Problema 1.a

Parte 1

Encontrando la primera parte de la frase:

$$A = \begin{pmatrix} 52 & 29 & 29 & 16 & 45 & 33 & 4 & 22 & 21 & 38 & 29 & 16 & 20 & 29 & 12 \\ 86 & 100 & 80 & 15 & 57 & 76 & 23 & 86 & 99 & 100 & 80 & 15 & 27 & 47 & 3 \\ -16 & 14 & 5 & -11 & -19 & -3 & 5 & 14 & 19 & 5 & 5 & -11 & -11 & -6 & -11 \end{pmatrix}$$

```
{{52, 29, 29, 16, 45, 33, 4, 22, 21, 38, 29, 16, 20, 29, 12},
 {86, 100, 80, 15, 57, 76, 23, 86, 99, 100, 80, 15, 27, 47, 3},
 {-16, 14, 5, -11, -19, -3, 5, 14, 19, 5, 5, -11, -11, -6, -11}}
```

$$B = \text{Inverse}\left[\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{pmatrix}\right]$$

```
{{-2, 1, -3}, {3, -1, 3}, {-2, 1, -2}}
```

Desencriptando:

```
MatrixForm[B.A]
```

$$\begin{pmatrix} 30 & 0 & 7 & 16 & 24 & 19 & 0 & 0 & 0 & 9 & 7 & 16 & 20 & 7 & 12 \\ 22 & 29 & 22 & 0 & 21 & 14 & 4 & 22 & 21 & 29 & 22 & 0 & 0 & 22 & 0 \\ 14 & 14 & 12 & 5 & 5 & 16 & 5 & 14 & 19 & 14 & 12 & 5 & 9 & 1 & 1 \end{pmatrix}$$

El primer grupo de números es:
30,22,14,0,29,14,7,22,12,16,0,5,24,21,5,19,14,16,0,4,5,0,21,19,9,29,14,7,22,12,16,0,5,20,0,9,7,22,1,12,0,1

quedando la primera parte de la frase: ("un ángulo externo de un triángulo es igual a)

Parte 2

Encontrando el complemento de la frase:

$$F = \begin{pmatrix} 1 & 22 & 26 & 8 & 44 & 14 & 26 & 32 & 39 & 24 & 32 & 28 & 27 & 7 & 33 & 70 \\ 13 & 42 & 1 & 5 & 36 & 43 & 34 & 20 & 35 & 33 & 20 & 16 & 29 & 8 & 26 & 57 \\ 12 & 20 & -12 & 1 & 4 & 29 & 15 & 4 & 5 & 14 & 4 & 2 & 3 & 2 & 7 & 7 \end{pmatrix}$$

```
{{1, 22, 26, 8, 44, 14, 26, 32, 39, 24, 32, 28, 27, 7, 33, 70},
 {13, 42, 1, 5, 36, 43, 34, 20, 35, 33, 20, 16, 29, 8, 26, 57},
 {12, 20, -12, 1, 4, 29, 15, 4, 5, 14, 4, 2, 3, 2, 7, 7}}
```

$$G = \text{Inverse}\left[\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}\right]$$

```
{{1, -1, 1}, {1, -1, 2}, {-1, 2, -2}}
```

Desencriptando:

MatrixForm[G.F]

$$\begin{pmatrix} 0 & 0 & 13 & 4 & 12 & 0 & 7 & 16 & 9 & 5 & 16 & 14 & 1 & 1 & 14 & 20 \\ 12 & 20 & 1 & 5 & 16 & 29 & 22 & 20 & 14 & 19 & 20 & 16 & 4 & 3 & 21 & 27 \\ 1 & 22 & 0 & 0 & 20 & 14 & 12 & 0 & 21 & 14 & 0 & 0 & 25 & 5 & 5 & 30 \end{pmatrix}$$

El segundo grupo de números es:
0,12,1,0,20,22,13,1,0,4,5,0,12,16,20,0,29,14,7,22,12,16,20,0,9,14,21,5,19,14,16,20,0,14,16,0,1,4,25,1,3,5,14,21,5,20,27,30

quedando la segunda parte de la frase: (la suma de los ángulos internos adyacentes")

Mensaje desencriptado: "un ángulo externo de un triángulo es igual a la suma de los ángulos internos adyacentes".

Problema 1.b

a.)

Matriz código inversa, en terminos de p, w, k, y s:

M = Inverse $\left[\begin{pmatrix} p & w \\ k & s \end{pmatrix}\right]$

$$\left\{\left\{\frac{s}{p\,s-k\,w}, -\frac{w}{p\,s-k\,w}\right\}, \left\{-\frac{k}{p\,s-k\,w}, \frac{p}{p\,s-k\,w}\right\}\right\}$$

b.)

Planteando ecuaciones:

$$H = \begin{pmatrix} 64 & 54 & 24 & 56 & 8 & 17 & 11 & 53 & 20 & 14 & 24 & 56 & 8 & 31 & 69 \\ 158 & 124 & 48 & 128 & 16 & 39 & 27 & 127 & 60 & 32 & 48 & 128 & 16 & 67 & 167 \end{pmatrix}$$

{{64, 54, 24, 56, 8, 17, 11, 53, 20, 14, 24, 56, 8, 31, 69},
{158, 124, 48, 128, 16, 39, 27, 127, 60, 32, 48, 128, 16, 67, 167}}

Realizando el producto:

M.H

$$\left\{\left\{\frac{64\,s}{p\,s-k\,w}-\frac{158\,w}{p\,s-k\,w},\ \frac{54\,s}{p\,s-k\,w}-\frac{124\,w}{p\,s-k\,w},\ \frac{24\,s}{p\,s-k\,w}-\frac{48\,w}{p\,s-k\,w},\right.\right.$$
$$\frac{56\,s}{p\,s-k\,w}-\frac{128\,w}{p\,s-k\,w},\ \frac{8\,s}{p\,s-k\,w}-\frac{16\,w}{p\,s-k\,w},\ \frac{17\,s}{p\,s-k\,w}-\frac{39\,w}{p\,s-k\,w},\ \frac{11\,s}{p\,s-k\,w}-\frac{27\,w}{p\,s-k\,w},$$
$$\frac{53\,s}{p\,s-k\,w}-\frac{127\,w}{p\,s-k\,w},\ \frac{20\,s}{p\,s-k\,w}-\frac{60\,w}{p\,s-k\,w},\ \frac{14\,s}{p\,s-k\,w}-\frac{32\,w}{p\,s-k\,w},\ \frac{24\,s}{p\,s-k\,w}-\frac{48\,w}{p\,s-k\,w},$$
$$\left.\frac{56\,s}{p\,s-k\,w}-\frac{128\,w}{p\,s-k\,w},\ \frac{8\,s}{p\,s-k\,w}-\frac{16\,w}{p\,s-k\,w},\ \frac{31\,s}{p\,s-k\,w}-\frac{67\,w}{p\,s-k\,w},\ \frac{69\,s}{p\,s-k\,w}-\frac{167\,w}{p\,s-k\,w}\right\},$$
$$\left\{-\frac{64\,k}{p\,s-k\,w}+\frac{158\,p}{p\,s-k\,w},\ -\frac{54\,k}{p\,s-k\,w}+\frac{124\,p}{p\,s-k\,w},\ -\frac{24\,k}{p\,s-k\,w}+\frac{48\,p}{p\,s-k\,w},\ -\frac{56\,k}{p\,s-k\,w}+\frac{128\,p}{p\,s-k\,w},\right.$$
$$-\frac{8\,k}{p\,s-k\,w}+\frac{16\,p}{p\,s-k\,w},\ -\frac{17\,k}{p\,s-k\,w}+\frac{39\,p}{p\,s-k\,w},\ -\frac{11\,k}{p\,s-k\,w}+\frac{27\,p}{p\,s-k\,w},\ -\frac{53\,k}{p\,s-k\,w}+\frac{127\,p}{p\,s-k\,w},$$
$$-\frac{20\,k}{p\,s-k\,w}+\frac{60\,p}{p\,s-k\,w},\ -\frac{14\,k}{p\,s-k\,w}+\frac{32\,p}{p\,s-k\,w},\ -\frac{24\,k}{p\,s-k\,w}+\frac{48\,p}{p\,s-k\,w},\ -\frac{56\,k}{p\,s-k\,w}+\frac{128\,p}{p\,s-k\,w},$$
$$\left.\left.-\frac{8\,k}{p\,s-k\,w}+\frac{16\,p}{p\,s-k\,w},\ -\frac{31\,k}{p\,s-k\,w}+\frac{67\,p}{p\,s-k\,w},\ -\frac{69\,k}{p\,s-k\,w}+\frac{167\,p}{p\,s-k\,w}\right\}\right\}$$

la primera palabra es ("por) = (30,17,16,19), tomando los valores de cada letra, igualando y resolviendo:

$$\frac{64\,s}{p\,s-k\,w}-\frac{158\,w}{p\,s-k\,w}=30$$

$$\frac{158\,p}{p\,s-k\,w}-\frac{64\,k}{p\,s-k\,w}=17$$

$$\frac{54\,s}{p\,s-k\,w}-\frac{124\,w}{p\,s-k\,w}=16$$

$$\frac{124\,p}{p\,s-k\,w}-\frac{54\,k}{p\,s-k\,w}=19$$

$$\textbf{Solve}\left[\left\{\frac{64\,s}{p\,s-k\,w}-\frac{158\,w}{p\,s-k\,w}==30,\ \frac{158\,p}{p\,s-k\,w}-\frac{64\,k}{p\,s-k\,w}==17,\ \frac{54\,s}{p\,s-k\,w}-\frac{124\,w}{p\,s-k\,w}==16,\ \frac{124\,p}{p\,s-k\,w}-\frac{54\,k}{p\,s-k\,w}==19\right\},\ \{p,w,k,s\}\right]$$

$$\{\{p\to 1,\ w\to 2,\ k\to 3,\ s\to 4\}\}$$

c.)

la matriz código es igual a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Y la matriz inversa es:

S = Inverse$\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]$

$\left\{\{-2, 1\}, \left\{\frac{3}{2}, -\frac{1}{2}\right\}\right\}$

d y e .)

Encontrando la primera parte de la frase:

Desencriptando:

MatrixForm[S.H]

$$\begin{pmatrix} 30 & 16 & 0 & 16 & 0 & 5 & 5 & 21 & 20 & 4 & 0 & 16 & 0 & 5 & 29 \\ 17 & 19 & 12 & 20 & 4 & 6 & 3 & 16 & 0 & 5 & 12 & 20 & 4 & 13 & 20 \end{pmatrix}$$

El primer grupo de números es:

30,17,16,19,0,12,16,20,0,4,5,6,5,3,21,16,20,0,4,5,0,12,16,20,0,4,5,13,29,20

quedando la primera parte de la frase: (“por los defectos de los demás)

Encontrando la segunda parte de la frase:

$$\mathbf{J} = \begin{pmatrix} \mathbf{10} & \mathbf{12} & \mathbf{22} & \mathbf{20} & \mathbf{16} & \mathbf{35} & \mathbf{57} & \mathbf{23} & \mathbf{5} & \mathbf{44} & \mathbf{20} & \mathbf{55} & \mathbf{50} & \mathbf{41} & \mathbf{80} \\ \mathbf{20} & \mathbf{36} & \mathbf{64} & \mathbf{42} & \mathbf{48} & \mathbf{73} & \mathbf{133} & \mathbf{55} & \mathbf{15} & \mathbf{100} & \mathbf{60} & \mathbf{127} & \mathbf{116} & \mathbf{91} & \mathbf{180} \end{pmatrix}$$

{{10, 12, 22, 20, 16, 35, 57, 23, 5, 44, 20, 55, 50, 41, 80},
{20, 36, 64, 42, 48, 73, 133, 55, 15, 100, 60, 127, 116, 91, 180}}

Desencriptando:

MatrixForm[S.J]

$$\begin{pmatrix} 0 & 12 & 20 & 2 & 16 & 3 & 19 & 9 & 5 & 12 & 20 & 17 & 16 & 9 & 20 \\ 5 & 0 & 1 & 9 & 0 & 16 & 19 & 7 & 0 & 16 & 0 & 19 & 17 & 16 & 30 \end{pmatrix}$$

El primer grupo de números es:

0,5,12,0,20,1,2,9,16,0,3,16,19,19,9,7,5,0,12,16,20,0,17,19,16,17,9,16,20,30

quedando la segunda parte de la frase: (el sabio corrige los propios”)

Mensaje desencriptado: “por los defectos de los demás el sabio corrige los propios”

Problema 2.a

i.) Exprese la integral como la suma de dos integrales impropias.

$$\int_2^\infty \frac{1}{x\sqrt{x^2-4}}\,dx = \int_2^3 \frac{1}{x\sqrt{x^2-4}}\,dx + \int_3^\infty \frac{1}{x\sqrt{x^2-4}}\,dx = \lim_{t\to 2^+}\int_t^3 \frac{1}{x\sqrt{x^2-4}}\,dx + \lim_{t\to\infty}\int_3^t \frac{1}{x\sqrt{x^2-4}}\,dx$$

ii.) Evalúe a mano.

$$\lim_{t\to 2^+}\int_t^3 \frac{1}{x\sqrt{x^2-4}}\,dx + \lim_{t\to\infty}\int_3^t \frac{1}{x\sqrt{x^2-4}}\,dx$$

Encontrando la integral.

$$\int \frac{1}{x\sqrt{x^2-4}}\,\mathrm{dx} \to \text{sustituyendo } x = 2\sec\theta,\ \ \mathrm{dx} = 2\sec\theta\tan\theta\,\mathrm{d}\theta =$$

$$\int\left(\frac{1}{2\sec\theta\sqrt{(2\sec\theta)^2-4}}\right).2\sec\theta\tan\theta\,\mathrm{d}\theta\mathrm{dx} = \int\left(\frac{1}{\sqrt{4\sec^2\theta-4}}\right).\tan\theta\,\mathrm{d}\theta = \int\left(\frac{1}{\sqrt{4\left(\sec^2\theta-1\right)}}\right).\tan\theta\,\mathrm{d}\theta$$

$$= \int\left(\frac{1}{\sqrt{4\tan^2\theta}}\right).\tan\theta\,\mathrm{d}\theta = \int\left(\frac{1}{2\tan\theta}\right).\tan\theta\,\mathrm{d}\theta = \int\frac{1}{2}\,\mathrm{d}\theta = \left[\frac{1}{2}\theta\right] + c\,,$$

$$\text{regresando } a \text{ términos de } x \to x = 2\sec\theta,\ \ \theta = \sec^{-1}(x/2) = \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)$$

Evaluando los limites.

$$\lim_{t\to 2^+}\left[\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)\right]_t^4 + \lim_{t\to\infty}\left[\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)\right]_4^t$$

$$= \lim_{t\to 2^+}\left[\frac{1}{2}\sec^{-1}\left(\frac{4}{2}\right) - \frac{1}{2}\sec^{-1}\left(\frac{t}{2}\right)\right] + \lim_{t\to\infty}\left[\frac{1}{2}\sec^{-1}\left(\frac{t}{2}\right) - \frac{1}{2}\sec^{-1}\left(\frac{4}{2}\right)\right]$$

$$= \left[\frac{1}{2}\sec^{-1}(2) - \frac{1}{2}\sec^{-1}(1)\right] + \left[\frac{1}{2}\sec^{-1}\left(\frac{\infty}{2}\right) - \frac{1}{2}\sec^{-1}(2)\right]$$

$$= \left[\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{2}(0)\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{\pi}{6} + \frac{\pi}{4} - \frac{\pi}{6}$$

$$= \frac{\pi}{4}$$

iii.) Evalúe con ayuda de un sistema computacional (evalúe la integral en la computadora) de manera directa.

$$\int_2^\infty \frac{1}{x\sqrt{x^2-4}}\,dx$$

$$\frac{\pi}{4}$$

iv.) Comente

Problema 2.b

Encuentre un valor aproximado para $\int_{\pi/2}^{3\pi/2} \frac{\text{senx}}{x} dx$ usando:

i.) El método de Simpson con n=24.

```
f[x_] := Sin[x]/x
```

```
n = 24
```

24

```
a = π / 2
```

$\frac{\pi}{2}$

```
b = 3 π / 2
```

$\frac{3\pi}{2}$

```
Δx = (b - a)/n
```

$\frac{\pi}{24}$

```
S = Δx / 3 (f[a] + 4 f[a + Δx] + 2 f[a + 2 Δx] + 4 f[a + 3 Δx] + 2 f[a + 4 Δx] + 4 f[a + 5 Δx] +
    2 f[a + 6 Δx] + 4 f[a + 7 Δx] + 2 f[a + 8 Δx] + 4 f[a + 9 Δx] + 2 f[a + 10 Δx] +
    4 f[a + 10 Δx] + 2 f[a + 11 Δx] + 4 f[a + 12 Δx] + 2 f[a + 13 Δx] + 4 f[a + 14 Δx] +
    2 f[a + 15 Δx] + 4 f[a + 16 Δx] + 2 f[a + 17 Δx] + 4 f[a + 18 Δx] + 2 f[a + 19 Δx] +
    4 f[a + 20 Δx] + 2 f[a + 21 Δx] + 4 f[a + 22 Δx] + 2 f[a + 23 Δx] + 4 f[b]) // N
```

0.232033

ii.) El método de punto medio con n=20.

```
Δx = (b - a)/20
```

$\frac{\pi}{20}$

$$m = \Delta x \left(f\left[\frac{a + (a + \Delta x)}{2}\right] + f\left[\frac{(a + \Delta x) + (a + 2\,\Delta x)}{2}\right] + f\left[\frac{(a + 2\,\Delta x) + (a + 3\,\Delta x)}{2}\right] + \right.$$
$$f\left[\frac{(a + 3\,\Delta x) + (a + 4\,\Delta x)}{2}\right] + f\left[\frac{(a + 4\,\Delta x) + (a + 5\,\Delta x)}{2}\right] + f\left[\frac{(a + 5\,\Delta x) + (a + 6\,\Delta x)}{2}\right] +$$
$$f\left[\frac{(a + 6\,\Delta x) + (a + 7\,\Delta x)}{2}\right] + f\left[\frac{(a + 7\,\Delta x) + (a + 8\,\Delta x)}{2}\right] + f\left[\frac{(a + 8\,\Delta x) + (a + 9\,\Delta x)}{2}\right] +$$
$$f\left[\frac{(a + 9\,\Delta x) + (a + 10\,\Delta x)}{2}\right] + f\left[\frac{(a + 10\,\Delta x) + (a + 11\,\Delta x)}{2}\right] + f\left[\frac{(a + 11\,\Delta x) + (a + 12\,\Delta x)}{2}\right] +$$
$$f\left[\frac{(a + 12\,\Delta x) + (a + 13\,\Delta x)}{2}\right] + f\left[\frac{(a + 13\,\Delta x) + (a + 14\,\Delta x)}{2}\right] + f\left[\frac{(a + 14\,\Delta x) + (a + 15\,\Delta x)}{2}\right] +$$
$$f\left[\frac{(a + 15\,\Delta x) + (a + 16\,\Delta x)}{2}\right] + f\left[\frac{(a + 16\,\Delta x) + (a + 17\,\Delta x)}{2}\right] + f\left[\frac{(a + 17\,\Delta x) + (a + 18\,\Delta x)}{2}\right] +$$
$$\left. f\left[\frac{(a + 18\,\Delta x) + (a + 19\,\Delta x)}{2}\right] + f\left[\frac{(a + 19\,\Delta x) + (a + 20\,\Delta x)}{2}\right]\right) \mathbin{//} N$$

0.237147