A PROPOSED OPTIMISATION FRAMEWORK FOR FOREIGN AID ALLOCATION IN KENYA

DEMO DAY SUBMISSION

Eric Ayivor

Non Trivial Fellow e.ayivor@icloud.com

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ABSTRACT

This project develops a simulation-based optimisation framework for the strategic allocation of foreign aid across six development sectors in Kenya—healthcare, education, agriculture, infrastructure, technology, and transportation. The model tracks sectoral resilience, governance quality, absorptive capacity, and national income (GNI per capita, PPP) over time, aiming to maximise long-term development while ensuring efficient use of limited aid resources. Key features include sector-specific effectiveness coefficients, elasticities linking sector improvements to GNI, dynamic absorptive capacity, and governance-linked performance adjustments. A Monte Carlo simulation generates thousands of plausible aid pathways to identify optimal strategies under uncertainty. Key simulation outputs include: minimum aid required to graduate from dependency, time to graduation under fixed budgets, optimal annual sector allocations and results from the simulation indicate that

1 Introduction

Foreign aid remains one of the most prominent policy tools for addressing global inequality, poverty, and underdevelopment. Since the 1960s, global official development assistance (ODA) has increased nearly sixfold, reaching over US\$230 billion in 2024 (ONE, 2024). Yet, this vast investment has yielded underwhelming results in recipient countries, with many nations showing only modest improvements in key indicators like the Human Development Index (HDI), GNI per capita, or sector-specific resilience (UNDP, 2023). For example, Ghana's HDI has increased just 32.6% since 1990, despite decades of foreign assistance (Oxfam, 2023). Numerous studies have highlighted the inefficiencies and fragmentation in aid allocation: resources are often misaligned with recipient needs, donor coordination is poor, and sectoral allocation is driven more by politics than evidence (Easterly, 2006; Moyo, 2009; Deaton, 2013).

1.1 Literature review and related works

Recent research has tried to correct this using sector-specific analysis. Michaelowa & Weber (2007) and Dreher et al. (2008) find positive but often modest returns to education aid, while Hussain (2016) estimates aid elasticity for multiple sectors and income groups. Meanwhile, Feeny (2024) employs constrained optimisation to model COVID-19 aid distribution, offering a proof of concept for policy-based quantitative methods. However, few if any studies systematically integrate cross-sector resilience, economic growth elasticities, and governance dynamics into a coherent optimisation framework for aid allocation within countries. Likewise, while simulations and regressions are widely used in development economics, the use of constrained optimisation combined with dynamic simulation remains underdeveloped for foreign aid applications.

1.2 Novel contribution and project aims

This project proposes a new framework for maximising the welfare gains of foreign aid by modelling its optimal allocation across key economic sectors. Unlike most aid studies that focus either on country-to-country flows or single-sector interventions, this model takes a bottom-up, sectoral approach within one recipient country – Kenya – and uses constrained optimisation, economic elasticity modelling, and Monte Carlo simulation to determine how aid should be distributed to enable full "graduation" from foreign aid dependency.

Specifically, the model integrates six core sectors: healthcare, education, agriculture, infrastructure, technology, and transport. For each, a sectoral resilience index is calculated based on composite World Bank and WHO indicators, scaled from 0 to 1. Each sector also has an associated aid effectiveness coefficient, a domestic progress constant, and an elasticity (hat relates resilience improvements to growth in GNI per capita. The model incorporates absorptive capacity constraints and governance responsiveness, with the latter both affecting and being affected by aid and economic growth. The entire framework is embedded in a simulation that determines how aid, constrained by an annual or total budget, should be distributed to ensure sectors reach resilience thresholds as efficiently as possible.

The novelty lies in three areas:

- The integration of sector-specific aid effectiveness, diminishing returns, and absorptive constraints within a single, formal optimisation and simulation framework.
- The dynamic updating of governance and population, allowing for endogenous feedback loops between institutional quality, effective aid absorption, and economic returns.
- The use of real-world estimated aid elasticities and historical indicator data to calibrate the model and validate simulation outputs against plausible country trajectories.

1.3 Theory of change

This project is built on the central belief that foreign aid should be a temporary accelerator, not a perpetual subsidy. The ultimate objective is to guide Kenya, and later other countries, toward full graduation from aid dependency by strategically allocating resources to sectors that most effectively build long-term resilience and national income.

Problem

Despite decades of foreign assistance, many low-income countries, including Kenya, remain reliant on aid. Poor allocation decisions – driven by politics or donor priorities rather than sector effectiveness – result in wasted opportunities and suboptimal development trajectories.

Assumptions

- Aid effectiveness varies significantly across sectors and over time.
- Governance quality and absorptive capacity constrain the real impact of aid.
- Long-term economic self-sufficiency requires sectoral resilience, not just short-term outputs.
- Improvements in key sectors (healthcare, education, infrastructure, etc.) causally influence economic growth and institutional quality.

Inputs

- A fixed or dynamic aid budget (e.g., \$150 million/year)
- Historical estimates of sector effectiveness, governance responsiveness, and sector-to-GNI elasticities

Initial sectoral resilience scores and demographic-economic baselines

Activities

- Allocate aid across six priority sectors based on an optimisation framework and stochastic simulations
- Update sector resilience annually based on aid effectiveness, absorptive constraints, and governance
- Model interactions between aid, sector growth, governance improvement, and national income
- Apply a graduation threshold: when all sectors are self-sustaining (e.g. resilience ≥ 0.9), aid is no longer needed

Outputs

- Time series projections of resilience, GNI per capita, and governance scores
- Optimal aid allocation strategy that minimises time to graduation
- Scenario analyses showing trade-offs between rapid graduation and sectoral balance

Intermediate Outcomes

- Increased sectoral resilience and public service reliability
- Rising governance quality through better-performing institutions
- Accelerated GNI growth driven by more productive and resilient sectors

Outcome

Kenya reaches self-sufficiency in all six core sectors and surpasses a minimum GNI per capita threshold or average sector resilience threshold, enabling it to graduate from foreign aid and finance its development sustainably through domestic revenue, private investment, and regional integration.

Why This Theory Matters

Unlike traditional aid studies that focus on short-term KPIs (e.g. school enrolment, vaccination rates), this model links sector resilience to structural economic growth, ensuring that aid interventions build lasting systems rather than temporary improvements. It also includes feedback loops: better governance boosts absorptive capacity, which in turn amplifies aid effectiveness. This theory of change makes clear that success is not measured by dollars spent, but by the speed and sustainability of a country's transition off aid.

1.4 Dependent variable choice

The choice of dependent variable is a key part of this project, and a distinguishing factor from other related research which often seeks to maximise or examine the impact of aid on GDP or GNI growth exclusively, or alternatively chooses a single indicator to examine the impact of.

At the heart of this research is the question of how foreign aid can be optimally distributed to maximise real, long-term improvements in welfare. To answer this, the model must do more than track short-term outputs like enrolment numbers or mortality rates; it must target structural transformation within the recipient country. This is where sectoral resilience becomes central to the optimisation problem.

Sectoral resilience is defined as the capacity of each sector to deliver services reliably, adapt to shocks, and continue improving without ongoing external support. In other words, resilience is a proxy for sustained progress and self-sufficiency. By focusing on increasing resilience, the model moves beyond stopgap interventions and instead prioritises investment strategies that enables recipient countries to eventually reduce or eliminate their dependence on aid. Resilience plays a central role in this project because it captures not just current capacity, but the ability of sectors to sustain progress and absorb shocks over time. Unlike short-term output indicators, resilience provides a structural measure of whether a sector can continue functioning and improving without continual external support. This makes it a natural intermediate target for optimisation: by maximising sectoral resilience across the economy, we indirectly maximise long-run self-sufficiency and reduce the future need for foreign aid.

Moreover, sector-specific resilience scores allow aid to be dynamically reallocated toward the most fragile or impactful sectors at each time step. This is especially useful in constrained environments, where strategic prioritisation is essential. By integrating resilience into the optimisation objective, we ensure the model aligns with both donor goals (cost-effectiveness, graduation) and recipient needs (sustainability, stability).

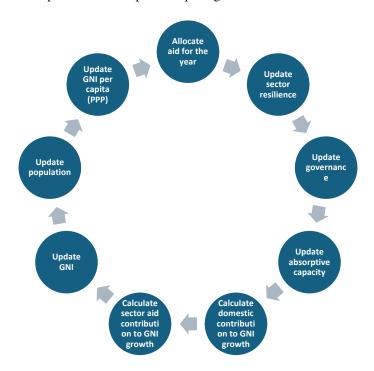
Mathematically, resilience serves as an intermediate variable connecting aid flows to macroeconomic outcomes like Gross National Income (GNI) per capita. Sector-specific elasticities translate changes in resilience into expected changes in economic output, creating a mechanism through which foreign aid can be compared and evaluated in a more traditional manor with other studies or broader policy comparisons. Moreover, the model introduces a clear graduation point after which aid is no longer needed by the introduction of resilience thresholds which can be customised, aligning with the policy goal of minimising total aid while maximising lasting impact.

2 Mathematical model

To give a high-level overview of what is going on in the model, the following steps are taken



Once sectoral aid proportions are randomly drawn for the year as detailed later, the model runs the year-by-year update loop described in step 4 above. The update loop's logic is described below:



2.1 Estimating aid budget

The model supports 3 methods for the initial and annual aid budgets. For determining the initial budget, depending on the goal one of the following scenario-based strategies can be applied:

- 1. Constant aid budget, the same amount is allocated every year for a fixed time period
- 2. Decreasing budget, simulating donor fatigue (Awadari, 2020)
- 3. Dynamic budget, responding to changes in governance and absorptive capacity

For determining the annual aid budget available for each simulation iteration if dynamic budget is chosen:

1. Governance-weighted budget, the budget is results-based and as governance and absorptive capacity increases, the country receives more aid (while it is counter intuitive that better performing countries receive more aid at the expense of struggling ones, this is how many real budgets tend to be structured)

2. Capped budget, the budget is dynamically allocated each year from a fixed budget pool agreed upon at the start of the program

Table 1: Symbol Table of Key Notation for the Mathematical Model

Symbol	Description
\overline{S}	The set of all sectors available to receive foreign aid (healthcare, education,
	agriculture, infrastructure, technology, transport)
i	Each individual sector available to receive foreign aid
$A_{ m total}$	The total aid budget available to the recipient country (USD)
$a_i(t)$	The aid proportion dedicated to sector i at time t
$R_i(t)$	The resilience of sector i at time t (resilience indicator points)
R*	The resilience threshold after which aid is no longer required and the sector is considered graduated
α	Macroeconomic absorptive capacity (% of received aid converted into usable, effective aid; efficiency)
β_i	Aid effectiveness coefficient in sector i (% increase in resilience from \$1 million aid)
$\gamma(t)$	The governance score of the entire economy at time t
δ_i	The domestic progress constant for sector i (expected points increase in resilience per year in the absence of aid)
E_i	The elasticity of $\%$ change in GNI to a 1-point increase in resilience of sector i
θ	The elasticity of % change in GNI to a 1-point increase in governance score
T	The total time of the simulation until graduation (years)
n	Number of Monte Carlo iterations

2.2 Initialising the model

Sectors

The set of sectors available to receive aid in this model comprises healthcare, agriculture, infrastructure, technology, transportation, and education. These sectors were selected because they collectively underpin long-term socioeconomic development and national self-sufficiency, particularly in the context of low- and middle-income countries like Kenya. Each sector plays a dual role: they are immediate drivers of human welfare (e.g. through health outcomes, food security, or basic literacy) and structural enablers of economic resilience, enabling individuals and firms to adapt, grow, and contribute productively to the economy.

Empirical research and historical case studies support their inclusion. For example, World Bank development indicators consistently show that improvements in education and infrastructure have among the highest elasticities with respect to GDP growth in sub-Saharan Africa (Calderón & Servén, 2010). Similarly, agriculture remains the largest employer in Kenya and a critical source of rural livelihoods and food security, meaning that even marginal improvements can translate into widespread welfare gains. Technology and transportation systems, while less mature, are foundational to long-term productivity and integration into global markets.

Sectors such as energy, financial services, and the environment were carefully considered but excluded from this initial model. The decision was primarily due to data limitations and the need to maintain a strategically scoped model that prioritises the most direct and quantifiable channels for improving GNI per capita and sectoral resilience. Many of these omitted sectors are important and often interlinked with those selected (e.g. energy access is critical for education and healthcare outcomes), but are treated here as enabling constraints rather than primary aid targets. Future model iterations could integrate these sectors where robust, comparable metrics of resilience and aid effectiveness are available. The sectors included here directly affect human capital, productivity, and economic resilience, making them priority targets for aid allocation.

Sector resilience

The resilience of each sector i is defined as R_i and is the target variable in the optimisation problem. Sector resilience refers to the ability of each sector to deliver reliable services and improve in the long-run, as well as withstand and recover from disruptions and shocks. Each sector i has a resilience score $R_i(t)$ at time t ranging from 0 (no capacity) to 1 (fully self-sustaining). To illustrate, the technology sector would have a resilience index of 0 in a country with no basic technology, internet access, and low-to-no R&D presence (most of Sub-Saharan

Africa pre-2000. A score of 1 is a country with reliable tech infrastructure, strong internet penetration, regular innovation output, etc.

To determine sector resilience in a useful way, each sector is given a composite index score based on several World Bank World Development indicators, indices, and other macroeconomic variables, which is then normalised on a 0 to 1 scale for easy comparison, using data from 2000 to 2021 where available.

Indicators are generally normalised in one of the following three ways:

- 1. **1+**: the indicator is transformed from its original scale to a 0 to 1 scale, and a greater number represents a higher score
- 2. **1-**: the indicator is such that a lower number represents a higher score, so the indicator is transformed to a 0 to 1 scale and 1 minus the result is taken
- 3. **Min-max(target_country):** for indicators where achieving a score of 100% is not necessary to demonstrate strong sector resilience (e.g. the UK and USA have internet penetration rates of 95.3% and 97.1% respectively and are not seeking to improve), the strongest comparable country in the region is used as a target score, and the indicator is normalised so the max value is the target country's score (or min value for negative indicators).

For many (not all) min-max indicators, Seychelles is used as the max value, due to its low dependence on foreign aid and strong scores across a range of economic, quality of life, and governance scores compared to other African countries.

Healthcare

The healthcare resilience index is an average of the following indicators:

- Universal Health Coverage service index (UHC), which combines 14 tracers of healthcare service coverage into 1 indicator. The highest score in Africa is Seychelles with 75 (2021), used as the max value
- Newborns protected against tetanus (%), which is a simple 1+ indicator where 100% protection is the target
- Life expectancy (years), where Seychelles is used as the max value with 73.4 years (2021)
- Infant mortality rate (per 1000 live births), which is a negative indicator where Seychelles is again used as the target value with an infant mortality of 13.5 per 1000 live births.

<u>Agriculture</u>

• Agriculture, forest, and fishing, value added (% of GDP), with Ethiopia used as the target value of 37.58% (2021) due to its strong agricultural presence, having the highest number employed in the sector in Africa as of 2024

<u>Infrastructure</u>

- Africa Infrastructure Development Index (AIDI), which received the highest weight in the composite indicator and is a 0-100 index of infrastructure development available from 2003 onwards
- Access to electricity (%), which is a simple 1+ indicator where 100% access is the target
- Access to clean fuels and technologies for cooking (%), again a simple 1+ indicator
- Renewable electricity output (% of total electricity output), a simple 1+ indicator
- Account ownership at a financial institution or with a mobile-money-service provider (% of population ages 15+), a simple 1+ indicator
- World Bank Starting a business score, which is already normalised from 1-100

Technology

- **Individuals using the Internet (%),** where Seychelles is once again the target score at 87.3% internet penetration (2021)
- Secure internet servers (per 1 million people), where the UK is used as the target score as most other sub-Saharan African countries are too low to be a target, and Seychelle's low (~150,000) population causes an overestimate of servers per million people
- **AIDI ICT composite index**, a subindex from the Africa Infrastructure Development Index focused on ICT indicators

Transport

• **AIDI Transport composite index**, a subindex from the Africa Infrastructure Development Index focused on transport-related indicators

Education

• UN HDI Education Index, a subindex from the Human Development Index which tracks educational attainment by measuring average adult years of schooling and expected years of schooling for students under the age of 25

Governance score

The governance of a country clearly affects the effectiveness of foreign aid; corrupt governments and inefficient ones are much less likely to make effective use of each dollar of aid received. The governance of a country is estimated using a simple average of 2 World Bank World Governance Indicators, adjusted (originally -2.5 to +2.5, translated to 0 to 5 scale, normalised to 0-1 scale).

- Government Effectiveness
- Control of Corruption

From this point, where the phrase "index point" is used, it refers to 0.01 of the index, which are all normalised from 0 to 1.

GNI per capita, PPP adjusted

Gross National Income per capita (PPP) is the preferred indicator of standard of living within recipient countries and is freely and widely available. Per capita and PPP adjustments are to control for population size and costs of living, respectively, but GNI is chosen as a preferred indicator to GDP for the following reasons:

- GNI includes income from abroad (e.g. remittances, foreign aid inflows, income earned by citizens overseas minus income sent to foreigners working in the country) whereas GDP only the value of all final goods and services produced within the country. This matters as countries like Kenya often have large remittance flows, foreign aid, and income from overseas workers, which significantly raises the money available to citizens
- **GNI reflects income, not just output:** GDP includes foreign firms' profits operating locally (e.g. Shell in Nigeria), but GNI excludes profits repatriated abroad and thus only measures income available to citizens and government.

Macroeconomic absorptive capacity

The theory of absorptive capacity states that there are diminishing returns to foreign aid, and that there is a constraint or threshold after which foreign aid is at best pointless and at worst disruptive. Macroeconomic absorptive capacity is the ability of a country to effectively and sustainably use external financial resources (e.g. foreign aid) without generating economic distortions, such as inflation, exchange rate overvaluation, or aid dependency. It reflects how much aid the economy can absorb before aid starts causing diminishing returns or negative spillovers. Really, this should be a threshold such that as the aid starts to experience diminishing returns as it approaches the threshold, but for simplicity, it is implemented in this model as a multiplicative effect reducing the total amount of aid to an available effective amount.

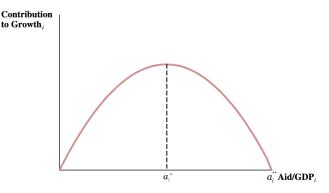


Fig. 1. The relationship between foreign aid and economic growth. Source: <u>Feeny and McGillivray (2011)</u>.

Estimates of absorptive capacity vary wildly from roughly 85% - "The losses attributed to this factor are put at 15 percent of the aggregate volume of foreign aid" (The economist, 1967) – to much lower, but to estimate Kenya's absorptive capacity, we use the CIAC1 framework proposed by Feeny, de Silva (2012). This index is composed of capital factors, government factors, and donor factors as outline in the paper. While criticised slightly for equal weightings of different components, the method is effective enough for the first iteration of this model.

Following this method (where indicators are readily available) leads to $\alpha \approx 24.47797299$ on a normalised 0 to 100 scale. Noting the slight unreliably in an evenly weighted approach, but given that certain indicators using by Feeny and de Silva are unable to be found and would take significant time to manually compute, and given that some indicators were from 5-15 years ago, we propose to use lpha=0.3 as a reasonable starting point for the model (albeit much less than the predicted value from the Economist). Details of this calculation can be found in the Github.

This is in line with many of the sub-Saharan African CIAC scores e.g. Liberia 0.28, Sierra Leone 0.28, Rwanda 0.25, Tanzania 0.33.

Graduation

A sector is determined to have graduated when its resilience is equal to or exceeds a target self-sustainable resilience R*, which is defined in this model to be 0.9. When all sectors in a country meet or exceed R*, the country is determined to have graduated and aid to the country ceases in the optimal scenario.

Estimating parameters

Details of justifications for the methods used to estimate these parameters can be found in 2.6.

Sector elasticity

The elasticity of each sector i is defined as E_i and represents the responsiveness of PPP-adjusted GNI per capita (abbreviated from this point as simply GNI) to a change in the sector resilience of i. Elasticity allows us to measure the sensitivity of GNI growth to improvements in each sector's resilience, which is represented by a composite indicator normalised on a 0-1 scale as defined above. We want this in the most easily interpretable form: a 1-point increase in the sector resilience indicator should correspond to an X% increase in GNI, leading us to a log-linear regression model, assuming there is a linear correlation between each sector's resilience and GNI. For this project, we propose the following method to estimate the elasticity GNI to each sector:

 E_i gives the % change in GNI from a 1-point increase in resilience so:

$$E_i \approx \frac{\Delta R_i}{R_i}$$

 $E_i \approx \frac{\Delta R_i}{R_i}$ This leads to the following equation, which is demonstrated to be consistent in Appendix 1.

$$\ln(GNI) = \alpha + \sum_{i} E_{i} \cdot R_{i} + \epsilon \tag{1}$$

We can now use this equation to estimate the elasticities of each sector by running an Ordinary Least Squares regression of the GNI data collected about Kenya from 2000 to 2021 and the composite indicators determined in 2.2 for the same time period. Details of the Python script are given in the linked Github, but the following results are determined.

Running the OLS regression gives the following results for sector elasticities:

Sector	Elasticity, E_i (4dp)
Healthcare	-0.1157
Agriculture	0.6616
Infrastructure	-0.6465
Technology	0.5770
Transport	0.7902
Education	3.2929

These results are puzzling, seeming quite unrealistic. The most impactful sector being education is somewhat reasonable as it clearly boosts human capital, but a 3.29% increase in GNI from just 1 percentage point increase in education seems ludicrously high. Similarly, technology is at the forefront of most significant economic activity today and it should likely have a higher number. Other traditional sectors which might be expected to have a much stronger effect in developing, less urban economies have relatively low or even negative elasticities such as infrastructure and transport. Analysis of the regression outputs reveal an abnormally high condition number of 209 which implies strong multicollinearity.

This is not unsurprising as many of these sectors are linked, and one would expect strong collinearity. As the education industry strengthens, for example, naturally healthcare will improve as medical professionals are more skilled, have higher quality information, and research output increases. Similarly, technology is both influenced by and influences education, as well as having strong correlations with transport and infrastructure.

The mode has likely overfit the data, leading to large swings in coefficient estimates from any small changes in input data, causing the implausible elasticities. One method to avoid this was to combine related variables, but a key part of this project was examining sectors individually to see which sector allocations produce the most beneficial results, so dropping/combing variables is slightly annoying.

To get around this, we propose the use of ridge regression which is useful in scenarios with highly correlated independent variables. Following the adjustment, the following, more reasonable results are outputted:

Sector, i	Elasticity, E_i (4dp)
Healthcare	0.0136
Agriculture	0.0339
Infrastructure	0.0083
Technology	0.0397
Transport	0.0100
Education	0.0612

Technology remains a strong contributor, and as expected education remains the single highest elasticity, but agriculture and transport have stronger elasticities, which is more reasonable for a developing country as they are likely to both have high employment in the agricultural sector and very poor transport systems so much space to grow without suffering diminishing returns.

Aid effectiveness coefficient

While sector elasticity models how an increase in sector resilience affects GNI, the aid effectiveness coefficient for a sector, β_i , shows how the sector resilience responds to each marginal dollar of financial aid.

Using the fixed effects regression for low-income countries from Hussain (2016), the coefficient between agricultural aid and log agricultural productivity is 0.01693. This means a \$1 million increase in agricultural aid is associated with a 0.01693 unit increase in log productivity, which we can use as a proxy measure for agricultural sector resilience to save compute time.

Taking the exponent on both sides gives 1.01707, implying that there is a 1.707% increase in productivity per \$1 million aid, assuming ceteris paribus applies for all other factors – plausible in the short run for targeted interventions, but this may represent an upper-bound estimate depending on context, governance, and baseline saturation. We will use this as the effectiveness coefficient for agriculture – $\beta_{\rm agriculture}$ is 0.01707.

Similarly for the education sector, Dreher et al. (2008) show that an additional dollar of education aid per capita increases primary school enrolment by approximately 0.3%. Furthermore, Lim (2021) estimated that a \$1000 increase in per capita aid to education would lead to a 1.22% increase in primary enrolment rate, a 2.54% increase in gross intake rate, a 1.98% increase in completion rate, and a 0.181% increase in secondary enrolment rate, but the regression found a negative correlation with repetition ratee (-0.45%). Taking the average of Lim's 5 indicators gives a 1.42% increase in educational outcomes for \$1000 dollars of per capita aid. The study ran from 1970 – 2013, so given Kenya's population of 44.99 million in 2013, this estimates \$44.99 billion dollars of aid would be required for a 1.42% increase, so beta is extremely low for education, in the range of 0.00003%. This is contrasted with the results of simply running a linear regression on HDI educational index over the past

25 years and aid amounts. We elected to use the second one as it gave us a slightly more promising (still extremely low) coefficient of 0.0065.

Based on empirical studies and reasonable proxies, we estimate the sector aid effectiveness coefficients as follows: Healthcare: 0.9%, Agriculture: 1.7%, Infrastructure: 1.1%, Technology: 0.8%, Transport: 1.0%, and Education: 0.0065%. The healthcare estimate is supported by Mishra & Newhouse (2009), who find that targeted health aid reduces child mortality and improves outcomes by 0.4–0.9% per \$1M, which we adopt as a proxy for resilience improvement. Infrastructure and transport estimates draw on Donaubauer & Nunnenkamp (2017), whose difference-in-differences analysis shows that infrastructure aid leads to about a 1% gain in related output metrics, which we use as a resilience proxy. Technology's coefficient is set slightly lower due to longer time horizons for impact and capacity limitations.

Sector, i	Aid effectiveness
	coefficient, $\beta_i(4dp)$
Healthcare	0.9000
Agriculture	1.7070
Infrastructure	1.1000
Technology	0.8000
Transport	1.0000
Education	0.0065

Domestic progress constants

Domestic progress constants, denoted as δ_i for sector i, represent the natural year-on-year rate of improvement in sector resilience in the absence of foreign aid. They capture the underlying domestic momentum in each sector driven by local government spending, private investment, policy reforms, or organic growth unrelated to international transfers. For instance, if Kenya's education sector is expanding due to domestic curriculum reforms or rising urbanisation, that baseline growth, independent of any aid, is embedded in the domestic progress constant for education. These constants are essential to the model because they prevent the over-attribution of progress to aid interventions and allow the simulation to remain realistic.

To determine them, we again turn to Seychelles as a country that has received minimal (although not zero) foreign aid in the past 25 years, receiving aid amounts in the order of magnitude of \$10 million per year, compared with ~\$5 billion for Kenya and roughly similar amounts for most of sub-Saharan Africa.

The simple method is to plot Seychelle's indicators over time and take the gradient as the domestic progress constant, neglecting the impact of the low levels of aid received. For agriculture, Seychelles is not an appropriate example due to its low reliance on the agriculture sector, so instead South Africa is used as a recipient of low foreign aid with a large agricultural sector.

By this approach we get the following results, which are a constant measure of resilience points expected to increase per year:

Sector, i	Domestic progress
	constant, δ_i (4dp)
Healthcare	0.0064
Agriculture	0.7799
Infrastructure	0.0313
Technology	0.0345
Transport	0.0039
Education	0.0077

Within the framework of this model, domestic progress constants are offset by the recipient country's governance score before being applied. Because the sector resiliencies are normalised from 0 to 1, each point is 0.01 on the index.

Governance elasticity

Governance elasticity θ measures the expected percentage increase in GNI following a 1-point increase in governance index. A recent panel study of 36 sub-Saharan African countries by Ayana & Demissie (2023) estimated an interactive coefficient of 0.019 between fiscal policy and the government effectiveness indicator, and a coefficient of 0.0046 between fiscal policy and the corruption control indicator. Given that our governance score is a simple average of those two indicators, we could take governance elasticity to be the average of the two, as these are "the major factors that encourage growth in SSA countries" (Ayana & Demissie, 2023). This would lead to a coefficient of 0.0118 or a 1.18% increase in GNI following a 1-point increase in governance score. This finding is also in line with the work of Mossadak (2017) who found that "a change in the governance index of a unit is likely to produce an increase of 1.7% in real GDP". Governance is clearly one of the strongest correlators with growth, which is to be expected in a region such as SSA with generally poor governance (average -0.76 on a scale of -2.5 to 2.5)

$$\theta = 1.18$$

Governance sensitivity to effective aid

Governance sensitivity ψ measures the expected increase in governance score per \$1 million effective aid received. By collecting data on the two governance indicators as well as total foreign aid received by Kenya and running a linear regression, we see that the coefficient between effective aid and governance score is 3.421e-12. Adjusting this to the common format of points change per million dollars of aid gives:

$$\psi = 3.421 \cdot 10^{-6}$$

2.4 The update equations

The update equations are the set of equations that determine how the model changes the key dependent variables (governance, sector resilience, GNI per capita (PPP), etc.) following each iteration of the model and are defined as follows:

The amount of aid available is determined by the choice of method as detailed in 2.1. The amount of useful aid available is determined by the macroeconomic absorptive capacity.

2.4.1 Aid budget equations

Constant annual budget

$$A_{\text{total}}(t+1) = A_{\text{total}}(t) \tag{2}$$

Fixed pool / capped budget

$$A_{\text{total}}(t+1) = \frac{A_{\text{total pool}} - A_{\text{used}}(t)}{T - t}$$
(3)

Governance-weighted budget

$$A_{\text{total}}(t+1) = A_{\text{total}}(t) \cdot (1 + \zeta \cdot \gamma(t)) \tag{4}$$

where ζ is a decided or dynamic budget growth multiplier based on stronger governance to capture the policy of rewarding improving institutions with more aid.

2.4.2 Updating variables each year

Raw sectoral aid allocation

$$A_{\rm i}({\bf t}) = a_i(t) \cdot A_{\rm total}(t) \mbox{ where } \sum_{i \in S} a_i(t) = 1 \eqno(5)$$

where $a_i(t)$ is determined randomly by the Monte Carlo simulation, and set to 0 when resilience exceeds the resilience threshold

Apply macroeconomic absorptive capacity

$$A_{\text{i,effective}}(t) = \alpha(t) \cdot A_{\text{i}}(t)$$
 (6)

which captures the fact that effective/usable aid in the sector is limited by the macroeconomic absorptive capacity

Sector resilience update

$$R_i(t+1) = \min\left(1, R_i(t) + \left(1 - R_i(t)\right) \cdot \beta_i \cdot A_{\text{i.effective}}(t)\right) \tag{7}$$

which describes sector resilience as previous resilience plus the effectiveness coefficient multiplied by the effective aid amount, capped at 1, with a simple diminishing returns multiplier such that as the sector resilience approaches 1 it becomes harder and harder to achieve returns.

Governance update

$$\gamma(t+1) = \min\left(1, \gamma(t) + \psi \cdot \frac{\sum_{i \in S} A_{i, \text{effective}}(t)}{1000000}\right)$$
(8)

which describes governance as previous governance plus the governance sensitivity to effective aid multiplied by the effective aid amount for that year

Absorptive capacity update

$$\alpha(t+1) = \min(1, \alpha_0 + \lambda \cdot \gamma(t+1)) \tag{9}$$

which describes absorptive capacity as previous absorptive capacity plus sensitivity of absorptive capacity to governance (lambda is a function, which for the purposes of this study will be assumed to be a linear multiple for simplicity).

2.4.3 Translating variables into GNI growth

GNI growth from sectors

$$\Delta R_i(t) = R_i(t+1) - R_i(t)$$

$$\Delta \text{GNI}_{\text{sector}}(t) = \sum_{i \in S} (1 + E_i) \cdot \Delta R_i(t)$$
 (10)

so GNI growth is a function of sector elasticity and change in sector resilience, summed over all sectors

GNI growth from domestic progress

$$\phi(t) = \sum_{i \in S} (1 + E_i) \cdot (\frac{\delta_i}{100}) \cdot (1 + \frac{\gamma(t)}{100})$$
(11)

as GNI is likely to be growing at a small rate already, independent of any foreign aid as societies naturally advance. Domestic progress rate is the domestic progress constant estimated using Seychelles, multiplied by the governance score as poorer governments are likely to have worse rest economic growth and will be more dependent on aid. This is then multiplied by elasticity to convert from domestic sector resilience points to GNI growth.

GNI growth from governance

$$\Delta \text{GNI}_{\text{governance}}(t) = \left(1 + \frac{\theta}{100}\right) \cdot \left[\gamma(t+1) - \gamma(t)\right] \tag{12}$$

where the rate of GNI growth due to governance is the elasticity multiplied by the change in governance score over the year.

Total GNI update

$$GNI(t+1) = GNI(t) \cdot [1 + \phi(t) + \Delta GNI_{sector}(t) + \Delta GNI_{governance}(t)]$$
(13)

where the rate of GNI growth due to governance is the elasticity multiplied by the change in governance score over the year.

Population update

$$P_{t+1} = P_t + r \cdot P_t \cdot \left(1 - \frac{P_t}{K}\right) \tag{14}$$

where the population growth follows a logistic distribution. The logistic growth model is well-suited for modelling population dynamics because it captures the natural constraints on growth imposed by limited resources and environmental factors. Unlike normal growth, which assumes unlimited expansion, logistic growth introduces a carrying capacity – a maximum population size that the environment can sustain. This creates an S-shaped curve where the population initially grows rapidly when numbers are low but slows down as it approaches the carrying capacity, reflecting real-world bottlenecks such as resource scarcity, habitat limitations, and social factors. For this study, K (carrying capacity is about 60 million, typical for a SSA country of Kenya's size, and r (growth rate is about 0.0311) (see Appendix 1).

GNI per capita

$$GNIpc(t+1) = \frac{GNI(t+1)}{P(t+1)}$$
(15)

2.5 Monte Carlo simulation

To estimate the most effective sectoral aid allocation, we use a Monte Carlo simulation that generates thousands of random allocation strategies and evaluates their long-term impact on Kenya's development trajectory. Each year, a sectoral allocation vector is sampled from a Dirichlet distribution, which is the optimal distribution for generating valid probability vectors. It ensures that:

- All sector shares are positive
- The shares sum to 1 exactly
- The distribution can be parameterised to control concentration, allowing us to explore both evenly spread and highly skewed allocation scenarios. Most scenarios within the context of this project are relatively even, but it leaves the opportunity for further investigation.

This makes the Dirichlet far more appropriate than naïve approaches like drawing from np.random.rand() and dividing by the sum. Such methods are prone to edge cases (e.g., near-zero weights), introduce numerical instability, and implicitly favour more uniform distributions unless carefully tuned. In contrast, the Dirichlet is statistically rigorous and well-suited to modelling constrained budget allocations.

Each simulation run proceeds as follows:

- Aid is allocated to sectors using the Dirichlet-sampled proportions.
- Effective aid is calculated by applying macro-level absorptive capacity.
- Sector resilience, governance, absorptive capacity, GNI, and population are dynamically updated each
 year using the model's equations.
- The simulation continues until Kenya "graduates" reaching the resilience threshold across all sectors and surpassing the GNI per capita target.

By repeating this process across n iterations, we identify sectoral allocation strategies that minimise graduation time, maximise national income, or optimise aid efficiency. The stochasticity of this approach allows us to approximate the space of plausible strategies under uncertainty.

2.6 Assumptions and justifications

Like all modelling efforts in complex social systems, this research relies on a number of simplifications and structural assumptions in order to make the optimisation tractable and policy-relevant. This section outlines the key assumptions, justifications for their use, and recognised limitations.

Most core update equations assume linear or log-linear relationships between variables (e.g., sector resilience and GNI, governance and absorptive capacity, effective aid and sector improvement). This is a deliberate modelling decision to ensure transparency, interpretability, and computational tractability during simulation.

- The log-linear form of the sector-GNI regression in Section 2.3 assumes that the elasticity of GNI to resilience is approximately constant over the range observed. This is supported by its widespread use in economic growth literature (e.g., Barro, 1991; Sachs & Warner, 1995).
- However, we acknowledge that real-world relationships are often nonlinear or subject to threshold
 effects and interaction terms. For instance, education's impact on growth may exhibit diminishing
 returns at higher levels, or only materialise after infrastructure passes a critical threshold. These are not
 currently captured.
- To partially mitigate this, we introduce diminishing returns within the sectoral resilience update function via a multiplier and recently t logarithmic penalties to reduce explosive growth. However, full nonlinear system dynamics (e.g., sigmoid responses, sectoral synergies) are reserved for future model extensions due to complexity and identification issues.

In Section 2.3, we use a log-linear Ordinary Least Squares (OLS) regression to estimate the elasticities between sector resilience and GNI. This approach assumes:

- Log-transformation of the dependent variable (GNI) linearises multiplicative relationships.
- Sector resilience indicators can be treated as continuous inputs influencing GNI additively.

We selected this approach because:

- It allows for intuitive interpretation of elasticities: a 1-point increase in sector resilience is associated with a fixed percent increase in GNI.
- It aligns with similar models in development macroeconomics.
- OLS is robust and transparent, with diagnostics (e.g., multicollinearity) easily observable and mitigatable via ridge regression, as we applied.

However, there are a few limitations, mainly that the linearity assumption neglects potential complementarities and interaction effects between sectors. As addressed above, linear relationships were much easier in order to make the project tractable during the 8-week period, and attempts have been made to introduce diminishing or logarithmic returns where easy to do so. Furthermore, during the proof-of-concept example, Kenya at the time of intervention (2000) has many extremely low variables due to its status as an underdeveloped country. This means that diminishing returns are unlikely to be a priority for several iterations, and that linearity is not a significant oversimplification at this stage.

• Some estimated elasticities were initially implausible due to collinearity, which we addressed through ridge regularisation.

Sectors such as energy, environment, and financial services were excluded to maintain strategic focus and due to the lack of standardised, comparable resilience indicators across years. These sectors may play critical enabling roles, but were omitted from this version of the model in order to:

- Avoid overfitting with too many interdependent regressors;
- Focus on the sectors most directly tied to welfare and economic resilience (e.g., education, agriculture, healthcare);
- Ensure that each sector included has a clearly defined, time-series trackable resilience metric.

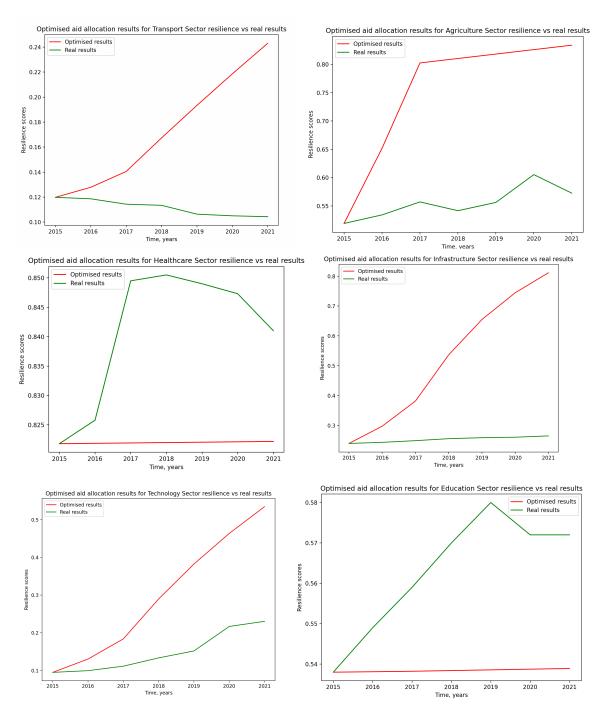
Future model versions hope to include additional sectors and more variables to strengthen the reliability of the relationships that we claim in this paper, but more computational time would be required. This is something to explore in future iterations of the model.

3 Proof of concept via Kenya simulation

Time simulation was run using the constant aid per year mode for simplicity. Given that Kenya has received in the ballpark of \$130m - \$150m from the UK in the years since 2015, we initialise the model with \$100m dollar of aid per year to compare. Despite receiving less aid than in real life, we expect that the optimised nature of the aid distributions should create much more efficient results.

In the basic \$100 million dollars per year allocation, all iterations of the simulation graduate within 25 years at most and 22 years at least (starting from 2015 data as initial variables). The data implies that current aid allocation is severely sub-optimised, as the best case iteration of the model performs significantly better than the

real-world case across all sectors bar healthcare(2021 is where resilience data ends but the sectors continue until majority graduation by 2037).



4 Discussion and future directions

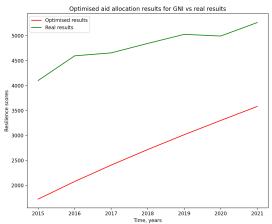
4.1 Limitations and future implementations

Despite the strengths of the current aid optimisation framework, there are important methodological and implementation limitations that constrain its predictive accuracy and real-world applicability.

PPP-adjustment

A major limitation is the inability to perform full dynamic PPP adjustment during the simulation. While the model uses a PPP-adjusted GNI per capita baseline, it would have to assume that subsequent economic growth translates directly into proportional PPP gains, which is unrealistic. This simplification would inflate long-term

welfare projections, particularly in an economy where inflation and external price shifts can significantly impact real income such as Kenya. To avoid this, the simulation test case used raw (not PPP-adjusted) GNI data, but that leads to the result below where GNI per capita seemingly underperforms the real-life case.



Parameter Estimates and Contextual Validity

Aid effectiveness coefficients and domestic progress constants, as well as other constants and coefficients, are derived from cross-country regressions or historical meta-studies, many of which are dated or only tangentially applicable to Kenya. As such, parameter bias is likely, and outcomes may diverge from reality—especially if local policy environments or institutional conditions differ from the data sources. Furthermore, even when regression is performed on direct Kenyan data, past assumptions inherently have a bias and are not perfectly applicable to the changing economic environment of Kenya today.

In the future, it would be excellent to partner with direct stakeholders such as Kenyan (and other large foreign aid recipient country) agencies e.g. Ministry of Planning, National Treasury) to generate sector-specific regressions using recent administrative data, and to consider methods of future-proofing the simulation by incorporating some stochasticity, or using AI to generate shocks at random intervals and study their effect on the aid transmission mechanism.

Linear relationships and elasticities

The model employs log-linear regression to link sector resilience with GNI growth and assumes linear proportionality between aid inputs and resilience change. This is a strong simplification. Real-world sector impact is likely nonlinear and may include threshold effects, diminishing marginal returns, or sectoral synergies that the model cannot yet capture.

Sectoral relationships

As of now, the model treats sectors as individual, as highlighted during the elasticity regression section. This is of course, not true, and many sectors have strong collinearity. For the purposes of this initial study, which represents a first step in this direction, we felt it more necessary to preserve data points for individual sectors and given that the point of the model is to guide aid allocation into target sectors, we felt that keeping sectors distinct was more important than reducing noise by combining sectors, given that examining cross-dependencies between sectors was a difficult task beyond the scope of this project.

The next immediate step to improve the model would be to try and incorporate some of the sector relationships, first in terms of measuring elasticities and other coefficients, and then in terms of the synergies as aid to one sector affects outcomes in others.

Diminishing returns to Aid

Although diminishing returns are partially captured via a logarithmic modifier and the dampening factor, the current implementation is uniform across all sectors. This ignores more realistic constraints like institutional bottlenecks, workforce shortages, or diminishing absorptive efficiency at scale.

Sectoral Absorptive capacity

While macroeconomic absorptive capacity is relevant, each sector is likely to have different volumes of aid that can be absorbed before diminishing returns, particularly in a country like Kenya where certain sectors will be highly established e.g. agriculture and nearing the threshold, whilst other sectors like technology and

transportation are relatively new and underdeveloped. It would improve the model if we could incorporate sectoral absorptive capacities to explain this.

Governance

Governance growth is currently treated as a smooth, deterministic function of effective aid, which likely overstates institutional improvement. In actuality, governance is often erratic, politically constrained, or exogenous to aid inputs in the short run, and likely to change with different political heads of state.

Sector Selection

The model only includes six core sectors: healthcare, education, agriculture, infrastructure, technology, and transport. Key areas like energy, financial services, and environmental resilience are excluded, not because they are unimportant but due to challenges with resilience measurement, beta estimation, or data availability. In the future we aim to build a second-generation model that includes these sectors using proxy indicators (e.g. electrification, forest loss, mobile banking usage), even if with wider confidence intervals.

Sectoral Underperformance: Healthcare

In multiple simulation runs, the healthcare sector consistently underperforms in achieving resilience gains relative to other sectors. This reflects a potential mismatch between its beta value, resilience measurement, and interaction with absorptive capacity. Given the sector's centrality to human development, this outcome raises concern. It was difficult to establish the aid effectiveness coefficient due to a lack of relevant literature.

Implementation Interface and Usability

The model currently runs in a programming environment and is not accessible to non-technical policy users

4.2 Future steps

Future Work and Stakeholder Engagement

This model is designed as a scalable tool for data-driven aid allocation, with potential applications for donors, recipient governments, and multilateral institutions. Expert advice is the direct next step, to quickly isolate flaws in the methodology before expanding it further, and to discuss the extremely low effectiveness coefficient of the healthcare sector, as well as the improvements highlighted above.

This work aims to serve as a first step for future research efforts by highlighting a new framework and proposing a more efficient method of aid allocation, but even existing optimised approaches are underutilised in policy due to conflicting objects such as political pressure, alternative motives, or simply personal disputes between heads of state. However, following improving the model and collaboration with other researchers if this project is given the chance to continue, theoretical next steps include:

- 1. Field calibration: Partner with Kenyan ministries and NGOs to refine parameter values.
- 2. Policy pilot: Propose aid allocation scenarios to FCDO or UNDP for simulated validation.
- 3. Multi-country adaptation: Build comparable models for Ghana, Uganda, or Nigeria.
- 4. Stakeholder platform: Create a real-time simulation interface for policy analysts and economists

Ultimately, this project proposes not just a technical model but a broader theory of aid governance with a direct and explicit focus on terminating aid as quickly and efficiently as possible and leaving behind a country resilient enough to stand on its own two feet.

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Appendix: Math

This appendix explains the maths behind the log-linear model in 2.3. We want

$$E_i \approx \frac{\Delta R_i}{R_i}$$

since elasticity is the percentage increase in GNI caused by a 1-point increase in sector resilience. We can show that this follows from our log-linear model as follows. Assume the model can be written in the following form where *Y* is used as shorthand for GNI:

$$ln(Y) = \alpha + \beta X + \epsilon$$

This is the standard form of a linear regression assuming that log of Y is the dependent variable and X represents the independent variables (sector resilience). Differentiating with respect to X gives:

$$\frac{\mathrm{dln}(Y)}{\mathrm{d}X} = \beta$$

And the following is true by the chain rule:

$$\frac{\mathrm{dln}(\mathbf{Y})}{\mathrm{dX}} = \frac{1}{Y} \cdot \frac{\mathrm{d}Y}{\mathrm{d}X}$$
$$\Longrightarrow \frac{\mathrm{dY}}{Y} = \beta \cdot \mathrm{d}X$$

When X (the independent variables) increase by 1 unit (dX = 1), the change in Y over Y is approximately equal to beta, which leads to the conclusion that we wanted to show was consistent – beta equals the change in Y over Y. In other words, beta is equal to the elasticity of X. Applying this with multiple independent variables gives us equation (1) used in 2.3 where we sum the elasticities of the independent variables.

Appendix 2: Data and Code

Almost all the data used in this project was from freely available sources, namely the UN, OECD, and World Bank. Where necessary, sources are linked in the References section. The code used in this project both for the simulation and for variable calculation is available at this Github repository.

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