ITS Institut Teknologi Sepuluh Nopember
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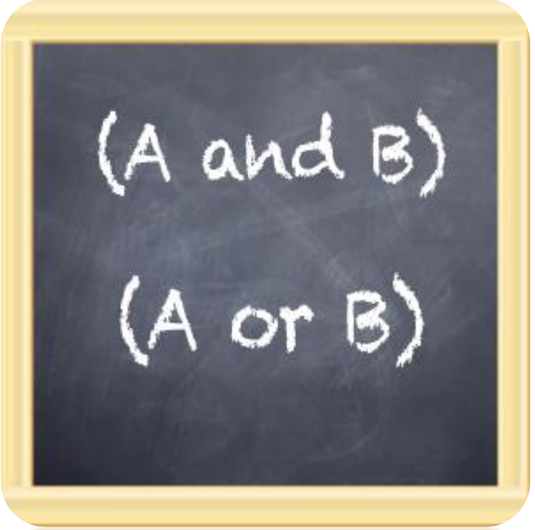
KS141203 MATEMATIKA DISKRIT
(*DISCRETE MATHEMATICS*)



LOGIC & PROPOSITIONAL EQUIVALENCE

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Outline

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Logic

Logic is the foundation of Mathematical reasoning (*penjabaran/penalaran Matematika*).

The rules of logic are used to distinguish between valid and invalid Mathematical arguments.

Logic has numerous application in Computer Sciences such as in the design of computer circuits, **the construction of computer programs, the verification of correctness of programs**, etc.

Proposition

Proposition is a statement that can be either true or false, but not both.

- "Washington DC is the capital of the USA."
- "Kuala Lumpur is a city in Indonesia."
- "3 = 2 + 1"
- "3 = 2 + 2"

Not propositions:

- "is Cameron Diaz a Prime Minister?"
- "x = 7"
- "I am good student"

Proposition (cont.)

The following are not proposition

- Instruction (*Kalimat perintah*)
- Question (*Kalimat pertanyaan*)
- Amazement (*Kalimat keheranan*)
- Expectancy (*Kalimat harapan*)

Propositional Variables

We use propositional variables to refer to propositions

- Usually are lower case letters starting with p (i.e. p, q, r, s, etc.)
- A propositional variable can have one of two values: true (T) or false (F)

A proposition can be...

- A single variable: p
- An operation of multiple variables: $p \wedge (q \vee \neg r)$

Logical Operators

Many mathematical statements are constructed by combining one or more propositions.

New propositions, called compound propositions *(pernyataan gabungan/ pernyataan majemuk)*, are formed from existing propositions using logical operators.

Logical Operator: Not

A “not” operation switches (negates) the truth value of a proposition.

Symbol: $\neg$ or ~

p = “Michael’s PC run Linux”

$\neg p$ = “Michael’s PC does not run Linux”

p	$\neg p$
T	F
F	T

Logical Operator: And

An “and” operation is true if both operands are true.

Symbol: $\wedge$

p = “Michael’s PC has more than 16 GB free HD space”

q = “The processor in Michael’s PC runs faster than 1 GHz”

$p \wedge q$ = “Michael’s PC has more than 16 GB free HD space and it’s processor runs faster than 1 GHz”

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Operator: Or

An “or” operation is true if either operands or both are true.

Symbol: $\vee$

p = “Michael’s PC has more than 16 GB free HD space”

q = “The processor in Michael’s PC runs faster than 1 GHz”

$p \vee q$ = “Michael’s PC has more than 16 GB free HD space or it’s processor runs faster than 1 GHz”

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Operator: XOR

An "xor" operation is true when exactly one of *p* and *q* is true, and false otherwise.

Symbol: $p \oplus q$

$p \oplus q$ = "Either Michael's PC has more than 16 GB free HD space or the processor in Michael's PC runs faster than 1 GHz (but not both)"

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Logical Operator: XOR (cont)

When two simple propositions are combined using 'or', context will often provide the clue as to whether the inclusive or exclusive sense is intended.

For instance, '*Tomorrow I will go swimming or play golf*' seems to suggest that I will not do both in the same time and therefore points to an exclusive interpretation.

Logical Operator: XOR (cont)

For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended.

1. Coffee or tea comes with dinner.

   Exclusive or: You get only one beverage.
2. A password must have at least three digits or be at least eight characters long.

   Inclusive or: Long passwords can have any combination of symbols.
3. The prerequisite for the course is a course in number theory or a course in cryptography.

   Inclusive or: A student with both courses is even more qualified.
4. You can pay using U.S. dollars or euros.

   Either interpretation possible; a traveler might wish to pay with a mixture of the two currencies, or the store may not allow that.

Logical Operator: Conditional

A conditional means "if *p* then *q*"

Symbol: $\rightarrow$

p = "You get 100 on the final exam"

q = "You will get an A"

$p \rightarrow q$ = "If you get 100 on the final exam, then you will get an A"

$p \rightarrow q = \neg p \vee q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Antecedent/ hypothesis/ premise

Consequence/ conclusion

Logical Operator: Conditional (cont)

Let p = "I pass my exams " and q = "I will go to the cinema"

I state: $p \rightarrow q$ = "If I pass my exams then I will go to the cinema"

Note that if p is false, then the conditional is true regardless of whether q is true or false.

The above statement says nothing about what I will do if I *don't* pass my exams. I may go to the cinema or I may not, but in either case you could not accuse me of having made a false statement.

The only circumstances in which I could be accused of uttering a falsehood is if I pass my exams and don't go to the cinema.

Logical Operator: Conditional (cont)

Alternate ways of stating a conditional:

- p implies q
- If p, q
- p only if q
- p is sufficient for q
- q if p
- q whenever p
- q is necessary for p

Logical Operator: Conditional (cont)

Conditional statement:

- If you use Ms. Word, then Windows is the operating system.
- Alternatively:
  - Ms. Word is sufficient for Windows
  - Windows is necessary for Ms. Word

Ms. Word adalah syarat cukup bagi Windows, sedangkan Windows adalah syarat perlu bagi Ms. Word

Ms. Word tidak dapat digunakan tanpa Windows tetapi Windows dapat digunakan tanpa Ms. Word

Logical Operator: Conditional (cont)

Propositions				Conditional	Inverse	Converse	Contrapositive
p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$q \rightarrow p$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

- Two contrapositive propositions have the same truth values (equivalent)
- Inverse and converse are the opposite of the conditional, both of them have the same truth values
- These rules are useful in proofing

Logical Operator: Bi-Conditional

A bi-conditional means "***p* if and only if *q***"

Symbol: $\leftrightarrow$

Alternatively, it means
"(if *p* then *q*) and (if *q* then *p*)"

Note that a bi-conditional has the opposite truth values of the exclusive or

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Operator: Bi-Conditional

Let p = "You can take the flight" and q = "You buy a ticket"

Then $p \leftrightarrow q$ means
"You can take the flight if and only if you buy a ticket"

Alternatively, it means "If you can take the flight, then you buy a ticket and if you buy a ticket then you take the flight"

Boolean Operator Summary

		not	not	and	or	xor	conditional	Bi-conditional
p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	F
F	T	T	F	F	T	T	T	F
F	F	T	T	F	F	F	T	T

Try to understand what they mean, don’t just memorize the table ☺

Presedence of Operators

Just as in algebra, operators have precedence

- 4+3*2 = 4+(3*2), not (4+3)*2

Precedence order (from highest to lowest):

$\neg \wedge \vee \rightarrow \leftrightarrow$

This means that $p \vee q \wedge \neg r \rightarrow s \leftrightarrow t$
yields: $(p \vee (q \wedge (\neg r)) \rightarrow s) \leftrightarrow (t)$

Not is *always* performed before any other operation

Translating English Sentences

Question 7 from Rosen, p. 17

- p = "It is below freezing"
- q = "It is snowing"

1. It is below freezing and it is snowing
2. It is below freezing but not snowing
3. It is not below freezing and it is not snowing
4. It is either snowing or below freezing (or both)
5. If it is below freezing, it is also snowing
6. It is either below freezing or it is snowing, but it is not snowing if it is below freezing
7. That it is below freezing is necessary and sufficient for it to be snowing

1. $p \wedge q$
2. $p \wedge \neg q$
3. $\neg p \wedge \neg q$
4. $p \vee q$
5. $p \rightarrow q$
6. $((p \oplus q) \wedge (p \rightarrow \neg q)$
7. $p \leftrightarrow q$

Translating English Sentences

- “I have neither given nor received help on this exam”
- Let p = “I have given help on this exam”
- Let q = “I have received help on this exam”

- $\neg p \land \neg q$

Translating English Sentences

- You can access the Internet from campus only if you are a computer science major or you are not a freshman.
- $a \rightarrow (c \vee \neg f)$

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.
- $(f \wedge \neg s) \rightarrow \neg r$
- $r \rightarrow (\neg f \vee s)$

Practice ☺

Let *p* and *q* be the propositions.

p: You drive over 65 miles per hour.

q: You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

1. You do not drive over 65 miles per hour.
2. You drive over 65 miles per hour, but you do not get a speeding ticket.
3. You will get a speeding ticket if you drive over 65 miles per hour.
4. If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
5. Driving over 65 miles per hour is sufficient for getting a speeding ticket.
6. You get a speeding ticket, but you do not drive over 65 miles per hour.
7. Whenever you get a speeding ticket, you are driving over 65 miles per hour.