Water Resour Manage (2013) 27:3663–3678
DOI 10.1007/s11269-013-0372-6

Multi-Objective Design Optimization of Branched Pipeline Systems with Analytical Assessment of Fire Flow Failure Probability

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Received: 16 February 2012 / Accepted: 15 May 2013 /
Published online: 28 May 2013

Abstract This paper develops a multi-objective optimization approach for incorporating the conditional probability of fire flow failure into the design of branched water networks. To this end, a new analytical probabilistic model was developed to quantify the conditional probability of fire flow failure in branched networks and incorporated into the non-dominated sorting genetic algorithm (NSGA-II). The optimization sought to minimize capital cost through pipe diameter and pump selection and to minimize the conditional probability of fire flow failure. The NSGA-II was applied to two branched networks to generate Pareto-optimal solutions. Results indicated a strategic allocation of pipe and pump capacity with limited fiscal resources and with a reduction in uncertainty of fire flow failure. Interestingly, optimization results for a real branched network supported the industry practice of using a minimum 150 mm distribution main sizing to provide fire flow protection.

Keywords Water distribution · Failure probability · Uncertainty · Fire flow · Optimization · Design

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1 Introduction

Water distribution systems (WDSs) are designed and operated to consistently and economically deliver water from source to consumer in sufficient quantity, of acceptable quality, and at appropriate pressure. Traditionally, after choosing a set of critical loading conditions – typically the greater of peak-hour demand or peak-day demand and a chosen fire flow – optimization methods are applied to select the most economical set of pipe, pump, and tank sizes that produces the desired range of pressures in the network. The rationale underlying the economical design is that, by selecting the smallest possible component sizes set to minimize the overall cost, pressures are marginally above an acceptable level for the specified assumed deterministic design loading conditions. However, because both the design loads and fire flow conditions are often subject to uncertainty, the satisfaction of the minimum required pressures is not certain; as a direct result, the conventional deterministic system design could be inappropriate under unexpected/unusual demand conditions, either by being over-designed (easily meeting the required conditions but at too great a cost) or under-designed (failing to meet the required conditions).

One important source of uncertainty in hydraulic design of networks originates from the estimation of needed fire flow. Needed fire flow is defined as the fire flow necessary to control a fire in a specific building type. This is determined by various estimation methods, typically through guidelines developed for this purpose (e.g., Insurance Services Office 2009; Fire Underwriters Survey 1999; AWWA 1999). Generally, fire flow estimation methods calculate needed fire flow from information on construction type, building area, combustibility of building occupancy, inter-building fire exposure and communication, and the presence of fire protection measures (e.g., automatic fire sprinkler systems). However, all of these methods include uncertainty as to whether the needed fire flow estimated is sufficiently large to control the fires that may actually occur in the network. For example, the Insurance Services Office method commonly used in the USA has a documented uncertainty of ±8 L/s on estimated fire flows of 32 L/s commonly used for residential construction (AWWA 1999). Fire Underwriters Survey and other methods have comparable levels of uncertainty. Uncertainty in needed fire flow means that designers have an (understandably) incomplete picture of fire protection and risk levels associated with the sizing of local distribution mains chosen in the hydraulic design of a network.

There is thus an incentive or potential value to incorporate explicit estimates of failure probabilities associated with unacceptable hydraulic capacity during fire flow conditions in the optimization of water distribution network design. Such a characterization of failure probability could be important for at least two reasons: i) hydraulic design of local water distribution mains (150–300 mm) is often governed by fire flow considerations, and ii) water utilities and their insurers could benefit by having access to information about fire flow failure probabilities associated with master planning decisions concerning the sizing of local distribution mains in new developments. Such knowledge is valuable in selecting the decision variables (e.g., pipe sizes) since it would provide a better basis for choice and, accordingly, reduce the overall uncertainty of the fire flow failure.

The aim of this paper is to develop a multi-objective optimization approach that incorporates the conditional probability of fire flow failure in branched water distribution networks. A new analytical probabilistic model is developed to quantify the conditional probability of fire flow failure in branched networks and incorporated into the non-dominated sorting genetic algorithm (NSGA-II) to minimize two conflicting objectives: capital cost and the conditional probability of fire flow failure. Capital cost and fire flow failure probability are balanced through the selection of pipe diameters and the number of pumps. In this study, the conditional probability of fire flow failure is solved analytically in a branched network. The non-dominated sorting genetic algorithm (NSGA-II) is used to produce a set of Pareto-optimal solution in the objective space of pipe and pump cost and the fire flow failure probability.

Of course the approach adopted here leaves the paper open to the obvious criticism that real networks are seldom built as in a tree-like topology. However, the theoretical treatment and full understanding of fire flow failure probability in branched networks are crucially important for three reasons. First, it represents one of the few theoretical optimization and probabilistic analyses that can be carried out "exactly" (within the framework of branched system) without using simplifying assumptions and approximations. An understanding of the method of analysis and the results obtained provides a foundation from which to carry out more complicated looped network analyses. To the authors' knowledge, no previous paper has presented an analytical assessment of the fire flow failure probability in pipeline systems. Second, compared to other existing approaches (Filion et al. 2007; Filion and Jung 2010), the analytical probabilistic approach is computationally efficient since it does not require any numerical approximation. This point is made clearer in the literature review that follows. Third, in practical applications, looped networks are often skeletonized into branched systems for a modelling simplicity. Here, skeletonization is widely applied to reduce a large water distribution network model to a manageable size ready for analysis. It makes use of database management and hydraulic equivalency theory to reduce excessive pipe segmentation while maintaining the hydraulic behaviour of the larger original model. For example, parallel pipes can be merged into a single hydraulically-equivalent pipe with the same carrying capacity; hence the original looped network can be skeletonized into a branched network in many cases and the resulting branched network is hydraulically equivalent to the larger original looped model so the fire flow failure probability can be quantified with the analytical approach of this paper. An example of such an application is presented in Case Study #2.

The paper is organized as follows. First, previous research in risk-based multi-objective optimization of water distribution networks is reviewed briefly. Analytical probabilistic approach is then developed to estimate fire flow failure probabilities in branched networks. Next, the multi-objective optimization approach that incorporates the fire flow failure probability is outlined. The non-dominated sorting genetic algorithm (NSGA-II) by Deb et al. (2002) is then adapted to include fire flow failure probability. Lastly, the NSGA-II algorithm is tested on a dual branch network and a more complex branch-skeletonized network. The results are discussed in the context of water main sizing for fire flow protection in real networks.

2 Research Advances in Optimization of Water Distribution Networks

Many optimization algorithms have been applied to solve the water distribution network design problem such as gradient-based mathematical optimization methods (Alperovits and Shamir 1977; Bhave 1985), Genetic Algorithm optimization (Simpson et al. 1994; Dandy et al. 1996; Kadu et al. 2008), Ant Colony Optimization (Maier et al. 2003; Ostfeld and Tubaltzev 2008), Shuffled Frog Leaping Algorithm (Eusuff and Lansey 2003), and Particle Swarm Optimization (Jung and Karney 2008 and 2009). These algorithms have been widely applied and have provided efficient computational procedures. These applications include: i) finding low-cost solutions in the design and/or expansion of new and existing water distribution networks (Alperovits and Shamir 1977; Bhave 1985; Simpson et al. 1994;

Dandy et al. 1996; Maier et al. 2003; Eusuff and Lansey 2003; Kadu et al. 2008), ii) minimizing energy consumption in pump operation (Ostfeld and Tubaltzev 2008), iii) calibrating a hydraulic network model against field data (Jung and Karney 2008), and iv) identifying the most severe transient loadings through simulation and choosing the most suitable surge protection strategy (Jung and Karney 2009). Recently, multi-objective optimization algorithms have been used to solve water distribution system problems with multiple, conflicting criteria or design objectives (Kapelan et al. 2003; Prasas et al. 2004; Prasas and Park 2004; Farmani et al. 2005; Jeong and Abraham 2006; Nicolini and Zovatto 2009; Giustolisi et al. 2009). Unlike in single-objective optimization where a single 'best' solution is found that minimizes (or maximizes) an objective function, multi-objective optimization produces a set of non-dominated or Pareto-optimal solutions that balance multiple objectives.

There has been a significant amount of research completed in risk-based optimization in the last twenty years. Tung (1986) first formulated the least-cost, chance-constrained design problem by considering uncertainty in demands, minimum required pressure head at nodes, and uncertainty in pipe roughness with known probability density functions (PDFs). Lansey et al. (1989) and Xu and Goulter (1999) applied the generalized reduced gradient method (GRG2) in combination with the first-order, second moment (FOSM) and first-order reliability method (FORM) approaches to solve the nonlinear chance-constrained problem. To overcome local minima and maxima search problems in the GRG2 algorithm, Tolson et al. (2004) combined a simple genetic algorithm (SGA) with FORM to increase the likelihood of finding the global, optimal solution in the nonlinear least-cost, chance-constrained design problem. To eliminate the need to compute derivatives in FORM, Babayan et al. (2005, 2006, and 2007) solved the nonlinear least-cost, chance-constrained design problem by combining an SGA with a new integration-based method. In a concurrent research effort, Kapelan et al. (2004) and Babayan et al. (2006) developed a robust chance constrained GA (rccGA) to perform uncertainty propagation of demand and pipe roughness to evaluate the hydraulic robustness of a network through stochastic sampling rather than through the quasi-analytical evaluation of a multiple integral. A number of new risk-based approaches have also been developed by Yannopoulos and Spiliotis (2013) and Christodoulou (2011) that offer the potential to assess the reliability and robustness of networks.

Up to this point, methodologies to quantify the risk and economic damages caused by low-pressure heads during emergency and/or fire flow conditions had not been developed. Kapelan et al. (2006) developed a risk measure to incorporate the consequences of hydraulic failure into the network optimization problem. Filion et al. (2007) developed a risk measure that computes the expected annual damages sustained during low- and high-pressure emergency events (including fires) in a water network with Monte Carlo Simulation (MCS). Building on the work of Jung and Karney (2008 and 2009) and Filion et al. (2007), Filion and Jung (2010) developed a Particle Swarm Optimization (PSO) algorithm that incorporates a new integration-based method that estimates the expected damages sustained during fire flow conditions in the least-cost design problem. The integration-based measure developed by Filion and Jung (2010) eliminates the need to perform computationally-expensive MCS to evaluate expected annual fire damages in networks. However, the integration-based method is still computationally-expensive to calculate the mean and standard deviation and only provides an approximation of fire damages. The accuracy of the integration method depends on its numerical integration interval, meaning the approximation is more accurate but more computationally-expensive as the numerical integration interval is finer.

3 Analytical Probabilistic Approach to Estimate Conditional Probability of Fire Flow Failure in Branched Networks

In this paper, an analytical probabilistic approach is developed to estimate the conditional probability of fire flow failure in a branched distribution network. Unlike the integration-based method of Filion and Jung (2010), the analytical approach is carried out exactly without using simplifying assumptions and approximations and it is computationally very efficient. A prototypical branched pipeline is indicated in Fig. 1. In this system, pumps draw water from a water source (e.g., reservoir, clearwell) with water level of H_s and apply a pumping head of h_p to convey water to downstream demand locations. The maximum day demand at node j is Q_j, and the needed fire flow at node j is Q_{fj} (Fig. 1). The available pressure head (H_j) at the time of a fire event at node j and the minimum required pressure head (H_{fj}) at node j during the fire flow are indicated in Fig. 1. The needed fire flow is modelled as a random variable to reflect its uncertainty and assumed to be normally distributed for the case studies follow. It is further assumed that a single fire demand occurs at any time in the system and that the maximum day demand exists at the time of the fire event. (In reality, there is a joint probability distribution of domestic and fire demands. This problem is the topic of subsequent research).

The available pressure head H_j during a fire at node j with needed fire flow Q_{fj} is calculated as

$$\begin{aligned} H_j &= H_s + h_p - h_{L1} \; \cdots \; - h_{Li} \\ &= H_s + A - B\Big/n^2\left(Q_{fj} + \sum_{k=1}^{N_n} Q_k\right)^2 - K_1\left(Q_{fj} + \sum_{k=1}^{N_n} Q_k\right)^2 \cdots - K_i\left(Q_{fj} + \sum_{k=j}^{N_n} Q_k\right)^2 \end{aligned} \tag{1}$$

where h_p=head supplied by n identical parallel pumps; A and B=coefficients of the quadratic pump characteristic curve expressed as $hp=A–B(Q/n)^2$; n=number of identical parallel pumps; N_n=number of nodes in the network; h_{Li}=head loss in pipe i; K_i= $f_iL_i/2gD_iA_i^2$ represents the hydraulic resistance of pipe i; f_i, L_i, D_i and A_i are Darcy-Weisbach friction factor, pipe length, the pipe diameter and the pipe cross-sectional area of pipe i, respectively. The pumping head value of h_p can be further extended to any configuration (series/parallel or combination) of pumps with identical pump curves.

The conditional probability of fire flow failure is defined as the probability that the available pressure head at node j (H_j) is below the minimum required pressure head (H_{fj}) during a fire at node j. The conditional probability means the probability that the fire is at node j given that there is a fire in the network. This is computed by evaluating the cumulative distribution function (CDF) of available pressure head at the minimum required residual pressure head H_{fj}, such that

$$F_{H_j}\left(H_{fj}\right) = P\left[H_j \le H_{fj}\right] \tag{2}$$

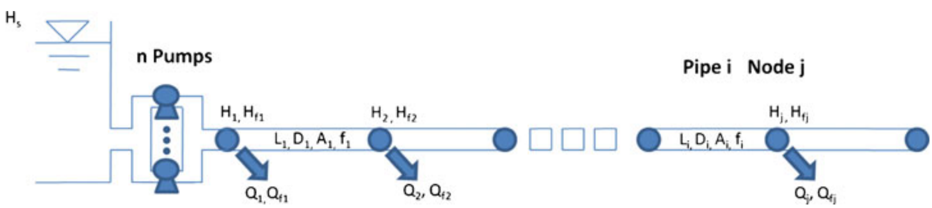

Fig. 1 Branched Network with Source, Parallel Pump Installation, Water Mains and Demand Locations

Combining Eq. 1 with 2 yields

$$P\left[-aQ_{fj}^2 - 2bQ_{fj} + c \leq H_{fj}\right] \tag{3}$$

where

$$a = B\Big/n^2 + \sum_{l=1}^{i} K_l,$$

$$b = B\Big/n^2 \sum_{k=1}^{N_n} Q_k + \sum_{l=1,m=2}^{i,j} K_l \sum_{k=m}^{N_n} Q_k$$

and

$$c = H_s + A - B\Big/n^2 \left(\sum_{k=1}^{N_n} Q_k\right)^2 - \sum_{l=1,m=2}^{i,j} K_l \left(\sum_{k=m}^{N_n} Q_k\right)^2$$

The solution of Eq. 3 for the fire flow Q_{fj} can be found using the quadratic formula, producing the following physical solution

$$P\left(Q_{fj} \geq \frac{-b + \sqrt{b^2 - a\left(H_{fj} - c\right)}}{a}\right) \tag{4}$$

Therefore, the CDF of pressure head is analytically solved as

$$F_{H_j}\left(H_{fj}\right) = 1 - P\left(Q_{fj} \leq \frac{-b + \sqrt{b^2 - a\left(H_{fj} - c\right)}}{a}\right) = 1 - F_{Q_{fj}}\left(\frac{-b + \sqrt{b^2 - a\left(H_{fj} - c\right)}}{a}\right) \tag{5}$$

The CDF of pressure head is used to compute the conditional probability of fire flow failure at node j of a branched network in an analytical manner. This is possible because the flow and headloss in each pipe depends in a straightforward way on all downstream demands and the fire flow in the system. In looped networks, multiple flow paths are possible and cannot be determined by simple inspection of the system; it is thus not possible to apply the analytical approach to looped networks.

4 Multi-Objective Optimization Formulation

The multi-objective optimization of a branched network under fire flow conditions is developed next. The first objective in (6) seeks to minimize the cost of pipes and pumps through the selection of pipe diameters and the number of pumps as the decision variables. The second objective seeks to minimize the fire flow probability of failure conditional on a fire flow event. All nodes are assumed to have an equivalent risk so the conditional probability is uniform. The conditional probabilities of fire flow failure at all fire flow nodes are calculated with (5) and summed up given that fire events are mutually exclusive (e.g., when fire occurs at node 1, then it does not occur at node 2 or at any other node in the system.) and uniformly distributed. The summation is normalized by the number of fire flow nodes and called “average conditional probability of fire flow failure” which can range from 0.0 to 1.0. A fire flow probability of zero represents a condition of no fire flow failure; the higher the probability, the higher the likelihood of fire flow failure.

$$\text{Minimize}\sum_{i=1}^{N_p} c(D_i) + n{\cdot}c_p \quad (\text{Pipe cost} + \text{Pump cost}) \tag{6}$$

$$\text{Minimize}\frac{1}{N_n}\sum_{j=1}^{N_n} F_{H_j}\left(H_{fj}\right)(\text{Average Conditional Probability of Fire Flow Failure}) \tag{7}$$

Subject to:

$$\sum Q_{in} - \sum Q_{out} = Q_j + Q_{fj}, j = 1, \dots, N_n \tag{8}$$

$$D_i \in \{\mathbf{D}\}, i = 1, \dots, N_p \tag{9}$$

$$H_j \geq H_{\min\ j}, j = 1, \dots, N_n, \tag{10}$$

$$V_i \leq V_i^{\max}, \quad i = 1, \dots, N_p \tag{11}$$

where D_i=discrete pipe diameters selected from the set of commercially-available pipe sizes $\{\mathbf{D}\}$; $c(D_i)$=cost of pipe i with diameter D_i; N_p=number of pipes; c_p=cost of single pump; $H_{min\ j}$=minimum required pressure during maximum hour demand condition. Equation 8 represents the continuity at all j=1, …, N_n nodes. Equation 10 requires that the nodal pressure H for any node j is equal to or greater than a specified minimum pressure $H_{\min}$ for the maximum hour demand condition. Maximum allowable fluid velocity (e.g., 3 m/s) can be applied through Eq. 11 to prevent pipe wall scouring (Walski et al. 2001).

Figure 2 depicts a flowchart of the multi-objective optimization approach. First, the NSGA-II initializes the pipe sizes and the number of pumps as decision variables, and the pipe cost is calculated. The hydraulic model then analyzes the system to check if the solution satisfies the required constraints in Eqs. 8, 9, 10 and 11. The analytical probabilistic model computes the second objective function of fire flow probability in Eq. 7. The optimization

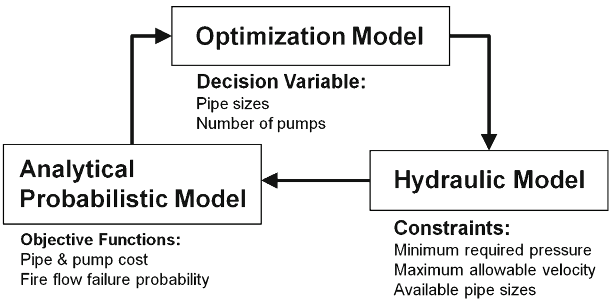

Fig. 2 Non-dominated Sorting Genetic Algorithm (NSGA-II) Multi-objective Optimization Approach

algorithm then evaluates the two objective functions and generates a new population of solutions for the next generation. This process is continued over successive generations until the optimal solution is found.

5 Case Study #1: Dual Branch Network

The multi-objective optimization approach was tested on the branched network illustrated in Fig. 3. The network comprises 1 reservoir with a fixed water elevation of 10 m, parallel pumps, 5 pipes, and 6 nodes at zero elevation. The rated flow, Q, and head gain, hp, of a single pump are set to 20 L/s and 100 m so the pump characteristic curve of the n parallel pumps is defined as $hp=133–0.083(Q/n)^2$. The length and Darcy-Weisbach roughness coefficient of all pipes are $L=1{,}000$ m and $f=0.02$, respectively. Each node provides water service to 100 single-family residences with an average 4-person occupancy and a per capita consumption rate of 400 L/day. The average day demand at each node is thus 1.9 L/s. With a maximum day demand peaking factor of 1.7, the maximum day demand at each node is 3.2 L/s. The needed fire flow to control a fire in a single-family residential unit is 32 L/s with a duration of 2 h (AWWA 1999). Needed fire flow is uncertain and is considered to be normally distributed (without loss of generality) with a mean of 32 L/s and a standard deviation of 8 L/s (coefficient of variation of 25 %). It was assumed that the reservoir storage volume is adequately sized for the 2-hour required fire flow duration. The minimum pressure required at all nodes during a fire flow event is 14 m, the maximum hour demand peaking factor is set to 3.4, and the minimum pressure required at all nodes during maximum hour demand is also 14 m.

The NSGA-II population size was set to 50 and the number of generations was set to 200. The probability of mutation was set to 0.025, the probability of uniform crossover was set to 0.9, and the length of each chromosome was set to 24. Pipe sizes were chosen from the commercially-available diameters with unit pipe costs indicated in Table 1. The cost of each pump unit was set to \$5,000. For this problem, 16 pipe sizes for five pipes and 16 maximum allowed number of pumps make up a solution space of 16^6 possible combinations. At the beginning of an optimization run, the NSGA-II initializes the population of solutions, where each solution is comprised of pipe diameters and the number of pumps in the pumping

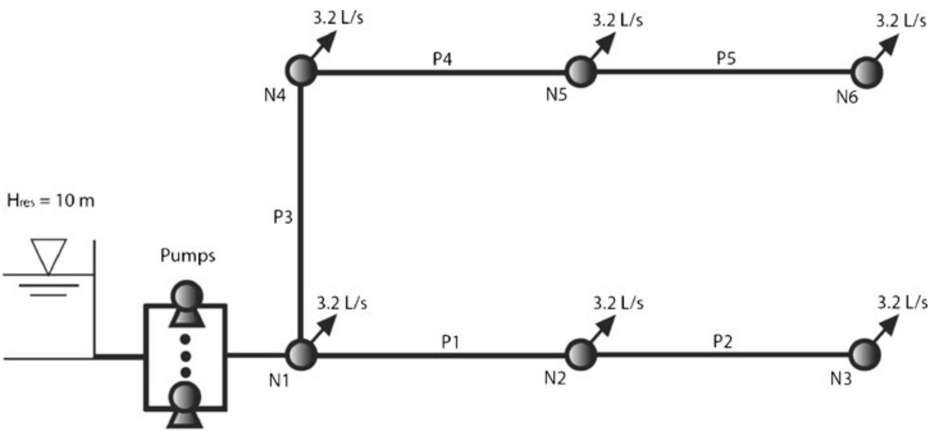

Fig. 3 Dual Branch Network with Maximum Day Nodal Demands of 3.2 L/s

Table 1 Commercially-available Pipe Diameters and Unit Costs

Pipe Diameter (mm)	Unit Cost ($/m)
50	20
100	39
150	59
200	79
250	98
300	118
350	138
400	157
450	177
500	197
550	217
600	236
650	256
700	276
750	295
800	305

station. The NSGA-II then calculates the cost of pipes and pumps and the fire flow failure probability of each solution as well as the constraint-violation errors to generate a new population in the next generation.

The solutions obtained after 200 generations are indicated in Fig. 4. The x-axis plots the total cost of pipes and pumps in Eq. 7 and the y-axis plots the fire flow failure probability in Eq. 8. Table 2 shows the optimal number of pumps, pipe diameters, total cost (pipe and pump) and fire flow failure probability for the 8 representative Pareto-optimal solutions indicated in Fig. 4. The interaction among the two objectives gives rise to a set of Pareto-optimal solutions. Each solution on the Pareto-optimal front of Fig. 4 and Table 2 is not dominated by any other solution. In going from one solution to another, it is not possible to

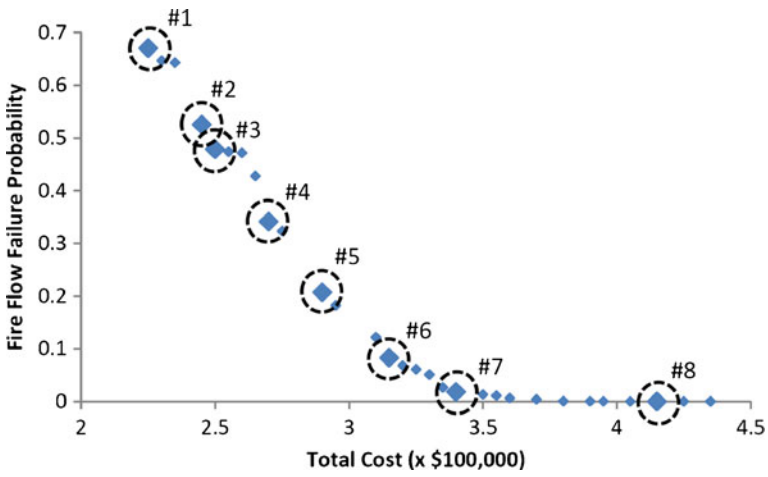

Fig. 4 Pareto-optimal Solutions for Case Study #1

Table 2 Optimal Number of Pumps, Pipe Diameters, Total Cost (pipe and pump) and Fire Flow Failure Probability for 8 Pareto-optimal Solutions

Solution	Number of pumps	Pipe diameter (mm)					Cost (x $100,000)	Fire Flow Failure Probability
		P1	P2	P3	P4	P5		
1	2	150	100	100	100	100	2.25	0.67
2	2	150	100	150	100	100	2.45	0.52
3	3	150	100	150	100	100	2.50	0.48
4	3	150	100	150	150	100	2.70	0.34
5	3	150	150	150	150	100	2.90	0.21
6	4	150	150	150	150	150	3.15	0.08
7	5	150	150	200	150	150	3.40	0.02
8	4	200	200	200	200	200	4.15	0.00

improve on the objective of minimizing the total cost without compromising the other objective of minimizing the fire flow failure probability. This trade-off relationship provides useful information on the cost-effectiveness of adding pipe and/or pump capacity to reduce the conditional probability of fire flow failures. The results in Table 2 indicate that when the total cost is increased from $225,000 to $270,000 (Solution #1 to #4), the 20 % additional investment in pipe and pump can achieve a 49 % reduction in fire flow failure probability (0.67 to 0.34). Similarly, when the pipe cost is increased from $270,000 to $415,000 (Solution #4 to #8), the 54 % additional investment in pipe and pump can reduce the fire flow probability to zero or near-zero. A decision-maker may use the information in Fig. 4 and Table 2 to evaluate the marginal rate of trade-off between system capacity/cost and fire flow probability.

Table 3 presents the conditional probability of fire flow failure at each node, calculated based on Eq. (5), for the 8 Pareto-optimal solutions indicated in Fig. 4. The results show that the conditional probability of fire flow failure increases with the distance from the upstream reservoir, as operating pressures are lower at these nodes. The lower fire flow probability thus leads the NSGA-II program to choose larger diameters (at an additional cost) near the upstream reservoir first to decrease the fire flow probability. In addition, the upstream pipe near the reservoir carries more

Table 3 Conditional Probability of Fire Flow Failure at each Node for 8 Pareto-optimal Solutions

Solution	Probability of Failure					
	N1	N2	N3	N4	N5	N6
1	0.00	0.10	0.94	0.98	1.00	1.00
2	0.00	0.10	0.94	0.14	0.97	0.99
3	0.00	0.00	0.91	0.01	0.96	0.99
4	0.00	0.00	0.91	0.01	0.18	0.94
5	0.00	0.00	0.11	0.01	0.18	0.94
6	0.00	0.00	0.05	0.00	0.10	0.34
7	0.00	0.00	0.03	0.00	0.00	0.08
8	0.00	0.00	0.00	0.00	0.00	0.00

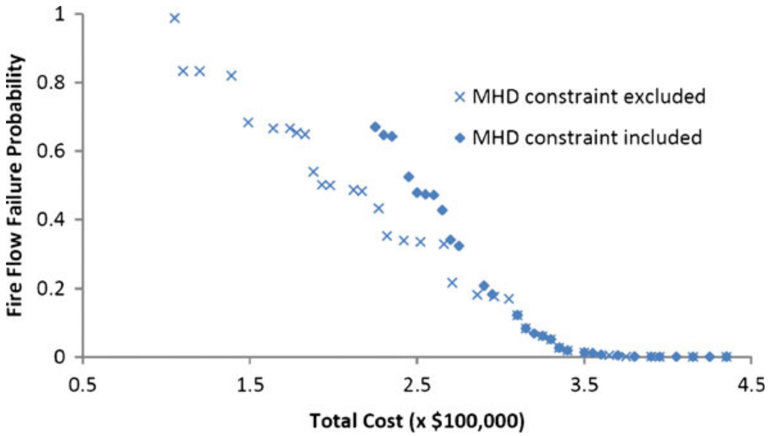

Fig. 5 Total Cost and Fire Flow Failure Probability for Pareto-front Solutions in Case Study #1 when Maximum Hour Demand (MHD) Constraint Included and Excluded from Optimization

maximum hour demand (MHD) than the downstream pipe; thus, pipe sizing in the upstream part of the system tends to be governed by the maximum hour demand. In pipes located farther away from the reservoir, fire flow in these locations is more significant relative to downstream maximum hour demands as the pipe conveys water to a smaller downstream service area. The influence of the maximum hour demand design condition on the cost and fire flow failure probability is indicated in Fig. 5. The results suggest that for a given level of fire flow failure probability, the inclusion of the maximum hour demand constraint increases total cost of the system since upstream pipes must be sized to meet the maximum hour demand design condition. For example, for a specified fire flow failure probability of 0.43 in Fig. 5, the inclusion of the maximum hour demand constraint leads to a total cost of $265,000 whereas the exclusion of the MHD constraint leads to a lower cost of $227,000.

6 Case Study #2: Branch-Skeletonized Network

The multi-objective optimization approach was applied to a larger and more complex branched network (Fig. 6). This network represents an actual water system and consists of 28 pipes, 29 junctions, parallel pumps and one source. This example was originally a looped network taken from the EPANET user's manual (Rossman 1993) and skeletonized into a branch system. Skeletonization is widely applied to reduce a large water distribution network model to a manageable size ready for analysis. Therefore, a looped network can often be skeletonized into a branched system for a modelling purpose. The resulting branched network is hydraulically-equivalent to the larger original looped model so the fire flow failure probability can be quantified in the network with the analytical approach of this paper. Table 4 summarizes the pipe and node data for the network. The 28 pipes selected for the optimization run were lumped into 13 distinct groups (Table 5) based on similar characteristics (e.g., location) to reduce the number of decision variables (13 pipe groups and a number of parallel pumps). Pipes within the same group were assigned identical pipe diameters in the optimization. The population size and the number of generations were increased to 200 and 400, respectively. The other system parameters (e.g., pump

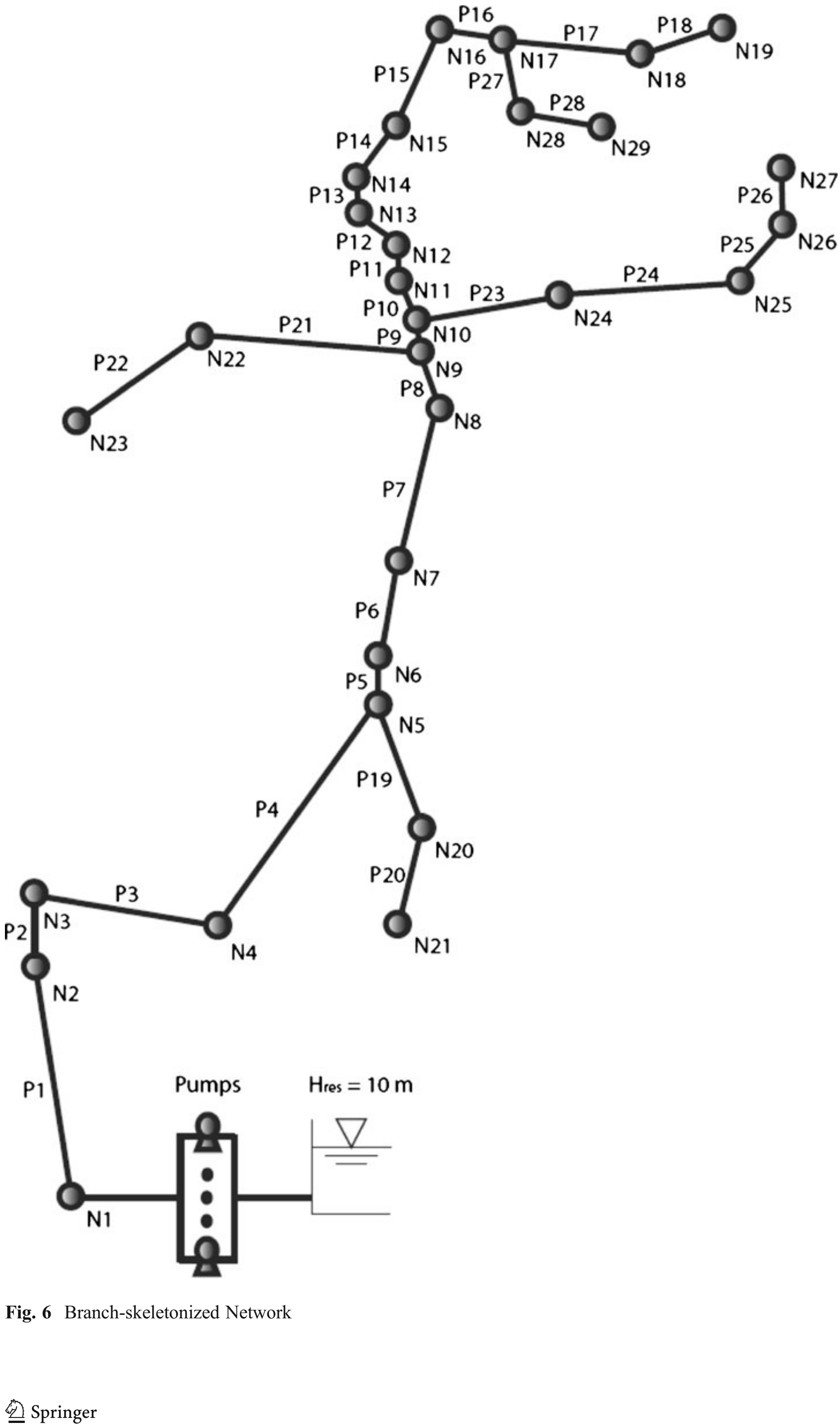

Fig. 6 Branch-skeletonized Network

Table 4 Pipe and node data for branch-skeletonized network in case study #2

Pipe Number	Length, m	Node Number	Elevation, m	Demand, L/s
P1	731.5	N1	15.2	0.0
P2	243.8	N2	30.5	1.4
P3	365.7	N3	30.5	1.0
P4	822.9	N4	38.1	0.3
P5	121.9	N5	48.8	0.3
P6	213.4	N6	54.9	0.9
P7	579.1	N7	56.4	2.2
P8	182.9	N8	64.0	1.0
P9	121.9	N9	64.0	0.1
P10	91.4	N10	61.0	0.1
P11	76.2	N11	57.9	0.1
P12	182.9	N12	57.9	0.7
P13	91.4	N13	70.1	0.5
P14	182.9	N14	70.1	0.4
P15	121.9	N15	57.9	1.1
P16	121.9	N16	39.6	0.5
P17	213.4	N17	33.5	0.4
P18	91.4	N18	33.5	0.0
P19	365.7	N19	33.5	0.1
P20	304.8	N20	33.5	0.6
P21	457.2	N21	39.6	0.3
P22	426.7	N22	45.7	2.5
P23	335.3	N23	45.7	2.7
P24	396.2	N24	51.8	1.7
P25	304.8	N25	61.0	1.1
P26	121.9	N26	54.9	0.1
P27	152.4	N27	57.9	0.1
P28	304.8	N28	33.5	0.0
		N29	39.6	0.2

characteristic curve) and optimization parameters (e.g., mutation probability) are identical to those chosen in Case Study #1.

The solutions obtained after 400 generations are indicated in Fig. 7. Table 5 indicates the optimal pipe diameters, number of pumps, total cost (pipe and pump) and fire flow failure probability for the 8 representative Pareto-optimal solutions indicated in Fig. 7. Similar to the results of the first case study, reducing total cost comes at the expense of increasing fire flow failure probability. This causes the total cost to increase from \$372,000 to \$687,000 and the fire flow failure probability to decrease from 0.61 to 0.00 between Solutions 1 to 8 in Table 5 and Fig. 7. The results in Table 5 suggest that the optimization approach allocates large pipe diameters (250–300 mm) near the water source to meet maximum hour demand conditions and then adds pipe capacity in downstream locations to reduce the conditional probability of fire flow failures in low-pressure sites located far away from the water source. This interaction between total

Table 5 Optimal Pipe Diameters, Number of Pumps, Total Cost (pipe and pump) and Fire Flow Failure Probability for 8 Pareto-optimal Solutions

Pipe diameter (mm)	Solutions							
	1	2	3	4	5	6	7	8
P1, P2	250	250	250	250	250	250	250	300
P3, P4	250	250	250	250	250	250	250	250
P5, P6	150	200	200	200	200	200	200	200
P7, P8	150	150	200	200	200	200	200	200
P9	150	150	150	150	200	200	150	200
P10, P11, P12, P13	150	150	150	150	150	150	150	200
P14, P15, P16	150	150	150	150	150	150	150	200
P17, P18	50	50	150	100	150	150	150	200
P19, P20	50	50	50	50	50	50	150	150
P21, P22	100	100	100	100	100	150	150	200
P23, P24	100	100	100	150	150	150	150	200
P25, P26	50	50	50	50	150	150	150	200
P27, P28	50	50	50	100	100	150	150	200
Number of pumps	4	6	4	5	4	4	5	8
Cost (x $100,000)	3.72	3.90	4.19	4.51	4.96	5.32	5.70	6.87
Fire flow failure probability	0.61	0.52	0.41	0.30	0.18	0.08	0.02	0.00

cost and fire flow failure probability is important for water utilities in making master planning decisions concerning the sizing of distribution mains in new development. Solutions #7 and #8 also suggest that a minimum 150 mm pipe diameter is required to reduce the fire flow failure probability to a near-zero level. The results support the industry practice of adopting a minimum 150 mm pipe diameter for local distribution mains to provide fire flow protection.

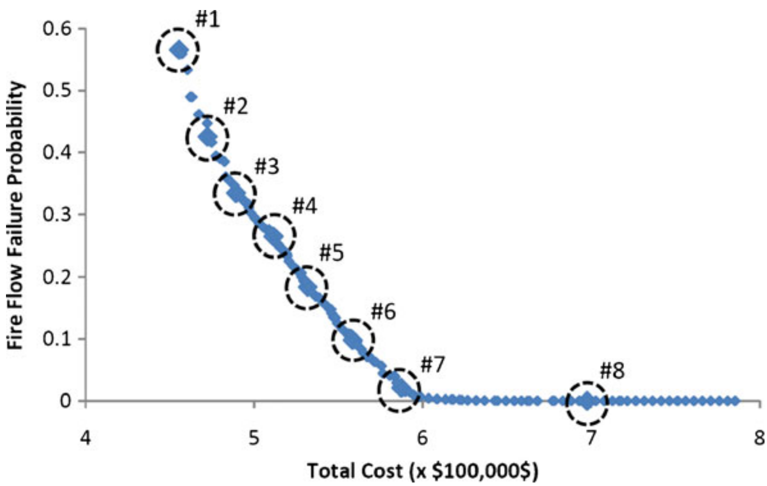

Fig. 7 Pareto-optimal Solutions for Case Study #2

7 Summary

This paper presents a multi-objective optimization approach to incorporate the conditional probability of fire flow failure with the cost of adding new pipes and pumps in branched water distribution networks. An analytical probabilistic model was developed to quantify the conditional probability of fire flow failure in branched networks and then incorporated into the non-dominated sorting genetic algorithm (NSGA-II). The probabilistic analysis is carried out exactly within the framework of a branched system without using simplifying assumptions and approximation so that it is computationally more efficient than other existing numerical approaches. Optimization objectives were formulated to minimize capital cost through pipe sizing and pump selection and to minimize the conditional probability of fire flow failure. The optimization program was applied to a 5-pipe dual branch network and a 28-pipe branch-skeletonized network to generate Pareto-optimal solutions for total cost and fire flow failure probability. The case studies demonstrated that fronts of Pareto-optimal solutions provides useful information to decision-makers on the cost-effectiveness of adding pipe and/or pump capacity to reduce fire flow failure probability in branched networks. A comparison of solutions where the maximum hour demand constraint was included and excluded suggested that the designs of upstream pipes near the source reservoir are governed by maximum hour demand design conditions and downstream pipes are governed by fire flow considerations. Finally, optimization results supported the industry practice of using a minimum 150 mm size for local distribution mains to provide fire flow protection.

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