) = \tan^{-1}(209.4/170) = 50.9^o$$

Solution

Step 3.

Axial velocity (c_{z1}) does not change and is equal to Vin = 170m/s. Calculate relative exit angle(β2), then calculate relative air speed (w2)



$$\beta_2 = \beta_1 - 15 = 35.9^o$$

$$w2 = c_{in}/\cos(\beta 2) = 209.9 \text{ m/s}$$

Note: you can get the angles without calculating w2, it is just shown for illustration

Step 4.

Calculate $c_{\theta 2}$

$$c_{\theta 2} = U - w2*\sin(\beta 2) = 209.4 – 209.9*\sin(35.9) = 85.5 \text{ m/s}$$

Solution

Step 4.

Calculate the shaft power contribution of the first stage

$$Ps = \dot{m}U\left[c_{\theta 2} - c_{\theta 1}\right]$$
$$Ps = 35 \times 209.4 * 85.5 = 626.8kW$$

Note: $c_{\theta 1} = 0$

Step 5.

Calculate the temperature rise of the first stage

$$From, P_s = \dot{m}c_p\left[T_{02} - T_{01}\right]$$
$$T_{02} - T_{01} = P_s / \dot{m}c_p$$

Stage temperature rise = 626.8 * 1000/(35*1005) = 17.8K

Solution

Step 6.

Calculate stage stagnation temperature ratio (given Tin and Vin, use enthalpy relationship)

$$T_{o1} := T_{in} + \frac{V_{in}^{\ 2}}{2 \cdot C_p} \qquad T_{o1} = 314.392\,K$$

Step 7.

Calculate the temperature rise of the first stage

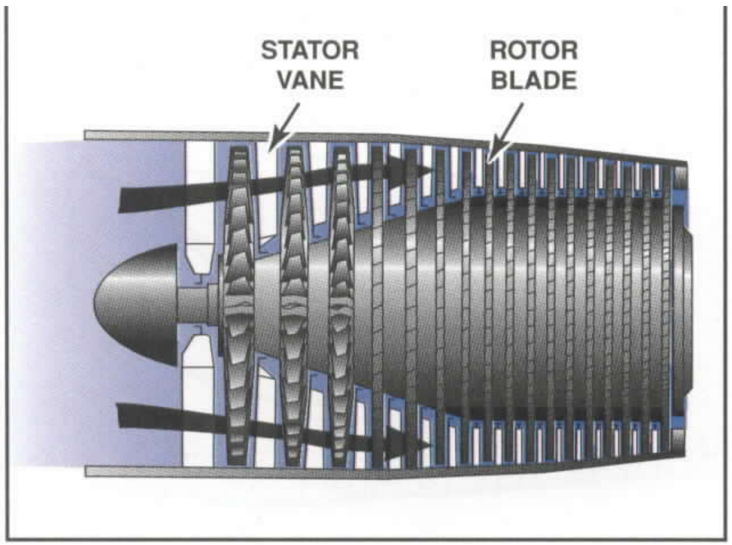
$$\boxed{\Pr_s = \frac{p_{03}}{p_{01}} = \left[1 + \eta_{st}\frac{\Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}}$$

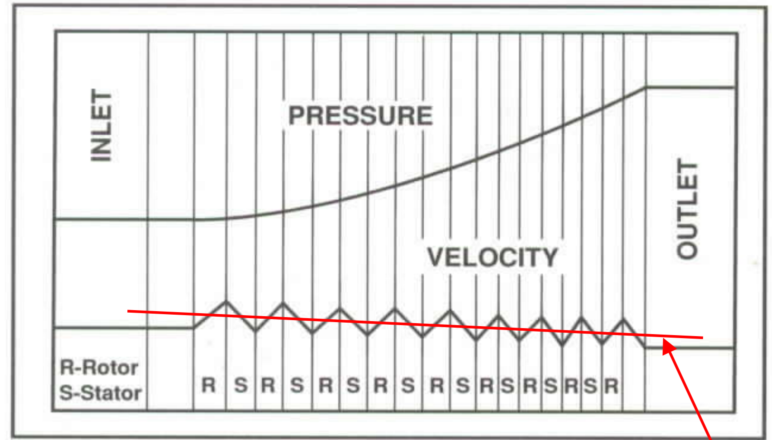
Therefore, $\Pr_s = (1+0.9*17.8/314.3)^\wedge(3.5)$

$= 1.19$

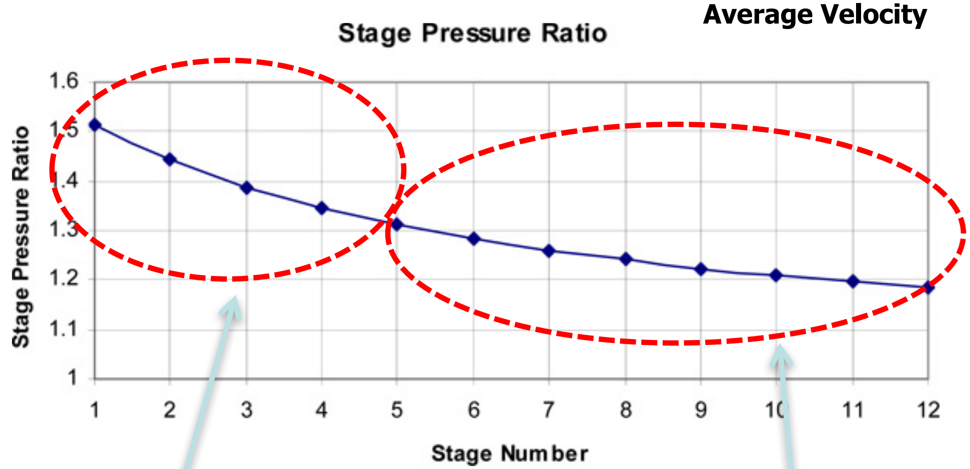
Axial Compressor Pressure Ratio

Axial compressors are designed with a casing and hub shape that allows the axial air velocity (c_z) to remain almost constant, while pressure gradually increases.





Average Velocity



Initial stages see a lot more variation than later stages

A&P Technician Powerplant Textbook published by Jeppesen Sanderson Inc., 1997

Overall compression ratio

The Overall compression ratio is defined as follows

- Assume P_{oe} is the final state and P_{oi} is the initial state, and there are j intermediate stages
- Then $P_{oe}/P_{oi} = P_{oe}/P_{oj} \times P_{oj}/P_{oj-1} \times P_{oj-1}/P_{oj-2} \ldots . \times P_{o1}/P_{oi}$
- If we assume approximately matched stages, then
    - $P_{oe}/P_{oi} = (P_{oj}/P_{oj-1})^{n,}$ where n is the total number of stages

Example Calculation

- A 16-stage compressor has an overall pressure ratio of 25, such that the pressure ratio of each stage is the same. The stage efficiency, measured experimentally, is 0.93.

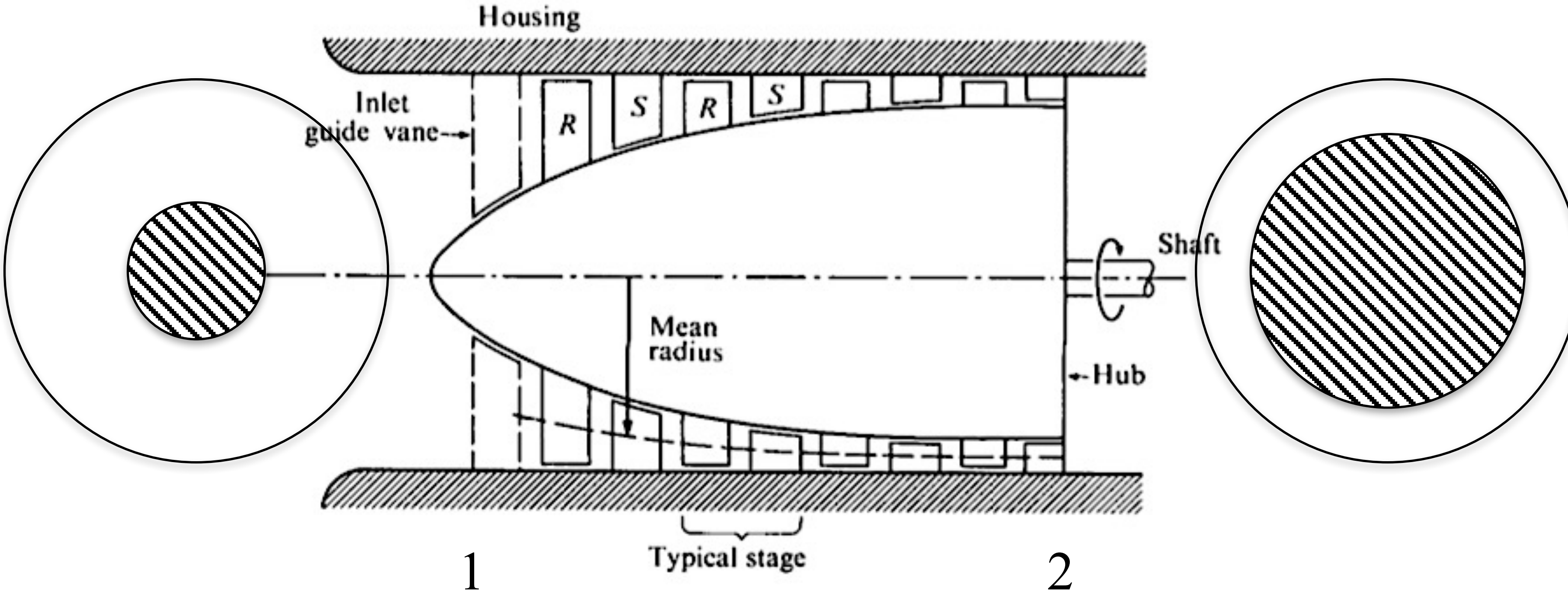
- What is the pressure ratio per stage?

$$\boxed{\begin{array}{l} Consider\ 16-stage\ compressor\ \pi_c = PR = 25 \\ PR_s = 25^{1/16} = 1.223 \end{array}}$$

- What is the overall adiabatic efficiency of the compressor? We will come back to this later

$$\boxed{\eta_{ad\atop comp} = \eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma\eta_p}} - 1} = \frac{25^{\frac{1}{3.5}} - 1}{25^{\frac{1}{3.5*0.93}} - 1} = \frac{1.508}{1.688} = 0.893}$$

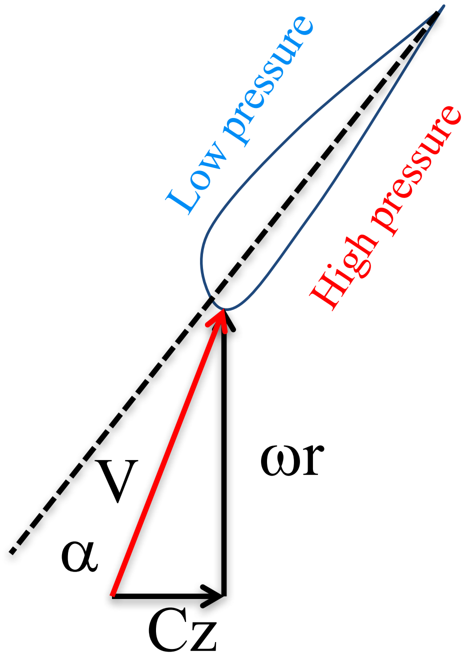
Axial variation in blade parameters

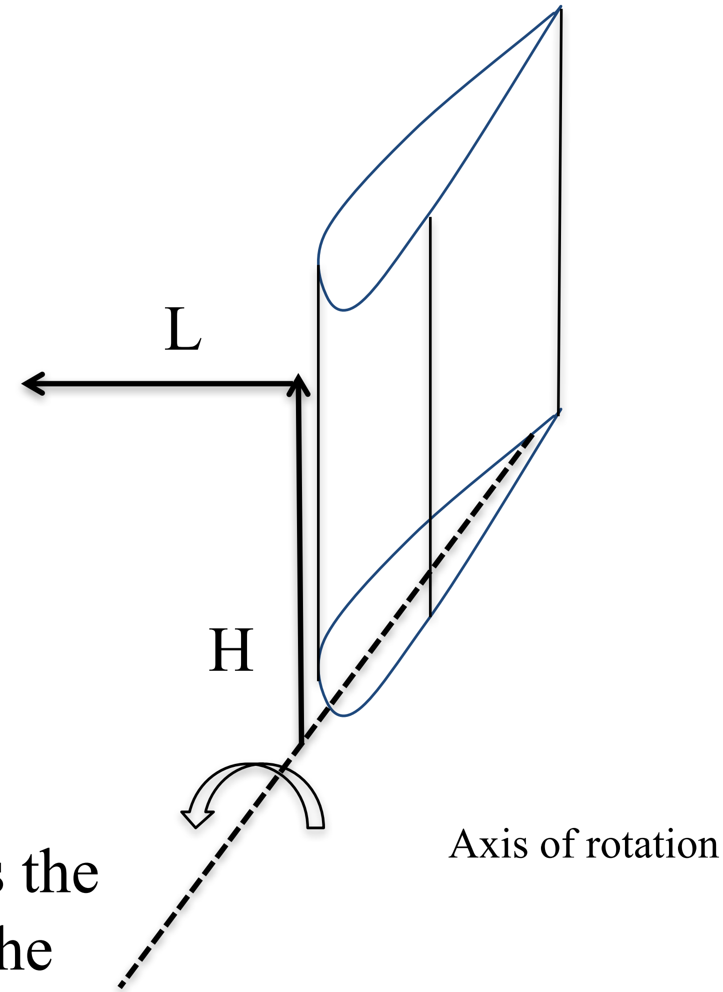


Mass conservation $\rho_1\, A_1\, Cz_1 = \rho_2\, A_2\, Cz_2$

Since $Cz_1 = Cz_2$, and since $\rho_2 > \rho_1$, $A_2 < A_1$

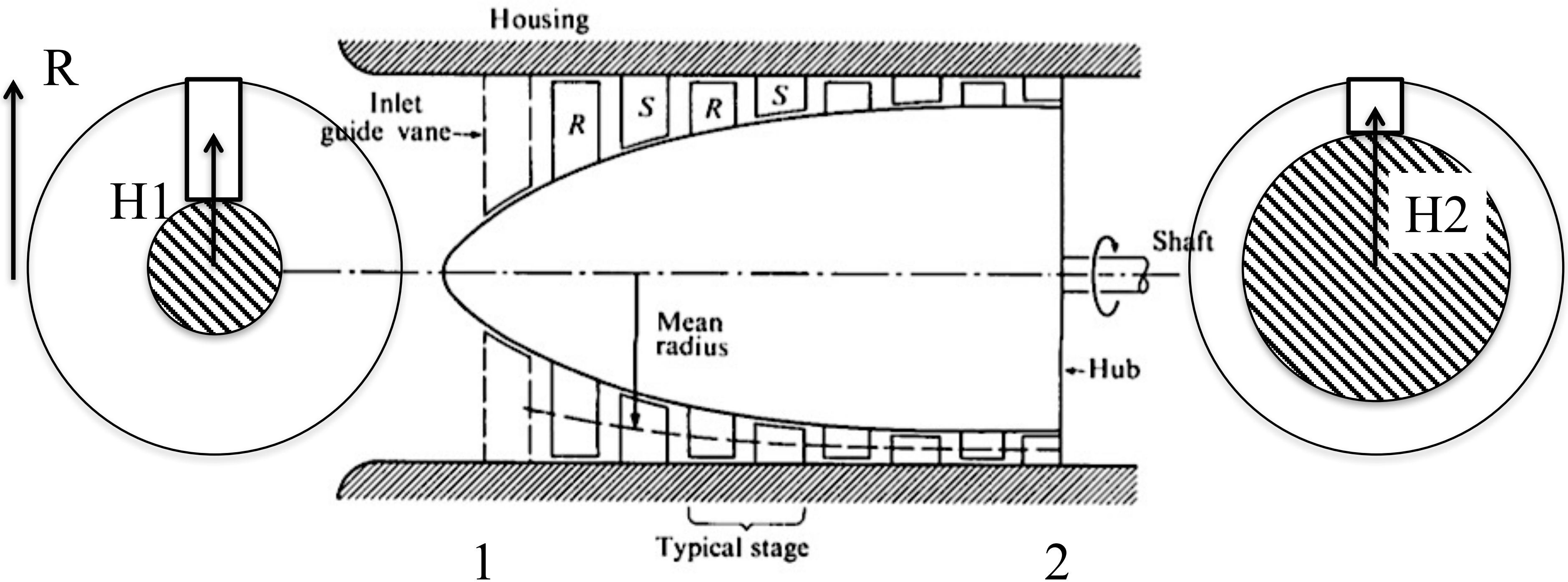
Axial variation in stage sizing





- Consider a rotor blade producing lift
- The Lift, $L = 0.5\ \rho\ V^2 C_L\ S$ where S is the span area of the rotor blade and U is the effective velocity at the blade
- The torque produced by a blade of average length H, is $L*H = 0.5\ \rho\ V^2 C_L\ S\ H$

Axial variation in stage sizing



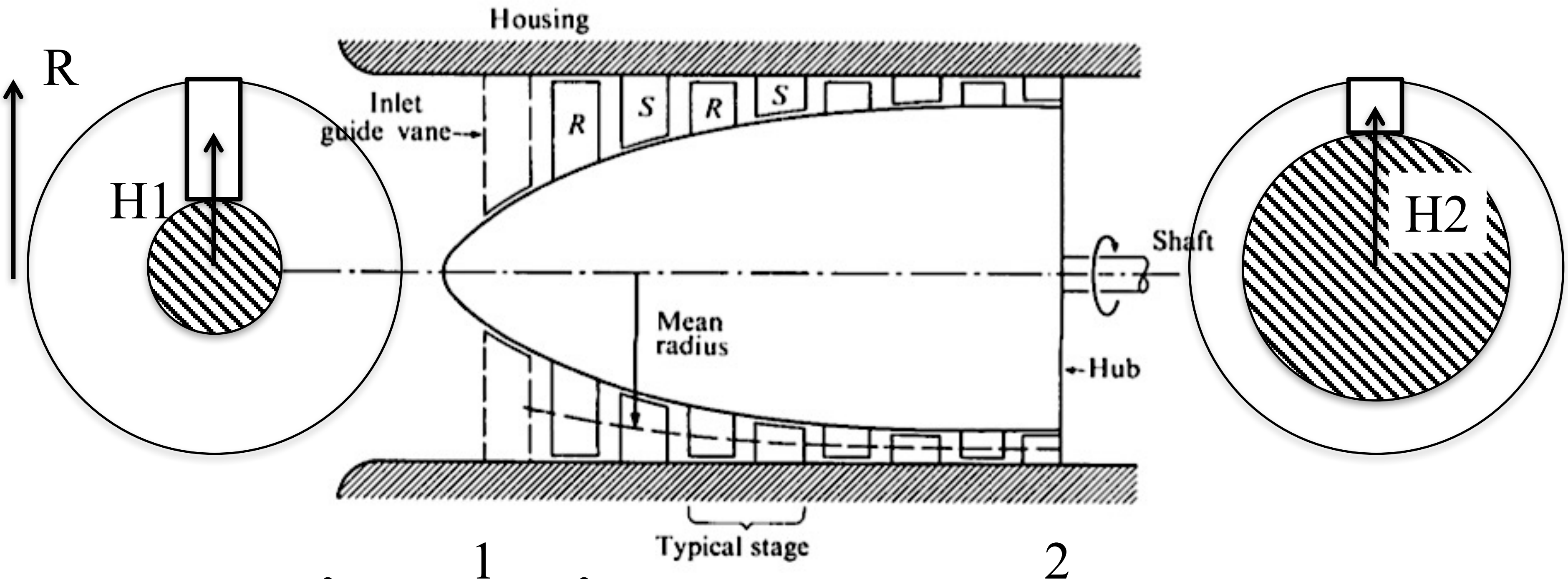
The need for producing uniform torque dictates

$$\boxed{L_1 H_1 = L_2 H_2}$$

Or, $$0.5\,\rho_1\,V_1^2\,C_{L1}\,S_1\,H_1 = 0.5\,\rho_2\,V_2^2\,C_{L2}\,S_2\,H_2$$

$C_{L1} \sim C_{L2}$ (Assume average lift coefficient is uniform across the stages

Axial variation in stage sizing



$\rho_1 \, S_1 \, H_1 \, V_1^2 = \rho_2 \, S_2 \, H_2 \, V_2^2$

$\rho_1 \, (R-H_1) \, C_1 \, H_1 \, V_1^2 = \rho_2 (R-H_2) \, C_2 \, H_2 V_2^2$ (Assuming rectangular blades)

$C_2/C_1 = \rho_1 \, V_1^2 / \rho_2 \, V_2^2 \, \underbrace{[H_1 \, (R-H_1)\,]/[(R-H_2) \;\; H_2]}_{\boxed{\sim 1}}$

Therefore,

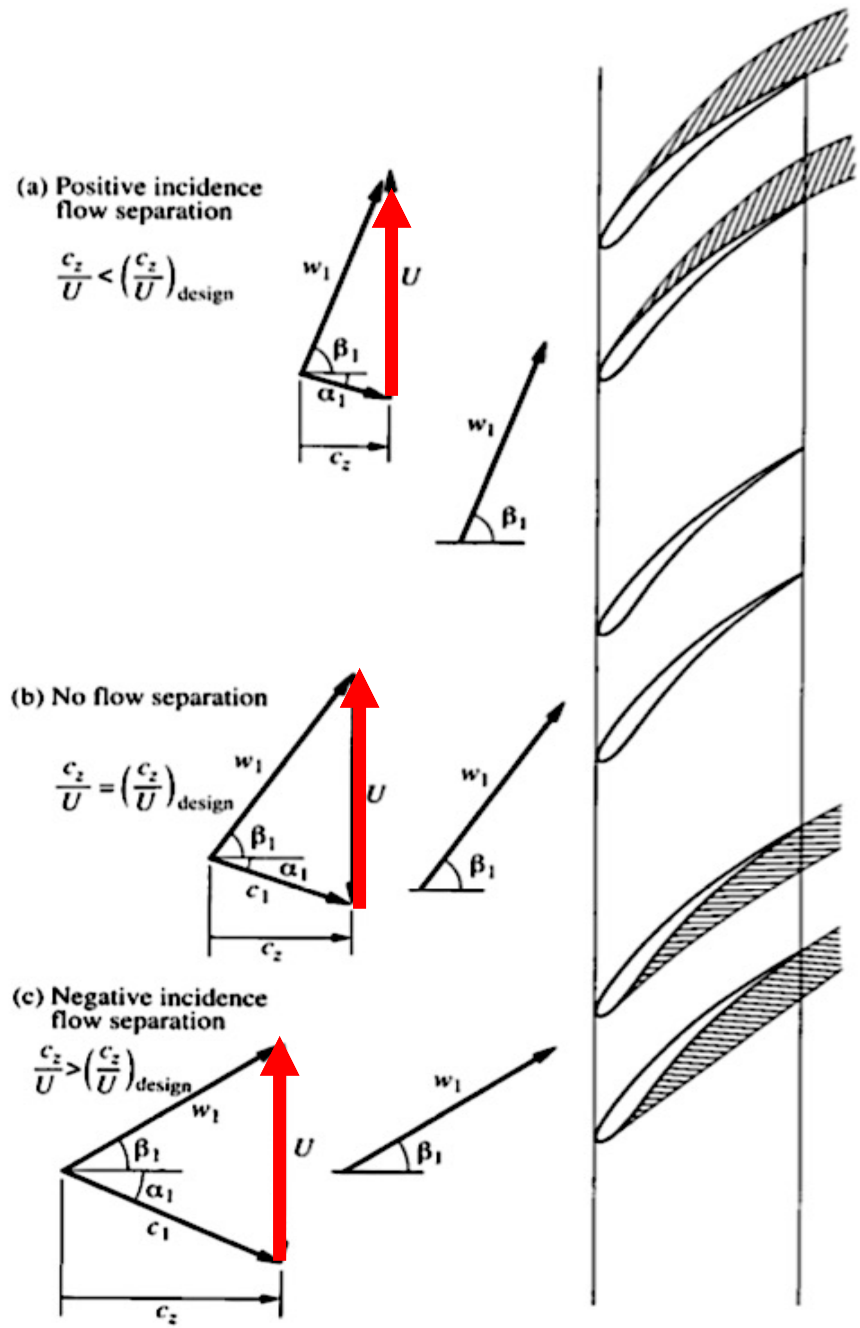
$C_2/C_1 \sim \rho_1 \, V_1^2 / \rho_2 V_2^2 < 1$ (i.e, Chords are smaller downstream)

Spanwise Effects

Rotor at higher than design speed
Flight speed lower than design speed

Rotor and flight speed at design condition

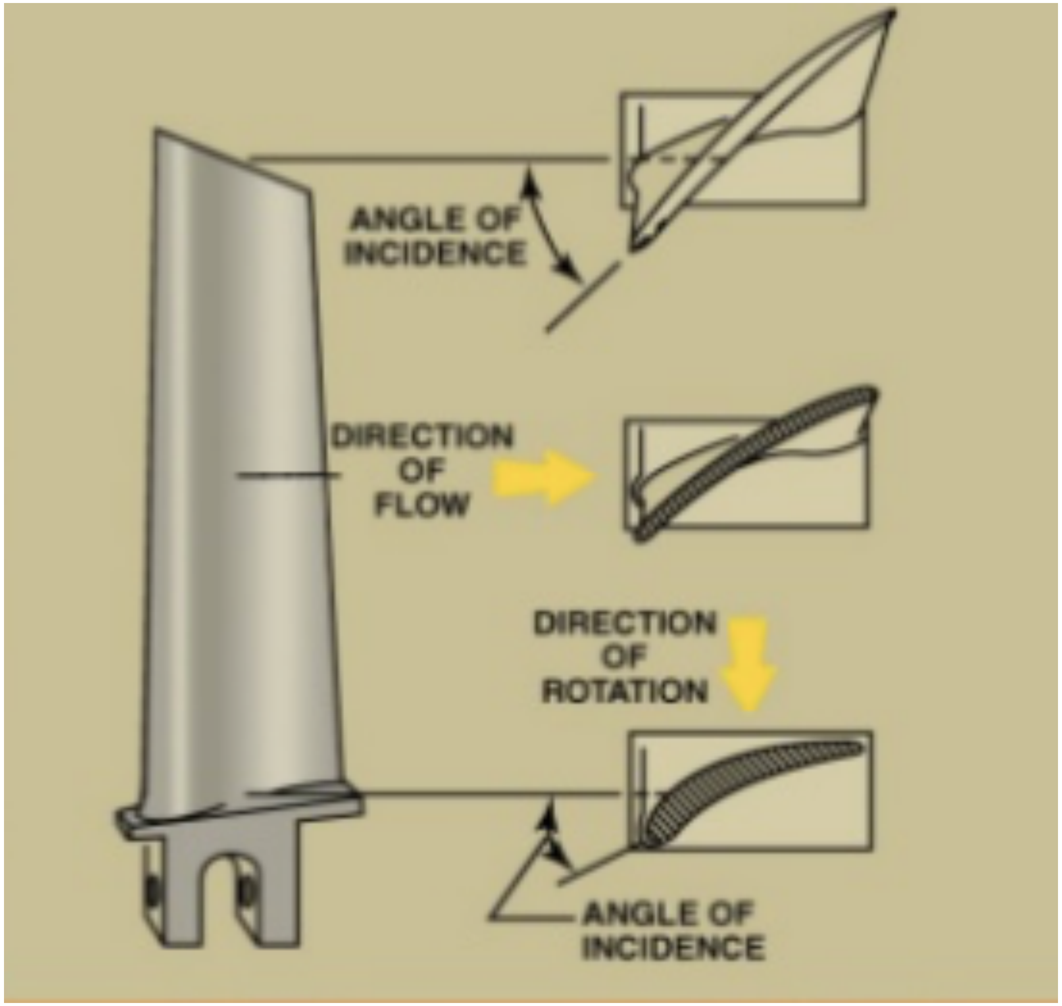
Rotor at lower than design speed
Flight speed higher than design speed



Blade twist

- The rotor blades of a compressor are twisted since tangential velocity varies from the root to the tip.

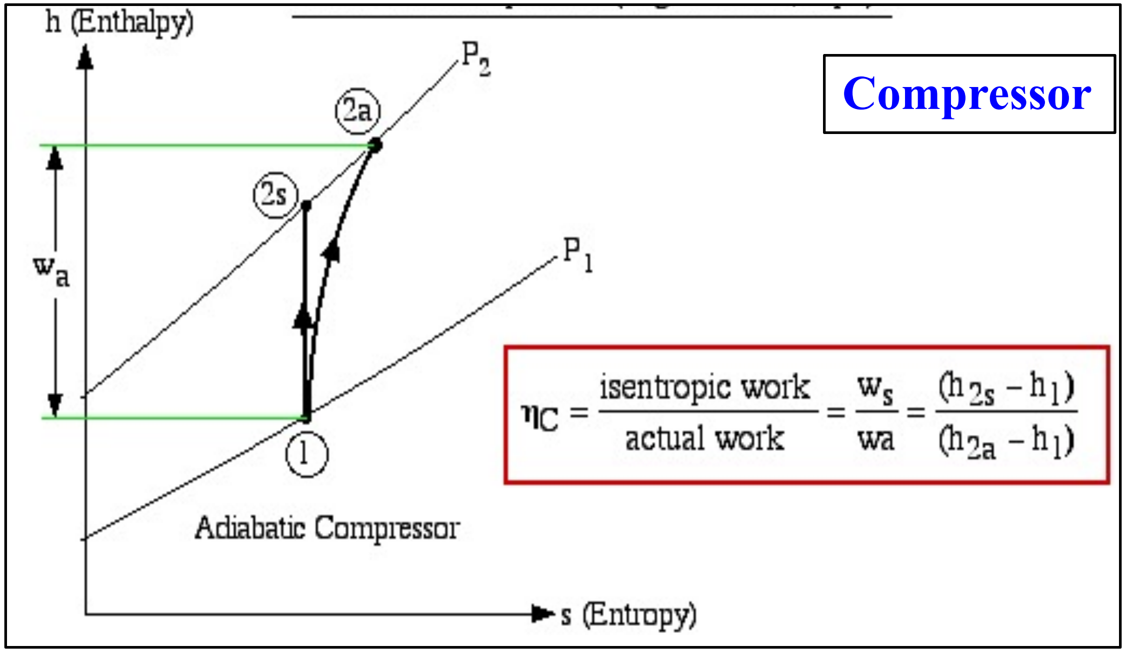
- Also, the blades are thicker at the root compared to the tip
- Provide more structural support near the root
- to reduce centrifugal forces at the tip.
- To reduce aerodynamic drag at the tip.



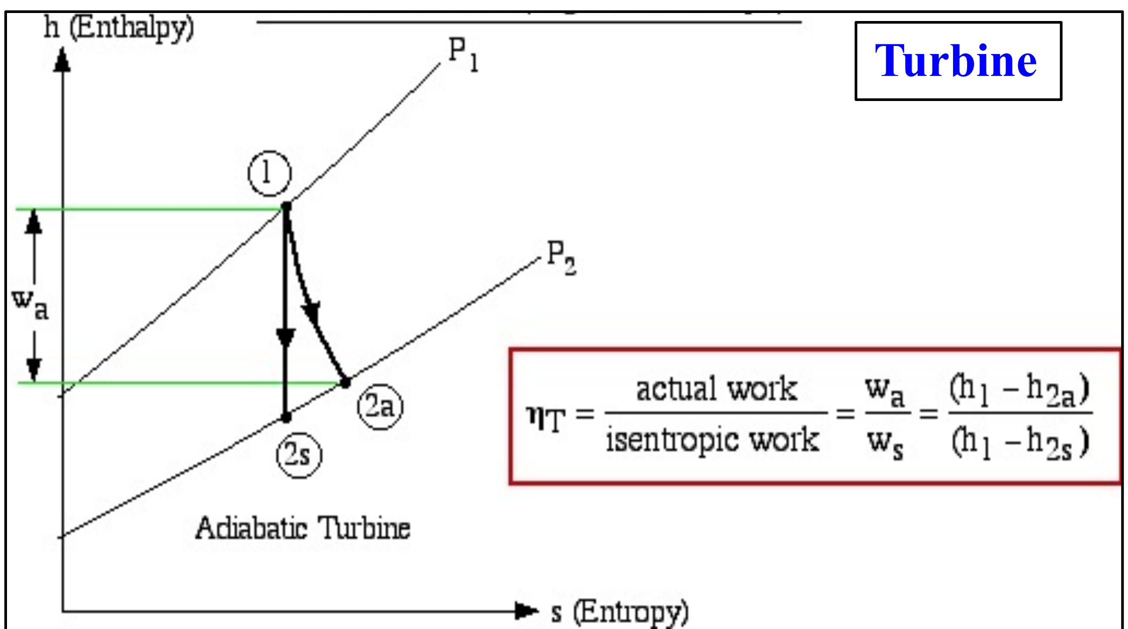
Compressor efficiency definitions

- Adiabatic efficiency, η_c : This is the ratio of the isentropic work to the actual work for the whole compressor.
- Stage efficiency, η_{st} : This is the ratio of the isentropic work to the actual work for a given stage.
- Polytropic efficiency, η_{pc} : This is the ratio of the ideal work to the isentropic work for an infinitesimal compression of the gas (also called small-stage efficiency)

Compressor Adiabatic Efficiency

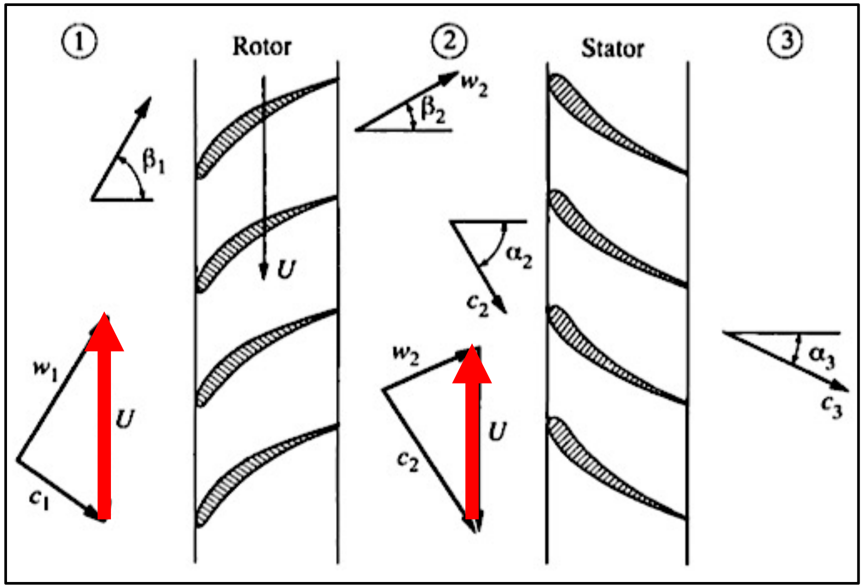


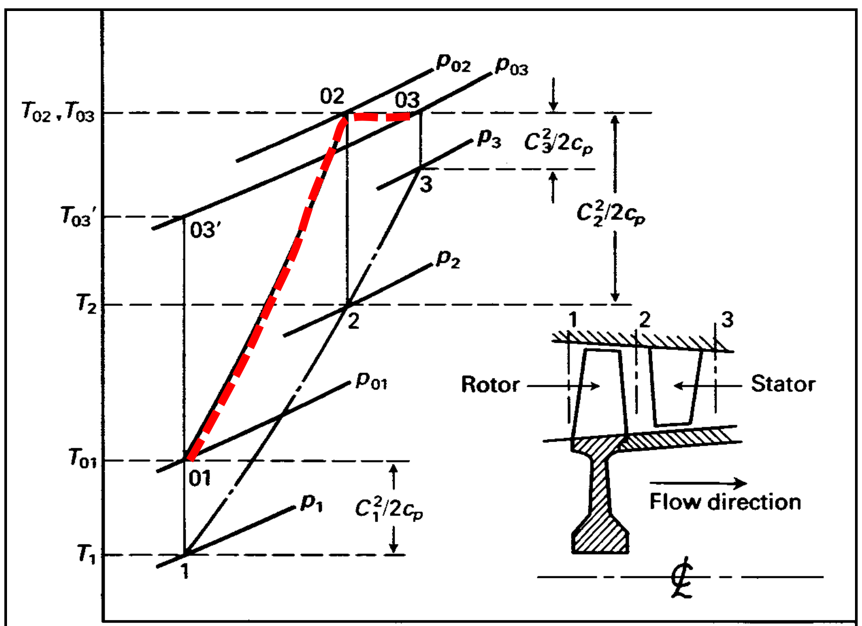
Work is being done to raise the pressure of the gases, i.e., by spinning the compressor



The pressure of the gases is being used to do extract work from the system – i.e., to spin the turbine

Compressor Stage Efficiency





$$\Pr_s = \frac{p_{03}}{p_{01}}$$

$$\eta_{isen} = \eta_{st} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}}$$

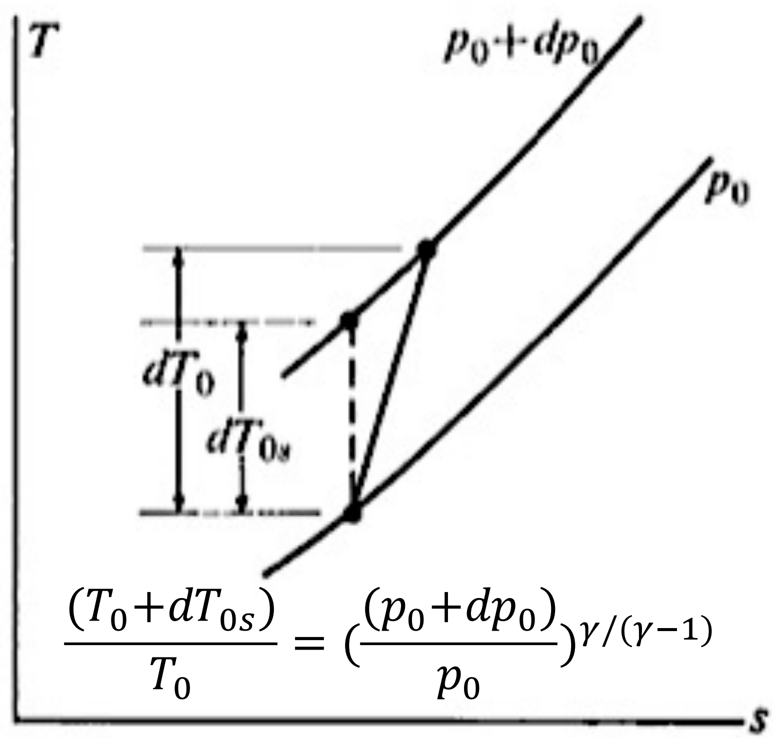
$$\frac{p_{03}}{p_{01}} = \left[1 + \eta_{st}\frac{\Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

- Key questions:
- How do we estimate stage efficiency?
- How do we relate overall adiabatic efficiency to stage efficiency?

Polytropic efficiency

Assume an infinitesimal compression from p0 to p0 +dP0

$$\eta_{pc} = \frac{dT_{0s}}{dT_0}.$$



$$\frac{(T_0+dT_{0s})}{T_0} = (\frac{(p_0+dp_0)}{p_0})^{\gamma/(\gamma-1)}$$

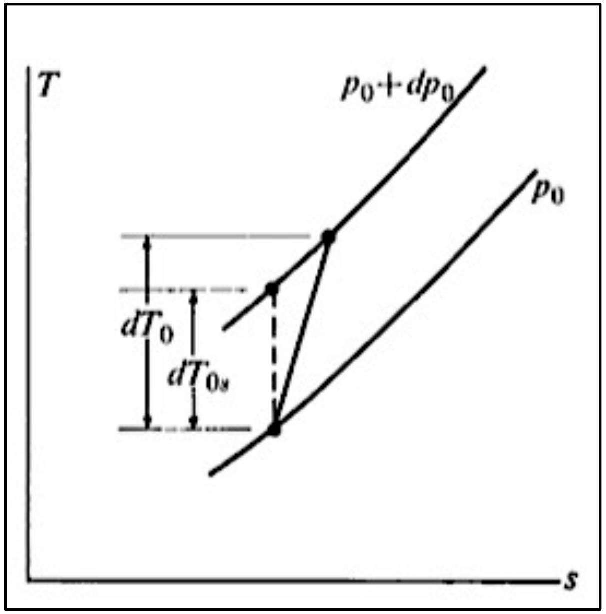
$$dT_{0s} = T_0\left[\left(\frac{p_0 + dp_0}{p_0}\right)^{(\gamma-1)/\gamma} - 1\right]$$

Using a series expansion and keeping only the linear term

$$\left(1 + \frac{dp_0}{p_0}\right)^{(\gamma-1)/\gamma} = 1 + \frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0}$$

$$\eta_{pc}\frac{dT_0}{T_0} = \frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0}$$

Polytropic efficiency



Integrating both sides of the equation in the red box from 01 to 02

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma\eta_{pc}}.$$

$$\eta_c = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{\dfrac{T_{02s}}{T_{01}} - 1}{\dfrac{T_{02}}{T_{01}} - 1} = \frac{\left(\dfrac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1}{\dfrac{T_{02}}{T_{01}} - 1}$$

$$\boxed{\eta_{pc}\frac{dT_0}{T_0} = \frac{\gamma-1}{\gamma}\frac{dp_0}{p_0}}$$

$$\boxed{\eta_c = \frac{\left(\dfrac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1}{\left(\dfrac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma\eta_{pc}} - 1}}$$

Example Calculation

- A 16-stage compressor has an overall pressure ratio of 25, such that the pressure ratio of each stage is the same. The stage efficiency, measured experimentally, is 0.93.

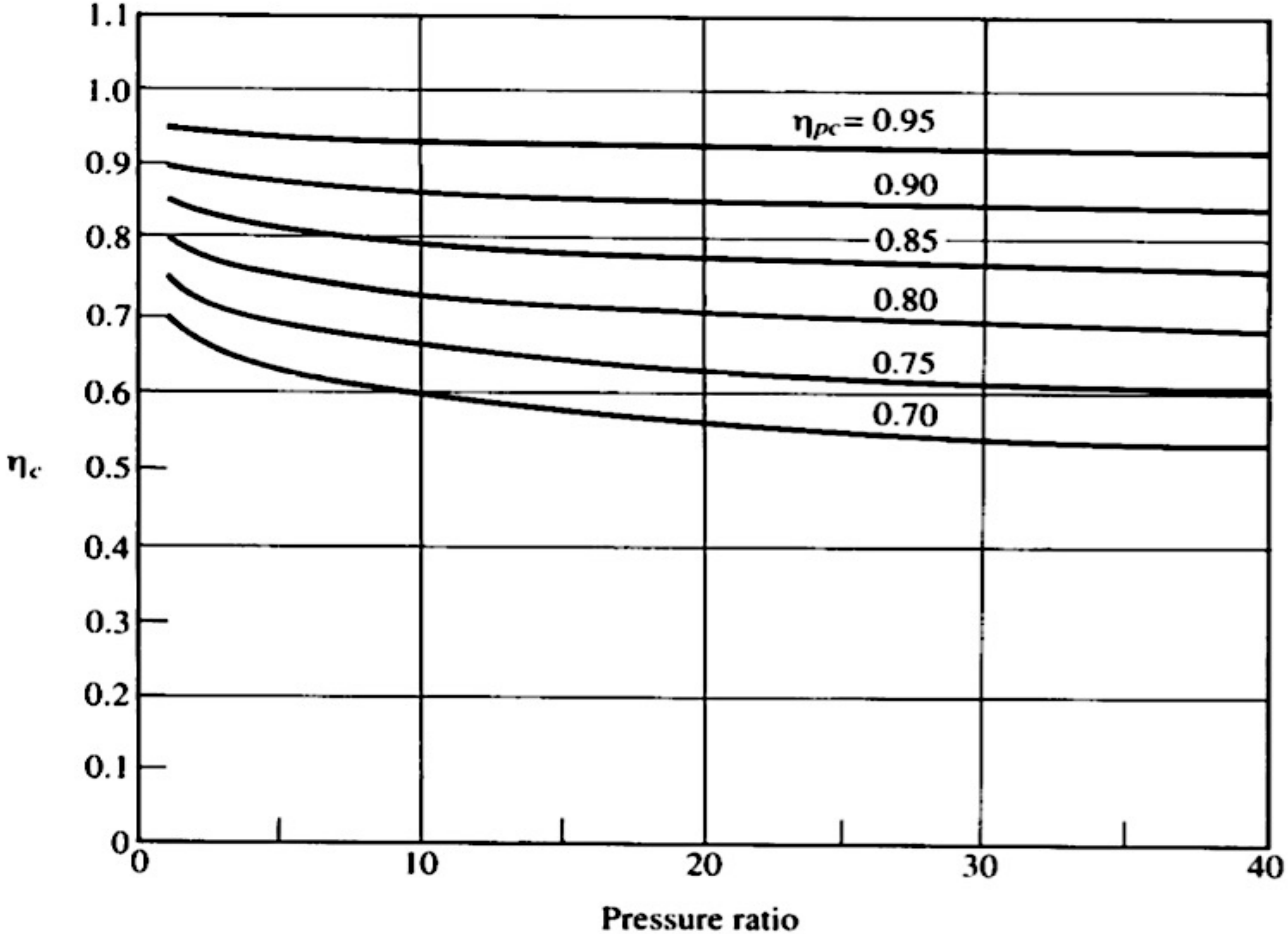
- What is the pressure ratio per stage?

$$\boxed{\begin{array}{l} \textit{Consider } 16-\textit{stage compressor } \pi_c = PR = 25 \\ PR_s = 25^{1/16} = 1.223 \end{array}}$$

- What is the overall adiabatic efficiency of the compressor?

$$\boxed{\eta_{\substack{ad \\ comp}} = \eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma\eta_p}} - 1} = \frac{25^{\frac{1}{3.5}} - 1}{25^{\frac{1}{3.5*0.93}} - 1} = \frac{1.508}{1.688} = 0.893}$$

Effect of pressure ratio(#stages)



- Compressor efficiency drops as you add more stages

Flow Coefficient

Defined as the ratio of the flow velocity to the blade speed

$$\boxed{\phi = \frac{c_z}{U}}$$

- Flow coefficient is low during take-off and high during cruise.
- It determines the flow angles.

Stage (Blade) Loading Coefficient

Defined as the ratio of the stagnation enthalpy change <u>across a stage</u> to the square of the blade speed:

$$\Psi = (h_{02}-h_{01})/U^2 = U(c_{\theta 2} - c_{\theta 1}) / U^2 = \Delta c_\theta / U$$

(where) $$h_{02} - h_{01} = U(c_{\theta 2} - c_{\theta 1})$$

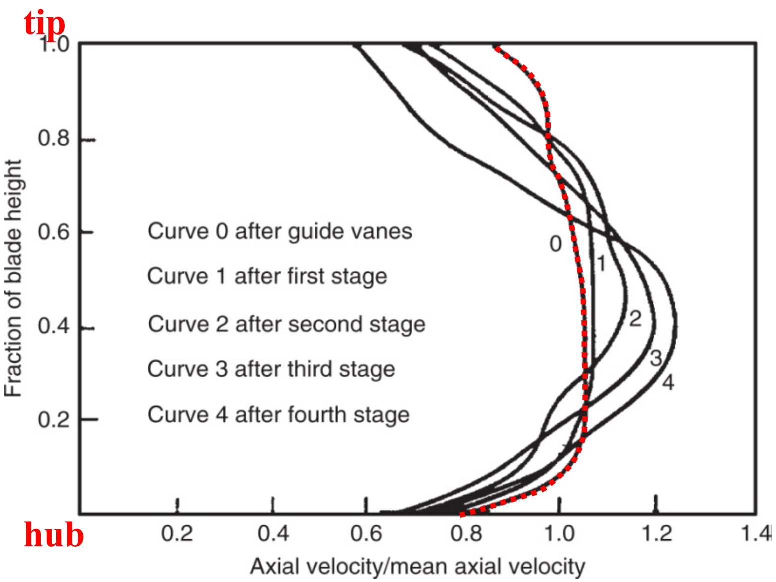
(From Slide 15)

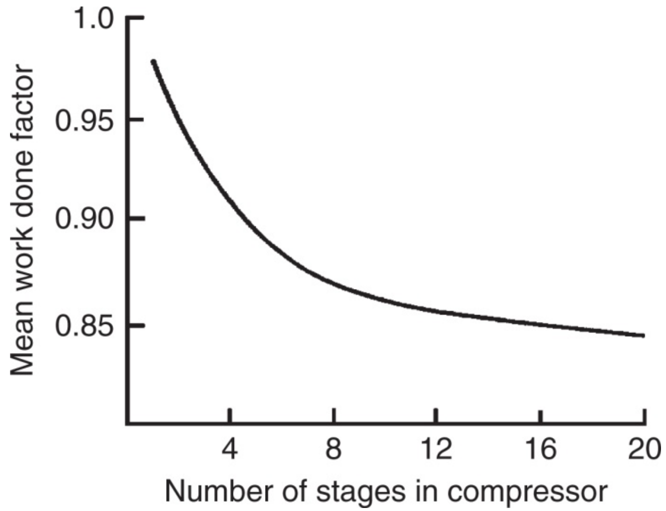
Empirical work factor

$$\psi_{ideal} = \phi(\tan\beta_1 - \tan\beta_2)$$

$$\psi_{actual} = \lambda\phi(\tan\beta_1 - \tan\beta_2)$$

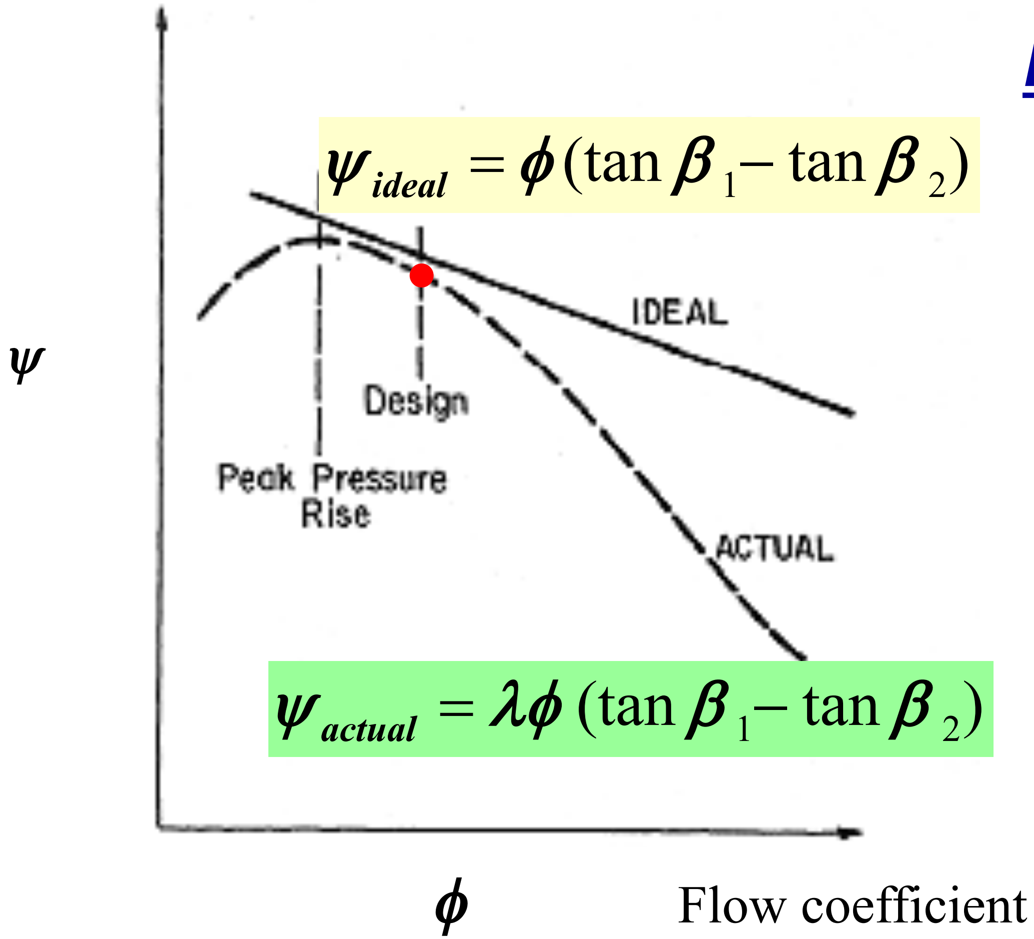
- λ = "work done factor" or just "work factor"
- lumps all losses into an empirical coefficient
- accounts for actual effects in a simple way





Loading Coefficient vs Flow Coefficient

(actual vs. ideal performance)



$$\psi_{ideal} = \phi(\tan\beta_1 - \tan\beta_2)$$

$$\psi_{actual} = \lambda\phi(\tan\beta_1 - \tan\beta_2)$$

Viscous Effects &
<u>BL growth result in</u>

- ***Non-uniform flow***
- ***turbulence***
- ***more mixing losses***
- ***3-D effects***
- ***stg-to-stg influence***
- ***end and hub effects***

(i.e., fluid kinematics issues)

Degree of Reaction of a Stage

Defined as the ratio of the change in <u>*static*</u> enthalpy through the rotor to the stage work (h_{o3} - h_{o1}).

$$R' = \frac{\Delta h_{rotor}}{\Delta h_{0\,stage}} = \frac{\Delta h_{rotor}}{w_{stage}} \approx \frac{\Delta p/\rho}{w_{stage}}$$

(Using)

$$T ds = 0 = dh - \frac{dp}{\rho}$$

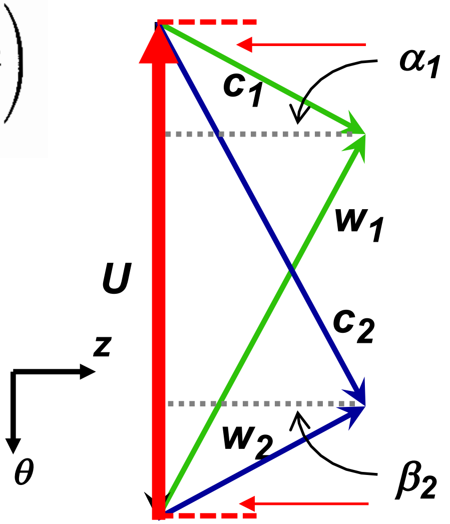
where: $w_{stage} = \lambda U_m \Delta c_\theta$

$$R = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{w_1^2 - w_2^2}{2U(c_{\theta 2} - c_{\theta 1})}$$

$$R = \frac{1}{2} - \frac{c_z}{U}\left(\frac{\tan\alpha_1 + \tan\beta_2}{2}\right) = -\frac{c_z}{U}\left(\frac{\tan\beta_1 + \tan\beta_2}{2}\right)$$

<u>R' = 0.5 (50%)</u>

- velocity diagram is symmetric ($\alpha_1 = \beta_2$)
- stage Δh and Δp equally distributed between rotor and stator
- Then stator and rotor blade shapes are similar and likelihood of BL separation is equally minimized in both the rotor and stator.



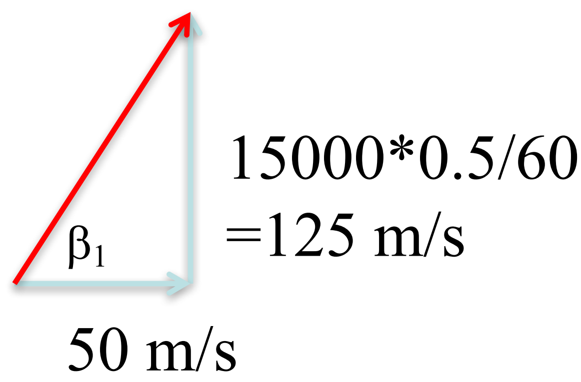
Note: all angles measured relative to axial direction

<u>*For a typical axial compressor, $R' \approx$ 0.4-0.6*</u>

Example Problem

- Calculate the stagnation pressure ratio across a stage at takeoff and cruise conditions of an aircraft as described below. The diameter of the engine is 2m and the hub is 1m.
- Also, if the engine has 25 stages, calculate the total pressure ratio of the engine, assuming equal pressure ratio in every stage. Work factor, $\lambda = 0.9$ and $\eta_{st} = 0.95$. Assume $\beta_2 = \beta_1 - 20$ degrees and $\alpha_1 = 0$.

- Takeoff
  - speed: 50 m/s
  - Engine RPM: 15000
  - Temperature 300K
- Cruise
  - Speed: 200 m/s
  - Engine RPM: 12000
  - Temperature = 270K



$$\boxed{\phi = \frac{c_z}{U}}$$

$$\psi_{ideal} = \phi(\tan\beta_1 - \tan\beta_2)$$

$\beta_1 = \tan^{-1}(125/50) = 68^o$

$\beta_2 = \beta_1 - 20 = 48^o$

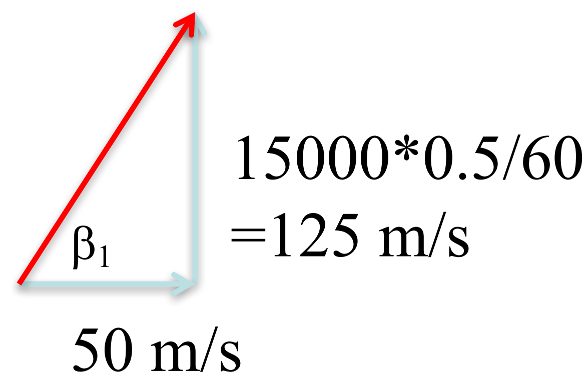
$\mathrm{Tan}(\beta_1)-\mathrm{Tan}(\beta_2) = 2.5-1.11 = 1.39$

$\phi = 50/125 = 0.4$ (flow coefficient)

$\Psi actual = 0.9 * 0.4 * 1.39 = 0.5$ (actual blade loading coefficient)

Example Problem

- Calculate the stagnation pressure ratio across a stage at <u>takeoff</u> and <u>cruise</u> conditions of an aircraft as described below. The diameter of the engine is 2m and the hub is 1m.
- Also, if the engine has 25 stages, calculate the total pressure ratio of the engine, assuming equal pressure rise in every stage. Work factor, λ = 0.9 and η_{st} = 0.95. Assume $\beta_2 = \beta_1$-20 degrees and α_1=0.
- Takeoff
  - speed: 50 m/s
  - Engine RPM: 15000
  - Temperature: 300K



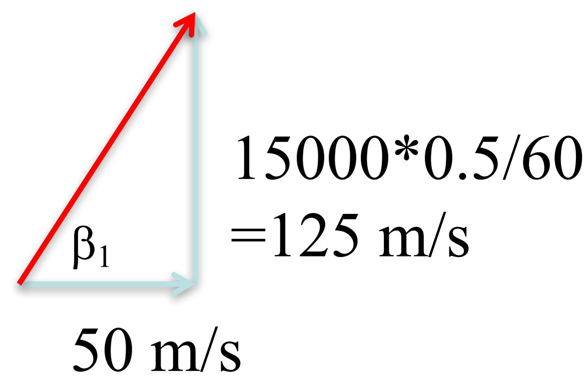
$$\Psi_{actual} = (h_{02}-h_{01})/U^2$$

$$\Delta h_0 = 0.5*125^2 = 7818$$
$$\Delta T_0 = 7818/1005 = 7.78K$$

$$M1 = 50/(sqrt(1.4*287*300) = 0.144$$
$$T01 = 300*(1+0.5*(1.4-1)*0.144^2) = 301K$$
$$T02 = T03 = 301+7.78 = 308.78K$$

Example Problem

- Calculate the stagnation pressure ratio across a stage at takeoff and cruise conditions of an aircraft as described below. The diameter of the engine is 2m and the hub is 1m.
- Also, if the engine has 25 stages, calculate the total pressure ratio of the engine, assuming equal pressure rise in every stage. Work factor, λ = 0.9 and η_{st} = 0.95. Assume $\beta_2 = \beta_1$-20 degrees and α_1=0.
- Takeoff
  - speed: 50 m/s
  - Engine RPM: 15000
  - Temperature: 300K



$$\mathrm{Pr}_s = \frac{p_{03}}{p_{01}}$$

$$\eta_{isen} = \eta_{st} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}}$$

$$\frac{p_{03}}{p_{01}} = \left[1 + \eta_{st}\frac{\Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

T03/T01=308.78/301= 1.026

$$p03/p01 = (1+0.95*7.78/301)^{3.5} = 1.089$$

$$\mathrm{Pr} = \mathrm{Pr}_s^{\,n} = (1.089)^{25} = 8.35$$