Asymptotic Analysis of Expressions
Basic Problems 2: Homework

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April 7, 2025

E1: Graphical Representation of Expressions

We analyze the following expressions in terms of Θ-order and display their graphical growth.
Expressions:

1. $n \log n$

2. n^{-1}

3. $\log n$

4. $n^{\log n}$

5. $10n + n^{3/2}$

6. π^n

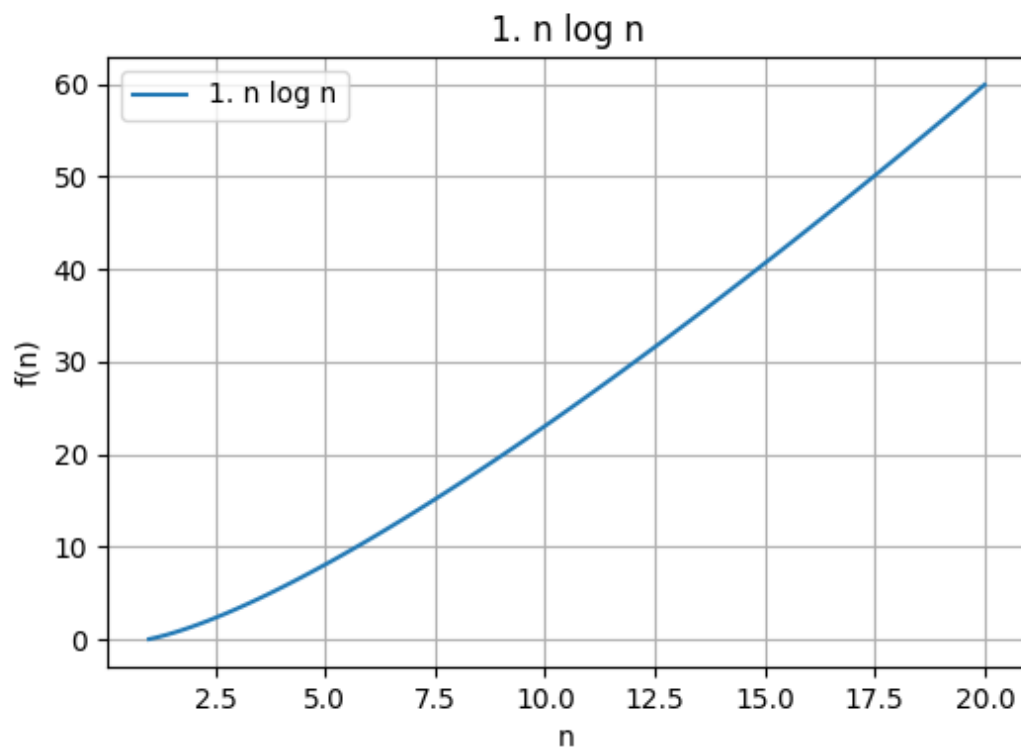7. 2^n

8. $2^{\log n}$

9. $2^{\log^2 n}$
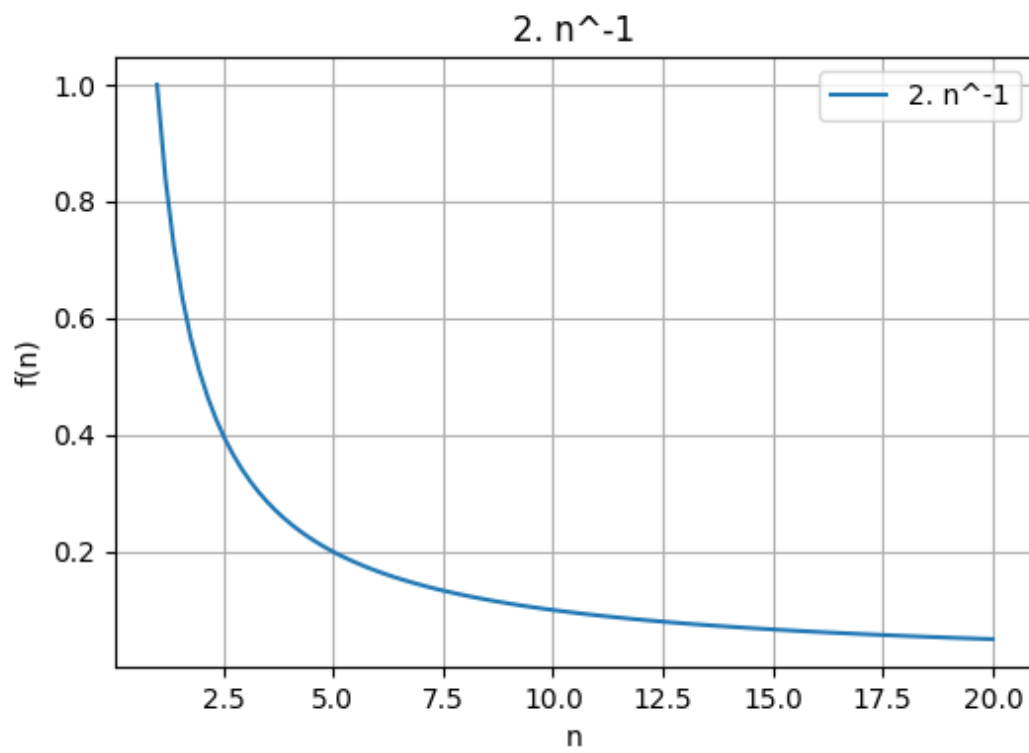
10. $\log n!$

Figure 1: Graphical comparison of n log n.
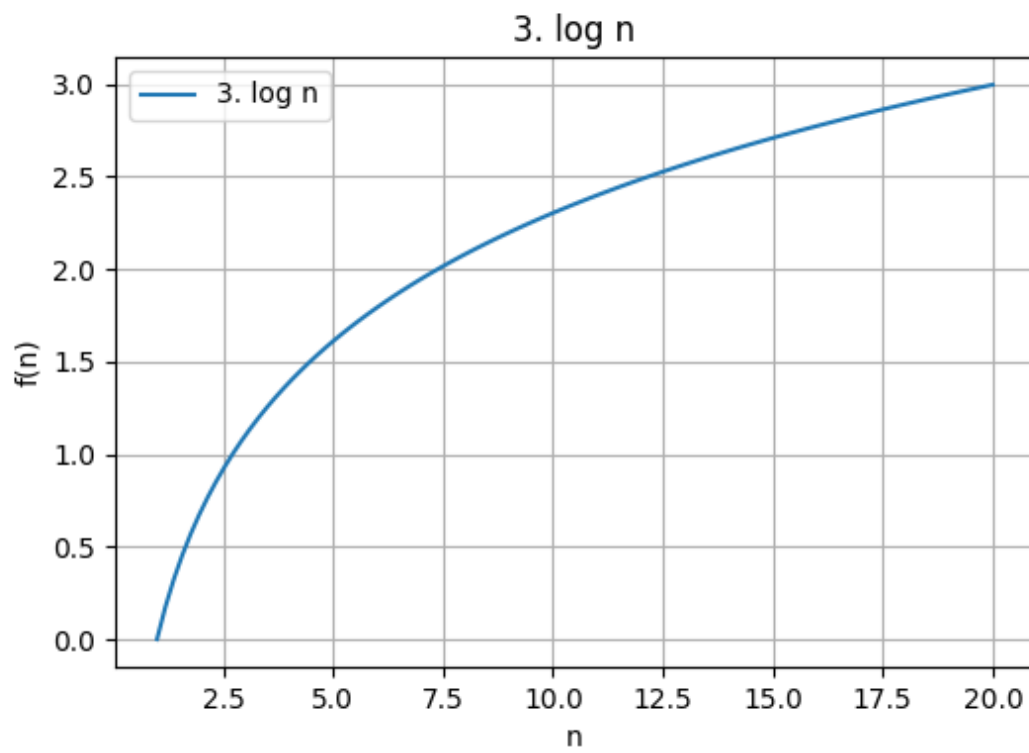
Figure 2: Graphical comparison of n^{-1}.
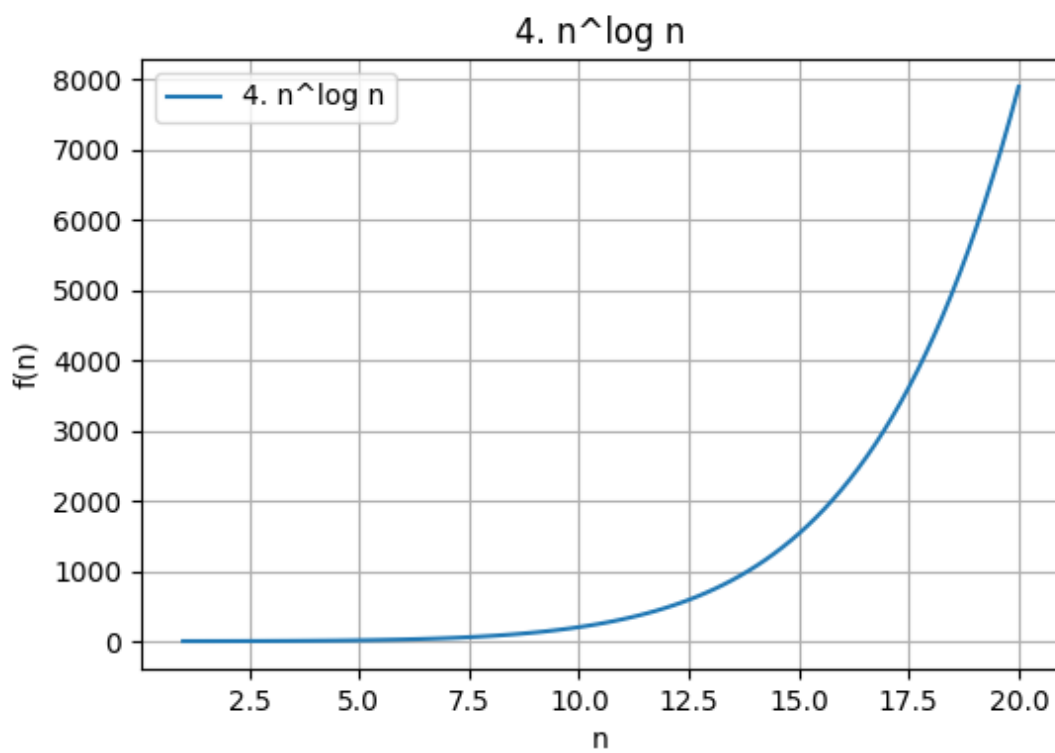
Figure 3: Graphical comparison of log n.
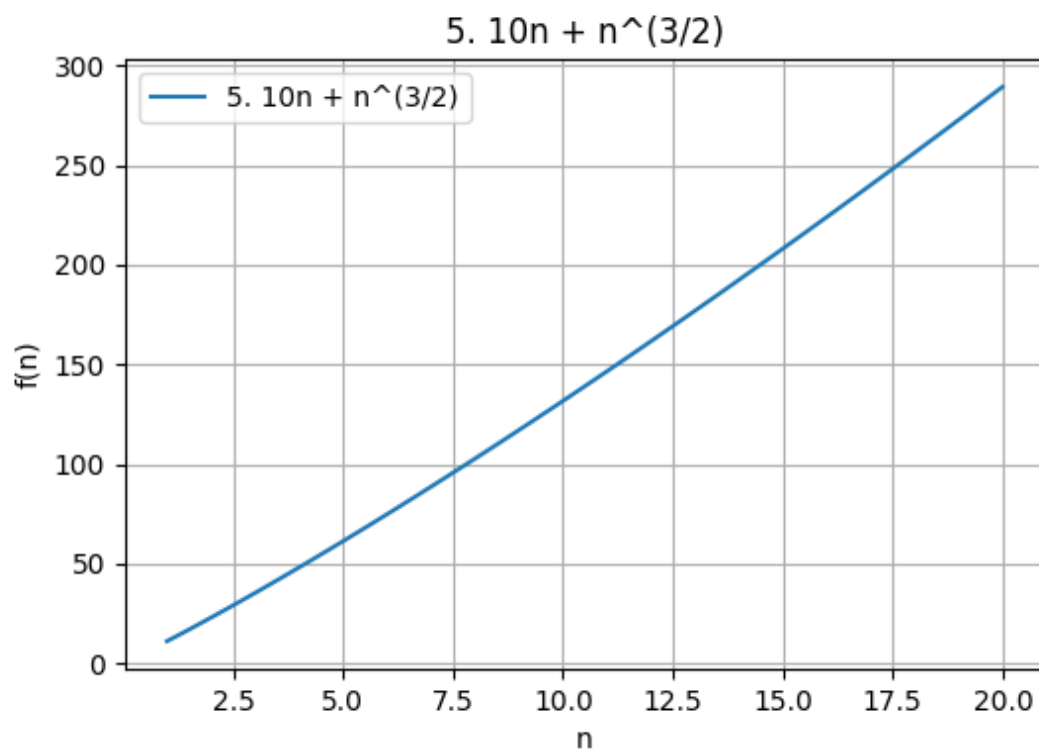
Figure 4: Graphical comparison of n^{logn}.
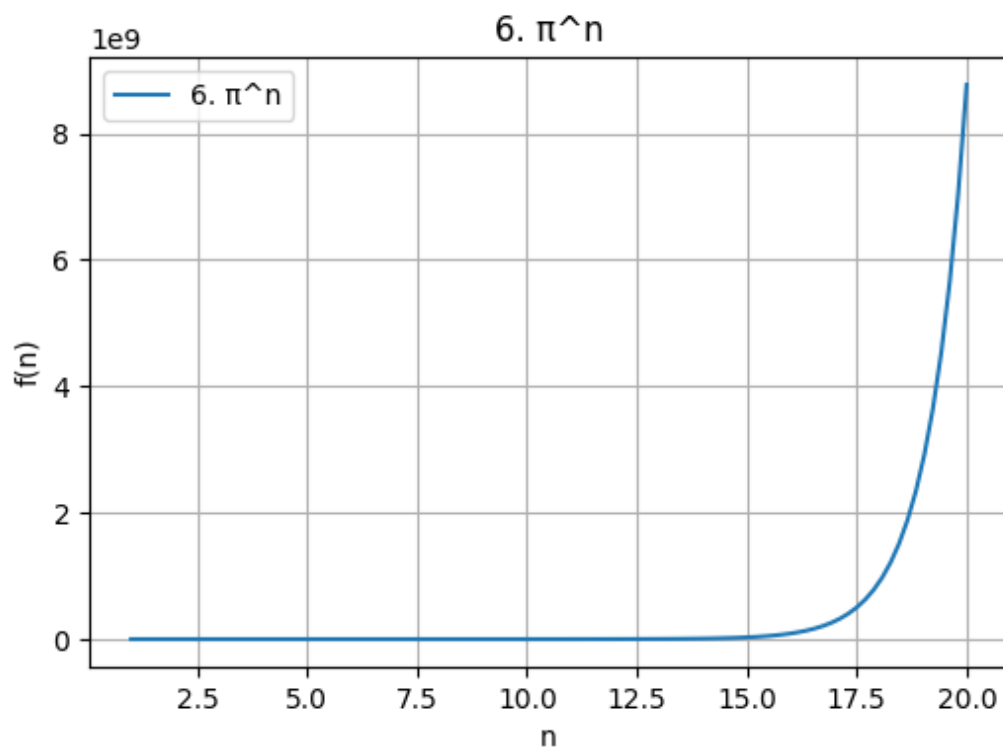
Figure 5: Graphical comparison of $10n + n^{3/2}$.
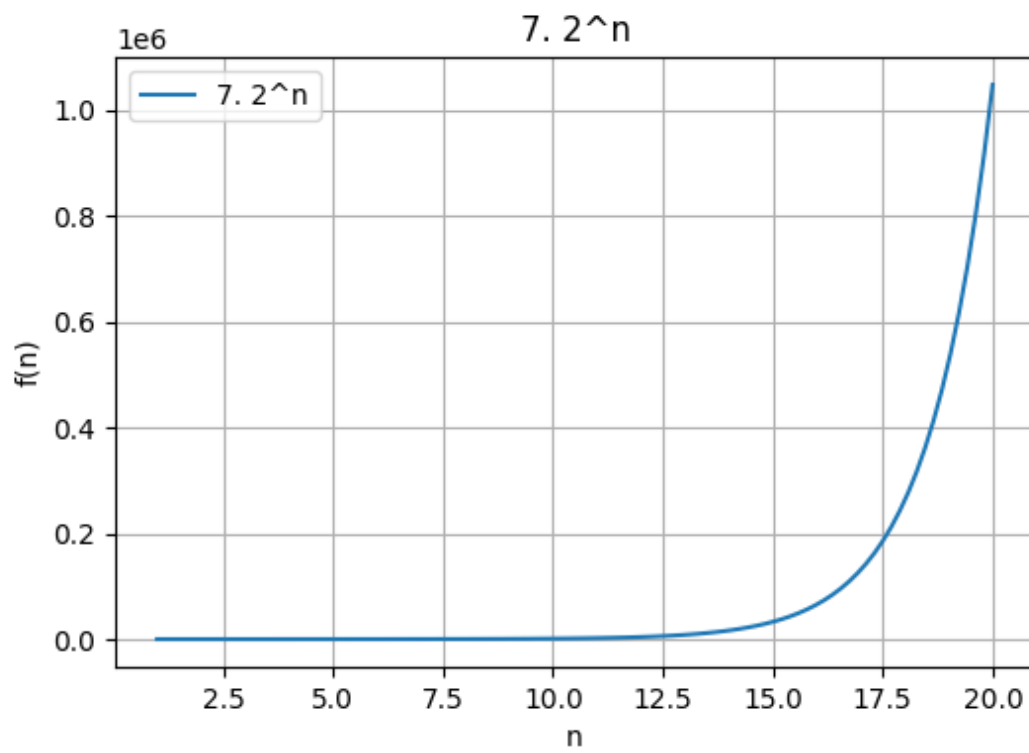
Figure 6: Graphical comparison of π^n .
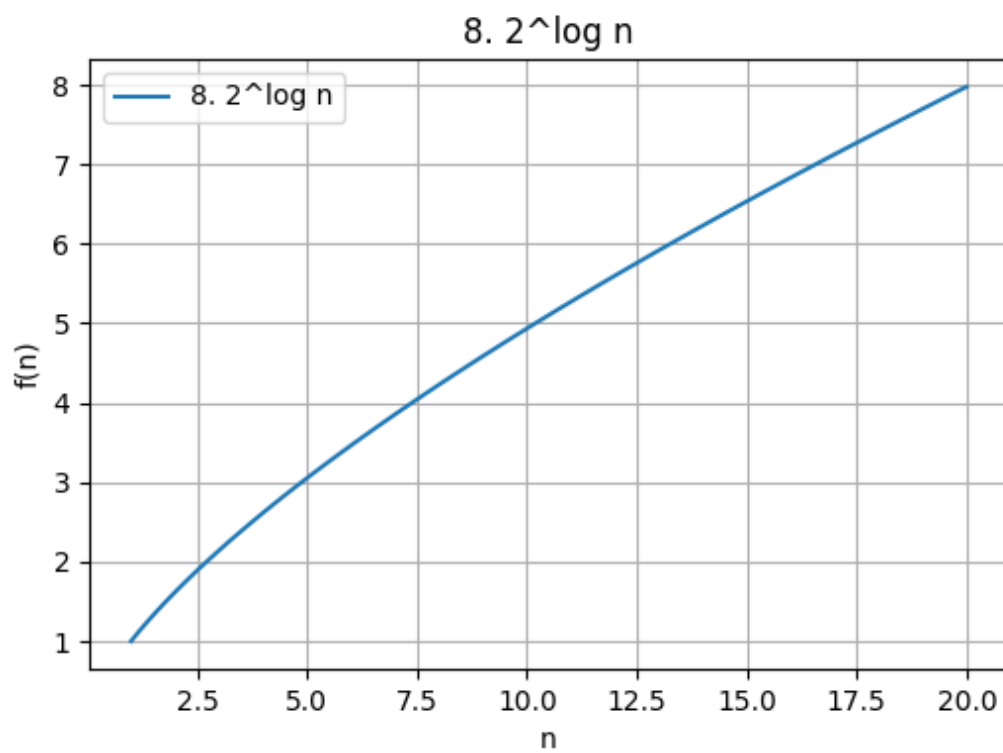
Figure 7: Graphical comparison of 2^n .
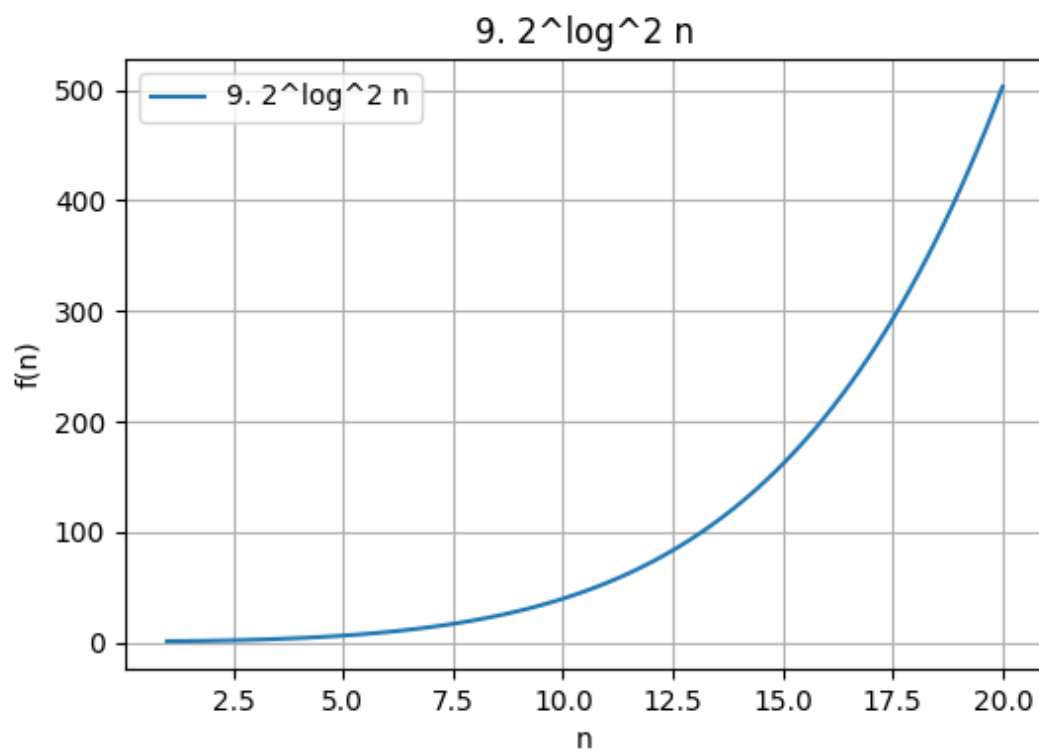
Figure 8: Graphical comparison of 2^{logn} .
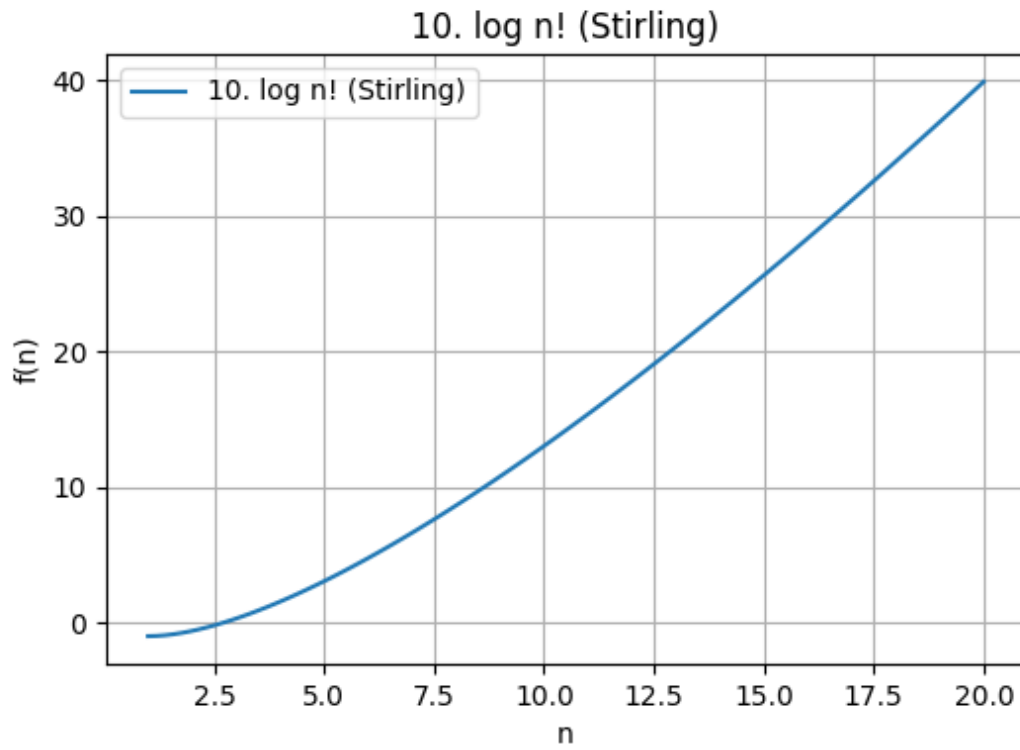
Figure 9: Graphical comparison of $2^{2^{log n}}$.

Figure 10: Graphical comparison of $log\ n!$.

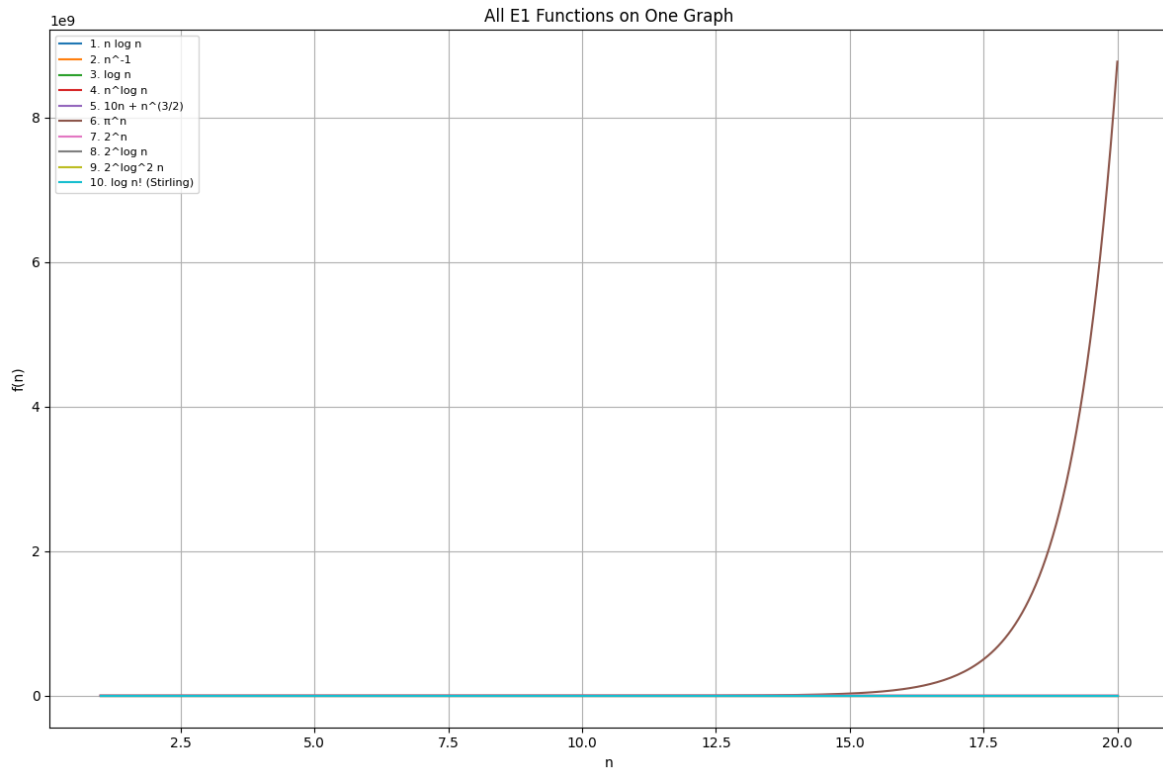Note: $\log n!$ is approximated using Stirling's approximation: $\log n! \approx n \log n - n$.

Figure 11: Graphical comparison of E1 expressions.

E2: Ordering Expressions by Θ-Growth

We have:

1. 2^{2^n}

2. 2^{n^2}

3. $n^2 log\ n$

4. n

5. n^{2n}

Increasing Order of Growth

By asymptotic comparison, the increasing order of growth is:

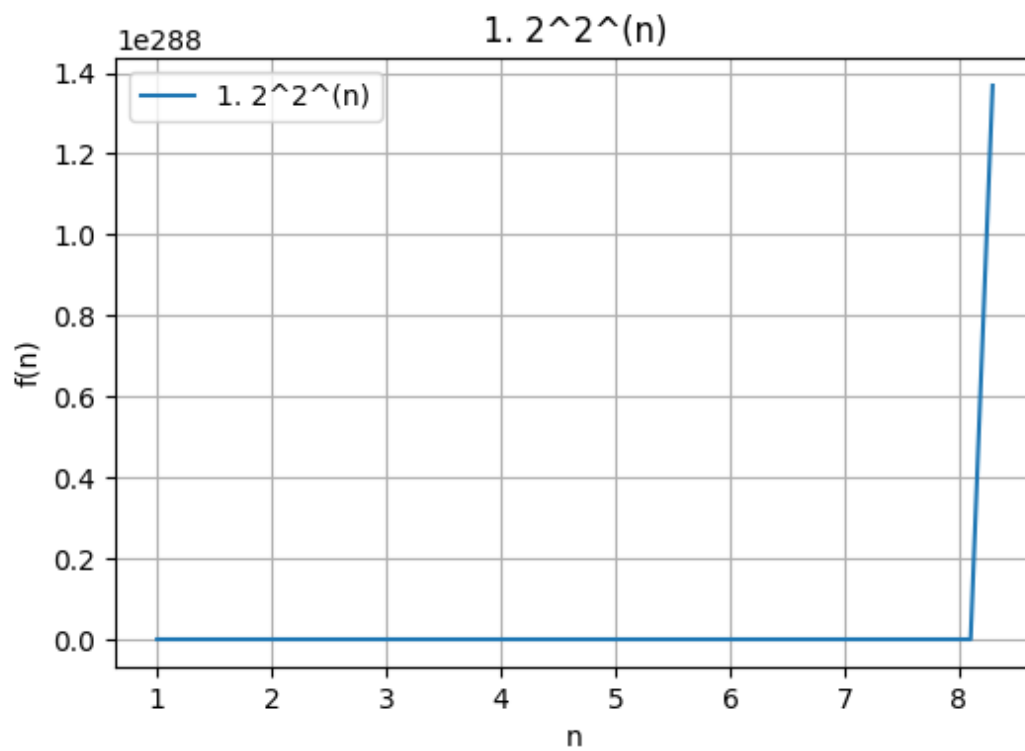$$n \prec n^{2\log n} \prec 2^n \prec 2^{n^2} \prec n^{2n}$$

Graphical Representation

Figure 12: Graphical growth comparison of 2^{2^n}.
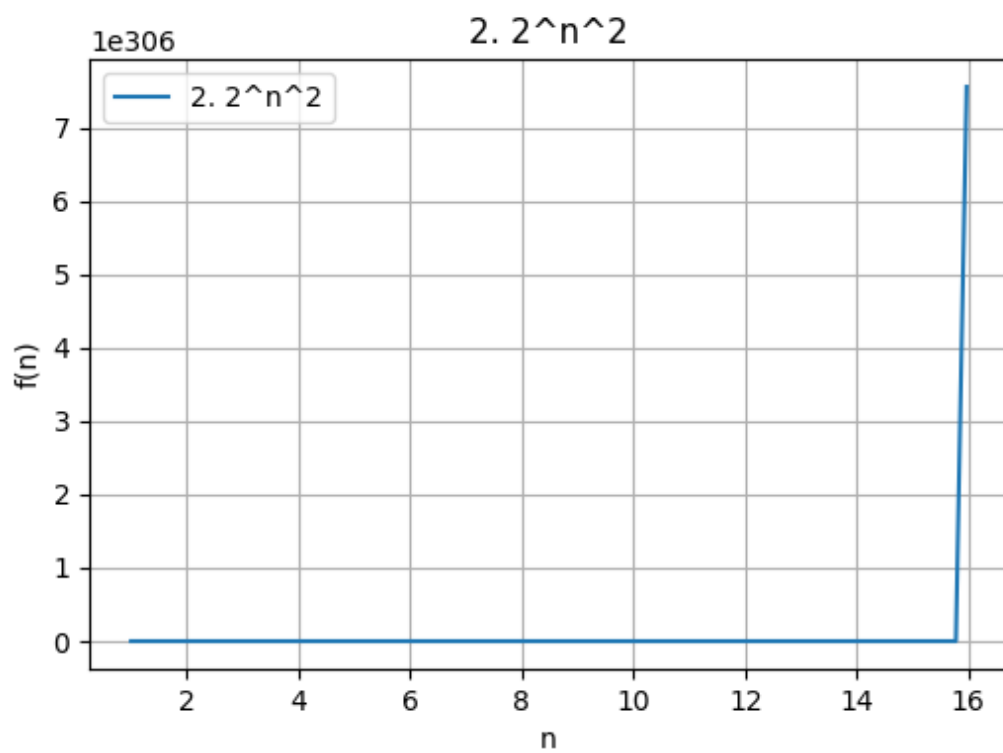
Figure 13: Graphical growth comparison of 2^{n^2}.
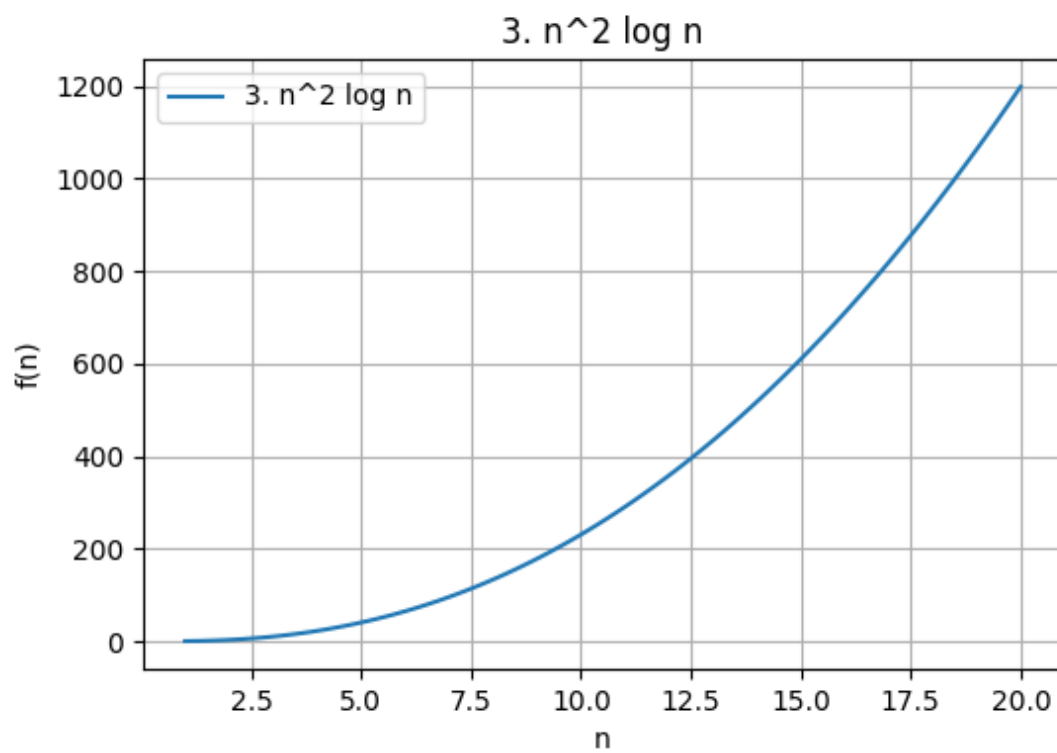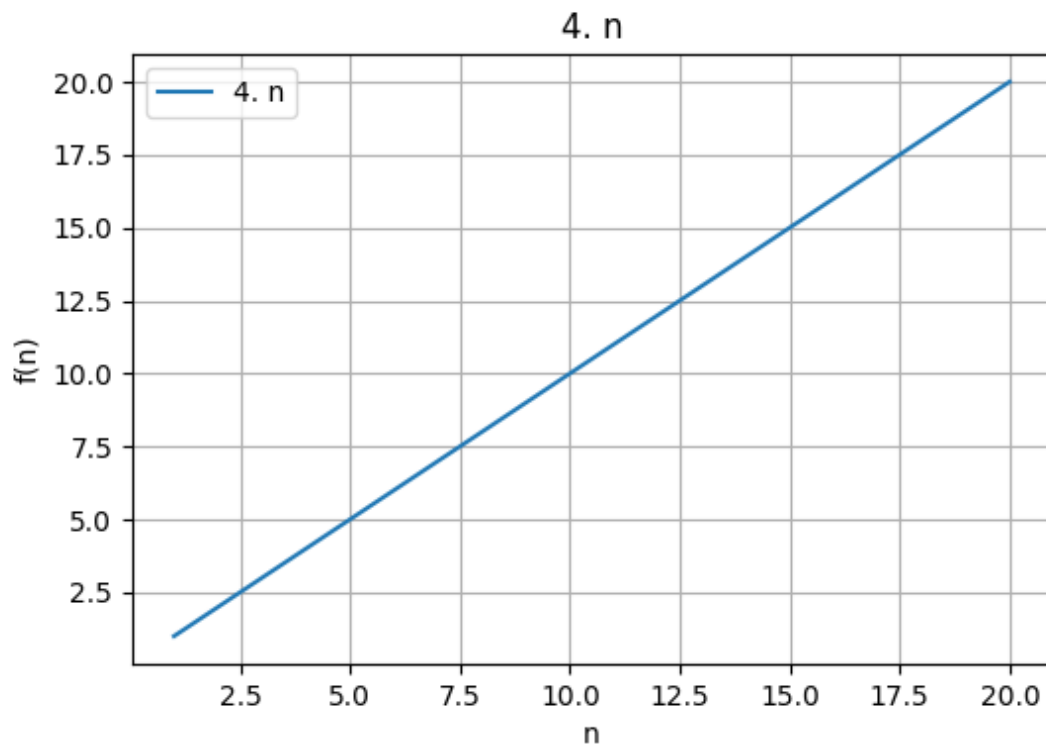
Figure 14: Graphical growth comparison of $n^2 \ log \ n$.
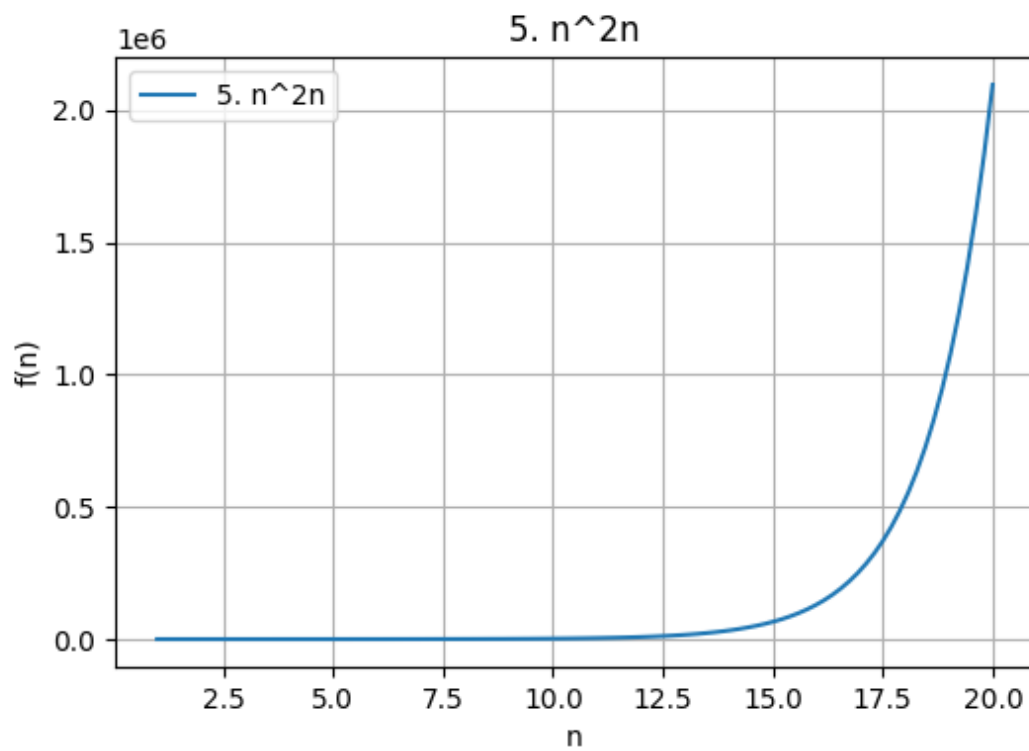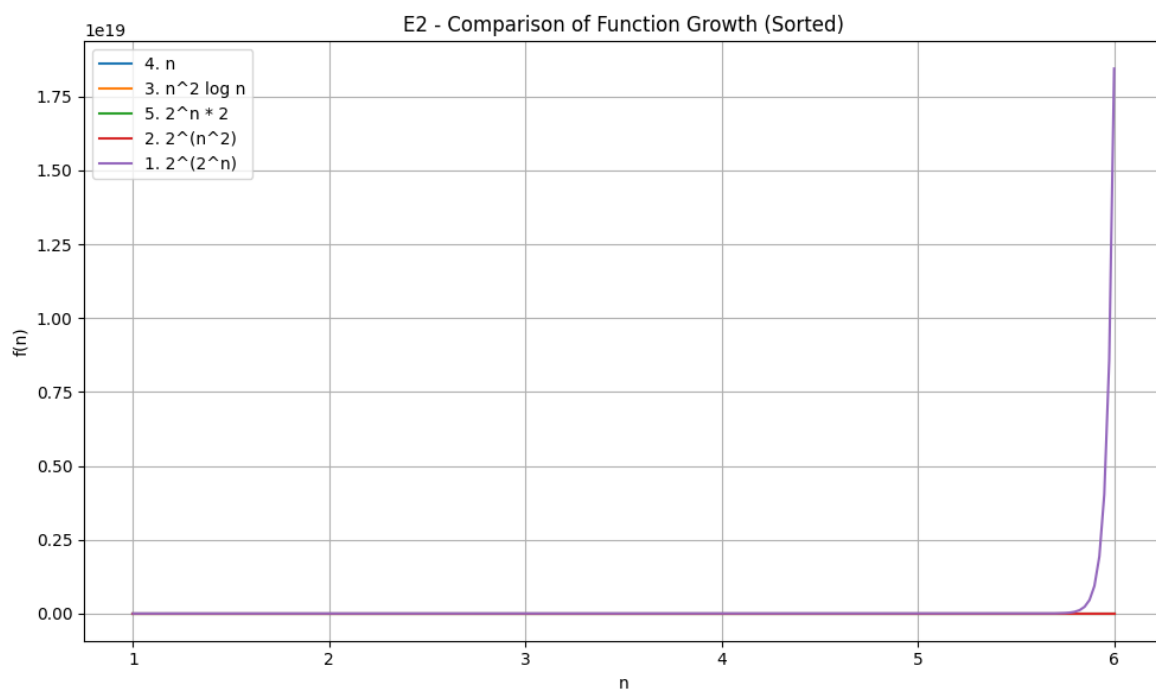
Figure 15: Graphical growth comparison of n.

Figure 16: Graphical growth comparison of n^{2n}.

Figure 17: Graphical growth comparison of E2 expressions.

Conclusion

This document illustrates the visual comparison of various functions used in asymptotic analysis. The plots support our theoretical growth comparisons using Θ-notation.