- 53 Uma firma de embalagem necessita fabricar caixas retangulares de $64cm^3$ de volume. Se o material da parte lateral custa a metade do material a ser usado para a tampa e para o fundo da caixa, determinar as dimensões da caixa que minimizam o custo.
  Supondo a caixa com dimensões da base igual a a e b, com altura c.
  Supondo também que o custo da tampa e fundo é igual a x:
  min $2abx + \frac{x}{2}(2bc + 2ac)$
  sujeito a $abc = 64$
  temos

$$C = 2abx + bcx + acx$$

  Utilizando Lagrange

$$L = 2abx + bcx + acx - \alpha(abc - 64)$$

  Temos as seguintes derivadas:
  $\frac{\varphi L}{\varphi a} = 2bx + cx - \alpha bc = 0$
  $\frac{\varphi L}{\varphi b} = 2ax + cx - \alpha ac = 0$
  $\frac{\varphi L}{\varphi c} = bx + ax - \alpha ab = 0$
  $\frac{\varphi L}{\varphi \alpha} = 64 - abc = 0$
  Tentando resolver com matrizes

$$a = \frac{64}{bc}$$

  $\frac{\varphi L}{\varphi a} = 2bx + cx - \alpha bc = 0$
  $\frac{\varphi L}{\varphi b} = 2x.\frac{64}{bc} + cx - \alpha.\frac{64}{b} = 0$
  $\frac{\varphi L}{\varphi c} = bx + \frac{64}{bc}.x - \alpha.\frac{64}{c} = 0$
  $x.(2b + c) - \alpha bc = x.(\frac{128}{bc} + c) - \alpha.\frac{64}{b} = x.(\frac{64}{bc} + b) - \alpha.\frac{64}{c}$

$0 = a.(0) + b.(2x - \alpha c) + c.(x)$
$0 = a.(2x - \alpha c) + b.(0) + c.(x)$
$0 = a.(x - \alpha b) + b.(x) + c.(0)$

$$\begin{pmatrix} 0 & 2x - \alpha c & x \\ 2x - \alpha c & 0 & x \\ x - \alpha b & x & 0 \end{pmatrix} \tag{1}$$

$0 + (2 - \alpha c).(x - \alpha b) + x^2.(2x - \alpha c)$

$$2x - 2\alpha b - x.\alpha c + \alpha^2 bc + 2x^3 - x^2.\alpha c$$

ou
$0 = a.(0) + b.(2x) + c.(x) - \alpha.(bc)$
$0 = a.(2x) + b.(0) + c.(x) - \alpha.(ac)$
$0 = a.(x) + b.(x) + c.(0) - \alpha.(ab)$
ou

$0 = ax.(0) + bx.(2) + cx.(1) + \alpha.(-bc)$
$0 = ax.(2) + bx.(0) + cx.(1) + \alpha.(-ac)$
$0 = ax.(1) + bx.(1) + cx.(0) + \alpha.(-ab)$

$$\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \alpha bc \\ \alpha ac \\ \alpha ab \end{pmatrix} \tag{2}$$

$D = 0 + 2 + 2 = 4$

$$\begin{pmatrix} \alpha bc & 2 & 1 \\ \alpha ac & 0 & 1 \\ \alpha ab & 1 & 0 \end{pmatrix} = 2.\alpha ab + \alpha ac - \alpha bc \tag{3}$$

$\dfrac{d_{ax}}{D} = \dfrac{\alpha.(ab + ac)}{4}$

$$\begin{pmatrix} 0 & \alpha bc & 1 \\ 2 & \alpha ac & 1 \\ 1 & \alpha ab & 0 \end{pmatrix} = \alpha bc + 2\alpha ab - \alpha ac \tag{4}$$

$\dfrac{d_{bx}}{D} = \dfrac{\alpha.(bc + 2ab - ac)}{4}$

$$\begin{pmatrix} 0 & 2 & \alpha bc \\ 2 & 0 & \alpha ac \\ 1 & 1 & \alpha ab \end{pmatrix} = 2\alpha ac + 2\alpha bc - 4\alpha ab \tag{5}$$

$\dfrac{d_c x}{D} = \dfrac{\alpha(bc - 2ab + ac)}{2}$

Resultado dará $P(a,b,c) = (\sqrt[3]{32}, \sqrt[3]{32}, 2\sqrt[3]{32})$

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$$\frac{1}{4}.\int_0^4 e^{-x^2}.x.dx$$

sendo $u = -x^2$, $du = -2x.dx$

$$\frac{1}{4}.\int_0^4 e^u.\frac{-du}{2}$$

$$\frac{1}{4}.\frac{-1}{2}.\int_0^4 e^u.du$$

- 21

$\int\int_R (1+x+y).dx.dy$
$= \int_1^2 \int_{-2x+3}^{-3x+5} (1+x+y).dy.dx$
$= \int_1^2 [y + xy + \frac{y^2}{2}|_{-2x+3}^{-3x+5}].dx$
$= \int_1^2 [(-3x+5)+x(-3x+5)+\frac{(-3x+5)^2}{2}-(-2x+3)-x.(-2x+3)-\frac{(-2x+3)^2}{2}].dx$
$\int_1^2 [-3x+5-3x^2+5x+\frac{9x^2-30x+25}{2}+2x-3+2x^2-3x-\frac{4x^2-12x+9}{2}].dx$
$\int_1^2 [-3x+5x+2x-3x+5-3-3x^2+2x^2+\frac{9x^2-30x+25}{2}-\frac{4x^2-12x+9}{2}].dx$
$\int_1^2 [-x^2+x+2+\frac{5x^2-18x+12x+16}{2}].dx$
$\int_1^2 [\frac{-2x^2+2x+4+5x^2-18x+12x+16}{2}].dx$
$\int_1^2 [\frac{3x^2-16x+20}{2}].dx$
$\frac{1}{2}.\int_1^2 (3x^2-16x+20).dx$
$\frac{1}{2}[\frac{3x^3}{3}-\frac{16x^2}{2}+20x]|_1^2$
$= \frac{1}{2}[(\frac{3.8}{3}-\frac{16.4}{2}+20.2)-(\frac{3.1}{3}-\frac{16.1}{2}+20.1)]$
$= \frac{1}{2}.[8-32+40-1+8-20] = \frac{3}{2}$

- 21 (Errei e vou refazer)
$\int\int_R (1+x+y).dx.dy$
$= \int_1^2 \int_{-2x+3}^{-3x+5} (1+x+y).dy.dx$
$= \int_1^2 [y + xy + \frac{y^2}{2}]|_{-2x+3}^{-3x+5}.dx$
$= \int_1^2 [(-3x+5)+x(-3x+5)+(\frac{9x^2-30x+25}{2})-(-2x+3)-x.(-2x+3)-$

$(\frac{4x^2 - 12x + 9}{2})].dx$

$\int_1^2 [-3x + 5 + 2x - 3 + x(-3x + 2x + 5 - 3) + \frac{9x^2 - 4x^2 - 30x + 12x + 25 - 9}{2}].dx$

$\int_1^2 [2 - x^2 + x + \frac{5x^2 - 18x + 16}{2}].dx$

$\int_1^2 [\frac{3x^2 - 16x + 20}{2}].dx$

$\frac{1}{2}[\frac{3x^3}{3} - \frac{16x^2}{2} + 20x]|_1^2$

$\frac{1}{2}.[8 - 1 - 8.(4 - 1) + 20.(2 - 1)]$

$\frac{7 - 24 + 20}{2}$

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$\int_0^2 \int_0^{3-\frac{3}{2}.d} (1 - \frac{x}{2} - \frac{y}{3}).dy.dx$

$\int_0^2 [y - \frac{xy}{2} - \frac{y^2}{6}]|_0^{3-\frac{3x}{2}} .dx$

$\int_0^2 [\frac{6 - 3x}{2} - \frac{x}{2}.\frac{6 - 3x}{2} - \frac{1}{6}.[9 - 9x + \frac{9x^2}{4}].dx$

$\int_0^2 \frac{72 - 36x - 36x + 18x^2 - 36 + 36x - 9x^2}{24}.dx$

$\int_0^2 \frac{36 - 36x + 9x^2}{24}.dx = \int_0^2 \frac{3x^2 - 12x + 12}{8}.dx$

$\frac{1}{8}.[\frac{3x^3}{3} - \frac{12x^2}{2} + 12x]|_0^2$

$\frac{x^3}{8} - \frac{3x^2}{4} - \frac{3x}{2}|_0^2$

$1 - 3 + 3$

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$\int_0^4 [-x^2.(8 - 2x) + 4x.(8 - 2x) - \frac{x}{4}.(4x^2 - 32x + 64)].dx$

$\int_0^4 [2x^3 - 8x^2 - 8x^2 + 32x - \frac{x^3 - 8x^2 + 16x}{1}].dx$

$\int_0^4 [x^3 - 8x^2 + 16x].dx$