

Traitement d'images

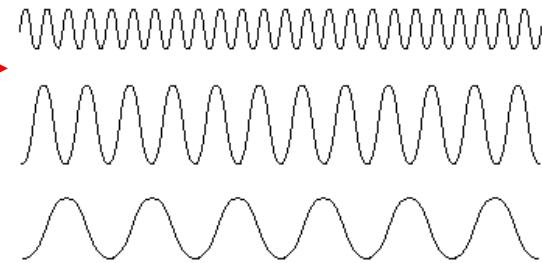
Traitements fréquentiels

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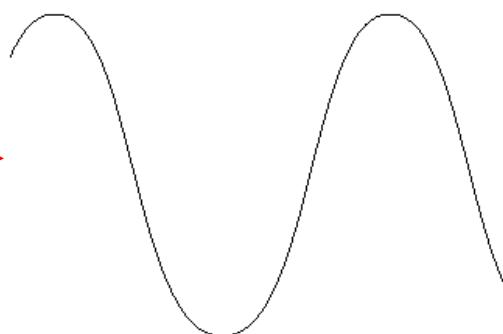
Notions de fréquences dans un signal

Signal haute fréquence →

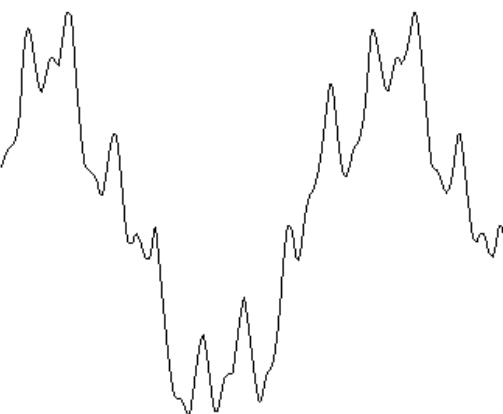


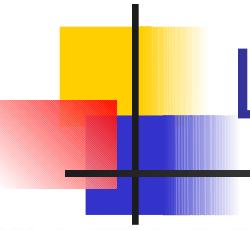
...

Signal basse fréquence →

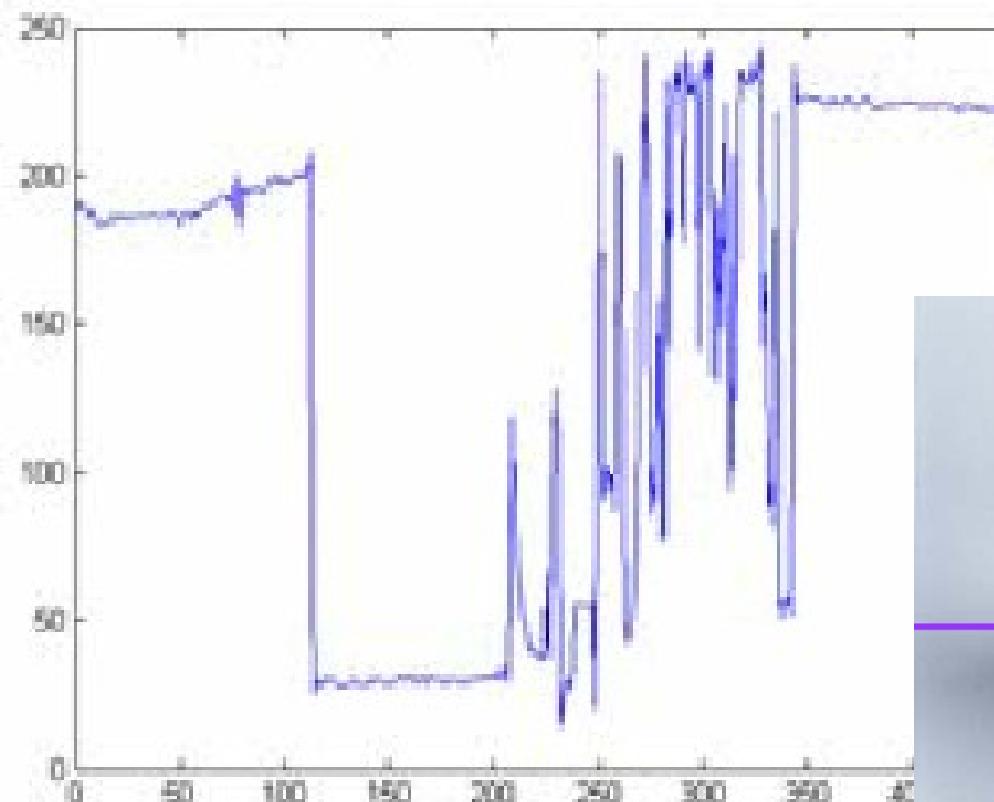


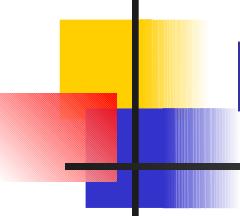
*Ce signal représente
la somme des quatre
signaux ci-dessus* →





Les fréquences dans l'image





Les fréquences dans l'image

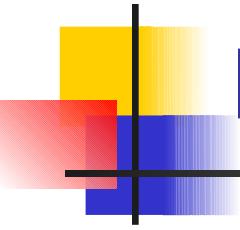
- Qu'est-ce qu'une fréquence dans une image ?
 - Fréquence = changement d'intensité
 - Basses fréquences : régions homogènes, flou
 - Hautes fréquences : contours, changement brusque d'intensité, bruit



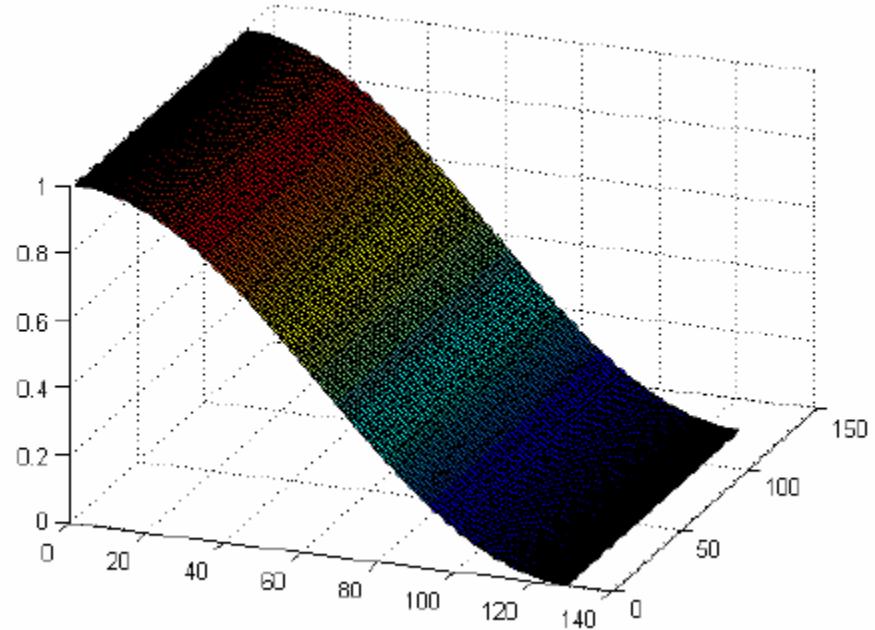
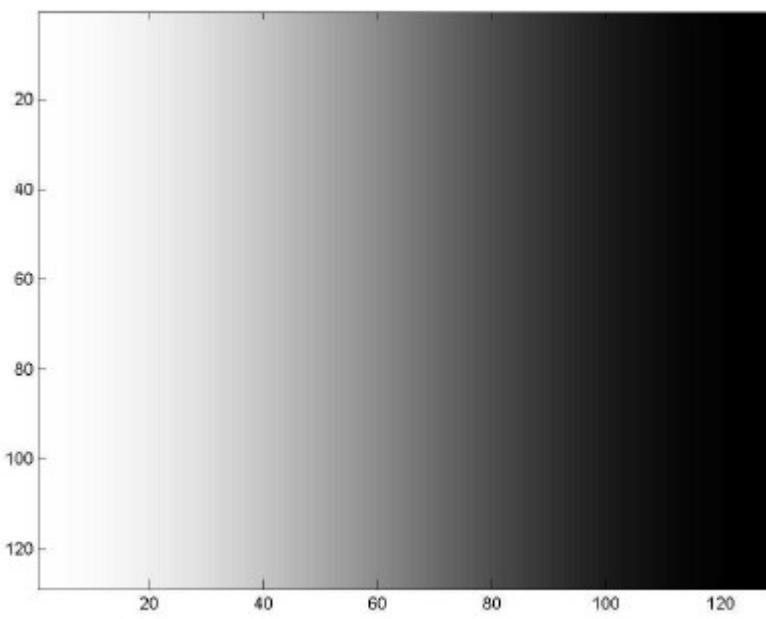
Haute fréquence

Basse fréquence

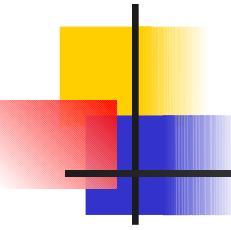
La plus grande partie de l'énergie d'une image se situe dans les basses fréquences.



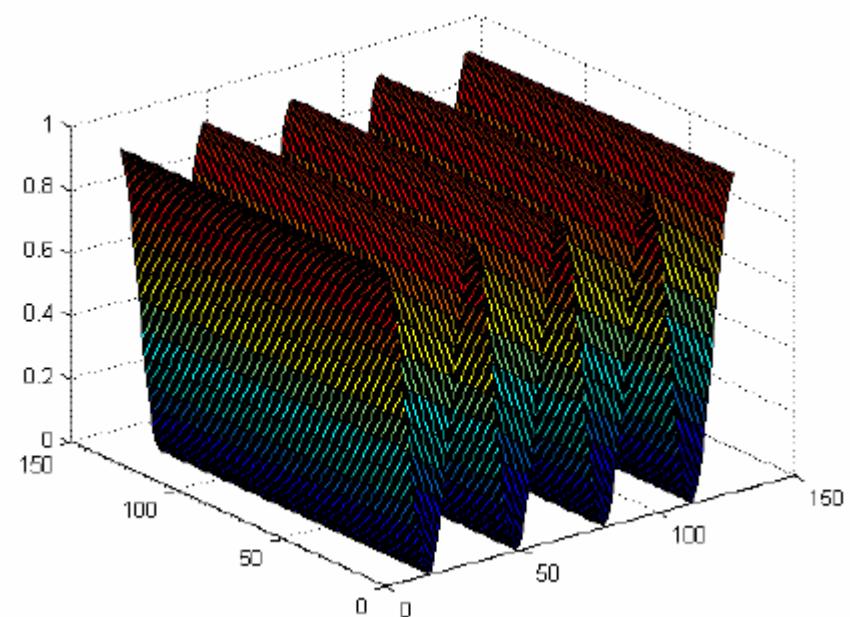
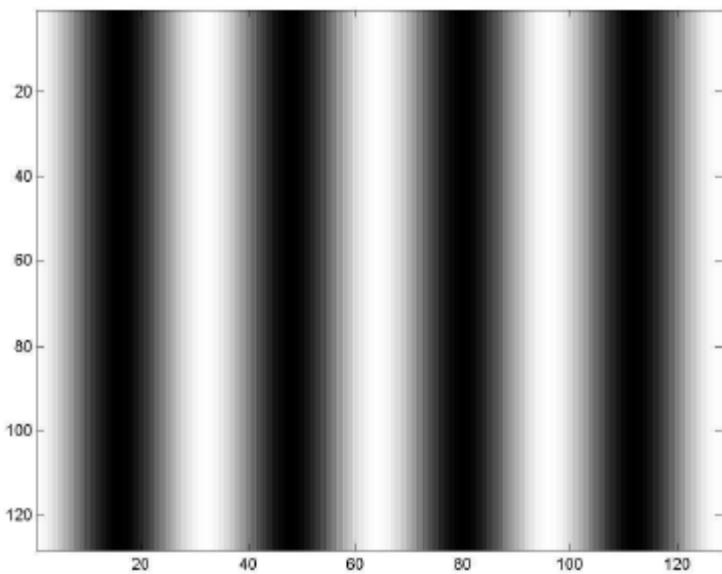
Basses fréquences



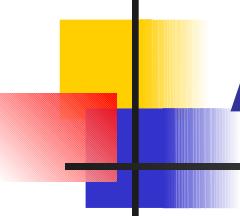
Les basses fréquences correspondent à des changements d'intensité lents



Hautes fréquences



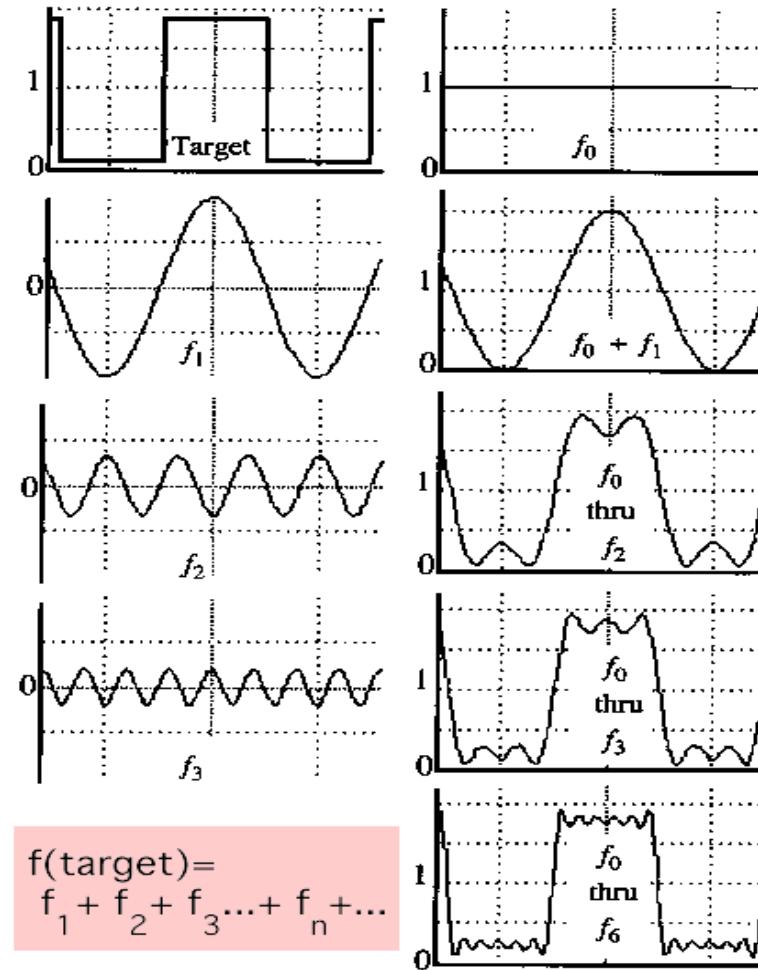
Les hautes fréquences correspondent à des changements d'intensité rapides

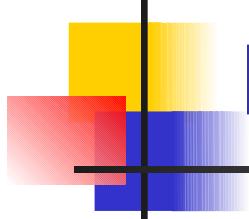


Analyse spectrale d'une image

- Une image est avant tout un signal (visuel)
 - *Tout comme le son est un signal audio*
- On peut analyser les fréquences de ce signal
 - On parle de **fréquences spatiales** (image) au lieu de **fréquences temporelles** (audio)
 -
- Pour cela, on crée un nouvel « histogramme » (graphique) qui va représenter les fréquences de l'image
 - L'outil de base pour cela est la **Transformée de Fourier**
- On parle de **domaine fréquentiel**, par opposition au **domaine spatial** (de l'image)

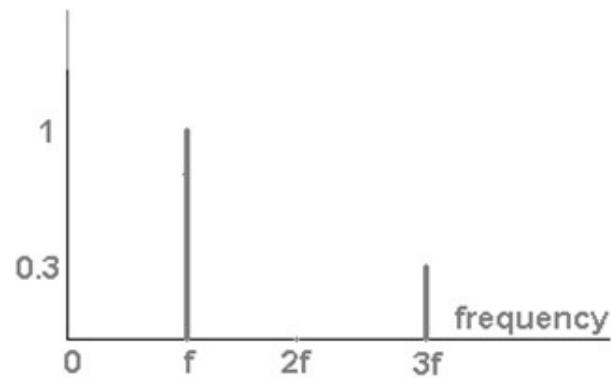
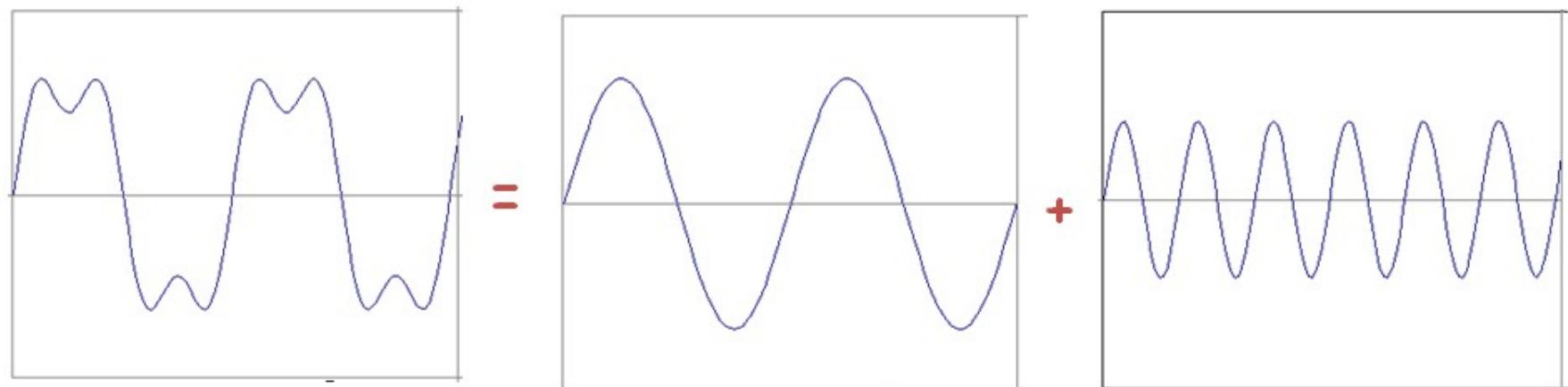
Exemple – La somme des sinus

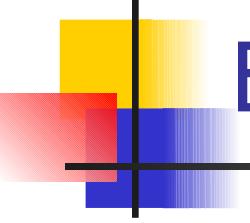




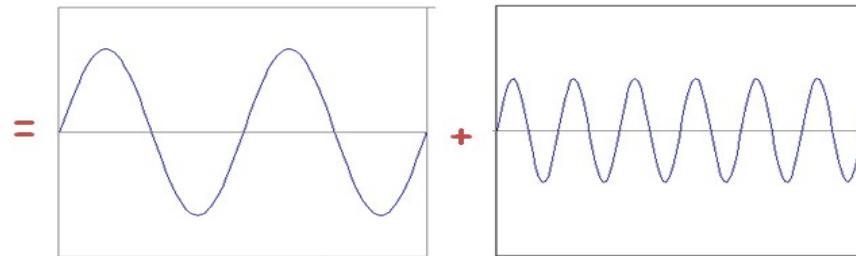
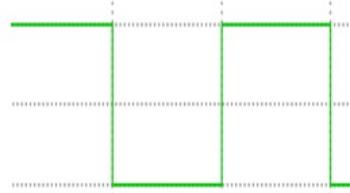
Exemple

$$g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi(3f) t)$$

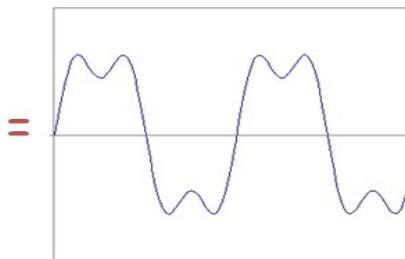
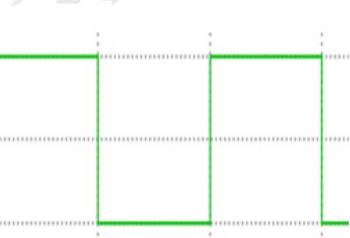
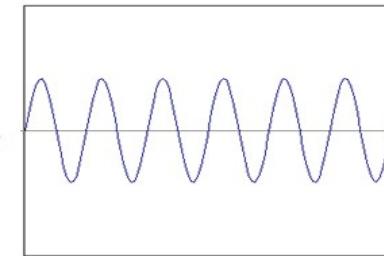




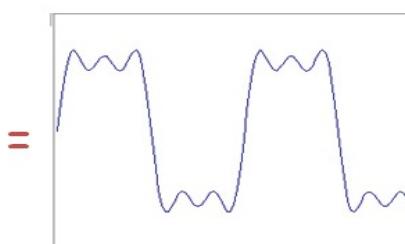
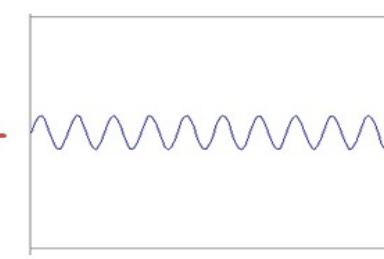
Exemple



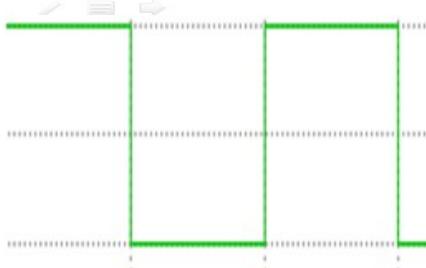
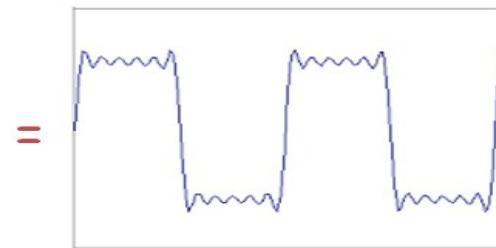
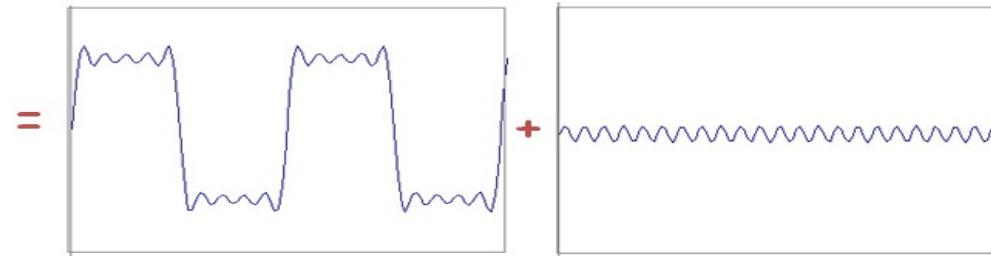
+



+

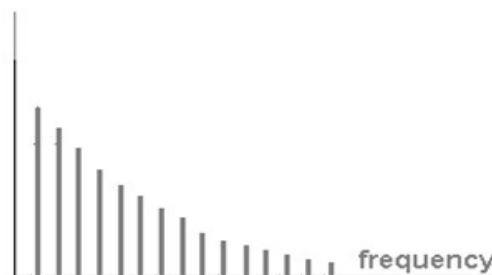


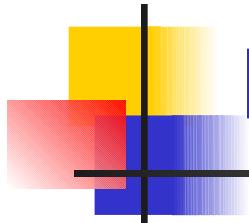
Exemple



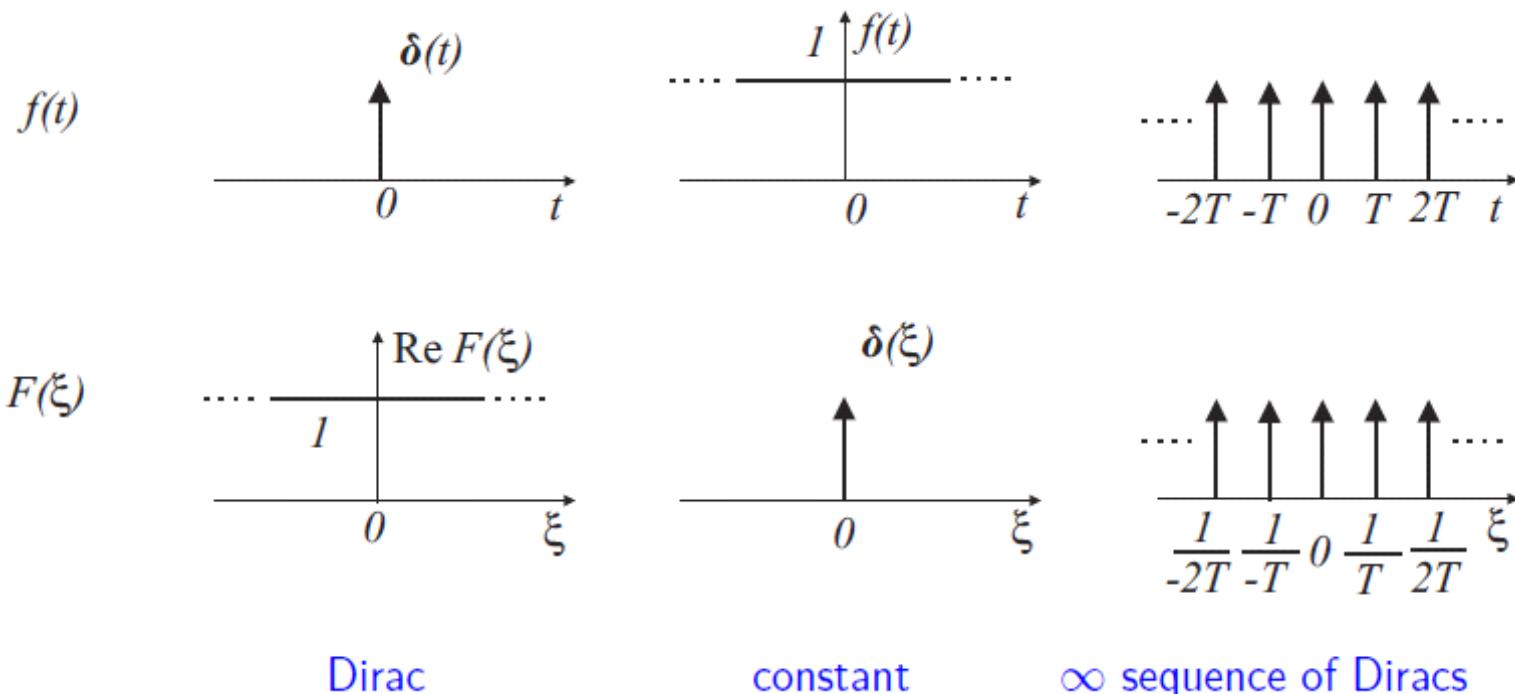
=

$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kt)$$

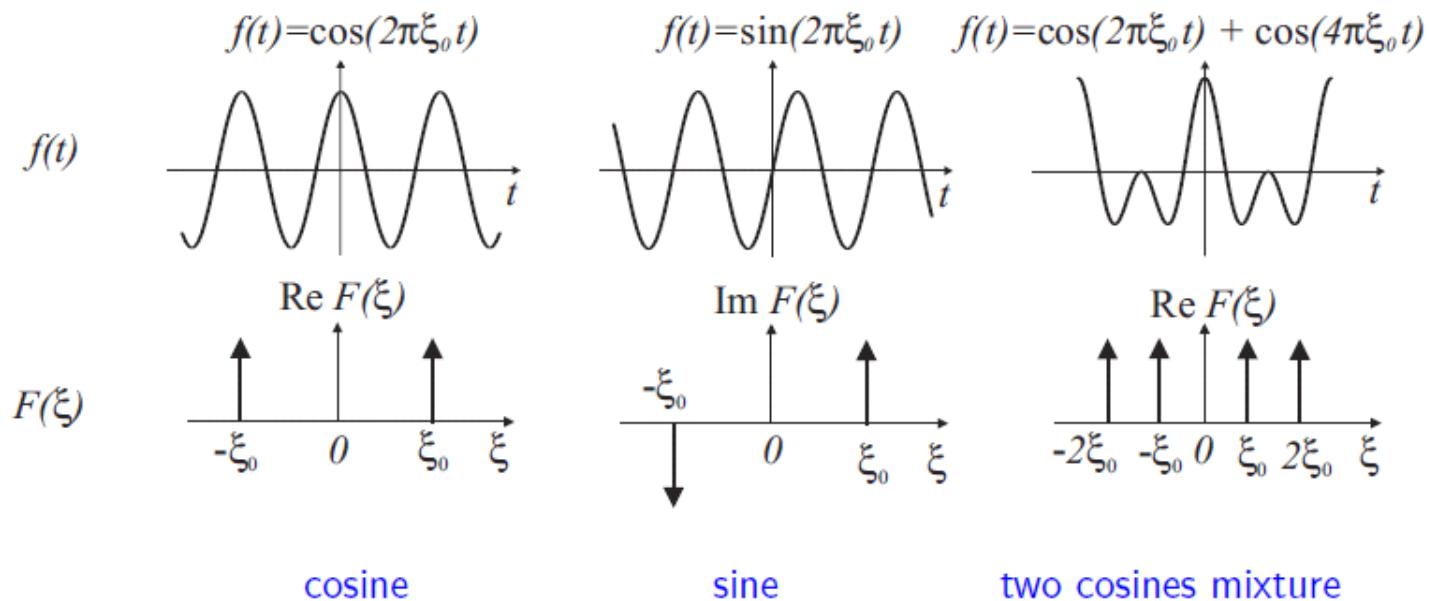




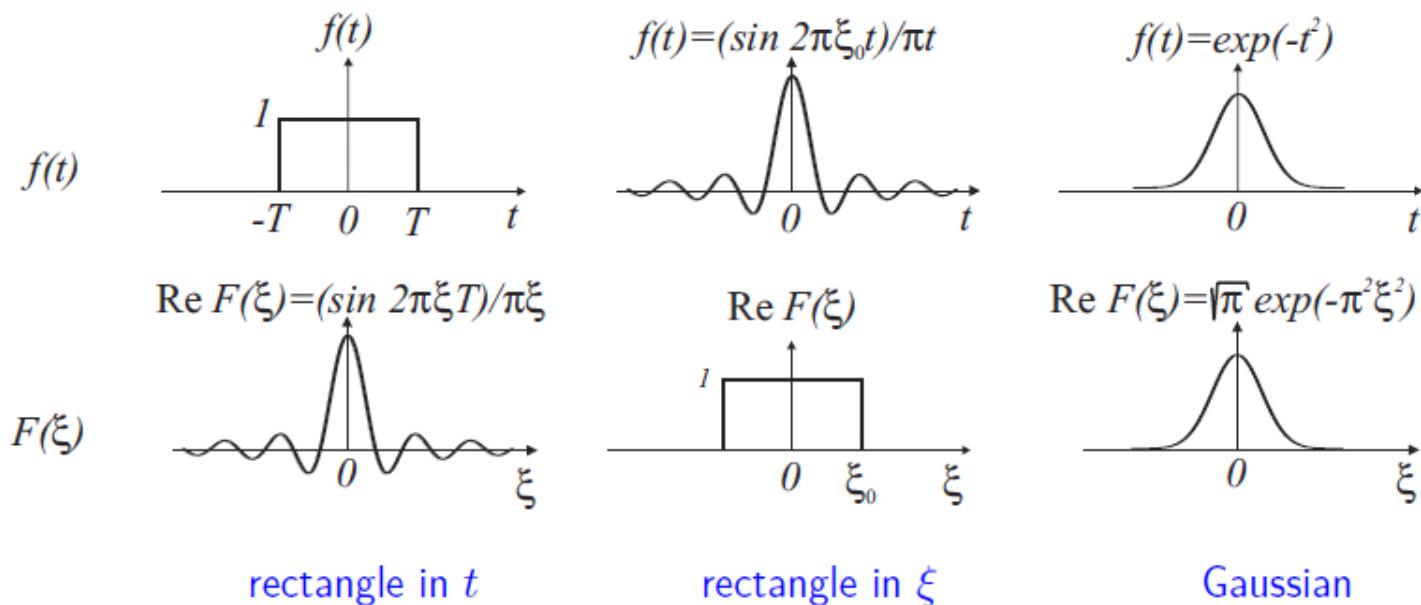
Exemple 1D TF



Exemple 1D TF



Exemple 1D TF



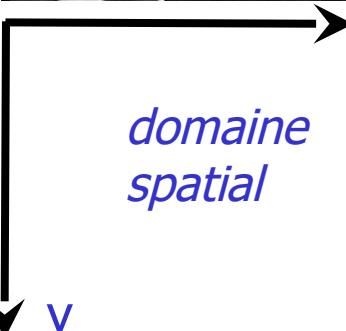
Transformée de Fourier (TF)

Image originale



domaine
spatial

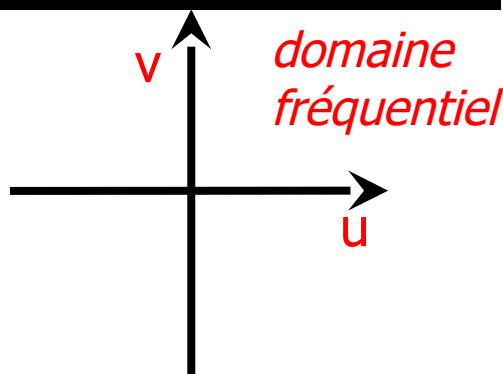
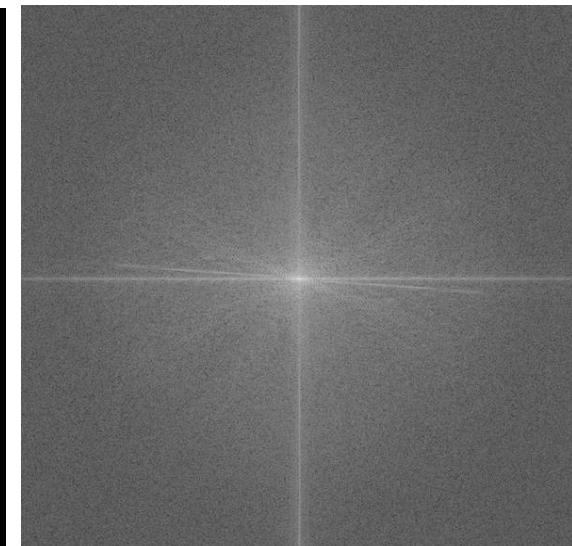
x

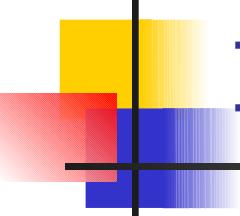


Spectre de Fourier
 $|F(u,v)|$



Spectre rehaussée
 $\log(1 + |F(u,v)|)$

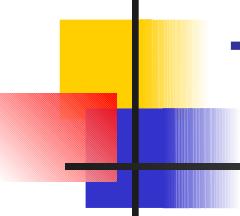




Interprétation de la Transformée 2D

- **Hautes fréquences** : loin du centre de la TF
- **Basses fréquences** : proche du centre de la TF
- **Composante continue (DC)** : centre de l'image
 - *fréquence zéro = moyenne de l'image*

$$F(k, l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i, j) e^{-\imath 2\pi (\frac{ki}{N} + \frac{lj}{N})}$$



Transformée en nombres complexes

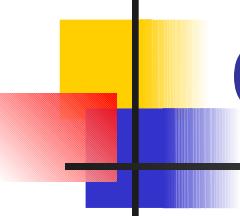
- La Transformée de Fourier d'une fonction réelle donne une fonction complexe
 - **Entrée** : une image
 - **Sortie** : image(partie réelle) + image(partie imaginaire)

- Rappel sur la définition d'un nombre complexe :

$$z = x + iy \quad \text{où} \quad i = \sqrt{-1}$$

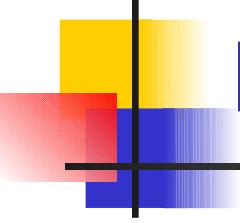
- On utilise beaucoup la norme de la Transformée :

$$\text{Norme} = |F(u, v)| = \sqrt{\text{Réel}^2 + \text{Imag}^2}$$



Codage de la transformée de Fourier

- L'algorithme le plus connu et le plus utilisé est la FFT
 - FFT = Fast Fourier Transform
- Deux implémentations très souvent utilisées sont
 - « Numerical Recipies », www.nr.com, chapitre 12
 - FFTW, www.fftw.org
- On le retrouve dans la majorité des librairies existantes
 - *incluant Torch3Vision et LTI-Lib*
 - *mais pas dans OpenCV qui possède une autre implémentation via la fonction cvDFT*

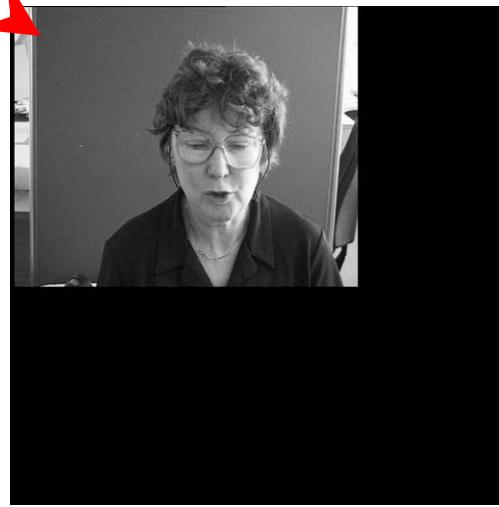


Fonctionnement (pratique) de la TF



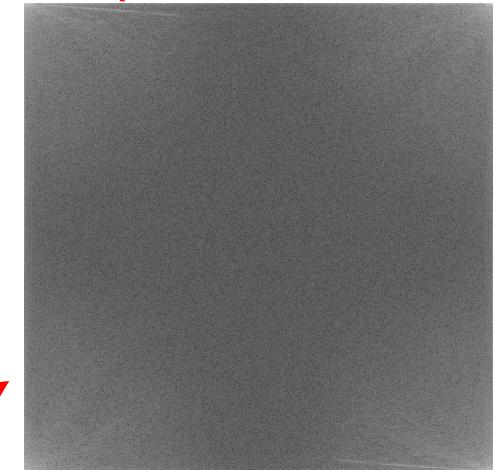
*Image originale
(niveau de gris)*

Pré-traitement

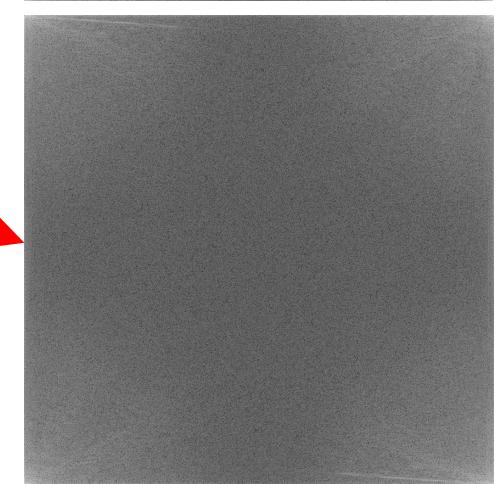


*Changement de taille
pour avoir des tailles
X et Y en puissance
de 2 (on remplit avec
des zéros).*

TF partie réelle



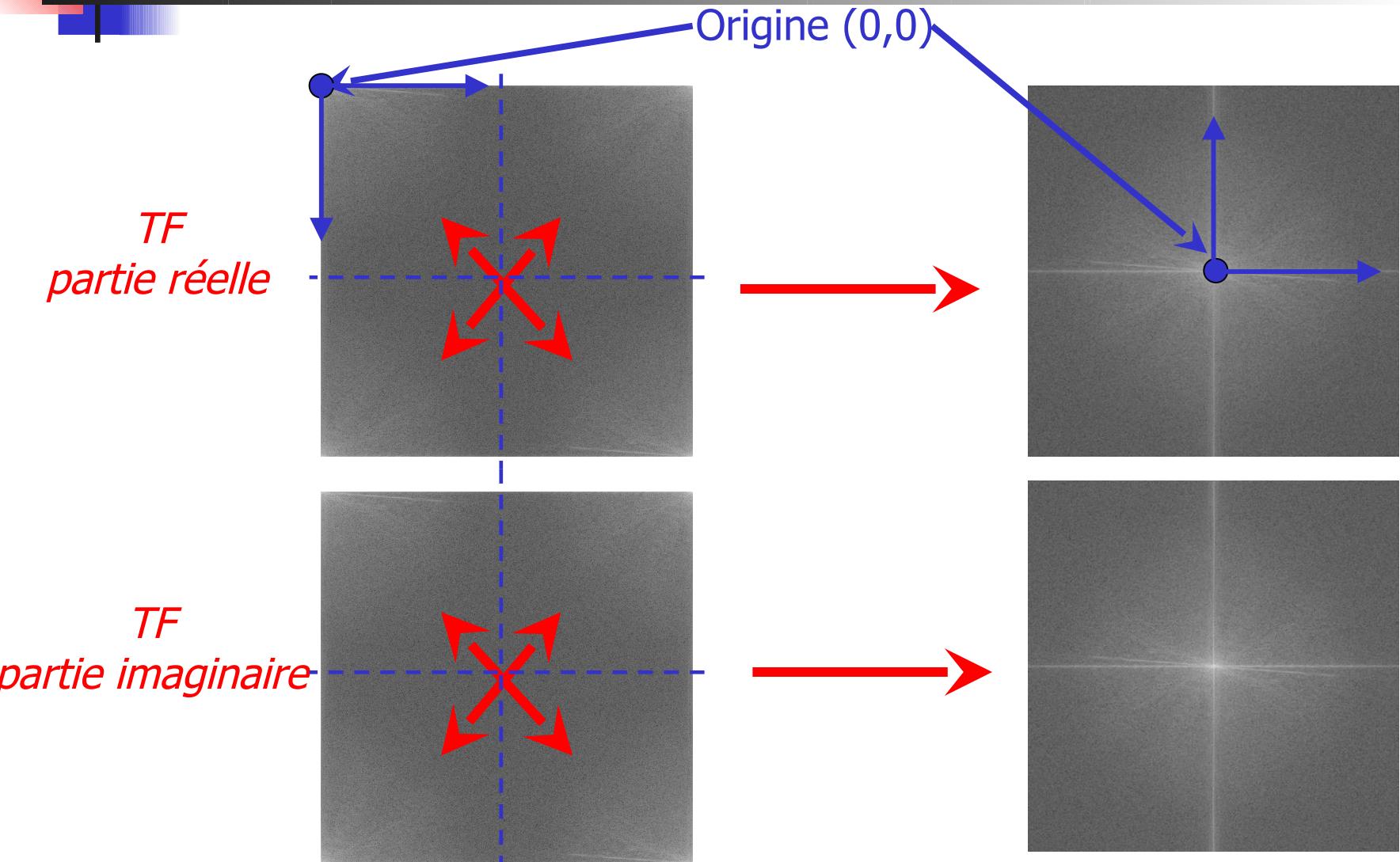
*FFT
ou
DFT*

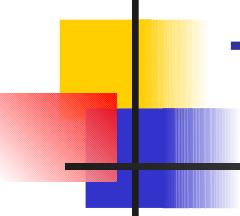


TF partie imaginaire

Inversion des quadrants de la TF

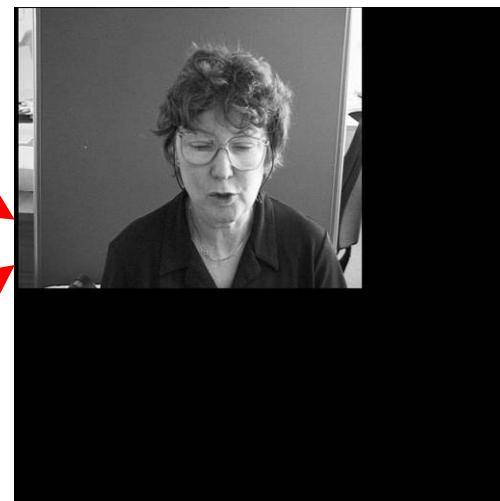
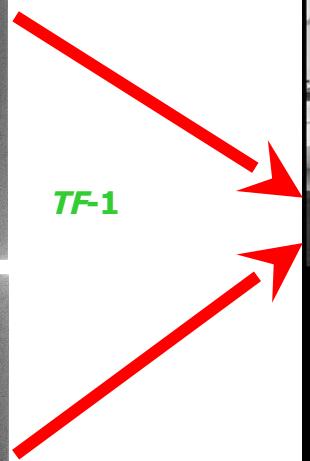
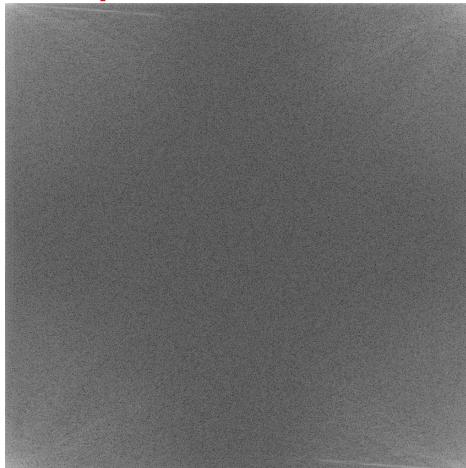
(selon les librairies)



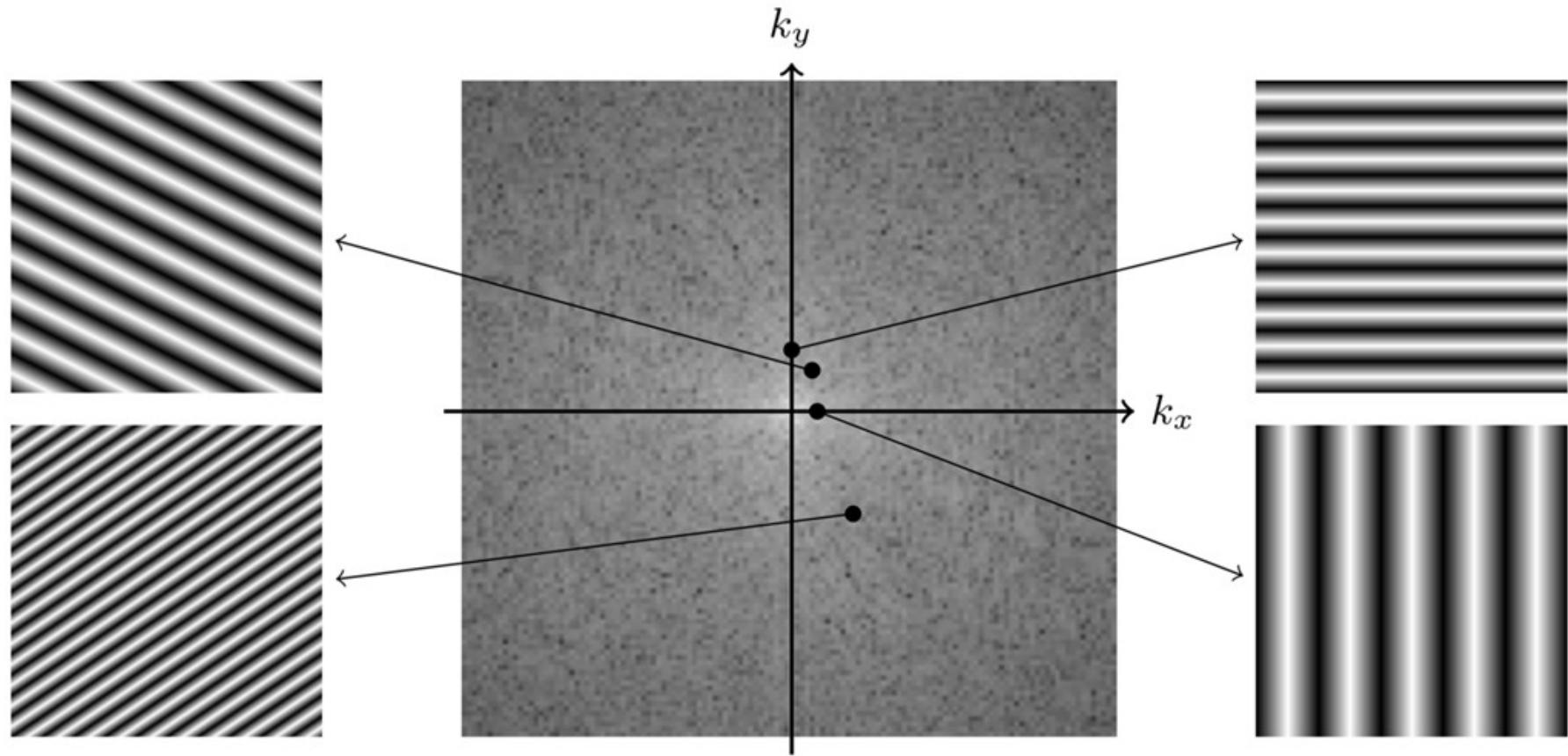
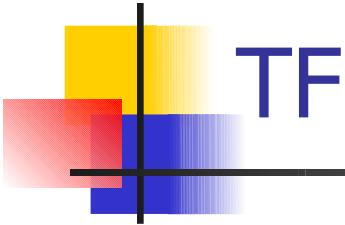


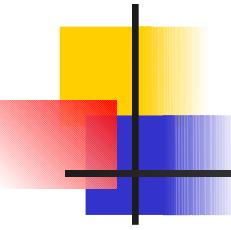
TF inverse

TF partie réelle

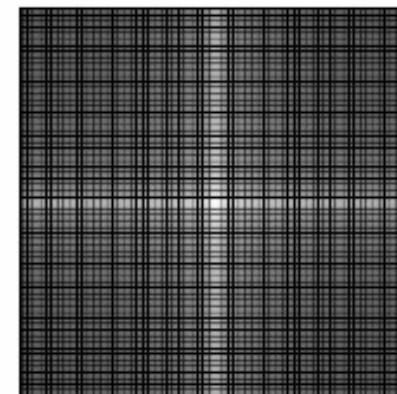
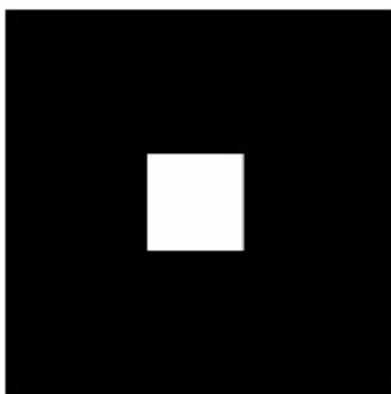
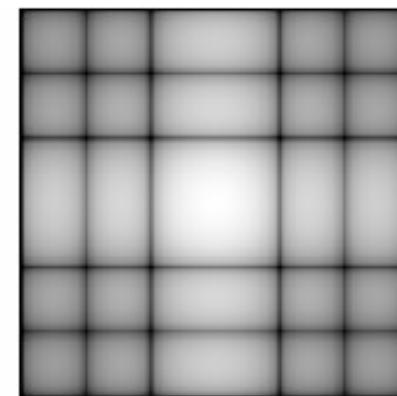
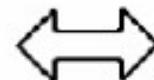
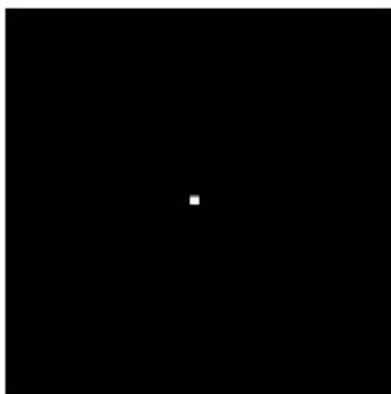


TF partie imaginaire





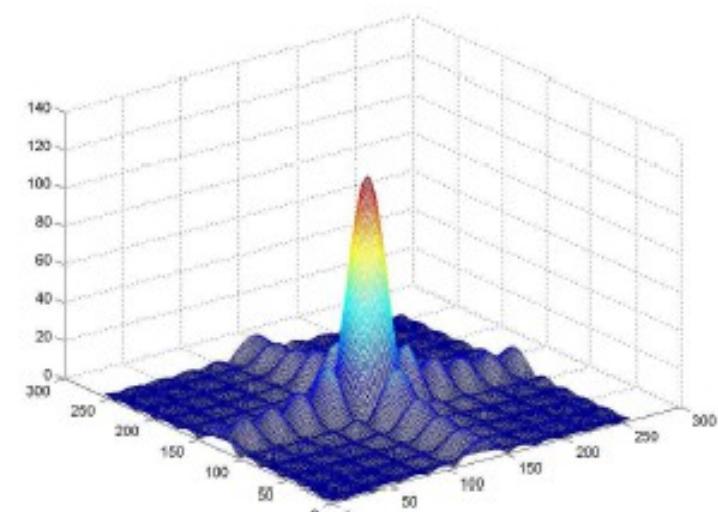
Exemples de Transformée de Fourier

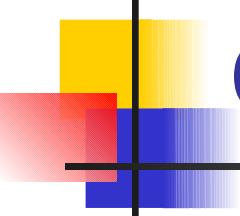


Images originales

TF

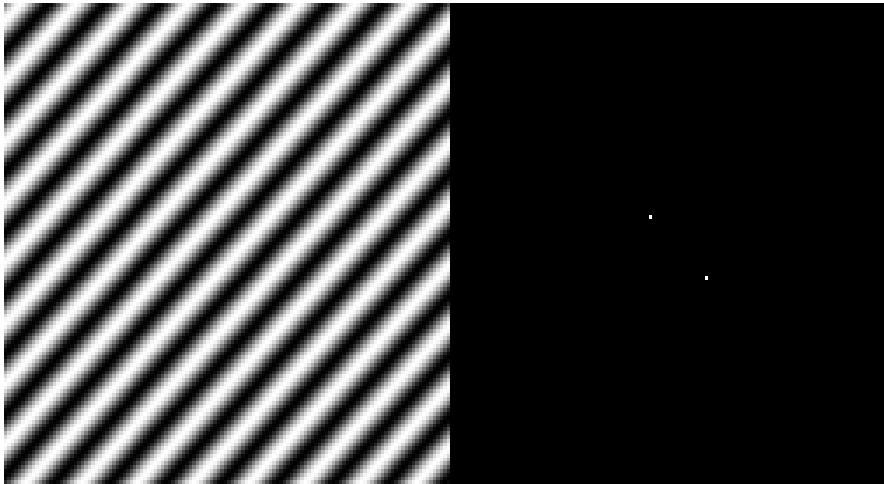
Vue 3D de la TF



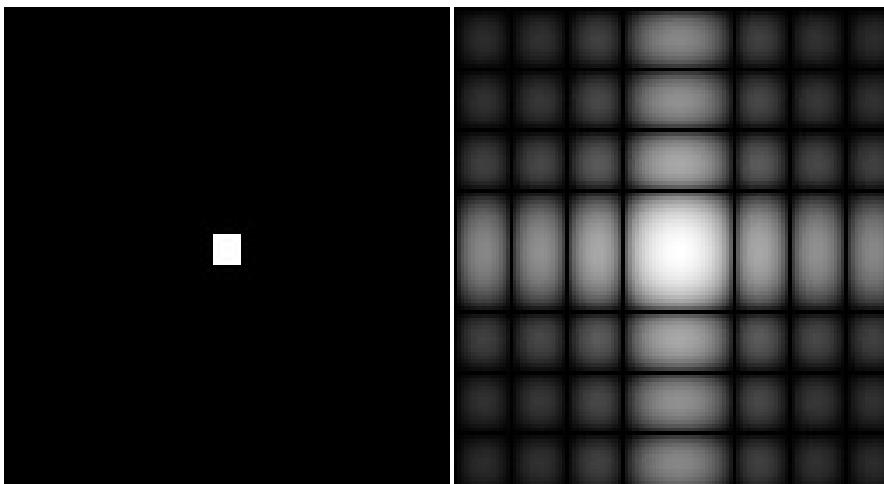
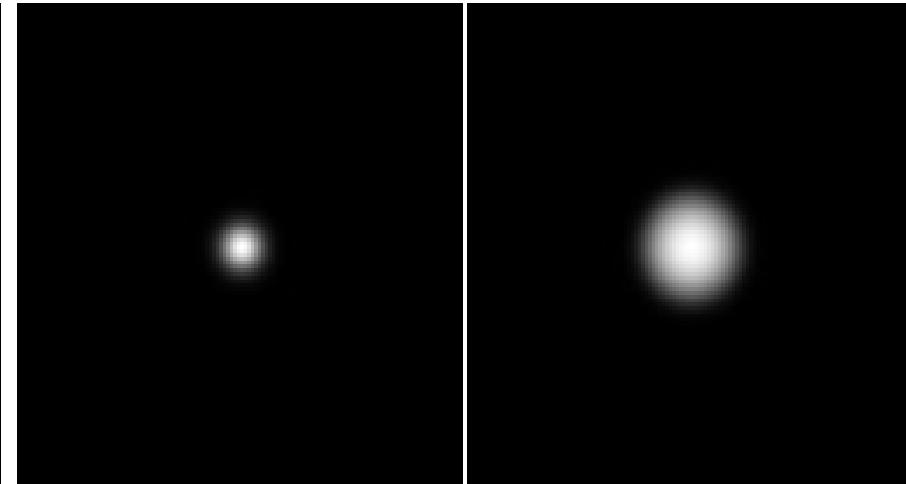


Quelques TF de base

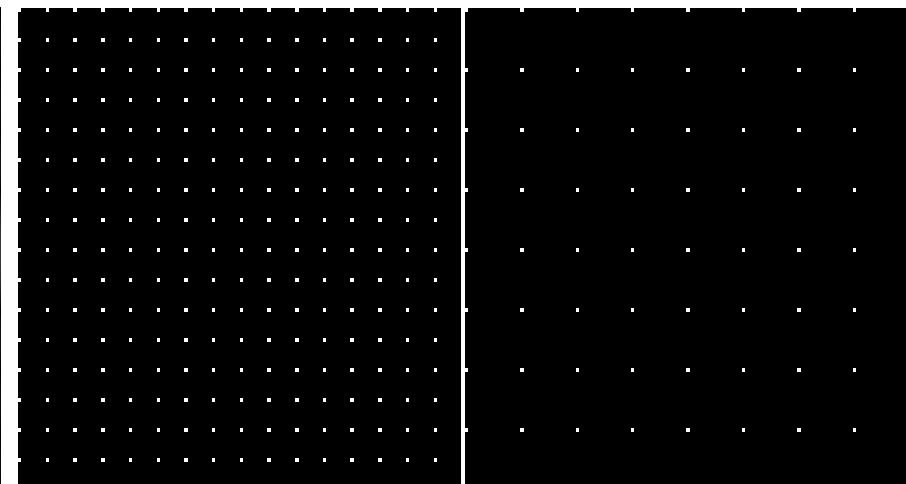
Sinus



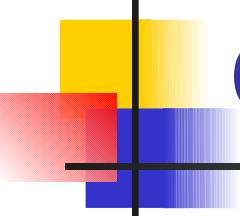
Gaussienne



Carré

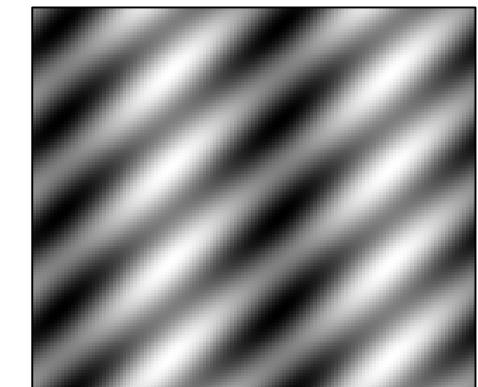
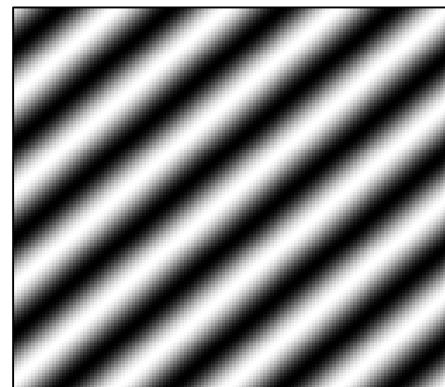
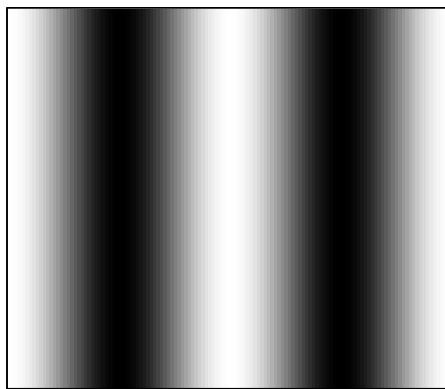


Impulsions

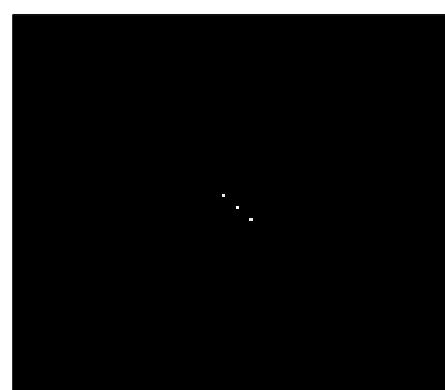
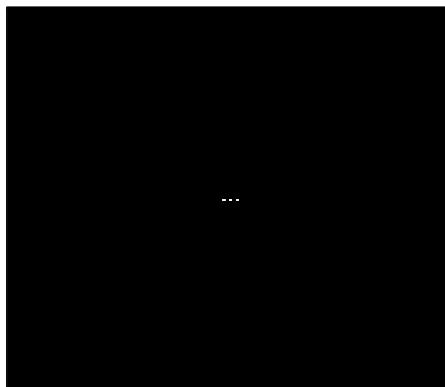


Quelques TF de base

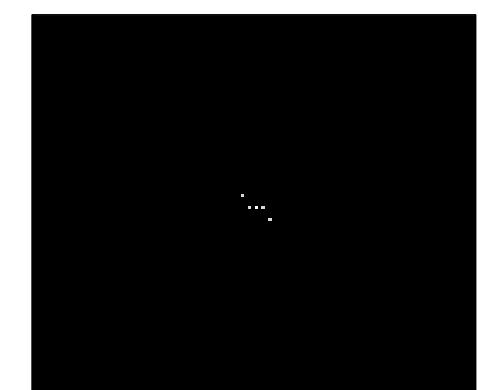
Images (domaine spatial)



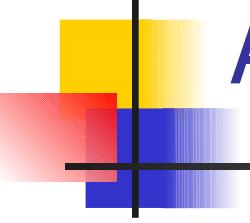
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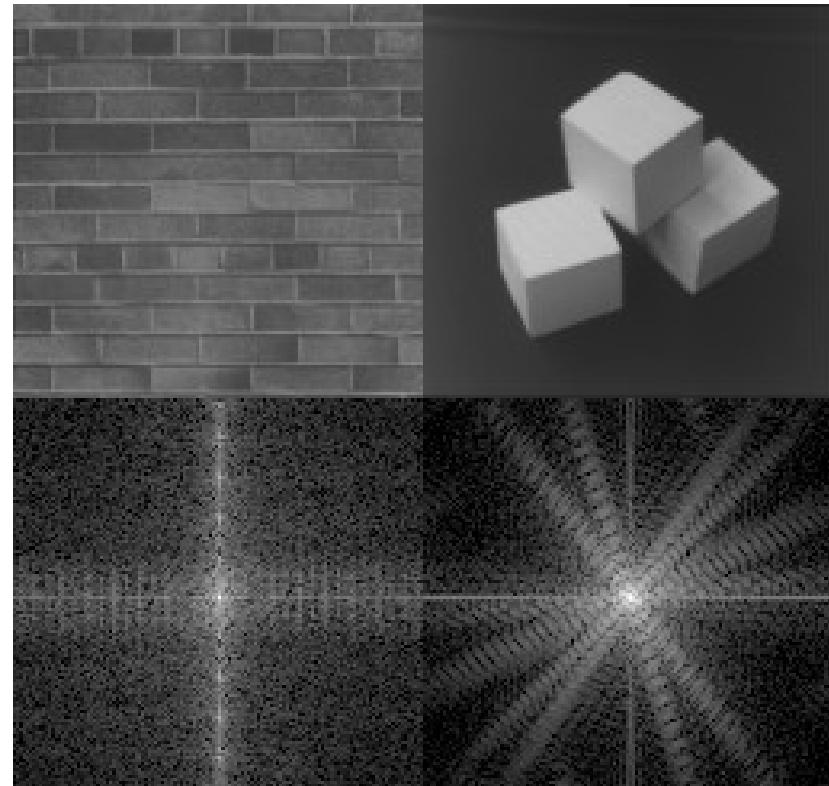


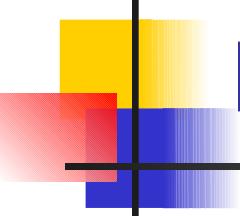
Spectre



Autres exemples (contours)

- Regardez les lignes dans ces deux images
- L'image de gauche possède des **lignes horizontales/verticales** qu'on retrouve dans sa transformée
- L'image de droite possède des **lignes dans toutes les directions** qu'on retrouve aussi dans sa transformée

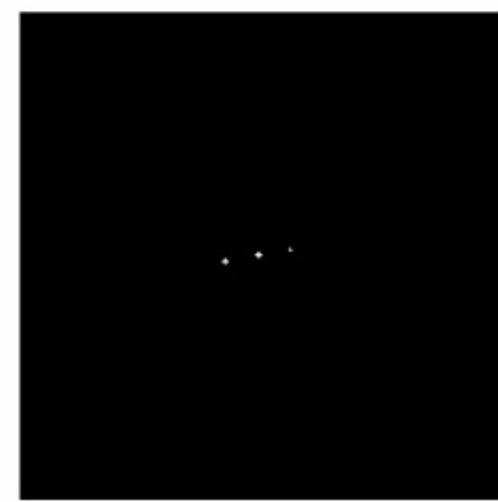
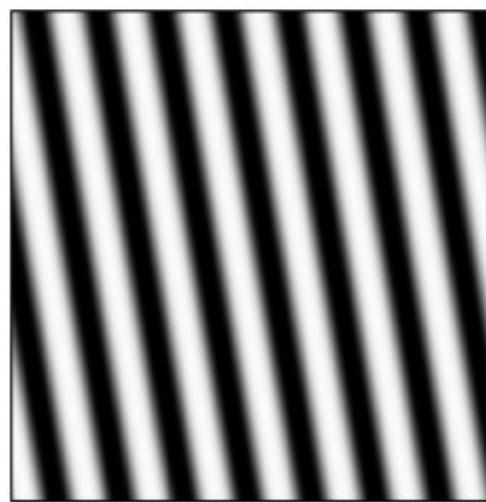
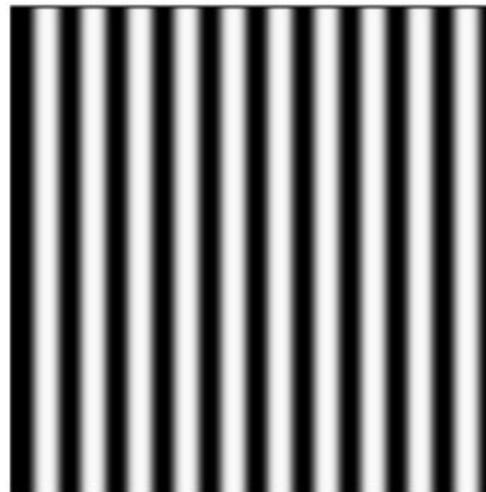


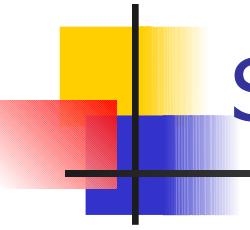


Rotation d'images

Rotation d'images

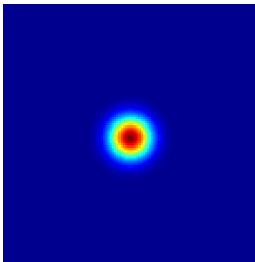
→ rotation de la
TF (même angle)



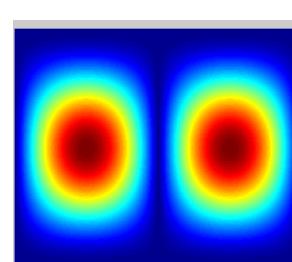


Spectres et images correspondantes?

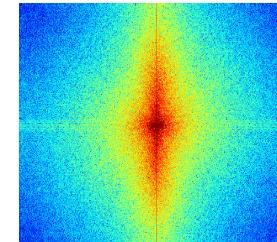
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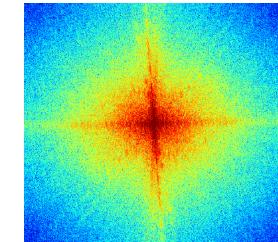
3



4

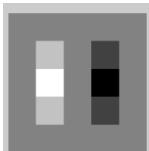


5



B

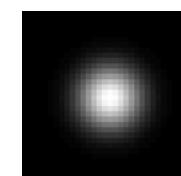
A



C

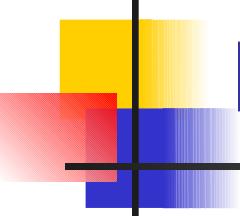


D

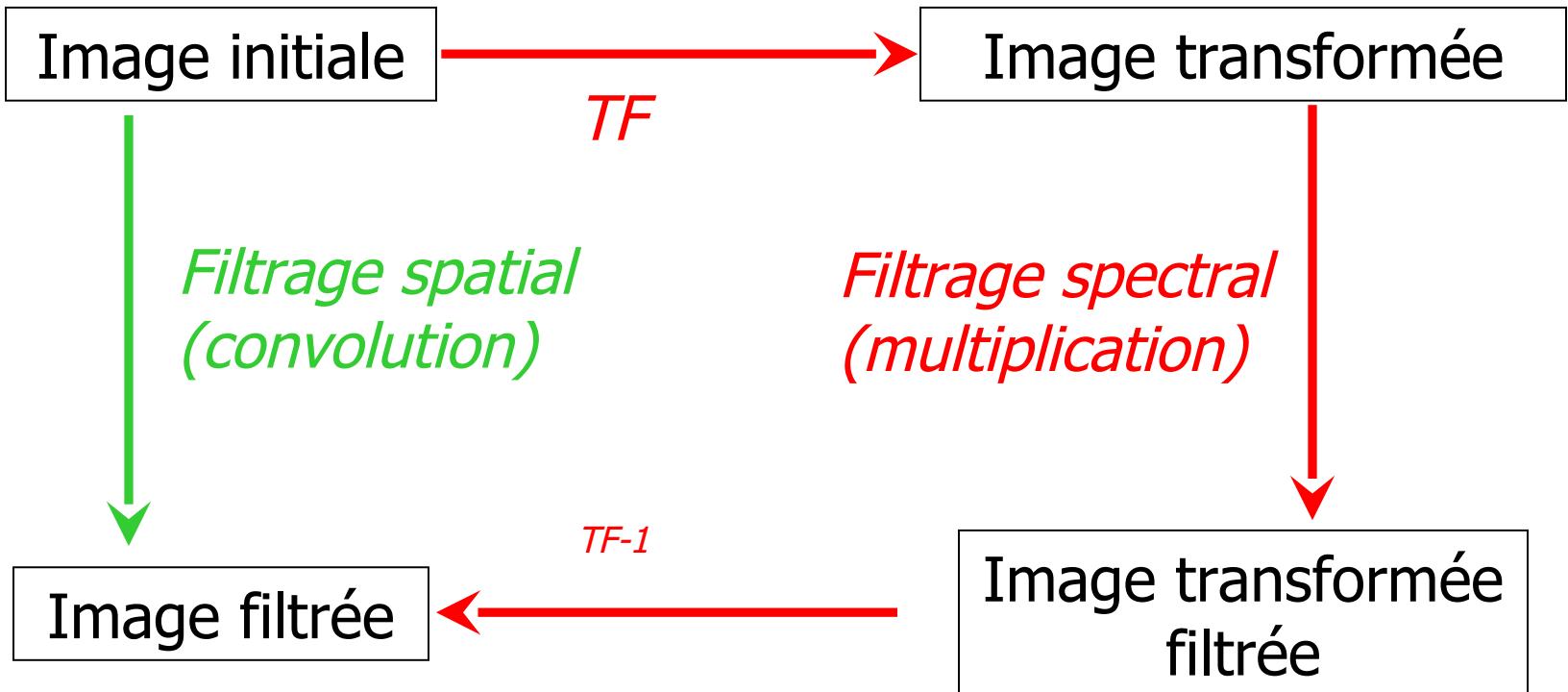


E



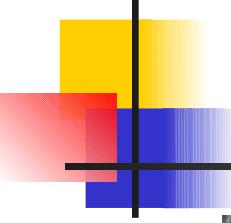


Filtrage dans le domaine spectral

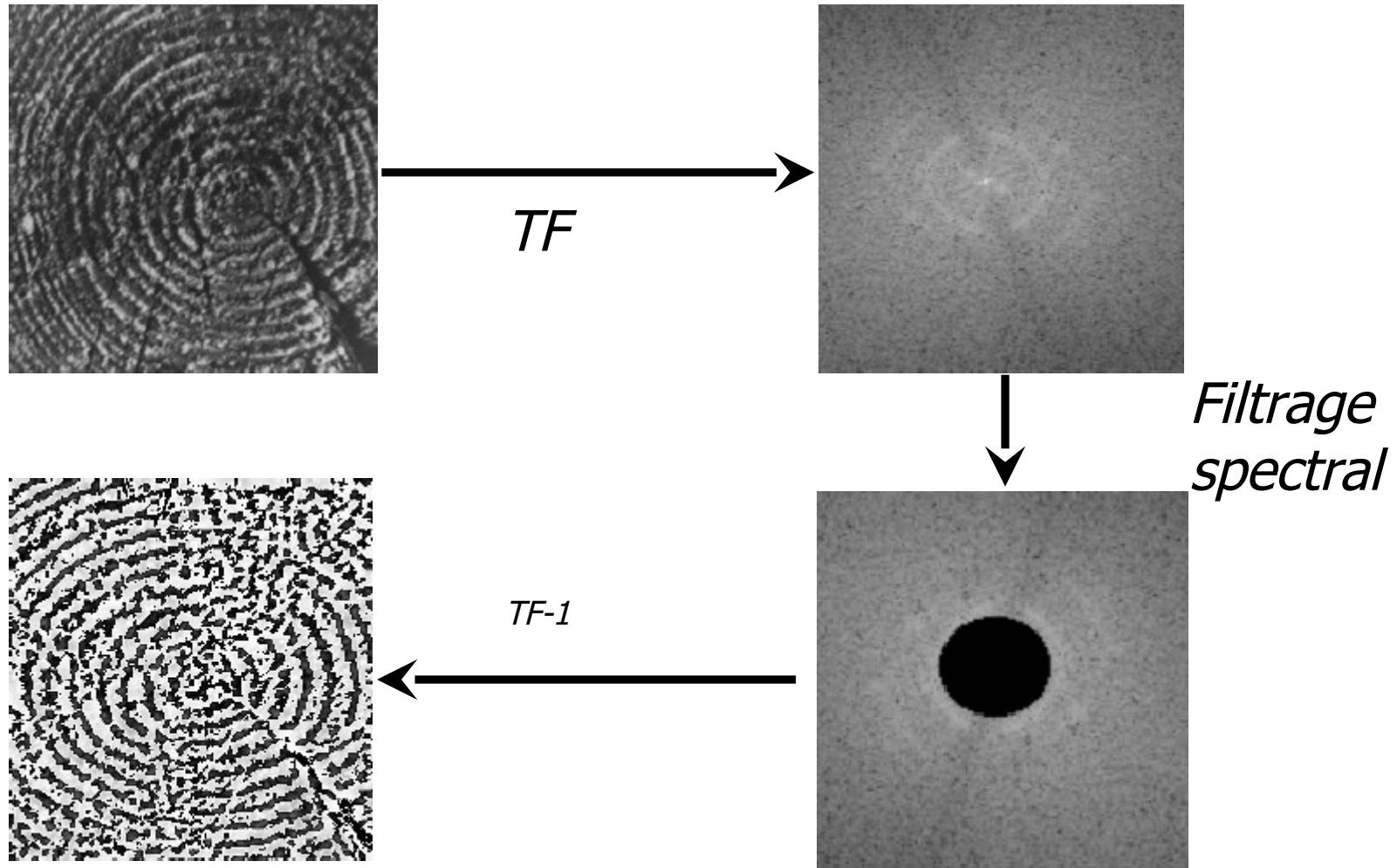


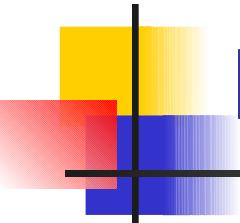
Dans le **domaine spatial**, le filtrage se fait par **convolution**. Dans le **domaine spectral** (ou **fréquentiel**), il se fait par **multiplication** (ou **masquage** de l'image).

Dans le cas des filtres dans le domaine fréquentiel non-multiplicatif, on ne peut pas obtenir le même résultat par convolution dans le domaine spatial.



Filtrage dans le domaine spectral



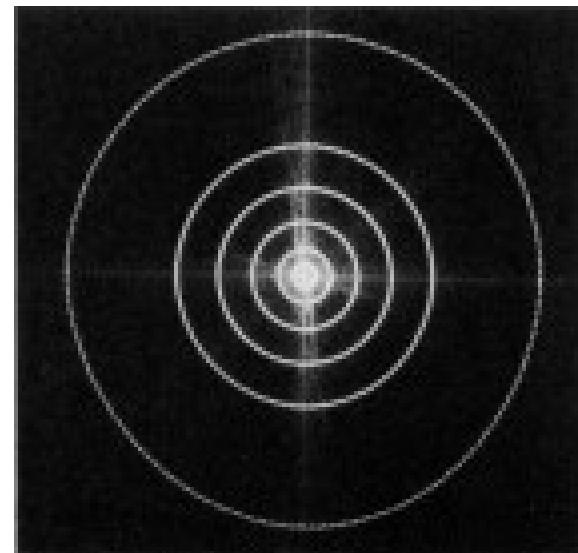


Bandes de fréquences

Image

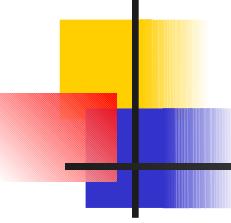


Spectre de Fourier



Pourcentage de l'information de l'image inclus dans les cercles (plus petit vers le plus grand) :

90%, 95%, 98%, 99%, 99.5%, 99.9%



Filtrage passe-bas

90%



98%



99.5%



95%



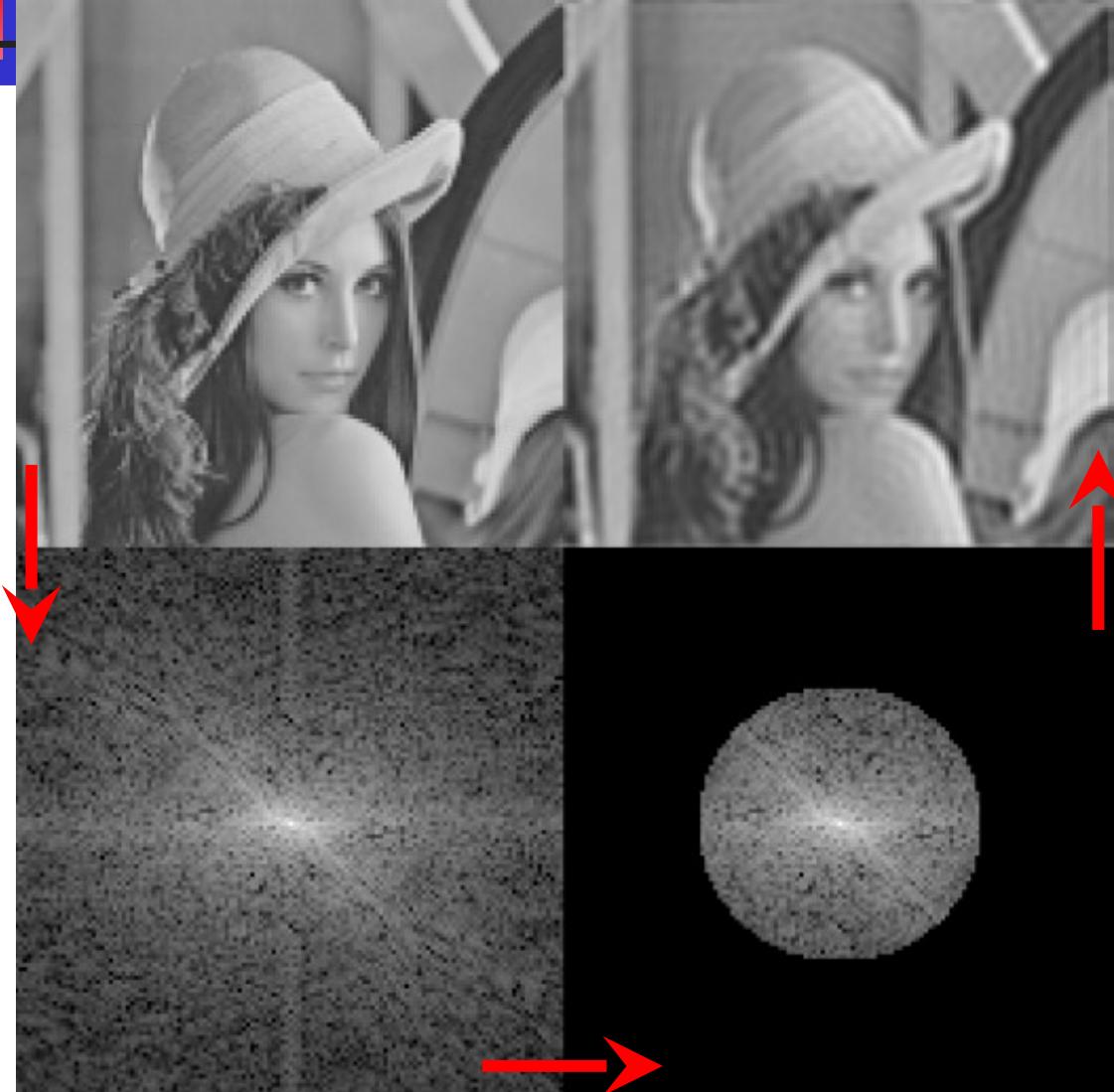
99%



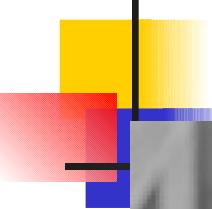
99.9%



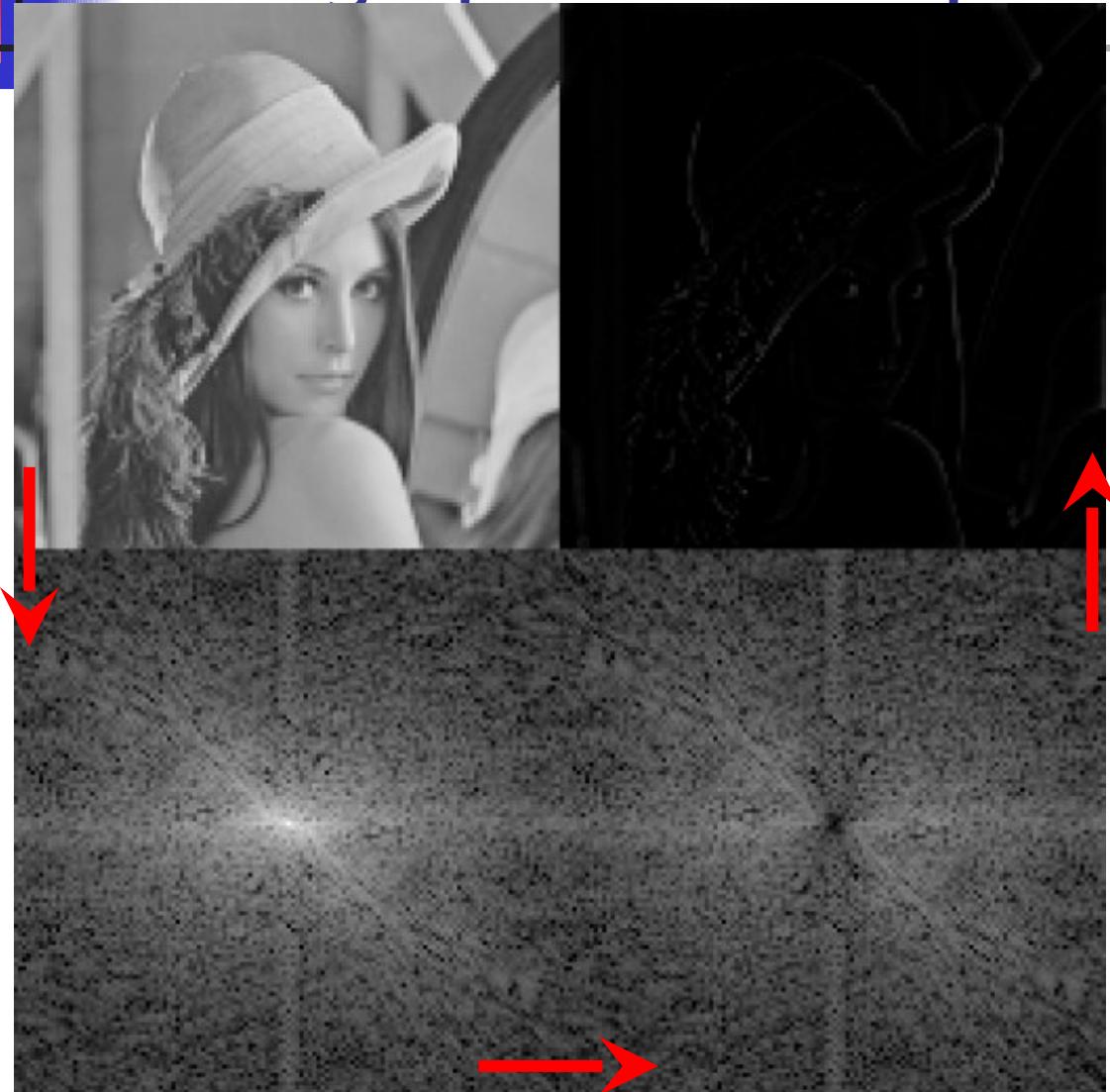
Filtrage passe-bas par TF



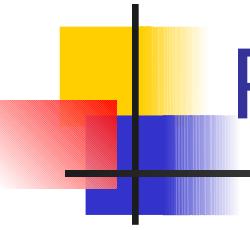
On efface les hautes fréquences de la TF en mettant les pixels loin du centre à zéro



Filtrage passe-haut par TF

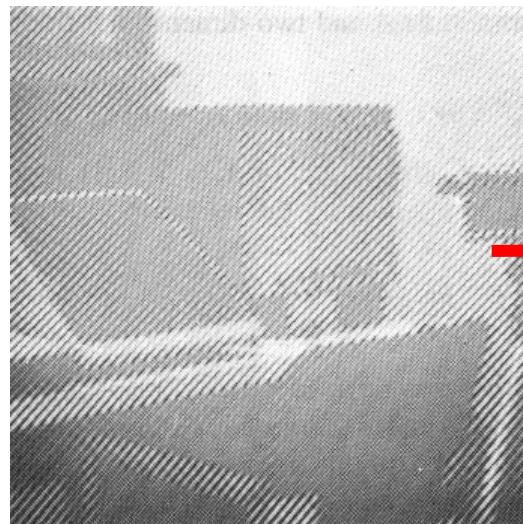


On efface les basses fréquences de la TF en mettant les pixels au centre à zéro

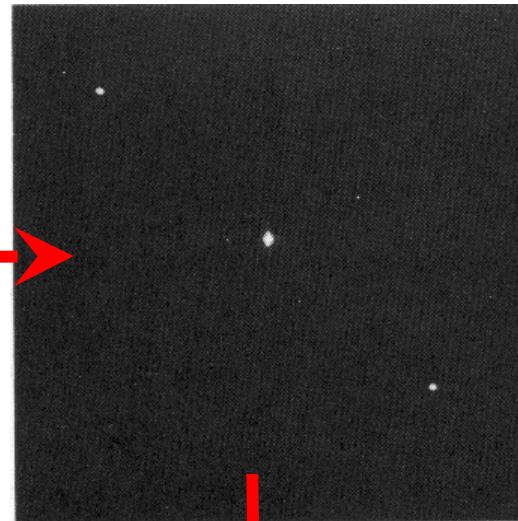


Réduction du bruit dans une image

Image bruitée
(bruit sinus)

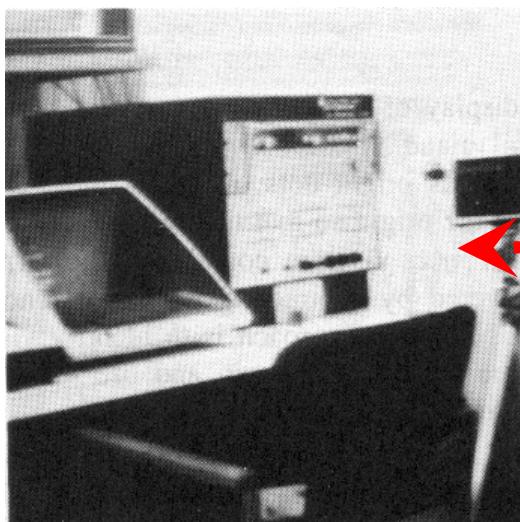


FFT

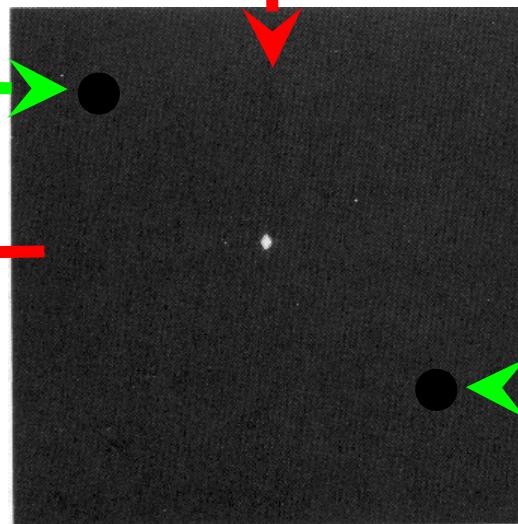


Spectre de Fourier (*DC + sinus visibles*)

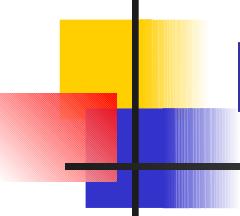
Image filtrée



FFT⁻¹



Mise à zéro des fréquences du sinus (*notch*)



Réduction du bruit dans une image

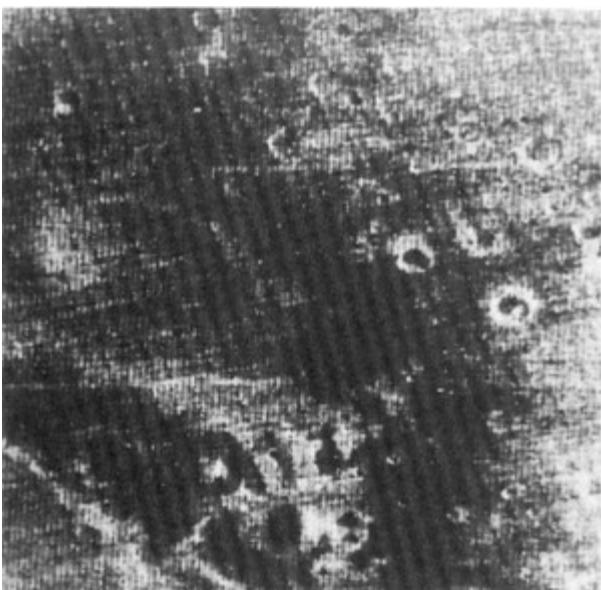
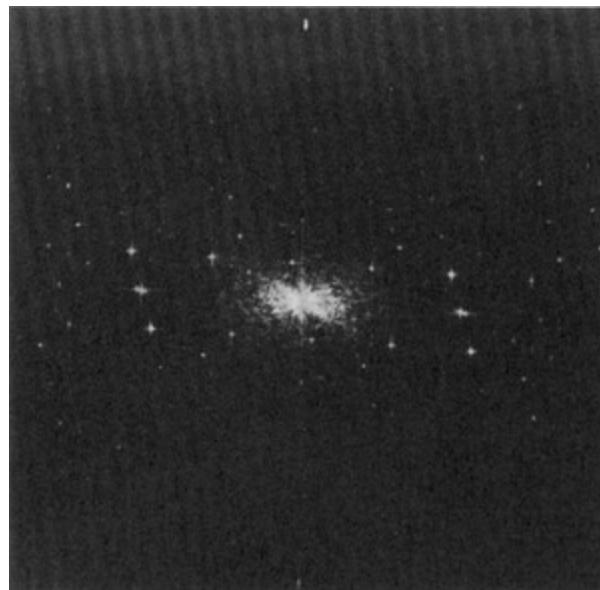


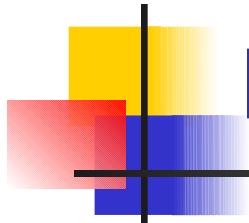
Image bruitée



Spectre de Fourier



Image filtrée



Exemple de filtrage passe-haut (1)

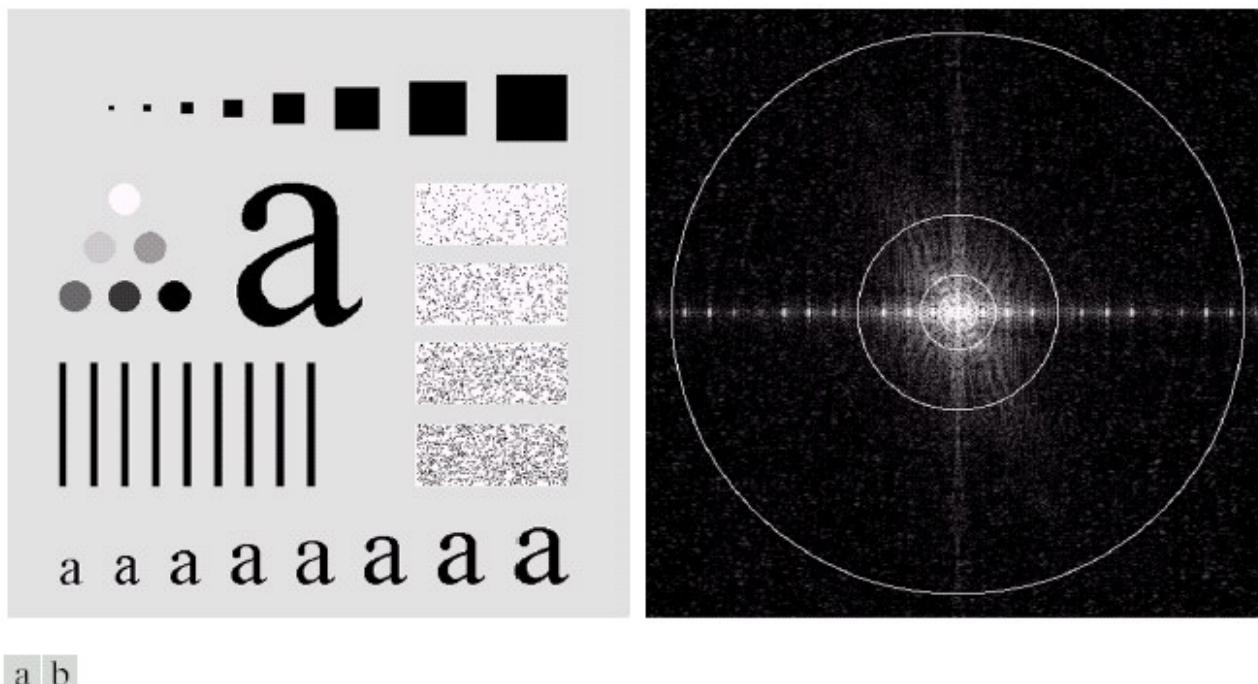
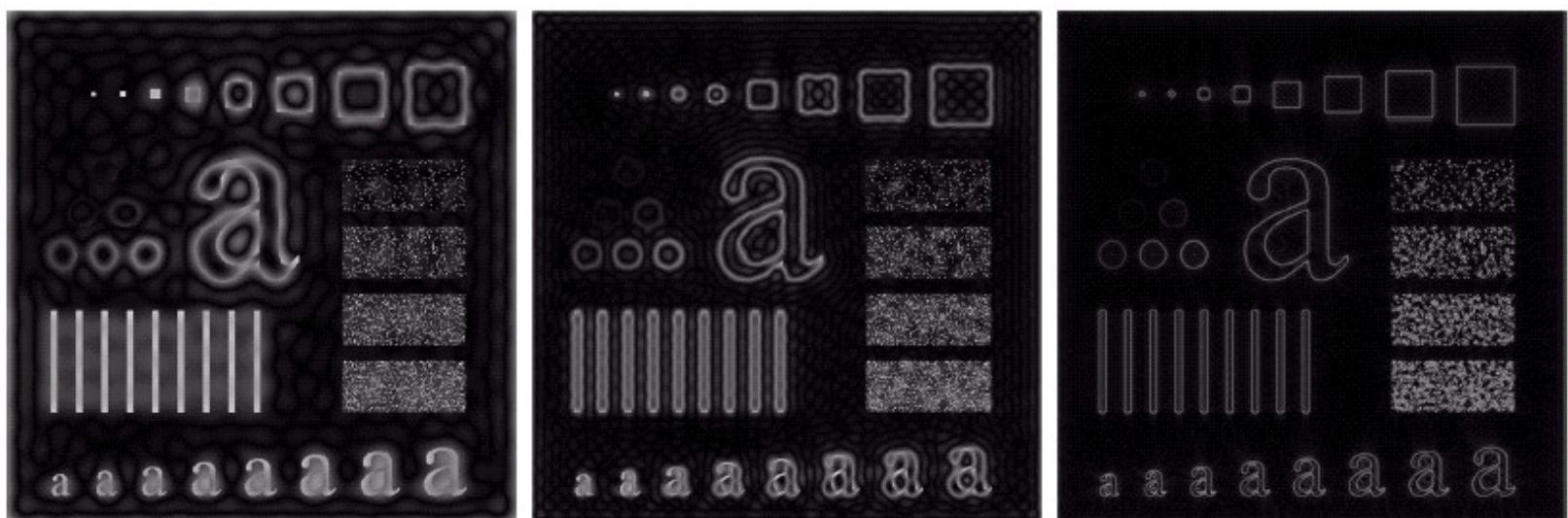


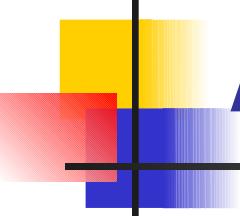
FIGURE 4.11 (a) An image of size 500×500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Exemple de filtrage passe-haut (2)

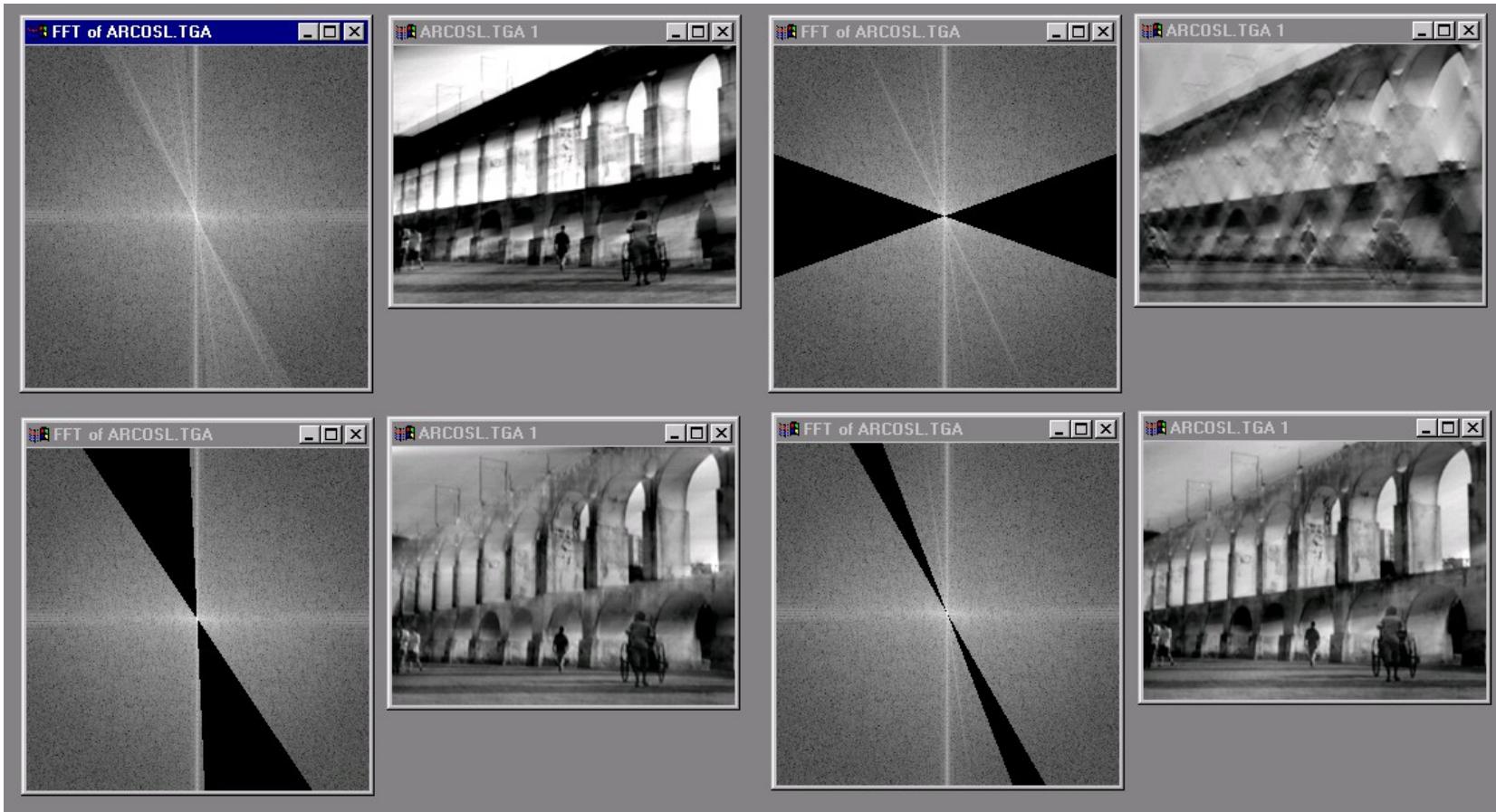


a | b | c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15, 30, and 80 , respectively. Problems with ringing are quite evident in (a) and (b).$



Autres examples



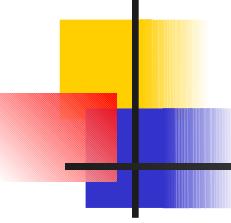


Image hybride

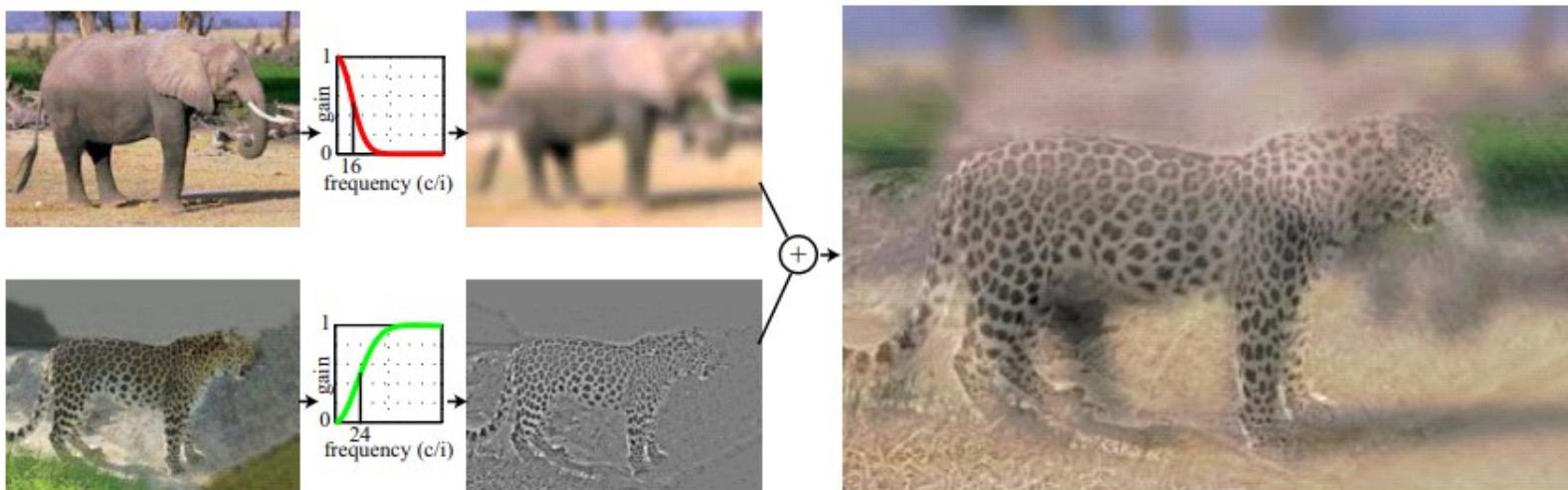
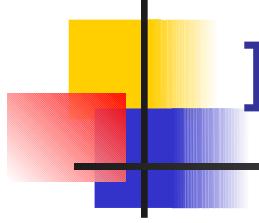
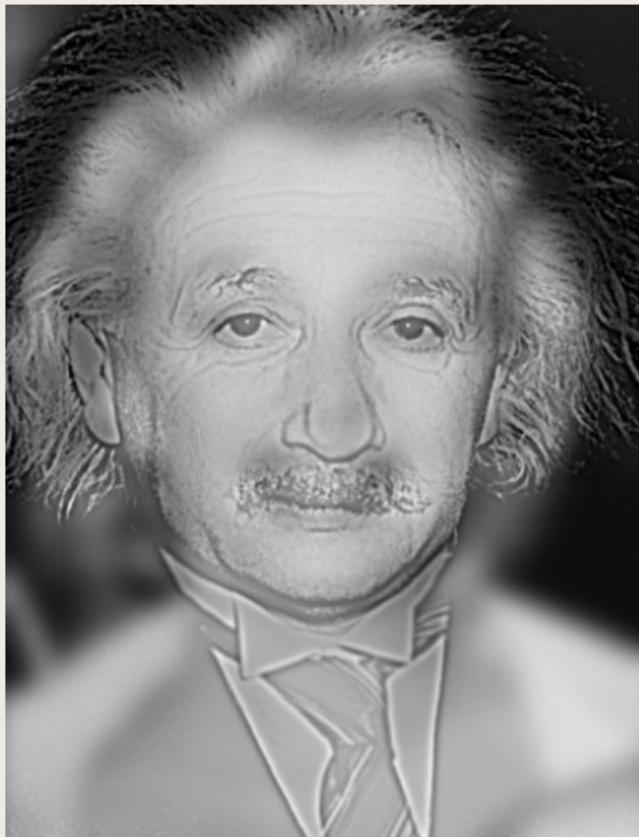


Figure 2: hybrid images are generated by superimposing two images at two different spatial scales: the low-spatial scale is obtained by filtering one image with a low-pass filter, and the high spatial scale is obtained by filtering a second image with a high-pass filter. The final hybrid image is composed by adding these two filtered images.



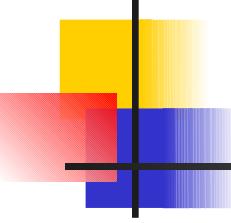
Marylin Einstein



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When you look at the image above, whose face do you see? At normal screen viewing distance you should see the face of the great scientist Albert Einstein. Now squint your eyes or take a few steps back from the image. Does a certain Hollywood personality pop into view? The Marylin Einstein hybrid image was created by Dr. Aude Oliva for the March 31st 2007 issue of New Scientist magazine.



Références

- **Livres**
 - Introduction au Traitement d'Images (Lingrand), sections 4.3 et 4.4
 - Digital Image Processing 2ed (Gonzalez & Woods), chapitres 3 et 4
- John M. Brayer. Introduction to Fourier transforms for image processing.
Université du Nouveau Mexique (USA).
 - <http://www.cs.unm.edu/~brayer/vision/fourier.html>
- <http://homepages.inf.ed.ac.uk/rbf/HIPR2/fourier.htm>