$$AX - 3I = (AB^t - 2BA^t)^t$$

$$AX - 3I = (AB^t)^t - 2(BA^t)^t$$

$$AX - 3I = BA^t - 2AB^t$$

$$AX + 2AB^t = BA^t + 3I$$

$$A(X + 2B^t) = BA^t + 3I$$

$$A^{-1}A(X + 2B^t) = A^{-1}(BA^t + 3I)$$

$$X + 2B^t = A^{-1}(BA^t + 3I)$$

$$X = A^{-1}(BA^t + 3I) - 2B^t$$

$$\boxed{\begin{array}{l}(A+B)^t = A^t + B^t \\ (AB)^t = B^t + A^t \\ (A^t)^t = A\end{array}}$$

$$\boxed{A^{-1}A = AA^{-1} = I}$$

Calculamos A^{-1}:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-f_3} \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \xrightarrow[f_3+f_2]{f_3+f_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \xrightarrow{-f_2+f_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right)$$

$\underline{A^{-1}}$

$$X = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix}}_{A^{-1}} \left[\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & -1 \end{pmatrix}}_{A^t} + \underbrace{\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{3I} \right] - \underbrace{\begin{pmatrix} 2 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}}_{2B^t}$$

$$X = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \left[\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \right] - \begin{pmatrix} 2 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 4 & -2 & 0 \\ -1 & 2 & -3 \\ -1 & 0 & -2 \end{pmatrix} - \begin{pmatrix} 2 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -4 & -2 \\ -3 & 2 & -5 \\ -3 & -2 & -4 \end{pmatrix}$$

2) $A = 2\begin{pmatrix} 1 & 0 & -h \\ 0 & -h & 0 \\ 0 & 0 & h \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -h & 0 & h \\ 0 & -h & 0 \\ 1 & 0 & 0 \end{pmatrix}^{-1}$

$$\boxed{\begin{aligned} (AB)^{-1} &= B^{-1}A^{-1} \\ (ABC)^{-1} &= C^{-1}B^{-1}A^{-1} \\ (A^{-1})^{-1} &= A \end{aligned}}$$

$A^{-1} = \left[2\begin{pmatrix} 1 & 0 & -h \\ 0 & -h & 0 \\ 0 & 0 & h \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -h & 0 & h \\ 0 & -h & 0 \\ 1 & 0 & 0 \end{pmatrix}^{-1} \right]^{-1}$

$A^{-1} = 2^{-1} \begin{pmatrix} -h & 0 & h \\ 0 & -h & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & -h \\ 0 & -h & 0 \\ 0 & 0 & h \end{pmatrix}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\frac{1}{2}f_2]{-f_3+f_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$

$A^{-1} = \frac{1}{2} \begin{pmatrix} -h & 0 & h \\ 0 & -h & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & -h & 0 \\ 0 & 0 & h \end{pmatrix}$

$A^{-1} = \frac{1}{2} \begin{pmatrix} -h & 0 & 2h \\ 0 & -h/2 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -h \\ 0 & -h & 0 \\ 0 & 0 & h \end{pmatrix}$

$A^{-1} = \frac{1}{2} \begin{pmatrix} -h & 0 & 3h^2 \\ 0 & h^2/2 & 0 \\ 1 & 0 & -2h \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -h/2 & 0 & 3/2\, h^2 \\ 0 & h^2/4 & 0 \\ \frac{1}{2} & 0 & -h \end{pmatrix}$

) $(X-A)(A+I) = I$, $A^2 = 0$

$$XA + XI - A\cdot A - AI = I$$
$$XA + XI - \underbrace{\cancel{A^2}}_{0} - A = I$$
$$XA + XI = A + I$$
$$X(A+I) = A+I$$
$$X(A+I)(A+I)^{-1} = (A+I)(A+I)^{-1}$$
$$X = I$$

4) $(AB - I)(AB+I) - AX = AXB - I$

$$AB\cdot AB + AB\cdot I - I\cdot AB - I\cdot I - AX = AXB - I$$
$$AB\cdot AB + \cancel{AB} - \cancel{AB} - \cancel{I} - AX = AXB - \cancel{I}$$
$$AXB + AX = AB\cdot AB$$
$$AX(B+I) = AB\cdot AB$$
$$\underbrace{A^{-1}A}X(B+I)(B+I)^{-1} = \underbrace{A^{-1}A}BAB(B+I)^{-1} \qquad AA^{-1} = A^{-1}A = I$$
$$X = BAB(B+I)^{-1}$$

$$(B+I)^{-1}: \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{-f_2+f_1} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{-f_1+f_2} \underbrace{\left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right)}_{(B+I)^{-1}}$$

$$X = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

5)

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix}^{-1}$$

$$A^{-1} = \left(\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix}^{-1} \right)^{-1}$$

$$A^{-1} = \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{pmatrix}^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 2 & 5 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[-f_1+f_3]{-f_1+f_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow[-2f_3+f_2]{-3f_3+f_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow{-f_2+f_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix} \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3a+1 & -a+1 & -a-2 \\ a-1 & a & -2a+1 \\ -a & 0 & a \end{pmatrix}$$

6) a.) $3I_2 - A = 3\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}2&1\\0&2\end{pmatrix} = \begin{pmatrix}3&0\\0&3\end{pmatrix} - \begin{pmatrix}2&1\\0&2\end{pmatrix} = \begin{pmatrix}1&-1\\0&1\end{pmatrix}$

$(3I_2-A)^{-1} = \left(\begin{array}{cc|cc}1&-1&1&0\\0&1&0&1\end{array}\right) \xrightarrow{f_2+f_1} \left(\begin{array}{cc|cc}1&0&1&1\\0&1&0&1\end{array}\right) \Rightarrow \boxed{(3I_2-A)^{-1} = \begin{pmatrix}1&1\\0&1\end{pmatrix}}$

b) $3X^t - B^t = (C + X^t)A$

$3X^t - B^t = CA + X^tA$

$3X^tI_2 - X^tA = CA + B^t$

$X^t(3I_2 - A) = CA + B^t$

$X^t(3I_2-A)(3I_2-A)^{-1} = (CA+B^t)(3I_2-A)^{-1}$

$X^t = (CA+B^t)(3I_2-A)^{-1}$

$(X^t)^t = [((A+B^t)(3I_2-A)^{-1})]^t$

$X = [(CA+B^t)(3I_2-A)^{-1}]^t$

$X = \left(\left[\underbrace{\begin{pmatrix}0&-2\\-2&0\end{pmatrix}}_{C}\underbrace{\begin{pmatrix}2&1\\0&2\end{pmatrix}}_{A} + \underbrace{\begin{pmatrix}3&4\\3&1\end{pmatrix}}_{B^t}\right]\underbrace{\left[\begin{pmatrix}3&0\\0&3\end{pmatrix} - \begin{pmatrix}2&1\\0&2\end{pmatrix}\right]^{-1}}_{(3I_2-A)^{-1}}\right)^t$

$X = \left(\left[\begin{pmatrix}0&-4\\-4&-2\end{pmatrix} + \begin{pmatrix}3&4\\3&1\end{pmatrix}\right]\begin{pmatrix}1&1\\0&1\end{pmatrix}\right)^t$

$X = \left(\begin{pmatrix}3&0\\-1&-1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}\right)^t$

$X = \begin{pmatrix}3&3\\-1&-2\end{pmatrix}^t$

$X = \begin{pmatrix}3&-1\\3&-2\end{pmatrix}$