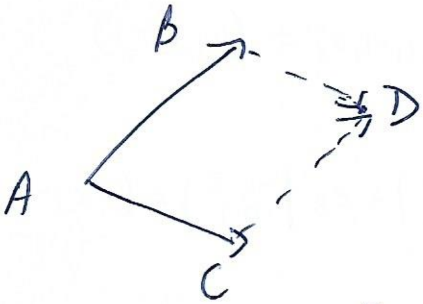
1) Considere los puntos $A=(-1,0,3)$, $B=(0,1,1)$, $C=(-1,-3,0)$.

a) Encuentre un punto $D\in\mathbb{R}^3$ tq $ABDC$ sea un paralelogramo.




$$\vec{AB} = \vec{CD}$$

$$B-A = D-C$$

$$D = B-A+C$$

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$$D=(0,1,1)-(-1,0,3)+(-1,-3,0)$$

$$D=(0,-2,-2)$$

b) $A=\|\vec{AB}\times\vec{AC}\|$ $\quad \vec{AB}=B-A=(0,1,1)-(-1,0,3)=(1,1,-2)$

$\vec{AC}=C-A=(-1,-3,0)-(-1,0,3)=(0,-3,-3)$

$$\vec{AB}\times\vec{AC}=\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & -2 \\ 0 & -3 & -3 \end{vmatrix} = e_1\begin{vmatrix} 1 & -2 \\ -3 & -3 \end{vmatrix} - e_2\begin{vmatrix} 1 & -2 \\ 0 & -3 \end{vmatrix} + e_3\begin{vmatrix} 1 & 1 \\ 0 & -3 \end{vmatrix}$$

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$$=(-9,3,-3)$$

$$\Rightarrow A=\|\vec{AB}\times\vec{AC}\| = \|(-9,3,-3)\| = 3\sqrt{11}$$

c) $$\mathrm{Proy}_{\vec{AC}}\vec{AB} = \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AC}\|^2}\cdot\vec{AC}$$

$$=\frac{(1,1,-2)\cdot(0,-3,-3)}{\|(0,-3,-3)\|^2}\cdot(0,-3,-3)$$

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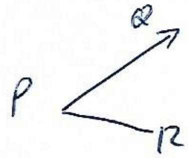
$$=\frac{3}{18}\cdot(0,-3,-3)$$

$$=\frac{1}{6}\cdot(0,-3,-3)$$

$$=\left(0,\frac{-1}{2},\frac{-1}{2}\right)$$

2) $P=(1,-1,0)$, $Q=(0,1,0)$, $R=(1,0,-1)$

a) 

$\vec{PQ} = Q-P = (0,1,0)-(1,-1,0) = (-1,2,0)$

$\vec{PR} = R-P = (1,0,-1)-(1,-1,0) = (0,1,-1)$

$$\vec{n}_1 = \vec{PQ} \times \vec{PR} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & 2 & 0 \\ 0 & 1 & -1 \end{vmatrix} = e_1 \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} - e_2 \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} + e_3 \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = (-2,-1,-1)$$

$\Pi_1$: $x\vec{n}_1 = P\vec{n}_1$

$(x,y,z)(-2,-1,-1) = (1,-1,0)(-2,-1,-1)$

$-2x-y-z=-1$

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b) $\Pi_2 \,//\, \Pi_1$ pasa por $(0,0,0)$ 5p

$\vec{n}_2 = \vec{n}_1 = (-2,-1,-1)$

$x\vec{n}_2 = P\vec{n}_2$

$(x,y,z)(-2,-1,-1) = (0,0,0)(-2,-1,-1)$

$-2x-2y-2z=0$

3) $\Pi_1: 2x-y+z=1$, $\Pi_2: 3x+y+z=2$

a) $\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 3 & 1 & 1 & 2 \end{array}\right) \xrightarrow{-f_1+f_2} \left(\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 1 & 2 & 0 & 1 \end{array}\right) \xrightarrow{-2f_2+f_1} \left(\begin{array}{ccc|c} 0 & -5 & 1 & -1 \\ 1 & 2 & 0 & 1 \end{array}\right) \xrightarrow[f_1 \leftrightarrow f_2]{-\frac{1}{5}f_1} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & -1/5 & 1/5 \end{array}\right)$

$\xrightarrow{-2f_2+f_1} \left(\begin{array}{ccc|c} 1 & 0 & 2/5 & 3/5 \\ 0 & 1 & -1/5 & 1/5 \end{array}\right)$

$z=t$

$x + 2/5\, z = 3/5$

$y - 1/5\, t = 1/5$

$x = -\frac{2}{5}t + \frac{3}{5}$, $y = \frac{1}{5}t + \frac{1}{5}$

8p

$(x,y,z) = \left(-\frac{2}{5}t+\frac{3}{5}, \frac{1}{5}t+\frac{1}{5}, t\right) = (3/5, 1/5, 0) + t(-2/5, 1/5, 1)$

b) $d(\Pi_2, P) = \dfrac{|3\cdot 2 - 1\cdot 0 + 1\cdot 7 - 2|}{\|(3,1,1)\|} = \dfrac{11}{\sqrt{11}}$ 5p

4a) $\begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 3 & 2 & 1 & 2 & 4 \\ 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 3 \end{pmatrix} \xrightarrow[\substack{-2f_1+f_3 \\ -f_1+f_4}]{-3f_1+f_2} \begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 0 & -1 & -2 & -1 & -5 \\ 0 & -1 & -2 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[\substack{-f_2+f_3 \\ -f_2}]{f_2+f_1} \begin{pmatrix} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & 2 & 1 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{-f_3+f_2} \begin{pmatrix} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$x_1 - x_3 - 2x_5 = 0 \qquad x_2 + 2x_3 + 5x_5 = 0 \qquad x_4 = 0$

$x_1 = x_3 + 2x_5 \qquad x_2 = -2x_3 - 5x_5$

8p

$(x_1, x_2, x_3, x_4, x_5) = (x_3 + 2x_5, -2x_3 - 5x_5, x_3, 0, x_5)$

$= x_3(1,-2,1,0,0) + x_5(2,-5,0,0,1)$

$N(A) = \{(1,-2,1,0,0), (2,-5,0,0,1)\}$.

b) $F_A = \{(1,0,-1,0,-2), (0,1,2,0,5), (0,0,0,1,0)\}$ 3

c) Pivotes 1: columna 1, columna 2, columna 4.

$C_A = \left\{ \begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \right\}$ 3