

- 53 Uma firma de embalagem necessita fabricar caixas retangulares de 64cm^3 de volume. Se o material da parte lateral custa a metade do material a ser usado para a tampa e para o fundo da caixa, determinar as dimensões da caixa que minimizam o custo.

Supondo a caixa com dimensões da base igual a a e b , com altura c .

Supondo também que o custo da tampa e fundo é igual a x :

$$\min 2abx + \frac{x}{2}(2bc + 2ac)$$

sujeito a $abc = 64$

temos

$$C = 2abx + bcx + acx$$

Utilizando Lagrange

$$L = 2abx + bcx + acx - \alpha(abc - 64)$$

Temos as seguintes derivadas:

$$\frac{\partial L}{\partial a} = 2bx + cx - \alpha bc = 0$$

$$\frac{\partial L}{\partial b} = 2ax + cx - \alpha ac = 0$$

$$\frac{\partial L}{\partial c} = bx + ax - \alpha ab = 0$$

$$\frac{\partial L}{\partial \alpha} = 64 - abc = 0$$

Resultado dará $P(a, b, c) = (\sqrt[3]{32}, \sqrt[3]{32}, 2\sqrt[3]{32})$

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$$\frac{1}{4} \cdot \int_0^4 e^{-x^2} \cdot x \cdot dx$$

sendo $u = -x^2, du = -2x \cdot dx$

$$\frac{1}{4} \cdot \int_0^4 e^u \cdot \frac{-du}{2}$$

$$\frac{1}{4} \cdot \frac{-1}{2} \cdot \int_0^4 e^u \cdot du$$

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$$\int \int_R (1 + x + y) \cdot dx \cdot dy$$

$$= \int_1^2 \int_{-2x+3}^{-3x+5} (1 + x + y) \cdot dy \cdot dx$$

$$= \int_1^2 \left[y + xy + \frac{y^2}{2} \right]_{-2x+3}^{-3x+5} \cdot dx$$

$$= \int_1^2 \left[(-3x+5) + x(-3x+5) + \frac{(-3x+5)^2}{2} - (-2x+3) - x(-2x+3) - \frac{(-2x+3)^2}{2} \right] \cdot dx$$

$$\begin{aligned}
& \int_1^2 [-3x + 5 - 3x^2 + 5x + \frac{9x^2 - 30x + 25}{2} + 2x - 3 + 2x^2 - 3x - \frac{4x^2 - 12x + 9}{2}] . dx \\
& \int_1^2 [-3x + 5x + 2x - 3x + 5 - 3 - 3x^2 + 2x^2 + \frac{9x^2 - 30x + 25}{2} - \frac{4x^2 - 12x + 9}{2}] . dx \\
& \int_1^2 [-x^2 + x + 2 + \frac{5x^2 - 18x + 12x + 16}{2}] . dx \\
& \int_1^2 [\frac{-2x^2 + 2x + 4 + 5x^2 - 18x + 12x + 16}{2}] . dx \\
& \int_1^2 [\frac{3x^2 - 16x + 20}{2}] . dx \\
& \frac{1}{2} \cdot \int_1^2 (3x^2 - 16x + 20) . dx \\
& \frac{1}{2} [\frac{3x^3}{3} - \frac{16x^2}{2} + 20x]_1^2 \\
& = \frac{1}{2} [(\frac{3.8}{3} - \frac{16.4}{2} + 20.2) - (\frac{3.1}{3} - \frac{16.1}{2} + 20.1)] \\
& = \frac{1}{2} . [8 - 32 + 40 - 1 + 8 - 20] = \frac{3}{2}
\end{aligned}$$

- 21 (Errei e vou refazer)

$$\begin{aligned}
& \int \int_R (1 + x + y) . dx . dy \\
& = \int_1^2 \int_{-2x+3}^{-3x+5} (1 + x + y) . dy . dx \\
& = \int_1^2 [y + xy + \frac{y^2}{2}]_{-2x+3}^{-3x+5} . dx \\
& = \int_1^2 [(-3x + 5) + x(-3x + 5) + (\frac{9x^2 - 15x + 25}{2}) - (-2x + 3) - x.(-2x + 3) - (\frac{4x^2 - 12x + 9}{2})] . dx \\
& \int_1^2 [-3x + 5 + 2x - 3 + x(-3x + 2x + 5 - 3) + \frac{9x^2 - 4x^2 - 15x + 12x + 25 - 9}{2}] . dx \\
& \int_1^2 [2 - x^2 + x + \frac{5x^2 - 3x + 16}{2}] . dx \\
& = \int_1^2 \frac{3x^2 - x + 20}{2} . dx = \frac{1}{2} \int_1^2 (3x^2 - x + 20) . dx \\
& \frac{1}{2} [\frac{3x^3}{3} - \frac{x^2}{2} + 20x]_1^2 \\
& = \frac{1}{2} [8 - 1 - \frac{4 - 1}{2} + 40 - 20] = \frac{1}{2} \cdot \frac{14 - 3 + 40}{2} = \frac{51}{4} \text{ serasetacerto}
\end{aligned}$$