P1

Proof

n tennis players.
2 are buddies if they've had a match together.

Nobody can have zero buddies, at the same time,
nobody can have n buddies.

Players can have 0 buddies. **-3**

each person can have $\{1, \dots n-1\}$ buddies

By pigeonhole principle

$$\left\lceil \frac{n}{n-1} \right\rceil = 2$$

meaning there are at least 2 people with the same buddy.

P2

(a) $3n$ guests.
Number of ways seated is $3n!$

$$(3n) \times (3n-1) \times (3n-2) \dots 1 = 3n!$$

$$= \frac{3n(3n-1)!}{3n}$$

Correct **2**

$$= (3n-1)!$$

(b) each family can be seated in 2 ways:

either $P_1 C P_2$ or $P_2 C P_1$ (P_1 means Paret one, C means child, P_2 is parent two)

The number of ways to seat n things on a bench is $n!$

on a circular table though, it's $(n-1)!$

Thus, multiply the two

$2^n \times (n-1)!$

Correct 3

P3

$S(t) = k_B \log W(t)$

$k_B > 0$

n particles. even.

$W(t) = \binom{n}{t}$ "up")

$t \in$ (up or down)

$\binom{n}{t_{up} / t_{up} + t_{down}}$

(i)

$W(t) = \binom{n}{t_{up}}$

(ii)

You need to show the number of microstates for $\binom{n}{n-t}$, THEN show why they are equal. -0.5

no part ii or iii -4

P4

(1) using a counting argument.

step 1:

say, we want to choose 4 apples out of a total of 2n apples.

we can do that in $\binom{2n}{4}$ ways.

step 2

say, out of the total, we want to choose 4 assuming there are n green apples and n red apples.

we can get different outcomes.

outcome 1 : we get all 4 green apples $\binom{n}{4}$ ways.

outcome 2 : all red apples $\binom{n}{4}$ ways

outcome 3: 1 red, 3 green $\binom{n}{3} n$

outcome 4: 1 green, 3 red

$$\binom{n}{3} n$$

outcome 5: 2 red, 2 green

$$\binom{n}{2}$$

outcome 6 : 2 green, 2 red

$$\binom{n}{2}$$

when we sum up all the outcomes, we get:

$$2\binom{n}{4} + 2n\binom{n}{3} + \binom{n}{2} \times \binom{n}{2}$$

CONCLUSION

Since step 1 and step 2 count the same items hence

$$\binom{2n}{4} = 2\binom{n}{4} + 2n\binom{n}{3} + \binom{n}{2} \times \binom{n}{2}$$

Hence Proved

Good

P4

(2) $2\binom{n}{4} + 2n \times \binom{n}{3} + \binom{n}{2} \times \binom{n}{2}$

$$\frac{2\,n!}{(n-4)!\,4!} + \frac{2n\;n!}{(n-3)!\,3!} + \frac{n!}{4(n-2)(n-2)(n-3)!}$$

solve.

$$= \frac{n!\,((n+1)(n+2)+\dots 2n)}{24\,(n-4)!\,(n-3)(n-2)\dots(2n-4)}$$

$$= \frac{2n!}{(2n-4)\,4!}$$

$$= \cancel{\tfrac{2n}{4}} \quad = \binom{2n}{4}$$

hence solved

Not the binomial theorem **-2.5**

Please make sure your scans are straight or use latex