A. Bunde et al., *The Science of Disasters*

11. Crowd Disasters and Simulation of Panic Situations

Dirk Helbing, Illés J. Farkas, and Tamás Vicsek

One of the most tragic collective behaviors is a panic stampede [11.1–11.9], as it often leads to the death of people who are either crushed or trampled down by others. While this behavior is comprehensible in life-threatening situations like fires in crowded buildings [11.10, 11.11], it is hardly understood in cases of a rush for good seats at a pop concert [11.12], or without any obvious reasons. Unfortunately, the frequency of such disasters is increasing [11.12], as growing population densities combined with easier transportation lead to greater mass events like pop concerts, sporting events, and demonstrations. Nevertheless, systematic studies of panics [11.8] are rare [11.5, 11.10, 11.12]. Moreover, there is a scarcity of quantitative theories capable of predicting the dynamics of human crowds [11.13–11.15]. Here we show that simulations of pedestrian behavior can give valuable insights into the mechanisms and preconditions of panic, jamming, and the observed 'faster-is-slower effect'. We also provide clues to practical ways of minimizing the related tragedies. Furthermore, we identify an optimal strategy for collective problem solving in crisis situations, corresponding to a suitable mixture of individualistic and herding behavior.

11.1 Introduction

With some exceptions, panics are observed in cases of scarce or dwindling resources [11.8, 11.10], which are either required for survival or anxiously desired. They are usually distinguished into escape panics ('stampedes', bank, or stock market panics) and acquisitive panics ('crazes', speculative manias) [11.5, 11.6], but in some cases this classification is questionable [11.12].

◀ **Fig. 11.0.** Panicking football fans trying to escape the football stadium in Sheffield. Hardly anybody manages to pass the open door, because of the clogging effect occurring in crowds at high pressures

It is believed that panicking people are obsessed by short-term personal interests uncontrolled by social and cultural constraints [11.5, 11.10]. This is possibly a result of the reduced attention in situations of fear [11.10], which also causes alternatives like side exits to be mostly ignored [11.11]. It is, however, often attributed to social contagion [11.1–11.10, 11.12], i.e. a transition from individual to mass psychology, in which individuals transfer control over their actions to others [11.6], leading to conformity [11.16]. This 'herding behavior' (regarding the herding behavior of stock markt dealers cf. Chap. 14) is irrational, as it often leads to bad overall results like dangerous overcrowding and slower escape [11.6, 11.11, 11.12]. In this way, herding behavior increases the fatalities, or, more generally, the damage in the crisis faced.

The various socio-psychological theories for this contagion assume hypnotic effects, rapport, mutual excitation of a primordial instinct, circular reactions, social facilitation (see the summary by Brown [11.4]), or the emergence of normative support for selfish behavior [11.7]. Brown [11.4] and Coleman [11.6] add another explanation related to the prisoner's dilemma [11.17] or common goods dilemma (an example is discussed in Chap. 6) [11.18], showing that it is reasonable to make one's subsequent actions contingent upon those of others, but the socially favorable behavior of orderly walking is unstable, which normally gives rise to rushing by everyone. These thoughtful considerations are well compatible with many aspects discussed above and with the classical experiments by Mintz [11.8], which showed that jamming in escape situations depends on the reward structure ('payoff matrix').

Nevertheless and despite of the frequent reports in the media and many published investigations of crowd disasters (see Table 11.1), a quantitative understanding of the observed phenomena in panic stampedes is still lacking. Here, we add another aspect to the explanation of panics by simulating a computer model for the crowd dynamics of pedestrians. Approaches to social phenomena in the spirit of statistical physics are quite promising as they have led to a number of exciting discoveries [11.19–11.23]. We are at a point where the exact methods of physics combined with the potentials of computers start to produce relevant results on society.

During our study we managed to find answers to questions like the following:

1. Why do people get ahead more slowly when they are trying to escape fast?
2. Why does the pedestrian flow through exits become irregular in panic situations?
3. Do wide spaces along corridors and escape routes increase the efficiency of leaving?
4. Why are alternative exits overlooked or not efficiently used in escape situations?
5. Why is it possible that panics are sometimes triggered without any apparent reason?

Table 11.1. Incomplete list of major crowd disasters after Smith and Dickie [11.24], `http://ourworld.compuserve.com/homepages/G_Keith_Still/disaster.htm`, `http://SportsIllustrated.CNN.com/soccer/world/news/2000/07/09/stadium_disasters_ap/`, and other Internet sources. The number of injured people was usually a multiple of the fatalities

Date	Place	Venue	Deaths	Injured	Reason
1863	Santiago, Chile	Church	2000		
1881	Vienna, Austria	Theatre	570		
1883	Sunderland, UK	Theatre	182		
1902	Ibrox, UK	Stadium	26	517	Collapse of West Stand
1903	Chicago, USA	Theatre	602		
1943	London, UK	Underground Station	173		Stampede while air raid
1946	Bolton, UK	Stadium	33	400	Collapse of a wall
1955	Santiago, Chile	Stadium	6		Fans trying to force their way into the stadium
1961	Rio de Janeiro, Brazil	Circus	250		
1964	Lima, Peru	Stadium	318	500	Goal disallowed
1967	Kayseri, Turkey	Stadium	40		
1968	Buenos Aires, Argentina	Stadium	75	150	Fans fleeing from fire
1970	St. Laurent-du-Pont, France	Dance Hall	146		
1971	Ibrox, UK	Stadium	66	140	Collapse of barriers
1971	Salvador, Brazil	Stadium	4	1500	Fight and wild rush
1974	Cairo, Egypt	Stadium	48		Crowds break barriers
1976	Port-au-Prince, Haiti	Stadium	2		Firecracker
1979	Nigeria	Stadium	24	27	Light failure
1979	Cincinatti, USA	Stadium	11		Fans trying to force their way into the stadium
1981	Piraeus, Greece	Stadium	24		Rush of leaving fans
1981	Sheffield, UK	Stadium		38	Crowd surge
1982	Cali, Columbia	Stadium	24	250	Provocation by drunken fans
1982	Moscow, USSR	Stadium	340		Re-entering fans after last minute goal
1985	Bradford, UK	Stadium	56		Fire in wooden terrace section
1985	Mexico City, Mexico	Stadium	10	29	Fans trying to force their way into the stadium
1985	Brussels, Belgium	Stadium	38	> 400	Riots break out and wall collapses
1987	Tripoli, Libya	Stadium	2	16	Collapse of a wall
1988	Katmandu, Nepal	Stadium	93	> 100	Stampede due to hailstorm
1989	Hillsborough, Sheffield, UK	Stadium	96		Fans trying to force their way into the stadium
1990	Mecca, Saudi Arabia	Pedestrian Tunnel	1425		Overcrowding
1991	Orkney, South Africa	Stadium	> 40		Fans trying to escape fighting
1991	New York, USA	Stadium	9		Overcrowding at concert
1992	Rio de Janeiro, Brazil	Stadium		50	Part of the fence giving way
1992	Bastia, Corsica	Stadium	17	1900	
1994	Mecca, Saudi Arabia		270		Rush at 'stoning the devil' ritual

continued on next page

Table 11.1 (*Continued*)

Date	Place	Venue	Deaths	Injured	Reason
7/31/1996	Tembisa, South Africa	Railway Station	15	> 20	Electric cattle prods used by security guards
10/17/1996	Guatemala City, Guatemala	Stadium	80	180	Fans trying to force their way into the stadium
6/29/1997	Las Vegas, USA	Hotel	1	50	Gunshot
7/2/1997	Düsseldorf, Germany	Stadium	1	> 300	Overcrowding at concert
3/23/1998	Dhaka, Bangladesh	Multi-Storey Building	1	15	Fire stampede
4/9/1998	Mecca, Saudi Arabia	Holy Place (Meda)	107		Pilgrim stampede at 'stoning the devil' ritual
4/19/1998	Harare, Zimbabwe	Stadium	4	10	Spectators scrambled for seats
8/22/1998	Manila, Phillipines	Presidential Action Centre	2		Large crowd waiting for jobs and housing
12/1/1998	Chervonohrad, Ukraine	Cinema	4		Stampede due to in- and outcoming children
12/25/1998	Lima, Peru	Disco	9	7	Tear gas
5/31/1999	Minsk, Belarus	Subway Station	51	150	Heavy rain at rock concert
1/15/1999	Kerala, India	Hindu Shrine	> 50		Collapse of parts of the shrine
10/7/1999	Benin, Nigeria	Religious Place	14		Stampede at a Christian revivalist rally
12/5/1999	Innsbruck, Austria	Stadium	5	25	Fans re-entering the stadium?
3/3/2000	Kaloroa, Bangladesh	Examination Place	5		Stampede to enter an examination hall
3/12/2000	Mecca, Saudi Arabia	Holy Place	2	4	Pilgrims were crushed by the crowd
3/24/2000	Durban, South Africa	Disco	13	44	Tear gas
3/24/2000	Chiaquelane, Mozambique	Chiaquelane Camp	5	10	Aid chaos
4/17/2000	Lisbon, Portugal	Nightclub	7	65	Poisonous gas bombs
4/21/2000	Seville, Spain			30	Spectators panicked in a crowd during Good Friday procession
4/23/2000	Monrovia, Liberia	Stadium	3		Fans trying to force their way into the stadium
5/26/2000	Lahore, Pakistan	Circus	8	3	Guards used batons
6/5/2000	Addis Ababa, Ethiopia	Memorial Place	14		Children trying to cover from a rainstorm
6/30/2000	Roskilde, Denmark	Stadium	8	25	Failure of loudspeakers
7/9/2000	Harare, Zimbabwe	Stadium	12		Tear gas
Dec. 2000	São Januário, Brazil	Stadium		200	Oversold stadium

6. How is it possible to estimate the number of casualties in emergency situations?
7. What would be the best escape strategy from a smoky room, when the exits are not visible?

One may, therefore, speak of a breakthrough in the understanding of the special dynamic phenomena observed in crowds under emergency conditions.

11.2 Observations

After having carefully studied the related socio-psychological literature [11.4, 11.8, 11.9, 11.25], reports in the media, available video materials (see `http://angel.elte.hu/~panic/`), empirical investigations [11.2, 11.10, 11.11, 11.26], and engineering handbooks [11.27, 11.28], we can summarize the following characteristic features of escape panics:

1. People move or try to move considerably faster than normal [11.27].
2. Individuals start pushing, and interactions among people become physical in nature.
3. Moving and, in particular, passing of a bottleneck becomes incoordinated [11.8].
4. At exits, arching and clogging are observed [11.27].
5. Jams are building up [11.25].
6. The physical interactions in the jammed crowd add up and cause dangerous pressures up to 4 450 N/m [11.11, 11.24], which can bend steel barriers or tear down brick walls.
7. Escape is further slowed down by fallen or injured people turning into 'obstacles'.
8. People show a tendency of mass behavior, i.e. to do what other people do [11.9, 11.10].
9. Alternative exits are often overlooked or not efficiently used in escape situations [11.10, 11.11].

To give a more personal impression of situations when panic strikes, we add some quotations:

1. "*They just kept pushin' forward and they would just walk right on top of you, just trample over ya like you were a piece of the ground.*" (After the panic at 'The Who Concert Stampede' in Cincinatti.)
2. "*People were climbin' over people ta get in ... an' at one point I almost started hittin' 'em, because I could not believe the animal, animalistic ways of the people, you know, nobody cared.*" (After the panic at 'The Who Concert Stampede'.)

3. "*Smaller people began passing out. I attempted to lift one girl up and above to be passed back ... After several tries I was unsuccessful and near exhaustion.*" (After the panic at 'The Who Concert Stampede'.)
4. "*I couldn't see the floor because of the thickness of the smoke.*" (After the 'Hilton Hotel Fire' in Las Vegas.)
5. "*The club had two exits, but the young people had access to only one*", said Narend Singh, provincial minister for agriculture and environmental affairs. However, the club's owner, Rajan Naidoo, said the club had four exits, and that all were open. "*I think the children panicked and headed for the main entrance where they initially came in*", he said. (After the 'Durban Disco Stampede'.)

11.3 Generalized Force Model of Pedestrian Motion

The above observations have encouraged us to model the collective phenomenon of escape panic in the spirit of self-driven many-particle systems. Our computer simulations of the crowd dynamics of pedestrians are based on a generalized force model [11.20,11.29–11.40], which is particularly suited to describe the fatal build up of pressure observed during panics [11.2, 11.11, 11.12, 11.24]. We assume a mixture of socio-psychological [11.20, 11.29, 11.31] and physical forces influencing the behavior in a crowd: each of N pedestrians i of mass m_i likes to move with a certain desired speed v_i^0 in a certain direction $\boldsymbol{e}_i^0$, and therefore tends to correspondingly adapt his or her actual velocity $\boldsymbol{v}_i$ within a certain characteristic time τ_i. Simultaneously, he or she tries to keep a velocity-dependent distance to other pedestrians j, walls W, and fire fronts F. This can be modeled by 'interaction forces' $\boldsymbol{f}_{ij}$, $\boldsymbol{f}_{iW}$, and $\boldsymbol{f}_{iF}$, respectively. Additionally, we take into account fluctuations $\boldsymbol{\xi}_i(t)$ of amplitude η_i. In mathematical terms, the change of velocity in time t is then given by the acceleration equation

$$m_i \frac{d\boldsymbol{v}_i}{dt} = m_i \frac{v_i^0(t)\boldsymbol{e}_i^0(t) - \boldsymbol{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \boldsymbol{f}_{ij} + \sum_W \boldsymbol{f}_{iW} + \boldsymbol{f}_{iF} + \boldsymbol{\xi}_i(t)\,, \qquad (11.1)$$

while the change of position $\boldsymbol{r}_i(t)$ is given by the velocity $\boldsymbol{v}_i(t) = d\boldsymbol{r}_i/dt$. We describe the *psychological tendency* of two pedestrians i and j to stay away from each other by a repulsive interaction force $A_i \exp[(r_{ij} - d_{ij})/B_i]\,\boldsymbol{n}_{ij}$, where A_i and B_i are constants. $d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$ denotes the distance between the pedestrians' centers of mass, and $\boldsymbol{n}_{ij} = (n_{ij}^1, n_{ij}^2) = (\boldsymbol{r}_i - \boldsymbol{r}_j)/d_{ij}$ is the normalized vector pointing from pedestrian j to i. If their distance d_{ij} is smaller than the sum $r_{ij} = (r_i + r_j)$ of their radii r_i and r_j, the pedestrians touch each other. In this case, we assume two additional forces inspired by granular interactions [11.41, 11.42], which are essential for understanding the particular effects in panicking crowds: a *'body force'* $k(r_{ij} - d_{ij})\,\boldsymbol{n}_{ij}$ counteracting body compression and a *'sliding friction force'* $\kappa(r_{ij} - d_{ij})\,\Delta v^t_{ji}\,\boldsymbol{t}_{ij}$ impeding *relative* tangential motion, if pedestrian i comes close to j. Herein, $\boldsymbol{t}_{ij} = (-n^2_{ij}, n^1_{ij})$ means the tangential direction and $\Delta v^t_{ji} = (\boldsymbol{v}_j - \boldsymbol{v}_i)\cdot\boldsymbol{t}_{ij}$ the tangential velocity difference, while k and κ represent large constants. In summary, we have

$$\boldsymbol{f}_{ij} = \{A_i \exp[(r_{ij} - d_{ij})/B_i] + k\Theta(r_{ij} - d_{ij})\}\,\boldsymbol{n}_{ij} + \kappa\Theta(r_{ij} - d_{ij})\Delta v^t_{ji}\,\boldsymbol{t}_{ij}\,, \tag{11.2}$$

where the function $\Theta(x)$ is zero if the pedestrians do not touch each other ($d_{ij} > r_{ij}$); otherwise it is equal to the argument x.

The interaction with the walls is treated analogously, i.e. if d_{iW} means the distance to wall W, $\boldsymbol{n}_{iW}$ denotes the direction perpendicular to it, and $\boldsymbol{t}_{iW}$ the direction tangential to it, the corresponding interaction force with the wall reads

$$\begin{aligned}\boldsymbol{f}_{iW} = \{A_i \exp[(r_i - d_{iW})/B_i] + k\Theta(r_i - d_{iW})\}\,\boldsymbol{n}_{iW} \\ - \kappa\Theta(r_i - d_{iW})(\boldsymbol{v}_i \cdot \boldsymbol{t}_{iW})\,\boldsymbol{t}_{iW}\,.\end{aligned} \tag{11.3}$$

Fire fronts are treated similarly to the walls, but have a stronger psychological effect. People reached by the fire front become injured and immobile ($v_i = 0$), which replaces the physical interaction effect ($k = 0 = \kappa$).

The model parameters have been specified as follows: with a mass of $m_i = 80\,\mathrm{kg}$, we represent an average soccer fan. The desired velocity v^0_i can reach more than $5\,\mathrm{m/s}$ (up to $10\,\mathrm{m/s}$) [11.28], but the observed free velocities for leaving a room correspond to $v^0_i \approx 0.6\,\mathrm{m/s}$ under relaxed, $v^0_i \approx 1\,\mathrm{m/s}$ under normal, and $v^0_i \lesssim 1.5\,\mathrm{m/s}$ under nervous conditions [11.27]. A reasonable estimate for the acceleration time is $\tau_i = 0.5\,\mathrm{s}$. With $A_i = 2 \times 10^3\,\mathrm{N}$ and $B_i = 0.08\,\mathrm{m}$ one can reflect the distance kept at normal desired velocities [11.28] and fit the measured flows through bottlenecks [11.28], amounting to 0.73 persons per second for an effectively 1 m wide door under conditions with $v^0_i \approx 0.8\,\mathrm{m/s}$. The parameters $k = 1.2 \times 10^5\,\mathrm{kg\,s^{-2}}$ and $\kappa = 2.4 \times 10^5\,\mathrm{kg\,m^{-1}s^{-1}}$ determine the obstruction effects in cases of physical interactions. Although, in reality, most parameters are varying individually, we chose identical values for all pedestrians to minimize the number of parameters for reasons of calibration and robustness, and to exclude irregular outflows because of parameter variations. However, to avoid model artefacts (gridlocks by exactly balanced forces in symmetrical configurations), a small amount of irregularity of almost arbitrary kind is needed. This irregularity was introduced by uniformly distributed pedestrian diameters $2r_i$ in the interval $[0.5\,\mathrm{m}, 0.7\,\mathrm{m}]$, approximating the distribution of shoulder widths of soccer fans.

11.4 Simulation Results

Based on the above model assumptions, we will now simulate several essential phenomena of escape panic, which are insensitive to reasonable parameter variations, but fortunately become less pronounced for broader corridors and wider exits. We will separately treat the following characteristic behaviors observed in panic situations or cases of emergency evacuation:

1. People are getting nervous, resulting in a higher level of fluctuations.
2. They are trying to escape from the source of panic, which can be reflected by a significantly higher desired velocity.
3. Individuals in complex situations, who do not know what is the right thing to do, orient at the actions of their neighbors, i.e. they tend to do what other people do. We will describe this by an additional herding interaction, but attractive interactions have probably a similar effect.

11.4.1 'Freezing by Heating' Instead of Lane Formation

For normal, relaxed situations with small fluctuation amplitudes η_i, our microsimulations of counterflows in corridors reproduce the empirically observed *formation of lanes* consisting of pedestrians with the same desired walking direction, see Fig. 11.1a [11.29–11.38, 11.43, 11.44]). If we do not assume periodic boundary conditions, these lanes are dynamically varying. Their number depends on the width of the street [11.31, 11.36], on pedestrian density, and on the noise level. Interestingly, one finds a *noise-induced ordering* [11.44, 11.45]: compared to small noise amplitudes, medium ones result in a more pronounced segregation (i.e. a smaller number of lanes), while large noise amplitudes lead to a 'freezing by heating' effect (see Fig. 11.1b).

The conventional interpretation of lane formation assumes that pedestrians tend to walk on the side which is prescribed in vehicular traffic. However, the above model can explain lane formation even without assuming a preference for *any* side [11.39, 11.44]. The most relevant point is the higher relative velocity of pedestrians walking in opposite directions. As a consequence, they have more frequent interactions until they have segregated into separate lanes. The resulting collective pattern of motion minimizes the frequency and strength of avoidance manoeuvres, if fluctuations are weak. Assuming identical desired velocities $v_i^0 = v_0$, the most stable configuration corresponds to a state with a minimization of interactions

$$-\frac{1}{N}\sum_{i\neq j}\tau \boldsymbol{f}_{ij}\cdot \boldsymbol{e}_i^0 \approx \frac{1}{N}\sum_{i}(v_0 - \boldsymbol{v}_i\cdot \boldsymbol{e}_i^0) = v_0(1-E)\,. \qquad (11.4)$$

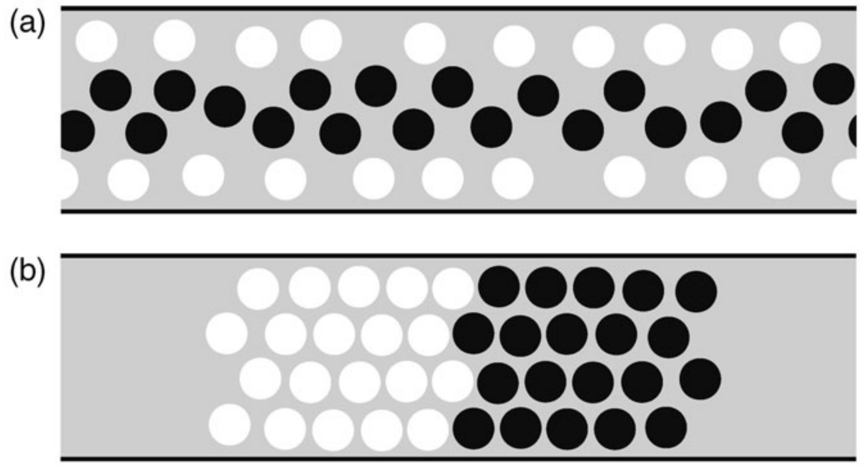


Fig. 11.1. (**a**) Formation of lanes in initially disordered pedestrian crowds with opposite walking directions and small noise amplitudes η_i (after [11.38, 11.39, 11.43]; cf. also [11.29–11.37, 11.44]). White disks represent pedestrians moving from left to right, black ones move the other way round. Lane formation does not require the periodic boundary conditions applied above; see the Java applet `http://www.helbing.org/Pedestrians/Corridor.html`. (**b**) For sufficiently high densities and large fluctuations, we observe the noise-induced formation of a crystallized, 'frozen' state (after [11.38, 11.39, 11.43])

It is related to a maximum efficiency

$$E = \frac{1}{N} \sum_i \frac{\boldsymbol{v}_i \cdot \boldsymbol{e}_i}{v_0} \tag{11.5}$$

of motion corresponding to *optimal self-organization* [11.44]. The efficiency E with $0 \le E \le 1$ (where $N = \sum_\alpha 1$ is the respective number of pedestrians α) describes the average fraction of the desired speed v_0 with which pedestrians actually approach their destinations. That is, lane formation 'globally' maximizes the average velocity into the respectively desired direction of motion, although the model does not even assume that pedestrians would try to optimize their behavior *locally*. This is a consequence of the symmetrical interactions among pedestrians with opposite walking directions. One can even show that a large class of driven many-particle systems, if they self-organize at all, tend to globally optimize their state [11.44].

To reflect the effect of getting nervous in panic situations, one can assume that the individual level of fluctuations is given by

$$\eta_i = (1 - n_i)\eta_0 + n_i \eta_{\max} , \tag{11.6}$$

where n_i with $0 \le n_i \le 1$ measures the nervousness of pedestrian i. The parameter η_0 means the normal and $\eta_{\max}$ the maximum fluctuation strength.

It turns out that, at sufficiently high pedestrian densities, lanes are destroyed by increasing the fluctuation strength (which is analogous to the temperature). However, instead of the expected transition from the 'fluid' lane state to a disordered, 'gaseous' state, a solid state is formed [11.39]. It is characterized by a blocked situation with a regular (i.e. 'crystallized' or 'frozen') structure so that we call this paradoxical transition *freezing by heating* (see Fig. 11.1b). Notably enough, the blocked state has a *higher* degree of order, although the internal energy is *increased* and the resulting state is *metastable* with respect to structural perturbations such as the exchange of oppositely moving particles. Therefore, 'freezing by heating' is just opposite to what one would expect for equilibrium systems, and different from fluctuation-driven ordering phenomena in metallic glasses and some granular systems [11.46–11.48], where fluctuations lead from a disordered *metastable* to an ordered *stable* state. A rather general model for related *noise-induced ordering* processes has been recently developed [11.45].

The preconditions for the unusual freezing-by-heating transition are the additional driving term $v_i^0 \boldsymbol{e}_i^0/\tau_i$ and the dissipative friction $-\boldsymbol{v}_i/\tau_i$, while the sliding friction force is not required. Inhomogeneities in the channel diameter or other impurities which temporarily slow down pedestrians can further this transition at the respective places. Finally note that a transition from fluid to blocked pedestrian counter flows is also observed, when a critical particle density is exceeded [11.39, 11.49].

11.4.2 Transition to Incoordination due to Clogging

At bottlenecks such as doors, one observes jamming and, in cases of counterflows, additional oscillations of the passing direction (see Fig. 11.8). The simulated outflow from a room is well-coordinated and regular, if the desired velocities $v_i^0 = v_0$ are normal. However, for desired velocities above 1.5 m/s, i.e. for people in a rush, we find an irregular succession of arch-like blockings of the exit and avalanche-like bunches of leaving pedestrians, when the arches break (see Fig. 11.2a, b). This phenomenon is compatible with the empirical observations mentioned above and comparable to intermittent clogging found in granular flows through funnels or hoppers [11.41, 11.42] (although this has been attributed to *static* friction between particles without remote interactions, and the transition to clogging has been observed for small enough openings rather than for a variation of the driving force).

11.4.3 'Faster-is-Slower Effect' due to Impatience

Since clogging is connected with delays, trying to move faster (i.e. increasing v_i^0) can cause a smaller average speed of leaving, if the friction parameter κ is large enough (see Fig. 11.2c, d). This 'faster-is-slower effect' is particularly

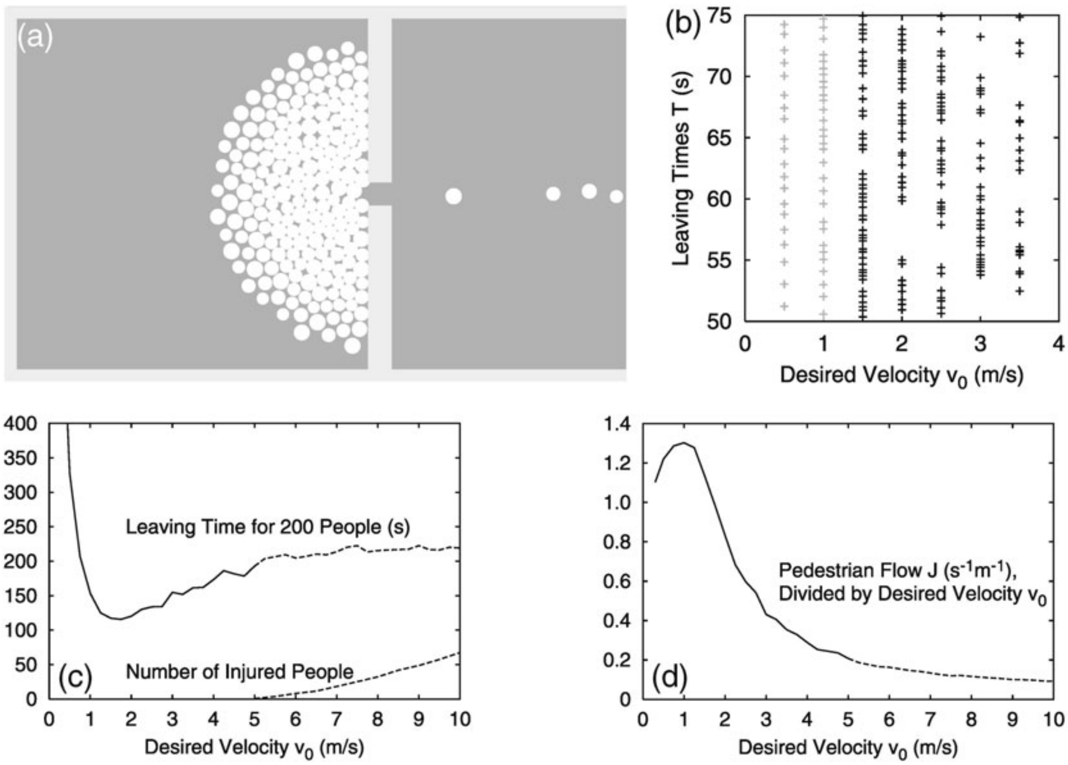


Fig. 11.2. Simulation of pedestrians moving with identical desired velocity $v_i^0 = v_0$ towards the 1 m wide exit of a room of size 15 m × 15 m (from [11.40]). (**a**) Snapshot of the scenario. Dynamic simulations are available at `http://angel.elte.hu/~panic/`. (**b**) Illustration of leaving times of pedestrians for various desired velocities v_0. Irregular outflow due to clogging is observed for high desired velocities ($v_0 \geq 1.5$ m/s, see dark plusses). (**c**) Under conditions of normal walking, the time for 200 pedestrians to leave the room decreases with growing v_0. (**d**) Desired velocities higher than 1.5 m/s reduce the efficiency of leaving, which becomes particularly clear when the outflow J is divided by the desired velocity. This is due to pushing, which causes additional friction effects. Moreover, above a desired velocity of about $v_0 = 5$ m/s (- -), people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of 1 600 N/m [11.24]

tragic in the presence of fires, where fleeing people reduce their own chances of survival. The related fatalities can be estimated by the number of pedestrians reached by the fire front (see Fig. 11.3).

Since our friction term has, on average, no deceleration effect in the crowd, if the walls are sufficiently remote, the arching underlying the clogging effect requires a *combination* of several effects:

1. slowing down due to a bottleneck such as a door and
2. strong inter-personal friction, which becomes dominant when pedestrians get too close to each other.

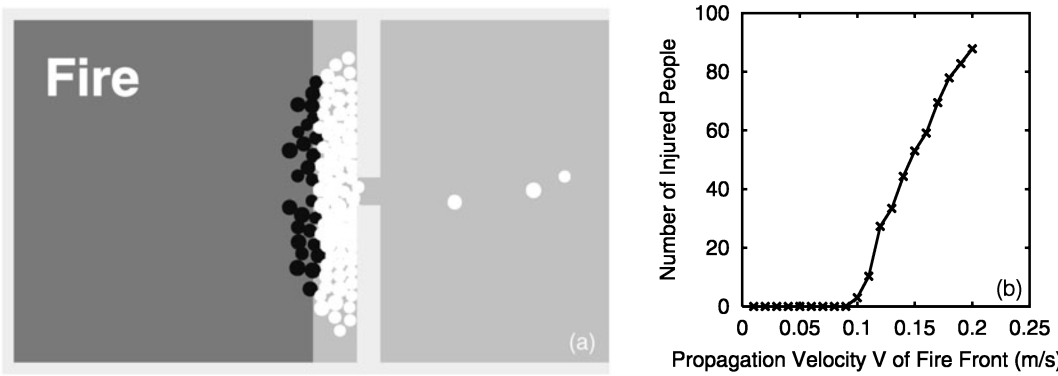


Fig. 11.3. Simulation of $N = 200$ individuals fleeing from a linear fire front, which propagates from the left to the right wall with velocity V, starting at time $t = 5\,\mathrm{s}$ (for a Java simulation applet, see `http://angel.elte.hu/~panic/`). (**a**) Snapshot of the scenario for a $15\,\mathrm{m} \times 15\,\mathrm{m}$ large room with one door of width 1 m. The fire is indicated by dark gray color. Pedestrians reached by the fire front are injured and symbolised by black disks, while the white ones are still active. The psychological effect of the fire front is assumed 10 times stronger than that of a normal wall ($A_F = 10A_i$). (**b**) Number of injured persons (casualities) as a function of the propagation velocity V of the fire front, averaged over 10 simulation runs. Up to a critical propagation velocity V_{crit} (here: about 0.1 m/s), nobody is injured. However, for higher velocities, we find a fast increase of the number of casualties with increasing V. The transition is continuous

Consequently, the danger of clogging can be minimised by avoiding bottlenecks in the construction of stadia and public buildings. Notice, however, that jamming can also occur at widenings of escape routes! This surprising result is illustrated in Fig. 11.4 and originates from disturbances due to pedestrians, who expand in the wide area because of their repulsive interactions or try to overtake each other. These squeeze into the main stream again at the end of the widening, which acts like a bottleneck and leads to jamming. *Significantly improved outflows can be reached by columns placed asymmetrically in front of the exits, which also prevent the build up of fatal pressures* (see Fig. 11.5 and the Java applets at `http://angel.elte.hu/~panic/`).

11.4.4 'Phantom Panics'

Sometimes, panics have occured *without* any comprehensible reasons such as a fire or another threatening event (e.g. in Moscow, 1982; Innsbruck, 1999). Due to the 'faster-is-slower effect', panics can be triggered by small pedestrian counterflows [11.11], which cause delays to the crowd intending to leave. Consequently, stopped pedestrians in the back, who do not see the reason for the temporary slowdown, are getting impatient and pushy. In accordance with observations [11.29, 11.36], one may describe this by increasing the desired velocity, for example, by the formula

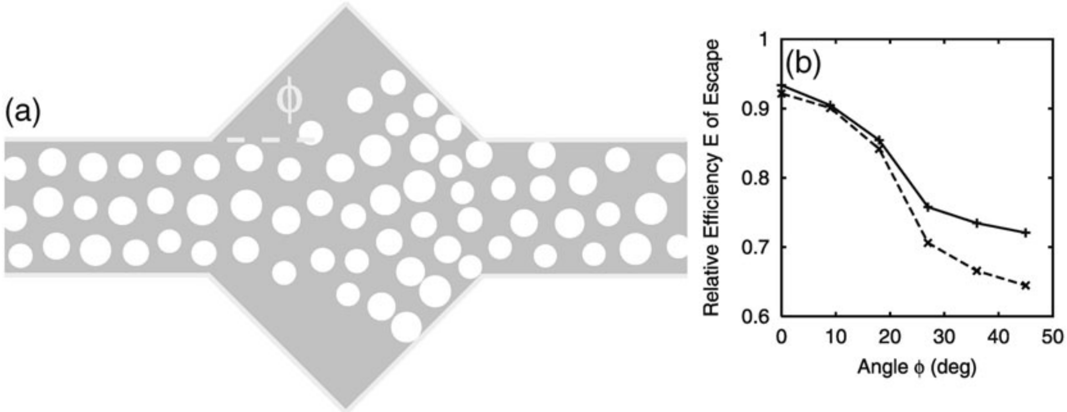


Fig. 11.4. Simulation of an escape route with a wider area (from [11.40], see also the Java applets supplied at `http://angel.elte.hu/~panic/`). (**a**) Illustration of the scenario with $v_i^0 = v_0 = 2\,\mathrm{m/s}$. The corridor is 3 m wide and 15 m long, the length of the triangular pieces in the middle being $2{\times}3\,\mathrm{m} = 6\,\mathrm{m}$. Pedestrians enter the simulation area on the left-hand side with an inflow of $J = 5.5\,\mathrm{s}^{-1}\mathrm{m}^{-1}$ and flee towards the right-hand side. (**b**) Efficiency of leaving as a function of the angle ϕ characterizing the width of the central zone, i.e. the difference from a linear corridor. The relative efficiency $E = \langle \boldsymbol{v}_i \cdot \boldsymbol{e}_i^0 \rangle / v_0$ measures the average velocity along the corridor compared to the desired velocity and lies between 0 and 1 (—). While it is almost one (i.e. maximal) for a linear corridor ($\phi = 0$), the efficiency drops by about 20%, if the corridor contains a widening. This becomes comprehensible, if we take into account that the widening leads to disturbances by pedestrians, who expand in the wide area due to their repulsive interactions or try to overtake each other, and squeeze into the main stream again at the end of the widening. Hence, the right half of the illustrated corridor acts like a bottleneck and leads to jamming. The drop of efficiency E is even more pronounced, (i) in the area of the widening where pedestrian flow is most irregular (- -), (ii) if the corridor is narrow, (iii) if the pedestrians have different or high desired velocities, and (iv) if the pedestrian density in the corridor is higher

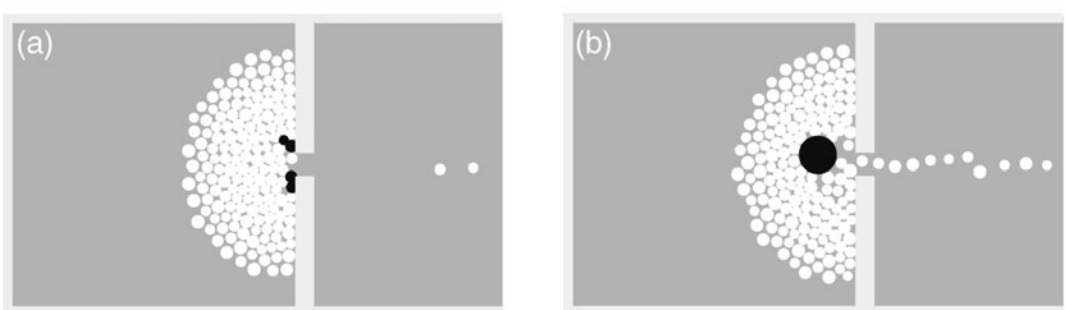


Fig. 11.5. (**a**) In panicking crowds, high pressures build up due to physical interactions. This can injure people (black disks), who turn into obstacles for other pedestrians trying to leave (see also the lower curve in Fig. 11.2c). (**b**) A column in front of the exit (large black disk) can avoid injuries by taking up pressure from behind. It can also increase the outflow by 50%. In large exit areas used by many hundred people, several randomly placed columns are needed to subdivide the crowd and the pressure. An asymmetric configuration of the columns is most efficient, as it avoids equilibria of forces which may temporarily stop the outflow

$$v_i^0(t) = [1 - n_i(t)]v_i^0(0) + n_i(t)v_i^{\max} . \tag{11.7}$$

Herein, $v_i^{\max}$ is the maximum desired velocity and $v_i^0(0)$ the initial one, corresponding to the expected velocity of leaving. The time-dependent parameter

$$n_i(t) = 1 - \frac{\overline{v}_i(t)}{v_i^0(0)} \tag{11.8}$$

reflects the nervousness, where $\overline{v}_i(t)$ denotes the average speed into the desired direction of motion. Altogether, long waiting times increase the desired velocity, which can produce inefficient outflow. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger so high pressures that people are crushed or falling and trampled. It is, therefore, imperative, to have sufficiently wide exits and to prevent counterflows, when big crowds want to leave [11.40].

11.4.5 Mass Behavior

Finally, we investigate a situation in which pedestrians are trying to leave a smoky room, but first have to find one of the invisible exits (see Fig. 11.6a). Each pedestrian i may either select an individual direction $\boldsymbol{e}_i$ or follow the average direction $\langle \boldsymbol{e}_j^0(t)\rangle_i$ of his neighbors j in a certain radius R_i [11.50,11.51], or try a mixture of both. We assume that both options are weighted with the nervousness n_i:

$$\boldsymbol{e}_i^0(t) = \mathcal{N}\left[(1 - n_i)\,\boldsymbol{e}_i + n_i\,\langle \boldsymbol{e}_j^0(t)\rangle_i\right] , \tag{11.9}$$

where $\mathcal{N}(\boldsymbol{z}) = \boldsymbol{z}/\|\boldsymbol{z}\|$ denotes normalization of a vector $\boldsymbol{z}$. As a consequence, we have individualistic behavior if n_i is low, but herding behavior if n_i is high.

Our model suggests that neither individualistic nor herding behavior performs well (see Fig. 11.6b). Pure individualistic behavior means that each pedestrian finds an exit only accidentally, while pure herding behavior implies that the complete crowd is eventually moving into the same and probably blocked direction, so that available exits are not efficiently used, in agreement with observations. According to Figs. 11.6b, c we expect optimal chances of survival for a certain mixture of individualistic and herding behavior, where individualism allows some people to detect the exits and herding guarantees that successful solutions are imitated by the others. If pedestrians follow the walls instead of 'reflecting' at them, we expect that herd following causes jamming and inefficient use of doors as well (see Fig. 11.2), while individualists moving in opposite directions obstruct each other.

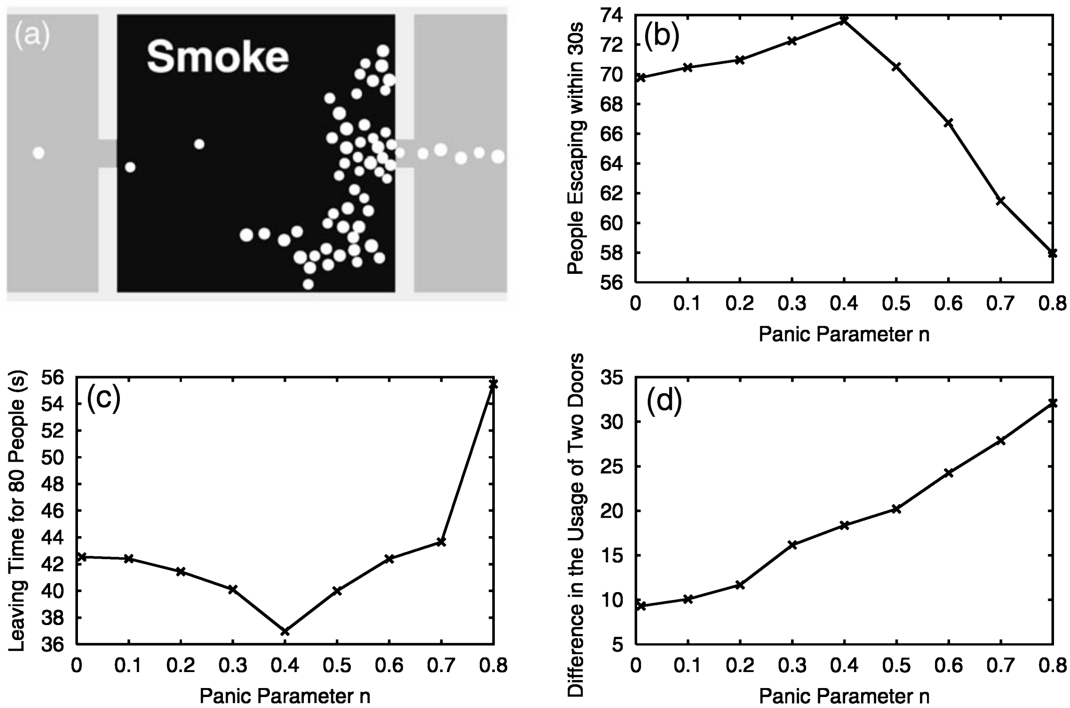


Fig. 11.6. Simulation of $N = 90$ pedestrians trying to escape a smoky room of area $A = 15\,\mathrm{m} \times 15\,\mathrm{m}$ (black) through two smoke-hidden doors of 1.5 m width, which have to be found with a mixture of individualistic and herding behavior (from [11.40]). Java applets are available at `http://angel.elte.hu/~panic/`. (**a**) Snapshot of the simulation with $v_i^0 = v_0 = 5\,\mathrm{m/s}$. Initially, each pedestrian selects his or her desired walking direction randomly. Afterwards, a pedestrian's walking direction is influenced by the average direction of the neighbors within a radius of, for example, $R_i = R = 5\,\mathrm{m}$. The strength of this herding effect grows with increasing nervousness parameter $n_i = n$ and increasing value of $h = \pi R^2 \rho$, where $\rho = N/A$ denotes the pedestrian density. When reaching a boundary, the direction of a pedestrian is reflected. If one of the exits is closer than 2 m, the room is left. (**b**) Number of people who manage to escape within 30 s as a function of the nervousness parameter n. (**c**) Illustration of the time required by 80 individuals to leave the smoky room. If the exits are relatively narrow and the degree n of herding is small or large, leaving takes particularly long, so that only some of the people escape before being poisoned by smoke. Our results suggest that the best escape strategy is a certain compromise between following of others and an individualistic searching behavior. This fits well into experimental data on the efficiency of group problem solving [11.52–11.54], according to which groups normally perform better than individuals, but masses are inefficient in finding new solutions to complex problems. (**d**) Absolute difference $|N_1 - N_2|$ in the numbers N_1 and N_2 of persons leaving through the left exit or the right exit as a function of the degree n of herding. We find that pedestrians tend to jam up at one of the exits instead of equally using all available exits, if the nervousness is large

11.4.6 Problem Solving Behavior

In contrast to the results presented in the previous paragraphs, the discussion in the following two paragraphs will be partly speculative in nature.

The above herding model could be viewed as paradigm for problem-solving behavior in science, economics, and politics, where new solutions to complex problems have to be found in a way comparable to finding the exit of a smoky

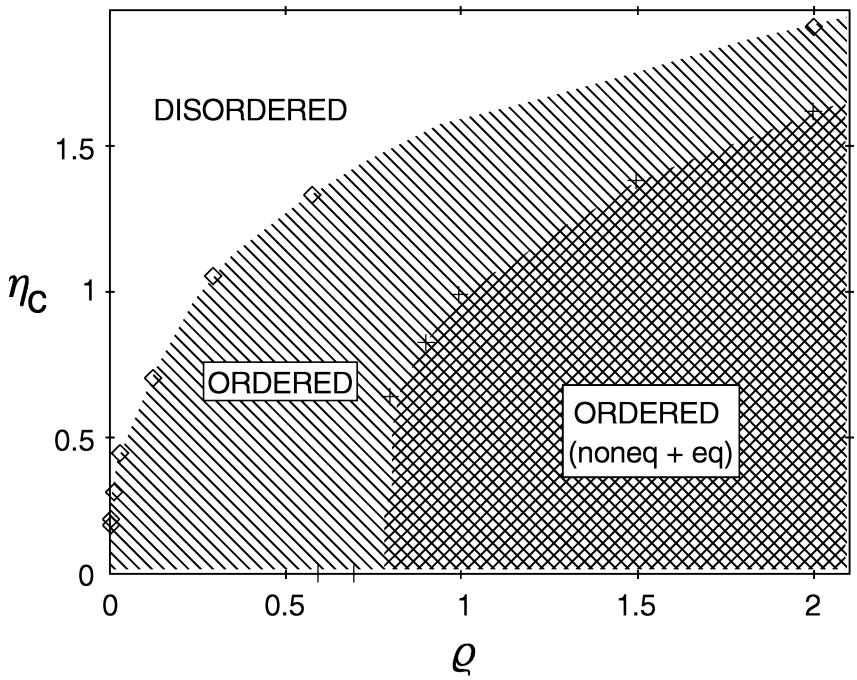


Fig. 11.7. Phase diagram of a self-propelled particle (SPP) model and the corresponding equilibrium system. The nonequilibrium SPP system with $v_i^0 = v_0 > 0$ becomes ordered in the whole region below the curve connecting the diamonds. In the static equilibrium case with $v_i^0 = v_0 = 0$, the ordered region extends only up to a finite 'percolation' density, see the beginning of the area given by the curve connecting the plus signs (after [11.51])

room. From the simulation results, we may conclude that people will not manage to cope with a sequence of challenging situations, if everyone sticks to his own idea egocentrically, but they will also fail, if everyone follows the same idea, so that the possible spectrum of solutions is not adequately explored. Therefore, the best strategy appears to be pluralism with a reasonable degree of readiness to follow good ideas of others, while a totalitarian regime would probably not survive a series of crises. This fits well into experimental data on the efficiency of group problem solving [11.52–11.54], according to which groups normally perform better than individuals, but masses are inefficient in finding new solutions to complex problems.

Considering the phase diagram in Fig. 11.7, we may also conjecture that, in the presence of a given level $\eta_i = \eta$ of fluctuations, there is a certain critical density above which people tend to show (ordered) mass behavior, and below which they behave individualistically (disordered). In fact, the tendency of mass behavior is higher in dense populations than in dilute ones.

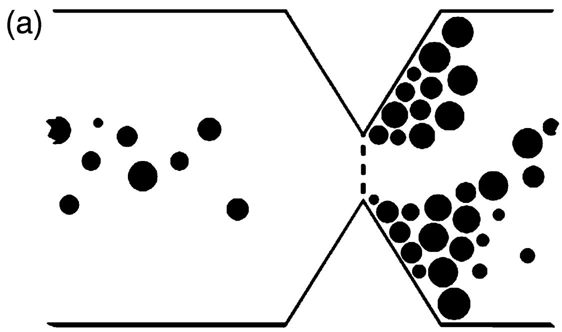
11.4.7 Comparison with Stock Markets

Although the situation seems to be more difficult [11.55], our model might also contribute to a better understanding of other forms of panics like the ones observed at 'crashes' of stock markets [11.5, 11.10, 11.23, 11.56–11.63] (regarding this topic see also Chaps. 12-14). Such cases may again be viewed as situations with dwindling resources, in which herding behavior occurs and the collective run for a reasonable return (corresponding to the 'exit') decreases the chances of everyone to achieve it, because of competitive (analogous to 'repulsive') interactions. In other words, the reinforcement of buying and selling decisions has sometimes features of herding behavior, which is reflected in bubbles and crashes of stock markets (see, e.g. [11.61–11.63]). Remember that herding behavior is frequently found in complex situations, where individuals do not know what is the right thing to do. Everyone counts on collective intelligence then, believing that a crowd is following someone who has identified the right action. However, as has been shown for panicking pedestrians seeking for an exit, such mass behavior can have undesirable results, since alternatives are not efficiently exploited.

Another analogy between stock markets and self-driven many-particle systems are the irregular oscillations of stock prices and of pedestrian flows at bottlenecks (see Fig. 11.8). Presently it is unknown whether this analogy is just by chance, but there could be similar mechanisms at work: at the stock exchange market we have a competition of two different groups, optimistic traders ('bulls') and pessimistic ones ('bears'). The optimists count on growing stock prices and increase the price by their orders. In contrast, pessimists speculate on a decrease in the price and reduce it by their selling of stocks. Hence, traders belonging to the same group enforce each others' (buying or selling) actions, while optimists and pessimists push (the price) into opposite directions. Consequently, the situation is partly comparable with opposite pedestrian streams pushing at a bottleneck, which is reflected in a roughly similar dynamics, see Fig. 11.8b [11.38, 11.43]. The mechanism leading to alternating pedestrian flows is the following: once somebody is able to pass the narrowing, pedestrians with the same walking direction can easily follow. Hence, the number and 'pressure' of waiting and pushing pedestrians becomes less than on the other side of the narrowing where, consequently, the chance to occupy the passage grows. This leads to a deadlock situation which is followed by a change in the passing direction.

11.5 Conclusions

In summary, we have developed a continuous pedestrian model based on plausible interactions, which is, due to its simplicity, robust with respect to parameter



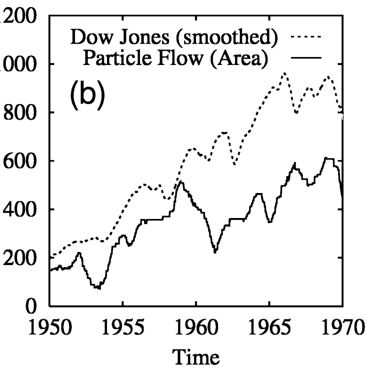


Fig. 11.8. (**a**) In pedestrian counterflows, one observes oscillations of the passing direction at bottlenecks (after [11.38,11.43], see also [11.30–11.37]). (**b**) The time-dependent difference in the area of particles that have passed the bottleneck from the left-hand side or right-hand side, respectively, looks similar to the dynamics of the Dow Jones index. The question whether this is just by chance or due to a deeper relationship of both systems, is subject to present research

variations and suitable for drawing conclusions about the possible mechanisms beyond escape panic (regarding an increase of the desired velocity, strong friction effects during physical interactions, and herding). After having calibrated the model parameters to available data on pedestrian flows, we managed to reproduce many observed phenomena including

1. the breakdown of fluid lanes ('freezing by heating'),
2. the build up of pressure,
3. clogging effects at bottlenecks,
4. jamming at widenings,
5. the 'faster-is-slower effect',
6. 'phantom panics' triggered by counterflows and impatience, and
7. inefficient use of alternative exits.

We are also able to simulate situations of dwindling resources and estimate the casualties (see Figs. 11.2c and 11.3). Therefore, the model can be used to test buildings for their suitability in emergency situations. It accounts for the considerably different dynamics both in normal and panic situations just by changing a single parameter $n_i = n$. In this way, we have proposed a consistent theoretical approach allowing a continuous switching between seemingly incompatible kinds of human behavior (individualistic, rational behavior vs. irrational herding behavior).

Moreover, the model can serve as an example linking collective behavior as a phenomenon of mass psychology (from the socio-psychological perspective) to the view of an emergent collective pattern of motion (from the perspective of physics). Our simulations suggest that the optimal behavior in escape situations is a suitable mixture of individualistic and herding behavior. This conclusion is probably transferable to many cases of problem solving in new and complex situations, where standard solutions fail. It may explain why both individualistic and herding behaviors are common in human societies. The crucial point, however, is to find the optimal mixture between them.

Finally, we point out that conclusions from findings for self-driven many-particle systems reach far into the realm of the social, economic, and psychological sciences. One of the reasons is that the competition of moving particles for limited space is analogous to the situation in various socio-economic and biological systems, where individuals or other entities compete for limited resources as well.

Acknowledgements. The authors are grateful to the Collegium Budapest-Institute for Advanced Study for the warm hospitality and the excellent scientific working conditions. D.H. thanks the German Research Foundation (DFG) for financial support by the Heisenberg scholarship He 2789/1-1. T.V. and I.F. are grateful for partial support by OTKA and FKFP. Last but not least, Tilo Grigat has helped with formatting this manuscript.

References

11.1 G. LeBon, *The Crowd* (Viking, New York, 1960).
11.2 B.D. Jacobs and P. 't Hart, in *Hazard Management and Emergency Planning*, edited by D.J. Parker and J.W. Handmer (James and James Science, London, 1992), Chap. 10.
11.3 N.J. Smelser, *Theory of Collective Behavior* (The Free Press, New York, 1963).
11.4 R. Brown, *Social Psychology* (The Free Press, New York, 1965).
11.5 D.L. Miller, *Introduction to Collective Behavior* (Wadsworth, Belmont, CA, 1985), Fig. 3.3 and Chap. 9.
11.6 J.S. Coleman, *Foundations of Social Theory* (Belkamp, Cambridge, MA, 1990), Chaps. 9 and 33.
11.7 R.H. Turner and L.M. Killian, *Collective Behavior*, 3rd edn. (Prentice Hall, Englewood Cliffs, 1987).
11.8 A. Mintz, J. Abnorm. Norm. Soc. Psychol. **46**, 150 (1951).
11.9 E. Quarantelli, Sociol. and Soc. Res. **41**, 187 (1957).
11.10 J.P. Keating, Fire J., 57+147 (May/1982).
11.11 D. Elliott and D. Smith, Ind. Env. Crisis Q. **7**(3), 205 (1993).
11.12 N.R. Johnson, Soc. Probl. **34**(4), 362 (1987).
11.13 K.H. Drager, G. Løvås, J. Wiklund, H. Soma, D. Duong, A. Violas, and V. Lanèrès, in *Proc,1992 Emergency Manage. Eng. Conf.* (Society for Computer Simulation, Orlando, Florida, 1992), p. 101.
11.14 M. Ebihara, A. Ohtsuki, and H. Iwaki, Microcomput. Civ. Eng. **7**, 63 (1992).
11.15 G.K. Still, Fire **84**, 40 (1993).
11.16 J.L. Bryan, Fire J., 27+86 (Nov./1985).
11.17 R. Axelrod and D. Dion, Science **242**, 1385 (1988).
11.18 N.S. Glance and B.A. Huberman, Sci. Am. **270**, 76 (1994).
11.19 W. Weidlich, Phys. Rep. **204**, 1 (1991).
11.20 D. Helbing, *Quantitative Sociodynamics. Stochastic Methods and Models of Social Interaction Processes* (Kluwer, Dordrecht, 1995).
11.21 W. Weidlich, *Sociodynamics* (Harwood, Amsterdam, 2000).
11.22 D. Helbing, H.J. Herrmann, M. Schreckenberg, and D.E. Wolf (eds.), *Traffic and Granular Flow '99: Social, Traffic, and Granular Dynamics* (Springer, Berlin, 2000).
11.23 F. Schweitzer and D. Helbing (eds.), *Economic Dynamics from the Physics Point of View*, Physica A **287**, 339 (2000).

11.24 R.A. Smith and J.F. Dickie (eds.), *Engineering for Crowd Safety* (Elsevier, Amsterdam, 1993).
11.25 H.H. Kelley, J.C. Condry Jr., A.E. Dahlke, and A.H. Hill, J. Exp. Soc. Psychol. **1**, 20 (1965).
11.26 D. Canter (ed.), *Fires and Human Behaviour* (David Fulton, London, 1990).
11.27 W.M. Predtetschenski and A.I. Milinski, *Personenströme in Gebäuden, Berechnungsmethoden für die Projektierung* (Rudolf Müller, Köln-Braunsfeld, 1971).
11.28 U. Weidmann, *Transporttechnik der Fußgänger* (Institut für Verkehrsplanung, Transporttechnik, Straßen- und Eisenbahnbau (IVT), ETH Zürich, 1993).
11.29 D. Helbing, Behav. Sci. **36**, 298 (1991).
11.30 D. Helbing, P. Molnár, and F. Schweitzer, in *Evolution of Natural Structures* (Sonderforschungsber. 230, Stuttgart, 1994), p. 229
11.31 D. Helbing and P. Molnár, Phys. Rev. E **51**, 4282 (1995).
11.32 D. Helbing, in *Traffic and Granular Flow*, edited by D.E. Wolf, M. Schreckenberg, and A. Bachem (World Scientific, Singapore, 1996), p. 87.
11.33 P. Molnár, *Modellierung und Simulation der Dynamik von Fußgängerströmen* (Shaker, Aachen, 1996).
11.34 P. Molnár, in *Social Science Microsimulation*, edited by J. Doran, N. Gilbert, U. Mueller, and K. Troitzsch (Springer, Berlin, 1996).
11.35 D. Helbing and P. Molnár, in *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, edited by F. Schweitzer (Gordon and Breach, London, 1997), p. 569.
11.36 D. Helbing, *Verkehrsdynamik* (Springer, Berlin, 1997).
11.37 D. Helbing, P. Molnár, I. Farkas, and K. Bolay, Env. Planning B **28**, 361 (2001).
11.38 D. Helbing, Rev. Mod. Phys. **73**(4), 1067 (2001).
submitted to Rev. Mod. Phys. (2000).
11.39 D. Helbing, I. Farkas, and T. Vicsek, Phys. Rev. Lett. **84**, 1240 (2000).
11.40 D. Helbing, I. Farkas, and T. Vicsek, Nature **407**, 487 (2000).
11.41 G.H. Ristow and H. J. Herrmann, Phys. Rev. E **50**, R5 (1994).
11.42 D.E. Wolf and P. Grassberger (eds.), *Friction, Arching, Contact Dynamics* (World Scientific, Singapore, 1997).
11.43 D. Helbing, Phys. Bl. **57**, 27 (2001).
11.44 D. Helbing and T. Vicsek, New J. Phys. **1**, 13.1 (1999).
11.45 D. Helbing and T. Platkowski, Int. J. Chaos Theory Appl. **5**, 25 (2000).
11.46 J. Gallas, H.J. Herrmann, and S. Sokołowski, Phys. Rev. Lett. **69**, 1371 (1992).
11.47 P.B. Umbanhowar, F. Melo, and H.L. Swinney, Nature **382**, 793 (1996).
11.48 A. Rosato, K.J. Strandburg, F. Prinz, and R.H. Swendsen, Phys. Rev. Lett. **58**, 1038 (1987).
11.49 M. Muramatsu, T. Irie, and T. Nagatani, Physica A **267**, 487 (1999).
11.50 T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Phys. Rev. Lett. **75**, 1226 (1995).
11.51 A. Czirók, M. Vicsek, and T. Vicsek, Physica A **264**, 299 (1999).
11.52 N.H. Anderson, J. Soc. Psychol. **55**, 67 (1961).
11.53 H.H. Kelley and J.W. Thibaut, in *The Handbook of Social Psychology*, Vol. 4, edited by G. Lindzey and E. Aronson (Addison-Wesley, Reading, 1969).
11.54 P.R. Laughlin, N.L. Kerr, J.H. Davis, H.M. Halff, and K.A. Marciniak, J. Personality Soc. Psychol. **31**, 522 (1975).
11.55 L. Mann, T. Nagel, and P. Dowling, Sociometry **39**(3), 223 (1976).
11.56 J.-P. Bouchaud, A. Matacz, and M. Potters, The leverage effect in financial markets: retarded volatility and market panic (2001); [available from `arXiv.org/abs/cond-mat/0101120v2`].
11.57 R.N. Mantegna and E. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, England, 1999).
11.58 J.-P. Bouchaud and M. Potters, *Theory of Financial Risk: From Statistical Physics to Risk Management* (Cambridge University Press, Cambridge, England, 2000).
11.59 H. Levy, M. Levy, and S. Solomon, *Microscopic Simulation of Financial Markets* (Academic, San Diego, 2000).
11.60 B.B. Mandelbrot, *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk* (Springer, New York, 1997).
11.61 M. Youssefmir, B.A. Huberman, and T. Hogg, Comp. Econ. **12**, 97 (1998).
11.62 J.D. Farmer, Market force, ecology, and evolution, submitted to J. Econ. Behav. Org. (2000); [available from `arXiv.org/abs/adap-org/9812005`].
11.63 T. Lux and M. Marchesi, Nature **397**, 498 (1999).