1) Calcule $\begin{vmatrix} 2a+2b & 2b+2c & 2c+2a \\ 2b+2c & 2c+2a & 2a+2b \\ 2c+2a & 2a+2b & 2b+2c \end{vmatrix}$ si $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3$

$$\begin{vmatrix} 2a+2b & 2b+2c & 2c+2a \\ 2b+2c & 2c+2a & 2a+2b \\ 2c+2a & 2a+2b & 2b+2c \end{vmatrix} = 2^3 \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} \qquad D(\alpha A) = \alpha^n D(A)$$

$$= 8\left[\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} b & c & a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}\right] \qquad \text{linealidad en fila 1}$$

$$= 8\left[\xrightarrow{-f_1+f_3} \begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ c & a & b \end{vmatrix} + \xrightarrow{-f_1+f_2} \begin{vmatrix} b & c & a \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix}\right]$$

$$= 8\left[\xrightarrow{-f_3+f_2} \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \xrightarrow{-f_2+f_3} \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}\right]$$

$$= 8\left[3 - (f_1 \leftrightarrow f_3) \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}\right] \qquad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3$$

$$= 8\left[3 + (f_2 \leftrightarrow f_3) \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\right]$$

$$= 8[3+3] = 48$$

/

2) Sea $A=\begin{pmatrix} x+1 & x & x \\ x & x+1 & x \\ x & x & x+1 \end{pmatrix}$, determine los valores de x para los cuales A es invertible.

Para que A sea invertible, se debe cumplir que $|A| \neq 0$

$$\Rightarrow \begin{vmatrix} x+1 & x & x \\ x & x+1 & x \\ x & x & x+1 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} x+1 & x & x \\ x & x+1 & x \\ x & x & x+1 \end{vmatrix} = \begin{matrix} -f_2+f_1 \\ -f_2+f_3 \end{matrix} \begin{vmatrix} 1 & -1 & 0 \\ x & x+1 & x \\ 0 & -1 & 1 \end{vmatrix} = -xf_1+f_2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 2x+1 & x \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 2x+1 & x \\ -1 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow 2x+1+x \neq 0 \;\Rightarrow 3x+1 \neq 0 \;\Rightarrow x \neq \tfrac{-1}{3} \;\Rightarrow \text{A es invertible } \forall x \in \mathbb{R} - \left\{\tfrac{-1}{3}\right\}$$

3) Sean A y P matrices de orden 3, con $\det(A)=3$ y $\det(P)=5$. Si $B=P^{-1}AP$. Calcule $\det(-3B^t)$.

$$\det(B) = d\left(P^{-1}AP\right) \qquad\qquad D(AB)=D(A)\cdot D(B)$$

$$= \det\left(P^{-1}\right)\cdot\det(A)\det(P) \qquad\qquad D\left(A^{-1}\right)=\frac{1}{D(A)}$$

$$= \frac{1}{\det(P)}\cdot\det(A)\cdot\det(P)$$

$$= \det(A) = 3$$

Ahora:

$$\det\left(-3B^t\right) = (-3)^3\det\left(B^t\right) \qquad\qquad D(\alpha A)=\alpha^n D(A)$$

$$= -27\det(B)$$

$$= -27 \bullet 3 = -81$$

4) Sean A, B y C tres matrices de orden nxn tal que $|A|=3,\ |C|=-6$ y con B invertible. Use propiedades de los determinantes para calcular el determinante de $\left(B^t(3A)B^{-1}C^{-1}\right)^t$.

$$\left|\left(B^t(3A)B^{-1}C^{-1}\right)^t\right| = \left|B^t(3A)B^{-1}C^{-1}\right| \qquad D(A)=D\left(A^t\right),\ D(AB)=D(A)\cdot D(B)$$

$$= \left|B^t\right|\left|3A\right|\left|B^{-1}\right|\left|C^{-1}\right| \qquad D\left(A^{-1}\right)=\frac{1}{D(A)}$$

$$= |B||3A|\cdot\frac{1}{|B|}\cdot\frac{1}{|C|} \qquad D(\alpha A)=\alpha^n D(A)$$

$$= \frac{3^n|A|}{|C|} = \frac{3^n\cdot 3}{-6} = \frac{-3^n}{2}$$

5) Verifique que: $\begin{vmatrix} 1 & a+b & b+c \\ 1 & c+a & a+b \\ 1 & b+c & c+a \end{vmatrix} = -\begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix}$

$$\begin{vmatrix} 1 & a+b & b+c \\ 1 & c+a & a+b \\ 1 & b+c & c+a \end{vmatrix} = \begin{vmatrix} 1 & a & b+c \\ 1 & c & a+b \\ 1 & b & c+a \end{vmatrix} + \begin{vmatrix} 1 & b & b+c \\ 1 & a & a+b \\ 1 & c & c+a \end{vmatrix} \qquad \text{linealidad en la columna 2}$$

$$= \left(c_2+c_3\right)\begin{vmatrix} 1 & a & a+b+c \\ 1 & c & a+b+c \\ 1 & b & a+b+c \end{vmatrix} + \left(-c_2+c_3\right)\begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & c & 1 \\ 1 & b & 1 \end{vmatrix} - \left(f_1 \leftrightarrow f_2\right)\begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix} \qquad D(\alpha A)=\alpha^n D(A)$$

$$= -\begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix}$$