

### KUNCI JAWABAN REMIDI UTS MATDIS

- The inhabitants of Joy Island consist of knights and knave. Knights always tell the truth while knaves always lie. You encounter two people  $A$  and  $B$ . Determine if possible, what  $A$  and  $B$  if they address you in the ways described:  $A$  says "The two of us are both knight" and  $B$  says " $A$  is a knave."

$A$  says "The two of us are both knight" and  $B$  says " $A$  is a knave."

$P(x)$ :  $x$  is a knight

$\neg P(x)$ :  $x$  is a knave

Suppose  $A$  is a knight.

$P(A) \leftrightarrow T$

What  $A$  says must be true

$P(A) \wedge P(B) \leftrightarrow T$

$P(B) \leftrightarrow T$

**Impossible**

However,  $B$  says

$\neg P(A) \leftrightarrow T$

$P(A) \leftrightarrow F$

$A$  is a knave and what  $A$  says is false.

$\neg P(A) \leftrightarrow T$

$P(A) \wedge P(B) \leftrightarrow F \wedge P(B) \leftrightarrow F$

$B$  is a knight because his statement ( $A$  is a knave) is true.

**Answer:**

$A$  is a knave.

$B$  is a knight.

- Show that  $\neg p \leftrightarrow \neg q$  is logically equivalent with  $p \leftrightarrow q$  by using Table of Equivalence Laws.

$p \leftrightarrow q$

$\equiv \neg p \leftrightarrow \neg q$

$= (p \rightarrow q) \wedge (q \rightarrow p)$

Definition of Bi conditional

$= (\neg p \vee q) \wedge (\neg q \vee p)$

Definition of Implication

$= (p \vee \neg q) \wedge (q \vee \neg p)$

Commutative Law

$= (\neg p \rightarrow \neg q) \wedge (\neg q \rightarrow \neg p)$

Definition of Implication

$= \neg p \leftrightarrow \neg q$

$\equiv \neg p \leftrightarrow \neg q$

- A) yes      B) no      C) no

- Determine whether  $A \times B \times C$  is equivalent to  $(A \times B) \times C$  and explain why.  
They aren't equivalent since the elements of  $A \times B \times C$  consist of 3 tuples  $(a, b, c)$  where  $a \in A$ ,  $b \in B$ , and  $c \in C$ ; whereas the elements of  $(A \times B) \times C$  look like  $((a, b), c)$  - ordered pairs, the first coordinate is again an ordered pair.

- Prove that if  $x$  and  $y$  are real numbers, then  $\max(x, y) + \min(x, y) = x + y$ .  
If  $x < y$ , then  $\max(x, y) + \min(x, y) = y + x = x + y$ .  
If  $x \geq y$ , then  $\max(x, y) + \min(x, y) = x + y$ .  
Because these are the only two cases, the equality always holds.