Refreshment Quiz

Let P(x), Q(x), and R(x) be the statements “*x is a professor*,” “*x is ignorant*,” and “*x is vain*,” respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), and R(x), where the domain consists of **all people**.

a) No professors are ignorant.

b) All ignorant people are vain.

c) No professors are vain.

Answer

a) $\forall x(P(x) \rightarrow \neg Q(x))$

b) $\forall x(Q(x) \rightarrow R(x))$

c) $\forall x(P(x) \rightarrow \neg R(x))$

Refreshment Quiz

Let P(x), Q(x), R(x), and S(x) be the statements *"x is a baby," "x is logical," "x is able to manage a crocodile,"* and *"x is despised,"* respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), and S(x).

a) Babies are illogical.

b) Nobody is despised who can manage a crocodile.

c) Illogical persons are despised.

d) Babies cannot manage crocodiles.

Answer

a) $\forall x(P(x) \rightarrow \neg Q(x))$

b) $\forall x(R(x) \rightarrow \neg S(x))$ / $\forall x(S(x) \rightarrow \neg R(x))$

c) $\forall x(\neg Q(x) \rightarrow S(x))$

d) $\forall x(P(x) \rightarrow \neg R(x))$

KS141203 MATEMATIKA DISKRIT
(*DISCRETE MATHEMATICS*)

RULES OF INFERENCE

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Outline

sistem informasi
fakultas teknologi informasi

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Valid Arguments

An argument in propositional logic is a sequence of propositions.

All but the final proposition are called premises.

The final proposition is called conclusion.

An argument is valid if the truth of all premises implies that the conclusion is true.

- i.e. $(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$ is a tautology.

Modus Ponens

Consider $(p \wedge (p \rightarrow q)) \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q))$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$$

Modus Ponens Example

Assume you are given the following two statements:

- "you are in this class"
- "if you are in this class, you will get a grade"

Let p = "you are in this class"

Let q = "you will get a grade"

$$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$$

By Modus Ponens, you can conclude that you will get a grade

Modus Tollens

Assume that we know: $\neg q$ and $p \to q$

- Recall that $p \to q \equiv \neg q \to \neg p$ (*contrapositive*)

Thus, we know $\neg q$ and $\neg q \to \neg p$

We can conclude $\neg p$

$$\begin{array}{l} \neg q \\ \underline{p \to q} \\ \therefore \neg p \end{array}$$

Modus Tollens Example

Assume you are given the following two statements:

- “you will not get a grade”
- “if you are in this class, you will get a grade”

Let *p* = “you are in this class”

Let *q* = “you will get a grade”

$$\begin{array}{l} \neg q \\ \underline{p \to q} \\ \therefore \neg p \end{array}$$

By Modus Tollens, you can conclude that you are not in this class

Addition & Simplification

Addition: If you know that p is true, then $p \vee q$ will ALWAYS be true

$$\underline{p}$$

$$\therefore\ p \vee q$$

Simplification: If $p \wedge q$ is true, then p will ALWAYS be true

$$\underline{p \wedge q}$$

$$\therefore\ p$$

Example

We have the hypotheses:

- "It is not sunny this afternoon and it is colder than yesterday"
- "We will go swimming only if it is sunny"
- "If we do not go swimming, then we will take a canoe trip"
- "If we take a canoe trip, then we will be home by sunset"

Does this imply that "we will be home by sunset"?

$((\neg p \wedge q) \wedge (r \rightarrow p) \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)) \rightarrow t$???

- When
  - p = "It is sunny this afternoon"
  - q = "it is colder than yesterday"
  - r = "We will go swimming"
  - s = "we will take a canoe trip"
  - t = "we will be home by sunset"

Example

1.	$\neg p \wedge q$	1st hypothesis
2.	$\neg p$	Simplification using step 1
3.	$r \to p$	2nd hypothesis
4.	$\neg r$	Modus tollens using steps 2 & 3
5.	$\neg r \to s$	3rd hypothesis
6.	s	Modus ponens using steps 4 & 5
7.	$s \to t$	4th hypothesis
8.	t	Modus ponens using steps 6 & 7

We showed that:

- $[(\neg p \wedge q) \wedge (r \to p) \wedge (\neg r \to s) \wedge (s \to t)] \to t$
- That when the 4th hypothesis is true, then the implication is true (the above is a tautology!)

More Rules of Inference

Conjunction: if *p* and *q* are true separately, then $p \wedge q$ is true

$$\begin{array}{l} p \\ \underline{q\qquad} \\ \therefore p \wedge q \end{array}$$

Disjunctive syllogism: If $p \vee q$ is true, and *p* is false, then *q* must be true

$$\begin{array}{l} p \vee q \\ \underline{\neg p\qquad} \\ \therefore q \end{array}$$

Resolution: If $p \vee q$ is true, and $\neg p \vee r$ is true, then $q \vee r$ must be true

$$\begin{array}{l} p \vee q \\ \underline{\neg p \vee r} \\ \therefore q \vee r \end{array}$$

Hypothetical syllogism: If $p \rightarrow q$ is true, and $q \rightarrow r$ is true, then $p \rightarrow r$ must be true

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

Summary: Rules of Inference

Modus ponens	p $p \to q$ $\therefore q$	Modus tollens	$\neg q$ $p \to q$ $\therefore \neg p$
Hypothetical syllogism	$p \to q$ $q \to r$ $\therefore p \to r$	Disjunctive syllogism	$p \vee q$ $\neg p$ $\therefore q$
Addition	p $\therefore p \vee q$	Simplification	$p \wedge q$ $\therefore p$
Conjunction	p q $\therefore p \wedge q$	Resolution	$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$

Proofing Example

"If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on"

- $(\neg r \lor \neg f) \rightarrow (s \land d)$

"If the sailing race is held, then the trophy will be awarded"

- $s \rightarrow t$

"The trophy was not awarded"

- $\neg t$

Can you conclude: "It rained"?

- r

Proofing Example

1.	$\neg t$	3rd hypothesis
2.	$s \rightarrow t$	2nd hypothesis
3.	$\neg s$	Modus tollens using steps 2 & 3
4.	$(\neg r \vee \neg f) \rightarrow (s \wedge d)$	1st hypothesis
5.	$\neg(s \wedge d) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of step 4
6.	$(\neg s \vee \neg d) \rightarrow (r \wedge f)$	DeMorgan's law and double negation law
7.	$\neg s \vee \neg d$	Addition from step 3
8.	$r \wedge f$	Modus ponens using steps 6 & 7
9.	r	Simplification using step 8

Fallacy of Affirming the Conclusion

Consider the following:

$$\begin{array}{l} q \\ \underline{p \to q} \\ \therefore p \end{array} \qquad \begin{array}{l} q \\ \underline{\neg q \to \neg p} \\ \therefore p \end{array}$$

Is this true?

p	q	$p \to q$	$q \wedge (p \to q))$	$(q \wedge (p \to q)) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Not a valid rule!

Fallacy Example 1

Assume you are given the following two statements:

- “you will get a grade”
- “if you are in this class, you will get a grade”

Let p = “you are in this class”

Let q = “you will get a grade”

$$\begin{array}{l} \quad q \\ \quad \underline{p \rightarrow q} \\ \therefore p \end{array}$$

You CANNOT conclude that you are in this class

- You could be getting a grade for another class

Fallacy of denying the hypothesis

Consider the following:

$$\neg p$$
$$\underline{p \rightarrow q}$$
$$\therefore \neg q$$

Is this true?

p	q	$p \rightarrow q$	$\neg p \wedge (p \rightarrow q))$	$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Not a valid rule!

Fallacy Example 2

Assume you are given the following two statements:

- “you are not in this class”
- “if you are in this class, you will get a grade”

$$\begin{array}{l} \neg p \\ p \to q \\ \hline \therefore \neg q \end{array}$$

Let p = “you are in this class”

Let q = “you will get a grade”

You CANNOT conclude that you will not get a grade

- You could be getting a grade for another class

Rules of Inference for Universal Quantifier

Assume that we know that $\forall x\, P(x)$ is true

- Then we can conclude that $P(c)$ is true
  - Here c stands for some specific constant
- This is called "universal instantiation"

Assume that we know that $P(c)$ is true for any value of c

- Then we can conclude that $\forall x\, P(x)$ is true
- This is called "universal generalization"

Rules of Inference for Existential Quantifier

Assume that we know that $\exists x\, P(x)$ is true

- Then we can conclude that $P(c)$ is true for some value of c
- This is called "existential instantiation"

Assume that we know that $P(c)$ is true for some value of c

- Then we can conclude that $\exists x\, P(x)$ is true
- This is called "existential generalization"

Proofing Example 1

Given the hypotheses:

- "Linda, a student in this class, owns a red convertible." $C(\text{Linda})$ $R(\text{Linda})$
- "Everybody who owns a red convertible has gotten at least one speeding ticket" $\forall x\,(R(x) \rightarrow T(x))$

Can you conclude: "Somebody in this class has gotten a speeding ticket"? $\exists x\,(C(x) \wedge T(x))$

Proofing Example 1

1.	$\forall x\ (R(x) \rightarrow T(x))$	3rd hypothesis
2.	$R(\text{Linda}) \rightarrow T(\text{Linda})$	Universal instantiation using step 1
3.	$R(\text{Linda})$	2nd hypothesis
4.	$T(\text{Linda})$	Modes ponens using steps 2 & 3
5.	$C(\text{Linda})$	1st hypothesis
6.	$C(\text{Linda}) \wedge \mathrm{T}(\text{Linda})$	Conjunction using steps 4 & 5
7.	$\exists x\ (C(x) \wedge T(x))$	Existential generalization using step 6

Thus, we have shown that "Somebody in this class has gotten a speeding ticket"

Proofing Example 2

Given the hypotheses:

- "There is someone in this class who has been to France"
- "Everyone who goes to France visits the Eiffel"

$\exists x\, (C(x) \wedge F(x))$

$\forall x\, (F(x) \rightarrow E(x))$

Can you conclude: "Someone in this class has visited the Eiffel"?

$\exists x\, (C(x) \wedge E(x))$

Proofing Example 2

1.	$\exists x\,(C(x) \wedge F(x))$	1st hypothesis
2.	$C(y) \wedge F(y)$	Existential instantiation using step 1
3.	$F(y)$	Simplification using step 2
4.	$C(y)$	Simplification using step 2
5.	$\forall x\,(F(x) \rightarrow E(x))$	2nd hypothesis
6.	$F(y) \rightarrow E(y)$	Universal instantiation using step 5
7.	$E(y)$	Modus ponens using steps 3 & 6
8.	$C(y) \wedge E(y)$	Conjunction using steps 4 & 7
9.	$\exists x\,(C(x) \wedge E(x))$	Existential generalization using step 8

Thus, we have shown that “Someone in this class has visited the Eiffel”

Proofing Example 3

Show that these premises: “A student in this class has not read the book” and “Everyone in this class passed the first exam” have the conclusion: “Someone who passed the first exam has not read the book”

Let:

- C(x): “x is in the class”
- B(x) : “x has read the book”
- P(x) : “x passed the first exam”

Premises:

- $\exists x\ (C(x) \land \neg B(x))$
- $\forall x\ (C(x) \rightarrow P(x))$

Conclusion: $\exists x\ (P(x) \land \neg B(x))$

Proofing Example 3

1	$\exists x\,(C(x) \wedge \neg B(x))$	Premise 1
2	$C(a) \wedge \neg B(a)$	Existential instantiation from (1)
3	$C(a)$	Simplification from (2)
4	$\forall x\,(C(x) \rightarrow P(x))$	Premise 2
5	$C(a) \rightarrow P(a)$	Universal instantiation from (4)
6	$P(a)$	Modus ponens from (3) and (5)
7	$\neg B(a)$	Simplification from (2)
8	$P(a) \wedge \neg B(a)$	Conjunction from (6) and (7)
9	$\exists x\,(P(x) \wedge \neg B(x))$	Existential generalization from (8)

Proofing Example 4

Explain which rules of inference are used for each step

"David, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get high-paying job. Therefore, someone in this class can get a high-paying job."

Let:

- C(x): "x is in the class"
- J(x): "x knows how to write programs in JAVA"
- H(x): "x can get high-paying job"

Premises:

- C(David); J(David); $\forall x\,(J(x) \rightarrow H(x))$

Conclusion: $\exists x\,(C(x) \wedge H(x))$

Proofing Example 4

1	$\forall x\ (J(x) \to H(x))$	Premise 3
2	$\forall x\ (J(David) \to H(David))$	Universal instantiation from (1)
3	$J(David)$	Premise 2
4	$H(David)$	Modus ponens from (2) and (3)
5	$C(David)$	Premise 2
6	$C(David) \wedge H(David)$	Conjunction from (4) and (5)
7	$\exists x\ (C(x) \wedge H(x))$	Simplification from (2)

Proofing Example 5

"Somebody in this class enjoy whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

Let:

- C(x): "x is in the class"
- W(x): "x enjoys whale watching"
- P(x): "x cares about ocean pollution"

Premises:

- $\exists x\,(C(x) \wedge W(x))$
- $\forall x\,(W(x) \rightarrow P(x))$

- Conclusion: $\exists x\,(C(x) \wedge P(x))$

Proofing Example 5

1	$\exists x\ (C(x) \wedge W(x))$	Premise 1
2	$(C(a) \wedge W(a))$	Existensial instantiation from (1)
3	$W(a)$	Simplification from (2)
4	$\forall x\ (W(x) \rightarrow P(x))$	Premise 2
5	$W(a) \rightarrow P(a)$	Universal instantiation from (4)
6	$P(a)$	Modus Ponens from (3) and (5)
7	$C(a)$	Simplification from (2)
8	$(C(a) \wedge P(a))$	Conjunction fro (6) and (7)
9	$\exists x\ (C(x) \wedge P(x))$	Existensial generalization from (8)

How do you know which one to use?

Experience!

In general, use quantifiers with statements like “for all” or “there exists”