P1

The probability of an edge is $p = \frac{1}{2}$

Each vertex has a total of $(n-1)$ possible edges

The probability that a vertex is isolated is

$$(1-p)^{n-1} = \left(\frac{1}{2}\right)^{n-1}$$

~~Thus~~ The choice of each vertice is $\binom{n}{2}$

So, the total sample space $= 2^{\binom{n}{2}}$

The probability of having no isolated vertices is $1 - n\left(\frac{1}{2}\right)^{n-1}$

$$P(\text{edge btn 2 vertices}) = \frac{1}{2}\binom{n}{2}$$

hence

$$\frac{1}{2}\binom{n}{2} - \frac{5}{2}\binom{n}{2}^{\frac{1}{2}} \le X \le \frac{1}{2}($$

did not compute variance -1

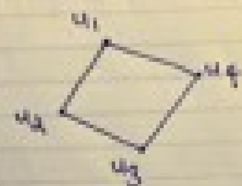
did not explain why t=5 in the equation -1

did not show work for Chebyshev's inequality -2

P2

Assume we have a subgraph of G, called H.

H contains a closed walk as follows
$(u_1, u_2, u_3, u_4, u_1)$



we also see a simple cycle in the said graph as follows $(u_1, u_2, u_3, u_4, u_1)$

in this subset (which sticks to the rules of graph G as offered in the question), we see that if there is a closed walk, then there is a simple cycle in the graph as well.

You have only shown that this singular example of a closed walk is a simply cycle but you haven't proven it for all closed walks -5

Make sure your submissions are clear

#4

P3

(a) average degree = $\dfrac{\text{sum of degrees}}{\text{Number of degrees}}$

sum of degrees = twice the number of edges $= 2m$

Number of deg $= n$

Therefore:

Average $= \dfrac{2m}{n}$

(b) vertices of degree $< \dfrac{m}{n}$ as per hint

~~total degree < m~~

Total vertices is n

Thus, the total degree $< \dfrac{m}{n} \times n < m$

But the numb[illegible]

there is a contradiction.

Total degree of what? Why is this contradictory to having m edges? -4

Hence there is a subset of students in which each of them have at least $\frac{m}{n}$ friends among themselves.

PROBLEM 4

Assume G is an undirected graph and suppose that it's disconnected.

To prove that $\overline{G}$ is connected, assume there are 2 vertices **A** and **B** in $\overline{G}$ that are adjacent to each other and an edge connects both of these vertices. If **A** and **B** are adjacent and connected in $\overline{G}$, then we know that they can't be adjacent and connected in G.

Now, suppose that **A** and **B** are adjacent to each other in G and belong to the same component. Let **C** be a vertex from another component in G. This means that edges **AC** and **BC** do not exist in G, but will exist in G's complement $\overline{G}$.

Therefore, we have proved that there exists an edge between any 2 vertices of $\overline{G}$. So either G or $\overline{G}$ has to be connected.