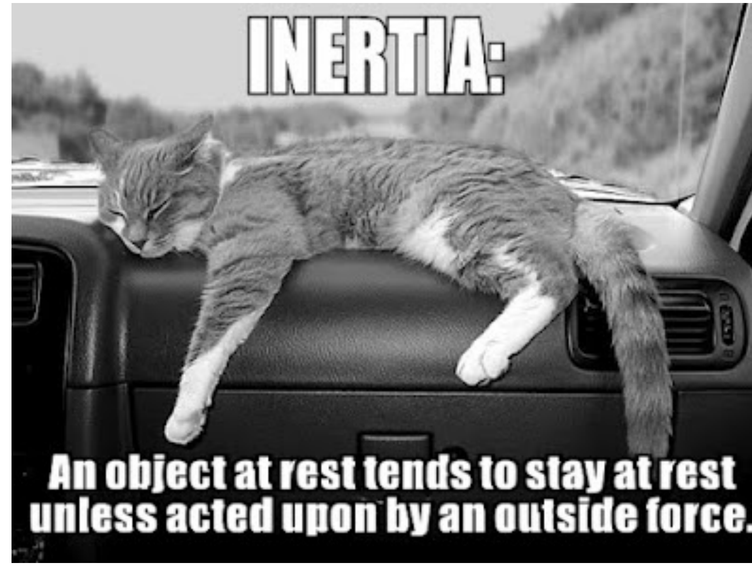
Electromechanical Systems
ASE 375

Lecture 16/17: Frequency Response, Shakers

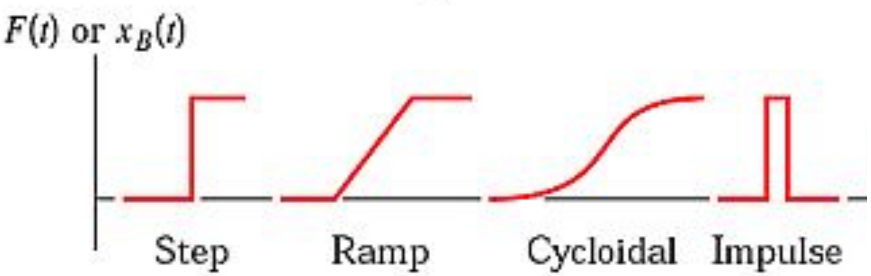
Input and Response

- What is Inertia?
  - Inertia is the behavior of a system which makes it want to stay the way it is currently
  - Newton's first law: A body will continue to stay in a state of rest or motion unless acted on by an external force



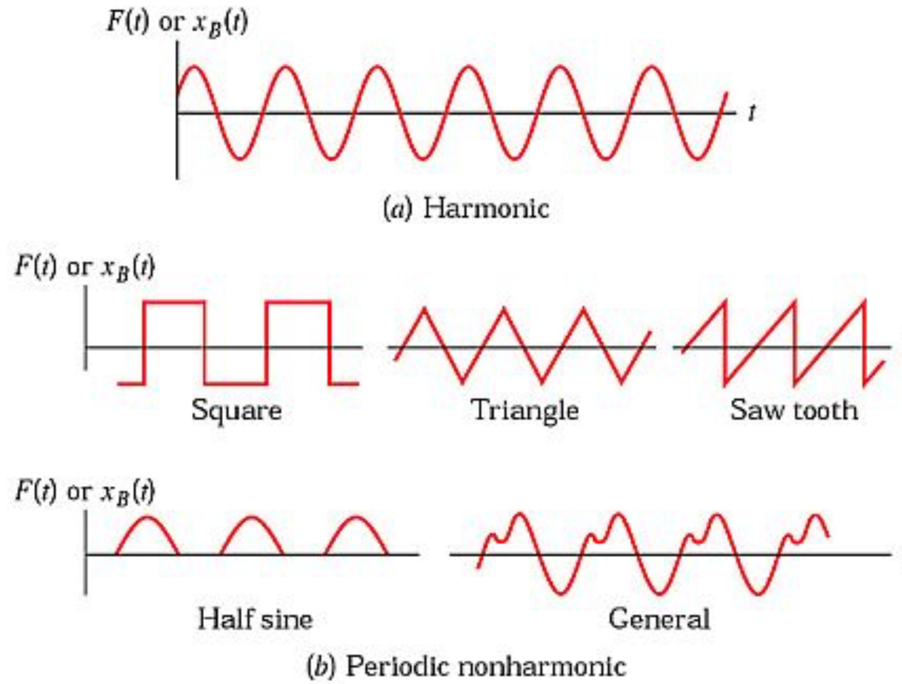
- The external force is the "input" to the system
  - Input could be steady (constant thrust from an engine to maintain steady level flight) or unsteady (gust of wind)
- The way the system behaves to the input is called its "response"
  - Response can be steady or unsteady, and does not need to mirror the input

Impulse Input vs Periodic Input



(c) Nonperiodic

- Sudden gust of wind
- Single tap of a piano key
- Hitting a baseball

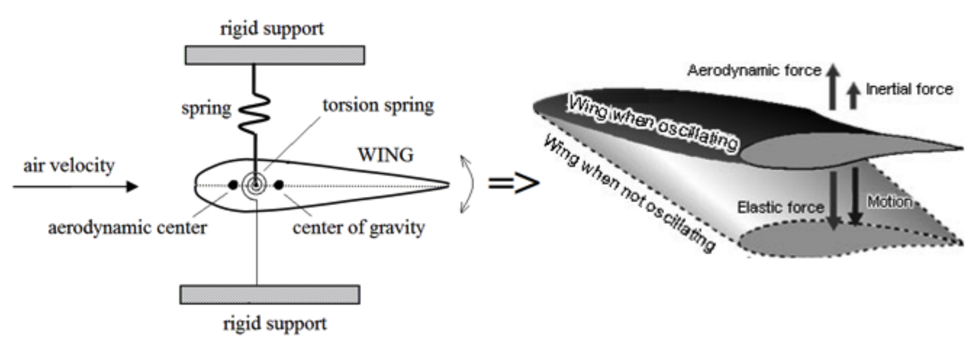


- Waves in the ocean
- Pushing someone on a swing
- Sun’s energy on a solar panel

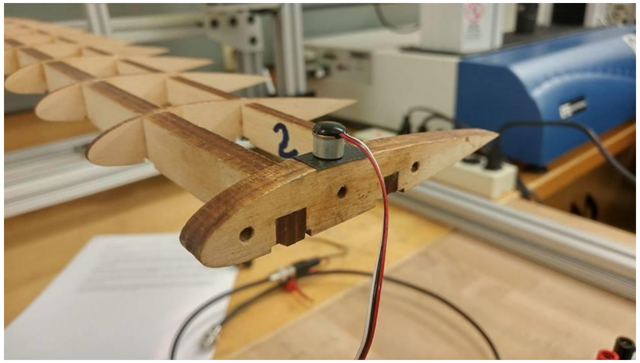
Examples of unsteady input (forcing)

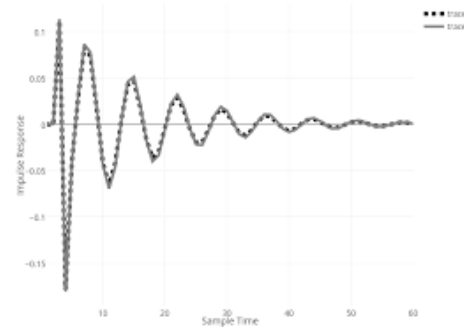
- There are many examples for unsteady forcing functions
  - Airflow passing over an aircraft tail – unsteady eddies from wing create unsteady pressure forces causing flutter
  - Aircraft engine vibrations (due to unsymmetrical loads)
  - Oncoming turbulent wind against a building – flow separation behind buildings causes unsteady pressures on buildings
  - Rocket vehicle liftoff vibration – engine thrust creates structural vibrations
  - Earthquake excitation of a building
- The response to these vibrations could result in mechanical and electrical failure

Wing/Tail Flutter



Lab 6: Impulse Response





In this lab, you will explore the dynamic response of a built-up wing. You measured the static response of this wing in Lab 3.

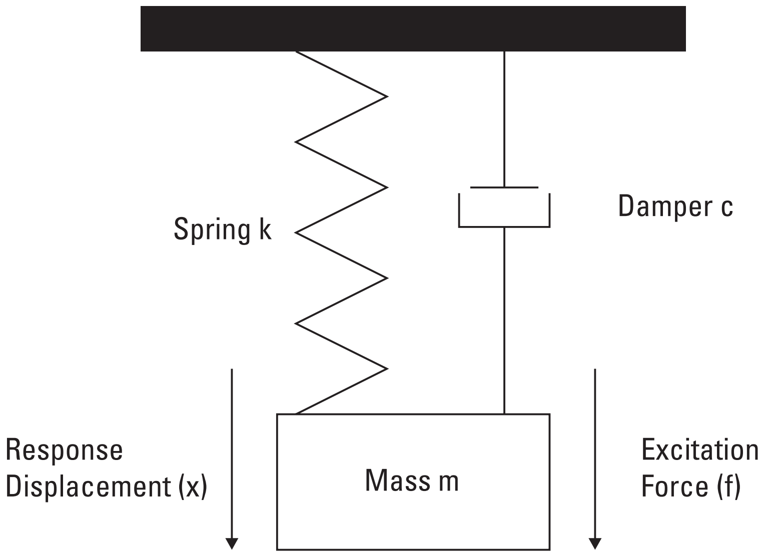
1. Attach the piezoelectric accelerometer (IMI 660) to the tip of the wing (use a small piece of wax). Gently tap the wing and record the output of the accelerometer over 3 seconds. Use a sampling frequency of 1 kHz. Do this for several taps so that you can average the measurements.

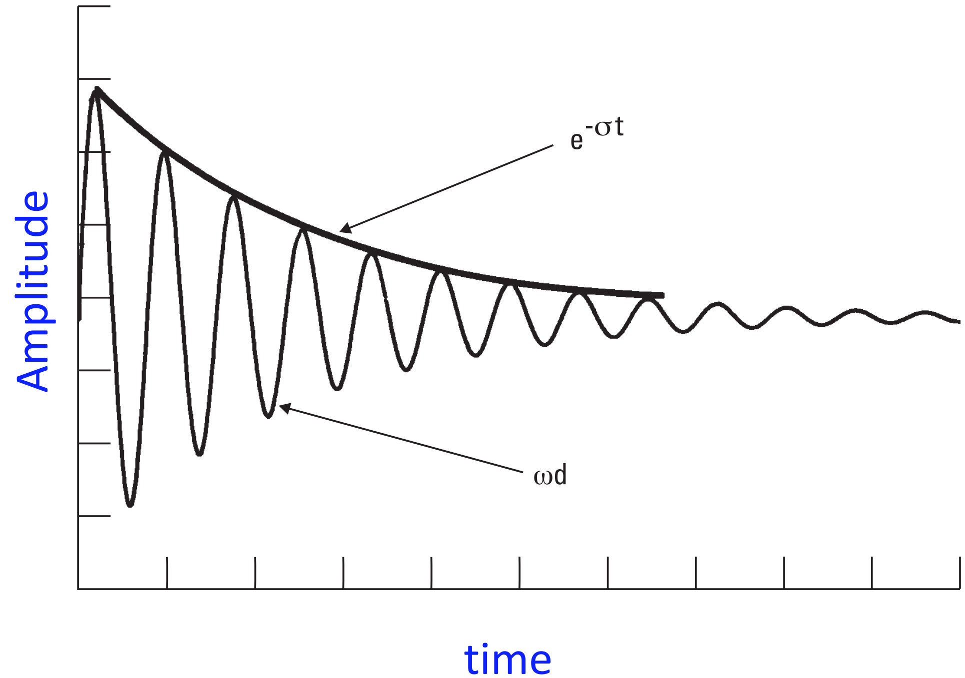
2. Repeat the above experiment using the MEMS accelerometer (MMA 7361L). In this case, tap the wing at an angle so that you excite both the out-of-plane and in-plane bending vibration. Measure the accelerations in both these directions.

3. Plot the measured accelerations and compare the two sensors. Use the acceleration to calculate the tip displacement as a function of time. Describe the errors inherent in these measurements.

4. Plot the power spectrum of the measured acceleration using (a) 3 seconds of data (b) the last 1 second of data. Identify the natural frequencies of the wing. What are the differences between the power spectrum of (a) and (b) ?

Single Degree of Freedom System



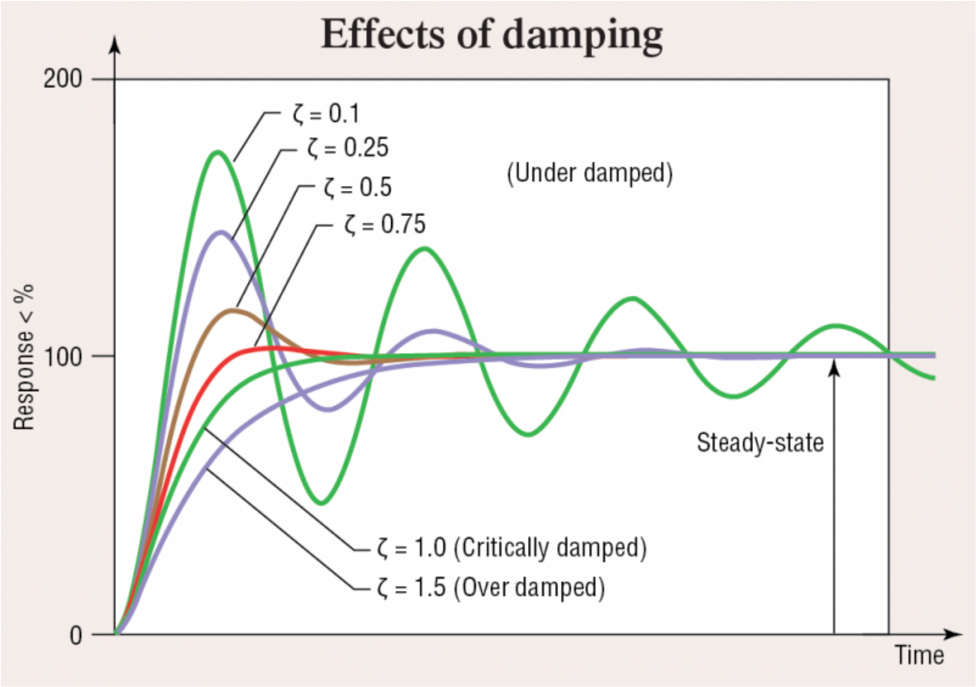


$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\omega_n^2 = \frac{k}{m}, \quad 2\zeta\omega_n = \frac{c}{m} \quad \text{or } \zeta = \frac{c}{\sqrt{2km}} \qquad \omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

- Numerous real world systems can be modeled using a spring-mass-damper model
- A one spring, one mass, one damper system has one degree of freedom (displacement x) and a natural resonance
- The impulse response of the system looks at the decay of displacement with time for an impulse input
- A system could be over-damped or underdamped, depending on the value of zeta (ζ)

Impulse Response - Single Degree of Freedom System

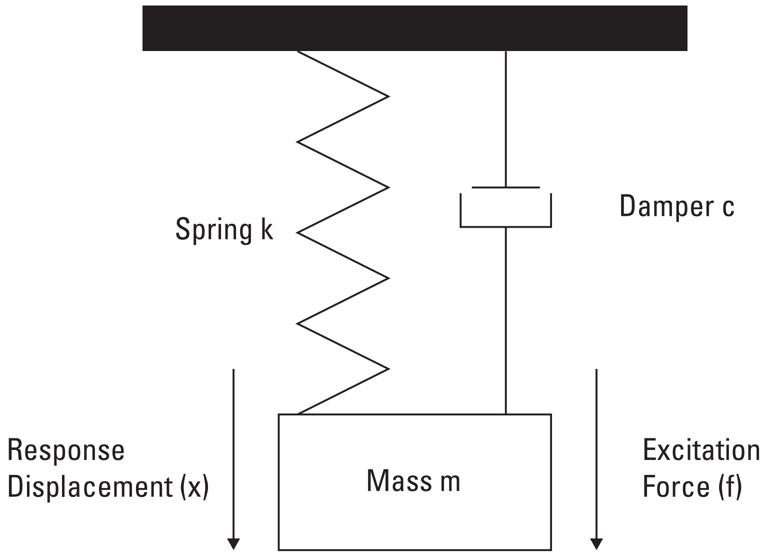


$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\omega_n^2 = \frac{k}{m}, \quad 2\zeta\omega_n = \frac{c}{m} \quad \text{or } \zeta = \frac{c}{\sqrt{2km}}$$

- The key responses are of two-types, under-damped or over-damped
- For zeta ($\zeta < 1$), the system is underdamped and reaches steady state via oscillations
- For zeta ($\zeta >= 1$), the system is over-damped and reaches steady state without oscillations
- Common example: Swing doors vs dashpot doors

Single Degree of Freedom System - Example



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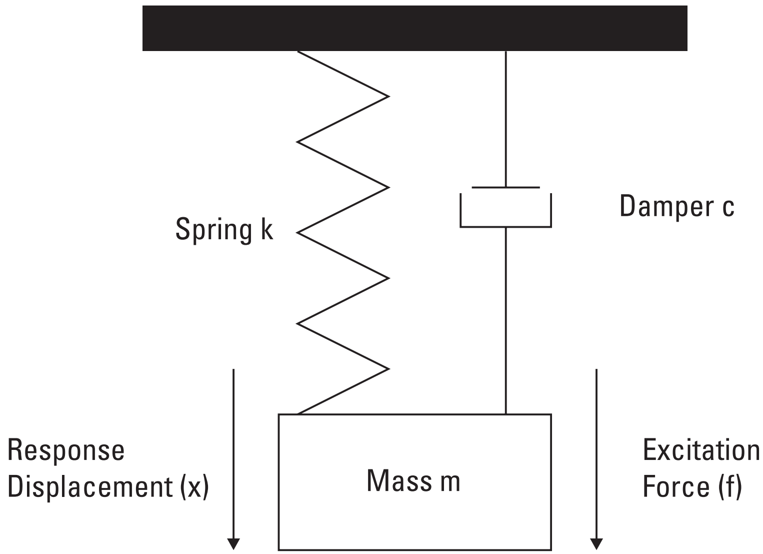
- A tow truck can carry vehicles up to a mass of 5000kg. The damper on the flatbed has to be designed to prevent oscillations of the flatbed when the truck hits a pothole (which leads to fatigue failure). Assuming this can be analyzed as a SDOF system, what must the relationship between stiffness and damping be to prevent oscillations?
- For the same problem, a designer mistaken uses a damping coefficient equal to 1/5th of the prescribed value at critical damping, where the stiffness. What is the damped natural frequency of the system, if the stiffness of the springs is 500kN/m?

Single Degree of Freedom System - Example



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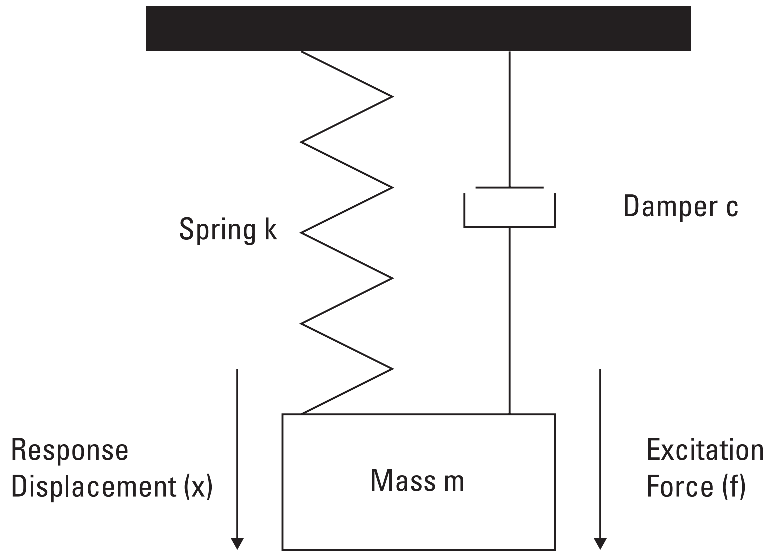
$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

- We need, $\zeta \geq 1$
- $c/\sqrt{2km} \geq 1$
- $\frac{c^2}{2km} \geq 1$
- $C^2 \geq 2k * 5000$
- $C^2 \geq 10000k \text{ or } c \geq 100\sqrt{k}$

Single Degree of Freedom System - Example



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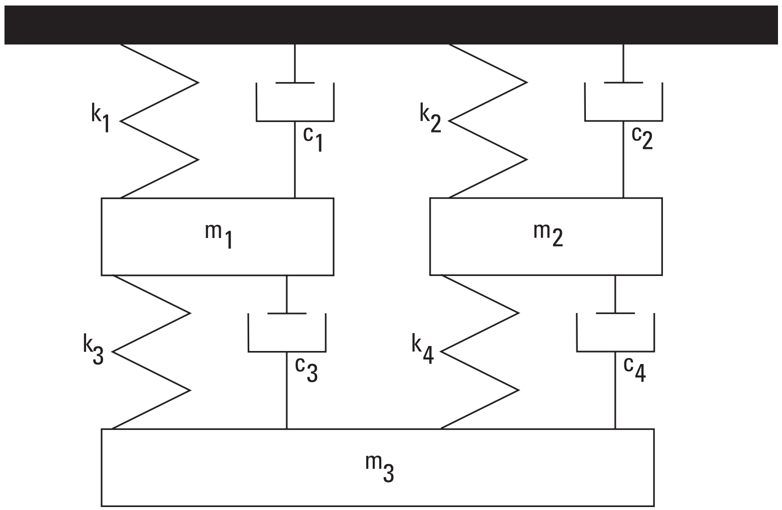
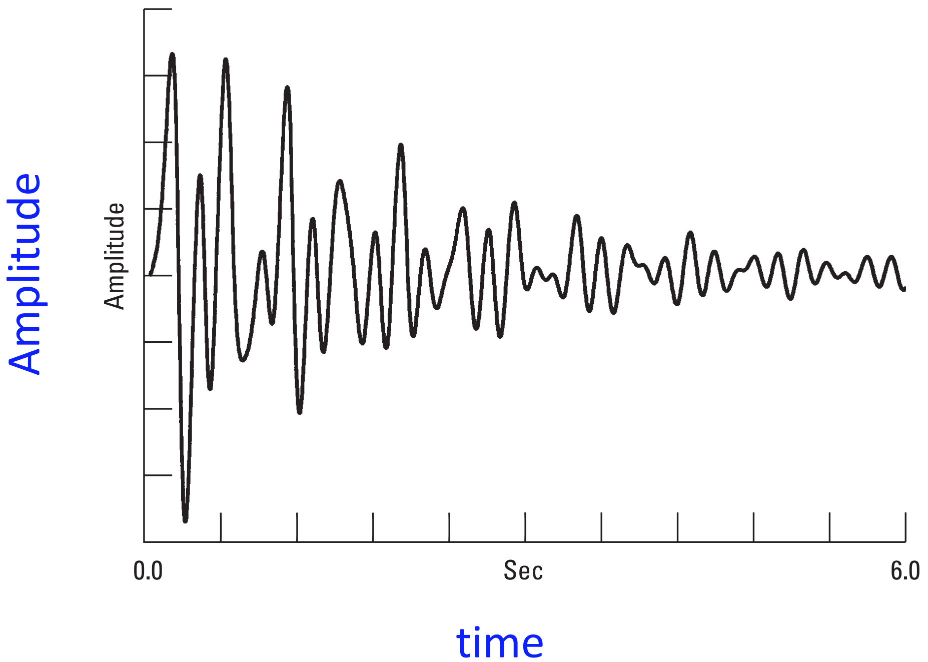
$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$$

- For a stiffness of k = 500kN/m, prescribed critical damping coefficient,
- $c = 100 \ * \sqrt{500000}$ =70710 Ns/m
- *c* chosen by designer is 14142 Ns/m
- $\zeta = 14142/\sqrt{2 * 500000 * 5000}$= 0.5
- $\omega_d = \omega_n * \sqrt{1-\zeta^2} = \sqrt{\frac{k}{m}} * \sqrt{1-\zeta^2}$ = $\sqrt{\frac{500000}{5000} * (1-0.25)}$ = 8.7Hz

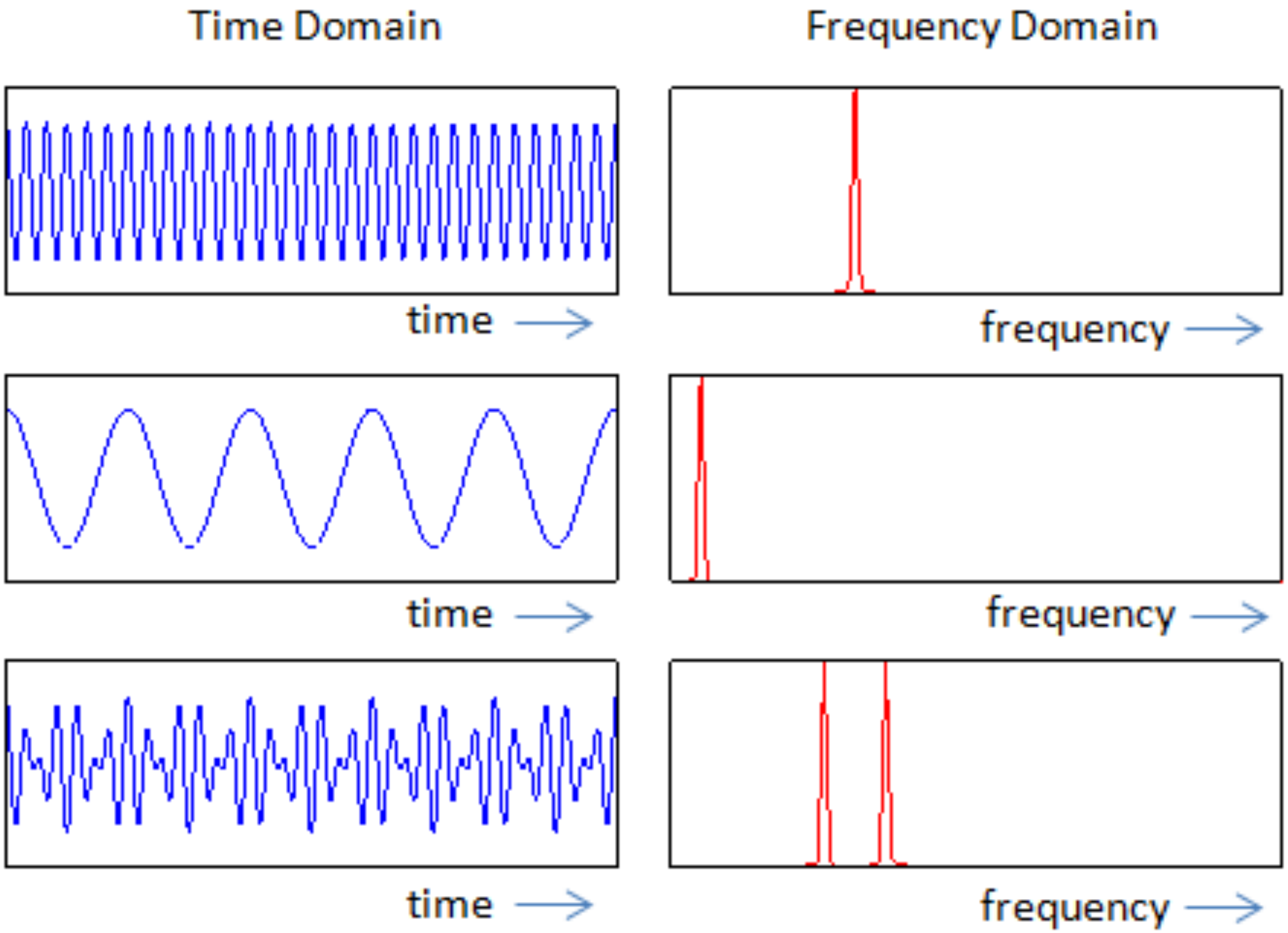
Impulse Response - Multi Degree of Freedom System





- A multi spring, mass, damper system has more than one degree of freedom and multiple resonances
- The impulse response of the system also has a decay characteristic, but it is not easy to isolate the decay of different frequencies in this system
- This is where frequency response becomes really useful – most real world systems are multi-degree of freedom systems

Time vs Frequency Domain



- The frequency content of a signal is as important as its magnitude
- The frequency content of a physical phenomenon show the key features in that phenomenon
- Fourier transforms are used to convert from one domain to another

Fourier Transform

- We want to understand the frequency content of our signal. So, let's reparametrize the signal by ω instead of *x*:

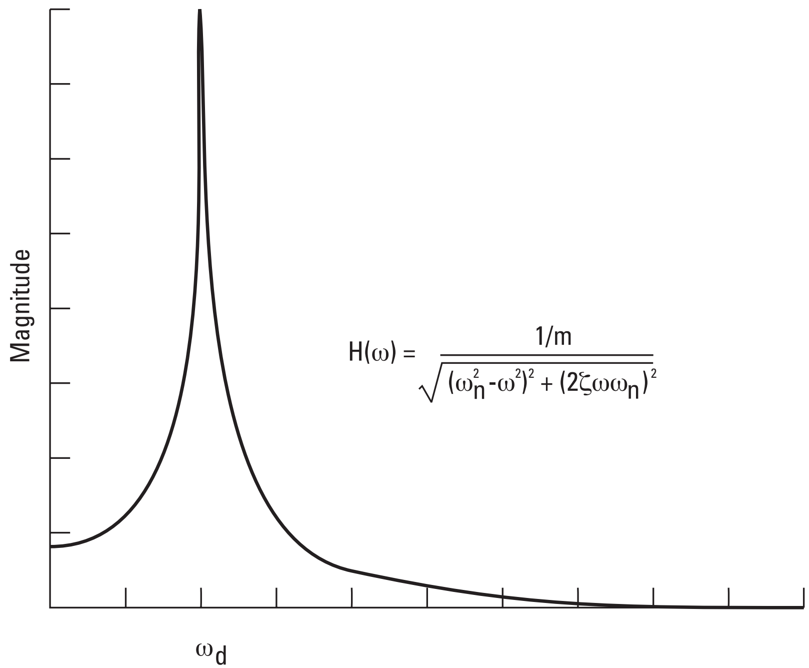
$$f(x) \longrightarrow \boxed{\text{Fourier Transform}} \longrightarrow F(\omega)$$

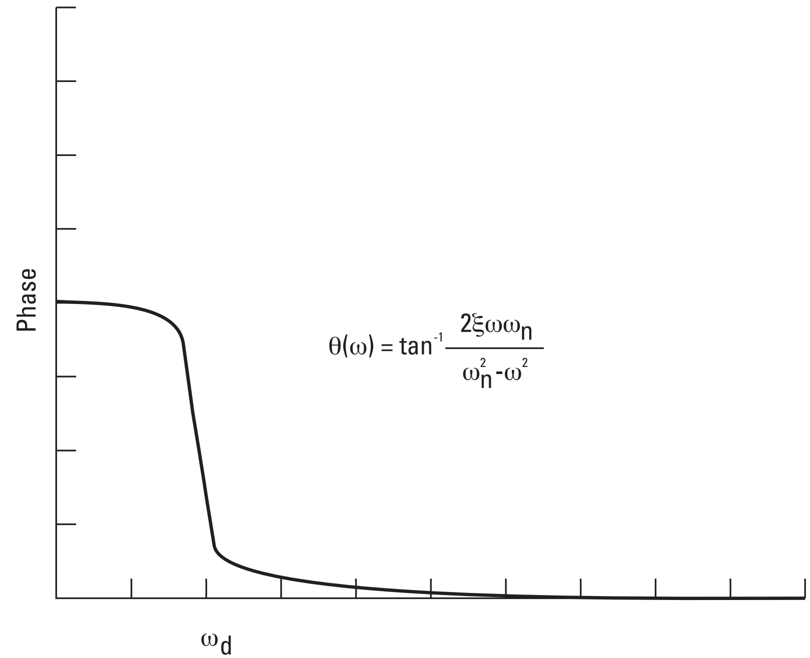
- For every ω from 0 to inf, ***F(ω)*** contains the amplitude *A* and phase ϕ of the corresponding sine $A\sin(\omega x + \phi)$

$$A = \pm\sqrt{R(\omega)^2 + I(\omega)^2} \qquad \phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$$

$$F(\omega) \longrightarrow \boxed{\text{Inverse Fourier Transform}} \longrightarrow f(x)$$

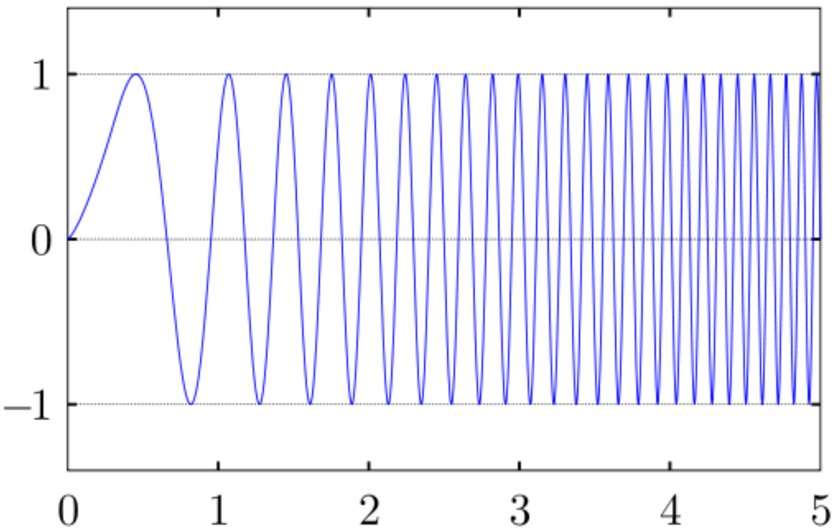
Frequency Response – Bode Plots



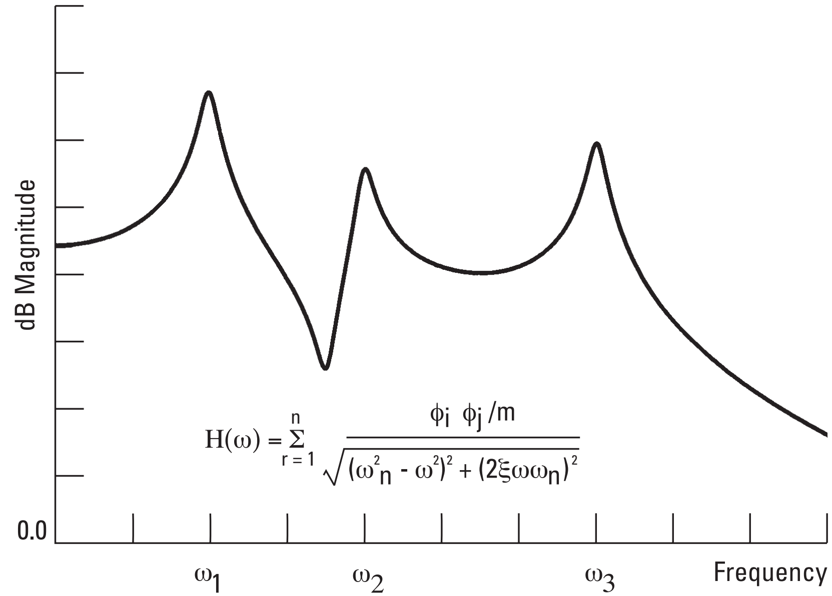


- The frequency response of the system shows how the system responds for different input frequencies
- The phase corresponds to the phase difference between the input signal and output response
- This type of response testing is called Modal Testing
- The combination of the Magnitude and Phase plots are Bode Plots

Frequency Response – Chirp Testing



Input

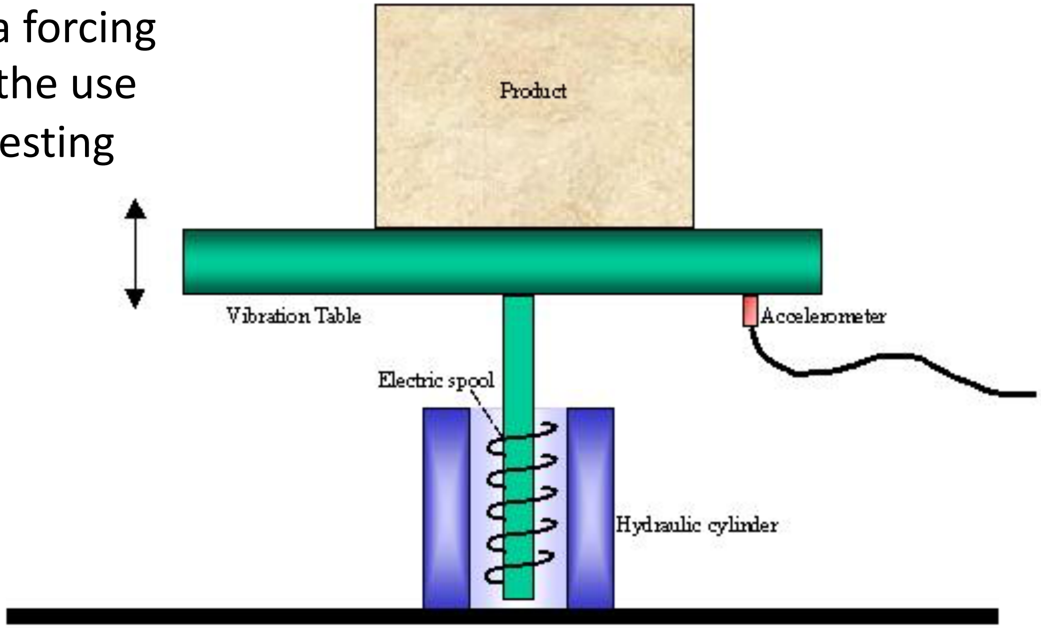


Output

- A chirp is a signal in which the frequency increases (up-chirp) or decreases (down-chirp) with time.
- Chirp testing shows the different frequencies at which different modes get activated

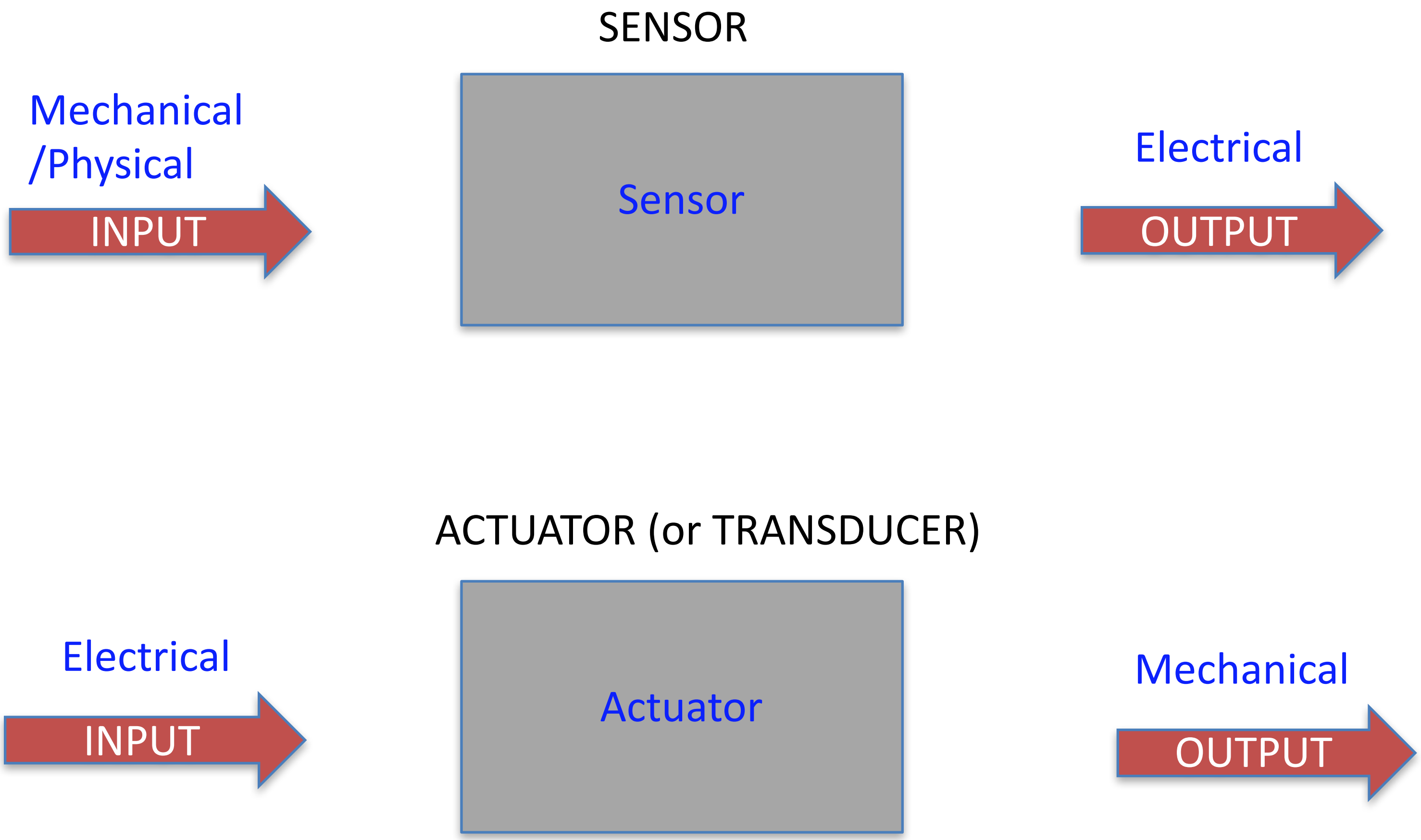
Shock and Vibration Testing

Vibration testing is done to introduce a forcing function into a structure, usually with the use of a vibration test shaker or vibration testing machine.



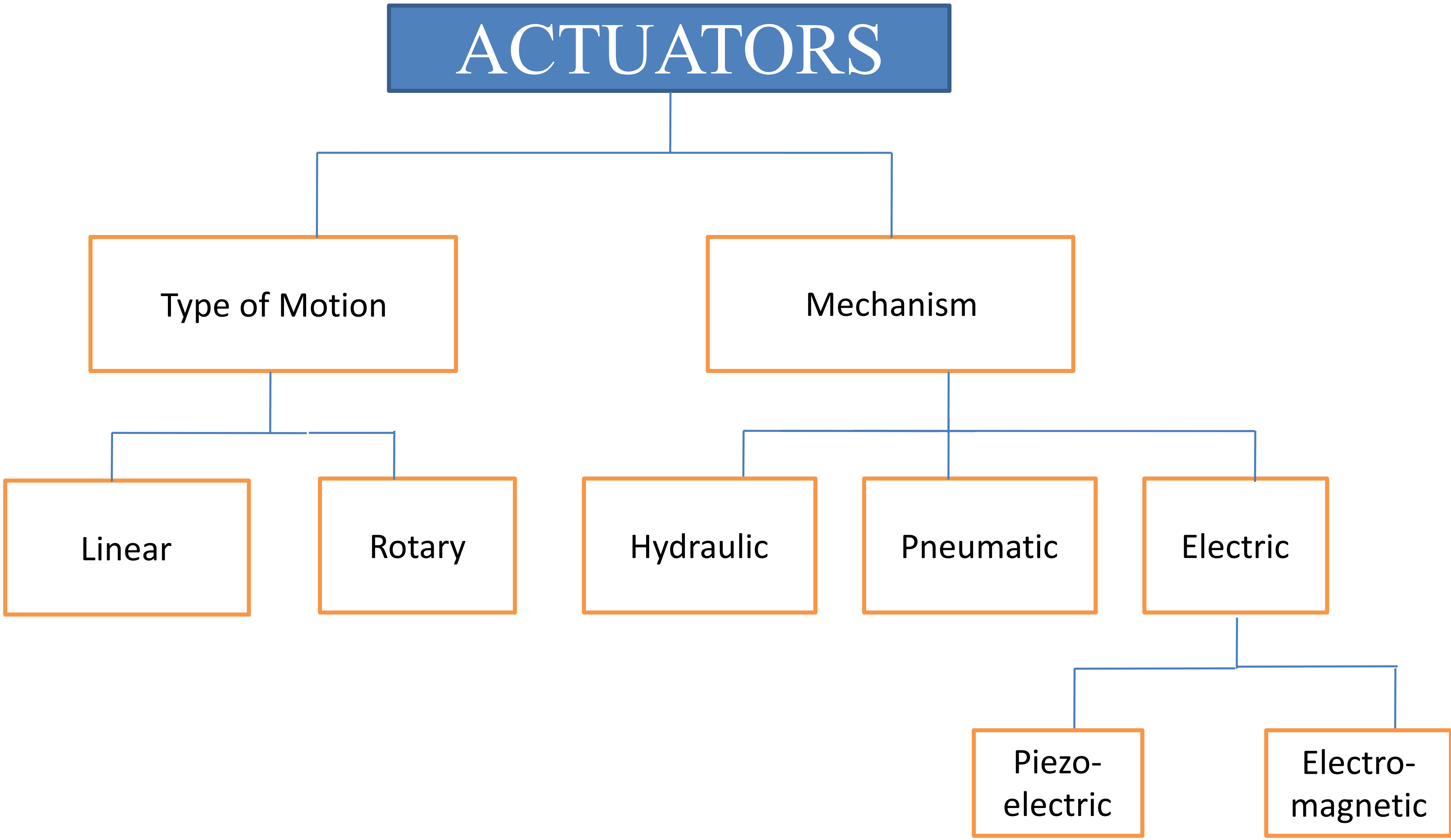
- These induced vibrations, vibration tests, or shaker tests are used in the laboratory or production floor for many reasons:
  - Qualifying products during design
  - Meeting standards
  - Regulatory qualifications (e.g. MIL-STD 810, etc.)
  - Fatigue testing
  - Screening products for failure modes
- Most often electrodynamic and hydraulic shakers (or actuators) are used, depending on the frequency range and displacement required.

Sensors vs Actuators



- Sensors and actuators are both electromechanical devices
- Most complex experimental setups and real world systems have both actuators and sensors

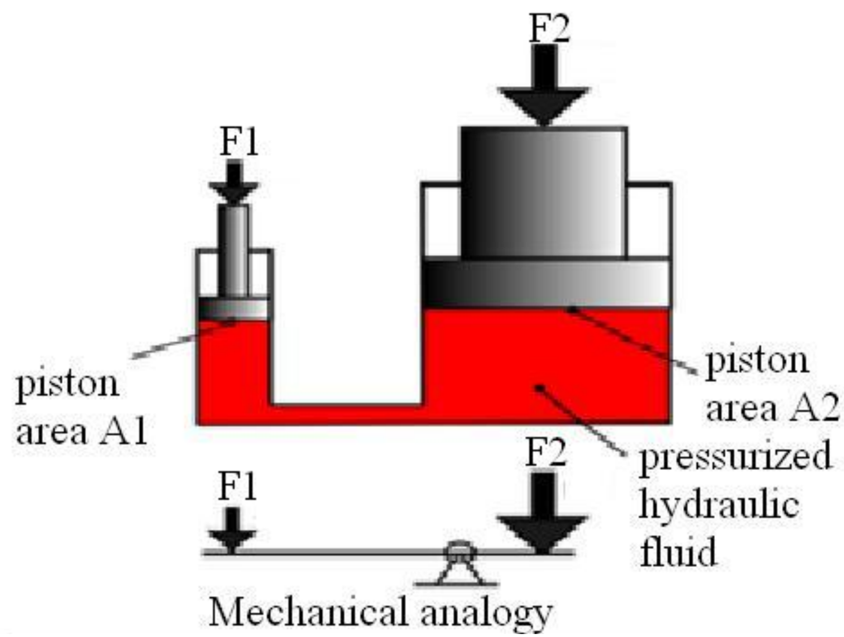
Actuator Classification



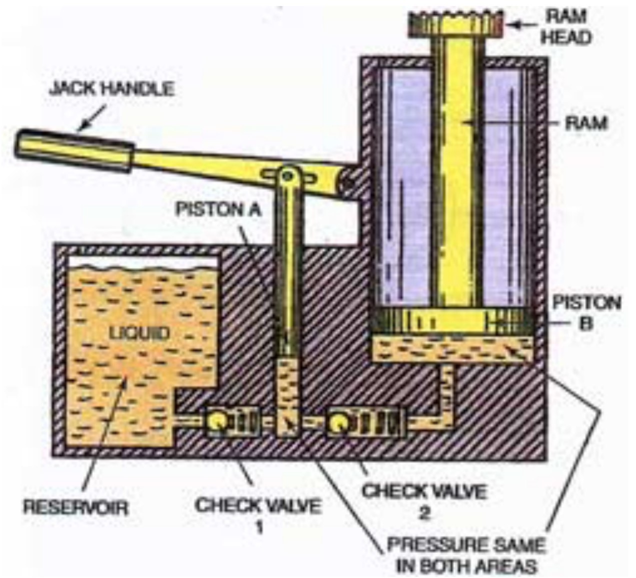
Hydraulic and Pneumatic Actuators

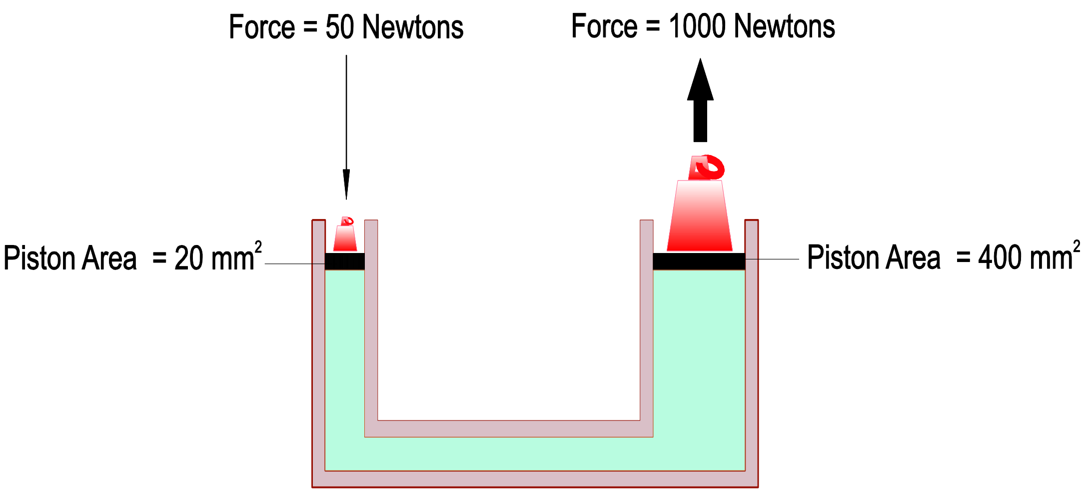
- Hydraulic/Pneumatic Actuators are used in a variety of applications, employ fluid pressure to drive an output member.
- Aircraft simulators, jackhammers
- Automotive breaking

Hydraulics principle



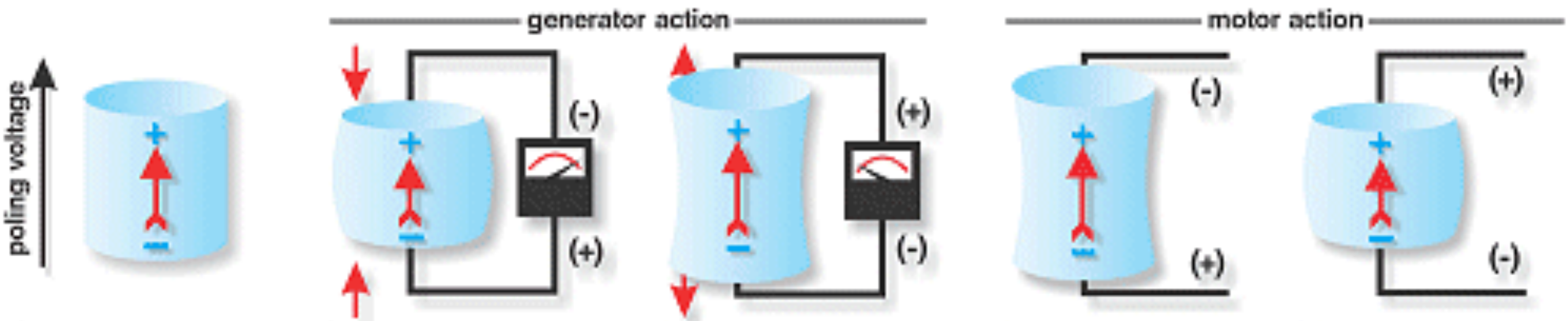
- Principle : Pascal's Law
- Pressure exerted anywhere in a confined incompressible fluid is transmitted equally in all directions throughout the fluid, acts upon every part of the confining vessel at right angles to its interior surfaces.
- F = PxA





Piezoelectric actuators

- Certain crystals can be polarized permanently by applying an electric field
- Piezoelectricity is the appearance of an electric potential across certain faces of the crystal when it is subjected to mechanical pressure
- The word originates from the greek word “piezein”, which means “to press”
- Discovered in 1880 by Pierre Curie in quartz crystals.
- Conversely, when an electric field is applied to one of the faces of the crystal it undergoes mechanical distortion.
- Examples --- Quartz, Barium Titanate, Boron Silicate



(a) disk after polarization (poling)

(b) disk compressed: generated voltage has same polarity as poling voltage

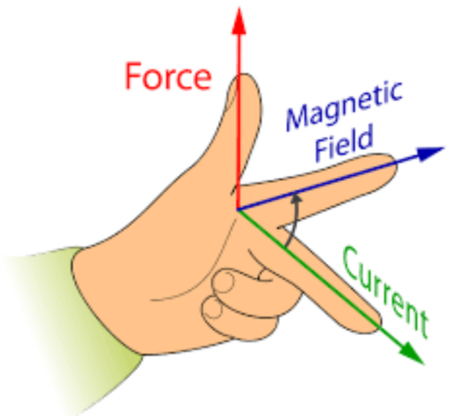
(c) disk stretched: generated voltage has polarity opposite that of poling voltage

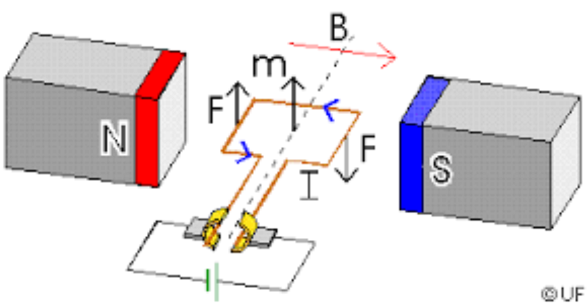
(d) applied voltage has same polarity as poling voltage: disk lengthens

(e) applied voltage has polarity opposite that of poling voltage: disk shortens

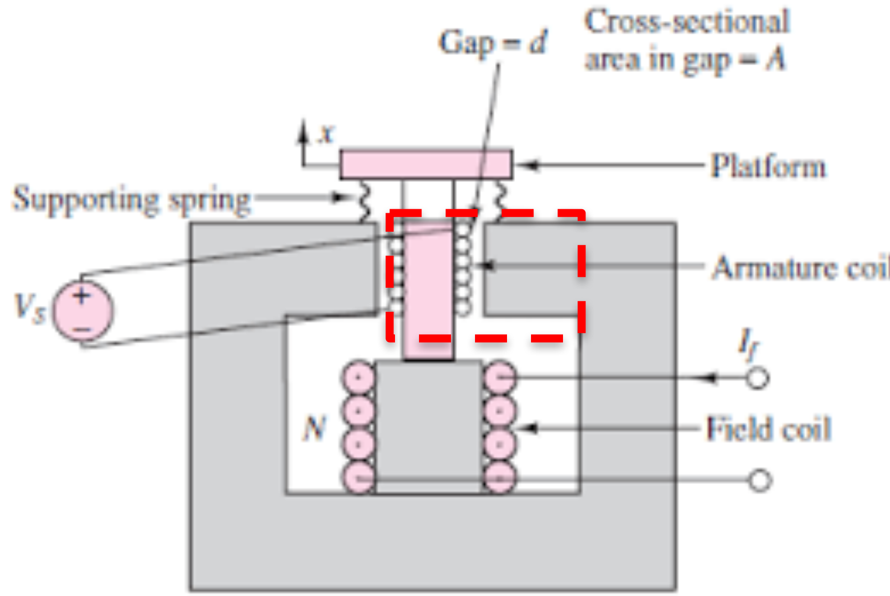
Electromagnetic Shaker

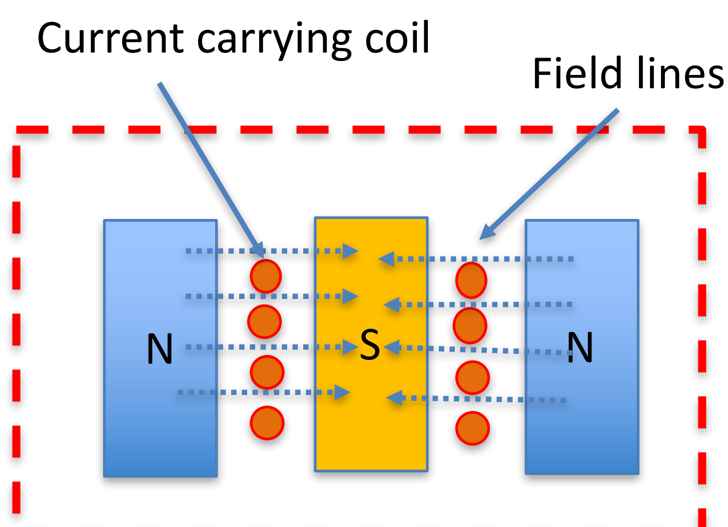
- Based on the principle of force on a current carrying coil in a magnetic field





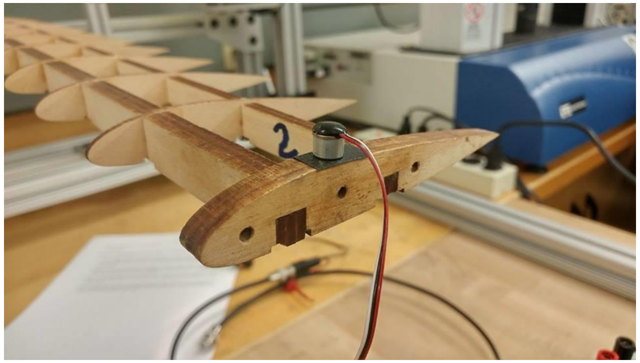
Electric Motor

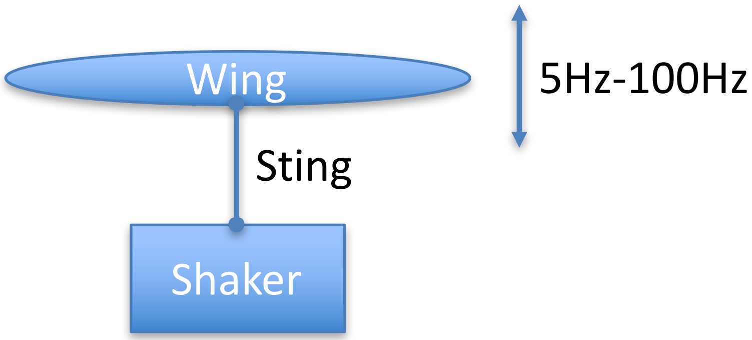




- Force on current carrying coil is up or down depending on direction of current
- An alternating current creates an oscillatory motion

Lab 7: Vibration Testing





The goal of this lab is to familiarize yourself with a load cell and electromagnetic shaker. You will explore the dynamic response of a built-up wing. You measured the static response of this wing in Lab 3 and performed rap testing on this wing in Lab 6.

1. Calibrate the load cell using the set of known masses. You will need atleast four data points to fit a straight line. Keep in mind that a larger number of data points will improve confidence in your calibration factor.

2. Attach the shaker to the front spar of the wing (near the tip). Excite the shaker with a sinusoidal voltage. Measure the force required to excite the wing and the tip acceleration (use the MEMS accelerometer). Do this for a frequency range between 5 Hz-100 Hz. Use a sampling frequency of 1 kHz.

3. Plot the magnitudes of tip acceleration, tip displacement and excitation force as a function of frequency. Also plot the magnitude of tip displacement divided by the magnitude of the excitation force at each frequency (a frequency response function).

4. Repeat this test by attaching the shaker to the aft spar of the wing (near the tip). Compare the results of the two tests. Also compare the resonant frequencies with those you measured in Lab 6.