KUNCI JAWABAN UTS MATDIS SEMESTER GANJIL 2015/2016

Solution: Let p and q be the statements that A is a knight and B is a knight, respectively, so that $\neg p$ and $\neg q$ are the statements that A is a knave and B is a knave, respectively.

We first consider the possibility that A is a knight; this is the statement that p is true. If A is a knight, then he is telling the truth when he says that B is a knight, so that q is true, and A and B are the same type. However, if B is a knight, then B's statement that A and B are of opposite types, the statement $(p \wedge \neg q) \vee (\neg p \wedge q)$, would have to be true, which it is not, because A and B are both knights. Consequently, we can conclude that A is not a knight, that is, that p is false.

If A is a knave, then because everything a knave says is false, A's statement that B is a knight, that is, that q is true, is a lie. This means that q is false and B is also a knave. Furthermore, if B is a knave, then B's statement that A and B are opposite types is a lie, which is consistent with both A and B being knaves. We can conclude that both A and B are knaves. ◀

1.

2. first door : tiger ; second door: lady

3. **Answer:**
   (a) There is no counterexample.
   (b) x =0
   (c) x >1 atau x<1

4. The set of students who are computerscience majors but not mathematics majors or who are mathematics majors but not computer science majors

5. Proof by contradictions:
   We begin by letting x and y be arbitrary real numbers. We then suppose that the conclusion is false, that is, that $\sim(x \geq 1 \ \vee \ y \geq 1)$ *is True. By De Morgan's laws of logic*
   $\sim(x \geq 1 \ \vee \ y \geq 1) \equiv \ \sim(x \geq 1) \ \wedge \ \sim(y \geq 1) \equiv (x < 1) \wedge (y < 1)$
   In words, we are assuming that x<1 and y<1. Then: $x + y < 1 + 1 < 2$
   At this point, we have derived a contradiction: $+y \geq 2$ *and* $x + y < 2$ *. Thus, we conclude that for all real numbers x and y, if* $x + y \geq 2$*, then either* $x \geq 1$ *or* $y \geq 1$

6. A ∩ B = B - (B - A)
   (A ∩ B) ∪ (B^c ∩B)
   B ∩ (A∪ B^c)
   B ∩ (A^c∩ B)^c
   B - (B ∩ A^c)
   B - (B – A

   A ∩ B = B - (B - A)
   Proof B - (B - A) = B - (B ∩ A^c)
   = B ∩ (B ∩ A^c)^c
   = B ∩ (B^c ∪ A)
   = (B ∩ B^c) ∪ (B ∩ A)
   = ∅ ∪ (B ∩ A)
   = A ∩ B commutative law

   atau

7. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent
   $(p \rightarrow r) \wedge (q \rightarrow r) \ \equiv (p \vee q) \rightarrow r$
   $(\backsim p \vee r) \wedge (\backsim q \vee r) \ \equiv \ (p \vee q) \rightarrow r$
   $(\backsim p \wedge \backsim q) \vee r \ \equiv (p \vee q) \rightarrow r$
   $\backsim (p \vee q) \vee r \ \equiv (p \vee q) \rightarrow r$
   $(p \vee q) \rightarrow r \ \equiv (p \vee q) \rightarrow r$

8. Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

Using Direct Proof:

$(p \to q) \wedge (q \to r) \to (p \to r)$

[1]	p	Assumption
[2]	$p \to q$	Premise 1
[3]	q	1 & 2 via modus ponens
[4]	$q \to r$	Premise 2
[5]	r	3 & 4 via modus ponens
[6]	$p \to r$	if p then r, direct method of proof

We assume P, in order to see what follows from P. In this case, after two applications of modus ponens, we see that R follows from that assumption, together with the hypotheses.
So we have proven if P, then R. (Which we state in line (6). In symbols, we have proven P→R. We haven't proven P. But we have proven P implies R. If P is false, the implication is true (any implication with a false antecedent is true). However, we know that if P is true, then so must be R.

9. For integers m and n, if mn is even then m is even or n is even.

Use Indirect Proof
Rephrased: If mn is even, m is even or n is even**,** for all integers
$\sim$q $\to$ $\sim$p so the statement become : if m is odd and n is odd, then mn is odd

Assume : both of m and n are odd are True,
So m =2k+1, and n = 2j+1
Proof mn is odd:
mn = (2k+1)(2j+1) = 4kj + 2k + 2j +1 = 2 (2kj + k + j) +1 -> Odd, where k = (2kj + k + j)
since ¬q→¬p is True, thus p→q is also True, The Theorem is also PROVEN

10. p: the chargers get a good linebacker
q: the chargers can beat the broncos
r: the chargers can beat the jets
s: the chargers can beat the dolphins
Then the hypotheses are: (1) p → q, (2) q → r, (3) q → s, and (4) p

[1]	$p \to q$	Premise 1
[2]	$q \to r$	Premise 2
[3]	$p \to r$	Hypothetical syllogism from 1 & 2
[4]	p	Premise 4
[5]	r	Modus ponens from 3 & 4
[6]	$q \to s$	Premise 3
[7]	$p \to s$	Hypothetical syllogism from 1 & 6
[8]	s	Modus ponens from 4 & 7
[9]	$r \wedge s$	Conjunction from 5 & 8

Since $r \wedge s$ represent the chargers can beat the jets nd the chargers can beat the dholphins, we conclude that the conclusion does follow from the hypotheses.