1 Sistema de Tipos da Linguagem Funcional

1.1 Sintaxe

$$E ::= n \mid v \mid b \mid E \ op \ E$$
$$\mid \texttt{If } E \texttt{ then } E \texttt{ else } E$$
$$\mid E \ E \mid \texttt{Fun } x : \texttt{T} \Rightarrow E$$
$$\mid \texttt{let } x : \texttt{T} = E \texttt{ in } E$$

$$n \in \mathbb{Z}$$
$$b \in \{\texttt{true}, \texttt{false}\}$$
$$op \in \{+, -, *, /\}$$
$$T ::= \text{Int}|\text{Bool}|\texttt{T} \to \texttt{T}$$
$$v ::= n|b|\texttt{Fun } x : \texttt{T} \Rightarrow E$$

$$\Delta, \texttt{x} : \texttt{T} \Leftrightarrow \Delta \cup \{\langle \texttt{x}, \texttt{T} \rangle\}$$

1.2 Semântica Estática

(TNum)
$$\overline{\Delta \vdash \texttt{n} : \texttt{Int}}$$

(TBool)
$$\overline{\Delta \vdash \texttt{b} : \texttt{Bool}}$$

(TVar)
$$\overline{\Delta \vdash \texttt{x} : \Delta(x)}$$

(TIf)
$$\frac{\Delta \vdash \texttt{E}_1 : \texttt{Bool} \quad \Delta \vdash \texttt{E}_2 : \texttt{T} \quad \Delta \vdash \texttt{E}_3 : \texttt{T}}{\Delta \vdash \texttt{If } E_1 \texttt{ then } E_2 \texttt{ else } E_3 : \texttt{T}}$$

(TSoma)
$$\frac{\Delta \vdash \texttt{E}_1 : \texttt{Int} \quad \Delta \vdash \texttt{E}_2 : \texttt{Int}}{\Delta \vdash E_1 + E_2 : \texttt{Int}}$$

(TAp)
$$\frac{\Delta \vdash \texttt{E}_1 : \texttt{T}_1 \to \texttt{T}_2 \quad \Delta \vdash \texttt{E}_2 : \texttt{T}_1}{\Delta \vdash E_1 \ E_2 : \texttt{T}_2}$$

(TFun)
$$\frac{\Delta, x : \texttt{T} \vdash \texttt{E} : \texttt{T}'}{\Delta \vdash \texttt{fun } x : \texttt{T} \Rightarrow \texttt{E} : \texttt{T} \to \texttt{T}'}$$

(TLet)
$$\frac{\Delta \vdash \texttt{E}_1 : \texttt{T} \quad \Delta, x : \texttt{T} \vdash \texttt{E}_2 : \texttt{T}'}{\Delta \vdash \texttt{let } x : \texttt{T} = \texttt{E}_1 \texttt{ in } \texttt{E}_2 : \texttt{T}'}$$