Introduction to Mathematics of Quantum Computing

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March 13, 2024

Quantum mechanics describes states of systems at microscopic scales, which generally means that only speaking in terms of probabilities is allowed. The state in a quantum system can be described as an element of a vector space. It is possible that the state of the system can be written as a linear combination of a basis of the vector space: superposition. The observables of a quantum system are those physical quantities that in principle can be determined simultaneously and with a precision constraint: Heisenberg's uncertainty principle. In other words, they can be measurable.

Measurements on objects prepared "in the same way" are distributed with a relative frequency around a mean value.

$$\text{relative frequency} = \frac{\text{Number of measurements with result } a}{\text{Total number of measurements}}$$

$$\text{mean value} = \sum_{a \in Measurements} a \times (\text{relative frequency of measurements with result } a)$$

1 Hilbert Spaces

There are initial preparations whose state can be characterized with a vector in a Hilbert space (vector space).

1. States that can be characterized with a vector are called pure states.
2. Otherwise, we speak of "mixed states".

Definition 1.1: Hlbert Space

A Hilbert Space $\mathbb{H}$ is a complete complex vector space, that is,

$$\Psi, \Phi \in \mathbb{H} \text{ and } a, b \in \mathbb{C} \implies a\Psi + b\Phi \in \mathbb{H} \tag{1}$$

with a positive-definite scalar product:

$$\langle \cdot | \cdot \rangle : \mathbb{H} \times \mathbb{H} \to \mathbb{C} \tag{2}$$

such that $\forall \Psi, \Phi, \Psi_1, \Psi_2 \in \mathbb{H}$ and $a, b \in \mathbb{C}$

$$\langle \Psi | \Phi \rangle = \langle \Psi | \Phi \rangle$$