

HWQ1

Proof by contradiction

let:

$$ax + by = 1 \quad \text{for integers } x \text{ and } y$$

since n divides a , and b :

$$a = np, \quad \text{for integers } p \text{ and } q$$

$$b = nq$$

Thus:

$$1 = ax + by = anp + bnq$$

$$1 = n(ap + bq)$$

so we get n divides 1 which is a contradiction since $n \geq 2$

Q2

The given $T(n) = nT(n-1) + 1$, $T(0) = 0$

Base case should be $n = 1$ since we are told to prove for $n \geq 1$. This is not induction

Proof:

$$T(n) = n \cdot T(n-1) + 1$$

put $n = n-1$, $T(n-1) = (n-1)T(n-2) + 1$

substitute the value of $T(n-1)$

$$T(n) = n(n-1)T(n-2) + n + 1$$

put $n = n-2$

$$T(n-2) = (n-2)T(n-3) + 1$$

substitute the value of $T(n-2)$

$$T(n) = n(n-1)(n-2)T(n-3) + n(n-1) + n + 1$$

put $K \neq n$

$$T(n) = n(n-1)(n-2) \dots (n-(K-1))T(n-K) + n(n-1)(n-2) \dots$$

put $K = n$

$$T(n) = n(n-1)(n-2) \dots 2 \cdot 1 T(0) + n(n-1)(n-2) \dots 3 + \dots n(n-1) + n + 1$$

since $T(0) = 0$

$$T(n) = n! / 1! + n! / 2! + \dots n! / (n-2)! + n! / (n-1)! + n! / n!$$

$$r_0, \quad T(n) = \sum_{m=1}^n \frac{n!}{m!}$$

Q3

Proof by Induction

What is your predicate?

$$(1+3)^2 > \frac{2}{3} + 6$$

$$16 > \frac{20}{3}, \text{ hence it's true for } n=1$$

Let's assume it's true for $n=k$,

$$(1+3)^{2k} > \frac{2k}{2k+1} + 6k$$

for $n=k+1$,

$$(1+3)^{2k+2} > \frac{2k+2}{2(k+1)+1} + 6(k+1)$$

$$16(1+3)^{2k} > \frac{2k+2}{2k+3} + 6k + 6$$

hence proved!

You should use a stronger predicate: $(1+3)^{2k} \geq 1 + 6k$. -2

Unclear how you got to here from the line above. This is why using a stronger predicate would be helpful (easier inductive step). -2

Q4

Proof by Induction

Base case: $P(3)$ is true because there are 6 computers and we can turn on the 3 on the left and 3 on the right.

Assume $P(k)$ is true for some integer $k \geq 3$

$P(k+1) = 2(k+1) = 2k+2$ computers

Since k is odd, $2k+2$ is even.

Thus $P(k+1)$ is true by induction.

true for all odd integers $n \geq 3$

You never state what the predicate P is that you're proving

Performing in- 0

No invariant 0 on on wrong variable

Wrong base case 0

Wrong inductive step 0

You must perform induction on the number of switches, not the number of computers. The number of computers is constant.

