

Lista 6. Erickson - pg 147

1.) $F(x, y, z) = (x+z, y-2z)$ Base $B = \{(1, 2, 1), (0, 1, 1), (0, 3, -1)\}$
 Base $C = \{(1, 5), (2, -1)\}$

$F(1, 2, 1) = (1+1, 2-2) = (2, 0) \quad v_1$

$F(0, 1, 1) = (1, 1-2) = (1, -1) \quad v_2$

$F(0, 3, -1) = (-1, 3+2) = (-1, 5) \quad v_3$

$(x, y) = a \cdot (1, 5) + b \cdot (2, -1)$

$$\begin{cases} a+2b = x \\ 5a-b = y \end{cases} \quad \underline{b = 5a - y}$$

$a + 2 \cdot (5a - y) = x$

$\underline{a = \frac{x+2y}{11}}$

$F(v_1) = (2, 0)$

$F(v_2) = (1, -1)$

$F(v_3) = (-1, 5)$

$a_1 = \frac{2}{11}$

$a_2 = \frac{-1}{11}$

$a_3 = \frac{9}{11}$

$b_1 = 5 \cdot \frac{2}{11} = \frac{10}{11}$

$b_2 = 5 \cdot \frac{-1}{11} + \frac{11}{11} = \frac{6}{11}$

$b_3 = 5 \cdot \frac{9}{11} - 5 = \frac{-10}{11}$

$$(F)_B^C = \begin{pmatrix} \frac{2}{11} & \frac{-1}{11} & \frac{9}{11} \\ \frac{10}{11} & \frac{6}{11} & \frac{-10}{11} \end{pmatrix} = \frac{1}{11} \cdot \begin{pmatrix} 2 & -1 & 9 \\ 10 & 6 & -10 \end{pmatrix}$$

2-) a) $F(x, y, z) = (x+y, z)$

$\mathbb{R}^3 \rightarrow \mathbb{R}^2$ Bases Canônicas

Base Domínio = $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

Base Contradomínio = $\{(1, 0), (0, 1)\}$

$F(1, 0, 0) = (1+0, 0) = (1, 0)$

$F(0, 1, 0) = (0+1, 0) = (1, 0)$

$F(0, 0, 1) = (0+0, 1) = (0, 1)$

$F = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

* Cada coluna é um vetor

b) $F(x, y) = (x+y, x, x-y)$ $\mathbb{R}^2 \rightarrow \mathbb{R}^3$

Base Domínio = $\{(1, 0), (0, 1)\}$

Base Contradomínio = $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

$F(1, 0) = (1+0, 1, 1-0) = (1, 1, 1)$

$F(0, 1) = (0+1, 0, 0-1) = (1, 0, -1)$

$F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix}$

6-) Base Canônica = $\{(1,0), (0,1)\}$ $(F)_B = \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix}$
 Base B = $\{(1,0), (1,4)\}$

$$(F)_C = P \cdot (F)_B \cdot P^{-1}$$

$$P = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix} \xrightarrow{P^{-1}} \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{pmatrix} \rightarrow L_1 = L_1 - (L_2/4)$$

$$\begin{pmatrix} 1 & 0 & 1 & -\frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{pmatrix} \quad P^{-1} = \begin{pmatrix} 1 & -\frac{1}{4} \\ 0 & \frac{1}{4} \end{pmatrix} = \frac{1}{4} \cdot \begin{pmatrix} 4 & -1 \\ 0 & 1 \end{pmatrix}$$

$$P(F)_B = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 20 & 4 \end{pmatrix}$$

(Handwritten notes in blue: 1+5, 1+1, 0+20, 0+4)

$$\begin{pmatrix} 6 & 2 \\ 20 & 4 \end{pmatrix} \cdot P^{-1} = \begin{pmatrix} 6 & 2 \\ 20 & 4 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{4} \\ 0 & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ 20 & -4 \end{pmatrix}$$

(Handwritten notes in blue: 6+0, -6/4+2/4, 20+0, -20/4+4/4)

$$(F)_C = \begin{pmatrix} 6 & -1 \\ 20 & -4 \end{pmatrix}$$

8-1 Base $B = \{(1, 2), (0, 5)\}$ $(F)_B = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$

$$F(1, 2) = 3 \cdot (1, 2) + 2 \cdot (0, 5)$$

$$F(1, 2) = (3, 6) + (0, 10) = (3, 16)$$

$$F(0, 5) = 1 \cdot (1, 2) + 1 \cdot (0, 5)$$

$$F(0, 5) = (1, 2) + (0, 5) = (1, 7)$$

$$(x, y) = a \cdot (1, 2) + b \cdot (0, 5)$$

$$\begin{cases} a = x & b = \frac{y - 2x}{5} \\ 2a + 5b = y \end{cases}$$

$$2x + 5b = y$$

$$(x, y) = x \cdot (1, 2) + \left(\frac{y - 2x}{5} \right) \cdot (0, 5)$$

$$F(x, y) = x \cdot F(1, 2) + \left(\frac{y - 2x}{5} \right) \cdot F(0, 5)$$

$$F(x, y) = x \cdot (3, 16) + \left(\frac{y - 2x}{5} \right) \cdot (1, 7)$$

$$x = 3x + \frac{y - 2x}{5} = \frac{15x + y - 2x}{5} = \frac{13x + y}{5}$$

$$y = 16x + \left(\frac{y - 2x}{5} \right) \cdot 7 = 16x + \left(\frac{-3y + 16x}{5} \right) = \frac{86x - 3y}{5}$$

$$F(x, y) = \left(\frac{13x + y}{5}, \frac{86x - 3y}{5} \right) \quad \text{operator linear}$$

1-) a) $T(x, y) = (x+y, x-y)$

$$T(1,0) = (1, 1)$$

$$T(0,1) = (1, -1)$$

$$M = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \det M = -1 - 1 = -2$$

Autovetores característicos

$$\det(M - \lambda I) = 0$$

$$(1-\lambda)(-1-\lambda) - 1 \cdot 1 = 0$$

$$-(1-\lambda^2) - 1 = 0$$

$$-1 + \lambda^2 - 1 = 0$$

$$\lambda^2 - 2 = 0$$

$$\underline{\lambda_1 = \sqrt{2}} \quad \underline{\lambda_2 = -\sqrt{2}}$$

$$\lambda = \pm \sqrt{2}$$

$$\lambda_1 \begin{pmatrix} 1-\sqrt{2} & 1 \\ 1 & -1-\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x = (1+\sqrt{2}) \\ y = 1 \end{cases}$$

$$v_1 = (1+\sqrt{2}, 1)$$

$$1 \cdot x + (-1-\sqrt{2})y = 0 \quad \begin{aligned} x &= (1+\sqrt{2}) \cdot y \\ (1+\sqrt{2}) &= (1+\sqrt{2}) \cdot 1 \end{aligned}$$

$$\lambda_2 \begin{pmatrix} 1+\sqrt{2} & 1 \\ 1 & -1+\sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x = (1-\sqrt{2}) \\ y = 1 \end{cases}$$

$$v_2 = (1-\sqrt{2}, 1)$$

$$1 \cdot x + (\sqrt{2}-1)y = 0 \quad \begin{aligned} x &= (1-\sqrt{2}) \cdot y \\ (1-\sqrt{2}) &= (1-\sqrt{2}) \cdot 1 \end{aligned}$$