Lista 5 - Erickson G. Müller · PG 11D

1-) a) $F_1(x,y,z) = (x-y, x+y, 0)$ operadores lineares

$u=(x_1, y_1, z_1)$ $v=(x_2, y_2, z_2)$

adição $u+v=(x_1+x_2, y_1+y_2, z_1+z_2)$

$F(u+v)=F(u)+F(v)$ $F(u+v) = ([x_1+x_2]-[y_1+y_2], [x_1+x_2]+[y_1+y_2], 0)$

$$= \underbrace{(x_1-y_1, x_1+y_1, 0)}_{F(u)} + \underbrace{(x_2-y_2, x_2+y_2, 0)}_{F(v)}$$

Logo, $F(u+v) = F(u)+F(v)$

mult por escalar $K.u = (K.x, K.y, K.z)$

$F(K.u) = (Kx - Ky, Kx+Ky, 0)$

$= (K.[x-y], K.[x+y], K.0)$

$= K.(x-y, x+y, 0)$

Resposta: F_1 é operador linear

Logo, $K.F(u) = F(K.u)$

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1.) b) $F_2(x,y,z) = (2x - y + z, 0, 0)$

$u = (x_1, y_1, z_1)$ $v = (x_2, y_2, z_2)$

$u + v = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

• $F(u+v) = F(u) + F(v)$

$F(u) + F(v) = (2x_1 - y_1 + z_1, 0, 0) + (2x_2 - y_2 + z_2, 0, 0)$

$= (2x_1 + 2x_2 - y_1 - y_2 + z_1 + z_2, 0, 0)$

$= (2.[x_1 + x_2] - [y_1 + y_2] + [z_1 + z_2], 0, 0)$

$F(u+v) = (2.[x_1 + x_2] - [y_1 + y_2] + [z_1 + z_2], 0, 0)$

Logo, $F(u+v) = F(u) + F(v)$

• $F(K.u) = K.F(u)$

$F(K.u) = (2.[Kx] - [K.y] + [K.z], 0, 0)$

$= (K.[2.x - y + z], K.0, K.0)$

$= K.(2x - y + z, 0, 0)$

Logo, $K.F(u) = F(K.u)$

Resposta: F_2 é operador Linear

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2-) $F(x,y,z) = x.(2,3,1) + y.(5,2,7) + z.(-2,0,7)$
$= (2x, 3x, x) + (5y, 2y, 7y) + (-2z, 0, 7z)$
$= (2x + 5y - 2z, \; 3x + 2y, \; x + 7y + 7z)$

$u = (x_1, y_1, z_1) \quad v = (x_2, y_2, z_2)$

• $F(u+v) = F(u) + F(v)$

$u+v = (x_1 + x_2, \; y_1 + y_2, \; z_1 + z_2)$

$F(u+v) = (2[x_1+x_2] + 5.[y_1+y_2] - 2.[z_1+z_2],$
$3.[x_1+x_2] + 2[y_1+y_2],$
$[x_1+x_2] + 7.[y_1+y_2] + 7.[z_1+z_2])$

$= (2x_1 + 5y_1 - 2z_1, \; 3x_1 + 2y_1, \; x_1 + 7y_1 + 7z_1)$
$+ (2x_2 + 5y_2 - 2z_2, \; 3x_2 + 2y_2, \; x_2 + 7y_2 + 7z_2)$

Logo, $F(u+v) = F(u) + F(v)$

• $F.(K.u) = K.F(u)$

$F(K.u) = (2.Kx + 5.Ky - 2.Kz, \; 3.Kx + 2Ky, \; K.x + 7.K.y + 7.K.z$
$= (K.[2x + 5y - 2z], \; K.[3x + 2y], \; K.[x + 7y + 7z])$
$= K.(2x + 5y - 2z, \; 3x + 2y, \; x + 7y + 7z)$

Logo, $F(K.u) = K.F.(u)$

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8-) a) $F(x,y) = x.(2,1) + y.(1,4)$

$F(x,y) = (2x, x) + (y + 4y)$

$F(x,y) = (2x+y, x+4y)$

$F(2,4) = (2.2+4, 2+4.4) = (8, 18)$

b) $(2x+y, x+4y) = (2,3)$

$\begin{cases} 2x+y = 2 \\ x+4y = 3 \end{cases}$ $\begin{pmatrix} 2 & 1 & -2 & 0 \\ 1 & 4 & -3 & 0 \end{pmatrix} \to L_2 = L_1 - 2L_2$

$\begin{pmatrix} 2 & 1 & -2 & 0 \\ 0 & -7 & 4 & 0 \end{pmatrix}$ $y = \frac{4}{7}$ $2x + \frac{4}{7} = 2$

$x = \frac{5}{7}$ $x = \frac{2.7 - 4}{14} = \frac{10}{14}$

$(x,y) = \left(\frac{5}{7}, \frac{4}{7}\right)$

c) $F(1,0) = (2,1)$
$F(0,1) = (1,4)$

$M = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$ $\det(M) = 8 - 1 = 7$

$\det(M) \neq 0$

Logo, S.P.D → Sobrejetora

Ker(F)

$(2x+y, x+4y) = (0,0)$

$\begin{cases} 2x+y = 0 \\ x+4y = 0 \end{cases}$ $x = -\frac{y}{2}$ → $\cancel{-\frac{y}{2}.2 + y = 0}$

$x = -4y$ → $\cancel{-4y + 4y = 0}$

$\boxed{\begin{matrix} 2.-4y + y = 0 \\ y = 0 \end{matrix}}$ $\boxed{\begin{matrix} -\frac{y}{2} + 4y = 0 \\ y = 0 \end{matrix}}$ $Ker(F) = \{(0,0)\}$
Injetora

Lista 5 - Erickson - pg 121

1-) a) $F(x,y,z) = x+y-z$ $\quad F: \mathbb{R}^3 \to \mathbb{R}$ *

$F(x,y,z)=0 \quad x+y-z=0$

$z = x+y$

$Ker(F) = (x,y,x+y) = x.(1,0,1) + y.(0,1,1)$

Base do Núcleo $\{(1,0,1),(0,1,1)\}$

$Dim(Ker(F)) = 2$

Base da imagem $= \{(1)\}$

$Im = \mathbb{R} \quad dim(Im) \to 1$

b) $F(x,y) = (2x, x+y)$ $\quad F: \mathbb{R}^2 \to \mathbb{R}^2$

$(2x, x+y) = (0,0) \quad x=0 \quad y=0$

Base Núcleo $= \{\}$ dim núcleo $= 0$

$F(x,y) = x.(2,1) + y.(0,1)$ $\quad \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2 \neq 0 \to LI$

Base imagem: $\{(2,1),(0,1)\}$ dim imagem $= 2$

Lista 5 - Eridson PG. 121

1-) c) $F(x,y,z) = (x-y-z,\ x+y+z,\ 2x-y+z,\ -y)$ $F: \mathbb{R}^3 \to \mathbb{R}^4$

Ker:

$$\begin{cases} x-y-z=0 \\ x+y+z=0 \\ 2x-y+z=0 \\ -y=0 \end{cases} \quad \begin{pmatrix} 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad y=0$$

$$\begin{array}{c} \begin{array}{ccc} x & z & \end{array} \\ \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 0 \end{pmatrix} \end{array} > \begin{array}{l} 2x=0 \\ z=0 \end{array}$$

$Ker(F) = (0,0,0)$

Base do núcleo $= \{\}$

Dim núcleo $= 0$

Imagem

$$\begin{aligned} v &= F(1,0,0) = (1,1,2,0) \\ u &= F(0,1,0) = (-1,1,-1,-1) \\ w &= F(0,0,1) = (-1,1,1,0) \end{aligned} \to \begin{pmatrix} 1 & 1 & 2 & 0 \\ -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 0 \end{pmatrix} \begin{array}{l} \\ \to L_2 : L_2 + L_1 \\ \to L_3 : L_3 + L_1 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 2 & 3 & 0 \end{pmatrix} \begin{array}{l} \\ \\ \to L_3 : L_3 - L_2 \end{array} \quad \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

Base imagem $= \{(1,1,2,0), (0,2,1,-1), (0,0,2,1)\}$

dim imagem $= 3$