Homework # 1

1. Recall from thermodynamics that $c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p$, $c_v \equiv \left(\frac{\partial e}{\partial T}\right)_v$ and $c_p - c_v = R$ so $c_p > c_v$. Use Gibbs' equation $(Tds = dh - vdp = de + pdv)$ to derive expressions for the slope of constant pressure and constant volume lines on a *T-s* diagram and prove that the slope of the constant volume line through a point is greater than the slope of a constant pressure line through the same point.

Answer:

Need: (a) Expressions for constant *p* and *v* lines on *T-s* diagram
(b) Prove that slope of constant *v* line exceeds that of constant *p* line.

Given: Reminders of thermodynamics; Gibbs' equation $(Tds = dh - vdp = de + pdv)$

Solution:

The slope of the constant property line on a *T-s* diagram is $\left(\frac{\partial T}{\partial s}\right)_x$ where *x* is the property – either *p* or *v* in this case.

We can use Gibbs' equation to obtain these. From Gibbs' equation:

Assumptions:
(1) Simple compressible substance, i.e. *p-v* work is the only mode of work transfer. This is implicit in the stated form of Gibbs' equation.

$Tds = dh - vdp \Rightarrow ds = \frac{1}{T}dh - \frac{v}{T}dp \Rightarrow \left(\frac{\partial s}{\partial T}\right)_p = \frac{1}{T}\left(\frac{\partial h}{\partial T}\right)_p - 0$, where the last term vanishes for a constant pressure process. Inserting the definition $c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p$ and taking the reciprocal we get

$\boxed{\left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p}}$.

Similarly $Tds = de + pdv \Rightarrow ds = \frac{1}{T}de + \frac{p}{T}dv \Rightarrow \left(\frac{\partial s}{\partial T}\right)_v = \frac{1}{T}\left(\frac{\partial e}{\partial T}\right)_v = \frac{c_v}{T}$, using $c_v \equiv \left(\frac{\partial e}{\partial T}\right)_v$.

Finally $\boxed{\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v}}$.

Because $c_p > c_v$ we see that $\left(\frac{\partial T}{\partial s}\right)_v > \left(\frac{\partial T}{\partial s}\right)_p$, i.e. the slope of the constant *v* line exceeds that of constant *p* line.

2. An aircraft engine draws in 370kg/s of air. The fuel ejectors in its combustion chambers pump 30kg/s of Kerosene. Kerosene combusts according to the equation,

$$C_{12}H_{26} + 18.5\ O_2 + 74\ N_2 = 12\ CO_2 + 13\ H_2O + 74\ N_2$$

Answers

a) Is the fuel-air ratio stoichiometric? Is it rich or lean?

Using the stoichiometric equation above, for complete combustion, the mass of incoming fuel and air are as follows:

Kerosene: 12x12+26x1= 170gms.
Air: 18.5x32+74*28= 2664gms.

Resulting in a stoichiometric fuel/air ratio of 0.0638.

For 370kg/s of air coming in to the engine, the stoichiometric fuel injection needs to be 0.0638x370=23.6 Kg/s. Therefore the fuel-air mixture is not in a stoichiometric ratio. It is rich, since more fuel than required is being injected.

b) Calculate the mole fractions of the combustion products for complete, stoichiometric combustion.

The mole fractions of the combustion products are calculated as follows:
Total number of moles = 12+13+74 = 99
Mole fraction of CO2=12/99
Mole fraction of H2O = 13/99
Mole fraction of N2 = 74/99

c) Assuming the combustion chamber has a uniform pressure of 50MPa, what are the partial pressures of the combustion gases?
The partial pressures of the combustion gases are in the proportion of the mole fractions, so,
PP of CO2= 12/99*50 = 6.06 MPa
PP of H2O = 6.56 MPa
PP of N2 = 37.37 MPa

3. For problem 4, assuming a fuel input for stoichiometric combustion, and assuming that the pressure at the exit of the engine is equalized to the ambient pressure, compute the thrust of the engine. The engine sees an incoming speed of M=0.8. Speed of sound is 340m/s. Kerosene produces has a heat of combustion (energy output) of 45MJ/kg. Assume 100% energy conversion from chemical energy to kinetic energy of air.

Answer:

For this problem, assumptions of 1-D mass, momemtum and energy conservation are made.
Under the assumption of pexit = pambient
Thrust if given by the equation (from conservation of momentum)

Thrust = mass flow x velocity (exit) – mass flow x velocity (inlet)

From conservation of mass,

Mass flow at exit = Mass flow of air + mass flow of fuel = 370 +30 = 400 Kg/s

From conservation of energy,

Energy (exit) = Energy (inlet) + Energy (input)

0.5 * 400 * Vexit^2 = 0.5 * 370 * (0.8 * 340)^2 + 45 *1e6 * 30
Vexit^2 = (13.687*1e6+ 1350*1e6)*2/400
Vexit = 2611 m/s

Thrust = 400*2611-370*340*0.8= 941kN

4. A blunt-nosed vehicle is flying in air at Mach 5, at an altitude where density is 0.1 kg/cu.m, temperature is 180K. Assuming a detached normal shock, compute the stagnation pressure at the nose of the vehicle.

Given the static density and temperature indicated, assuming an ideal gas, and gas constant R for air of 287[J/kg-K], we get a static pressure of 5166[Pa].

According to the NS relations, for Mach 5, the pressure ratio across the shock is 29.00, so the pressure on the other side of the shock is 149,814[Pa], and the new Mach number is 0.4152.

To find the pressure at the nose of the body, we can assume that the fluid comes to rest approximately isentropically. For our Mach number of 0.4152, we get a static to stagnation pressure ratio of about 0.89. Thus, our stagnation pressure (the pressure at the nose of the vehicle) is 168,330[Pa].