Lista 2 · Erickson G. Müller

1-)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A.B = \begin{pmatrix} 1.4+0+0 & 1.0+0.2+0 & 1.0+0+0.1 \\ 0.4+2.0+0 & 0.0+2.2+0 & 0.0+2.0+0.1 \\ 0.4+0+4.0 & 0.0+0.2+4.0 & 0+0+4.1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$B.A = \begin{pmatrix} 4.1+0+0 & 4.0+0.2+0 & 4.0+0+0.4 \\ 0.1+2.0+0 & 0.0+2.2+0.0 & 0.0+2.0+0.4 \\ 0.\cancel{0}^{1}+0.0+1.0 & 0.0+0.2+1.0 & 0.0+0.0+1.4 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Resposta: Para as matrizes A e B, vale a propriedade comutativa da multiplicação, Contudo, isso não ocorre a todas as matrizes 3x3 $\in \mathbb{R}$. O motivo de essa propriedade ser valida é porque A e B são matrizes diagonais.

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3-)

$$A+B = \begin{pmatrix} 1+4 & 0 & 0 \\ 0 & 2+2 & 0 \\ 0 & 0 & 4+1 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$A-B = \begin{pmatrix} 1-4 & 0 & 0 \\ 0 & 2-2 & 0 \\ 0 & 0 & 4+1 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$(2x-y)+(x+y) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix} + \begin{pmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$y = A-B-x$$

$$3x = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix} \Rightarrow x = \frac{1}{3}\cdot\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$y = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{8}{3} \end{pmatrix} = \begin{pmatrix} -\frac{11}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$$S = \left\{ \left(x = \frac{1}{3}\cdot\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix},\ y = \frac{1}{3}\cdot\begin{pmatrix} -11 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \right\}$$

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6-) $AB = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2.1 & 2.2 & 2.1 \\ 1.1 & 1.2 & 1.1 \\ 1.1 & 1.2 & 1.1 \end{pmatrix}$

$$AB = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \left((1.2) + (2.1) + (1.1) \right) = 5$$

$$BA = (5)$$

$$A_{3\times 1} \cdot B_{(1\times 3)} = M_{3\times 3}$$

$$B_{1\times 3} \cdot A_{3\times 1} = M_{1\times 1}$$

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7-) $A^2 - 6A + 5I_n = 0$

$$A^2 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} (2.2)+(3.1) & (2.3+3.4) \\ (1.2)+(4.1) & (1.3)+(4.4) \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix}$$

$$6.A = \begin{pmatrix} 6.2 & 6.3 \\ 6.1 & 6.4 \end{pmatrix} = \begin{pmatrix} 12 & 18 \\ 6 & 24 \end{pmatrix}$$

$$5I = 5.\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 7-12+5 & 18-18+0 \\ 6-6+0 & 19-24+5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

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8-1

$$x = \begin{pmatrix} 1 & y^{-1} \\ y & 1 \end{pmatrix}, \quad y \neq 0$$

$$x^2 = \begin{pmatrix} 1 & y^{-1} \\ y & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & y^{-1} \\ y & 1 \end{pmatrix} = \begin{bmatrix} (1.1) + (y^{-1}.y) & (1.y^{-1}) + (y^{-1}.1) \\ (y.1) + (1.y) & (y.y^{-1}) + (1.1) \end{bmatrix}$$

$$x^2 = \begin{pmatrix} 1+1 & \frac{1}{y} + \frac{1}{y} \\ y+y & 1+1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{y} \\ 2y & 2 \end{pmatrix}$$

$$2x = 2.\begin{pmatrix} 1 & \frac{1}{y} \\ y & 1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{y} \\ 2y & 2 \end{pmatrix}$$

Logo $x^2 = 2x$

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12-) $A = \begin{pmatrix} x_a & y_a \\ w_a & z_a \end{pmatrix}$ $B = \begin{pmatrix} x_b & y_b \\ w_b & z_b \end{pmatrix}$

$x \cdot AB = x_a \cdot x_b + y_a w_b = 0$

$y AB = x_a \cdot y_b + y_a \cdot z_b = 0$

$w AB = w_a \cdot x_b + z_a \cdot w_b = 0$

$z AB = w_a \cdot y_b + z_a \cdot z_b = 0$

$x BA = x_b \cdot x_a + y_b \cdot w_a$

$y BA = x_b \cdot y_a + y_b \cdot z_a$

$w BA = w_b \cdot x_a + z_b \cdot w_a$

$z BA = w_b \cdot y_a + z_b \cdot z_a$

Condições para ser nula:

I $\cancel{x_a \cdot x_b} + y_a \cdot w_b = \cancel{x_b \cdot x_a} + y_b \cdot w_a$

II $x_a \cdot y_b + y_a \cdot z_b = x_b \cdot y_a + y_b \cdot z_a$

III $x_a \cdot x_b + z_a \cdot w_b = w_b \cdot x_a + z_b \cdot w_a$

IV $w_a \cdot y_b + \cancel{z_a \cdot z_b} = w_b \cdot y_a + \cancel{z_b \cdot z_a}$

Como as quatro condições não se anularam, a conclusão é falsa.

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12-) exemplo gráfico.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} (1 \cdot 0)+(0 \cdot 1) & (1 \cdot 0)+(0 \cdot 0) \\ (0 \cdot 0)+(0 \cdot 1) & (0 \cdot 0)+(0 \cdot 0) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} (0 \cdot 1)+(0 \cdot 0) & (0 \cdot 0)+(0 \cdot 0) \\ (1 \cdot 1)+(0 \cdot 0) & (1 \cdot 0)+(0 \cdot 0) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$BA \neq AB$$