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Airline crew pairing with fatigue: Modeling and analysis

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A B S T R A C T

Crew fatigue is one of the main causes of airline accidents. Regulatory authorities such as the Federal Aviation Agency constantly introduce new fatigue regulations, often in the form of hard constraints on the length of duty and rest periods. The complex nature of travel-related fatigue, however, makes it difficult to account for it indirectly through such constraints. Recent studies show that fatigue depends on human factors such as the homeostatic process and the circadian body clock as well as time-zone differences. In this work, we explicitly account for fatigue in crew pairing optimization through the Three Process Model of Alertness, one of the most comprehensive fatigue models available in the literature. We provide a mathematical model for the crew pairing problem that incorporates fatigue and solve it using a column generation approach. Numerical analysis on two real data sets reveals that the proposed approach is able to reduce the crew fatigue levels substantially with minimal impact on cost. In particular, it is shown that hard constraints on fatigue may still lead to high fatigue levels and that jet-lag and time-zone differences have a major impact. The results of the tests also show that some of the rules and regulations in practice may be omitted if the fatigue is accounted for directly.

1. Introduction

Although air transportation is one of the fastest and safest mode of transportation, in the rare event of an accident, losses - both humanitarian and financial - are devastating (Evans, 2003; Evans, 2003; Loeb et al., 1994). According to Caldwell (2005), the total cost of a single major civil aviation accident can exceed $500 million. Several investigations have identified fatigue-related factors as major causes of aviation accidents. According to Goode (2003), the number of accidents per pilot is proportional to the length of his/her duty periods and directly correlates with his/her fatigue level. Furthermore, using official data, Caldwell (2005) shows that in at least 4–8% of aviation accidents, fatigue is involved. For example, in the "Flight CRX 3597" accident, the Swiss Aircraft Accident Investigation Bureau found that fatigue has negatively affected the pilot's ability to concentrate and make appropriate decisions and impaired his capacity to analyze complex processes (BEAA, 2001). According to the US National Transportation Safety Board, more than 300 fatalities are attributed to fatigue, some of which are direct causes of long duty periods, circadian disruptions, and sleep loss (Avers and Johnson, 2011).

In light of the direct relation between crew fatigue and accidents, the U.S. Federal Aviation Administration (FAA) and similar regulatory authorities are constantly introducing and enforcing new rules and regulations to implicitly account for fatigue. These rules are often hard constraints on the length of duty periods, the minimum break time between duties, etc. Up until now, the research literature has sought to model existing regulations; which are hard constraints. The research

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literature has not sought to model fatigue directly, which would require more complex models. (Suraweera et al., 2013) uses a constraint learning system based on machine learning to infer minimum rest crew scheduling constraints based on historical data.

Although traditional models include these strict rules, judging by the increasing focus on fatigue-related safety, it is clear that the current rules are not sufficient to model the complex nature of fatigue. This paper is an attempt to model fatigue explicitly and incorporate it in crew pairing optimization. We present a model that captures the trade-off between operating costs and fatigue levels by explicitly accounting for fatigue in the objective function. Instead of assuming that fatigue is a function of duty periods, we use a comprehensive model to account for circadian body clock, time-zone differences, and homeostatic processes.

The airline crew pairing problem is the first step in a sequential two-step process in crew scheduling. The second step is crew rostering. The crew scheduling problem is tackled after an airline generates schedules, makes fleet assignment, and decides on maintenance routing. The airline crew pairing problem decomposes by fleet type, because cockpit crews are usually qualified to operate only one fleet type.

Barnhart et al. (2003) give a comprehensive survey of airline crew scheduling. They describe the terminology used in the industry, examine different cost structures, and provide a set partitioning formulation for the problem. Denoting the set of flight legs by L and the set of all possible feasible pairings by P, and using binary decision variables θ_p that is equal to 1 if pairing p is selected and 0 otherwise, the set partitioning formulation for the airline crew pairing problem is:

$$[P]: \min \quad \sum_{p \in P} c_p \theta_p$$

$$\text{s.t.} \quad \sum_{p \in P} a_{ip} \theta_p = 1 \quad i \in L \tag{1}$$

$$\theta_p \in \{0, 1\} \quad p \in P, \tag{2}$$

where a_{ip} is 1 if pairing p contains flight i. Column generation is commonly used to solve problem [P], where the pairing generation subproblem is a Resource Constrained Shortest Path (RCSP) problem. The subproblem can be solved using a label setting algorithm by considering the label of a node as a cost-resource vector (Desaulniers et al., 1997; Desaulniers et al., 1998 and Desrosiers et al., 1995). The subproblem includes constraints on the time spent away from base and minimum/-maximum duty and flight breaks that implicitly account for fatigue. In this work, however, we try to model fatigue explicitly.

According to Brown (1994), fatigue is defined as the "Reduced capacity to perform mentally or physically taxing work, or the subjective situation in which the person cannot perform a task anymore, and it results from inadequate sleep, circadian rhythm disruptions and/or time spent at work". Caldwell (2005) and Graeber (1988) showed that the reason that domestic and international pilots experience fatigue is largely because of sleep irregularities, night flights, early duty starts, and long duty periods. Furthermore, Graeber et al. (1986) argued that multiple time zone changes that are experienced during long flights can lead to the desynchronization of the circadian body clock. This desynchronization results from the difficulty of the 24-h sleep/wake cycle to swiftly readjust to the rapid change of "time givers", such as sunlight, that is caused by multiple time zone changes (Arendt et al., 2000 and Graeber, 1988). This desynchronization is shown to result in sleep disruption and decreased alertness (Caldwell and Caldwell, 2003 and Gander et al., 1998).

In a survey conducted by Bourgeois-Bougrine et al. (2003), international pilots identified night flights (59%) and jet-lag (45%) as the primary causes of fatigue. They also confirmed that fatigue results in reduced attention and increased difficulty in accomplishing certain tasks. A detailed literature on fatigue is given by Petrilli et al. (2006).

Although regulations are in place to minimize crew fatigue, Caldwell (2005) and Dawson and Fletcher (2001) argue that current fatigue-related regulations take duty time and sleep into account, but fail to account for the sleep levels before the start of a duty. According to Petrilli et al. (2006), higher levels of fatigue have been reported by commercial airline pilots at the end of international flights and the amount of sleep obtained in the 24 h prior to the end of the flights seems to be a significant indicator of fatigue. A study similar to Ahlstrom et al. (2013) to assess pilots' fitness for duty prior to each flight will help clarify the severity of fatigue that pilots experience.

The literature is rich with crew pairing formulations that include the strict rules and regulations provided by aviation agencies (e.g. Desrosiers et al., 1995; Desaulniers et al., 1997; Lu and Gzara, 2014, Gürkan et al., 2016). In a report by Jeppesen, it was shown that current aviation rules by themselves are ineffective to account for fatigue (Olbert and Klemets, 2011). In a report by Romig and Klemets (2009), Boeing and Jeppesen work on a fatigue measuring scheme. It is not clearly stated how they account for fatigue, but using an interface called CAPI (Common Alertness Prediction Interface), they were able to measure the fatigue level for flights and use it at the pairing selection stage. To our knowledge, there is no academic work that tries to explicitly tackle the fatigue issue in airline crew pairing. This work is an attempt in that direction.

To explicitly account for fatigue in a mathematical model, a quantitative approach is needed. There are several published models to calculate the fatigue level of a person. The Three Process Model of Alertness (TPM) created by Åkerstedt and Folkard (1990) is a comprehensive model that calculates alertness recovered during sleep, alertness lost during wake, and the effect of the 24-h circadian body clock. Since its introduction, several extensions were introduced. Åkerstedt et al. (2007) provided another function to account for the effect of the 12-h circadian body clock. Åkerstedt et al. (2008) provided a brake function that modifies the alertness recovery during sleep. According to this function, the sleep is broken into two parts: deep sleep and normal sleep. The recovery rate during deep sleep is higher than that during normal sleep. There is also a sleep generator function provided by Åkerstedt and Folkard (1996) that can be used to generate the times that a person falls asleep, transitions to normal sleep, and wakes up, when such data is not available. Lastly, Ingre et al. (2014) provided a function that introduces the effect of time-zone differences to the model, and conducted detailed tests to validate it. In this work, the complete and most up-to-date version of TPM is used. It accounts for both homeostatic and circadian processes, and includes the effect of jet-lag. In fact it is the only model of its kind that accounts for jet-lag.

The main contributions of this work are a comprehensive crew pairing optimization model with a realistic, explicit, and detailed account for fatigue; a solution methodology based on column generation where the pairing generation subproblem is a shortest path problem with fatigue that is solved using a label setting algorithm; and a detailed numerical analysis based on real data. The analysis shows that the proposed approach achieves lower fatigue levels compared with the current industry practice that is based on hard constraints, and that some of the hard constraints may be removed. In addition, it demonstrates that accounting for fatigue directly and removing some of the strict regulations leads to lower fatigue levels and financial costs.

The rest of the paper is organized as follows: The details of the Three Process Model of Alertness are discussed in Section 2. The mathematical model that explicitly models fatigue is discussed in Section 3. Section 4 presents a column-generation solution method for the crew pairing with fatigue model. The discussion about the selection of parameters and the comparison between different variations of the model are given in Section 5, together with the results of the computational experiments. Additionally, an analysis of the results and comments on how fatigue affects the building of pairings are given in the section. Concluding remarks are given in Section 6.

2. The fatigue function

We use the Three Process Model of Alertness due to Åkerstedt and Folkard (1990) to measure the accumulated fatigue endured by crew members. The model considers two body processes: the homeostatic and the circadian. The homeostatic body process, which is denoted by S, determines the alertness lost while a person is awake and the alertness regained during sleep. The circadian process depicts the effect of the body clock on alertness. This process is divided in two parts, a 24-h and a 12-h circadian process denoted C and U, respectively. The two processes are calculated over time using a set of equations (Åkerstedt and Folkard, 1990). The alertness level is given by $S + C + U$, which is transformed to fatigue using the linear transformation to sleepiness measurement standard, known as the Karolinska Sleepiness Scale (KSS) (Ingre et al., 2014):

$$f = a_f + b_f(S + C + U). \tag{3}$$

Parameters a_f and b_f are experimentally determined as 10.6 and -0.6, respectively. The details of calculating each process are given next.

The homeostatic process S relates to the change of alertness during a person being awake or asleep. It is separated in two parts. The first part calculates the effect of the homeostatic process while a person is awake (decay in alertness):

$$S(S_w, \Delta T_{awake}) = la + (S_w - la)e^{(d)(\Delta T_{awake})}.$$

where S_w refers to the value of the process S when the person wakes up, while ΔT_{awake} refers to the time elapsed since waking up. Parameters $d = -0.0353, la = 2.4$ and $ha = 14.3$ refer to the decay rate in alertness, the lowest and the highest levels of alertness, respectively.

The second is the homeostatic process while the person is sleeping (regain of alertness):

$$S'(S_s, \Delta T_{asleep}) = \begin{cases} S'_1(S_s, \Delta T_{asleep}), & \text{if } S \leqslant bl, \\ S'_2(\Delta T_{asleep}, bt), & \text{otherwise.} \end{cases} \tag{4}$$

It is broken down into two equations to calculate the different levels of fatigue recovery during different stages of sleep:

$$S'_1(S_s, \Delta T_{asleep}) = S_s + g(bl - ha)\Delta T_{asleep}, \tag{5}$$

$$S'_2(\Delta T_{asleep}, bt) = ha - (ha - bl)e^{(\Delta T_{asleep} - bt)g}, \tag{6}$$

$$bt(S_s) = (bl - S_s)/((bl - ha)g). \tag{7}$$

The process S'_1 depicts the higher recovery rate of fatigue in the early stages of sleep, while S'_2 depicts the lower recovery rate in the later stages. S_s is the value of S when the person falls asleep, while ΔT_{asleep} refers to the time the person spends sleeping. Parameter g is the fatigue recovery rate per unit time and its value is -0.3813. Parameter bl is the alertness level that breaks the function into two parts, and its value is 12.2. This means that when $S < bl = 12.2$, the person will recover from fatigue at a higher rate. The function bt provides the time point at which the sleep stage changes, i.e., the process used changes from S'_1 to S'_2, and is calculated using Eq. (7).

The impact of time-zone differences on alertness is calculated as a function of time spent in locations that have different time-zones using the following equations:

$$A_t(TZ_t, A_{t-1}) = A_{t-1} + (1 - (daily)^{(T_t - T_{t-1})})(TZ_t - A_{t-1}), \tag{8}$$

where T_t refers to the time, TZ_t refers to the time-zone of the location at time t, A_t is the jet-lag effect at time t, and *daily* refers to the daily rate of jet-lag recovery while the person is at the home-base. The value of *daily* is 0.5.

The effect of the body clock on the fatigue/alertness levels is calculated using:

$$C(A_t, T_t) = C_m + C_a cos((2\pi/24)(T_t - p - A_t)), \tag{9}$$
$$U(A_t, T_t) = U_m + U_a cos((2\pi/12)(T_t - p - A_t - 3)), \tag{10}$$

where (9) and (10) determine the 24-h and the 12-h circadian processes, respectively. C_m is the mesor (the circadian rhythm adjusted mean) and C_a is the amplitude of the process. They have the values of 0 and 2.5, respectively. U_m is the mesor and U_a is the amplitude, and their values are −0.5 and 0.5 respectively. Parameter p is the circadian phase and it has the value of 16.8. T_t is the time at which the processes are calculated.

It must also be noted that there are certain sleeping and waking up thresholds. In normal conditions, the person falls asleep when $S + C + U \leqslant 8.38$ and wakes up when $S + C + U \geqslant 11.38$. Note that although the transition between deep and normal sleep occurs when the homeostatic process (S) value is 12.2, the awakening lower point of 11.38 is due to the influence of the circadian processes C and U. The speed of a person's recovery of alertness is based on his/her homeostatic levels. However, based on the effect of the body clock, the person may wake up before the alertness recovery slows down, or even before deep sleep is initiated.

Fig. 1 shows the alertness function ($S + C + U$) of a person who can rest/sleep at any point in a 24 h interval from 8 AM to 8 AM the next day. In the case of a crew, if the person is on duty and cannot go to sleep at point b, the alertness level would continue to decline.

The time between points b and d is the sleeping time, with $[b, c]$ being deep sleep and $[c, d]$ being normal sleep. The awake time is $[d, a]$. Alertness keeps increasing during the first hours of the awake time because of the circadian body clock. In this graph, the minimum level of alertness is around 8.2, which corresponds to the sleeping threshold. The alertness function is nonlinear, because of the cosine functions in circadian processes C and U.

Fig. 2 shows the effect of 8-h jet-lag on a person who can sleep/rest at any point for 24 h from 8 AM to 8 AM the next day. The figure shows that the effects can be substantial. The lowest alertness level is reached around noon. The processes that jet-lag affects are the 12 and 24-h circadian rhythms. The effect can be seen in the different placements of the peak and low points of the function, compared to the case without jet-lag given in Fig. 1. In this case, the person goes to sleep at point a, as he/she can sleep at any time. Additionally, Fig. 3 shows the effect of an 8-h jet-lag on a person who has to continue working even if the sleep threshold is passed. It is clear that the alertness levels are notably lower, even compared to the case with the jet-lag. It is also alarming to see the sleep threshold being passed twice during the 24-h period.

The calculation of the fatigue function when deciding on the pairings is not straight-forward as the alertness level must be known to determine which homeostatic process function should be used. In other words, the alertness level must be known to determine if the person is in deep sleep, normal sleep or is awake. However, the rate of change in alertness is dependent on the specific S function that is being used as well. Therefore, the fatigue should be calculated in a step-by-step fashion with

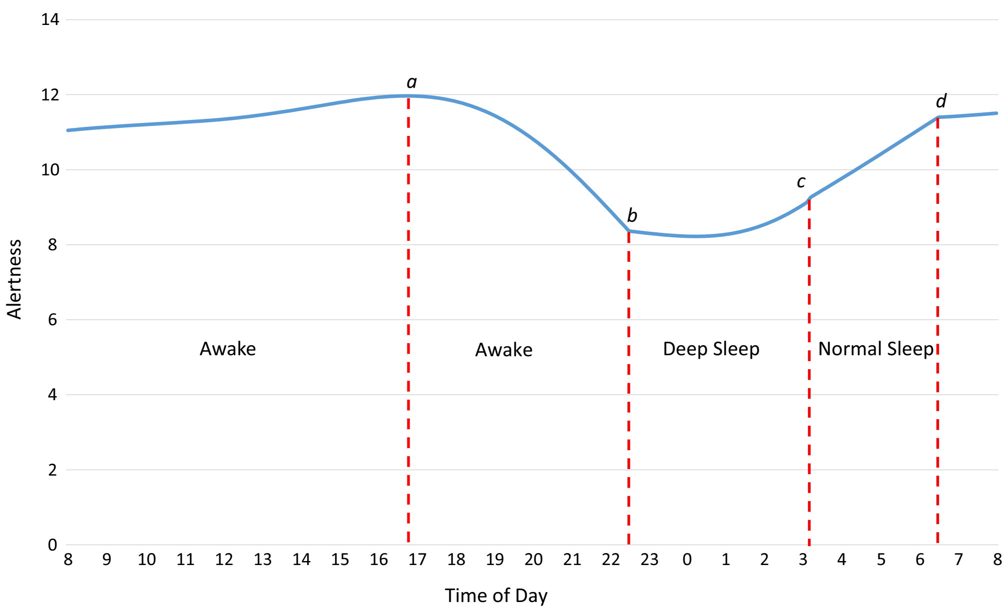

Fig. 1. The alertness function ($S + C + U$).

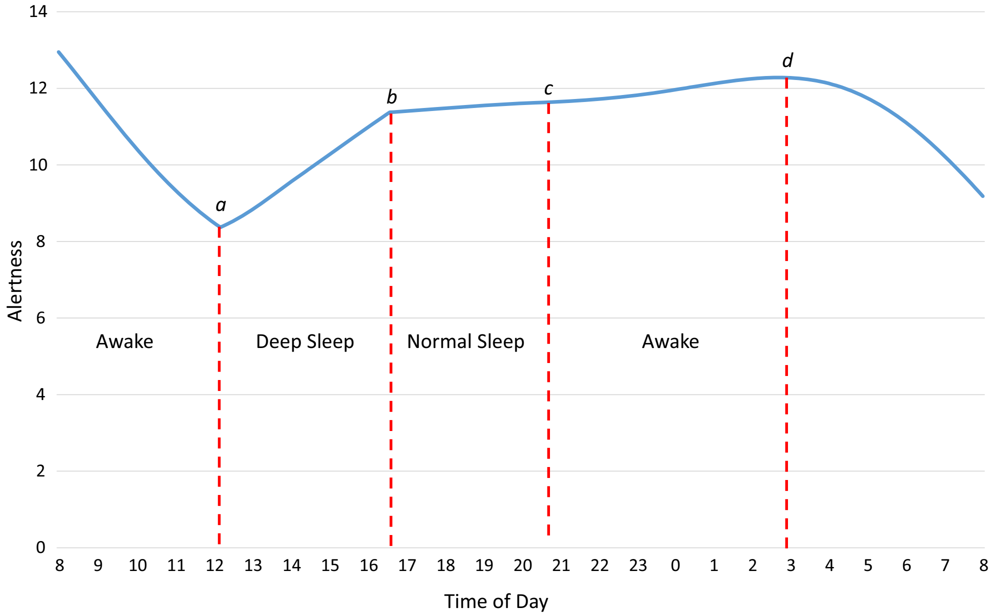

Fig. 2. The alertness function with 8-h of jet-lag ($S + C + U + A$).

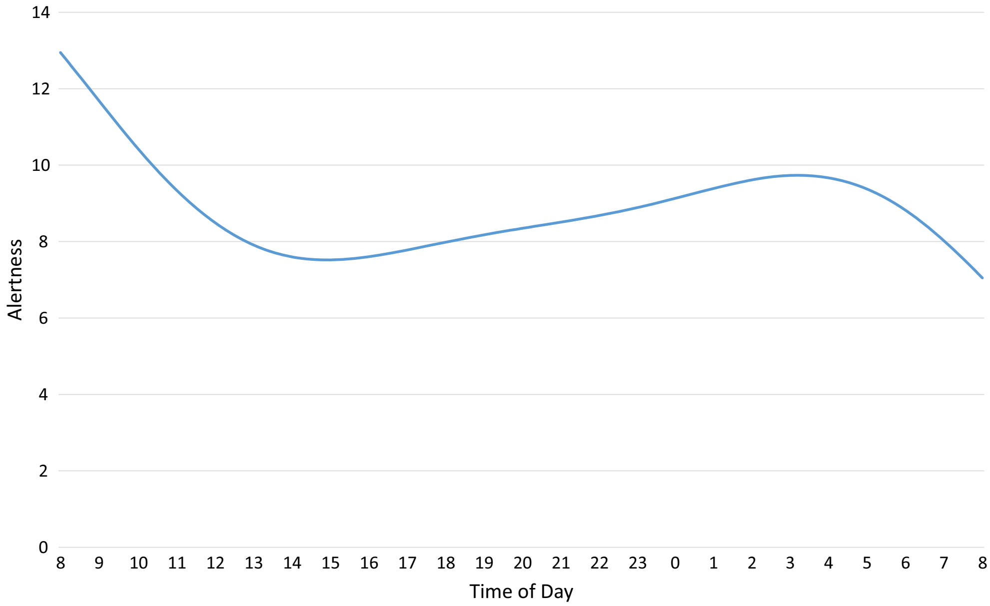

Fig. 3. The alertness function with 8-h of jet-lag ($S + C + U + A$) without sleep.

small time increments, as the fatigue value is updated in every step. In order to incorporate the fatigue function in the crew pairing problem, the following approach is used:

1. Divide the rest/flight period into 100 time intervals.
2. Calculate the values of the processes S, C and U and the alertness level ($S + C + U$) at every time interval.
3. If a specific threshold (falling asleep, transition between deep sleep and normal sleep, and waking up) is exceeded (e.g. if $S + C + U \leqslant 8.38$), change the S process used accordingly.
4. At the end of rest/flight period, obtain the fatigue level using Eq. (3).

In the third step of the algorithm, the S process is changed to deep sleep or normal sleep only if the person is in a rest period. During a flight period, only Eq. (2) is used. The mathematical model proposed next uses the fatigue function (3) and the algorithm above to evaluate the fatigue function at defined time intervals.

3. Mathematical model

The model extends the classical crew pairing formulation with resource constraints (Desaulniers et al., 1997) to explicitly account for fatigue. Let K be the set of crews to be assigned to the pairings and $G = (N, A)$ be a flight network where N is the set of nodes and A is the set of arcs. The set N has three types of nodes: Set O denotes the crew origin nodes, set D denotes the crew destination nodes, and set $N \setminus \{O, D\}$ denotes the flight nodes. The flight nodes are time and location dependent and are associated with the departure time and departing airport. The origin and destination nodes o^k and d^k denotes the base of crew k. There are three types of arcs in set A. The arcs (o^k, i) and (i, d^k) exist if crew k can start or end its schedule with flight i, respectively. The arc (i,j) exists if it is feasible for flight j to follow flight i. Graph G may be decomposed into subgraphs $G^k = (N^k, A^k)$ for each crew k, by removing the nodes and arcs that cannot be visited by crew k. Let c_{ij}^k be the cost of selecting arc (i,j) for crew k. The decision variables are:

$$x_{ij}^k = \begin{cases} 1, & \text{if arc } (i,j) \text{ is selected for crew } k \\ 0, & \textit{otherwise}. \end{cases}$$

$F_i^k =$ The fatigue level at node i for crew k.

Based on these definitions, the nominal crew pairing problem (CPP) is:

$$[CPP]: \quad \min \sum_{k \in K} \sum_{(i,j) \in A_f^k} c_{ij}^k x_{ij}^k \tag{11}$$

$$\text{s.t.} \sum_{k \in K} \sum_{i:(i,j) \in A_f^k} x_{ij}^k \geqslant 1 \quad j \in N^k \setminus \{o^k, d^k\} \tag{12}$$

$$\sum_{j:(o^k,j) \in A^k} x_{ij}^k = 1 \quad k \in K \tag{13}$$

$$\sum_{i:(i,d^k) \in A^k} x_{ij}^k = 1 \quad k \in K \tag{14}$$

$$\sum_{i:(i,j) \in A^k} x_{ij}^k - \sum_{j:(j,i) \in A^k} x_{ji}^k = 0 \quad k \in K, j \in N^k \tag{15}$$

$$x_{ij}^k \in X^k \quad (i,j) \in A^k, k \in K \tag{16}$$

$$x_{ij}^k \in \{0,1\} \quad (i,j) \in A_k, k \in K \tag{17}$$

The objective function (11) is the total crew pairing cost. Constraints (12) ensure that each flight is covered. Constraints (13)–(15) represent the flow balance constraints, which ensure that there is one path from origin o^k to destination d^k for each crew k. Constraints (17) define the domain of the variables.

Constraints (16) represent the rules and regulations enforced by aviation agencies (e.g. FAA) and the airline industry. The set X^k denotes the feasible set of connections for crew k. First of these rules is minimum/maximum break time between flight segments, which allows enough briefing/debriefing time for crews after each flight, and also enough time for the preparation of the planes for the next flight. The maximum break time also dictates that if a crew has to wait more than a certain amount of time, the duty would be finished without having more flights. The second is the minimum/maximum duty break time. These breaks are the most important elements that reduce fatigue throughout a pairing. The maximum duty break also enforces a duty in each day of the pairing. Another rule is the maximum duty flight time, which limits the number of hours a crew may be in flight throughout each duty. The aim is to reduce the in flight fatigue. The next set of rules is the minimum pairing flight time, which ensures that the crews will have a certain amount of credit hours for each pairing. The final rule is the maximum time spent away from base, which ensures the cyclic pattern of the pairings, and allows for sufficient break time between pairings. It also allows for crew members to spend enough time at home each week.

Based on $[CPP]$, the crew pairing problem with fatigue (CPPF) is given by

$$[CPPF]: \quad \min \alpha \sum_{k \in K} \sum_{i \in N^k:((i,j) \in A_f^k)} F_i^k + \sum_{k \in K} \sum_{(i,j) \in A_f^k} c_{ij}^k x_{ij}^k \tag{18}$$

$$\text{s.t.} (12)-(17)$$

$$F_j^k = f(x_{ij}^k, F_i^k) \quad (i,j) \in A^k, k \in K \tag{19}$$

$$F_i^k \geqslant 0 \quad i \in N_k, k \in K \tag{20}$$

The objective function (18) is the total crew pairing cost, which is calculated as a weighted sum of the pairing cost and the fatigue function value at each node in the pairing. The weight parameter α scales the fatigue and cost values and may be interpreted as the importance of fatigue relative to cost. Kuhn (2013) argues that a weighted sum approach in a bi-objective optimization model may miss some points on the pareto frontier due to the way the weights are incremented. We took this into account when deciding on the α values based on experimentation. Constraints (19) capture the fatigue level of crew k at node j, that is calculated using the fatigue function f given in Eq. (3) and described in detail in Section 2. The fatigue function value at node i experienced by crew k is accounted for in three different ways. The proposed model could be modified to impose a maximum fatigue level as a constraint instead of having it in the objective. The solution approach would only differ at the level of the subproblem.

The first way is to use the fatigue function value experienced at the beginning of each flight i. By minimizing this value, the goal is to have the most fit crew perform the flight. The second way is to use the fatigue function value experienced at the end of each flight i. Since the fatigue function value increases during a flight, minimizing this value aims to have fitter crews at the end of the flight. This way it may be assigned to more flights per duty. Also, even if a crew is fit at the beginning of a flight, it may get fatigued during the flight. More to this end, the third way is to use the maximum fatigue function value experienced by crew k during flight i. This aims to minimize the maximum fatigue levels experienced by any crew during any flight. Since a crew becomes impaired when the alertness is most reduced and fatigue is high, minimizing the maximum fatigue level during a flight aims to avoid reaching high fatigue levels that could lead to more accidents. In Section 5, we analyze the impact of the three ways to account for fatigue on fatigue levels and pairing solutions.

Constraints (16) and (19) are complex to handle, making [*CPPF*] not directly solvable by commercial software. Fortunately, it is possible to handle the constraints at the pairing generation subproblem. Due to the large number of pairings, the crew pairing problem is often solved by column generation. Our model is further complicated by a complex fatigue function that cannot be used explicitly within a crew pairing model as it does not have a closed-form expression. In Section 2, we developed an iterative algorithm that evaluated the fatigue function at defined break points. In the next section, we show how this algorithm could be integrated within a column generation formulation to solve practical size crew pairing problems.

4. Solution methodology

As typically done for crew pairing problems, we propose a column generation algorithm where the subproblem is a shortest path problem with fatigue. We present the approach from a Lagrangean relaxation perspective where constraints (12), the flight leg covering constraints, are relaxed with Lagrangean multipliers λ_j. The resulting subproblem is:

$$\begin{aligned} \min \quad & \alpha\sum_{k\in K}\sum_{i\in N^k:((i,j)\in A_f^k)} F_i^k + \sum_{k\in K}\sum_{(i,j)\in A_f^k} c_{ij}^k x_{ij}^k - \sum_{j\in N^k\setminus\{o^k,d^k\}} \lambda_j\Big(\sum_{k\in K}\sum_{i:(i,j)\in A_f^k} x_{ij}^k\Big) \\ \text{s.t.} \quad & (13){-}(17),\ (19),\ (20) \end{aligned}$$

which decomposes into $|K|$ subproblems, one for each crew $k \in K$. Since crews originating from the same base are equivalent, the corresponding subproblems are identical. Consequently, the number of subproblems can be reduced to $|B|$, one for each base $b \in B$. The pricing subproblem for base b is defined on graph $G^b = (N^b, A^b)$:

$$\begin{aligned} [SP_b] : \min \quad & \alpha \sum_{i\in N^b:((i,j)\in A_f^b)} F_i^b + \sum_{(i,j)\in A_f^b} c_{ij}^b x_{ij}^b - \sum_{i:(i,j)\in A_f^b} \sum_{j\in N^b\setminus\{o^b,d^b\}} \lambda_j x_{ij}^b \\ \text{s.t.} \quad & (13){-}(17),\ (19),\ (20) \end{aligned}$$

The solution of subproblem b provides the optimal variables $\bar{x}_{ij}^b$ for base $b \in B$. The variables $\bar{x}_{ij}^b$ define a pairing that a crew originating from base $b \in B$ may be assigned to. These pairings may be used to create a feasible solution to the crew pairing problem.

Let H^b be the set of feasible pairings for base $b \in B$. Define binary decision variable β_h^b which takes value 1 if pairing $h \in H^b$ for base b is selected, and recall $\bar{x}_{ij}^{bh}$ which is 1 if arc (i,j) is used in pairing h for base b. Let F_h be the fatigue level, and c_h be the financial cost of pairing $h \in H^b$. The set covering formulation for crew pairing with fatigue is:

$$[SC]: \quad \min \sum_{b\in B}\sum_{h\in H^b} (\alpha F_h + c_h)\beta_h^b$$

$$\text{s.t.} \sum_{b\in B}\sum_{h\in H^b} \bar{x}_{ij}^{bh}\beta_h^b \geqslant 1 \quad (i,j)\in A_f \tag{21}$$

$$\sum_{h\in H^b} \beta_h^b \leqslant |I_b| \quad b\in B \tag{22}$$

$$\beta_h^b \in \{0,1\} \quad h\in H, b\in B \tag{23}$$

where $|I_b|$ is the number of crews originating from base $b \in B$. This formulation is similar to the set covering formulation (1) and (2). The main differences is the inclusion of fatigue as a measure of the quality of a pairing, and the consideration of multiple bases. The linear relaxation of this problem is the Dantzig-Wolfe master problem.

Before the column generation algorithm is started, a feasible set of pairings is needed such that every flight is covered. To generate an initial set of pairings, the subproblems $[SP_b]$ for $b \in B$ are solved several times with random multipliers λ_j. Then the column generation algorithm proceeds by solving a restricted Dantzig-Wolfe master problem. Optimal dual variables λ_j are used to solve the shortest path subproblems $[SP_b]$ for $b \in B$ and to generate new pairings. The column generation algorithm stops when the subproblems cannot generate more pairings with negative reduced cost. At this point, the set covering problem $[SC]$ is solved using all of the generated pairings to obtain a feasible solution and an upper bound.

At each iteration of the column generation algorithm, the subproblem $[SP_b]$ is solved using a label setting algorithm on the flight network $G^b = (N^b, A^b)$. A duty network was also tested, but proved to be too difficult to solve because of the high number of feasible duties.

The shortest path algorithm that is designed is similar to the one described by Irnich and Desaulniers (2005). It is modified to integrate fatigue calculations. Let N be the set of sub-paths, and P the set of completed paths (pairings). For each base $b \in B$, all possible starting sub-paths using the arcs (o^b, i) are created and added to set N. Then until there are no non-dominated paths remaining, sub-paths in set N are extended using arcs A^b. When the path reaches base $b \in B$ it is added to P, otherwise it is added to N. For each extension of a sub-path using an arc A^b, the algorithm described in Section 2 is used to calculate fatigue (F_i^b), and the labels are updated. Dominated paths (paths that have the same starting and current points, with a higher weighted objective function value) are removed from the set N whenever they are encountered.

To calculate the cost c_h of a pairing, we use a realistic pricing scheme used by an airline industry software solutions company. The cost accounts for total credit hours, total time away from base, accommodation, the number of credit days, and the number of duty periods.

5. Computational results

In this section, we perform extensive testing on realistic data sets to investigate the impact of considering fatigue in constructing crew pairings.

The column generation and shortest path algorithms were implemented in MS Visual Studio 2013 IDE, using C++ programming language. The master problems are solved using CPLEX Concert Technology. The tests are performed on a computer running Microsoft Windows 7 64-Bit OS, with Intel i7 4790 3.40 Ghz processor with 8 threads and 8 GB of RAM.

Two real-life data sets are used to analyze the effects of fatigue on crew pairing solutions. Both data sets are obtained from an airline industry software solutions company. The first data set was used by Lu and Gzara (2014). This data set has 7 airports situated in Europe, one of which is a home-base for all crews. The airports span two time-zones. The data set has 378 flights in a weekly flight schedule. The longest and shortest flights in the set take 2.58 and 1.33 h, respectively. The second data set has 30 airports situated in Brazil, six of which are home-bases for crews. The airports span four time-zones. The data set has 470 flights in a weekly flight schedule. The longest and shortest flights in the set take 4.27 and 0.52 h, respectively. For both data sets, a minimum connection time of 30 min, and a maximum time away from base of four days are used.

The cost figures are obtained from an airline software solutions company, and are set to \$2200 per credit hour, \$3 per hour spent away from base, \$1000 per accommodation booked per night, \$4000 per day of work, and \$400 per duty period in the pairing.

For the weight parameter α, values in the interval [0,1000] with increments of 100, and values in the interval [1000,10000] with increments of 1000 were tested. The aim was to analyze the impact of fatigue on the optimal pairings as the weight associated with fatigue changes. Additionally, to see the impact of jet-lag on fatigue, two cases were tested on the 470-flight data set with all the α values. In the first case, the data set was used as is and in the second case, time-zones of all the airports in the set were set to the same value, to remove the effect of jet-lag.

As stated in Section 2, the Karolinska Sleepiness Scale (KSS), the most standard way of quantifying sleepiness, is used for fatigue. The KSS values and their interpretations are given in Table 1.

We first experiment with the three different approaches to account for fatigue, namely: the maximum fatigue experienced during a flight (MF), the fatigue level at the beginning of a flight (BF), the fatigue level at the end of a flight (EF).

Table 1
Karolinska sleepiness scale (Åkerstedt et al., 2014).

KSS value	Meaning
1	Extremely alert
2	Very alert
3	Alert
4	Rather alert
5	Neither alert nor sleepy
6	Some signs of sleepiness
7	Sleepy, but no effort to keep awake
8	Sleepy, some effort to keep awake
9	Very sleepy, great effort to keep awake, fighting sleep

For comparison, we provide results where fatigue is not accounted for at all (NF). We solve the 470-flight data set four times, once for each fatigue calculation approach, and once with only rules and regulations. Once we obtain the crew pairing solutions, we record the fatigue and alertness levels. For these experiments, the value of α was set to 1000. Statistics on alertness and fatigue are given in Table 2.

In this table, the minimum (maximum) alertness refers to the minimum (maximum) alertness experienced by any crew at any point in time. The average alertness provides the average alertness experienced at any time over all crews. Fatigue range gives the minimum and maximum fatigue over all time points for all crews. The last two columns provide the number of flights where the fatigue function values exceed 5 and 7, respectively.

In light of the information given about KSS, Table 2 shows that the current rules and regulations by themselves do not achieve an ideal case in terms of fatigue, with fatigue levels going as high as 8 (crew members need to exert some effort to keep awake). It is also clear that all three ways to account for fatigue do much better than the application of the industry standards. Among these ways, MF provided the least number of times that fatigue levels exceed 5, and generally higher alertness values. Consequently, we use it for the rest of the testing and analysis.

We then run a set of experiments by varying the value of the weight parameter α and compare the crew pairing solutions in terms of financial costs, fatigue levels, and pairing characteristics. The results are given in Tables 3 and 4, where all solutions are within 0.1% of optimality. The percentage increase in financial costs, and decrease in fatigue levels are calculated relative to the base case with $\alpha = 0$. The other statistics reported in the tables are the number of pairings used to cover all of the flights, average flight legs per pairing, average pairing length in hours, and percentage time spent on flights and on breaks in these pairings, respectively.

The relationship between the increase in financial costs and the decrease in fatigue costs is shown in Figs. 4 and 5. As α increases, small increases in financial costs are converted into large improvements in fatigue levels until α hits a certain threshold (around 1000). For example from $\alpha = 0$ to $\alpha = 1000$, the increase in cost for the 378-flight case is 3.18%, while the decrease in fatigue is 10.99% (Table 3). When α increases another order of magnitude, however, there is only an additional 3.21% decrease in fatigue and a substantial (10.25%) increase in cost. When looking at the characteristics of the optimal pairings as α increases, more pairings (crews) are used to cover all flights. This in turn translates into a decrease in the average number of flights per pairing. For example, for the 378-flight case, and as α goes from 0 to 1000, the number of pairings used increases by 12.77% and the average legs per pairing decreases by 13%. When α increases another tenfold, the increase in the number of pairings used and the decrease in average legs per pairing are similar (13.2% more pairings used and 13% fewer legs per pairing). This shows that the marginal reduction of fatigue as α increases has a decreasing trend when α exceeds 1000. The percentage time spent on flight and on break changes inversely as α increases. The time spent on flight decreases by 3.92% when α increases from 0 to 1000 for the 378-flight case and by 2.57% for the 470-flight case.

To investigate the impact of accounting for jet-lag when calculating fatigue, we modify the 470 flight case by ignoring time zone differences to obtain two cases: "With Jet-Lag" and "Without Jet-Lag". Fig. 6 gives the number of pairings used for each of the 2 cases. It is observed that if jet-lag is incorporated, the model uses more crews to cover flights. The reason for this is that the jet-lag changes the time of day at which the peak rate of fatigue is experienced (as stated in Section 2). Consequently, this increases the amount of fatigue experienced during daytime, which happens often during the flight time. When α increases from 0 to 10, the number of pairings used increases by 2 in the case with jet-lag, while it increases by 1 in the case without jet-lag. Fig. 6 shows that this trend continues as α increases. The solution with jet-lag always has 1 or 2 more pairings compared to the case without jet-lag.

To further observe the impact of jet-lag on alertness, we solve the 470 flight case with time zones, once accounting for jet-lag using the Eq. (8) at the pairing generation and once without. In both cases we assess the corresponding alertness. The results are given in Table 5 where the minimum and maximum alertness levels, the fatigue range, and the number of flights in which the fatigue function value exceed 5 are reported using the three different ways of accounting for fatigue MF, BF, and EF.

The results show that not accounting for jet-lag leads to decreased alertness levels and increased extreme fatigue levels (times fatigue exceeds 5). Judging that the impact of jet-lag is obvious for this data set that has domestic flights in Brazil only, with only 11.4% of the flights spanning all four-time zones, it can be argued that the effect of jet-lag would only increase in long haul flights where the number of time-zones spanned is much larger. However, since long haul flights are usually scheduled separately with the possibility of pilot augmenting, the model could be extended to handle this case. If pilot augmenting is not possible, Fig. 3 indicates that the fatigue levels would continue to increase during the flight and the time needed to recover would increase.

Table 2
Comparison of fatigue calculation schemes.

	Alertness			Fatigue range	Times	Times
	Min	Average	Max	(min,max)	Fatigue > 5	Fatigue > 7
MF	7.22	10.51	13.48	(2,6)	25	0
BF	5.81	8.35	11.02	(4,7)	58	3
EF	6.87	10.06	12.85	(3,6)	32	0
NF	4.58	7.35	10.15	(5,8)	102	21

Table 3
Computational results for the 378-flight case.

α	Cost increase (%)	Fatigue decrease (%)	Nb. pairings used	Avg. legs per pairing	Avg. pairing length (hr)	Time spent on flights (%)	Time spent on break (%)
0	–	–	47	8.04	35.28	35.36	64.64
10	0.24	2.26	47	8.04	34.89	35.03	64.97
100	0.43	3.21	48	7.88	33.81	34.72	65.28
200	0.73	4.82	48	7.88	33.13	34.1	65.9
300	1	5.72	48	7.88	32.71	33.7	66.3
400	1.28	6.67	49	7.71	31.53	33.19	66.81
500	1.7	8.04	49	7.71	31.27	32.93	67.07
600	2.07	9.22	50	7.56	30.35	32.62	67.38
700	2.24	9.55	52	7.27	29.02	32.44	67.5
800	2.4	9.87	52	7.27	28.33	31.64	68.36
900	2.7	10.36	52	7.27	28.07	31.33	68.67
1000	3.18	10.99	53	7.13	27.63	31.44	68.56
2000	4.78	11.76	55	6.87	25.14	29.46	70.54
3000	6.43	12.69	56	6.75	24.68	29.44	70.56
4000	8.64	13.42	56	6.75	24.92	29.79	70.21
5000	10.65	13.9	58	6.52	23.18	28.45	71.55
10,000	13.43	14.2	60	6.3	22.27	28.22	71.78
Min	0.24	2.26	47	6.3	22.27	28.22	64.64
Max	13.43	14.2	60	8.04	35.28	35.36	71.78
Avg	3.87	9.17	51.76	7.34	29.19	31.99	68.01

Table 4
Computational results for the 470-flight case.

	Cost	Fatigue	Nb. pairings	Avg. legs	Avg. pairing	Time spent	Time spent
α	increase (%)	decrease (%)	used	per pairing	length (hr)	on flights (%)	on break (%)
0	–	–	68	6.91	34.64	30.16	69.84
10	0.085	1.88	68	6.91	33.84	29.19	70.81
100	0.23	3.24	68	6.91	33.62	28.94	71.06
200	0.32	4.92	69	6.81	33.31	28.93	71.07
300	0.51	5.54	69	6.81	32.3	28.7	71.3
400	0.54	6.89	69	6.81	32.1	28.62	71.38
500	0.59	7.82	69	6.81	31.38	28.58	71.42
600	0.64	8.7	70	6.71	31.14	28.22	71.78
700	0.68	9.76	70	6.71	30.18	28.03	71.97
800	0.75	10.29	71	6.62	29.98	28.02	71.98
900	0.8	10.91	71	6.62	29.48	27.98	72.02
1000	0.9	11.91	72	6.53	29.32	27.59	72.41
2000	1.6	12.55	73	6.44	29.28	27.41	72.59
3000	2.94	12.79	75	6.27	29.08	26.73	73.27
4000	4.09	13.22	76	6.18	28.73	26.58	73.42
5000	5.02	13.36	77	6.1	27.73	26.55	73.45
10,000	6.58	13.41	80	5.88	25.32	25.94	74.06
Min	0.09	1.88	68	5.88	25.32	25.94	69.84
Max	6.59	13.42	80	6.91	34.64	30.16	74.06
Avg	1.65	9.21	71.471	6.59	30.67	28.01	71.99

At the pairing generation, we considered industry rules on fatigue such as minimum/maximum break times between flight segments, minimum/maximum duty breaks, maximum duty flight time, minimum pairing flight time, and maximum time spent away from base. In order to see if accounting for fatigue explicitly in pairing generation can replace some of the industry rules, we relax a subset of the rules and compare. Table 6 displays the findings. In case **R**, all rules are used while in case **S**, rules related to maximum break time between flight segments and maximum break time between duties are dropped. Minimum break time between flight segments is kept, so that proper briefing, debriefing and preparation times are guaranteed. Some of the other rules are kept so that a duty based structure is kept (e.g. minimum duty breaks), while the others are kept such that the pairings will end properly (e.g. maximum time spent away from base).

It is interesting to observe that as the value of α changes from 0 to 1000, financial costs increase as some of the rules are neglected, and the fatigue level decreases. One would expect that as the industry rules are ignored, the financial cost would decrease and fatigue level increase. The results show that the opposite happens which is proof that the fatigue model effectively accounts for fatigue and could replace many of the industry rules. It is possible that both financial costs and fatigue levels decrease when the regulations are relaxed as seen when comparing $\alpha = 100$ with relaxed regulations (**S**) to $\alpha = 0$ with regulations (**R**). This shows that some rules may be too strict, and that explicitly accounting for fatigue may help reduce costs

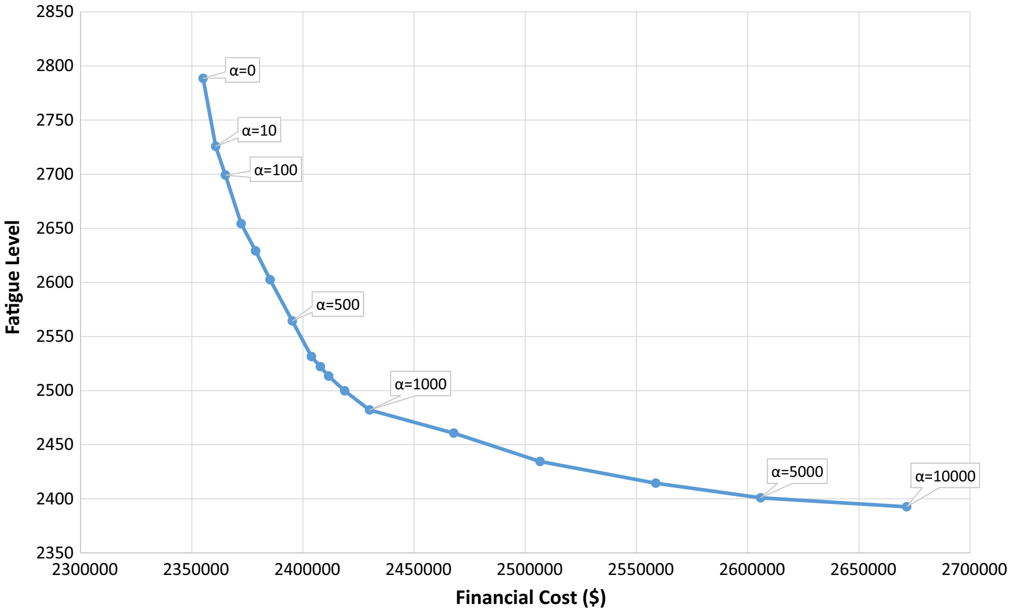

Fig. 4. Pareto Frontier for the 378-flight case.

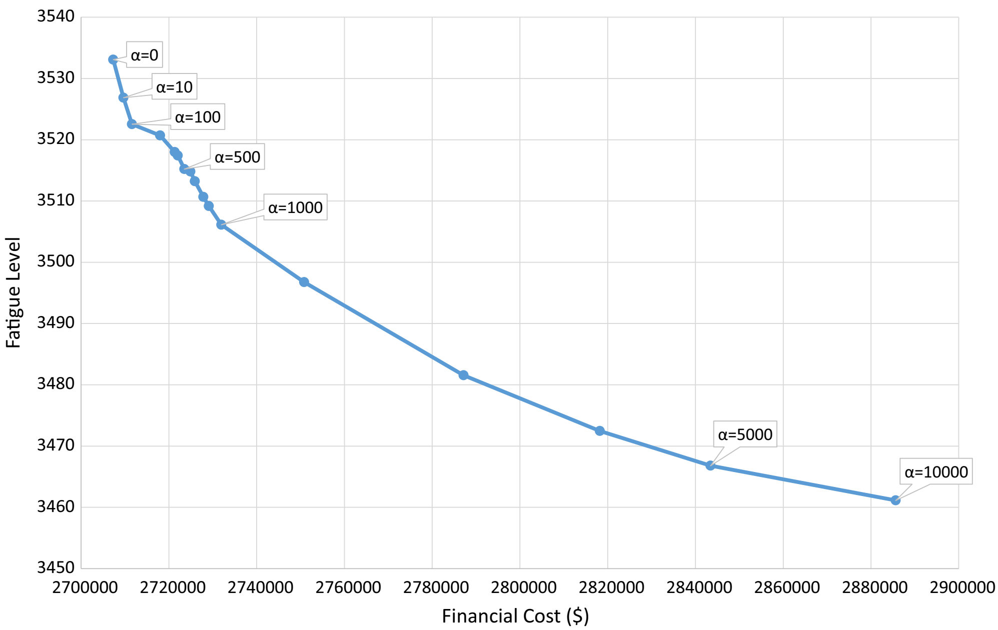

Fig. 5. Pareto Frontier for the 470-flight case.

without affecting the fatigue levels. The results also show that explicitly accounting for fatigue caters for the maximum break times between flights as well as between duties. When excluding other rules, we observed pairings that either have no duty based structure, do not have enough time between flights to prepare for the next one (e.g. minimum break between flight segments) or have a time away from base which is longer than 4 days.

Last, the CPU times of the solutions and their percentage distribution between subproblems and master problems are given in Table 7. In all tested instances, the results are within 0.1% of optimality. It is important to note that the solutions are obtained in meaningful times for the problem sizes. Since the fatigue calculation per arc described in Section 2 does not increase with the problem size, its effect on overall CPU times is very small. To illustrate this, the 378-flight and the

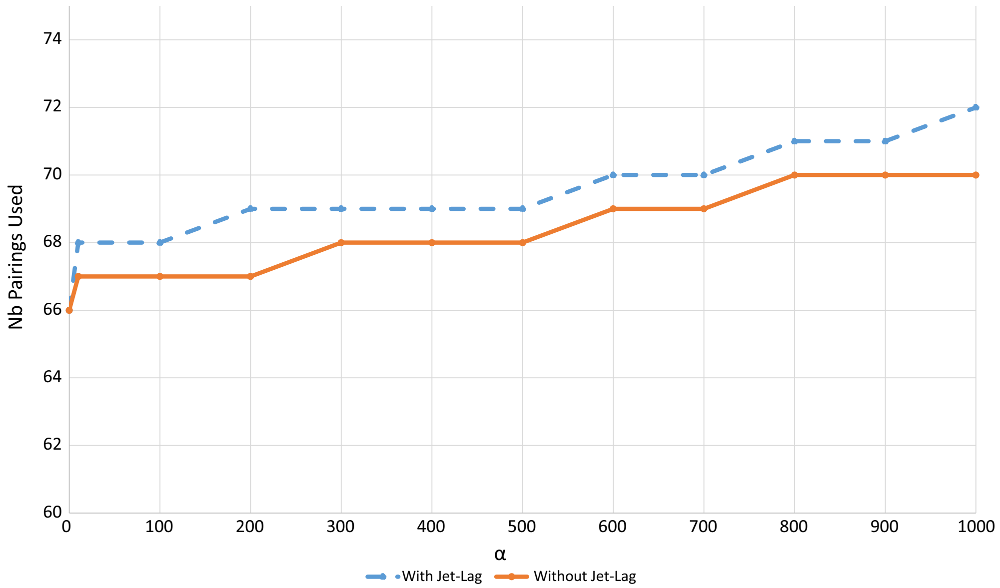

Fig. 6. The effect of Jet-Lag on the number of pairings used.

Table 5
Minimum/maximum alertness levels for jet-lag comparison.

	Pairings generated accounting for jet-lag				Pairings generated ignoring jet-lag			
	Min	Max	Fatigue range (min, max)	Times Fatigue > 5	Min	Max	Fatigue range (min,max)	Times fatigue > 5
MF	7.22	13.58	(2,6)	25	7.13	13.17	(3,6)	31
BF	5.81	11.02	(4,7)	58	5.47	10.54	(4,7)	72
EF	6.87	12.85	(3,6)	32	6.77	12.42	(3,7)	40

Table 6
Comparison of aviation rules.

	Financial cost	Fatigue level	Weighted objective	Pairings used	Avg. legs per pairing	Avg. pairing length (hr)	Time spent on flight (%)	Time spent on break (%)
α = 1000								
R	2,731,963	3506	6,237,963	72	6.53	29.32	27.59	72.41
S	2,736,652	3488	6,224,652	74	6.35	28.47	28.32	71.68
α = 100								
R	2,714,974	3521	3,067,074	68	6.91	33.62	28.94	71.06
S	2,703,436	3530	3,056,436	67	7.16	34.22	28.67	71.33
α = 0								
R	2,708,245	3534	2,708,245	68	6.91	34.64	30.16	69.84
S	2,691,548	3559	2,691,548	65	7.23	35.92	33.74	66.26

470-flight problems are solved as crew pairing problems with no fatigue calculation. The computational times are 240.7 and 1003.4 s, respectively, only an increase of 5.7 and 8.3 s from the computational times corresponding to $\alpha = 0$ in Table 7.

6. Conclusion

In this paper, we introduced and solved a crew pairing model that explicitly accounts for fatigue at the pairing generation. The Three Process Model of Alertness is used to model crew fatigue and alertness in a realistic and explicit manner. A column generation algorithm is proposed where the subproblem is a shortest path problem with fatigue that is solved using a label setting algorithm. Three fatigue-based objectives were tested. The maximum fatigue experienced during a flight was found to achieve the best results. We carried out detailed numerical analysis and testing based on two industry data sets. The results revealed the substantial effect of accounting for fatigue explicitly on the optimal pairings; and that notable decreases

Table 7
Comparison of CPU times.

α	378-Flight case			470-Flight case		
	CPU (s)	SP time (%)	MP time (%)	CPU (s)	SP time (%)	MP time (%)
0	246.4	27.37	72.63	1011.6	55.14	44.86
10	261.91	31.92	68.08	915.24	48.92	51.08
100	222.31	22.81	77.19	1128.05	52.73	47.27
200	261.36	33.46	66.54	1230.48	53.23	46.77
300	202.07	23.03	76.97	1061.88	53.55	46.45
400	228.69	28.62	71.38	1281.77	53.07	46.93
500	210.54	28.27	71.73	1138.86	51.48	48.52
600	273.79	28.53	71.47	1263.74	57.81	42.19
700	217.36	23.99	76.01	1273.28	53.29	46.71
800	213.84	30.04	69.96	918.9	46.35	53.65
900	206.36	30.76	69.24	1175.16	50.51	49.49
1000	189.2	27.15	72.85	1227.12	59.12	40.88
2000	215.6	38.21	61.79	1274.35	46.71	53.29
3000	259.16	31.49	68.51	1160.95	62.09	37.91
4000	263.67	31.91	68.09	1057.92	54.58	45.42
5000	264.11	30.65	69.35	1260.48	58.48	41.52
10,000	260.59	32.88	67.12	987.36	62.62	37.38
Min	189.2	22.81	61.79	915.24	46.35	37.38
Max	273.79	38.21	77.19	1281.77	62.62	53.65
Avg	235.12	29.48	70.52	1139.24	54.1	45.9

in fatigue levels can be achieved with minimum financial investments. It was also found that accounting for fatigue using hard constraints alone does not capture the full picture and may lead to high fatigue levels. The importance of jet-lag and time-zone differences was also assessed and found to lead to higher fatigue levels if ignored. Finally, the proposed model provided a tool to assess the fatigue levels associated with crew schedules, and could be used in union negotiations to support claims related to lower risks and improved quality of life.

The current policies of limiting crew fatigue through hard regulations accounts for fatigue indirectly and does not explicitly model crew fatigue. The proposed approach provides a mechanism to measure fatigue at the pairing level, paving the way for policies that could enforce this. What is meant here, is that given crew pairings, it is possible to calculate the fatigue associated with each, and therefore could be reported by airline companies to regulatory authorities.

A promising research direction could focus on the robustness of the sleep/wake-up times for crew members and the integration of schedule robustness and fatigue. The sleep/wake-up times are not necessarily rigid as assumed in this work and taking this flexibility into account would make the problem more realistic. Another future research direction could be the accounting of fatigue in crew recovery. Finally, adding a limit on the maximum fatigue level to the pairing generation subproblem as a constraint, rather than considering it in the objective is a viable alternative approach to tackle the problem.

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References

Ahlstrom, C., Nystrm, M., Holmqvist, K., Fors, C., Sandberg, D., Anund, A., Kecklund, G., Åkerstedt, T., 2013. Fit-for-duty test for estimation of drivers sleepiness level: eye movements improve the sleep/wake predictor. Transport. Res. Part C: Emerg. Technol. 26, 20–32.
Åkerstedt, T., Anund, A., Axelsson, J., Kecklund, G., 2014. Subjective sleepiness is a sensitive indicator of insufficient sleep and impaired waking function. J. Sleep Res. 23 (3), 242–254.
Åkerstedt, T., Axelsson, J., Kecklund, G., 2007. Individual validation of model predictions of sleepiness and sleep hours. Somnologie-Schlafforschung Schlafmedizin 11 (3), 169–174.
Åkerstedt, T., Folkard, S., 1990. A model of human sleepiness. In: Horne, J.A. (Ed.), Sleep '90, pp. 310–313.
Åkerstedt, T., Folkard, S., 1996. Predicting duration of sleep from the three-process model of alertness regulation. Occupational Environ. Med. 53, 136–141.
Åkerstedt, T., Ingre, M., Kecklund, G., Folkard, S., Axelsson, J., 2008. Accounting for partial sleep deprivation and cumulative sleepiness in the three-process model of alertness regulation. Chronobiol. Int. 25 (2–3), 309–319.
Arendt, J., Stone, B., Skene, D., 2000. Jet lag and sleep disruption. In: Kryger, M. (Ed.), Principles and Practice of Sleep Medicine. W.B. Saunders, Philadelphia, pp. 591–599.
Avers, K., Johnson, W.B., 2011. A review of federal aviation administration fatigue research: transitioning scientific results to the aviation industry. Aviation Psychol. Appl. Human Factors 1 (2), 87.
Barnhart, C., Cohn, A., Johnson, E., Klabjan, D., Nemhauser, G., Vance, P., 2003. Handbook of transportation science. In: Hall, R.W. (Ed.). Springer, US.
BEAA, 2001. Status of the Accident Investigation of Crosshair Flight CRX 3597 of 24 November 2001 near Bassersdorf/Zh. Press Release of 30 November, 2001. Tech. rep., BEAA.
Bourgeois-Bougrine, S., Carbon, P., Gounelle, C., Mollard, R., Coblentz, A., 2003. Perceived fatigue for short and long-haul flights: a survey of 739 airline pilots. Aviat. Space Environ. Med. 74 (10), 1072–1077.

Brown, I., 1994. Driver fatigue. Human Factors 36 (2), 298–314.
Caldwell, J.A., 2005. Fatigue in aviation. Travel Med. Infect. Disease 3 (2), 85–96.
Caldwell, J.A., Caldwell, J.L.J., 2003. Fatigue in Aviation: A Guide to Staying Awake at the Stick. Ashgate Publishing, Hants, England.
Dawson, D., Fletcher, A., 2001. A quantitative model of work-related fatigue: background and definition. Ergonomics 44 (2), 144–163.
Desaulniers, G., Desrosiers, J., Dumas, Y., Marc, S., Rioux, B., Solomon, M.M., Soumis, F., 1997. Crew pairing at air France. Eur. J. Oper. Res. 97 (2), 245–259.
Desaulniers, G., Desrosiers, J., Solomon, M.M., Soumis, F., Villeneuve, D., et al, 1998. A Unified Framework for Deterministic Time Constrained Vehicle Routing and Crew Scheduling Problems. Springer.
Desrosiers, J., Dumas, Y., Solomon, M.M., Soumis, F., 1995. Time constrained routing and scheduling. Handbooks in Operations Research and Management Science, 8, pp. 35–139.
Evans, L., 2003. Handbook of Transportation Science. Springer, New York.
Gander, P., Gregory, K., Miller, D., Graeber, R., Connell, L., Rosekind, M., 1998. Flight crew fatigue v: long-haul air transport operations. Avia. Space Environ. Med. 69 (9 Suppl.), B37–B48.
Goode, J.H., 2003. Are pilots at risk of accidents due to fatigue? J. Safety Res. 34 (3), 309–313.
Graeber, R., 1988. Fatigue and circadian rhythmicity. In: Weiner, E., Nagel, D. (Eds.), Aircrew Fatigue and Circadian Rhythmicity. Academic Press, New York, pp. 305–344.
Graeber, R., Lauber, J., Connell, L., Gander, P., 1986. International aircrew sleep and wakefulness after multiple time-zone flights: a cooperative study. Avia. Space Environ. Med. 57 (12), B3–B9.
Gürkan, H., Gürel, S., Aktürk, M.S., 2016. An integrated approach for airline scheduling, aircraft fleeting and routing with cruise speed control. Transport. Res. Part C: Emerg. Technol. 68, 38–57.
Ingre, M., Van Leeuwen, W., Klemets, T., Ullvetter, C., Hough, S., Kecklund, G., Karlsson, D., Åkerstedt, T., 2014. Validating and extending the three process model of alertness in airline operations. PLoS ONE 9 (10), e108679.
Irnich, S., Desaulniers, G., et al, 2005. Shortest path problems with resource constraints. Column Gener. 6730, 33–65.
Kuhn, K.D., 2013. Ground delay program planning: delay, equity, and computational complexity. Transport. Res. Part C: Emerg. Technol. 35, 193–203.
Loeb, P.D., Talley, W.K., Zlatoper, T.J., CT, W., 1994. Causes and Deterrents of Transportation Accidents: An Analysis by Mode. Quorum Books.
Lu, D., Gzara, F., 2014. The robust crew pairing problem: model and solution methodology. J. Global Optim. 62 (1), 29–54.
Olbert, Å., Klemets, T., 2011. A Comprehensive Investigation of Flight and Duty Time Limitations and their Ability to Control Crew Fatigue. Jeppesen.
Petrilli, R., Roach, G., Dawson, D., Lamond, N., 2006. The sleep, subjective fatigue, and sustained attention of commercial airline pilots during an international pattern. Chronobiol. Int. 23 (6), 1347–1362.
Romig, E., Klemets, T., 2009. Fatigue risk management in flight crew scheduling. Aviat. Space Environ. Med. 80 (12), 1073–1074.
Suraweera, P., Webb, G.I., Evans, I., Wallace, M., 2013. Learning crew scheduling constraints from historical schedules. Transport. Res. Part C: Emerg. Technol. 26, 214–232.