Lista 3. Erickson Müller

2-)

$$\det\begin{pmatrix}1&0&0\\a&1&0\\b&c&1\end{pmatrix} = 1^3 + a.c.0 + 0.0.b - 0.1.b - 0.1.c - 0.1.a$$

$\det(A) = 1 \neq 0$, Logo A é inversível $\forall\, a,b,c \in \mathbb{R}$

$$A \cdot A^{-1} = I_n$$

$$\begin{pmatrix}1&0&0\\a&1&0\\b&c&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\-a&1&0\\ac-b&-c&1\end{pmatrix} = M$$

$$M = \begin{pmatrix}(1.1)+(0.-a)+(0.\overset{ac-b}{\cancel{a}}) & (1.0)+(0.1)+(0.-c) & (1.0)+(0.0)+(0.1)\\(a.1)+(1.-a)+(0.[ac-b]) & (a.0)+(1.1)+(0.-c) & (a.0)+(1.0)+(0.1)\\(b.1)+(c.-a)+(1.[ac-b]) & (b.0)+(c.1)+(1.-c) & (b.0)+(c.0)+(1.1)\end{pmatrix}$$

$$M = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = I_n$$

Logo A^{-1} é a inversa de A

$$\hookrightarrow \begin{pmatrix}1&0&0\\-a&1&0\\ac-b&-c&1\end{pmatrix}$$

Lista 3. Erickson Müller

3-1 a) $\det\begin{pmatrix}1 & 2\\ 2 & 2\end{pmatrix} = 2-4 = -2$, Logo A é inversível

$\det(A) \neq 0$

$A^{-1} = \begin{pmatrix}-1 & 1\\ 1 & -\frac{1}{2}\end{pmatrix}$

$$\left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 2 & 2 & 0 & 1\end{array}\right) \xrightarrow{L_2-2L_1} = \left(\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & -2 & -2 & 1\end{array}\right) \begin{array}{l}\to L_1+L_2\\ \to \frac{-L_2}{2}\end{array} = \left(\begin{array}{cc|cc}1 & 0 & -1 & 1\\ 0 & 1 & 1 & -\frac{1}{2}\end{array}\right)$$

b) $\det\begin{pmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 2 & 1\end{pmatrix} = 1+0+2-0-0-0 = 3$

$\det(B) \neq 0$, Logo $\exists\, B^{-1}$

$$\begin{pmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0 & 1 & 0\\ 0 & 2 & 1 & 0 & 0 & 1\end{pmatrix} \xrightarrow{L_2-L_1} \begin{pmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & 2 & 1 & 0 & 0 & 1\end{pmatrix} \begin{array}{l}\to L_2+L_3\\ \to -2L_2+L_3\end{array}$$

$$\begin{pmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 3 & 0 & -1 & 1 & 1\\ 0 & 0 & -3 & -2 & 2 & -1\end{pmatrix} \begin{array}{l}\to 3L_1+L_3\\ \\ \to -L_3\end{array} \left(\begin{array}{ccc|ccc}3 & 0 & 0 & 1 & 2 & -1\\ 0 & 3 & 0 & -1 & 1 & 1\\ 0 & 0 & 3 & 2 & -2 & 1\end{array}\right) \begin{array}{l}\div 3\\ \div 3\\ \div 3\end{array}$$

$$\begin{pmatrix}1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{-1}{3}\\ 0 & 1 & 0 & \frac{-1}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & 0 & 1 & \frac{2}{3} & \frac{-2}{3} & \frac{1}{3}\end{pmatrix} \quad B^{-1} = \frac{1}{3}\cdot\begin{pmatrix}1 & 2 & -1\\ -1 & 1 & 1\\ 2 & -2 & 1\end{pmatrix}$$

Lista 3. Erickson Müller

3-) c) Por Laplace

$$\det\begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & -1 \\ 0 & 2 & 0 & 3 \end{pmatrix} =$$

a_{11} $0 \cdot (-1)^2 \cdot \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix}$ $\quad 0$

$+$ $\quad +$

a_{21} $1 \cdot (-1)^3 \cdot \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix}$ $\quad 1 \cdot -1 \cdot (-2 - 2 - 3)$

$+$ $\quad +$

a_{31} $1 \cdot (-1)^4 \cdot \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix}$ $\quad 1 \cdot 1 \cdot (2)$

$+$ $\quad +$

a_{41} $0 \cdot (-1)^5 \cdot \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ $\quad 0$

$\det(C) = 7 + 2 = 9$

$\det(C) \neq 0$, logo $\exists\, C^{-1}$

Desenvolvi o cálculo dessa matriz inversa na última página desta lista.

Lista 3. Erickson Müller

4-) a) $\begin{cases} x-y=4 \\ x+y=0 \end{cases}$ b) $\begin{cases} x+y+z=2 \\ x-y+z=0 \\ y+2z=0 \end{cases}$ c) $\begin{cases} x-y+z+t=0 \\ x+y-z+t=1 \\ -x+y+z-t=0 \\ 2x-y-z+3t=1 \end{cases}$

a $\overset{A}{\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}} \cdot \overset{M}{\begin{pmatrix} 4 \\ 0 \end{pmatrix}}$ $x = \dfrac{\begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \cancel{\dfrac{0}{0}} \quad \dfrac{4}{1+1} = 2$

$S=\{(2,-2)\}$ $y = \dfrac{\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \cancel{\dfrac{0}{0}} \quad \dfrac{-4}{1+1} = -2$

$\det(A) = -2+0+1-0-1-2 = -4$

b $\overset{A}{\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{pmatrix}} \overset{M}{\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}}$ $x = \dfrac{\begin{vmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 2 \end{vmatrix}}{-4} = \dfrac{-4-2}{-4} = \dfrac{-6}{-4} = \dfrac{3}{2}$

$S=\{(\frac{3}{2}, 1, -\frac{1}{2})\}$

$y = \dfrac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{vmatrix}}{-4} = \dfrac{-4}{-4} = 1$ $z = \dfrac{\begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{vmatrix}}{-4} = \dfrac{2}{-4} = -\dfrac{1}{2}$

$\cancel{\det(A) = 3-2+1-1+2+1+1+3 =}$

↳ não dá pra usar Sarrus em $M_{4\times4}$

c $\overset{A}{\begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 2 & -1 & -1 & 3 \end{pmatrix}} \overset{M}{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}}$ $\to L_2 - L_1$, $\to L_3 + L_1$ $\to \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & 0 \\ 2 & -1 & -1 & 3 \end{pmatrix} \to L_4 - 2L_1 = (0\ 1\ -3\ 1) \to \cancel{L_2 + 2L_4}$

Correção $L_2 - 2L_4$

$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -4 & -2 \end{pmatrix} \to \cancel{L_4 + 2L_3}$ $\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Correção → $L_3 - \dfrac{L_4}{2}$

$\det(A) = 1.2.2.1 = 4$

□ multiplica o determinante por 2 e em seguida divide por 2.

Gauss

Lista 3. Erickson Müller

4 c) $\det(A) = 4$

$$x = \det\begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -1 & -1 & 3 \end{pmatrix} \cdot \frac{1}{4} = \frac{0}{4} = 0$$

usei o geogebra para calcular essas determinantes.

$$y = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & 0 & 1 & -1 \\ 2 & 1 & -1 & 3 \end{vmatrix} \cdot \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$z = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 0 & -1 \\ 2 & -1 & 1 & 3 \end{vmatrix} \cdot \frac{1}{4} = \frac{0}{4} = 0$$

$$t = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 0 \\ 2 & -1 & -1 & 1 \end{vmatrix} \cdot \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$S = \left\{\left(0, \frac{1}{2}, 0, \frac{1}{2}\right)\right\}$$

Lista 3. Erickson Müller

5-)

$$\overset{A}{\cancel{M}} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & m \end{pmatrix} \quad \det(\overset{A}{\cancel{M}}) = 0+2-2-0-4+m$$

$$\det(\overset{A}{\cancel{M}}) = m-4 \neq 0$$

$$m \neq 4$$

$$\overset{A}{\cancel{M}} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{\det(\overset{x}{M_x})}{\det(A)} = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 2 & m \end{vmatrix}}{m-4} = \frac{0+0+2-0-8+m}{m-4} \qquad x = \frac{m-6}{m-4}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & m \end{vmatrix}}{m-4} = \frac{m+4+0-1-0-2m}{m-4} \qquad y = \frac{-m+3}{m-4}$$

$$z = \frac{\begin{vmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}}{m-4} = \frac{0-1+4-0-2-0}{m-4} = \frac{1}{m-4} \qquad z = (m-4)^{-1}$$

Logo, o sistema é de Cramer $\forall m \in \mathbb{R} \,/\, m \neq 4$

com solução $S = \left\{ \left(\frac{m-6}{m-4}, \frac{-m+3}{m-4}, \frac{1}{m-4} \right) \right\}$

Lista 3. Erickson G. Miller

7-) $A.(B^{-1}.X) = C^{-1}.A$

$A^{-1}.A.(B^{-1}.X) = A^{-1}.C^{-1}A$

$I.(B^{-1}.X) = A^{-1}.C^{-1}.A$

$B.(B^{-1}.X) = (B.B^{-1}).X = B.A^{-1}.C^{-1}.A$

$I.X = B.A^{-1}.C^{-1}.A$

Logo

$$X = B.A^{-1}.C^{-1}.A$$

Lista 3. Erickson Müller

9-) $v_1 = (1, 0, 0)$

$v_2 = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$v_3 = (x, y, z)$

$v_1 \cdot v_3 = 1 \cdot x + 0 \cdot y + 0 \cdot z = 0$

$\underline{x = 0}$

$v_2 \cdot v_3 = 0 \cdot x + \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$

$\underline{y = -z}$

$v_3 \cdot v_3 = x^2 + y^2 + z^2 = 1$

$0^2 + y^2 + (-y)^2 = 1$

$y^2 + y^2 = 1$

$y = \pm \frac{1}{\sqrt{2}}$

$S_1 = \{(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})\}$

$S_2 = \{(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\}$

Resolução da matriz inversa da questão 3.C

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \to L_1 + L_3 \\ \to L_3 - L_2 \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 & -1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ \to L_1 - L_3 \\ \to L_4 - 2L_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 7 & 0 & 2 & -2 & 1 \end{array}\right) \begin{array}{l} \to L_1 - L_2 \\ \to L_2 - L_3 \\ \\ \to \cancel{L_1 + L_4}\ L_4 + 2L_3 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 2 & 2 & -2 & 1 \end{array}\right) \begin{array}{l} \to L_1 - L_3 \\ \to L_2 + \frac{L_4}{3} \\ \to L_3 - \frac{L_4}{9} \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{7}{9} & -\frac{2}{9} & \frac{2}{9} & -\frac{1}{9} \\ 0 & 0 & 0 & 9 & 2 & 2 & -2 & 1 \end{array}\right) \begin{array}{l} \to L_1 - \frac{L_4}{9} \\ \\ \\ \to \frac{L_4}{9} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{2}{9} & \frac{7}{9} & \frac{2}{9} & -\frac{1}{9} \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{7}{9} & -\frac{2}{9} & \frac{2}{9} & -\frac{1}{9} \\ 0 & 0 & 0 & 9 & \frac{2}{9} & \frac{2}{9} & -\frac{2}{9} & \frac{1}{9} \end{array}\right)$$

$$C^{-1} = \frac{1}{9} \cdot \begin{pmatrix} -2 & 7 & 2 & -1 \\ -3 & -3 & 3 & 3 \\ 7 & -2 & 2 & -1 \\ 2 & 2 & -2 & 1 \end{pmatrix}$$