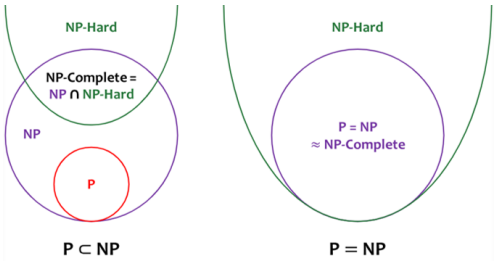
EECS 376: Foundations of Computer Science
Discussion 8 (Sec 21) PNPotpourri

Instruction: For each blank, fill in with *yes*, *no*, or *depends/unknown*.

Important Theorems:

1. $\mathsf{P} \subseteq \mathsf{NP}$. Moreover, all languages in NP are decidable.
2. If $A \leq_p B$, then $B \in P \implies A \in \mathsf{P}$. Contrapositive: $A \notin P \implies B \notin \mathsf{P}$.
3. If $A \leq_p B$, then $A \in \mathsf{NPH} \implies B \in \mathsf{NPH}$.
4. $\mathsf{P} = \mathsf{NP} \iff \mathsf{NPC} \subseteq \mathsf{P} \iff \mathsf{NPH} \cap \mathsf{P} \neq \emptyset \iff \mathsf{NP} \setminus \{\Sigma^*, \emptyset\} \subseteq \mathsf{NPH}$.
5. If $A \leq_p B$, then $A \leq_T B$. (The converse is not true.)



1 Closure Properties

Determine whether the following complexity classes are close under complement, union, and intersection.

Classes	Complement	Union	Intersection
P			
NP			
NPC			
NPH			

2 Union and Intersection

Suppose A and B are languages. Based on the membership of A and B in the first two columns, determine whether $A \cup B$ and $A \cap B$ are necessarily in (or not in) $\mathsf{P}, \mathsf{NP}, \mathsf{NPC}, \mathsf{NPH}$.

A	B	$A \cup B$				$A \cap B$			
		P	NP	NPC	NPH	P	NP	NPC	NPH
P	NP								
P	NPC								
P	NPH								
NP	NPC								
NP	NPH								
NPC	NPH								

3 Poly-Time Mapping Reduction

Suppose L is a language. Based on the membership of L in the first column, determine whether L' is necessarily in (or not in) $\mathsf{P}, \mathsf{NP}, \mathsf{NPH}, \mathsf{NPC}$ if $L \leq_p L'$ and $L' \leq_p L$.

L	$L \leq_p L'$				$L' \leq_p L$			
	P	NP	NPC	NPH	P	NP	NPC	NPH
P								
NP								
NPC								
NPH								

Remark

Please do **not** ask Eric for the solution of this worksheet. It's structured to help you create your own cheatsheet and solidify your understanding. Working through it on your own is essential. For guidance on specific concepts, feel free to ask on Piazza or during office hours. (Also to be honest, I don't have the full solution.)