Si $B=\left\{u_1=(1,0,1)^t, u_2=(2,0,0)^t, u_3=(-1,1,0)^t\right\}$ una base de $\mathbb{R}^3$, utilice el proceso de G-S para calcular una base ortonormal B_{orton} con B.

$$a)\ v_1=\frac{(1,0,1)}{\|(1,0,1)\|}=\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)\ \Rightarrow S_1=c\ell\{v_1\}$$

$$b)\ v_2=\frac{u_2-proy_{S_1}^{u_2}}{\left\|u_2-proy_{S_1}^{u_2}\right\|}$$

$$proy_{S_1}^{u_2}=(u_2\bullet v_1)v_1=\left[(2,0,0)\bullet\left(\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)\right)\right]\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)=\tfrac{2}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)=(1,0,1)$$

$$u_2-proy_{S_1}^{u_2}=(2,0,0)-(1,0,1)=(1,0,-1)$$

$$v_2=\frac{(1,0,-1)}{\|(1,0,-1)\|}=\left(\tfrac{1}{\sqrt{2}},0,-\tfrac{1}{\sqrt{2}}\right)\qquad\Rightarrow S_2=c\ell\{v_1,v_2\}$$

$$c)\ v_3=\frac{u_3-proy_{S_2}^{u_3}}{\left\|u_3-proy_{S_2}^{u_3}\right\|}$$

$$proy_{S_2}^{u_3}=(u_3\bullet v_1)v_1+(u_3\bullet v_2)v_2$$

$$=\left[(-1,1,0)\bullet\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)\right]\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)+\left[(-1,1,0)\bullet\left(\tfrac{1}{\sqrt{2}},0,-\tfrac{1}{\sqrt{2}}\right)\right]\left(\tfrac{1}{\sqrt{2}},0,-\tfrac{1}{\sqrt{2}}\right)$$

$$=-\tfrac{1}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)+\tfrac{-1}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}},0,-\tfrac{1}{\sqrt{2}}\right)$$

$$=\left(\tfrac{-1}{2},0,\tfrac{-1}{2}\right)+\left(\tfrac{-1}{2},0,\tfrac{1}{2}\right)=(-1,0,0)$$

$$u_3-proy_{S_2}^{u_3}=(-1,1,0)-(-1,0,0)=(0,1,0)$$

$$v_3=\frac{(0,1,0)}{\|(0,1,0)\|}=(0,1,0)$$

$$\Rightarrow B_{orton}=\left\{\left(\tfrac{1}{\sqrt{2}},0,\tfrac{1}{\sqrt{2}}\right)^t,\left(\tfrac{1}{\sqrt{2}},0,\tfrac{-1}{\sqrt{2}}\right)^t,(0,1,0)^t\right\}$$