problem 1

let $b = (b_1, b_2, b_3 \dots b_n) \in A$.

Prove that $f(a) = f(b)$ iff $a = b$.

Suppose $a \neq b$, $a_k \neq b_k$ for some $k \in S$.
hence $S(a_1), S(a_2) \neq S(b_1), S(b_2)$ since
$S(a_i) \neq S(b_1)$ hence it is injective.

Prove that it is surjective

Suppose $(S_1, S_2, S_3) \in B$, $a = (a_1, a_2, a_3 \dots a_n)$

where $a_i = a$ iff $i \in S_1$
$a_i = c$ iff $i \in S_2 - S_1$
$\dots 3 - S_2,$
$\dots S_3$

$\dots S_3)$ hence surjective

This is a bit confusing, is this the mapping of the function or the inverse? Although I can assume what you meant

Hence there is a bijection

P2

(a) 2^k

Incorrect for part a

(b) since the total websites can't be a set larger than ~~both~~ all servers, then it's implied that $m(C_1 .. C_k)$ is a subset of Total servers $|C_1| + |C_2| + .. |C_k|$

P3

$n^{\log 10}$, $(\log 10)^n$, $n!$, $\binom{n}{2}$, n^{2n}

$$\lim_{n\to\infty} \frac{n^{\log 10}}{(\log 10)^n} = \frac{\infty}{1} = \infty$$

hence

$\lim_{n\to\infty} (\log 10)^n = \infty$ **-1**

$(\log 10)^n$

$$\lim_{n\to\infty} \frac{n!}{(\log 10)^n} = \infty,$$

$n! = \omega\,(\log 10)^n$

Why? **-1**

This equality is not obvious at all, in the future this needs to be clarified or points can be taken off.

$$\lim_{n\to\infty} \frac{n^{2n}}{n^{\log 10}}$$

$n^{2n} = \omega(n^{\log 10})$

$$\lim_{n\to\infty} \frac{\binom{n}{2}}{n!} = \frac{\frac{n!}{2(n-2)!}}{n!} = \frac{1}{2(n-2)!} = 0$$

Put all this Together.

$$n^{2n} > n! > \binom{n}{2} > n^{\log 10} > (\log 10)^n$$

This is the order of growth from the largest To smallest.

This order is not correct, it should go $n^{2n} > n! > (\log 10)^n > n^{\log 10} > \binom{n}{2}$. -1

P4

$2^{\log n}$, $2^{3\log n}$, $2^{n\log n}$,

$2^{\sqrt{\log n}}$, 2^n

Ignore bases because they are common:

$\log n$, $3\log n$, $n\log n$, $\sqrt{\log n}$, n

We use limits:

$$\lim_{n\to\infty} \frac{3\log n}{\log n} = 3.$$ hence $3\log n = \theta \log n$

$$\lim_{n\to\infty} \frac{n\log n}{3\log n} = \infty$$ hence $n\log n = \omega\, 3\log n$

$$\lim_{n\to\infty} \frac{\sqrt{\log n}}{n\log n} = 0$$ $\therefore \sqrt{\log n} = o\; n\log n$

$$\lim_{n\to\infty} \frac{n}{\sqrt{\log n}} = \infty$$ $\therefore\; n = \omega \sqrt{\log n}$

$$\lim_{n\to\infty} \frac{3\log n}{\sqrt{\log n}} = \infty \quad \text{hence} \quad 3\log n = \omega\sqrt{\log n}$$

$$\lim_{n\to\infty} \frac{n}{\log n} = \infty, \quad n = \omega \log n$$

Putting it all together:

$$2^{3\log n} > 2^{n\log n} > 2^{n} > 2^{\sqrt{\log n}} > 2^{\log n}.$$

This is the order of Growth rate from the largest to smallest

wrong order, some justifications are also not right (i.e. 3logn vs nlogn)