MATH 132 PROBLEM SET I

04/10/2023

Exercise 1.5.

1. *P, Q, and R are the points* $(5,-3)$, $(-6,1)$, *and* $(1,8)$ *respectively. Show that triangle PQR is isosceles, and find the coordinates of the midpoints of the base.*

2. *A and B are the points* $(-1,-6)$ *and* $(5,-8)$, *respectively. Which of the following points lie on the perpendicular bisector of AB?*
   (a) $P(3,-4)$ (b) $Q(4,0)$ (c) $R(5,2)$ (d) $S(6,5)$

3. *A and B are the points* $(12,0)$ *and* $(0,-5)$, *respectively.Find the length of AB, and the length of the median, through the origin O, of the triangle OAB.*

4. *Find the equations of straight line of given gradients passing through the given points:*
   (a) $4,(1,3)$ (b) $\frac{1}{3},(2,5)$ (c) $\frac{2}{3},(1,7)$.

5. *Find the equations of the straight lines joining the following pairs of points*
   (a) $(1,6)$ *and* $(8,1)$ (b) $(3,2)$ *and* $(7,-3)$.

6. *Write down the gradient of the straight line joining* (a,b) *and* (p,q). *Write down the two conditions that these points should lie on the line* $y = 7x - 3$. *From these two conditions deduce the actual gradient of the line.*

7. *Find the equation of the straight line*

   (a) *through* $(1,4)$, *parallel to* $2x - 5y - 7 = 0$

   (b) *through* $(-2,-3)$, *perpendicular to* $4x + 3y - 5 = 0$

   (c) *where the slope is* -2, *and the* $x-$*intercept is* 4.

8. *Three consecutive vertices of a parallelogram are* $(-4,1)$, $(2,3)$, *and* $(8,9)$. *Find the coordinates of the fourth vertex.*

9. *For each of the following sets of three points, determine if the points are on a line*
   (a) $(2,3)$, $(-4,-7)$, $(5,8)$ (b) $(-3,6)$, $(3,2)$, $(9,-2)$ (c) $(4,6)$, $(1,2)$, $(-5,-4)$.

10. *Show that the triangle with vertices* $P(6,-7)$, $Q(11,-3)$ *and* $R(2,-2)$ *is a right -angled triangle using (i) Pythagoras theorem; (ii) using slopes (gradients).*

11. *Find an equation that must be satisfied by the coordinates of any point whose distance from the point* $(5,3)$ *is always two units greater than its distance from the point* $(-4,-2)$.

12. *Prove that the points* $A(6,-13)$, $B(-2,2)$, $C(13,10)$, *and* $D(21,-5)$ *are the verices of a square. Find the length of one of the diagonals.*

13. *Given* l_1, *having the equation* $2x - 3y = 12$, *and line* l_2, *having the equation* $4x + 3y = 6$, *draw a sketch of each of the lines. Then find the coordinates of the point of intersection of* l_1 *and* l_2.

14. *Show by means of slopes that the points* $(-4,-1)$, $(3,\frac{8}{3})$, $(8,-4)$, *and* $(2,-9)$ *are the vertices of a trapezoid.*

15. *Find the equations of the perpendicular bisectors of the sides of the triangle having vertices* $A(-1,-3)$, $B(5,-3)$, *and* $C(5,5)$, *and prove that they all meet at a point.*

16. *If two vertices of an equilateral triangle are* $(-4,3)$ *and* $(0,0)$, *find the other vertex.*

17. *Find the length of the segment cut off by the coordinate axes from the line whose equation is* $7x - 24y + 168 = 0$.

18. *A triangle has vertices* $A(-2, 1)$, $B(2, 3)$, $C(-2, -4)$. *Find (a)* $\langle ABC$ *(b)* $\langle ACB$

19. *Find the equations of the bisectors of the angles between the lines* $4x + 3y - 12 = 0$ *and* $y = 3x$.

20. *Find the equations of the tangent to the circle* $x^2 + y^2 - 4x - 2y - 8 = 0$, *which are parallel to the line* $3x + 2y = 0$.

21. *Given the line* L *having the equation* $2y - 3x = 4$ *and the point* $P(1, -3)$, *find (a) an equation of the line through* P *and perpendicular to* L; *(b) the shortest distance from* P *to* L.

22. *Let* L_1 *be the line having the equation* $A_1x + B_1y + C_1 = 0$, *and let* L_2 *be the line having the equation* $A_2x + B_2y + C_2 = 0$. *If* L_1 *is not parallel to* L_2 *and if* k *is any constant, the equation* $A_1x + B_1y + C_1 + k(A_2x + B_2y + C_2) = 0$ *represents an unlimited number of lines. Prove that each of these lines contains the point of intersection of* L_1 *and* L_2.

23. *If* r_1 *and* r_2 *are positive integers, prove that the coordinates of the point* $P(x, y)$, *which divides the line* P_1P_2 *in the ratio* $\frac{r_1}{r_2}$, *that is,* $\frac{|P_1P|}{|P_1P_2|} = \frac{r_1}{r_2}$, *are given by*

$$x = \frac{(r_2 - r_1)x_1 + r_1x_2}{r_2} \qquad \text{and} \qquad y = \frac{(r_2 - r_1)y_1 + r_1y_2}{r_2}$$

24. *In the following find the coordinates of the point that divides the segment from* P_1 *to* P_2 *in the given ratio:*

    (a) $P_1(4, -2)$, $P_2(8, 6)$, $3 : 1$
    (b) $P_1(5, 4)$, $P_2(2, -2)$, $-3 : 2$