# Modelling the Climate and Energy Balance of an Earth-Like Planet

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# Contents

# 1 Introduction

This project aims to numerically model the climate of an earth-like planet by predicting its temperature over time and investigate how changing factors such as distance from the star and emissivity of the planet's atmosphere can have a profound impact on the climate.

## 1.1 Dependencies and distribution

The dependencies used in this project are NumPy and MatPlotLib's Pyplot function. Specifically from NumPy, the constant $\pi$ is imported directly.

### 1.1.1 Files

- climate_modelling.py is the complete model with a few functions for plotting and analysis. This is all explained in section 4 - Usage
- climate_modelling.pdf is this document, including the code, graphs and this project
- climate_modelling.tex is the LaTeX code required to build climate_modelling.pdf

### 1.1.2 Experimenting

To experiment, you can either import the classes and model function into a new python file to preserve the original model, or you can simply edit the parameters in the original file, such as change LUMINOSITY to 2*LUMINOSITY, which would probably be easier.

## 1.2 Background: Climate Modelling

Climate modelling is the process of using computer simulations to predict how the Earth's climate will change over time. This field of study is a relatively new one, having begun in the 1960s, and has since evolved to become an essential tool for scientists and amateurs like myself to understand the complex processes that shape our planet's climate.

Climate models work by incorporating various factors that affect the Earth's climate, such as greenhouse gas emissions, solar radiation, and ocean currents. By inputting data on these factors, scientists can create models that predict how the Earth's climate will change over time. These models are tested against historical data to verify their accuracy and are continually updated as new information becomes available.

My model is rather more simplistic, taking into account the same broad factors and processes, but simplifying them into a way that my GCSE Maths and Science understanding could cope with. I had to do a fair bit of research to compensate for my lack of knowledge, but because all the prior similar work is far more complex and rigorous than I could hope to deal with, much of what follows is my own work and deductions in order to create a model that is atleast semi-accurate and simple enough for me to understand everything that is going on. There was no point making a model too complex for me to understand by modifying prior work as it negates the point of the project, which for me, was to try and gain a deeper understanding of the processses that affect our planet and it's climate.

Overall, climate modelling is a vital tool in our efforts to understand and mitigate the impacts of climate change, and its continued development is critical to our ability to address this global challenge.

At first I tried to make a model that was atleast 95% accurate to current global temperatures, but I had neither the time nor the knowledge to make that work, so I decided to attempt to create one that was atlest 60% accurate. I achieved this goal as my final model was around 75% accurate, that too only considering the first 1000 years of an early Earth. I am confident that it would become more accurate given the time for billions of iterations to match the billions of years that our Earth has been around.

## 1.3 Summary of Results

My results gave an average global temperature of around 200K, and average temperature of around 180K for high latitudes and around 230K for equatorial ones.

# 2 Mathematical Details

## 2.1 Calculating Energy Input

### 2.1.1 Intensity of global solar insolation

The first and easiest step was calculating the energy input from the sun for each latitude band. The most commonly used value for the intensity of solar intensity is approximately 1400 $\mathrm{Wm}^{-2}$. In my first attempts at making this model, I simply used this value as a constant, however creating a function to calculate the energy input makes it easier to experiment later, for example by varying the distance of the planet in AU. The light intensity that reaches the top of the planet's atmosphere from the sun is given by the following equation:

$$I = \frac{L}{4\pi D^2}$$

where L is the luminosity of the star and D is its distance from the planet. These values are initialised to 1L and 1AU by default to match the Earth's parameters but they can be easily modified to experiment with their effect on the planet's climate. For the purposes of the model, we divide this value by 4 because the circular area that is actually exposed to solar radiation at any one time ($\pi r^2$) is 4 times smaller than the surface area of the planet that could be exposed to solar radiation ($4\pi r^2$). This gives us the value of approximately 1361 $\mathrm{Wm}^{-2}$ in total, and approximately 340 $\mathrm{Wm}^{-2}$ reaching the surface.

### 2.1.2 Intensity of local solar insolation

The exact light intensity that reaches the Earth's surface is dependent on 2 additional factors: latitude, and albedo. Because the earth's surface is spherical, the energy reaching the polar latitudes is spread out over a larger surface area, reducing the intensity. Furthermore, the earth is not a perfect blackbody so the albedo means that a proportion of the received radiation will be reflected. This leads to the following equation:

$$j = I cos\theta A$$

where j is the light intensity in $\mathrm{Wm}^{-2}$ at each latitude, $\theta$ is the average angle of the latitude band (for example between 85 and 90 degrees, $\theta = 87.5$), and A is the albedo of the latitude band. Albedo is calculated based on the composition of the latitude band in terms of water, ice / snow, and dry land. At the start of the model, every latitude band is initialised in the same way reflecting the approximate composition of the earth. Every latitude band starts off as 71% water, 12% permanent ice and 19% dry land, approximately in line with the average composition of the planet [1]. As time progresses and the model evolves, the composition changes. For example, the model assumes that every latitude with a temperature below -10° C (263K) is snow covered, thus affecting the albedo for the next iteration of the model.

## 2.2 Calculating Meridional Heat Transport

Probably the most challenging part of the model was calculating the heat transport to and from neighbouring latitude bands, known as meridional heat flow. In order to calculate this, we have to use the concepts of heat flux and heat flux density.

Heat flux ($\phi$) is defined as the rate of heat energy transfer through a given service (W) and heat flux density ($\phi_q$) is heat flux per unit area ($\mathrm{Wm}^{-2}$ [2]). We are trying to calculate heat flux density, which is given by the following equation [3]:

$$\phi_q = -k\nabla t$$

where the negative sign denotes that heat always travels from higher to lower temperature zones, $k$ is the thermal conductivity of the material (in this case the ocean or the atmosphere) in $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$ and $\nabla t$ is the temperature gradient in $\mathrm{Km}^{-1}$.

$$\nabla t = \frac{\Delta t}{d}$$

The change in temperature $t$ can be calculated using the Stefan-Boltzmann Law using $j$ which was calculated in the previous section. Then, we consider

the greenhouse effect to find $t$ for each latitude band before meridional heat transport. The Stefan-Boltzmann Law [4] is as follows:

$$j = \sigma T^4$$

Rearranging for surface temperature in kelvin gives:

$$T_{surface} = \sqrt[4]{\frac{j}{\sigma}}$$

This temperature is much colder than expected, however, due to the lack of consideration for the greenhouse effect. The effects of the greenhouse effect can be summarised in the following equation [5]:

$$T_{greenhouse} = T_{surface} \left[ \frac{1}{1 - \frac{\epsilon}{2}} \right]^{\frac{1}{4}}$$

$$T_{greenhouse} = \sqrt[4]{\frac{j}{\sigma}} \times \left[ \frac{1}{1 - \frac{\epsilon}{2}} \right]^{\frac{1}{4}}$$

Therefore $\Delta$ t is as follows:

$$\Delta t = T_{greenhouse_1} - T_{greenhouse_2}$$

The distance d between any two latitude bands can be approximated as the distance along the Earth's surface between the midpoint angle of the two latitude bands. For example, between the latitude bands (85° - 90°) and (80° - 85°), $d$ can be approximated as the distance along the Earth's surface between the latitude line at 87.5°N and the latitude line at 82.5°N. In the general case:

$$d = r \times \Delta\theta \times cos(\theta)$$

where r is the radius of the earth in m, and $\theta$ is the angle between the 2 latitude bands in radians, which for the purposes of this model will always

be $\frac{5\pi}{180}$ because this model deals with discrete latitude bands in 5-degree increments.

Meridional heat flow is primarily composed of contributions from oceanic and atmospheric circulation (Enderton, 2009) [6] in the approximate ratio of 3:7 (Wunsh, 2007) [7]. Thus, we create two new constants representing the proportional contributions: $P_{\text{AIR}} = 0.7$ & $\text{P}_{\text{OCEAN}} = 0.3$
Substituting accordingly gives:

$$\phi_q = \frac{-k\Delta t}{r \times \Delta\theta \times cos(\theta)}$$

Accounting for the relative contributions of the ocean and atmosphere and then factorising gives:

$$\phi_q = \frac{\Delta t(P_{\text{AIR}} \times -k_{\text{AIR}} + (P_{\text{OCEAN}} \times -k_{\text{OCEAN}})}{r \times \Delta\theta \times cos(\theta)}$$

$$\text{where } \text{k}_{\text{AIR}} = 0.025 \text{ and } \text{k}_{\text{OCEAN}} = 0.592 \text{ [8]}$$

which is the final equation form that I use to calculate meridional heat transport in $\text{Wm}^{-2}$.

## 2.3 Updating latitude composition

After each year of the simulation, the albedo must be updated to reflect changes in temperature. The way I handled this was as follows:

For every time increment, for every latitude band, if the average temperature of that latitude band is less than 263.15 K (-10° C), the percentage of ice will increase by a certain percentage and the percentage of water will decrease accordingly. This will continue until the percentage of water becomes too low, at which point the ice is set to 100% and the ice band is 100% iced over.

The only mathematical part of this process was determining by what percentage the ice should increase. I couldn't find any suitable data in my research, so I decided to estimate an appropriate parameter.

I found that the Antarctic Ice Sheet contains about 26.5 million cubic kilometres of ice and is losing about 150 billion tons of ice per year.

The density of ice is 916.7 kg/m$^3$ and to convert between km$^3$ and m$^3$ you simply multiply by $10^9$. Density is mass divided by volume, so the mass of the Antarctic Ice Sheet is approximately $2.429 \times 10^{19}$ kg or $2.429 \times 10^{16}$ tons (which is 99.6% accurate to the value on Wikipedia). This means that the Antarctic Ice Sheet is losing $6.175 \times 10^{-6}$% every year. Given this is accelerated by human activities quite significantly and assuming that the ice sheet would gain ice at approximately the same rate that it loses ice, in my model, for every year that the temperature of the latitude band remains below 263.15 K, the percentage of ice increases by $5 \times 10^{-6}$% and is therefore multiplied by 1.000005. The value of water decreases appropriately to accommodate this.

### 2.3.1 Other details

On a similar but not entirely related note, the Earth's climactic processes do not remain exactly the same throughout it's orbit around the sun. Because the earth's orbit is very circular but still partially elliptical, variations in the Earth's orbit called Milkankovitch cycles must be considered. In total, Milkankovitch cycles can cause variations of up to 25% of the insolation reaching the Earth's surface [11]. Variations in eccentricity (how close to a circle Earth's orbit is) occur in cycles of around 100,000 years [12] and variations in obliquity (the Earth's tilt) take place in cycles of around 41,000 years. [13]. Also, every few thousand years, $CO_2$ levels naturally fluctuate, changing the emissivity of the atmosphere. Over the past several hundred thousand years, $CO_2$ and other greenhouse gas concentrations hjave been on average rising. Because this is a very simplistic model, I included all of these effects in a simple *if* statement. Every 50,000 years, the emissivity increases by 2%, the global insolation increases or decreases by 25%, and obliquity changes apparently have no discernible effect on insolation [14]. Given more time and a better understanding of the complexity of Earth's natural cycles and patterns I would love to more accurately implement these processes, but this will have to do for now.

# 3 Overview of Model

My simulation uses a fixed time step assumed to be 1 year on Earth and can be summarised simply as follows:

1. Initialise a planet and it's relationship with its main star
2. Calculate the temperature after meridional heat transfer etc.
3. Repeat all steps as many times as required (time is initialised to 500,000 365 day years

## 3.1 Flow

### 3.1.1 Initialisation

At the start of the program, a class called Constants is created which is inherited by the main class of the model. This contains the constants which are initialised at the start of the model. The values in the constants class will never change and should ideally not be experimented with, though it is possible. These are:

```
STEFAN = 5.67e-8        # Stefan's Constant (Wm-2K-4)
ALBEDO_LAND = 0.2       # average albedo of dry land
ALBEDO_ICE = 0.85       # average albedo of ice / snow covered land
ALBEDO_WATER = 0.4      # average albedo of open water (oceans / seas)
K_AIR = 0.025           # thermal conductivity of air
K_OCEAN = 0.592         # thermal conductivity of (sea)water
```

## 3.2 Parameters

The model takes in certain parameters which are used to set up the Planet class. These are luminosity of the star, distance from the star, radius of the planet emissivity of the atmosphere, and the relative contributions of air and ocean to meridional heat transport.

```
LUMINOSITY = 3.828e26     # luminosity of the star (W) - initialised to 1
↪ L
DISTANCE = 1495978707e2   # distance from planet to star (m) - initialised
↪ to 1 AU
RADIUS = 63781e2          # radius of the planet (m)
```

```
4  EMISSIVITY = 0.78          # radius of the planet (m)
5  P_AIR = 0.7                # atmospheric contribution to meridional heat
   ↪  transport
6  P_OCEAN = 0.3              # oceanic contribution to meridional heat
   ↪  transport
7  YEARS = 500000             # length of simulation in years
```

## 3.3 Things not worth including

If you change the length of the time step in months (that is, if you make every year longer, there will be no discernible effect of results except that the temperatures of each latitude band will get closer due to more meridional heat transfer but there will be no other change in results. Therefore, even if you change the distance between the planet and the star, there is no point changing the length of the year.

# 4 Usage

```
### Libraries
import numpy as np                    # NumpPy
from matplotlib import pyplot as plt # MatPlotLib
from numpy import pi                  # Pi

class Constants:
    STEFAN = 5.67e-8        # Stefan's Constant (Wm-2K-4)
    ALBEDO_LAND = 0.2       # average albedo of dry land
    ALBEDO_ICE = 0.85       # average albedo of ice / snow covered land
    ALBEDO_WATER = 0.4      # average albedo of open water (oceans / seas)
    K_AIR = 0.025           # thermal conductivity of air
    K_OCEAN = 0.592         # thermal conductivity of (sea)water

    def __init__(self) -> None:
        pass

class Planet(Constants):

    def __init__(self, luminosity: float, distance: float, radius: float,
    ↪  emissivity: float, pAIR: float, pOCEAN: float) -> None:
        super().__init__()
        self.luminosity = luminosity
        self.distance = distance
        self.radius = radius
        self.emissivity = emissivity
        self.P_AIR = pAIR
        self.P_OCEAN = pOCEAN

        self.global_intensity = (self.luminosity / (4 * pi * self.distance
        ↪  ** 2)) / 4

        self.latitudes = np.arange(87.5, -92.5, -5)

        self.composition = [[0.65, 0.19, 0.16] for i in
        ↪  range(int(len(self.latitudes)/3))]
        self.composition.extend([[0.81, 0.0, 0.19] for i in
        ↪  range(int(len(self.latitudes)/3))])
        self.composition.extend([[0.65, 0.19, 0.16] for i in
        ↪  range(int(len(self.latitudes)/3))])

        self.albedos = [(i[0] * self.ALBEDO_WATER + i[1] * self.ALBEDO_ICE
        ↪  + i[2] * self.ALBEDO_LAND) for i in self.composition]
```

```
37          self.distances = [(self.radius * 5 * pi / 180 *
            ↪  np.cos(np.radians(theta * pi / 180)))/1000 for theta in
            ↪  self.latitudes]
38
39      def local_intensity(self):
40          latitudes = list(self.latitudes)
41          local_intensity_array = [(self.global_intensity *
            ↪  np.cos(np.radians(theta)) *
            ↪  self.albedos[latitudes.index(theta)]) for theta in latitudes]
42
43          return np.array(local_intensity_array)
44
45      def surface_temperatures(self, intensity_array: list):
46          surface_temperatures_array = [((j / self.STEFAN) ** 0.25) for j in
            ↪  intensity_array]
47
48          return np.array(surface_temperatures_array)
49
50      def delta_temperatures(self, temperatures_array: list):
51          temperatures_array = list(temperatures_array)
52          delta_temps = []
53          one_to_eighteen = []
54          nineteen_to_thirty_five = []
55
56          for i in range(1, 19):
57              delta_t = temperatures_array[i] - temperatures_array[i-1]
58              one_to_eighteen.append(delta_t)
59
60          for i in range(18, 36):
61              delta_t = temperatures_array[i-1] - temperatures_array[i]
62              nineteen_to_thirty_five.append(delta_t)
63
64          delta_temps.append(one_to_eighteen)
65          delta_temps.append(nineteen_to_thirty_five)
66
67          return np.array(delta_temps)
68
69      def meridional_heat_transport(self, delta_temperatures_array: list):
70          delta_temperatures_array = list(delta_temperatures_array)
71          meridional_heat_transport = []
72          lst1 = []
73          lst2 = []
74
75          for t in delta_temperatures_array[0]:
```

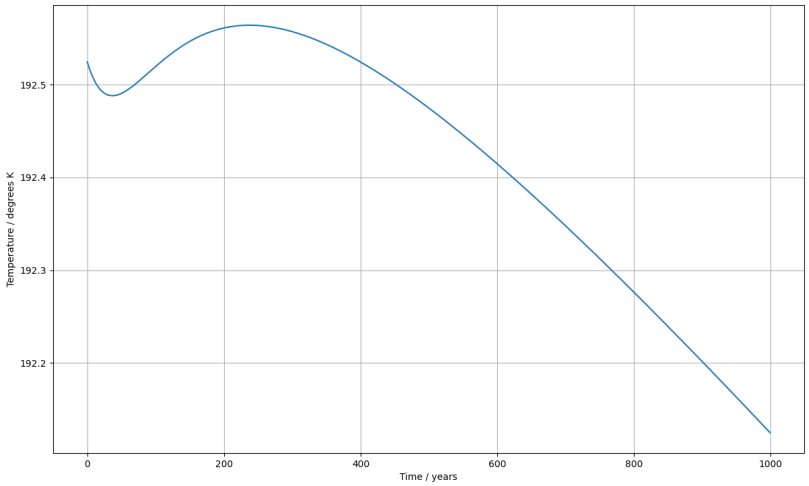
```
76              distance =
                ↪  self.distances[list(delta_temperatures_array[0]).index(t)]
77              phi = (t * ((self.P_AIR * -1 * self.K_AIR) + (self.P_OCEAN *
                ↪  -1 * self.K_OCEAN)))/distance
78              lst1.append(phi)
79
80          for t in delta_temperatures_array[1]:
81              distance =
                ↪  self.distances[list(delta_temperatures_array[1]).index(t)]
82              phi = (t * ((self.P_AIR * -1 * self.K_AIR) + (self.P_OCEAN *
                ↪  -1 * self.K_OCEAN)))/distance
83              lst2.append(phi)
84
85          meridional_heat_transport.extend(lst1)
86          meridional_heat_transport.extend(lst2)
87
88          return np.array(meridional_heat_transport)
89
90      def greenhouse_temperatures(self, temperatures_array: list):
91          greenhouse_temperatures = [temp * ((1 / (1 - self.emissivity / 2))
            ↪  ** 0.25) for temp in temperatures_array]
92
93          return np.array(greenhouse_temperatures)
94
95      def update_intensity(self, input_array: list,
        ↪  meridional_transfer_array: list):
96          input_array = list(input_array)
97          for i in range(len(input_array) - 1):
98              if i == 1:
99                  input_array[i] -= meridional_transfer_array[i]
100             elif i <= int(len(input_array) / 2):
101                 input_array[i] += meridional_transfer_array[i-1]
102                 input_array[i] -= meridional_transfer_array[i]
103             elif i > int(len(input_array) / 2) and i < len(input_array):
104                 input_array[i] += meridional_transfer_array[i+1]
105                 input_array[i] -= meridional_transfer_array[i]
106             elif i == len(input_array):
107                 input_array[i] -= meridional_transfer_array[i]
108
109         return input_array
110
111     def update_albedos(self, latitude_temperatures_dictionary: dict):
112         ice_list = []
113         for latitude in latitude_temperatures_dictionary:
```

```
114             if latitude_temperatures_dictionary[latitude][-1] > 263.15:
115                 ice = False
116             else:
117                 ice = True
118             ice_list.append(ice)
119         for i in self.composition:
120             if not ice_list[self.composition.index(i)]:
121                 i[0] = 0.81
122                 i[1] = 0.0
123             else:
124                 update = i[1] * 0.00005
125                 if latitude_temperatures_dictionary[latitude][-1] >
                    ↪  latitude_temperatures_dictionary[latitude][-2]:
126                     i[0] += update
127                     i[1] -= update
128                 else:
129                     i[0] -= update
130                     i[1] += update
131
132 def plot_latitude_temp(time: list, latitude_temps: dict, latitude):
133     plt.plot(time, latitude_temps[latitude])
134     plt.xlabel("Time / years")
135     plt.ylabel("Temperature / degrees K")
136     plt.grid(True)
137     plt.show()
138
139 def plot_global_temp(time: list, average_temps: np.ndarray):
140     plt.plot(time, average_temps)
141     plt.xlabel("Time / years")
142     plt.ylabel("Temperature / degrees K")
143     plt.grid(True)
144     plt.show()
145
146 def model():
147     LUMINOSITY = 3.828e26       # luminosity of the star (W) - initialised
        ↪  to 1 L
148     DISTANCE = 1495978707e2     # distance from planet to star (m) -
        ↪  initialised to 1 AU
149     RADIUS = 63781e2            # radius of the planet (m)
150     EMISSIVITY = 0.78           # radius of the planet (m)
151     P_AIR = 0.7                 # atmospheric contribution to meridional
        ↪  heat transport
152     P_OCEAN = 0.3               # oceanic contribution to meridional heat
        ↪  transport
```

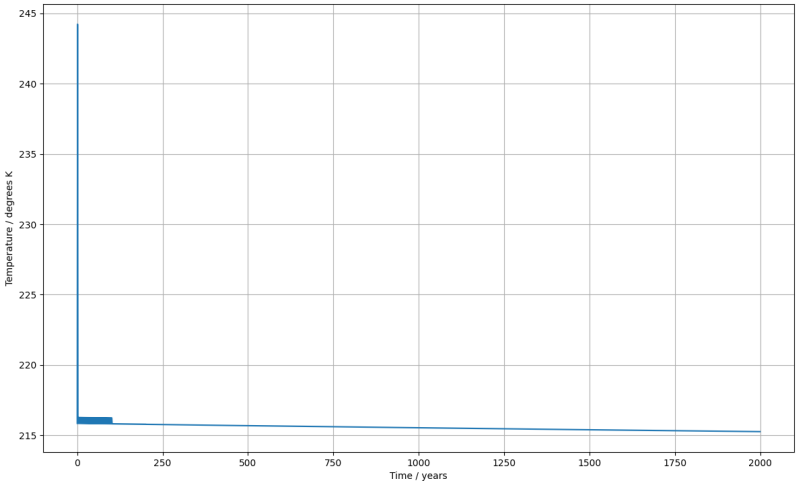
```
153     YEARS = 1000                # length of simulation in years
154
155     earth = Planet(luminosity=LUMINOSITY,
156                    distance=DISTANCE,
157                    radius=RADIUS,
158                    emissivity=EMISSIVITY,
159                    pAIR=P_AIR,
160                    pOCEAN=P_OCEAN)
161
162     average_temps_array = []
163     latitude_temps_dictionary = {i:[] for i in (np.arange(87.5, -92.5,
        ↪  -5))}
164
165     local_insolation_array = earth.local_intensity()
166
167     year = 0
168
169     while year < YEARS:
170         surface_temps_array =
            ↪  earth.surface_temperatures(intensity_array=local_insolation_
171         array)
172         greenhouse_temps_array =
            ↪  earth.surface_temperatures(intensity_array=local_insolation_
173         array)
174
175         average_temps_array.append(np.mean(greenhouse_temps_array))
176
177         for i in earth.latitudes:
178             latitude_temps_dictionary[i].append(greenhouse_temps_array
179             [list(earth.latitudes).index(i)])
180
181         for i in range(12):
182             delta_temps_array =
                ↪  earth.delta_temperatures(temperatures_array=greenhouse_
183             temps_array)
184
185             meridional_heat_transport_array =
                ↪  earth.meridional_heat_transport(delta_temperatures_array
186             =delta_temps_array)
187
188             local_insolation_array =
                ↪  earth.update_intensity(input_array=local_insolation_array,
189             meridional_transfer_array=meridional_heat_transport_array)
190
```

```
191         surface_temps_array =
            ↪  earth.surface_temperatures(intensity_array=local_insolation_
192         array)
193         greenhouse_temps_array =
            ↪  earth.greenhouse_temperatures(temperatures_array=surface_temps_
194         array)
195
196         for i in earth.latitudes:
197             latitude_temps_dictionary[i].append(greenhouse_temps_array
198             [list(earth.latitudes).index(i)])
199
200         earth.update_albedos(latitude_temperatures_dictionary=latitude_
201         temps_dictionary)
202
203         change_intensity = 0.25 * earth.global_intensity
204
205         if year % 50 == 0 and (year / 50) % 2 == 1:
206             earth.emissivity *= 0.02
207             earth.global_intensity += change_intensity
208         elif year % 50 == 0 and (year / 50) % 2 == 0:
209             earth.emissivity *= 0.02
210             earth.global_intensity += change_intensity
211
212         year += 1
213
214     return np.array(average_temps_array), latitude_temps_dictionary
215
216 plot_global_temp(time=range(1000),
217                  average_temps=model()[0])
218
219 plot_latitude_temp(time=range(2000),
220                    latitude_temps=model()[1],
221                    latitude=42.5)
```

# 5 Data Analysis




Global Average Temperature




Average Temperature of 2.5 degrees North latitude band

Clearly the results produced by the model aren't wildly implausible but they are definitely not accurate to the current planet. The temperature seems to be consistently decreasing with occasional spikes despite the increeases in

insolation caused by variations in eccentricity. I would like to think that 1 or 2 thousnad years is simply too small a timescale, and given longer to run (which I unfortunately don't have enough time for), the temperatures such correct themselves and eventually arrive at around 280 - 290K, which is the current average temperature.

However, it could also be that this model is simply too simplistic and lacks enough detail to make it viable. The results are correct to the nearest order of magnitude (which is a large jump from 200K to 280K in terms of life-sustaining temperatures), but perhaps there have simply been too many shortcuts and assumptions taken regarding the maths and modelling.

The data is around 70 - 75% accurate. In terms of machine learning, that would be considered a great model [15], however I think that even working within the constraints of this prize, the model could be improved a lot to make its accuracy more reliable. A longer runtime capacity would significantly aid this.

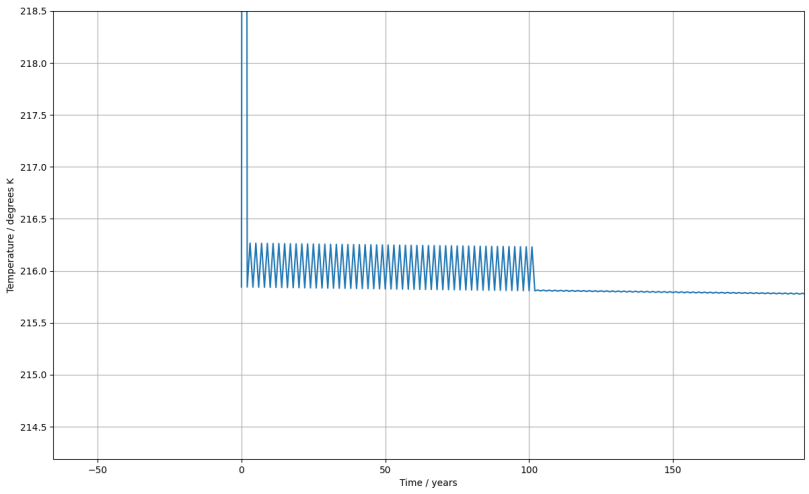
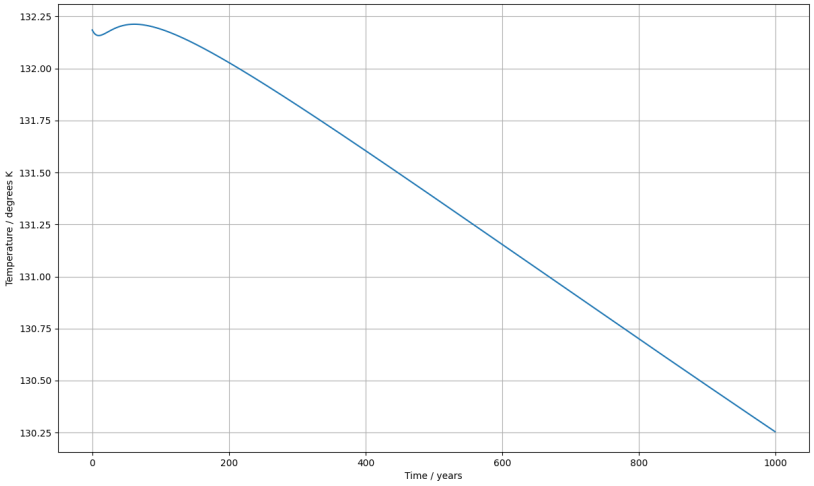
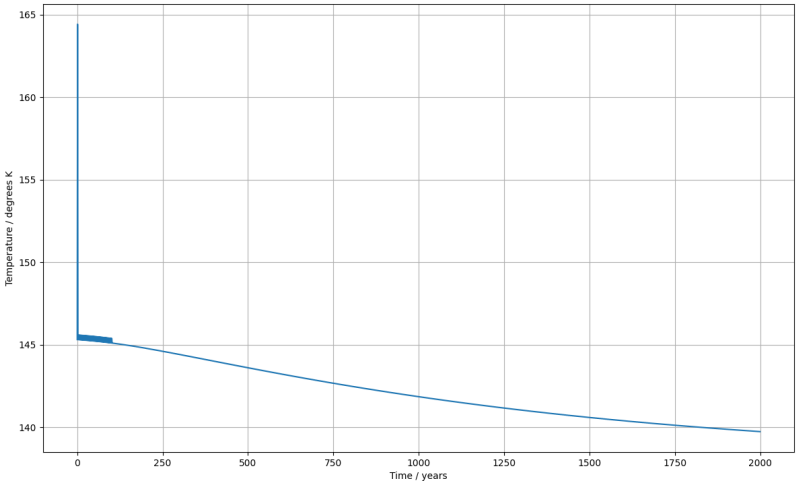


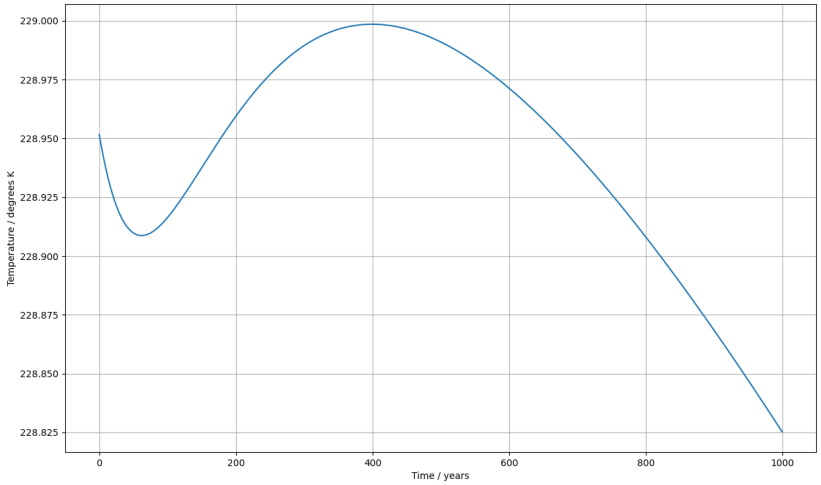

Figure 2 zoomed in

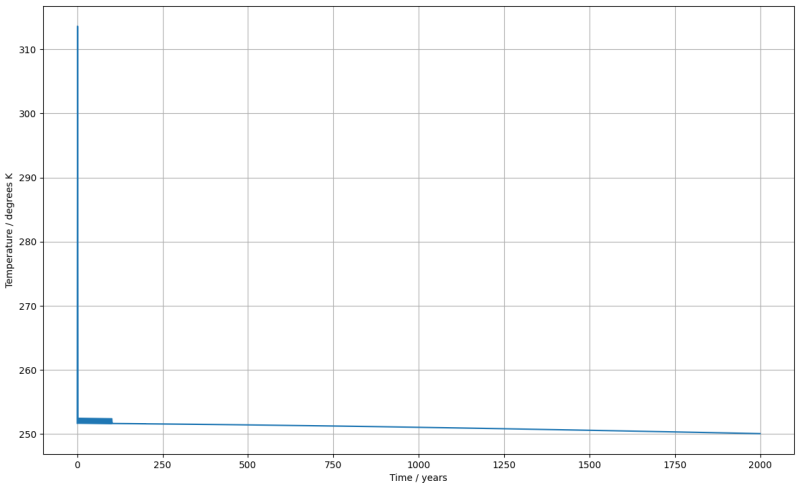
## 5.1 Varying Parameters




When the luminosity is doubled, but the distance is tripled, and the emissivity increases by 25% the above results are produced




When the luminosity is doubled and the emissivity increases by 25% the above results are produced

# 6 Conclusion and Final Thoughts

In any case, the results this model produces are fairly accurate, but the trends, at least with the limited runtime are not looking promising. The average global temperature at the end of 1 run of the simulation is around 73% accurate, however, I believe that the backbone and mathematics behind the model is all sound, and simply small errors in implementation or the assumptions which I took to avoid the model becoming too complex to handle have limited its success. Perhaps working with a partner would yield better results as 2 minds to deal with the complexity and implementation would have probably eliminated some minor errors and together, I and a partner could perhaps have come up with better ways to implement this project.

Given my limited knowledge of Python and best practices, there were probably better ways of implementation but I think that this model performs satisfactorily to well in that 75% accuracy is pretty high given how many assumptions and simplifications I made. I would hope that the model produces better results when run for longer, but there is always the possibility that the downward trend continues and the accuracy decreases. However, global temperatures have reached lows of 150 to 160 Kelvin in prehistory, so perhaps comparing my results of an earth that is only 1000 years old to today's earth of 4.6 billion years creates an unfair margin. This project could be improved but given my limited knowledge of python, geography and physics, I believe that this was a good first attempt at modelling the climate, an undoubtedly very complex task. Given more time, I would like to further investigate the effects of time and also change certain parameters to get a better understanding of their effect on the climate.

# 7 Bibliography and Notes

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https://education.seattlepi.com/percent-earth-permanently-covered-snow-ice-4666.html

[2] This definition is taken and paraphrased from https://www.sciencedirect.com/topics/chemical-engineering/heat-flux

[3] This equation is the differential form of Fourier's law, according to https://en.wikipedia.org/wiki/Thermal_conduction#Fourier's_law

[4] https://en.wikipedia.org/wiki/Stefan–Boltzmann_law

[5] The full derivation of this equation can be found at https://en.wikipedia.org/wiki/Idealized_greenhouse_model

[6] Contributions from the cryosphere and lithosphere are much lesser according to Enderton, 2009 and are therefore considered negligible in my model to decrease the complexity and make the model easier to deal with. https://core.ac.uk/download/pdf/4412312.pdf

[7] "Ocean circulation is probably responsible for 30% of the poleward transport of heat, leaving a contribution of 70% for the atmosphere (Wunsch, 2005), quoted in Rummel, 2020

[8] https://en.wikipedia.org/wiki/List_of_thermal_conductivities

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