Q1

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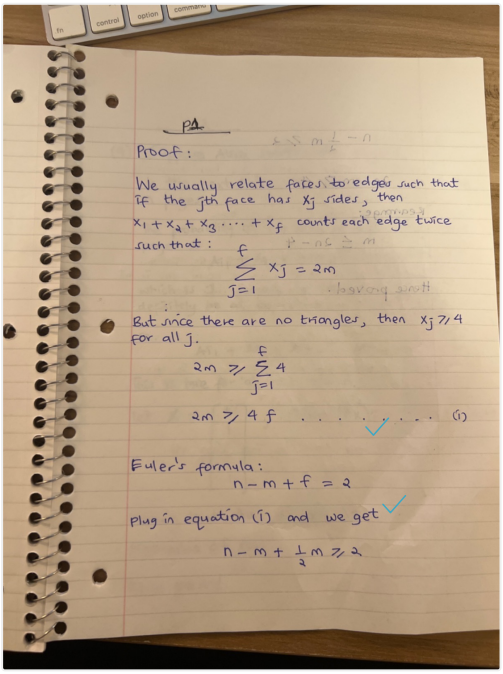
See [PS7.pdf] (https://canvas.northwestern.edu/files/15108068/download?download_frd=1) for Question 1 about planar graphs.



PA

Proof:

We usually relate faces to edges such that if the jth face has x_j sides, then $x_1 + x_2 + x_3 \cdots + x_f$ counts each edge twice such that:

$$\sum_{j=1}^{f} x_j = 2m$$

But since there are no triangles, then $x_j \geq 4$ for all j.

$$2m \geq \sum_{j=1}^{f} 4$$

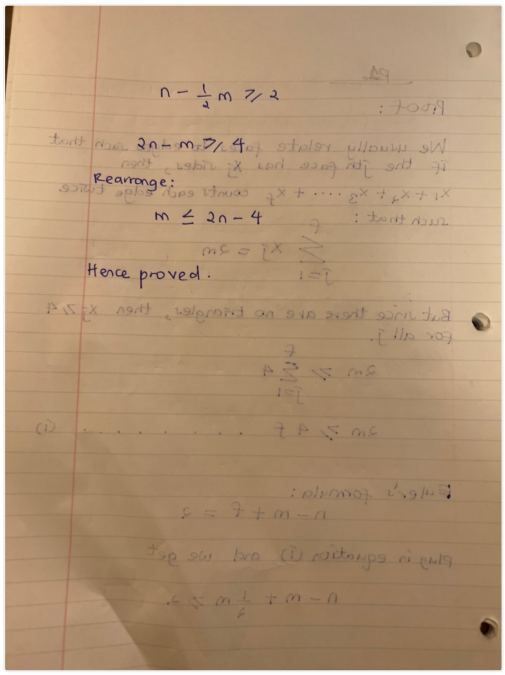
$$2m \geq 4f \quad \ldots\ldots\ldots \quad (1)$$

Euler's formula:

$$n - m + f = 2$$

Plug in equation (1) and we get

$$n - m + \frac{1}{2}m \geq 2$$



$n - \frac{1}{2}m \geq 2$

$2n - m \geq 4$

Rearrange:

$m \leq 2n - 4$

Hence proved.

Q2

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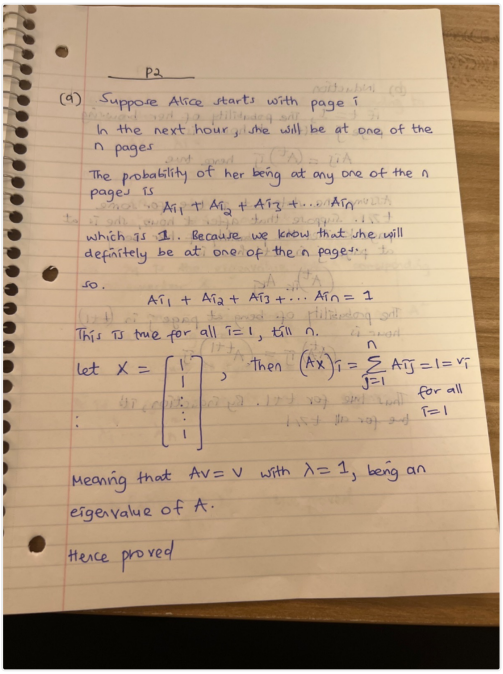
See [PS7.pdf] (https://canvas.northwestern.edu/files/15108068/download?download_frd=1) for Question 2

website visits.



P2

(a) Suppose Alice starts with page i

In the next hour, she will be at one of the n pages

The probability of her being at any one of the n pages is

$A_{i1} + A_{i2} + A_{i3} + \dots A_{in}$

which is 1. Because we know that she will definitely be at one of the n pages.

so.

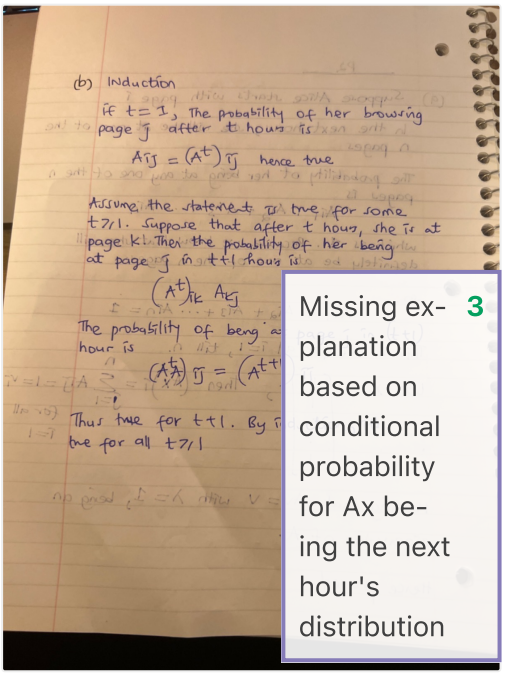
$$A_{i1} + A_{i2} + A_{i3} + \dots A_{in} = 1$$

This is true for all $i = 1$, till n.

let $X = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$, Then $(Ax)_i = \sum_{j=1}^{n} A_{ij} = 1 = v_i$ for all $i = 1$

Meaning that $Av = v$ with $\lambda = 1$, being an eigenvalue of A.

Hence proved


(b) Induction
if t=1, the probability of her browsing
page j after t hours is
$A_{ij} = (A^t)_{ij}$ hence true
Assume the statement is true for some
t≥1. Suppose that after t hours, she is at
page k. Then the probability of her being
at page j in t+1 hours is
$(A^t)_{ik} A_{kj}$
The probability of being a
hour is
$(A^t A)_{ij} = (A^{t+1}$
Thus true for t+1. By
true for all t≥1
Missing explanation based on conditional probability for Ax being the next hour's distribution
3

Q3

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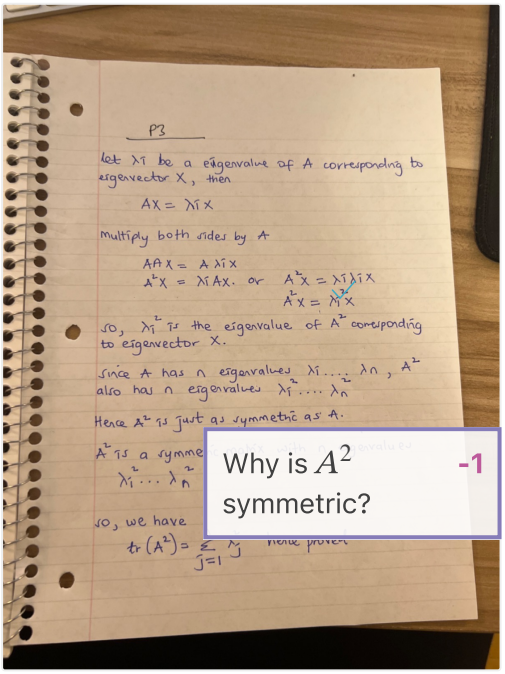
See [PS7.pdf] (https://canvas.northwestern.edu/files/15108068/download?download_frd=1) for Question 3 about trace and eigenvalues.


P3
let λi be a eigenvalue of A corresponding to eigenvector X, then
AX = λiX
multiply both sides by A
AAX = A λiX
A²X = λiAX. or A²X = λiλiX
A²X = λi²X
so, λi² is the eigenvalue of A² corresponding to eigenvector X.
since A has n eigenvalues λi.... λn, A² also has n eigenvalues λi².... λn²
Hence A² is just as symmetric as A.
A² is a symme
λi².... λn²
so, we have
tr(A²) = Σ λj
j=1
Why is A^2 symmetric? -1

Q4

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See [PS7.pdf] (https://canvas.northwestern.edu/files/15108068/download?download_frd=1) for Question 4 about linear algebra and graphs.

This question wasn't answered