Homework Assignment
Discrete Mathematics (3 Credits)
Information Systems Department FTIf – ITS

1. The inhabitants of Jeju Island consist of knights and knaves. Knights always tell the truth while knaves always lie. You encounter two people *A* and *B*. Determine if possible, what *A* and *B* if they address you in the ways described: *A* says "*B is a knight*" and *B* says "*The two of us are opposite types*"?

2. Encode the following statement using quantifiers and propositional functions (of one or two variables). Clearly label what your propositional functions and variables represent.

   "*There is a building on campus of some colleges in Indonesia in which every room is painted white*."

3. Show that each of these conditional statements is tautology without using truth table.
   a) $[\neg p \wedge (p \vee q)] \rightarrow q$
   b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
   c) $[p \wedge (p \rightarrow q)] \rightarrow q$
   d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

4. Consider the following statements.
   a) If Dominic goes to the racetrack, then Helen will be mad.
   b) If Ralph plays cards all night, then Carmela will be mad.
   c) If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified.
   d) Veronica has not heard from either of these two clients.

   From these, can we conclude that *Dominic didn't make it to the racetrack* and *Ralph didn't play cards all night*?
   Write each of these statements in symbolic form. Clearly label what your variables represent! Then establish the validity of the conclusion. You must clearly label which rule of inference is used for each step.

5. An old man famous for his bizarre sense of humor and love of logic puzzles left the following clues as to the location of the treasure. The treasure can only be in one place.
   a) If the house is next to a lake, then the treasure is in the kitchen
   b) If the house is not next to a lake or the treasure is buried under the flagpole, then the tree in the front yard is an elm and the tree in the back yard is not an oak
   c) If the treasure is in the garage, then the tree in the back yard is not an oak
   d) If the treasure is not buried under the flagpole, then the tree in the front yard is not an elm
   e) The treasure is not in the kitchen

Using rules of inference, determine where the treasure is hidden. You must clearly state what your propositional variable represent and which rule of inference is used for each step.

6. Consider the statement: "The sum of any even integer and any odd integer is odd"
   a) Restate this (in English) as a conditional
   b) Prove it via an indirect proof XOR a proof by contradiction

7. Prove that if n is an integer, these four statements are equivalent:
   (i) n is even, (ii) n + 1 is odd,
   (iii) 3n + 1 is odd, (iv) 3n is even.

8. Suppose that a and b are odd integers with $a \neq b$. Show that there is a unique integer c such that $|a - c| = |b - c|$.

9. The **symmetric difference** of A and B, denoted by $A \oplus B$ is the set containing those elements in either A or B, but not in both A and B.
   a) Find the symmetric difference of the set of Information Systems majors and the set of Informatics majors at this school.
   b) Draw a Venn diagram for the symmetric difference of the sets A and B.
   c) Show that $A \oplus B = (A \cup B) - (A \cap B)$.

10. Determine whether each of these statements is true or false.
    a) $0 \in \varnothing$
    b) $\varnothing \in \{0\}$
    c) $\{0\} \subset \varnothing$
    d) $\varnothing \subset \{0\}$
    e) $\{0\} \in \{0\}$
    f) $\{0\} \subset \{0\}$
    g) $\{\varnothing\} \subseteq \{\varnothing\}$