1.

$$X - E + (X^t B)^t = -CA^t X$$

$$X - E + B^t X = -CA^t X$$

$$X + B^t X + CA^t X = E$$

$$I \cdot X + B^t X + CA^t X = E$$

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$$(I + B^t + CA^t) X = E$$

$$(I + B^t + CA^t)^{-1} (I + B^t + CA^t) X = E (I + B^t + CA^t)^{-1} E$$

$$X = (I + B^t + CA^t)^{-1} E$$

②

$$\begin{cases} x - y - mz = 1 \\ mx + y - z = 1 \\ -mx + 2y + z = -1 \end{cases}$$ Reducimos:

a)

$$\left(\begin{array}{ccc|c} 1 & -1 & -m & 1 \\ m & 1 & -1 & 1 \\ -m & 2 & 1 & -1 \end{array}\right) \xrightarrow[f_2+f_3]{-mf_1+f_2} \left(\begin{array}{ccc|c} 1 & -1 & m & 1 \\ 0 & m+1 & m^2-1 & -m+1 \\ 0 & 3 & 0 & 0 \end{array}\right) \xrightarrow{f_2 \leftrightarrow f_3} \left(\begin{array}{ccc|c} 1 & -1 & m & 1 \\ 0 & 3 & 0 & 0 \\ 0 & m+1 & (m+1)(m-1) & -m+1 \end{array}\right)$$

tenemos: $\left(\begin{array}{ccc|c} 1 & -1 & m & 1 \\ 0 & 3 & 0 & 0 \\ 0 & m+1 & (m+1)(m-1) & -m+1 \end{array}\right)$; entonces

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Si $m=-1$ $\left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right)$ ⊗ el sistema es inconsistente.

Si $m=1$ $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) \xrightarrow{\cdots} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ el sistema tiene infinitas soluciones.

Si $m \neq \pm 1$ el sistema tiene solución única:

b) Si $m=0 \Rightarrow \begin{cases} x - y = 1 \\ y - z = 1 \\ 2y + z = -1 \end{cases}$ es el nuevo sistema

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$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3 \quad , \quad b = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$x = \frac{\begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}}{3} \overset{\substack{-f_1+f_2 \\ f_1+f_3}}{=} \frac{\begin{vmatrix} 1 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}}{3} = \frac{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}}{3} = \frac{3}{3} = 1$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{vmatrix}}{3} = \frac{\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}}{3} = 0$$

$$z = \frac{\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{vmatrix}}{3} = \frac{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}}{3} = \frac{-3}{3} = -1$$

$$\Rightarrow S: \{(1,0,-1)\}.$$

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a) $A = \begin{pmatrix} x & -1 & 1 \\ 1 & 2 & -x \\ 1 & 2 & -1 \end{pmatrix}$ $\quad$ $Ran(A) < 3 \rightarrow |A| = 0$.

$$\begin{vmatrix} x & -1 & 1 \\ 1 & 2 & -x \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$\xrightarrow{-f_2+f_3} \begin{vmatrix} x & -1 & 1 \\ 1 & 2 & -x \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

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$$(x-1) \begin{vmatrix} x & -1 \\ 1 & 2 \end{vmatrix} = 0$$

$$(x-1)(2x+1) = 0 \iff x = 1 \vee x = -\frac{1}{2}$$

$\Rightarrow Ran(A) < 3$ si $x = 1, -\frac{1}{2}$.

b) $Adj(A) = \begin{pmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 2 & 0 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} \end{pmatrix}$

$\underline{x=0}$

$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{pmatrix}$ $\quad = \begin{pmatrix} -2 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

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$$|A| = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{vmatrix} \overset{-f_2+f_3}{=} \begin{vmatrix} 0 & -1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = -1$$

$$A^{-1} = \frac{1}{-1} \begin{pmatrix} -2 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$