

# Supplemental Material for: Probing Quantum Scrambling via OTOCs in Digital Circuits

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## I. IBM QUANTUM HARDWARE DETAILS

All hardware experiments were performed on `ibm_marrakesh`, a 156-qubit Heron processor accessed via IBM Quantum Platform. Table I summarizes the hardware parameters at the time of execution.

TABLE I. Hardware parameters for `ibm_marrakesh`.

Parameter	Value
Processor	Heron
Total qubits	156
Median $T_1$ ( $\mu$ s)	$\sim 250$
Median $T_2$ ( $\mu$ s)	$\sim 150$
Median 2Q gate error	$\sim 0.8\%$
Median readout error	$\sim 1.5\%$
Qiskit version	2.3.0
Primitive	Sampler (V2)
Shots per circuit	4096
Runs per configuration	5
Optimization level	3

Qubit selection was automatic via Qiskit transpilation. No manual qubit mapping or routing optimization was applied. All circuits used the native gate set of the Heron processor (CZ, RZ, SX, X).

## II. CIRCUIT STRUCTURE

Each OTOC measurement consists of the following steps:

- State preparation:** Apply  $X$  and  $H$  to qubit 0, preparing  $|\psi_0\rangle = |-\rangle \otimes |0\rangle^{\otimes(N-1)}$ .
- Forward evolution:** Apply  $d$  layers of the Floquet circuit  $U_F = U_{\text{kick}} \cdot U_{\text{Ising}}$ .
- Perturbation:** Apply  $X$  to qubit 0 (butterfly operator).
- Backward evolution:** Apply  $d$  layers of  $U_F^\dagger$ .
- Measurement:** Measure qubit 0 in the  $Z$  basis.

The OTOC signal is extracted as  $C(d) = P_0(d)$ , the probability of measuring  $|0\rangle$  on qubit 0. Equivalently,  $C(d) = 1 - P_1(d)$ . For a perfect echo ( $d = 0$ ),  $C(0) = |\langle 0|-\rangle|^2 = 1/2$  exactly, since the butterfly  $X_0$  acts as a global phase on  $|-\rangle$ .

For each depth  $d$ , a reference circuit without the butterfly operator is also run to verify protocol integrity.

The circuit depth (number of two-qubit gates) scales as:

- $d$  layers forward:  $d \times (N - 1)$  CNOT equivalents for Ising, plus  $N$  single-qubit gates for kick.
- $d$  layers backward: identical count.
- Total two-qubit gates:  $\approx 2d(N - 1)$ .

For  $N = 20$ ,  $d = 7$ : total  $\approx 266$  two-qubit gates, well beyond the coherence limit for faithful state evolution.

## III. MODEL PARAMETERS

Table II lists the parameters used for each model.

TABLE II. Model parameters used in all simulations and experiments.

Model	Parameters	Notes
Kicked Ising	$J = 0.9, h = 0.7$	Chaotic, PBC
Integrable	H + CNOT layers	Clifford, OBC
Floquet	$\theta = 0.8, \phi = 1.2, J = 0.9$	Prethermal, PBC
SYK	$J_{ij} \sim U[0.5, 1.5]$	50 seeds (exact), 9 (IBM)

For the Kicked Ising model, the Floquet operator is:

$$U_F = \exp\left(-ih \sum_j X_j\right) \exp\left(-iJ \sum_j Z_j Z_{j+1}\right) \quad (1)$$

with periodic boundary conditions ( $Z_{N+1} \equiv Z_1$ ).

For the Floquet prethermal model, each layer consists of  $RX(2\theta)$  and  $RY(2\phi)$  rotations on all qubits,  $RZZ(2J)$  coupling on nearest neighbors with periodic boundary conditions, and  $CZ$  gates on even-indexed pairs, with parameters  $\theta = 0.8, \phi = 1.2, J = 0.9$ .

For the SYK-inspired model at  $N = 4$ , the Hamiltonian consists of random all-to-all  $ZZ$  couplings:

$$H_{\text{SYK}} = \sum_{i < j} J_{ij} Z_i Z_j \quad (2)$$

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where  $J_{ij}$  are drawn independently from a uniform distribution on  $[0.5, 1.5]$ . Each disorder realization uses a different set of couplings. Time evolution is  $U(t) = e^{-iH_{\text{SYK}}t}$  with  $t = 1$  per Floquet step.

#### IV. IBM QUANTUM JOB INVENTORY

A total of 40 jobs were submitted to `ibm_marrakesh` between February 12–14, 2026. Table III provides a summary by model and system size.

TABLE III. Job summary by model and configuration.

Model	$N$	Runs	Depths	Points
Kicked Ising	4	5	11	55
Kicked Ising	8	5	11	55
Kicked Ising	12	5	11	55
Kicked Ising	20	5	11	55
Integrable	4	5	11	55
Floquet	4	5	11	55
SYK	4	9 seeds	11	99
<b>Total</b>				<b>429</b>

Depths measured:  $d \in \{1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14\}$  for all configurations (11 data depths per run, plus  $d = 0$  normalization reference).

Full job IDs are available in the data repository.

#### V. EXACT SIMULATION DETAILS

All exact simulations used statevector evolution (no sampling noise, no approximation). The simulation computes the full  $2^N$ -dimensional state vector at each Floquet step.

TABLE IV. Exact simulation inventory.

Model	$N$	Depths	Points
Kicked Ising	4, 8, 12, 20	11 each	44
Integrable	4	11	11
Floquet	4	11	11
SYK (50 seeds)	4	11 each	550
<b>Total</b>			<b>616</b>

For  $N = 20$ , the Hilbert space dimension is  $2^{20} = 1,048,576$ . Each Floquet step requires multiplication of the state vector by two unitary matrices (Ising and kick), implemented as sparse matrix operations. Total computation time:  $\sim 30$  minutes on Google Colab (single CPU).

The Lyapunov exponent  $\lambda_L = 3.12 \pm 0.1$  was extracted by fitting  $\ln C(d)$  vs  $d$  for  $d = 2$ –10 at  $N = 20$ , excluding  $d = 0$  (normalization) and  $d = 1$  (pre-scrambling, identical across all  $N$  due to locality).

#### VI. NOISE FLOOR ANALYSIS

The hardware noise floor for each  $N$  was estimated as the mean of  $C(d)$  for depths  $d \geq 6$ , where the exact signal has decayed significantly.

TABLE V. Noise floor estimates.

$N$	$\langle C \rangle_{\text{IBM}}^{d \geq 6}$	$1/2^N$	Ratio
4	0.059	0.0625	0.9×
8	0.005	0.0039	1.3×
12	0.0005	0.00024	1.9×
20	$< 0.0001$	0.000001	—

The noise floor is consistent with depolarization toward the maximally mixed state, for which  $C = 0$  (each qubit equally likely to be  $|0\rangle$  or  $|1\rangle$ ). The scaling  $\sim 1.5/2^N$  reflects the exponential suppression of any residual coherence with system size.

For  $N = 20$ , the shot noise floor ( $1/\sqrt{4096} \approx 0.016$ ) exceeds any possible signal beyond  $d = 2$ . This means IBM returns  $C \approx 0$  by decoherence, while exact simulation gives  $C \approx 0$  by scrambling—the same qualitative result via different mechanisms.

#### VII. SYK CONVERGENCE DATA

Table VI shows the convergence of the disorder-averaged OTOC statistics with the number of seeds.

TABLE VI. SYK  $N = 4$  convergence with number of disorder realizations.

Seeds	$\Omega$	$R_{\text{exp}}^2$	$R_{\text{pow}}^2$	$\Delta\Omega$
1	0.342	0.15	0.22	+97%
3	0.253	0.41	0.55	+46%
5	0.203	0.52	0.72	+17%
9	0.179	0.58	0.85	+3.4%
15	0.164	0.60	0.89	−5%
25	0.170	0.61	0.90	−2%
50	0.173	0.61	0.91	—

Individual seed statistics (50 seeds): median  $R_{\text{exp}}^2 = 0.27$ , mean = 0.35, only 2/50 with  $R^2 > 0.9$ . The exponential model is not intrinsic to individual SYK realizations at  $N = 4$ .

#### VIII. SCRAMBLING RESIDUAL $\Omega$ — FULL DATA

The maximum discrepancy  $|\Delta\Omega| = 0.089$  occurs for the Floquet model, attributable to decoherence at deep circuits where the near-integrable dynamics produce slow decay.

TABLE VII. Complete  $\Omega$  values for all models and system sizes.

Model	$\Omega_{\text{exact}}$	$\Omega_{\text{IBM}}$	$ \Delta\Omega $
KI $N = 4$	0.136	0.133	0.003
KI $N = 8$	0.009	0.016	0.007
KI $N = 12$	0.006	0.007	0.001
KI $N = 20$	0.005	0.004	0.001
Integrable $N = 4$	0.727	0.726	0.001
Floquet $N = 4$	0.357	0.268	0.089
SYK $N = 4$ (9s)	0.179	0.180	0.001
SYK $N = 4$ (50s)	0.173	—	—

## IX. CODE AND DATA AVAILABILITY

All simulation code is written in Python using:

- `numpy` and `scipy` for exact statevector evolution
- `qiskit 2.3.0` for circuit construction and IBM execution
- `matplotlib` for figure generation

The complete codebase, raw data (JSON format), and figure-generation scripts are available at <https://github.com/ErickPerez79/otoc-finite-size-scaling> and archived on Zenodo (DOI to be assigned upon publication). The repository includes:

- `paper1_analisis_ibm_v1.py`: IBM data collection, exact statevector simulation, and full experimental protocol.
- `paper1_figuras.py`: Data recovery, comparison with exact simulation, statistical analysis, and figure generation.
- `paper1_raw_data.json`: 616 exact simulation data points (all models, all  $N$ ).
- `paper1_recovered_ibm_data.json`: 429 IBM experimental data points with run-level statistics and full count distributions.
- `README.md`: Reproduction instructions for all figures and tables.