Problem 1

Probability of being a culprit $P(c) = 1/90000$

" " not being a culprit $P(c') = 1 - 1/90000$

$P(+ve \mid c) = 0.90$

$P(+ve \mid c') = 10^{-4}$.

Find $P[c' \mid +ve]$:

$\Rightarrow$ Use Bayes Theorem

$$\Rightarrow \frac{P(+ \mid c') \times P(c')}{P(+ \mid c') \times P(c') + P(+ \mid c) \times P(c)}$$

$$\Rightarrow \frac{10^{-4} \cdot (1 - 1/90{,}000)}{\left(10^{-4} \times 1 - \frac{1}{90000}\right) + \left(0.90 \times \frac{1}{90000}\right)}$$

$\Rightarrow 0.91$

P2

P(each client machine connects from one server), we'll call it P(A)

P(At least one machine connected to 2 servers), we'll call it P(B)

$P(B) \leq 1 - P(B')$

$P(B')$ = no machine connected to 2 servers

$$P(B') \geq 1 - \left(1 - P(B'|A)\right)^{k}$$

$$\geq 1 - e^{(-k/n)}$$

$$\geq 1 - n^{2\sqrt{n \ln n + 1}\,\cancel{+1}}$$

$$\geq 1 - 1/n^2$$

$$\geq 1/n^2$$

the probability of getting a untaken server should de- crease as you continue assigning servers

3.5

(1 - 1/n) * (1 - 2/n) * (1 - 3/n)... etc

P(3)

(a) $P(e_2) = \frac{1}{4}\left(\frac{3}{4}\right) + \frac{3}{4}\left(\frac{1}{4}\right) = \frac{3}{8}$

$P(e_1) = \frac{9}{64} \therefore \left(\left(\frac{3}{8}\right)^2\right)$

$P(e_1 \cap e_2) = \frac{12}{64}$

$P(e_1) \times P(e_2) = \left(\frac{3}{8} \times \frac{9}{64}\right)$

since $\frac{12}{64} \neq \left(\frac{3}{8} \times \frac{9}{64}\right)$, e_1 and e_2 are NOT independent.

(b) Probability at indices i and $i-1$

$$= \frac{3}{16} + \frac{3}{16} = \frac{3}{8}$$

$P(A) = P(B) = 3/8$

$P(A) \times P(B) = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$

$P(A \cap B) = 4 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{64}$

Since $P(A) \times P(B) = P(A \cap B)$, they ARE INDEPENDENT

P4

Pickaxe or Dynamite

\$ 100

-1000 \$

$\frac{1}{3}$ 12K

$\frac{2}{3}$ nothing

(a) $E(T) = \frac{1}{p}$, $p = 0.5$, so

$E(T) = \frac{1}{0.5} = 2$

Wrong value of p, prove that this is a geometric RV 1

(b) $E(R)$?

$E(R \mid \text{pickaxe}) = \$100$

$E(R \mid \text{dynamite}) =$ if $p = \frac{1}{3}$, $R = (12000 - 1000)\$$

else if $p = \frac{2}{3}$, $R = (-1000)\$$

$= \frac{1}{3}(11000) - \frac{2}{3}(1000)$

$= \frac{11000}{3} - \frac{2000}{3} = \$3000$

Profit for one day:

$= 0.5(100) + 0.5(3000)$

$= 1550$

but since $E[T] = 2$

Total profit $= 1550 \times 2$

$= \$3100$

Correct 3