Lista 4 - Erickson G. Müller

2-) $V = \{(x, y) \mid x, y \in \mathbb{R}\}$ regras: adição $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, 0)$; multip. $a.(x_1, y_1) = (ax_1, ay_1)$

adição

$u + v = v + u$

$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, 0)$

$(x_2, y_2) + (x_1, y_1) = (x_2 + x_1, 0)$

$(x_1 + x_2, 0) = (x_2 + x_1, 0)$

$u + (v + w) = (u + v) + w$

$(x_1, y_1) + [(x_2, y_2) + (x_3, y_3)]$

$(x_1, y_1) + (x_2 + x_3, 0)$

$\bullet (x_1 + x_2 + x_3, 0)$

$[(x_1, y_1) + (x_2, y_2)] + (x_3, y_3)$

$(x_1 + x_2, 0) + (x_3, y_3)$

$\bullet (x_1 + x_2 + x_3, 0)$

$u + \bar{0} = u$

$(x, y) + (a, b) = (x + a, 0)$

~~$(x, y) \neq (x, 0)$ apenas quando $y = 0$~~

$(x, y) \neq (x + a, 0)$

$u + (-u) = \bar{0}$

~~$(x, y) + [-(x, y)] = (x, y) + (-x, -y)$~~

~~$= (x - x, 0) = (0, 0) = \bar{0}$~~

$(x, y) + (w, z) = (0, 0)$

$(x, y) + (w, z) = (x + w, 0)$

$x + w = 0 \Rightarrow x = -w$

apenas quando x do inverso for $-x$

multiplicação $\alpha.(\beta u) = (\alpha.\beta).u$

$\alpha.(\beta x, \beta y) = (\alpha \beta x, \alpha.\beta y)$

$\alpha.\beta.(x, y) = (\alpha.\beta x, \alpha.\beta y)$

$(\alpha + \beta).u = \alpha.u + \beta.u$

$(\alpha + \beta).(x, y) = ((\alpha + \beta)x, (\alpha + \beta)y)$

$\alpha.(x, y) + \beta.(x, y) = (\alpha x, \alpha y) + (\beta x, \beta y)$

$(\alpha x, \alpha y) + (\beta x, \beta y) = (\alpha x + \beta x, 0)$

$(\alpha x + \beta x, 0) \neq (\alpha x + \beta x, \alpha y + \beta y)$

$\alpha(u + v) = \alpha u + \alpha v$

$\alpha[(x_1, y_1) + (x_2, y_2)] = \alpha.(x_1 + x_2, 0)$

$\bullet = (\alpha x_1 + \alpha x_2, 0)$

$\alpha(x_1, y_1) + \alpha(x_2, y_2) = (\alpha x_1, \alpha y_1) + (\alpha x_2, \alpha y_2)$

$\bullet = (\alpha x_1 + \alpha x_2, 0)$

$1.u = u$

$1.(x, y) = (1.x, 1.y) = (x, y)$

Portanto, V não é um espaço vetorial sobre $\mathbb{R}$ pois falha nos axiomas de elemento neutro e elemento inverso da adição e no axioma de distributividade da multiplicação.

Lista 4 - Erickson G. Müller

4 a) $V = \{(x,y) \mid x,y \in \mathbb{R}\}$ $\begin{cases} \text{add } (x_1,y_1)+(x_2,y_2) = (x_1+x_2, y_1+y_2) \\ \text{mul } a.(x,y) = (x, ay) \end{cases}$

adição $u+v = v+u$

$(x_1,y_1)+(x_2,y_2) = (x_1+x_2, y_1+y_2)$

$(x_2,y_2)+(x_1,y_1) = (x_2+x_1, y_2+y_1)$

$> (x_1+x_2, y_1+y_2) = (x_2+x_1, y_2+y_1)$

$u+(v+w) = (u+v)+w$

$(x_1,y_1)+[(x_2,y_2)+(x_3,y_3)] = (x_1,y_1)+(x_2+x_3, y_2+y_3)$

$= (x_1+x_2+x_3, y_1+y_2+y_3)$

$[(x_1,y_1)+(x_2,y_2)]+(x_3,y_3) = (x_1+x_2, y_1+y_2)+(x_3,y_3)$

$= (x_1+x_2+x_3, y_1+y_2+y_3)$

$u+\bar{0} = u$

$(x_1,y_1)+(a,b) = (x_1+a, y_1+b) = (x_1,y_1) \Rightarrow \begin{matrix} a=0 \\ b=0 \end{matrix} \Rightarrow \bar{0} = (0,0)$

$u+(-u) = \bar{0}$

$(x,y)+[-(x,y)] = (x,y)+(-x,-y) = (x-x, y-y) = (0,0) = \bar{0}$

multiplicação $\alpha.(\beta.u) = (\alpha.\beta).u$

$\alpha.[\beta.(x,y)] = \alpha.(x, \beta y) = (x, \alpha.\beta.y)$

$(\alpha.\beta).(x,y) = (x, (\alpha.\beta)y) = (x, \alpha\beta y)$

$(\alpha+\beta).u = \alpha u + \beta.u$

$(\alpha+\beta).(x,y) = \alpha.(x,y)+\beta.(x,y)$

$(\alpha+\beta).(x,y) = (x, (\alpha+\beta).y) = (x, \alpha y+\beta y)$

$\alpha.(x,y)+\beta.(x,y) = (x, \alpha y)+(x, \beta y) = (x+x, \alpha y+\beta y)$

$(x, \alpha y+\beta y) \neq (2x, \alpha y+\beta y)$

$\alpha(u+v) = \alpha u + \alpha v$

$\alpha.(x_1+x_2, y_1+y_2) = (x_1+x_2, \alpha.y_1+\alpha y_2)$

$\alpha.(x_1,y_1)+\alpha(x_2,y_2) = (x_1, \alpha y_1)+(x_2, \alpha y_2) = (x_1+x_2, \alpha y_1+\alpha y_2)$

$1.u = u$

$1.(x,y) = (x, y.1) = (x,y)$

O axioma de multiplicação distributiva não se aplica

Lista 4. Ericlson G. Müller

4-b) $V=\{(x,y) \mid x,y \in \mathbb{R}\}$ regras $\begin{cases} (x_1,y_1)+(x_2,y_2) = (x_1,y_1) \\ a.(x,y) = (ax, ay) \end{cases}$

adição $u+v=v+u$

$$\left.\begin{array}{l}(x_1,y_1)+(x_2,y_2)=(x_1,y_1)\\(x_2,y_2)+(x_1,y_1)=(x_2,y_2)\end{array}\right\rangle (x_1,y_1) \neq (x_2,y_2)$$ não se verifica

$u+(v+w)=(u+v)+w$

$$\left.\begin{array}{l}(x_1,y_1)+[(x_2,y_2)+(x_3,y_3)]=(x_1,y_1)+(x_2,y_2)=(x_1,y_1)\\ [(x_1,y_1)+(x_2,y_2)]+(x_3,y_3)=(x_1,y_1)+(x_3,y_3)=(x_1,y_1)\end{array}\right\rangle (x_1,y_1)=(x_1,y_1)$$

$u+\bar{0}=u$

$(x,y)+(a,b)=(x,y)$

$\bar{0}=(a,b)$; (a,b) não é um elemento único não se verifica

$u+(-u)=\bar{0}$

$(x,y)+(-x,-y)=(x,y)$

não existe $\bar{0}$ não se verifica

multiplicação $\alpha.(\beta u)=(\alpha.\beta).u$

$$\left.\begin{array}{l}\alpha.[\beta.(x,y)]=\alpha[(\beta x,\beta y)]=(\alpha\beta x,\alpha\beta y)\\(\alpha.\beta).(x,y)=((\alpha\beta).x,(\alpha\beta).y)\end{array}\right\rangle(\alpha\beta x,\alpha\beta y)=(\alpha\beta x,\alpha\beta y)$$

$(\alpha+\beta).u=\alpha u+\beta u$

~~$(\alpha+\beta).u=(\alpha\beta x,\alpha\beta y)$~~ $(\alpha+\beta).u=((\alpha+\beta)x,(\alpha+\beta).y)=(\alpha x+\beta x,\alpha y+\beta y)$

$\alpha.(x,y)+\beta(x,y)=(\alpha x,\alpha y)+(\beta x,\beta y)=(\alpha x,\alpha y)\neq(\alpha x+\beta x,\alpha y+\beta y)$

não se verifica

$\alpha.(u+v)=\alpha.u+\alpha.v$

$\alpha[(x_1,y_1)+(x_2,y_2)]=\alpha.(x_1,y_1)=(\alpha x_1,\alpha y_1)$

$\alpha.(x_1,y_1)+\alpha.(x_2,y_2)=(\alpha x_1,\alpha y_1)+(\alpha x_2,\alpha y_2)=(\alpha x_1,\alpha y_1)$

$(\alpha x_1,\alpha y_1)=(\alpha x_1,\alpha y_1)$

$1.u=u$

$1.(x,y)=(1.x,1.y)=(x,y)=(x,y)$

Lista 4. Erickson

5-) $V = \{(x, y) \mid x, y \in \mathbb{R}\}$ $\begin{cases} (x_1, y_1) + (x_2, y_2) = (2x_1 - 2y_1, -x_1 + y_1) \\ \alpha(x, y) = (3\alpha y, -\alpha x) \end{cases}$

adição $u + v = v + u$

$(x_1, y_1) + (x_2, y_2) = (2x_1 - 2y_1, -x_1 + y_1)$
$(x_2, y_2) + (x_1, y_1) = (2x_2 - 2y_2, -x_2 + y_2)$ $\neq$ não se aplica

$u + (v + w) = (u + v) + w$

$(x_1, y_1) + [(x_2, y_2) + (x_3, y_3)] = (x_1, y_1) + (2x_2 - 2y_2, -x_2 + y_2) = (2x_1 - 2y_1, -x_1 + y_1)$

$[(x_1, y_1) + (x_2, y_2)] + (x_3, y_3) = (2x_1 - 2y_1, -x_1 + y_1) + (x_3, y_3)$

$= (2.(2x_1 - 2y_1) - 2.(-x_1 + y_1), -(2x_1 - 2y_1) + (-x_1 + y_1))$

$= (4x_1 - 4y_1 + 2x_1 - 2y_1, -2x_1 + 2y_1 - x_1 + y_1)$

$= (6x_1 - 6y_1, -3x_1 + 3y_1) \neq (2x_1 - 2y_1, -x_1 + y_1)$ não se aplica

$u + \bar{0} = u$

$(x, y) + (a, b) = (2x - 2y, -x + y)$

$\nexists\ \bar{0} \in \mathbb{R}$ não se aplica

$u + (-u) = \bar{0}$

$(x, y) + (-x, -y) = (2x - 2y, -x + y)$

$\nexists\ \bar{0} \in \mathbb{R}$, portanto $\nexists$ $-u$ tal que $u + (-u) = \bar{0}$ não se aplica

multiplicação $\alpha.(\beta.u) = (\alpha.\beta).u$

$\alpha.[\beta.(x, y)] = \alpha.(3.\beta.y, -\beta x) = (3\alpha.(-\beta x), -\alpha.(3\beta y))$

$= (-3\alpha\beta x, -3.\alpha\beta y)$ não se aplica

$\alpha.\beta.(x, y) = (3.\alpha\beta y, -\alpha\beta x) \neq (-3\alpha\beta x, -3\alpha\beta y)$

$(\alpha + \beta).u = \alpha.u + \beta.u$

$(\alpha + \beta).u = (3.(\alpha + \beta).y, -(\alpha + \beta).x) = (3\alpha y + 3\beta y, -\alpha x - \beta x)$

$\alpha.(x, y) + \beta(x, y) = (3\alpha y, -\alpha x) + (3\beta y, -\beta x) = (2.3\alpha y + 2\alpha x, -3\alpha y - \alpha x)$

$(3\alpha y + 3\beta y, -\alpha x - \beta x) \neq (6\alpha y + 2\alpha x, -3\alpha y - \alpha x)$ não se aplica

Lista 4 . Erickson

5-) continuação

$\alpha.(u+v) = \alpha.u + \alpha.v$

$\alpha.[(x_1,y_1)+(x_2,y_2)] = \alpha.(2x_1 - 2y_1, -x_1 + y_1)$

$= (3.\alpha.(-x_1+y_1), -\alpha.(2x_1-2y_1)) = (-3\alpha x_1 + 3\alpha y_1, -2\alpha x_1 + 2\alpha y_1)$

$\alpha.(x_1,y_1) + \alpha.(x_2,y_2) = (3\alpha y_1, -\alpha x_1) + (3\alpha y_2, -\alpha x_2)$

$= (2.(3\alpha y_1) - 2.(-\alpha x_1), -3\alpha y_1 - \alpha x_1)$ não se aplica

$= (6\alpha y_1 + 2\alpha x_1, -3\alpha y_1 - \alpha x_1)$

$1.u = u$

$1.(x,y) = (3.y, -x) \neq (x,y)$ não se aplica

Lista 4 - Erickson PG 63

1 - a) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x = 0\}$

Condição vetor nulo $\quad w = (0, y, z)$, tal que $y \in \mathbb{R}$
$z \in \mathbb{R}$

portanto, $(0,0,0) \in W$.

Cond. Fechado sob adição $\quad u = (0, y_1, z_1)$, $v = (0, y_2, z_2)$ $\Rightarrow u+v = (0, y_1 + y_2, z_1 + z_2)$

$0 = 0$
$y_1 + y_2 \in \mathbb{R}$
$z_1 + z_2 \in \mathbb{R}$

Cond. Fechado sob multiplicação $\quad u = (0, y, z)$
$K \cdot u = (0, y \cdot K, z \cdot K)$

$0 \cdot K = 0$
$y \cdot K \in \mathbb{R}$
$z \cdot K \in \mathbb{R}$

Resposta: W é subespaço de $\mathbb{R}^3$.

Lista 4 - Erickson Pg 63

1-b) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x \in \mathbb{Z}\}$

V.N $(x, y, z) = \bar{0}$ $x = 0 \in \mathbb{Z}$
$y = 0 \in \mathbb{R}$
$z = 0 \in \mathbb{R}$

adição $u = (x_1, y_1, z_1)$
$v = (x_2, y_2, z_2)$ > $u+v = (x_1+x_2, y_1+y_2, z_1+z_2)$

$x_1 + x_2 \in \mathbb{Z}$ → a soma de dois números inteiros é sempre um número inteiro

$y_1 + y_2 \in \mathbb{R}$ $u+v \in W$
$z_1 + z_2 \in \mathbb{R}$

multiplicação $u = (x, y, z)$, $k \in \mathbb{R}$
$k \cdot u = (x \cdot k, y \cdot k, z \cdot k)$

$\underline{x \cdot k \in \mathbb{R}}$
$y \cdot k \in \mathbb{R}$
$z \cdot k \in \mathbb{R}$

Resposta: W não é um sub-espaço de $\mathbb{R}^3$ pois falha na condição de ser fechado por multiplicação por escalar real.

Lista 4 · Erickson - Pi 63

3.a) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x = 1\}$

vetor nulo

$\bar{0}$		W
$x = 0$	X	$x = 1$
$y = 0$	✓	$y \in \mathbb{R}$
$z = 0$	✓	$z \in \mathbb{R}$

Soma $u = (x_1, y_1, z_1)$ $v = (x_2, y_2, z_2)$ $u+v = (x_1+x_2, y_1+y_2, z_1+z_2)$

W		$u+v$
$x = 1$	X	$\cancel{2}$ $x_1 + x_2 = 2$
$y \in \mathbb{R}$	✓	$\cancel{y \in \mathbb{R}}$ $y_1 + y_2 \in \mathbb{R}$
$z \in \mathbb{R}$	✓	$\cancel{z \in \mathbb{R}}$ $z_1 + z_2 \in \mathbb{R}$

mult: $u = (x, y, z)$ $k \in \mathbb{R}$

W		$u \cdot k$
$x = 1$	X	$x \cdot k \in \mathbb{R}$
$y \in \mathbb{R}$	✓	$y.k \in \mathbb{R}$
$z \in \mathbb{R}$	✓	$z.k \in \mathbb{R}$

Resposta: W não é sub-espaço de $\mathbb{R}^3$ pois falha em todas as condições

Lista 4 . Erickson . Pg 63

3b) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y + z = 0\}$

Vetor nulo $u = (0, 0, 0)$

$0^2 + 0 + 0 = 0$

adição $u = (x_1, y_1, z_1) \quad v = (x_2, y_2, z_2)$

$u + v = (x_1 + x_2, y_1 + y_2, z_1 + z_2) =$

$$\begin{cases} x_1^2 + y_1 + z_1 = 0 \\ x_2^2 + y_2 + z_2 = 0 \\ (x_1 + x_2)^2 + (y_1 + y_2) + (z_1 + z_2) = 0 \end{cases}$$

$(x_1 + x_2)^2 - x_1^2 - x_2^2 = 0$

$x_1^2 + 2 \cdot x_1 \cdot x_2 + x_2^2 - x_1^2 - x_2^2 = 0$

$2 \cdot x_1 \cdot x_2 = 0$

multiplicação $u = (x, y, z)$

$K.u = (K.x, Ky. Kz)$

$$\begin{cases} x^2 + y + z = 0 \\ (Kx)^2 + y\cancel{K} + z.K = 0 \end{cases}$$

Resposta: W não é sub-espaço do $\mathbb{R}^3$ pois não é fechado sob adição e multiplicação.

$K^2 . x^2 + K.y + z.K = 0 \quad \div K$

$K.x + y + z = 0 \quad \Rightarrow \quad Kx + y + z - x^2 - y - z = 0 - 0$

$Kx \neq x^2$

$K.x + y + z \neq x^2 + y + z$

Lista 4 - Erickson - PG 63

9 - a) $U = \{(x,y,z) \mid x - 2y = 0\}$

$u = (2y, y, z) = (2y, y, 0) + (0, 0, z)$

$u = y \cdot (2, 1, 0) + z \cdot (0, 0, 1)$

Geradores de $U = \{(2,1,0), (0,0,1)\}$

b) $V = \{(x,y,z) \mid x + z = 0 \text{ e } x - 2y = 0\}$

$2y = x$

$x = -z$

$z = -2y$

$v = (2y, y, -2y)$

$v = y \cdot (2, 1, -2)\}$

$S = \{(2, 1, -2)\}$

c) $W = \{(x,y,z) \mid x + 2y - 3z = 0$

$x = -2y + 3z$

$w = (-2y + 3z, y, z)$

$w = (-2y, y, 0) + (3z, 0, z)$

$w = y \cdot (-2, 1, 0) + z \cdot (3, 0, 1)$

$W = \{(-2, 1, 0), (3, 0, 1)\}$

Lista 4 · Erickson Pg 63.

3·d) $U \cap V$

Condição U $x - 2y = 0$

Condição V $x - 2y = 0$ $U \subset V$

$x + z = 0$

$\begin{pmatrix} \text{Geradores} \\ \text{de } U \end{pmatrix} \cap \begin{pmatrix} \text{Geradores} \\ \text{do } V \end{pmatrix} \Rightarrow S = \{(2,1,-2)\}$

~~3·d~~

3·e) $U+V$

Geradores de V $\{(2,1,-2)\}$

Geradores de U $\{(-2,1,0),(3,0,1)\}$

$V+U = \Rightarrow S = \{(2,1,-2),(-2,1,0),(3,0,1)\}$

Lista 4. Ericason

12 -)

$$a \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \cdot \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} b & b \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ c & c \end{pmatrix} + \begin{pmatrix} 0 & d \\ d & 2d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} a + b = 0 \\ b + d = 0 \\ c + d = 0 \\ a + c + 2d = 0 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 \end{pmatrix} \rightarrow L_4 : L_4 - L_1 + L_2 - L_3$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}$$

Laplace

$$\det = (-1)^2 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} + (-1)^3 \cdot \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$\det = (2 - 1) \cdot 1 + (-1) \cdot -1 = 2$$

$$2 \neq 0 \quad LI$$

R: As matrizes são base para M_2

Lista 4 - Erickson pg 73

1-a) $\{(1,0,0), (0,1,0), (0,0,1), (2,3,5)\}$

$\quad\ \ a \qquad\quad b \qquad\quad c \qquad\quad d$

$d = 2a + 3b + 5c$

$2a + 3b + 5c - d = (0,0,0)$

Resposta: Linearmente Dependente

b) $\{(1,1,1), (1,0,1), (1,0,-2)\}$

det = 0 → LD
det ≠ 0 → LI

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -2 \end{vmatrix} = 0 + 1 + 0 - 0 - 0 + 2 = 3$$

Resposta: Linearmente Independente

Lista 4 · Erickson Pg 73.

2. a) $\{(1, x-1, x^2+2x+1, x^2)\}$

a b c d

$d = c - 2b - a$ Resposta: Linearmente Dependente

b) $\{(2x, x^2+1, x+1, x^2-1)\}$

a b c d

$b - d = 2$

$2 \cdot c = 2x + 2$

$a + (b - d) = 2x + 2$

$a + b - d - 2c = 0$ Resposta: Linearmente Dependente

Lista 4 - Ericksson pg 73

LD → det = 0
LI → det ≠ 0

7-) a) $\{(3, 5m, 1), (2, 0, 4), (1, m, 3)\}$

$$\begin{vmatrix} 3 & 5m & 1 \\ 2 & 0 & 4 \\ 1 & m & 3 \end{vmatrix} = 0 + 20m + 2m - 0 - 12m - 30m \neq 0$$

LI: $-20m \neq 0$

$m \neq 0$

b) $\{(1, 3, 5), (2, m+1, 10)\}$

$(2, m+1, 10) = K \cdot (1, 3, 5)$

$$LD: \begin{cases} 2 = K \\ m+1 = 3K \\ 10 = 5K \end{cases}$$

LI: $m \neq 5$

$m+1 = 6$

$m = 5$

Lista 4 - Erickson . pg. 88

1-) $x - y = y \Rightarrow x = 2y$

$2y - 3y + t = 0$

$t = y$

$V = (2y, y, z, y)$

$V = y.(2,1,0,1) + z.(0,0,1,0)$

Base $\{(2,1,0,1), (0,0,1,0)\}$

Dimensão: 2

2-) $\begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 5 & 2 & 1 \end{pmatrix} \begin{matrix} \\ \to L_2 + L_1 \\ \to L_3 + L_1 \end{matrix} \Rightarrow \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ 0 & 3 & 1 & 1 \end{pmatrix} \begin{matrix} \\ \\ \to L_3 - L_2 \end{matrix}$

$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\dim(W \cap U) = 2$

$\dim(A+B) = \dim(A) + \dim(B) - \dim(A \cap B)$

$\dim(W+U) = \underline{3 + 3 - 2 = 4}$

$\dim(W+U) = 4$

$W + U = \mathbb{R}^4$

Lista 4 · Erickson Pg 88

3. $B_W = \{(2,1,0,1), (0,0,1,0)\}$ $\dim(W) = 2$

$B_U = \{(1,2,1,3), (3,1,-1,4)\}$ $\dim(U) = 2$

B_{U+W}

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 3 & 1 & -1 & 4 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_2 - 3L_1} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -4 & -5 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_3 - 2L_1}$$

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -4 & -3 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_3 - L_2 - L_1} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -4 & -3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{L_4 - L_3}$$

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -4 & -3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$B_{U+W} = \{(1,2,1,3), (3,1,-1,4), (2,1,0,1), (0,0,1,0)\}$

$\dim(U+W) = 4$

$4 = 2 + 2 - \dim(U \cap W)$

R: $\dim(U \cap W) = 0$

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4-) $U = \{(x, y, z, \cancel{t}) \mid x - y = 0 \text{ e } x + 2y + t = 0\}$

$x = y$ $\quad$ $y + 2y + t = 0$

$t = -3y$

$U = (y, y, z, -3y) = y.(1, 1, 0, -3) + z.(0, 0, 1, 0)$

$B_U = \{(1, 1, 0, -3), (0, 0, 1, 0)\}$

$\dim U = 2$