Lista 3 Cálculo II Erickson G. Müller

Lista 5.1 (pg. 198)

27) $z = 4x^2 + 3xy + y^2 + 12x + 2y + 1$

$$\frac{\partial z}{\partial x} = 8x + 3y + 12 \qquad \frac{\partial z}{\partial y} = 3x + 2y + 2$$

$$\begin{cases} 8x + 3y + 12 = 0 \\ 3x + 2y + 2 = 0 \end{cases}$$

x_0:

$$\begin{cases} 16x + 6y + 24 = 0 \\ -9x - 6y - 6 = 0 \end{cases}$$
$$7x + 18 = 0 \qquad x_0 = \frac{-18}{7}$$

y_0:

$$\begin{cases} 24x + 9y + 36 = 0 \\ -24x - 16y - 16 = 0 \end{cases}$$
$$-7y + 20 = 0 \qquad y_0 = \frac{20}{7}$$

$$H(x,y) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} \qquad \frac{\partial^2 z}{\partial x^2} = 8 \qquad \frac{\partial^2 z}{\partial y^2} = 2$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = 3 \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 3$$

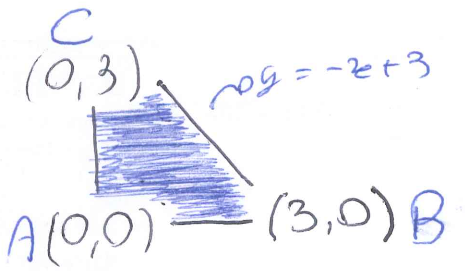
$$H(x,y) = \begin{bmatrix} 8 & 3 \\ 3 & 2 \end{bmatrix} = 16 - 9 = 7$$

$$H(x_0,y_0) > 0 \ \& \ \frac{\partial^2 f}{\partial x^2} > 0 \Rightarrow \left(\frac{-18}{7}, \frac{20}{7}\right) \text{ é ponto crítico}$$

é ponto de mínimo local

Lista 5.1 (pg. 199)

37-) $Z = x^2 + y^2 - 2x - 2y$



$\frac{\partial z}{\partial x} = 2x - 2 = 0 \Rightarrow x = 1$

$\frac{\partial z}{\partial y} = 2y - 2 = 0 \Rightarrow y = 1$

$H(x,y) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 4 - 0$

$\left.\begin{array}{l} \frac{\partial^2 z}{\partial x^2} = 2 > 0 \\ H(x_0, y_0) > 0 \end{array}\right\}$ (1,1) é ponto de mínimo

AB $\boxed{y=0}$, $0 \le x \le 3$

$\left.\begin{array}{l} Z = x^2 + 0^2 - 2x - 2.0 = x^2 - 2x \\ Z' = 2x - 2 = 0 \Rightarrow \boxed{x = 1} \end{array}\right\}$ (1,0)

BC $y = -x + 3$, $0 \le x \le 3$

$Z = x^2 + (-x+3)^2 - 2x - 2.(-x+3)$

$Z = x^2 + x^2 - 6x + 9 - \cancel{2x} + \cancel{2x} - 6$

$Z = 2x^2 - 6x + 3$

$\left.\begin{array}{l} Z' = 4x - 6 = 0 \Rightarrow \boxed{x = \frac{3}{2}} \\ y = -\frac{3}{2} + 3 \Rightarrow \boxed{y = \frac{3}{2}} \end{array}\right\}$ $\left(\frac{3}{2}, \frac{3}{2}\right)$

CA $\boxed{x=0}$, $0 \le y \le 3$

$\left.\begin{array}{l} Z = 0^2 + y^2 - 2.0 - 2y = y^2 - 2y \\ Z' = 2y - 2 \Rightarrow \boxed{y = 1} \end{array}\right\}$ (0,1)

P.C.	Posição	Z
(1,1)	interior	-2 → mínimo
(0,0)	fronteira	0
(3,0)	fronteira	3 → máximo
(0,3)	fronteira	3
(1,0)	fronteira	-1
$\left(\frac{3}{2}, \frac{3}{2}\right)$	fronteira	$-\frac{3}{2}$
(0,1)	fronteira	-1

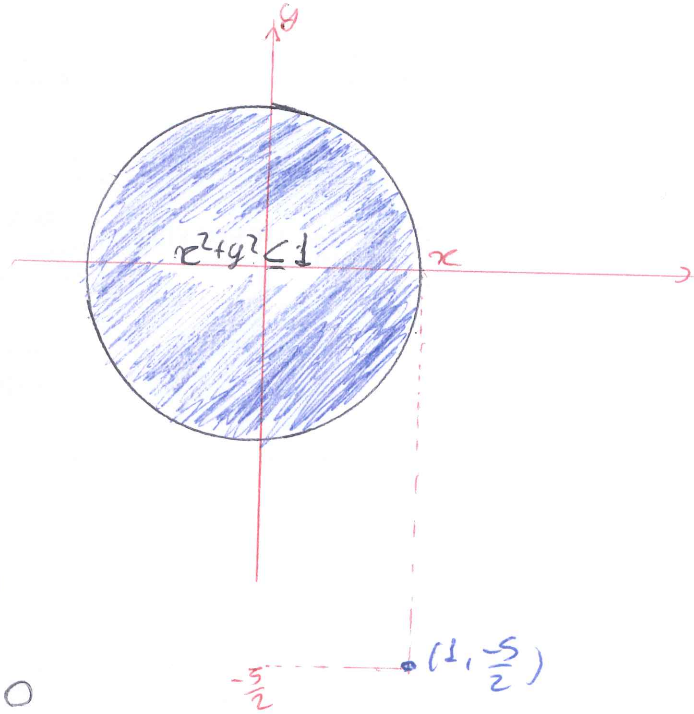
Valor máximo = 3
Valor mínimo = -2

$\frac{9}{2} - 2.3 = \frac{9}{2} - \frac{12}{2} = -\frac{3}{2}$

Lista 5.1 (pg. 199)

$T(x,y) = \underbrace{x^2+y^2}_{1} - 2x + 5y - 10$

46-)



$\frac{\partial T}{\partial x} = 2x - 2 = 0 \Rightarrow x = 1$

$\frac{\partial T}{\partial y} = 2y + 5 = 0 \Rightarrow y = -\frac{5}{2}$

$T = 1 - 2x + 5\sqrt{1-x^2} - 10 = -2x + 5\sqrt{1-x^2} - 9$

$T' = -2 + 5 \cdot -2x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} = -2 - \frac{5x}{\sqrt{1-x^2}} = 0$

$-2 = \frac{+5x}{\sqrt{1-x^2}} \Rightarrow 4 = \frac{25x^2}{1-x^2} \Rightarrow 4 - 4x^2 = 25x^2 \Rightarrow x = \frac{-2}{\sqrt{29}}$

$y = \sqrt{1 - \left(\frac{-2}{\sqrt{29}}\right)^2} = \sqrt{\frac{29-4}{29}} = \frac{5}{\sqrt{29}}$ $\quad P_1 = \left(\frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\right)$ $P_2 = \left(\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$

$T = 1 - 2x + 5 \cdot -\sqrt{1-x^2} - 10 = -2x - 5\sqrt{1-x^2} - 9$

$T' = -2 - 5 \cdot -2x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} = -2 + \frac{5x}{\sqrt{1-x^2}} = 0$

P_1 = máximo local

P_2 = mínimo local

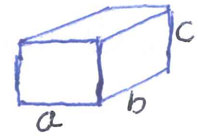
$5x = 2\sqrt{1-x^2} \Rightarrow x = +\frac{2}{\sqrt{29}}$ $\quad y = -\sqrt{1 - \left(\frac{2}{\sqrt{29}}\right)^2} = \frac{-5}{\sqrt{29}}$

$T_1 = 1 - 2 \cdot \frac{-2}{\sqrt{29}} + 5 \cdot \frac{5}{\sqrt{29}} - 10 = -9 + \frac{25+4}{\sqrt{29}} = \underbrace{-9 + \sqrt{29}}_{T\,max}$

$T_2 = 1 - 2 \cdot \frac{2}{\sqrt{29}} + 5 \cdot \frac{-5}{\sqrt{29}} - 10 = -9 \frac{-25-4}{\sqrt{29}} = \underbrace{-9 - \sqrt{29}}_{T\,min}$

Lista 5.1 (PG 200)

53-) $\begin{cases} \text{mínimo} \quad 2ab\alpha + \frac{\alpha}{2}(2bc + 2ac) \\ \text{Sujeito} \quad abc = 64 \end{cases}$



$C = 2ab\alpha + bc\alpha + ac\alpha$

$L = 2ab\alpha + bc\alpha + ac\alpha - \alpha(abc - 64)$

$\frac{\partial L}{\partial a} = 2b\alpha + c\alpha - \alpha bc = 0$

$\frac{\partial L}{\partial b} = 2a\alpha + c\alpha - \alpha ac = 0$

$\frac{\partial L}{\partial c} = b\alpha + a\alpha - \alpha ab = 0$

$\frac{\partial L}{\partial \alpha} = 64 - abc = 0$

Resolvendo esse sistema temos que $a = b = \frac{c}{2}$

$P(a,b,c) = (\sqrt[3]{32}, \sqrt[3]{32}, 2 \cdot \sqrt[3]{32})$

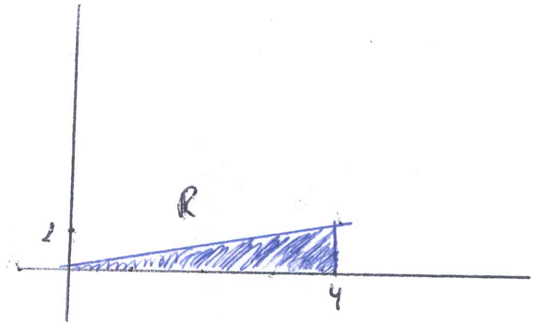
$\alpha \cdot \alpha \cdot 2\alpha = 64$

$\alpha^3 = 32$

$\alpha = \sqrt[3]{32}$

Lista 7.6 (PG 251)

14-) $\iint_R e^{-x^2} \, dx \, dy$



$$= \int_0^4 \left[\int_0^{\frac{x}{4}} e^{-x^2} \, dy \right] dx$$

$$= \int_0^4 \left[e^{-x^2} \cdot y \Big|_0^{\frac{x}{4}} \right] dx = \int_0^4 e^{-x^2} \cdot \frac{x}{4} \, dx$$

$$= \frac{1}{4} \cdot \int_0^4 e^{-x^2} \cdot x \, dx$$

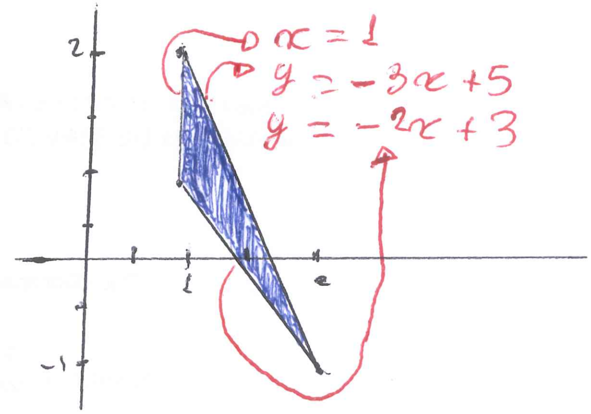
Sendo $u = -x^2$
$du = -2x \, dx$

$$I = \frac{1}{4} \cdot \int_0^4 e^u \cdot \frac{-du}{2} = \frac{1}{4} \cdot \frac{-1}{2} \cdot \int_0^4 e^u \, du$$

$$I = \frac{-1}{8} \left[e^{-x^2} \Big|_0^4 \right] = \frac{-1}{8} \cdot \left[e^{-16} - e^0 \right] = \frac{1}{8} - \frac{1}{8 \cdot e^{16}} = \frac{1}{8} \left(1 - e^{-16} \right)$$

Lista 7.6 (Pg 251)

21-) $\displaystyle\iint_R (1+x+y)\,dx\,dy$



$$= \int_1^2 \left[\int_{-2x+3}^{-3x+5} (1+x+y)\,dy \right] . dx$$

$$= \int_1^2 \left[\left(y + xy + \frac{y^2}{2} \right) \Bigg|_{-2x+3}^{-3x+5} \right] . dx$$

$$= \int_1^2 \left[(-3x+5) + x.(-3x+5) + \frac{(-3x+5)^2}{2} - (-2x+3) - x.(-2x+3) - \frac{(-2x+3)^2}{2} \right] . dx$$

$$= \int_1^2 \left[-3x+5-3x^2+5x+\frac{9x^2-30x+25}{2} + 2x - 3 + 2x^2 - 3x - \left(\frac{4x^2-12x+9}{2} \right) \right] . dx$$

$$= \int_1^2 \left[-x^2+x+2+\frac{5x^2-18x+16}{2} \right] . dx$$

$$= \int_1^2 \left(\frac{-2x^2+2x+4+5x^2-18x+16}{2} \right) . dx = \frac{1}{2} . \int_1^2 (3x^2-16x+20) . dx$$

$$= \frac{1}{2} . \left(\frac{3x^3}{3} - \frac{16x^2}{2} + 20x \right) \Bigg|_1^2$$

$$= \frac{1}{2} . \left[\left(\frac{3.8}{3} - \frac{16.4}{2} + 20.2 \right) - \left(\frac{3.1}{3} - \frac{16.1}{2} + 20.1 \right) \right]$$

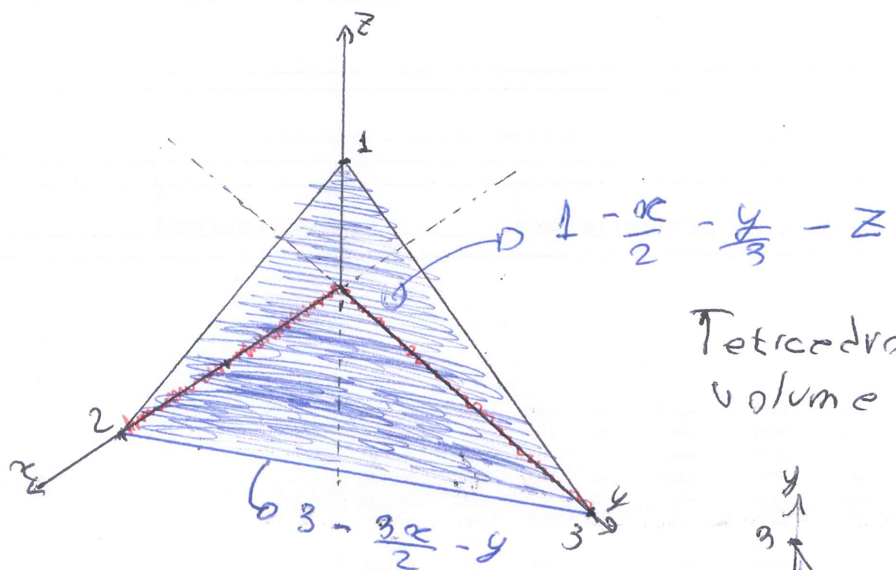
$$= \frac{1}{2} . \left[8 - 32 + 40 - 1 + 8 - 20 \right] = \frac{3}{2}$$

Lista 2.1 (Pg 278)

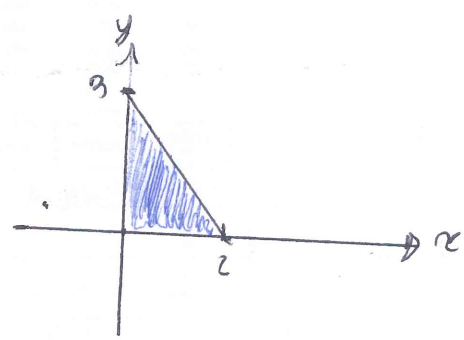
17-) $V = \int_0^2 \int_0^{3-\frac{3}{2}x} \left(1 - \frac{x}{2} - \frac{y}{3}\right) . dy . dx$

$V = \int_0^2 \int_0^{\left(3-\frac{3}{2}x\right)} \int_0^{\left(1-\frac{x}{2}-\frac{y}{3}\right)} dz . dy . dx$

$$\begin{cases} 0 \leq x \leq 2 \\ 0 \leq y \leq 3 - \frac{3x}{2} \\ 0 \leq z \leq 1 - \frac{x}{2} - \frac{y}{3} \end{cases}$$



Tetraedro de volume = 1 u.V



$V = \frac{3 . 2}{2} . \frac{1}{3} = 1$ ou

$V = \int_0^2 \left[y - \frac{xy}{2} - \frac{y^2}{6} \right]_0^{3-\frac{3x}{2}} . dx$

$V = \int_0^2 \left[\frac{6-3x}{2} - \frac{x}{2} . \left(\frac{6-3x}{2}\right) - \frac{1}{6} . \left(9 - 9x + \frac{9x^2}{4}\right) \right] . dx$

$V = \int_0^2 \frac{3x^2 - 12x + 12}{8} . dx = \frac{1}{8} . \left[\frac{3x^3}{3} - \frac{12x^2}{2} + 12x \right]_0^2$

$V = \frac{x^3}{8} - \frac{3x^2}{4} - \frac{3x}{2} \Big|_0^2 = 1 - 3 + 3 = 1$

Lista 8.5 (PG 290)

2-) $\iiint\limits_T x \, dv$

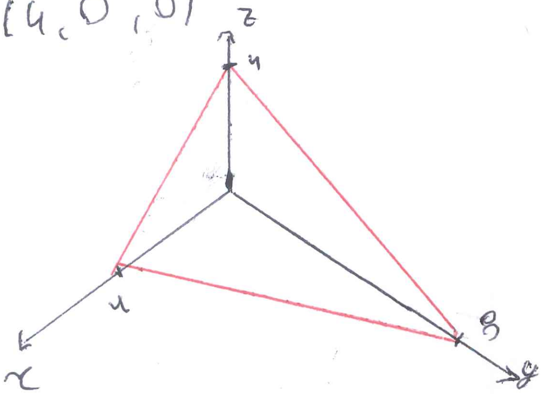
$$x + \frac{y}{2} + z = 4$$

$$0 + \frac{0}{2} + z = 4 \Rightarrow z = 4$$

$$0 + \frac{y}{2} + 0 = 4 \Rightarrow y = 8$$

$$x + \frac{0}{2} + 0 = 4 \Rightarrow x = 4$$

(0,0,4)
(0,8,0)
(4,0,0)



$$T \begin{cases} 0 \le z \le 4 - x - \frac{y}{2} \\ 0 \le y \le 8 - 2x \\ 0 \le x \le 4 \end{cases}$$

$$\int_0^4 \int_0^{8-2x} \int_0^{4-x-\frac{y}{2}} x \, dz \, dy \, dx = \int_0^4 \int_0^{8-2x} x \int_0^{4-x-\frac{y}{2}} dz \, dy \, dx$$

$$= \int_0^4 \int_0^{8-2x} x . z \Big|_0^{4-x-\frac{y}{2}} dy \, dx = \int_0^4 \int_0^{8-2x} x . \left(4 - x - \frac{y}{2}\right) dy \, dx$$

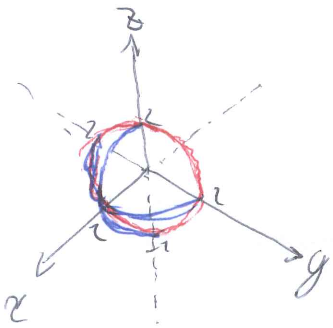
$$= \int_0^4 \int_0^{8-2x} -x^2 + 4x - \frac{xy}{2} \, dy \, dx = \int_0^4 \left[-x^2 y + 4xy - \frac{xy^2}{4}\right]_0^{8-2x} dx$$

$$\cancel{= \int_0^4 -y.(4x^2 - 32x + 64) + 4y.(8-2x)} = \int_0^4 \left[-x^2.(8-2x) + 4x.(8-2x) - \frac{x}{4}.(4x^2 - 32x + 64)\right] dx$$

$$= \int_0^4 [x^3 - 8x^2 + 16x] \, dx = \left[\frac{x^4}{4} - \frac{8x^3}{3} + \frac{16x^2}{2}\right]_0^4 = 64 - \frac{512}{3} + 128 = \frac{64}{3}$$

Lista 8.5 (pg. 291)

7-)



$$0 \le x \le \sqrt{4-y^2-z^2}$$

$$-\sqrt{4-z^2} \le y \le \sqrt{4-z^2}$$

$$-2 \le z \le 2$$

$$\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{0}^{\sqrt{4-y^2-z^2}} dx\cdot dy\cdot dz$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \left[\left[\sqrt{4-y^2-z^2}\right]-0\right]\cdot dy\,dz$$

sendo $y = 2\cdot sen(\alpha)$

e $dy = 2\cdot \cos(\alpha)\cdot d\alpha$

temos $sen\,\alpha = \frac{\sqrt{4-z^2}}{2}$

$$\cos^2\alpha = \frac{(1+\cos(2\alpha))}{2}$$

$$\alpha = \pm\, arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)$$

$$\int_{-2}^{2}\int_{-arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)}^{arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)} (1+\cos(2\alpha))\cdot d\alpha\cdot dz = \int_{-2}^{2} \frac{sen(2\alpha)}{2}\Bigg|_{arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)}^{arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)} \cdot dz$$

$$\int_{-2}^{2} sen\left(2\cdot arcsen\left(\frac{\sqrt{4-z^2}}{2}\right)\cdot 2\right)\cdot dz = \int_{-2}^{2}\left[\frac{\pi}{2} + z\cdot\sqrt{4-z^2}\right]\cdot dz$$

sendo $u = 4-z^2$

$du = -2z\cdot dz$

$$= \frac{\pi}{2}\cdot z\Big|_{-2}^{2} + \int_{-2}^{2}\frac{-\sqrt{u}}{2}\,du = \frac{\pi}{2}(2+2) - \frac{1}{2}\cdot\left[u^{\frac{3}{2}}\cdot\frac{2}{3}\right]_{-2}^{2} = 2\pi - \frac{1}{2}\,\frac{2}{3}\cdot\left((4-4)\cdot\sqrt{4-4} - (4-4)\cdot\sqrt{4-4}\right)$$

$$2\pi - \frac{1}{3}\cdot 0 = 2\pi$$