Refreshment Quiz

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1) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

2) Prove that if $m + n$ and $n + p$ are even integers, where m, n, and p are integers, then $m + p$ is even. What kind of proof did you use?

Answer

1) Proof the contraposition: If $x < 1$ and $y < 1$, then $x + y < 2$
   - Assume that $x < 1$ and $y < 1$ are true
   - Adding these two inequalities, we obtain $x + y <2$, which is the negation of $x + y \geq 2$.
   - Our proof by contraposition succeeded; we have proved the statement if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$

2) Direct proof:
   - Suppose that $m + n$ and $n + p$ are even.
   - Then $m + n = 2s$ for some integer s and $n + p = 2t$ for some integer t.
   - If we add these, we get $m + p + 2n = 2s + 2t$.
   - Subtracting $2n$ from both sides and factoring, we have

     $$m + p = 2s + 2t - 2n = 2(s + t - n).$$

   - Because we have written $m + p$ as 2 times an integer, we conclude that $m + p$ is even.

Refreshment Quiz

1) Use a direct proof to show that every odd integer is the difference of two squares?

2) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

Answer

1) Direct proof:
   - Assume that n is odd (p is true). Because n is odd, we can write $n = 2k + 1$ for some integer k.
   - Then $(k+1)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k + 1 = n$ (odd integer) – (q is true)
   - We have proved that every odd integer is the difference of two squares.

2) Proof the contraposition: If $x < 1$ and $y < 1$, then $x + y < 2$
   - Assume that $x < 1$ and $y < 1$ are true ($-q$ is true)
   - Adding these two inequalities, we obtain $x + y < 2$, which is the negation of $x + y \geq 2$ ($-p$ is true)
   - Our proof by contraposition succeeded; we have proved the statement if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$

Refreshment Quiz

1) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

2) Show that if n is an odd integer, then there is a unique integer k such that n is the sum of $k - 2$ and $k + 3$.

Answer

1) Proof the contraposition: If $x < 1$ and $y < 1$, then $x + y < 2$
   - Assume that $x < 1$ and $y < 1$ are true (-*q* is true)
   - Adding these two inequalities, we obtain $x + y < 2$, which is the negation of $x + y \geq 2$ (-*p* is true)
   - Our proof by contraposition succeeded; we have proved the statement if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$

2) Uniqueness proof:

   Constructive Existence proof:

   $n = (k - 2) + (k + 3)$
   $n = 2k + 1$
   $k = (n - 1)/2$
   Because n is odd, then $n - 1$ is even, so k is an integer.
   This is the one and only value of k that makes the equation true, in other words k is a unique solution for the equation.

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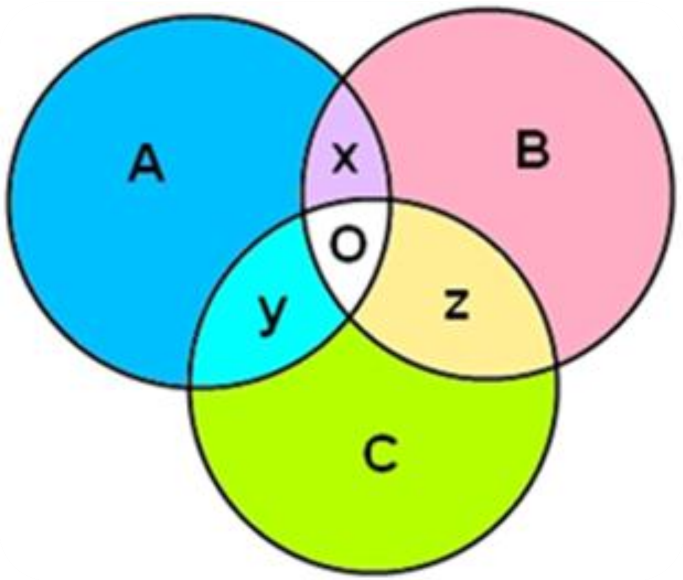

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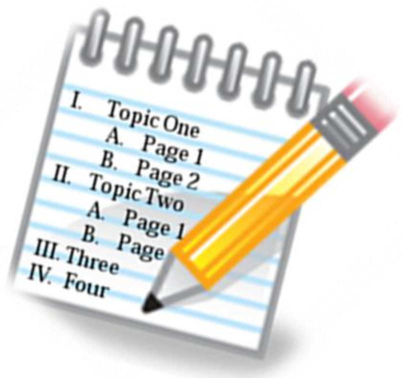
Outline

- What is a set?
- Set properties
- Specifying a set
- Often used sets
- The universal set
- Venn diagrams
- Sets of sets
- The empty set
- Set equality
- Subsets and Proper subsets
- Set cardinality
- Power sets
- Tuples
- Cartesian products
- Sets operation:
  - Union
  - Intersection
  - Disjoint
  - Difference
  - Symmtric difference
  - Complement
  - Set Identities
  - How to proof set identities



What is a set?

A set is a group of "objects"

- People in a class: { Alice, Bob, Chris }
- Colors of a rainbow: { red, orange, yellow, green, blue, purple }
- States of matter { solid, liquid, gas, plasma }
- States in the US: { Alabama, Alaska, Virginia, … }
- Sets can contain non-related elements: { 3, a, red, Virginia }

Although a set can contain (almost) anything, we will most often use sets of numbers

- All positive numbers less than or equal to 5: {1, 2, 3, 4, 5}
- A few selected real numbers: { 2.1, π, 0, -6.32, e }

Set properties

Order does not matter
- We often write them in order because it is easier for humans to understand it that way
- {1, 2, 3, 4, 5} is equivalent to {3, 5, 2, 4, 1}

Sets are notated with curly brackets { }

Sets do not have duplicate elements
- Consider the set of vowels in the alphabet.
  - It makes no sense to list them as {a, a, a, e, i, o, o, o, o, o, u}
  - What we really want is just {a, e, i, o, u}
- Consider the list of students in this class
  - Again, it does not make sense to list somebody twice

Note that a list is like a set, but order does matter and duplicate elements are allowed
- We won't be studying lists much in this class

Specifying a set

Sets are usually represented by a capital letter (A, B, S, etc.)

Elements are usually represented by an italic lower-case letter (*a*, *x*, *y*, etc.)

Easiest way to specify a set is to list all the elements: A = {1, 2, 3, 4, 5}

- Not always feasible for large or infinite sets

Can use an ellipsis (…) when general pattern of the elements is obvious: B = {0, 1, 2, 3, …}

- Can cause confusion.
  - Consider the set C = {3, 5, 7, …} What comes next?
  - If the set is all odd integers greater than 2, it is 9
  - If the set is all prime numbers greater than 2, it is 11

Specifying a set (cont.)

Can use set-builder notation

- D = {x | x is prime and $x > 2$}
- E = {x | x is odd and $x > 2$}
- The vertical bar means “such that”
- Thus, set D is read (in English) as: “all elements x such that x is prime and x is greater than 2”

A set is said to “contain” the various “members” or “elements” that make up the set

- If an element x is a member of (or an element of) a set S, we use then notation $x \in S$
  - $4 \in \{1, 2, 3, 4\}$
- If an element is not a member of (or an element of) a set S, we use the notation $x \notin S$
  - $7 \notin \{1, 2, 3, 4\}$
  - Virginia $\notin \{1, 2, 3, 4\}$

Often used sets

N = {0, 1, 2, 3, …} is the set of natural numbers

Z = {…, -2, -1, 0, 1, 2, …} is the set of integers

Z^+ = {1, 2, 3, …} is the set of positive integers (a.k.a whole numbers)

- Note that people disagree on the exact definitions of whole numbers and natural numbers

Q = $\{p/q \mid p \in Z, q \in Z, q \neq 0\}$ is the set of rational numbers

- Any number that can be expressed as a fraction of two integers (where the bottom one is not zero)

R is the set of real numbers

The universal set

U is the universal set – the set of all of elements (or the "universe") from which given any set is drawn

- For the set {-2, 0.4, 2}, ***U*** would be the real numbers
- For the set {0, 1, 2}, ***U*** could be the natural numbers (zero and up), the integers, the rational numbers, or the real numbers, depending on the context
- For the set of the students in this class, ***U*** would be all the students in the University (or perhaps all the people in the world)
- For the set of the vowels of the alphabet, ***U*** would be all the letters of the alphabet
- To differentiate ***U*** from $\cup$ (which is a set operation), the universal set is written in a different font (and in bold and italics)

Venn diagrams

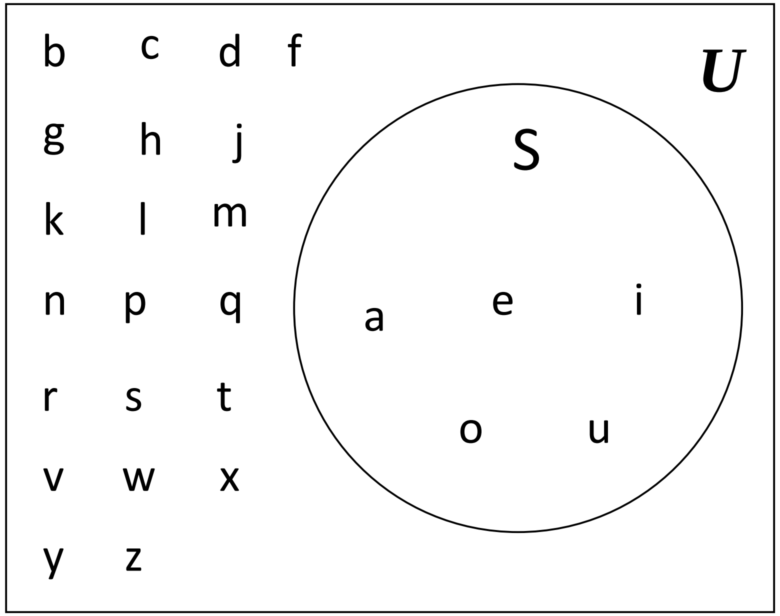
Represents sets graphically

- The box represents the universal set
- Circles represent the set(s)

Consider set *S*, which is the set of all vowels in the alphabet

The individual elements are usually

not written in a Venn diagram



Sets of sets

Sets can contain other sets

- S = { {1}, {2}, {3} }
- T = { {1}, {{2}}, {{{3}}} }
- V = { {{1}, {{2}}}, {{{3}}}, {{1}, {{2}}, {{{3}}}} }
  - V has only 3 elements!

Note that 1 ≠ {1} ≠ {{1}} ≠ {{{1}}}

- They are all different

The empty set

If a set has zero elements, it is called the empty (or null) set

- Written using the symbol $\varnothing$
- Thus, $\varnothing = \{ \}$ ← VERY IMPORTANT
- If you get confused about the empty set in a problem, try replacing $\varnothing$ by $\{ \}$

As the empty set is a set, it can be an element of other sets

- $\{ \varnothing, 1, 2, 3, x \}$ is a valid set

Note that $\varnothing \neq \{ \varnothing \}$

- The first is a set of zero elements
- The second is a set of 1 element (that one element being the empty set)

Replace $\varnothing$ by $\{ \}$, and you get: $\{ \} \neq \{ \{ \} \}$

- It's easier to see that they are not equal that way

Set equality

Two sets are equal if they have the same elements

- $\{1, 2, 3, 4, 5\} = \{5, 4, 3, 2, 1\}$
  - Remember that order does not matter!
- $\{1, 2, 3, 2, 4, 3, 2, 1\} = \{4, 3, 2, 1\}$
  - Since duplicate elements are not allowed!

Two sets are not equal if they do not have the same elements

- $\{1, 2, 3, 4, 5\} \neq \{1, 2, 3, 4\}$

Subsets

If all the elements of a set S are also elements of a set T, then S is a subset of T

- For example, if S = {2, 4, 6} and T = {1, 2, 3, 4, 5, 6, 7}, then S is a subset of T
- This is specified by $S \subseteq T$
  - Or by $\{2, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6, 7\}$

If S is not a subset of T, it is written as such: $S \not\subseteq T$

- For example, $\{1, 2, 8\} \not\subseteq \{1, 2, 3, 4, 5, 6, 7\}$

Note that any set is a subset of itself!

- Given set S = {2, 4, 6}, since all the elements of S are elements of S, S is a subset of itself
- This is kind of like saying 5 is less than or equal to 5
- Thus, for any set S, $S \subseteq S$

Subsets (cont.)

The empty set is a subset of *all* sets (including itself!)

- Recall that all sets are subsets of themselves

All sets are subsets of the universal set

A horrible way to define a subset:

- $\forall x\ (\ x \in A \rightarrow x \in B\ )$
- English translation: for all possible values of x, (meaning for all possible elements of a set), if x is an element of A, then x is an element of B

Proper Subsets

If S is a subset of T, and S is not equal to T, then S is a proper subset of T.

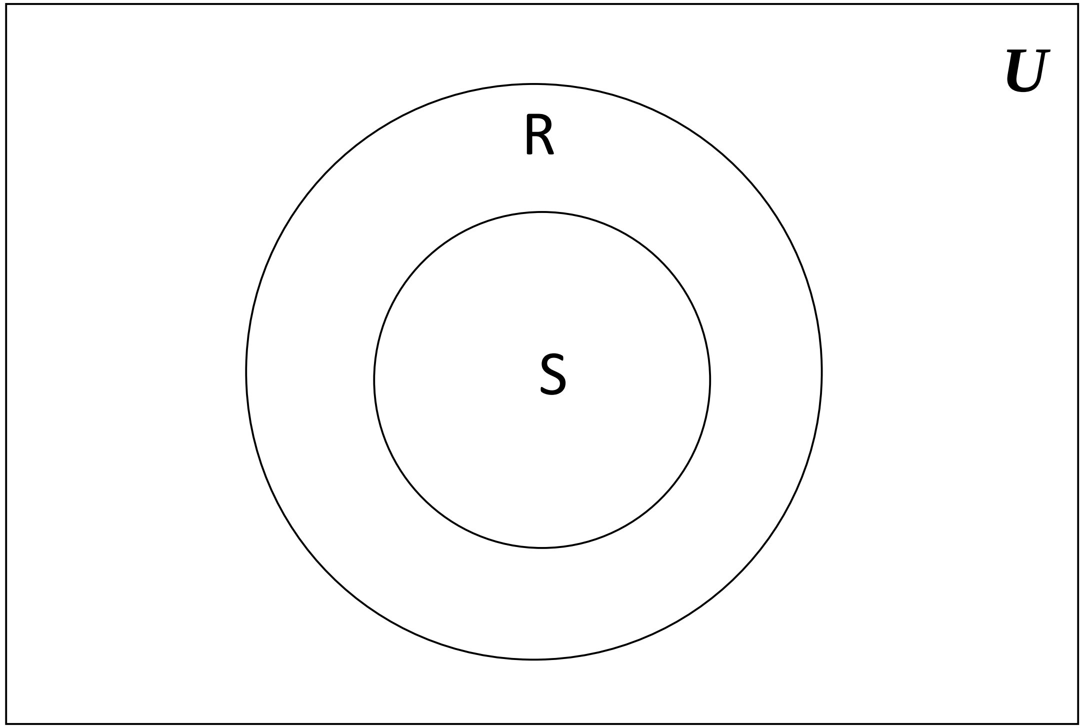
- Let T = {0, 1, 2, 3, 4, 5}
- If S = {1, 2, 3}, S is not equal to T, and S is a subset of T
- A proper subset is written as $S \subset T$
- Let R = {0, 1, 2, 3, 4, 5}. R is equal to T, and thus is a subset (but not a proper subset) of T
  - Can be written as: $R \subseteq T$ and $R \not\subset T$ (or just R = T)
- Let Q = {4, 5, 6}. Q is neither a subset or T nor a proper subset of T

The difference between "subset" and "proper subset" is like the difference between "less than or equal to" and "less than" for numbers.

The empty set is a proper subset of all sets other than the empty set (as it is equal to the empty set).

Proper subsets: Venn diagram

$$S \subset R$$



Set cardinality

The cardinality of a set is the number of elements in a set.

- Written as $|A|$

Examples

- Let $R = \{1, 2, 3, 4, 5\}$. Then $|R| = 5$
- $|\varnothing| = 0$
- Let $S = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then $|S| = 4$

This is the same notation used for vector length in geometry

A set with one element is sometimes called a singleton set

Power sets

Given the set S = {0, 1}. What are all the possible subsets of S?

- They are: ∅ (as it is a subset of all sets), {0}, {1}, and {0, 1}
- The power set of S (written as P(S)) is the set of all the subsets of S
- P(S) = { ∅, {0}, {1}, {0,1} }
  - Note that |S| = 2 and |P(S)| = 4

Let T = {0, 1, 2}. The P(T) = { ∅, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }

- Note that |T| = 3 and |P(T)| = 8

P(∅) = { ∅ }

- Note that |∅| = 0 and |P(∅)| = 1

If a set has *n* elements, then the power set will have 2^n elements

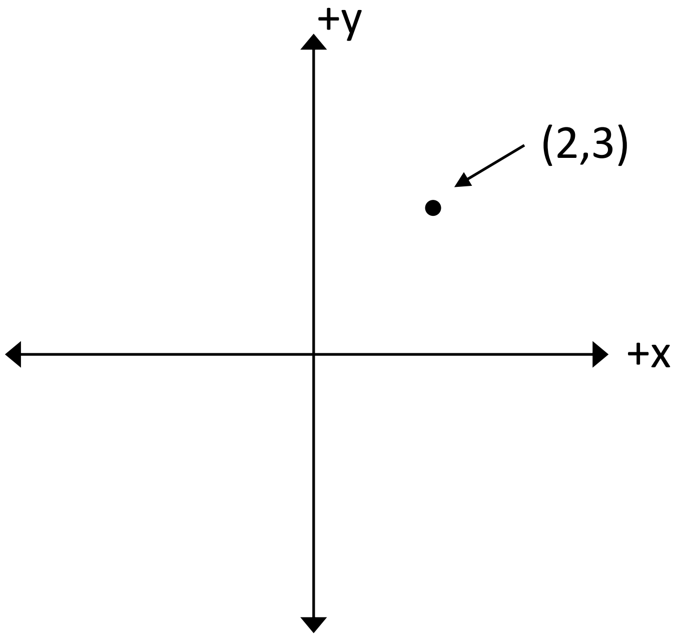
Tuples

In 2-dimensional space, it is a (x, y) pair of numbers to specify a location

In 3-dimensional (1,2,3) is not the same as (3,2,1) – space, it is a (x, y, z) triple of numbers

In n-dimensional space, it is a n-tuple of numbers

- Two-dimensional space uses pairs, or 2-tuples
- Three-dimensional space uses triples, or 3-tuples



Note that these tuples are **ordered**, unlike sets

- the x value has to come first

Cartesian products

A Cartesian product is a set of all ordered n-tuples where each "part" is from a given set

- Denoted by A x B, and uses parenthesis (not curly brackets)
- For example, 2-D Cartesian coordinates are the set of all ordered pairs **Z** x **Z**
  - Recall **Z** is the set of all integers
  - This is all the possible coordinates in 2-D space
- Example: Given A = { a, b } and B = { 0, 1 }, what is their Cartiesian product?
  - C = A x B = { (a,0), (a,1), (b,0), (b,1) }

Note that Cartesian products have only 2 parts in these examples (later examples have more parts)

Formal definition of a Cartesian product:

- $A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$

Cartesian products (cont.)

All the possible grades in this class will be a Cartesian product of the set S of all the students in this class and the set G of all possible grades

- Let S = { Alice, Bob, Chris } and G = { A, B, C }
- D = { (Alice, A), (Alice, B), (Alice, C), (Bob, A), (Bob, B), (Bob, C), (Chris, A), (Chris, B), (Chris, C) }
- The final grades will be a subset of this: { (Alice, C), (Bob, B), (Chris, A) }
  - Such a subset of a Cartesian product is called a relation (more on this later in the course)

There can be Cartesian products on more than two sets

A 3-D coordinate is an element from the Cartesian product of Z x Z x Z

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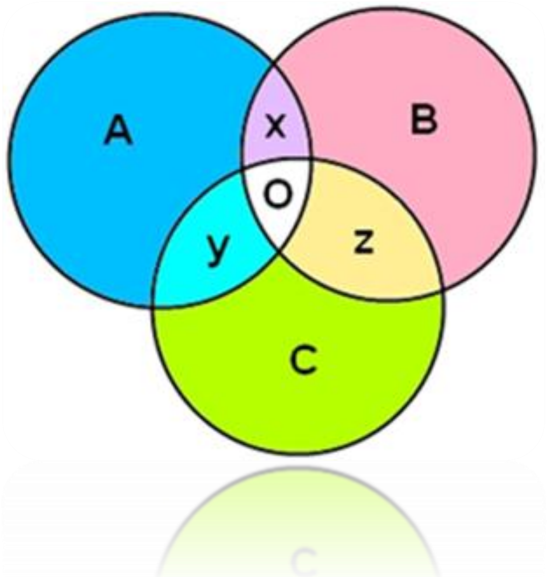

Sets Operations

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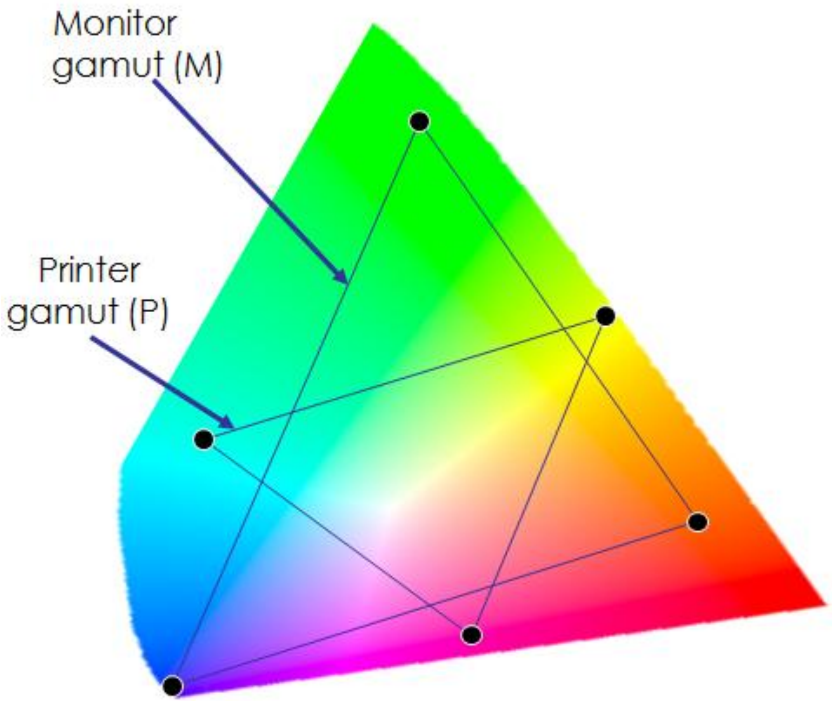
Sets of Colors

Pick any 3 “primary” colors

Triangle shows mixable color range (gamut) – the set of colors



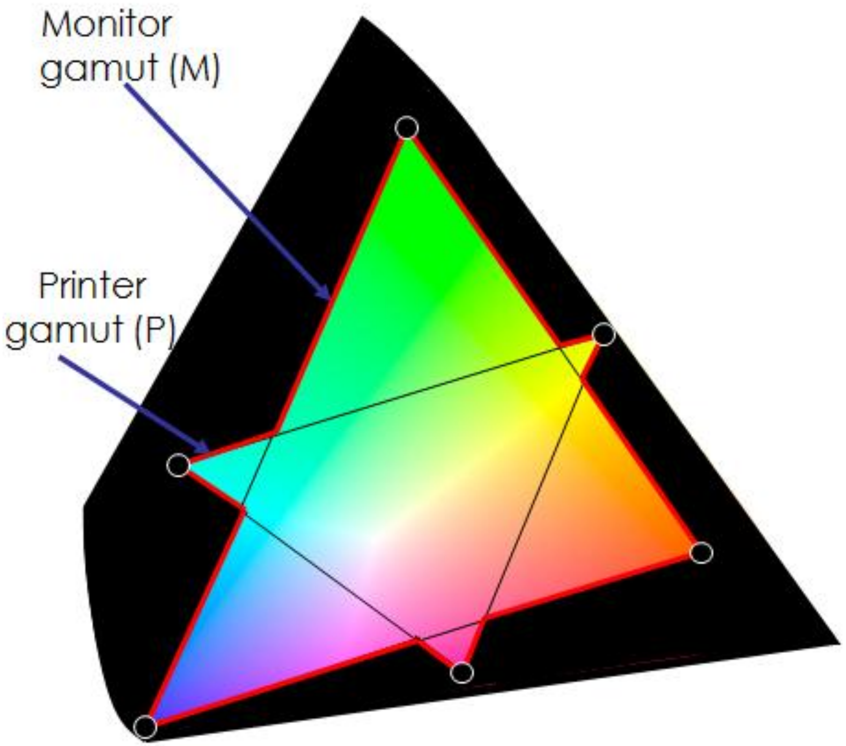
Set operations: Union (Gabungan)

A union of the sets contains all the elements in **EITHER** set

Union symbol is usually a ∪

Example:

- C = M ∪ P



Set operations: Union (cont.)

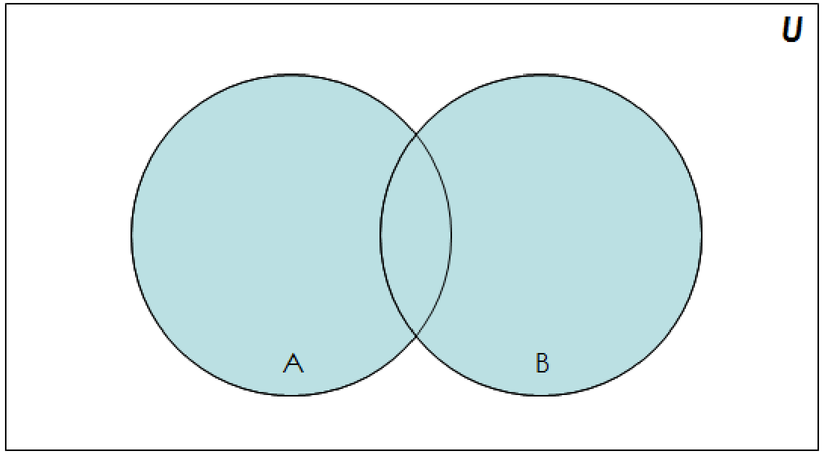
Formal definition for the union of two sets:

$A \cup B = \{ x \mid x \in A$ **or** $x \in B \}$

Further examples

- $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
- {New York, Washington} ∪ {3, 4} = {New York, Washington, 3, 4}
- $\{1, 2\} \cup \varnothing = \{1, 2\}$

A U B



Properties of the union operation

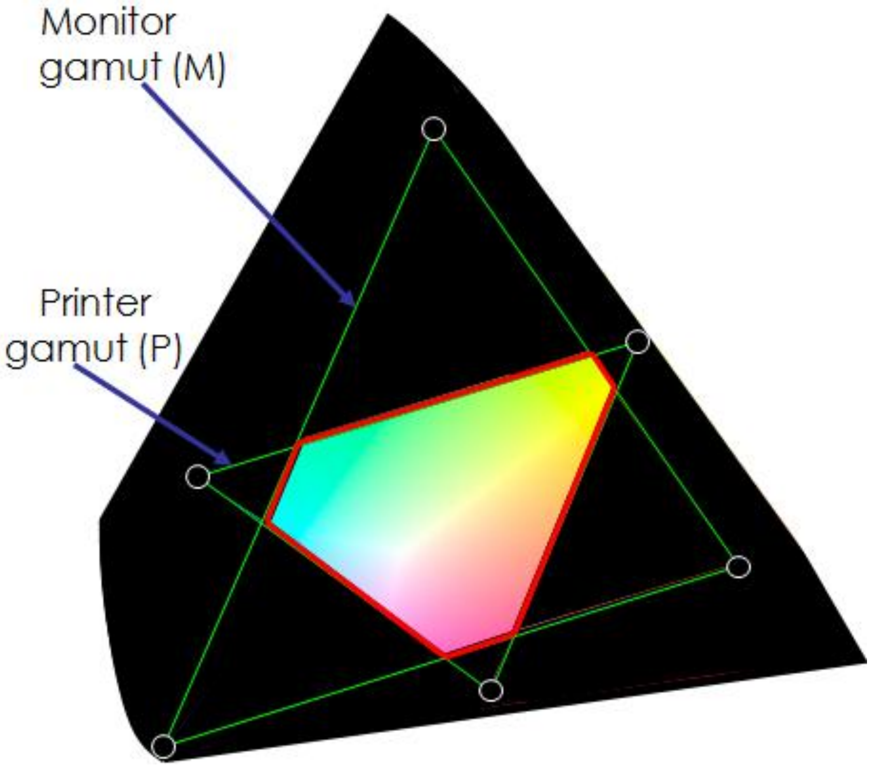
$A \cup \varnothing = A$	Identity law
$A \cup \boldsymbol{U} = \boldsymbol{U}$	Domination law
$A \cup A = A$	Idempotent law
$A \cup B = B \cup A$	Commutative law
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative law

Set operations: Intersection (Irisan)

An intersection of the sets contains all the elements in **BOTH** sets

Intersection symbol is a ∩

Example:
$C = M \cap P$

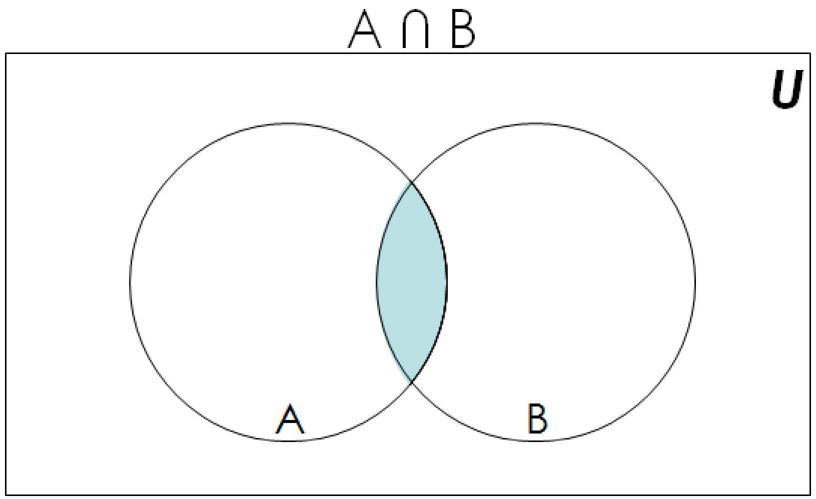


Set operations: Intersection

Formal definition for the intersection of two sets: $A \cap B = \{ x \mid x \in A \textbf{ and } x \in B \}$

Further examples

- $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
- $\{\text{New York, Washington}\} \cap \{3, 4\} = \varnothing$
  - No elements in common
- $\{1, 2\} \cap \varnothing = \varnothing$
  - Any set intersection
    with the empty set
    yields the empty set



Properties of the intersection operation

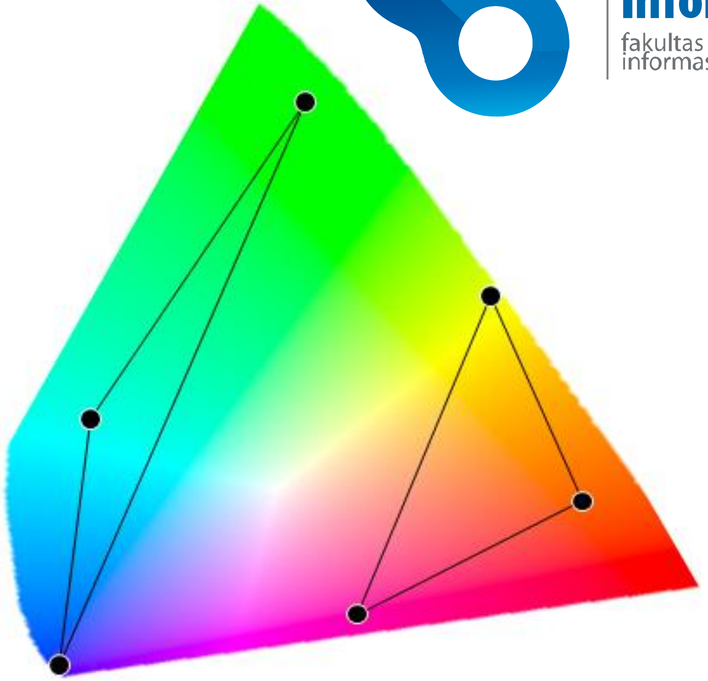
$A \cap \boldsymbol{U} = A$	Identity law
$A \cap \emptyset = \emptyset$	Domination law
$A \cap A = A$	Idempotent law
$A \cap B = B \cap A$	Commutative law
$A \cap (B \cap C) = (A \cap B) \cap C$	Associative law

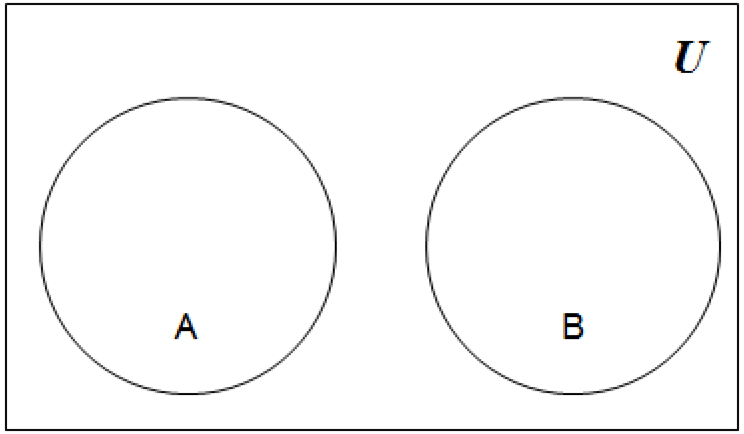
Disjoint sets

Two sets are disjoint if they have NO elements in common

Formally, two sets are disjoint if their intersection is the empty set

Another example: the set of the even numbers and the set of the odd numbers





Disjoint sets (cont.)

Formal definition for disjoint sets: two sets are disjoint if their intersection is the empty set

Further examples

- {1, 2, 3} and {3, 4, 5} are not disjoint
- {New York, Washington} and {3, 4} are disjoint
- {1, 2} and ∅ are disjoint
  - Their intersection is the empty set
- ∅ and ∅ are disjoint!
  - Their intersection is the empty set

Set operations: Difference (Selisih)

A difference of two sets is the elements in one set that are **NOT** in the other

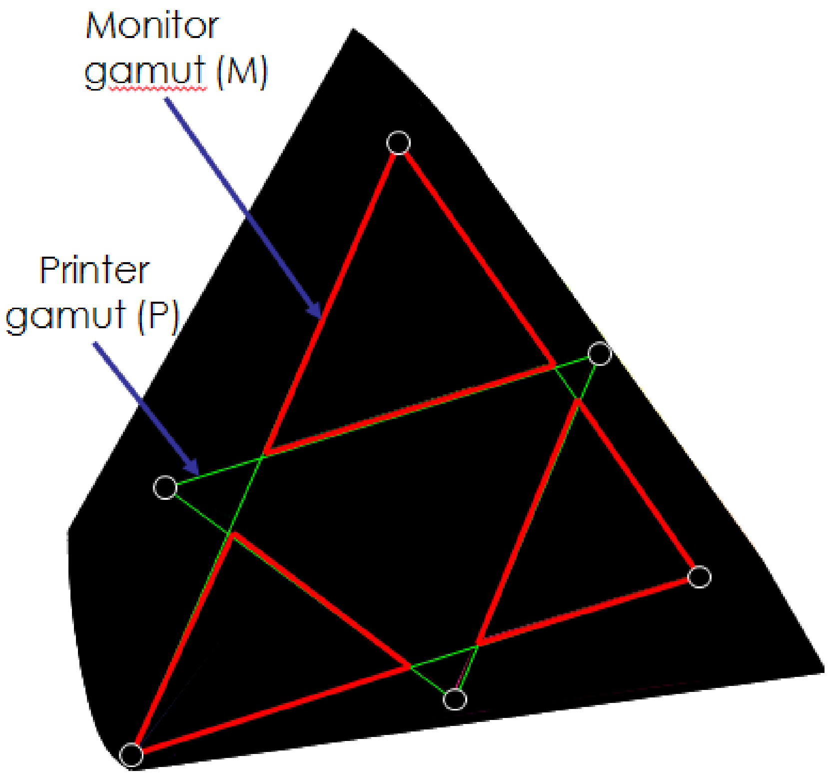
Difference symbol is a **minus sign**

Example:

- C = M – P

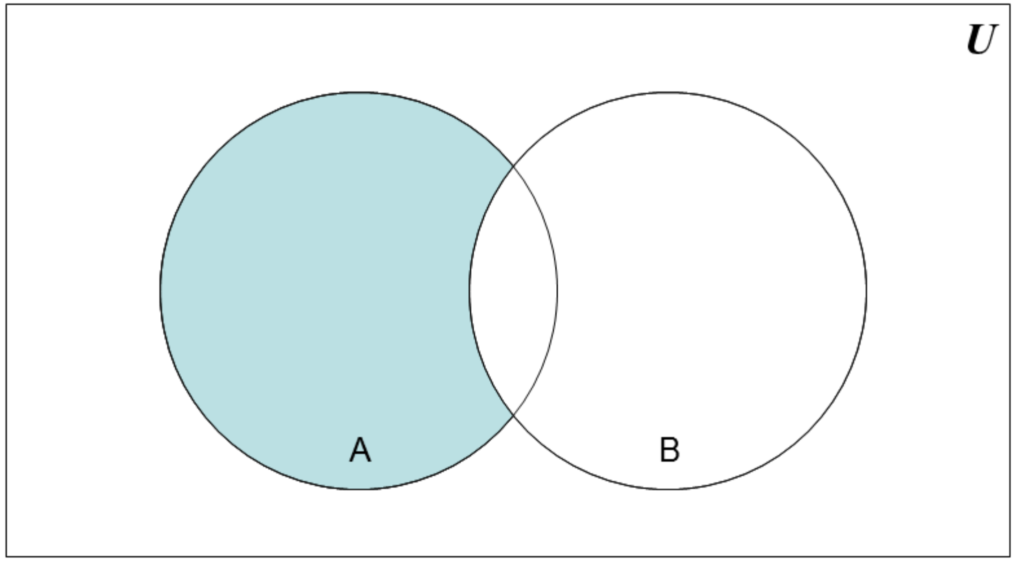
Also visa-versa:

- C = P – M

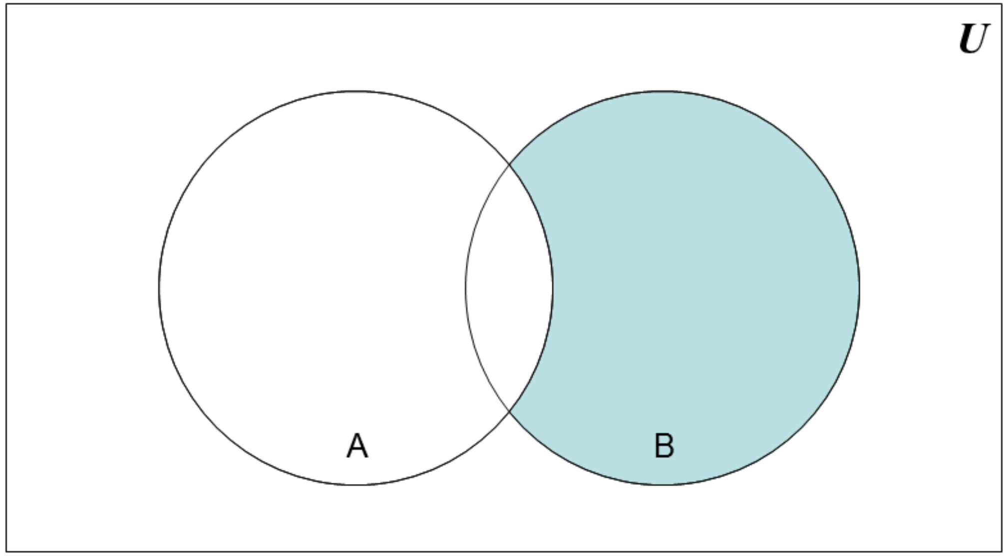


Set operations: Difference (cont.)

A - B



B - A



Set operations: Difference (cont.)

Formal definition for the difference of two sets:

$A - B = \{ x \mid x \in A \textbf{ and } x \notin B \}$

$A - B = A \cap \overline{B}$ ← Important!

Further examples

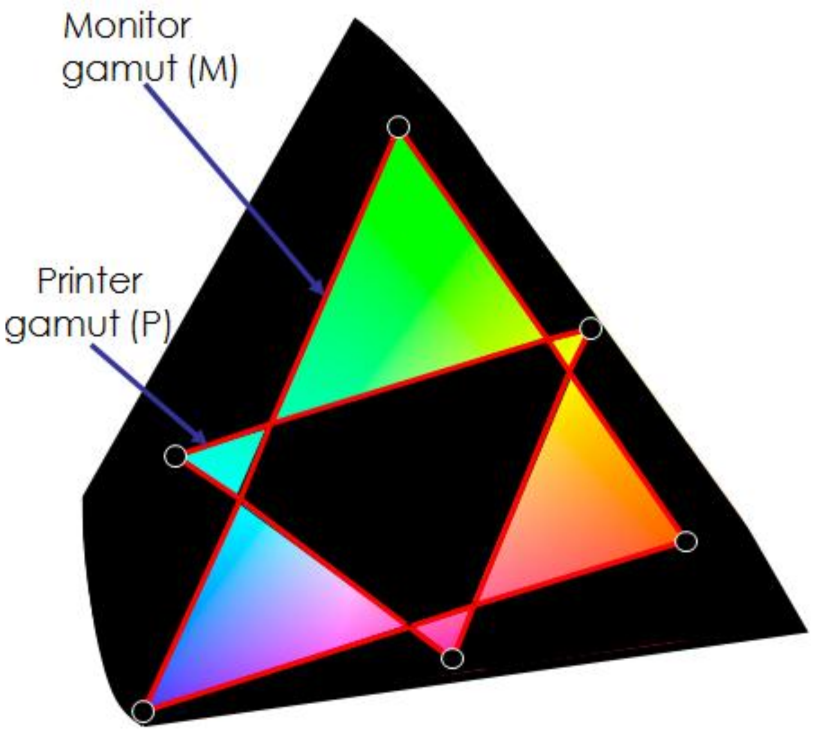
- {1, 2, 3} - {3, 4, 5} = {1, 2}
- {New York, Washington} - {3, 4} = {New York, Washington}
- {1, 2} - ∅ = {1, 2}
  - The difference of any set S with the empty set will be the set S

Set operations: Symmetric Difference

A symmetric difference of the sets contains all the elements in either set but **NOT both**

Symmetric diff. symbol is a ⊕

Example: $C = M \oplus P$



Set operations: Symmetric Difference

Formal definition for the symmetric difference of two sets:

$A \oplus B = \{ x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$

$A \oplus B = (A \cup B) - (A \cap B)$ ← Important!

Further examples

- $\{1, 2, 3\} \oplus \{3, 4, 5\} = \{1, 2, 4, 5\}$
- {New York, Washington} $\oplus$ {3, 4} = {New York, Washington, 3, 4}
- $\{1, 2\} \oplus \varnothing = \{1, 2\}$
  - The symmetric difference of any set S with the empty set will be the set S

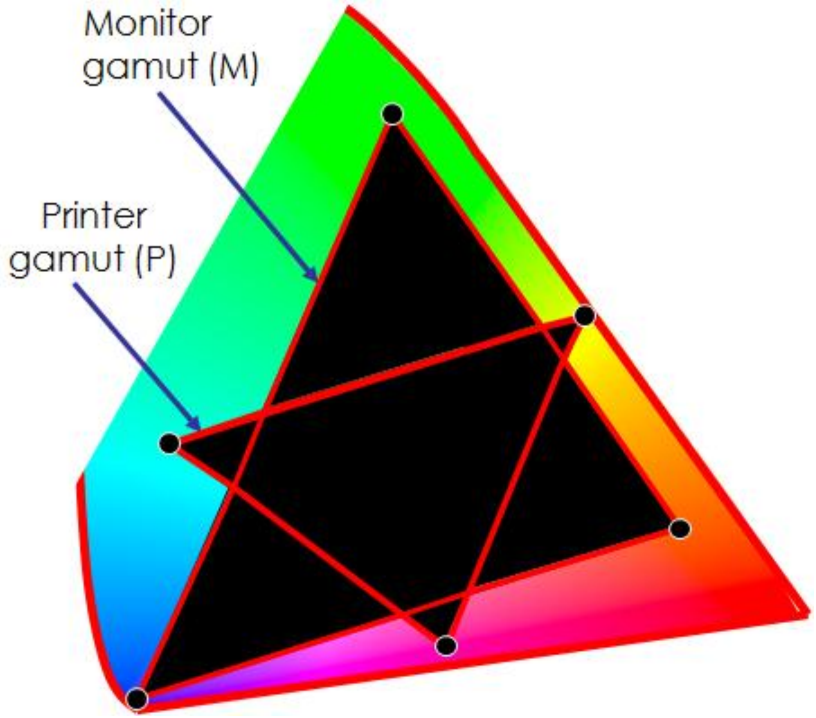
Complement sets

A complement of a set is all the elements that are NOT in the set

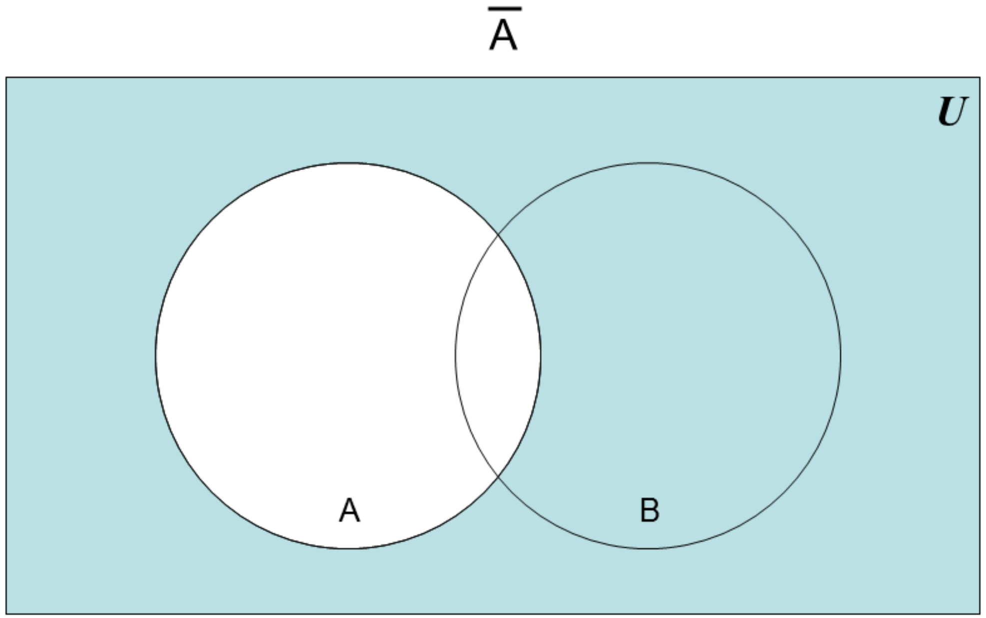
Complement symbol is a bar above the set name: $\overline{P}$ or $\overline{M}$

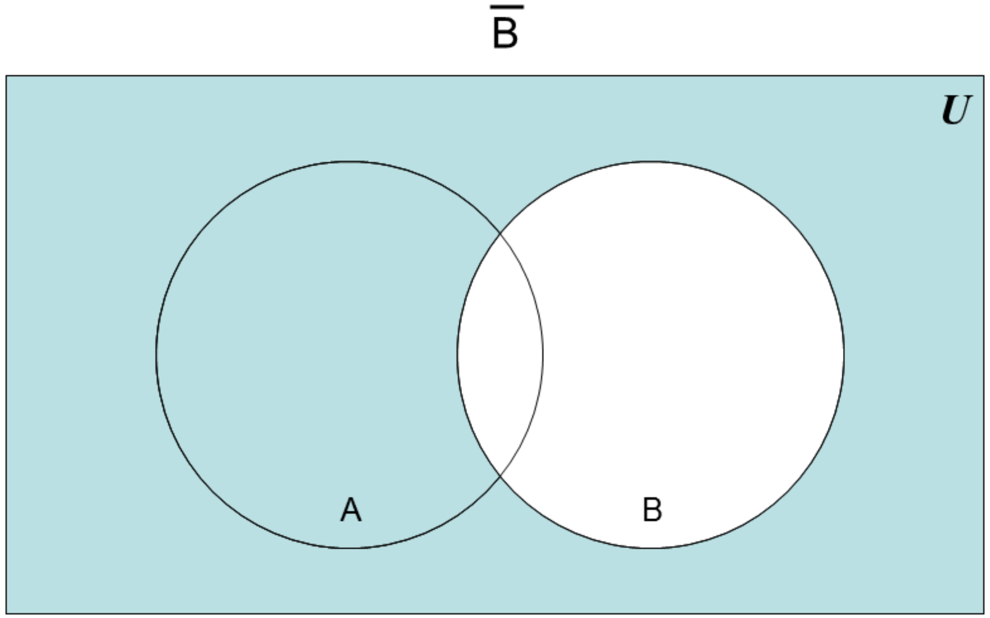
Alternative symbol:

- P^C or M^C



Complement sets (cont.)





Complement sets (cont.)

Formal definition for the complement of a set: $\overline{A} = \{ x \mid x \notin A \} = A^c$

- Or $\boldsymbol{U} - A$, where $\boldsymbol{U}$ is the universal set

Further examples (assuming $\boldsymbol{U} = \mathbf{Z}$)

- $\overline{\{1, 2, 3\}} = \{ \ldots, -2, -1, 0, 4, 5, 6, \ldots \}$

Properties of complement sets

- $\overline{\overline{A}} = A$ — Complementation law
- $A \cup \overline{A} = \boldsymbol{U}$ — Complement law
- $A \cap \overline{A} = \varnothing$ — Complement law

Set identities

Set identities are basic laws on how set operations work

- Many have already been introduced on previous slides

Just like logical equivalences!

- Replace $\cup$ with $\vee$
- Replace $\cap$ with $\wedge$
- Replace $\emptyset$ with F
- Replace $\boldsymbol{U}$ with T

Recap of set identities

$A \cup \varnothing = A$ $A \cap U = A$	Identity Law	$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent Law	$(A^c)^c = A$	Complement Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Law	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
$A \cup (B \cup C)$ $= (A \cup B) \cup C$ $A \cap (B \cap C)$ $= (A \cap B) \cap C$	Associative Law	$A \cap (B \cup C) =$ $(A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) =$ $(A \cup B) \cap (A \cup C)$	Distributive Law
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Law	$A \cup A^c = U$ $A \cap A^c = \varnothing$	Complement Law

How to prove a set identity?

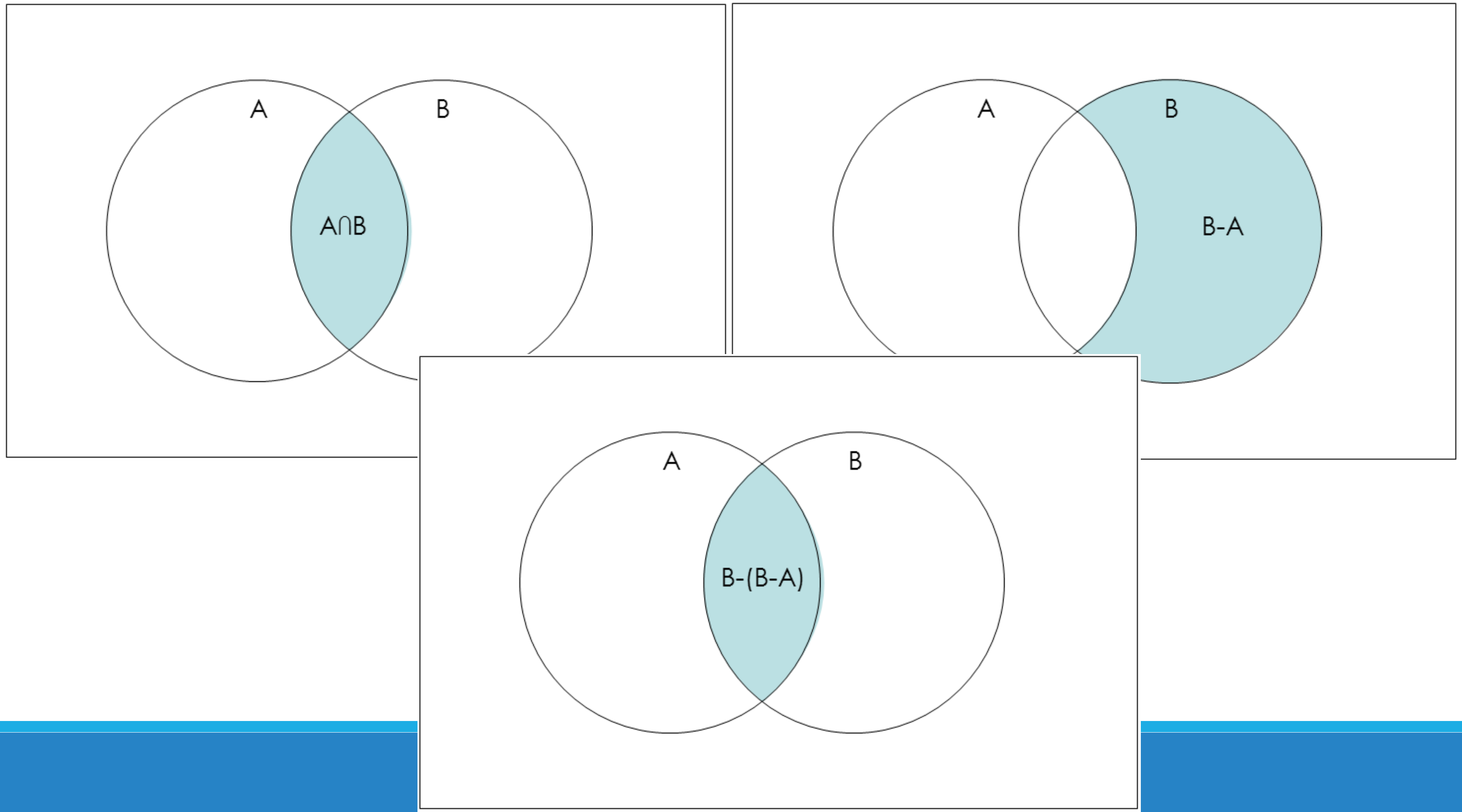
For example: $A \cap B = B - (B - A)$

There are four methods to prove:

- Use the basic set identities
- Use membership tables
- Prove each set is a subset of each other
  - This is like proving that two numbers are equal by showing that each is less than or equal to the other
- Use set builder notation and logical equivalences

What we are going to prove?

$$A \cap B = B - (B - A)$$



Proof by Set Identities

Prove that A ∩ B = B - (B - A)

$A \cap B = B\text{-}(B \cap \overline{A})$	Definition of difference
$= B \cap \overline{(B \cap \overline{A})}$	Definition of difference
$= B \cap (\overline{B} \cup \overline{\overline{A}})$	DeMorgan's law
$= B \cap (\overline{B} \cup A)$	Complementation law
$= (B \cap \overline{B}) \cup (B \cap A)$	Distributive law
$= \varnothing \cup (B \cap A)$	Complement law
$= (B \cap A)$	Identity law
$= A \cap B$	Commutative law

What is a membership table?

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A U B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

- The bottom row is all elements that belong to neither set A or set B
  - Thus, these elements are neither the union, the intersection, nor difference

Proof by membership tables

The following membership table shows that A ∩ B = B - (B - A)

A	B	A ∩ B	B-A	B-(B-A)
1	1	1	0	1
1	0	0	0	0
0	1	0	1	0
0	0	0	0	0

Because the two indicated columns have the same values, the two expressions are identical

This is similar to Propositional logic!

Proof by showing each set is a subset of the other

Assume that an element is a member of one of the identities

- Then show it is a member of the other

Repeat for the other identity

We are trying to show:

- $(x \in A \cap B \rightarrow x \in B-(B-A)) \wedge (x \in B-(B-A) \rightarrow x \in A \cap B)$
- This is the biconditional:
- $x \in A \cap B \leftrightarrow x \in B-(B-A)$

Not good for long proofs

Proof by showing each set is a subset of the other

Assume that $x \in B-(B-A)$

- By definition of difference, we know that $x \in B$ and $x \notin B-A$

Consider $x \notin B-A$

- If $x \in B-A$, then (by definition of difference) $x \in B$ and $x \notin A$
- Since $x \notin B-A$, then only one of the inverses has to be true (DeMorgan's law): $x \notin B$ or $x \in A$

So we have that $x \in B$ and ($x \notin B$ or $x \in A$)

- It cannot be the case where $x \in B$ and $x \notin B$
- Thus, $x \in B$ and $x \in A$
- This is the definition of intersection

Thus, if $x \in B-(B-A)$ then $x \in A \cap B$

Proof by showing each set is a subset of the other

Assume that $x \in A \cap B$

- By definition of intersection, $x \in A$ and $x \in B$

Thus, we know that $x \notin B-A$

- B-A includes all the elements in B that are also not in A not include any of the elements of A (by definition of difference)

Consider B-(B-A)

- We know that $x \notin B-A$
- We also know that if $x \in A \cap B$ then $x \in B$ (by definition of intersection)
- Thus, if $x \in B$ and $x \notin B-A$, we can restate that (using the definition of difference) as $x \in B-(B-A)$

Thus, if $x \in A \cap B$ then $x \in B-(B-A)$

Proof by set builder notation and logical equivalences

First, translate both sides of the set identity into set builder notation

Then modify one side to make it identical to the other

- Do this using logical equivalences

Proof by set builder notation and logical equivalences

$B-(B-A)$	Original statement
$=\{x \mid x\in B \wedge x \notin (B-A)\}$	Definition of difference
$=\{x \mid x\in B \wedge \neg(x \in (B-A))\}$	Negating “element of”
$=\{x \mid x\in B \wedge \neg(x \in B \wedge x \notin A)\}$	Definition of difference
$=\{x \mid x\in B \wedge (x \notin B \vee x \in A)\}$	DeMorgan’s Law
$=\{x \mid (x\in B \wedge x \notin B) \vee (x \in B \wedge x \in A)\}$	Distributive Law
$=\{x \mid (x\in B \wedge \neg(x \in B)) \vee (x \in B \wedge x \in A)\}$	Negating “element of”
$=\{x \mid F \vee (x \in B \wedge x \in A)\}$	Negation Law
$=\{x \mid x\in B \wedge x \in A\}$	Identity Law
$= A \bigcap B$	Definition of intersection

Example

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Solution:

$\overline{A \cap B} = \{x \mid x \notin A \cap B\}$	by definition of complement
$= \{x \mid \neg(x \in (A \cap B))\}$	by definition of does not belong symbol
$= \{x \mid \neg(x \in A \land x \in B)\}$	by definition of intersection
$= \{x \mid \neg(x \in A) \lor \neg(x \in B)\}$	by the first De Morgan law for logical equivalences
$= \{x \mid x \notin A \lor x \notin B\}$	by definition of does not belong symbol
$= \{x \mid x \in \overline{A} \lor x \in \overline{B}\}$	by definition of complement
$= \{x \mid x \in \overline{A} \cup \overline{B}\}$	by definition of union
$= \overline{A} \cup \overline{B}$	by meaning of set builder notation

Refreshment Test ☺

1. For each of the following sets, determine whether 2 is an element of that set.
   a) {x ∈ R | x is an integer greater than 1}
   b) {x ∈ R | x is the square of an integer}
   c) {2,{2}}
   d) {{2},{2,{2}}}
2. If a set has *n* elements, what is the cardinality of its power set?
3. What can you say about the sets A and B if A ⊕ B = A?
4. Let A, B, and C be sets. Show that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

   (Specify the law you used in every steps).

Answers

1. a) Yes b) No c) Yes d) No
2. 2^n elements
3. B =∅
4. $$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{(B \cap C)} && \text{by the first De Morgan law} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law} \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions.}
\end{aligned}$$

Refreshment Test ☺

1. What is the cardinality of :
   a) {{a,a}}
   b) {a, {a}}
   c) {a, {a}, {a, {a}} }
2. If a set has *n* elements, what is the cardinality of its power set?
3. What can you say about the sets A and B if A ⊕ B = A?
4. Let A, B, and C be sets. Show that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

   (Specify the law you used in every steps).

Answers

1. The cardinality:
   a) 1
   b) 2
   c) 3
2. 2^n elements
3. B =Ø
4.

$$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap (\overline{B \cap C}) && \text{by the first De Morgan law} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law} \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions.}
\end{aligned}$$