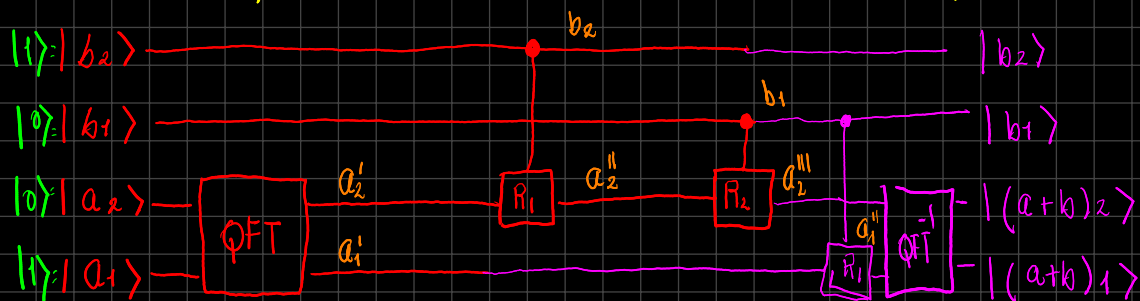


- Suma de 1 + 2

- Prerequisitos

$$QFT = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}, \quad R_2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^2}} \end{pmatrix}, \quad QFT^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}$$

Como vamos a representar nuestros números con 2 qbits, nuestro circuito queda



Primero representamos nuestros números en forma binaria

$$A = 1 = 01 \rightarrow \underbrace{|0\rangle}_{a_2} \otimes \underbrace{|1\rangle}_{a_1}$$

$$B = 2 = 10 \rightarrow \underbrace{|1\rangle}_{b_2} \otimes \underbrace{|0\rangle}_{b_1}$$

1. Realizamos el producto tensorial $a_2 \otimes a_1$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

2. Aplicamos QFT

$$QFT \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{a'_2} \otimes \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}}_{a'_1}$$

3. Realizamos p. tensorial con $b_2 \otimes a'_2$

$$(11 \otimes R_1) (b_2 \otimes a'_2) = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] (b_2 \otimes a'_2)$$

$$= (11 b_2) \otimes (R_1 a'_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \left[\begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right]$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{b_2} \otimes \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{a''_2}$$

4. Aplicamos producto tensorial $b_1 \otimes a_2''$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

5. Aplicamos la notación R_2

$$(1 \otimes R_2)(b_1 \otimes a_2'')$$

$$\Rightarrow (1 \otimes R_2)(b_1 \otimes a_2'')$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} b_1 \\ 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad a_2''''$$

6. Producto tensorial $b_1 \otimes a_1'$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix}$$

7. Aplicamos R_1

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}_{b_1} \otimes \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}}_{a_1''}$$

8. Producto tensorial $a_2''' \otimes a_1''$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ 1 \\ 1 \end{pmatrix}$$

9. Aplicación de QFT^{-1}

$$\frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & 1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ 1 \\ 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$a_2^* \otimes a_1^*$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ i \\ 1 \\ i \end{pmatrix}$$