EECS 376: Foundations of Computer Science
Discussion 7 (Sec 23) -ability Potpourri

Instruction: For each blank, fill in with *yes*, *no*, or *depends*. If your answer is *depends*, provide two counterexamples: one where the answer is *yes* and another where it is *no*.

Legend: D: Decidable; UD: Undecidable;
R: Recognizable; UR: Unrecognizable.

Language L	L		$\bar{L}$	
	D?	R?	D?	R?
Σ^* = Set of all finite-length strings	Yes	Yes	Yes	Yes
$L_{\text{BARBER}} = \{\langle M \rangle : M$ does not accept $\langle M \rangle\}$	No	No	No	Yes
$L_{\text{ACC}} = \{(\langle M \rangle, x) : M$ accepts $x\}$	No	Yes	No	No
$L_{\text{HALT}} = \{(\langle M \rangle, x) : M$ halts on $x\}$	No	Yes	No	No
$L_{\text{EQ}} = \{(\langle M_1 \rangle, \langle M_2 \rangle) : L(M_1) = L(M_2)\}$	No	No	No	No

1. The class of decidable languages is closed under union, intersection, and complement.
2. If $A \leq_T B$, then B decidable $\implies$ A decidable. Contrapositive: A undecidable $\implies$ B undecidable.
3. Any decidable language is Turing-reducible to any language.
4. Any language is Turing-reducible to itself and to its own complement.
5. L decidable $\implies$ L recognizable. Contrapositive: L unrecognizable $\implies$ L undecidable.
6. A language L and its complement $\bar{L}$ are both recognizable if and only if L is decidable. Contrapositive: L is undecidable if and only if at least one of L and $\bar{L}$ is unrecognizable.
7. The class of recognizable languages is closed under union and intersection.

1 Complement

Suppose L is a language and $\bar{L}$ is the complement of L. Based on the hypotheses in the first two columns, determine whether L and $\bar{L}$ are necessarily (un)decidable and necessarily (un)recognizable.

L	$\bar{L}$	L D?	L R?	$\bar{L}$ D?	$\bar{L}$ R?
D		Yes			
UD, R		No	Yes		
UD, UR		No	No		
R	R		Yes		Yes
R	UR		Yes		No
UR	UR		No		No

2 Union and Intersection

Suppose A and B are languages. Based on the hypotheses in the first two columns, determine whether $A \cap B$ and $A \cup B$ are necessarily (un)decidable and necessarily (un)recognizable.

A	B	$A \cap B$ D?	$A \cap B$ R?	$A \cup B$ D?	$A \cup B$ R?
D	D				
D	UD				
D	R				
D	UR				
UD	UD				
UD	R				
UD	UR				
R	R				
R	UR				
UR	UR				

3 Subset and Superset

Suppose L is a language. Based on the hypotheses in the first column, determine whether L' is (un)decidable and necessarily (un)recognizable if L' is a subset of L and if L' is a superset of L.

L	$L' : L' \subseteq L$ D?	$L' : L' \subseteq L$ R?	$L' : L' \supseteq L$ D?	$L' : L' \supseteq L$ R?
D				
R				
UD				
UR				

4 Turing Reduction

Suppose L is a language. Based on the hypotheses in the first column, determine whether L' is necessarily (un)decidable and necessarily (un)recognizable if $L' \leq_T L$ and if $L \leq_T L'$.

L	$L' : L' \leq_T L$ D?	$L' : L' \leq_T L$ R?	$L' : L \leq_T L'$ D?	$L' : L \leq_T L'$ R?
D				
R				
UD				
UR				

Remark

Please do **not** ask Eric for the solution of this worksheet. It's structured to help you create your own cheatsheet and solidify your understanding. Working through it on your own is essential. For guidance on specific concepts, feel free to ask on Piazza or during office hours.