Q1

0 / 5

See [PS6.pdf] (https://canvas.northwestern.edu/files/15040931/download?download_frd=1) for Question 1 about spanning trees.

This question wasn't answered

Q2

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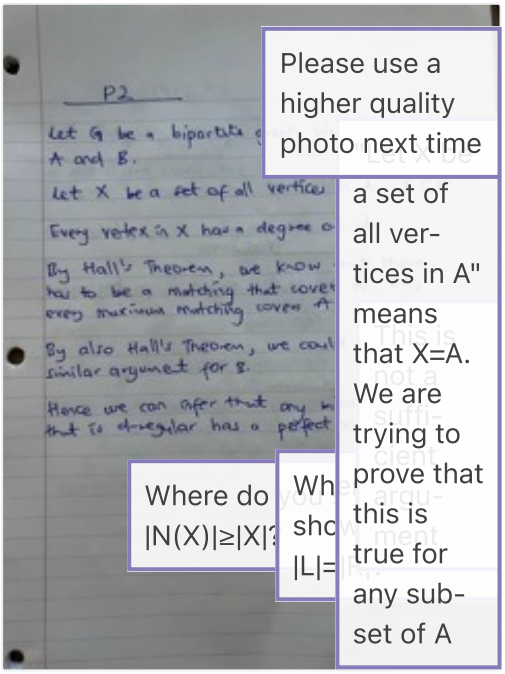
See [PS6.pdf] (https://canvas.northwestern.edu/files/15040931/download?download_frd=1) for Question 2 about matching.


P2
Let G be a bipartite
A and B.
Let X be a set of all vertice
Every vertex in X has a degree o
By Hall's Theorem, we know
has to be a matching that cover
every maximum matching covers A
By also Hall's Theorem, we coul
similar argument for B.
Hence we can infer that any b
that is d-regular has a perfect
Please use a higher quality photo next time
a set of all ver- tices in A" means that X=A. We are trying to prove that this is true for any sub- set of A
Where do
|N(X)|≥|X|?
Wh
sho
|L|=

Q3

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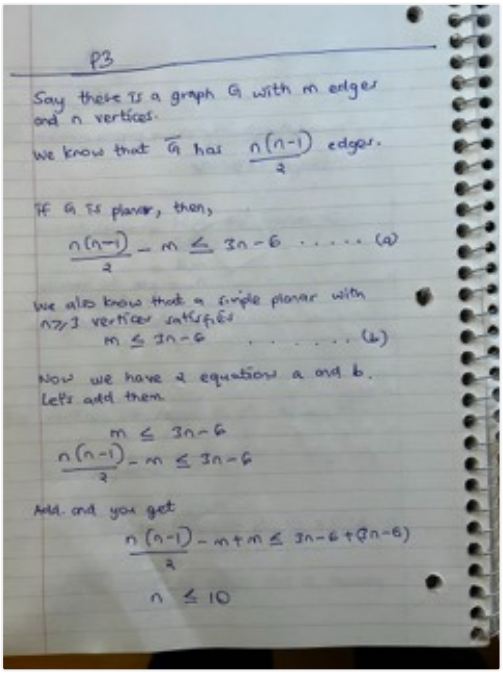
See [PS6.pdf] (https://canvas.northwestern.edu/files/15040931/download?download_frd=1) for Question 3 about planarity.



P3

Say there is a graph G with m edges and n vertices.

We know that $\overline{G}$ has $\frac{n(n-1)}{2}$ edges.

If G is planar, then,

$$\frac{n(n-1)}{2} - m \leq 3n - 6 \quad \ldots\ldots (a)$$

We also know that a simple planar with $n \geq 3$ vertices satisfies

$$m \leq 3n - 6 \quad \ldots\ldots (b)$$

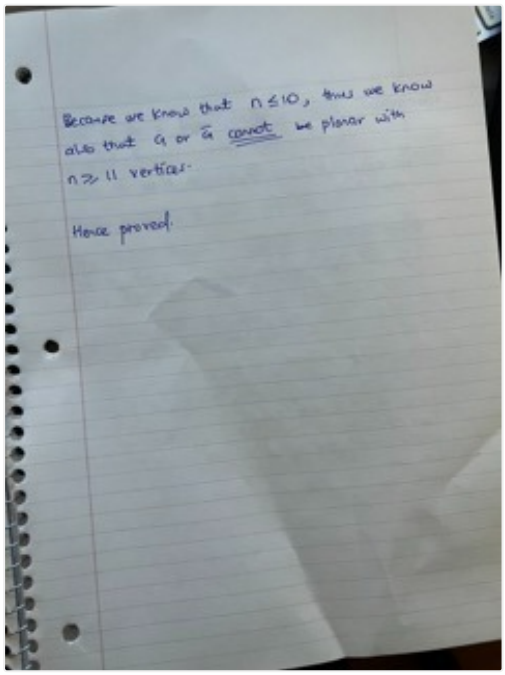
Now we have 2 equations a and b. Let's add them.

$$m \leq 3n - 6$$
$$\frac{n(n-1)}{2} - m \leq 3n - 6$$

Add. and you get

$$\frac{n(n-1)}{2} - m + m \leq 3n - 6 + (3n - 6)$$

$$n \leq 10$$


Because we know that $n \leq 10$, thus we know
also that G or $\bar{G}$ cannot be planar with
$n \geq 11$ vertices.
Hence proved.

Q4

2 / 5

See [PS6.pdf] (https://canvas.northwestern.edu/files/15040931/download?download_frd=1) for Question 4 about graph coloring.

PROBLEM 4

(a)

Suppose that **d** is the largest degree of any vertex in **G** (that is, all vertices have **d** or fewer edges attached, and **at least one** vertex has precisely **d** edges attached.

As we go about coloring, when we color any particular vertex **v**, it is attached to at most **d** other vertices, of which some may already be colored. Then there are at most **d** colors that we must avoid using. We use the lowest-numbered color not prohibited. That means that we use some color numbered **d + 1** or lower because at least one of the colors 1, 2, . . ., d + 1 is NOT prohibited. Therefore we will **never** need to use any color numbered higher than **d + 1**.

Hence, we have proved that if **d** is ... any simple graph **G,** then we ought to color the graph with **at m**...

Good

No part b -3