Orc 2018.x

Quantitative Guide

New quantguide

This document is an updated version of the old quant guide. Some sections are still missing in the new version and they can be found in the appendix.

Extensive documentation

The new quant guide covers many details which probably are of little interest to most readers. For the sake of readability such extensive documentation is printed in purple.

Example. This text is extensive documentation and can be skipped unless you are interested in this particular detail.

Contents

1. Notation and tools

1.1. Notation

Π	Theoretical price of a derivative contract.
B	Base price, i.e. the price of the base contract. See section 2.1.5.
S	Spot price of the underlying asset. See section 2.1.5.
X	Strike price of the derivative contract.
T	Time to expiry of the derivative contract.
r	The financing rate. See section 7.0.4.
q	Underlying rate or dividend yield payed continuously by underlying asset. See section 7.0.5.
σ	Volatility of the underlying asset.
b	Cost of carry.
D	Discrete dividend payed out by the asset. See section 7.0.6.
$P(S)$	The payoff function. Depends on S and X and possibly more parameters. Example (Vanilla call payoff): $P(S) = \max(S - X, 0)$

$$d_1(S, X, T, r, q, \sigma) = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{1}$$

$$d_2(S, X, T, r, q, \sigma) = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{2}$$

Φ is the 1-dimensional cumulative normal distribution function.

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt. \tag{3}$$

Φ_n is the n-dimensional cumulative normal distribution function.

1.2. Mathematical tools

1.2.1. Numerical derivatives

Here follow various methods for calculating the derivative of a function numerically. We consider a function $f(x)$ of a single variable x. For derivatives of functions of several variables see the section Derivatives of functions of several variables.

Symmetrized finite differences

The first derivative of a function $f(x)$ is approximated by

$$\frac{df}{dx} = \frac{f(x+\varepsilon) - f(x-\varepsilon)}{2\varepsilon}. \tag{4}$$

The second derivative of a function $f(x)$ is approximated by

$$\frac{d^2f}{dx^2} = \frac{f(x+\varepsilon) - 2f(x) + f(x-\varepsilon)}{\varepsilon^2}. \tag{5}$$

Symmetrized average rate of change

The symmetrized average rate of change takes the magnitude of the variable x into account when choosing the size of the interval for which the difference is calculated. For a fixed $\varepsilon > 0$ the function is evaluated on the interval $[-\varepsilon x, \varepsilon x]$.

The first derivative of a function $f(x)$ is approximated by

(6) $$\frac{df}{dx} = \frac{f((1+\varepsilon)x) - f((1-\varepsilon)x)}{2\varepsilon x}.$$

The second derivative of the function is apporximated by

(7) $$\frac{d^2f}{dx^2} = \frac{f((1+\varepsilon)x) - 2f(x) + f((1-\varepsilon)x)}{(\varepsilon x)^2}.$$

Derivatives of functions of several variables

In Orc Trader™ the symmetrized average rate of change calculates the total derivative of functions of several variables. Consider a function $f(x, y)$ where y is a function of x. Calculating the derivative with respect to x gives

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}.$$

Further numerical derivative methods

The following is a list of numerical derivatives methods which make strong use of the numerical method of the pricing model.

- Tree derivative approximation

2. Financial Mathematics

Valuation of derivatives is a mathematical science and some background theory is needed in order to explain the calculations performed by Orc Trader™ . There are many excellent text books on the subject of financial mathematics, see [Ha], [Hu]. The aim of the current section is to briefly explain necessary theory for calculations in Orc Trader™ and refer to the literature for more thorough discussions.

2.1. Fundamental concepts

2.1.1. Exercise type

Exercise, expiry and maturity

American options

For American vanilla options the payment occurs immediately as the option is exercised. For *deferred payment* options the payment is deferred until the expiration date of the option.

2.1.2. Moneyness

Moneyness describes whether the payoff of an option is positive or zero for the current spot price S of the underlying.

In-the-money (ITM): The payoff is positive for the current spot price.
At-the-money (ATM): The spot price S equals the strike X.
Out-of-the-money (OTM): The payoff is zero for the current spot price.

An option with strike equal to the forward price F of the underlying is called the *ATM Forward option*.

2.1.3. Call and put options

The notion of call and put options can be defined in terms of moneyness.

A *call option* is ITM when the spot price lies above the strike and it is OTM when the spot price lies below the strike.

A *put option* is ITM when the spot price lies below the strike and it is OTM when the spot price lies above the strike.

2.1.4. Present and future values of money

Present value factor for money

The present value of an asset x at a future date τ is calculated by multiplying x by the *present value factor* pv_T

$$PV(x) = \mathrm{pv}_T \cdot x.$$

For assets with no underlying rate, e.g. a certain payment in the domestic currency, the present value factor depends on

- The financing rate time to the payment T_{FR}
- The financing rate r at time T_{FR}

For continuous rate type the present value factor is calculated by

$$\mathrm{pv}_T = e^{-rT_{\mathrm{FR}}}. \tag{8}$$

For other rate types the present value factors are listed in table 3.

Present value factor for equity assets

For assets with non-zero underlying rate, e.g. stock (with no discrete dividend payments), the present value factor for a future date τ will depend on

- The financing rate time to the payment T_{FR}
- The financing rate r at time T_{FR}
- The plain time to the future date, T
- The underlying rate q

For continuous rate type the present value factor is calculated by

$$\mathrm{pv}_T = e^{-rT_{\mathrm{FR}}+qT}. \tag{9}$$

The future value at the date τ of the asset x is calculated as

$$FV(x) = \frac{1}{\mathrm{pv}_T} \cdot x.$$

2.1.5. Underlying asset

Base contract and underlying asset

The underlying asset is an input parameter to all theoretical calculations on derivative contracts. Some pricing models use spot value of the underlying as input and other use the forward value. In Orc Trader™ the concept of *base contract* is used to determine the spot or forward value of the underlying asset.

The *base price* B is configurable in Orc Trader™ (see Orc Trader manual). The default value is the average spread of the market bid and ask prices, but it can be set to bid, ask, last paid, opening price etc.

Forward and theoretical spot value of underlying

The forward or future value of the underlying asset is affected by the following parameters

- plain time to expiry of the forward contract T
- financing rate time to expiry T_{FR}
- continuous financing rate r
- underlying rate q
- dividends D_i
- plain time to dividend payment T_i
- financing rate time to dividend payment $T_{\mathsf{FR},i}$
- financing rate r_i at time $T_{\mathsf{FR},i}$

The forward value is given by

$$F = \frac{1}{\mathsf{pv}_T}\left(S - \sum_{i=1}^{N} \mathsf{pv}_{T_i} D_i\right), \tag{10}$$

where pv_T is given by (9). This formula is used for valuation of forwards and futures contracts in Orc Trader™ .

The theoretical spot price of of an asset is calculated from the forward or future contract F by

$$S_{\mathsf{theor}} = \mathsf{pv}_T F + \sum_{i=1}^{N} \mathsf{pv}_{T_i} D_i. \tag{11}$$

The relation between spot and forward prices can be modified by applying various base offset modes. The default mode is 'Not used'. For other alternatives see section 7.0.12.

Derivative of spot with respect to base

The derivative of the spot price with respect to the base price is useful for various calculations. It depends on the following parameters

- plain time to expiry of the base contract T
- financing rate time to expiry T_{FR}
- continuous financing rate r at time T_{FR}
- underlying rate q

It is calculated as

$$\frac{\partial S}{\partial B} = e^{-rT_{\mathsf{FR}}+qT}$$

The derivative of spot with respect to base depends base offset modes. The default mode is 'Not used'. For other alternatives see section 7.0.12.

The derivative of the base with respect to the spot is given by the inverse relation

$$\frac{\partial B}{\partial S} = \left(\frac{\partial S}{\partial B}\right)^{-1}$$

2.1.6. Settlement and final settlement days

The number of settlement days is the time between the trade date for the contract and the time when the seller must deliver what was bought and the purchaser must pay for what was bought. The settlement days refer to business days in relation to a specific settlement calendar. In Orc there are also final settlement days that are relevant for some contract types having an expiry date. Depending on the settlement type for vanilla options, i. e. cash or delivery, the settlement and final settlement days are applied slightly differently. When pricing a vanilla option there are typically two periods that need to be defined; the period for the theoretical forward calculation and the period corresponding to the present value calculation.

The theoretical forward calculation period reflects the lending/borrowing period associated with replication of the option. The following holds when the base contract of the option is a spot contract. If the option is cash settled the theoretical forward period will be from calculation date + base contract settlement days until expiry date + base contract settlement days. If the option is delivery settled the delivery of the underlying contract is allowed to be any number of days after the expiry of the option and for this Orc uses the final settlement days. Hence the theoretical forward period will be from calculation date + base contract settlement days until expiry date + option contract final settlement days. There is one exception to this; if the calculation date + contract settlement days is after the expiry date + final settlement days then the period end will be calculation date + contract settlement days. If underlying rates are used then the corresponding period for that factor is independent of settlement days and will be from the calculation date to the expiry date. The above description also holds for the theoretical prices of forward and future contracts (not in the context of an option).

If the base contract of the option is a future or forward contract Orc first calculates a theoretical spot value and then applies the above logic. The 'discount' period for the theoretical spot calculation starts at calculation date + base contract settlement days in both the cash and delivery cases. The end of the period will be base contract expiry date + base contract settlement days in the cash case and base contract expiry date + base contract final settlement days in the delivery case. If underlying rates are used then the corresponding period for that factor is independent of settlement days and will be from the base contract expiry date to the expiry date of the option.

The present value period is defined by the cash flow dates of the option. These are the payment date of the option price and the potential payment date corresponding to exercising the option. Independent of settlement type the present value period is from calculation date + option settlement days until expiry date + option final settlement days.

2.1.7. Volatility

Implied volatility

Implied volatility is a tool to calibrate the derivative pricing model to better match market data. Pricing models, such as Black-Scholes are not flawless models of the reality. For liquid contracts market prices are determined by supply and demand and these prices usually do not agree with the theoretical price.

The theoretical price of a vanilla option is given by a function $\Pi(S, X, T, r, q, \sigma)$ where the parameters S, X, T, r, q, σ are defined in section 1.1. All parameters except from volatility are easily read from market or contract specification. Given a market price Π_m and the parameters S, X, T, r, q we can look for the *implied volatility* σ_{imp} which makes the theoretical price and the market price agree. Hence, the implied volatility is the volatility σ_{imp} for which the following equality holds

$$\Pi_m = \Pi(S, X, T, r, q, \sigma_{\text{imp}}) \tag{12}$$

It is not possible to obtain a closed-form formula for the implied volatility, so (12) must be solved using numerical methods. The Orc Trader™ uses a combination of root bracketing, bisection, and inverse quadratic interpolation (see [**FPTV**] p 359–362).

The implied volatility is only well defined if the contract price is a monotone function of volatility. This is the case for most derivative contracts. However, there are exceptions such as binary options and caps or combinations call spread which do not have monotone value functions. Implied volatility is not available for these contracts in Orc Trader™ .

Not all market prices have a corresponding implied volatility. The lower bound is given by the theoretical price corresponding to volatility zero, i.e. $\Pi_{\text{low}} = \Pi(S, X, T, r, q, 0)$. If the market price is below the lower bound $\Pi_m < \Pi_{\text{low}}$ there exists no implied volatility.

Volatility surfaces

Volatility is a property of the underlying contract. There exist options with a range of strike prices X and time to expiry T which share the same underlying. For each option an implied volatility can be calculated. This gives us a set of implied volatilities as a function of the pair (X, T).

Given the set of implied volatilities for a range of strikes and maturities, a volatility surface can be constructed in various ways. The models available in Orc Trader™ are discussed in section 6.

Volatility time

The value of a derivative contract changes over time due to two factors: the financing rate and the volatility. In the Black -76 framework it is possible to separate out the contribution to the change due to volatility.

Consider a derivative contract with parameters (S, X, T, r, q, σ) as defined in section 1.1. According to (21) and (22) the theoretical price Π of the contract can be calculated by

$$\Pi = e^{-rT}\Pi_{\mathsf{F}}(F, X, T, \sigma)$$

where F is given by (18). As time moves forward the time to expiry changes to T_2 and the total change in the value of the contract δ_{tot} is

$$\delta_{\text{tot}} = e^{-rT_2}\Pi_{\mathsf{F}}(F_2, X, T_2, \sigma) - e^{-rT}\Pi_{\mathsf{F}}(F, X, T, \sigma)$$

where $F_2 = Se^{(r-q)T_2}$. Since the forward price function Π_{F} does not depend on r the volatility change in the value of the contract δ_{vol} is calculated by keeping the present value factor e^{-rT} and forward spot price F fixed

$$\delta_{\text{vol}} = e^{-rT} \left(\Pi_{\mathsf{F}}(F, X, T_2, \sigma) - \Pi_{\mathsf{F}}(F, X, T, \sigma) \right).$$

2.1.8. Time

Time t, expiration time τ and time to expiry T

The *time* variable is denoted t. Time is measured in years. In the theory on mathematical finance the time $t = 0$ usually denotes the time of issue of the contract. E.g. $t = 1/2$ is the point of time half a year from date of issue.

All derivative contracts have an *expiration time* τ which is set when the contract is issued. E.g. for a contract that expires three months after the date of issue the expiration time is $\tau = 1/4$.

Time to expiry T is the time left until the contract expires. When calculating the value of derivative contracts, time to expiry is the time variable of interest. A one year vanilla option issued nine months ago and an equivalent three months option issued today have the same value.

Plain, financing rate and volatility time to expiry

The time to expiry of a contract can be measured in different ways. Three variants of time to expiry are used for calculation purpose in Orc Trader™ .

Plain time to expiry, T, is the default method for measuring time to expiry. It is defined as the time between calculation date and the expiration date as given by the Actual/365 day count convention.

Financing rate time to expiry, T_{FR}, measures the time between the calculation date and the expiration date using the day count convention parameter of the yield curve.

Volatility time to expiry, T_σ, consists of the time between the calculation date and the expiration date plus additional volatility time. The time between the calculation date and expiration date is calculated using the volatility day count convention. The contract parameter volatility time mode determines how additional volatility time is calculated.

Time in Orc Trader™

In the theory of mathematical finance the value of contracts is usually considered to be a function of time $V(t)$ where $t = 0$ is the time of contract issue. Since time of issue is not essential to contract valuation, Orc Trader™ considers the value of contracts as a function of time to expiry $\Pi(T)$. Hence, in Orc Trader™ the value of a contract with time to expiration T corresponds to the value at time zero of a contract with expiration time $\tau = T$, i.e. $\Pi(T) = V(0)$.

One trading day

The length of one trading day measured in years is denoted $1_{\text{trading day}}$. For a contract with time to expiry T, the time to expiry the same time next trading day is $T - 1_{\text{trading day}}$.

The value varies for different day count conventions, e.g. for Actual/365 we have $1_{\text{trading day}} = 1/365$ while for Actual/360 we have $1_{\text{trading day}} = 1/360$.

2.2. Mathematical frameworks

A mathematical framework is a set of assumptions on the components needed primarily for calculating the theoretical price of a derivative contract. A framework consists of a set of general assumptions which are required in order to get consistent pricing. In addition there are model specific assumptions which will affect the outcome of the calculations.

Some mathematical frameworks share the same name as a pricing models. A mathematical framework is however only one component of a pricing model, which also consists of solution methods for calculating theoretical values and possibly other assumptions. For instance, with a few exceptions all pricing models assume the mathematical framework the Black-Scholes framework or equivalent Black -76 framework to hold.

2.2.1. Black-Scholes framework

The Black-Scholes model assumes that under the risk-neutral probability measure the underlying asset is modeled by a stochastic process $S(t)$ that follows a geometric Brownian motion

$$dS(t) = (r-q)S(t)dt + \sigma S(t)dW(t) \tag{13}$$

$$S(0) = S \tag{14}$$

where r, q and σ are defined in section 1.1 and S is the current spot price of the underlying asset.

Consider a derivative contract with strike price X, expiration time τ and payoff function $P(S)$, e.g. the payoff of a vanilla call option is given by $P(S) = \max(S - X, 0)$. The value at time t of this contract is given by the function $V(S,t)$ that solves the Black-Scholes partial differential equation (PDE)

$$\frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} - rV = 0 \tag{15}$$

$$V(S,\tau) = P(S). \tag{16}$$

The Black-Scholes theoretical price Π_{BS} calculated in Orc Trader™ is defined as

$$\Pi_{\mathrm{BS}}(S, X, T, r, q, \sigma) = V(S, 0) \tag{17}$$

where $V(S,t)$ solves (15)–(16). See section 2.1.8 for a comment on this definition.

The Black-Scholes pricing model covers how this framework is implemented in Orc Trader™ .

2.2.2. Black -76 framework

The Black -76 framework in equivalent with the Black-Scholes framework in the sense that the market assumptions are the same so results from theoretical calculations are identical for the two frameworks.

The Black -76 framework prices the derivative contract in terms of the *forward value* F of the underlying asset rather than the *current spot* price S of the underlying. Let $F(t)$ be the forward value of $S(t)$ at time T, where $S(t)$ solves (13)–(14). The stochastic differential equation solved by the forward process $F(t)$ is

$$dF(t) = \sigma F(t)dW(t)$$
$$F(0) = F$$

where r, q and σ are defined in section 1.1 and the initial forward value is

$$F = e^{(r-q)T}S. \tag{18}$$

Consider a derivative contract with strike price X, expiration time τ and payoff function $P(F)$, e.g. the payoff of a vanilla call option is given by $P(F) = \max(F - X, 0)$. The forward value at time t of this contract is given by the function $V_{\mathsf{F}}(F,t)$ that solves the following PDE

$$\frac{\partial V_{\mathsf{F}}}{\partial t} + \tfrac{1}{2}\sigma^2 F^2 \frac{\partial^2 V_{\mathsf{F}}}{\partial F^2} = 0 \tag{19}$$

$$V_{\mathsf{F}}(F,\tau) = P(F). \tag{20}$$

The Black -76 theoretical price $\Pi_{\text{B-76}}$ calculated in Orc Trader™ is defined as the present value of V_{F},

$$\Pi_{\text{B-76}}(F,X,T,r,\sigma) = e^{-rT} V_{\mathsf{F}}(F,0) \tag{21}$$

where $V_{\mathsf{F}}(F,t)$ solves (19)–(20). See section 2.1.8 for a comment on this definition.

The Black -76 pricing model covers how this framework is implemented in Orc Trader™ .

Relation between Black-Scholes and Black -76

The function $e^{-rT} V_{\mathsf{F}}(F,t)$ satisfies (15)–(16). Thus

$$\Pi_{\text{BS}}(S,X,T,r,q,\sigma) = \Pi_{\text{B-76}}(F,X,T,r,\sigma).$$

where F is given by (18).

PROOF. Set $\tilde{V}(S,t) = e^{-r(\tau-t)} V_{\mathsf{F}}(F(S,t),t)$. The equality $F(S,\tau) = S$ gives that $\tilde{V}(F(S,\tau),\tau) = P(S)$. Applying the Black-Scholes operator to $\tilde{V}(F(S,t),t)$ gives

$$\begin{aligned}
& \frac{\partial \tilde{V}(F,t)}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V(F,t)}{\partial S^2} + (r-q)S\frac{\partial \tilde{V}(F,t)}{\partial S} - rV(F,t) \\
= \; & e^{-r(T-t)} \left(rV_{\mathsf{F}} + \frac{\partial V_{\mathsf{F}}}{\partial t} + \frac{\partial F}{\partial t}\frac{\partial V_{\mathsf{F}}}{\partial F} + \frac{1}{2}\sigma^2 S^2 \left(\frac{\partial F}{\partial S}\right)^2 \frac{\partial^2 V_{\mathsf{F}}}{\partial F^2} + (r-q)S\frac{\partial F}{\partial S}\frac{\partial V_{\mathsf{F}}}{\partial F} - rV_{\mathsf{F}} \right) \\
= \; & e^{-r(T-t)} \left(\frac{\partial V_{\mathsf{F}}}{\partial t} + \frac{1}{2}\sigma^2 F^2 \frac{\partial^2 V_{\mathsf{F}}}{\partial F^2} \right)
\end{aligned}$$

□

Forward value of derivative contract

The function $V_{\mathsf{F}}(F,t)$ that solves (19)–(20) is the forward price of the derivative contract. It is denoted by

$$\Pi_{\mathsf{F}}(F,X,T,\sigma) = V_{\mathsf{F}}(F,0). \tag{22}$$

The forward derivative price is useful in the discussion on volatility time.

2.3. Measuring risk

This section covers general theory on how to measure risk. For risk measures available in Orc Trader™ see section 8.1.

2.3.1. The Greeks

The most common risk measures are often referred to as the Greeks. They measure the sensitivity of some quantity with respect to movement in some parameter. Consider the quantity y which depends on the parameter x. The risk measure μ equals the change in y when x increases by one unit, i.e.

$$y(x+1) = y(x) + \mu(x)$$

Greeks are most commonly calculated as the partial derivative of the quantity with respect to the parameter

$$\mu = \frac{\partial y}{\partial x}.$$

Some Greeks in Orc Trader$^{\text{TM}}$ are calculated as the difference $\mu(x) = y(x+1) - y(x)$.

Example. Delta Δ is the sensitivity of the theoretical price Π with respect to the underlying asset S. It is calculated as

$$\Delta = \frac{\partial \Pi}{\partial S}.$$

If the underlying asset moves up by one unit the option price will approximately change by Delta,

$$\Pi(S+1) \approx \Pi(S) + \Delta(S).$$

Note that if Δ is negative the option price decreases as S increases.

2.3.2. Bounds on the Greeks

Some risk measures are subject to bounds which limit the maximal and minimal theoretical values that they can assume.

For call options we have

$$0 \leq \Delta_{\text{call}} \leq e^{-qt}$$

and for put options

$$-e^{-qt} \leq \Delta_{\text{call}} \leq 0$$

where q is the dividend.

2.3.3. Skew

Skew occurs as a result of the volatility not being constant with respect to the underlying.

Example. Consider a contract with theoretical price Π which is a function of the underlying S and the volatility σ, where σ also is a function of S. Measuring how sensitive Π is to a change in S by calculating the partial derivative gives

$$\frac{\mathrm{d}}{\mathrm{d}S}\Pi(S, \sigma(S)) = \frac{\partial \Pi}{\partial S} + \frac{\partial \Pi}{\partial \sigma} \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}S}.$$

3. Numerical methods

3.1. Binomial methods

The spot tree

In the binomial model the underlying spot is assumed to move either up or down with given probabilities at discrete times. Suppose that the current spot price of the underlying S_0. In the next small time interval of length Δt it moves up with factor u with probability p and down with a factor d with probability $1-p$, where $0 < d < 1 < u$.

At time Δt there are two possible stock prices:	uS_0 and dS_0.
At time $2\Delta t$ there are three possible stock prices:	u^2S_0, udS_0 and d^2S_0.
At time $i\Delta t$ there are $i+1$ possible stock prices:	$u^jd^{i-j}S_0$ where $j = 0, 1, \ldots, i$.

The binomial spot tree is constructed by setting

$$S_{ij} = u^j d^{i-j} S_0. \tag{23}$$

The coefficients u, d and p must be chosen so that the mean and variance of the binomial walk match the mean and variance of the stock price. For the Black-Scholes framework we have from (13)–(14) that the mean of the stock is $S_0e^{(r-q)t}$ and the variance is $S_0^2e^{2(r-q)t}(e^{\sigma^2 t}-1)$. This gives us two equations

$$pu + (1-p)d = e^{(r-q)\Delta t} \tag{24}$$

$$pu^2 + (1-p)d^2 - (pu + (1-p)d)^2 = e^{2(r-q)\Delta t}(e^{\sigma^2\Delta t}-1). \tag{25}$$

The tree coefficients u, d and p are uniquely determined by imposing a third condition. For the CRR tree we impose $u = 1/d$ and for the JR tree we impose $p = 0.5$.

In Orc Trader[TM] the number of time steps N is determined by the precision parameter i_{prec}

$$N = (i_{\text{prec}} + 1) \cdot 20 \tag{26}$$

For a contract with time to expiry T the time step length is $\Delta t = T/N$. With lowest precision ($i_{\text{prec}} = 0$) then $\Delta t = T/20$ and highest precision ($i_{\text{prec}} = 5$) then $\Delta t = T/120$.

The option tree

The value Π of a contract with payoff function $P(S)$ is calculated as follows. Let Π_{ij} be the value of the contract at time $i\Delta t$ if the stock price is S_{ij}. At expiry the contract value is given by the payoff function

$$\Pi_{Nj} = P(S_{Nj}).$$

When early-exercise is not allowed the value for the nodes $0 \le i \le N-1$ and $0 \le j \le i$ is given by (see Non-smooth payoff for special treatment of the time step $i = N-1$)

$$\Pi_{ij} = e^{-r\Delta t}(p\Pi_{i+1,j+1} + (1-p)\Pi_{i+1,j}) \tag{27}$$

and when early-exercise is allowed

$$\Pi_{ij} = \max(e^{-r\Delta t}(p\Pi_{i+1,j+1} + (1-p)\Pi_{i+1,j}), P(S_{ij})). \tag{28}$$

The contract value is given by $\Pi = \Pi_{00}$.

Non-smooth payoff

The expressions (27) and (29) are very inaccurate near points of non-smoothness. Instead of these expressions, the Black-Scholes formula is applied to calculate $(N-1, k-1)$, $(N-1, k)$ and $(N-1, k+1)$ where k is the index of the strike, $S_{Nk} = X$. For contracts with no early-exercise we have

$$\Pi_{N-1,j} = \Pi_{BS}(S_{N-1,j}, X, \Delta t, r, q, \sigma) \qquad \text{for } j = k-1, k, k+1$$

and contracts with early-exercise

$$\Pi_{N-1,j} = \max(\Pi_{BS}(S_{N-1,j}, X, \Delta t, r, q, \sigma), P(S_{N-1,j})) \qquad \text{for } j = k-1, k, k+1. \tag{29}$$

The spot tree and dividends

When dividends are present we apply the *escrowed model* which assumes that the asset price minus the present value of all dividends to be paid until maturity follows a Geometric Brownian motion (see [**VN**]).

Assume the underlying pays out N dividends $D_1, \ldots, D_N$ with times to the ex-dividend dates $T_1, \ldots, T_N$. The dividends are accounted for when building the spot tree by removing the total present value of the dividends, $D_{\text{tot},T}$, from the current spot price

$$S_{\text{GBM},ij} = u^j d^{i-j}(S_0 - D_{\text{tot},T}). \tag{30}$$

where $D_{\text{tot},T}$ is given by (141). The spot tree is then constructed by for each time step i adding the forward value of all remaining dividends at time step i

$$D_{\text{remain},i} = e^{ri\Delta t}(D_{\text{tot},T} - D_{\text{tot},T_i}). \tag{31}$$

The spot tree is given by

$$S_{ij} = S_{\text{GBM},ij} + D_{\text{remain},i}. \tag{32}$$

The time to ex-dividend T_n typically occurs between two time steps $i\Delta t < T_n < (i+1)\Delta t$ for some i. This is handled by moving the dividend slightly and consider

$$D'_n = D_n e^{-r(T_n - i\Delta t)} \tag{33}$$

when doing the calculations.

Known problem with discrete dividends

Assume for ease of notation that interest rates, underlying rates etc. are zero in this section, $r = q = 0$. Note that by this assumption $D_{\text{remain},i} = D_{\text{remain},i+1}$. Let us denote $D = D_{\text{remain,i}}$.

For American call options with discrete dividends it may be optimal to exercise the option just before the dividend. In this case, if a dividend occurs between nodes i and $i+1$ then for high enough spot price S_{ij} we have the following equality from (29)

$$\Pi_{ij} = S_{ij} + D - X. \tag{34}$$

However, for low values of S_{ij}, i.e. far out of the money cases, the dividend drop is not sufficient motivate early exercise and we have

$$\Pi_{ij} = p\Pi_{i+1,j+1} + (1-p)\Pi_{i+1,j}. \tag{35}$$

A problem occurs for the index j_0 of the early exercise boundary, i.e. the index such that for $j < j_0$ we have (34) and for $j > j_0$ we have (35).

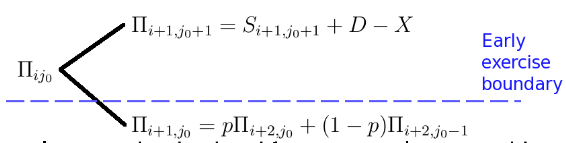

FIGURE 1. The value Π_{ij_0} is obtained from two values on either side of the early exercise boundary.

For all $j \neq j_0$ it is straight forward to show that we get the same value of Π_{ij} regardless if the dividend is just before node $i+1$ or just after node $i+1$. For $j > j_0$ we have that if the dividend is before $i+1$ then

$$\Pi_{ij} = S_{ij} + D - X$$

and if the dividend is after $i+1$ then

$$\begin{aligned}
\Pi_{ij} &= p\Pi_{i+1,j+1} + (1-p)\Pi_{i+1,j} \\
&= p(S_{i+1,j+1} + D - X) + (1-p)(S_{i+1,j} + D - X) \\
&= S_{ij} + D - X.
\end{aligned}$$

A similar calculation can be made for $j < j_0$. However, for j_0 we have that if the dividend is before $i+1$ then

$$\Pi_{ij_0} = S_{ij_0} + D - X$$

and if the dividend is after $i+1$ then

$$\begin{aligned}
\Pi_{i+1,j_0+1} &= S_{i+1,j_0+1} + D - X \\
\Pi_{i+1,j_0} &= p\Pi_{i+2,j_0} + (1-p)\Pi_{i+2,j_0-1} \\
&> S_{i+1,j_0} + D - X \\
\Pi_{ij_0} &= p\Pi_{i+1,j_0+1} + (1-p)\Pi_{i+1,j_0} > S_{ij_0} + D - X.
\end{aligned}$$

As time elapses the nodes i are pushed forward and dividends go from being just after a node to being just before the node. The effect is that the price of the option makes a negative jump. This is most noticeable when volatility time is enabled and the theoretical price makes a jump at the strike of an even half hour. Even though the jump may be just a few percent of the theoretical price, the risk value theta may change quite a lot since it is the difference between two theoretical prices with dividends on either side of a node.

3.1.1. CRR binomial method

All binomial methods in Orc Trader™ construct a spot and a option tree as described in the main section 3.1. The default binomial method used in Orc Trader™ is based on the tree proposed by Cox, Ross and Rubinstein (see [Hu]), which imposes the condition $u = 1/d$. By linearizing the term $e^{2(r-q)\Delta t}$ in (25) and neglecting first and higher order terms we get the three equations

$$\begin{aligned}
pu + (1-p)d &= e^{(r-q)\Delta t} \\
pu^2 + (1-p)d^2 - (pu + (1-p)d)^2 &= e^{\sigma^2\Delta t} - 1 \\
u &= \frac{1}{d}.
\end{aligned}$$

The solutions are given by

$$\begin{aligned}
u &= \tfrac{1}{2}e^{-(r-q)\Delta t}\left(1 + e^{(2(r-q)+\sigma^2)\Delta t} + \sqrt{(1 + e^{(2(r-q)+\sigma^2)\Delta t})^2 - 4e^{2(r-q)\Delta t}}\right) \\
d &= \frac{1}{u} \\
p &= \frac{e^{(r-q)\Delta t} - d}{u - d}.
\end{aligned}$$

where r, q and σ are defined as in section 1.1.

Dividends

Dividends are handled as described in the section on spot tree and dividends. Note that the contract financing rate is used for all present value calculations on the dividends. The time to ex-dividend dates $T_1, \ldots T_N$ are the actual number of days between the calculation date and the ex-dividend date. The binomial robust uses more accurate financing rates and time to ex-dividend dates.

3.1.2. Binomial Robust

The binomial robust method constructs a CRR binomial tree in the same way as the plain binomial methods. The main difference between the two methods is in the handling discrete dividends as described in the following section.

The CRR parameters

We consider first the case when no dividends are present. A forward value factor over a time step Δt is calculated by

$$\text{fv}_T = \left(\frac{F_{\text{no div},T}}{S}\right)^{\Delta t/T}$$

where $F_{\text{no div},T}$ is the forward value of the underlying excluding all dividends. The CRR parameters are given by

$$u = \frac{1}{2\text{fv}_T}\left(1 + \text{fv}_T^2 e^{\sigma^2\Delta t} + \sqrt{(1+\text{fv}_T^2 e^{\sigma^2\Delta t})^2 - 4\text{fv}_T^2}\right) \tag{36}$$

$$d = \frac{1}{u} \tag{37}$$

$$p = \frac{\text{fv}_T - d}{u-d}. \tag{38}$$

where r, q and σ are defined as in section 1.1.

If dividends are present the forward value over Δt for the time to first ex-dividend T_1 is given by

$$\text{fv}_{T_1} = \left(\frac{F_{\text{no div},T_1}}{S}\right)^{\Delta t/T_1}.$$

The up and down factors are given by (36)-(37) and a probability is calculated as

$$p_1 = \frac{\text{fv}_{T_1} - d}{u-d}.$$

The probability p_1 is used until the first dividend date, and the probability p given by (38) is used for the remaining nodes.

Dividend discounting

Discrete dividends are handled by applying the escrowed model as described in the section on spot tree and dividends. The calculation of the present value of future dividends is done more accurately by considering both the financing rate r_1 at the time of the first dividend payment T_1 and the financing rate r at the expiry of the contract T. We calculate two present value factors pv_{T_1} and pv_T by (9) and use a weighted average of these factors to calculate the present value of a future dividend

$$D_{\text{tot},T} = \sum_{n=1}^{N} D_n \left(\text{pv}_{T_1}{}^{T-T_n} \cdot \text{pv}_T{}^{T_n-T_1}\right)^{\frac{1}{T-T_1}} . \tag{39}$$

where $T_1, \ldots T_N$ are the time to ex-dividend dates. We use this formula to calculate the total present value of future dividends in equations (30) and (31). The spot tree is given by (32).

Financing rate time is used for calculating the present value factors at time to the first ex-dividend $T_{\text{FR},1}$ and time to expiry T_{FR}. For all other purposes volatility time is used. Hence (39) is actually given by

$$D_{\text{tot},T_\sigma} = \sum_{n=1}^{N} D_n \left(\text{pv}_{T_{\text{FR},1}}{}^{T_\sigma-T_{n,\sigma}} \cdot \text{pv}_{T_{\text{FR}}}{}^{T_{n,\sigma}-T_{1,\sigma}}\right)^{\frac{1}{T_\sigma-T_{1,\sigma}}} .$$

The volatility time to ex-dividend dates $T_{\sigma,n}$ is calculated using the volatility time logic where the dividend is assumed to be payed out at opening of the exchange, i.e. at the time specified by the volatility decrease start parameter.

Additional calculation before dividends

Consider the setup of the section on spot tree and dividends with N dividends, where dividend D_n is payed out at T_n. Assume that $i\Delta t < T_n < (i+1)\Delta t$ and set $\Delta t_n = T_n - i\Delta t$. In order not to disregard time value from $i\Delta t$ and T_n we include the following procedure. We introduce the financing rate $\bar{r}$ and spot forward rate $\bar{b}$ given by

$$\begin{aligned} \bar{r} &= \begin{cases} r_1 & \text{if } i\Delta t < T_1 \\ (rT - r_1 T_1)\frac{1}{T-T_1} & \text{otherwise} \end{cases} \\ \bar{b} &= \begin{cases} b_1 & \text{if } i\Delta t < T_1 \\ (bT - b_1 T_1)\frac{1}{T-T_1} & \text{otherwise} \end{cases} \end{aligned}$$

where T_1 is time to first dividend, T time to expiry, r_1 the financing rate at T_1, r the financing rate at T, $b_1 = \log(F_{T_1}/S)/T_1$ and $b = \log(F_T/S)/T$ and F_{T_1} and F_T are the forward prices assuming no dividends. The additional value at the time of the dividend is calculated as

$$e^{-\bar{r}\Delta t_n}\left(S_{\text{GBM},ij} e^{\bar{b}\Delta t_n} - (X - e^{\bar{r}\Delta t_n} D_{\text{remain},i})\right) \tag{40}$$

where $S_{\text{GBM},ij}$ is given by (30) and $D_{\text{remain},i}$ was defined in (31). The option value in the node i, j is given by the maximum of (40) and Π_{ij} from (29).

3.1.3. Binomial (relative) method

The binomial (relative) pricing model in Orc Trader™ uses a Jarrow-Rudd tree for derivative valuation (see [Hu]). We assert solutions $u = e^{\alpha\Delta t + \beta\sqrt{\Delta t}}$ and $d = e^{\alpha\Delta t - \beta\sqrt{\Delta t}}$ for some α and β and solve the equations (24)–(25) for small Δt. The equations are linearized, higher order terms are omitted and the additional condition $p = 1/2$ is imposed. For $\bar{u}(\alpha,\beta) = 1 + (\alpha + \frac{1}{2}\beta^2)\Delta t + \beta\sqrt{\Delta t}$ and $\bar{d}(\alpha,\beta) = 1 + (\alpha + \frac{1}{2}\beta^2)\Delta t - \beta\sqrt{\Delta t}$ we have the equations

$$\begin{aligned} \tfrac{1}{2}(\bar{u} + \bar{d}) &= 1 + (r - q)\Delta t \\ \tfrac{1}{2}(\bar{u}^2 + \bar{d}^2) - \tfrac{1}{4}(\bar{u} + \bar{d})^2 &= \sigma^2 \Delta t. \end{aligned}$$

We get the factors

$$\begin{aligned} u &= e^{(r-q-\sigma^2/2)\Delta t+\sigma\sqrt{\Delta t}} \\ d &= e^{(r-q-\sigma^2/2)\Delta t-\sigma\sqrt{\Delta t}} \\ p &= \frac{e^{\sigma^2\Delta t/2}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}. \end{aligned}$$

Limitations

As the assertion $p = 1/2$ is made on the linearized system, it is only valid for small Δt. The theoretical error from the linearisation is related to the term $\sigma^2\Delta t$ and when this term becomes large enough the model will break down. The theoretical limit is imposed by the condition $p < 1$ which introduces the limit $\sigma < 2/\sqrt{\Delta t}$. However, Orc Trader™ will cut volatilities in accordance with the limit

$$\sigma < \frac{1}{\sqrt{\Delta t}}$$

because u is not a monotone function of σ for volatilities beyond this limit. By formula (26) when precision is zero the limit is $\sigma < \frac{1}{\sqrt{T}} \cdot 447\%$ and when precision is five the limit is $\sigma < \frac{1}{\sqrt{T}} \cdot 1095\%$. The binomial relative pricing model should not be used for volatilities outside of these bounds.

3.1.4. Derivative from binomial tree

It is well known that the symmetrized average rate of change tends to give inaccurate estimates of the derivative for the binomial model (see [**PV**]). Instead, the derivative can be estimated from the binomial tree directly.

Note that $S_{21} = S_{00}$. By shifting the tree and time variable by two time steps the node $(2, 1)$ of the shifted tree corresponds to the node $(0, 0)$ of the unshifted tree. Hence $C_{21}^{\text{shifted}} = C_{00}^{\text{unshifted}}$. The first and second derivatives can be approximated by considering a parabola through the points $(S_{20}, C_{20}), (S_{21}, C_{21})$ and (S_{22}, C_{22}). The resulting formulas are

$$\frac{d}{dS}C \sim \frac{S_{20}(2S_{21}-S_{20})(C_{21}-C_{22})+S_{21}^2(C_{22}-C_{20})+S_{22}(2S_{21}-S_{22})(C_{20}-C_{21})}{(S_{21}-S_{20})(S_{22}-S_{20})(S_{22}-S_{21})} \tag{41}$$

and

$$\frac{d^2}{dS^2}C \sim \frac{S_{20}(C_{21}-C_{22})+S_{21}(C_{22}-C_{20})+S_{22}(C_{20}-C_{21})}{(S_{21}-S_{20})(S_{22}-S_{20})(S_{22}-S_{21})}. \tag{42}$$

3.2. Vellekoop-Nieuwenhuis

The Vellekoop-Nieuwenhuis model is named after the authors of [**VN**]. The main result of the article is a tree based modelling of the spot with discrete dividends retaining good performance. This approach is different from the other models in Orc as they use the escrowed model that, up to a discount factor, models the forward. The mathematical framework of the model is Black-Scholes and thus the parameters of the Vellekoop-Nieuwenhuis model are the same as for the other models.

In the presence of discrete dividends the Vellekoop-Nieuwenhuis model will, for a given volatility, give higher prices than the other models in Orc. This is because spot - discounted dividends (used in the escrowed model) is smaller than the spot before the last dividend time. The constant volatility will imply larger deviations in absolute terms and thus a higher price. This difference forces a different (recalibrated) volatility surface to be used for the Vellekoop-Nieuwenhuis model as compared to the other models. A volatility surface connected to a pricing model using the escrowed assumption will typically have jumps at the discrete dividend times. This is not the case for the Vellekoop-Nieuwenhuis model and it is therefore well suited for volatility interpolation between expiries. This can be especially useful for OTC-contracts.

As discussed above (The spot tree and dividends) tree models often suffer from big numerical errors near the optimal exercise boundary. This problem is particularly pronounced close to discrete dividends. To significantly decrease this problem the implementation of the Vellekoop-Nieuwenhuis model takes extra care in these regions. This is done by using more time steps locally around the dividends yielding improved accuracy, especially when it comes to risk numbers such as delta and gamma.

The Vellekoop-Nieuwenhuis model can be used for both for European and American options.

The use of the yield curve for the Vellekoop-Nieuwenhuis model is done as in the ordinary Binomial model; only the yield of the expiry date of the option is used.

4. Pricing models

A *pricing model* is a pair consisting of a mathematical model and a solution method. E.g. the pricing model "binomial" solves the Black-Scholes PDE using the binomial method.

5. Contract types and valuation

The theoretical price of a derivative contract is the fair value of the contract given a mathematical framework. Unless otherwise stated the valuation formulas stated in the sections below are derived in the Black-Scholes framework.

5.1. Forwards and futures contracts

To be able to exactly replicate values produced by Orc it is important to know how present value factors are calculated. How this is done is described in section Contract financing rate. A theoretical forward value is calculated as usual by multiplying the spot value with a forward factor and by subtracting the forward value of any dividends. The forward factors here will always include the underlying rate.

5.2. Vanilla options

5.2.1. Black 76

The default valuation of vanilla European call and put options are given by the Black 76 forward based formula:

$$\Pi_{\text{call}} = Fe^{-rT}\Phi(d_1) - Xe^{-rT}\Phi(d_2) \tag{43}$$

$$\Pi_{\text{put}} = Xe^{-rT}\Phi(-d_2) - Fe^{-qT}\Phi(-d_1) \tag{44}$$

$$d_1 = \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}} \tag{45}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{46}$$

Here F is the theoretical forward price for the expiry T. X, r, σ and Φ are defined in section 1.1. In the absence of discrete dividends the Black 76 formula is equivalent to the ordinary Black-Scholes formula.

5.2.2. The Bos-Vandermark approximation

When pricing equities with discrete dividends usually a spot based model is prefered instead of a forward based one. The Vellekoop-Niewenhuis model can be used for this purpose but for European options the analytic Bos-Vandermark approximation will be faster and typically give enough accuracy. The approximation is the ordinary Black-Scholes formula but with modified inputs. The modified spot price is

$$S' = S - \sum_i \frac{T - \tau_i}{T} \delta_i e^{-r_i \tau_i}. \tag{47}$$

The modified strike price is given as

$$X' = X - \sum_i \frac{\tau_i}{T} \delta_i e^{-r_i \tau_i}.$$

Here δ_i represent the ith dividend and τ_i, r_i the corresponding dividend time and discount rate. T as usual represents the time to maturity. The idea about this approximation is that dividends that are close in time will affect the spot more and dividends later in time will have greater effect on the strike price.

5.3. Binary options

There are only two possible payoff alternative of a binary (or digital) option: a fixed amount or nothing. A *cash-or-nothing* option pays off a fixed amount of cash if it is exercised ITM and a *asset-or-nothing* pays off a fixed amount of some asset. Binary options in Orc Trader™ are cash-or-nothing options. European asset-or-nothing options can be constructed synthetically from the relations (50) and (51)).

5.3.1. European binary options

European binary call options are ITM when the underlying spot price lies above the strike and OTM when the spot lies below the strike. If the spot lies above the strike at expiry the holder receives the fixed payment. European binary puts are ITM when the spot lies below the strike.

The value of a European cash-or-nothing option with the payoff one unit of cash is given by

$$\Pi_{\text{cash-or-nothing call}} = e^{-rT}\Phi(d_2) \tag{48}$$

$$\Pi_{\text{cash-or-nothing put}} = e^{-rT}\Phi(-d_2) \tag{49}$$

where S, X, T, r, q, σ and $d_2(S, X, T, r, q, \sigma)$ and Φ are defined in section 1.1.

A European vanilla option with strike X can be constructed synthetically by combining asset-or-nothing and cash-or-nothing options. For Π_{call} and Π_{put} given by (43) and (44) respectively the following relation holds.

$$\Pi_{\text{call}} = \Pi_{\text{asset-or-nothing call}} - X \cdot \Pi_{\text{cash-or-nothing call}} \tag{50}$$

$$\Pi_{\text{put}} = -\Pi_{\text{asset-or-nothing put}} + X \cdot \Pi_{\text{cash-or-nothing put}}. \tag{51}$$

5.3.2. American binary options

The American binary options available in Orc Trader™ are of knock-in type where the strike price acts as barrier. They start off OTM and are exercised immediately if the spot reaches the strike (ATM). The terminology from barrier options is applicable so American binary call options are referred to as up-and-in (UI) binary options and American binary put options are referred to as down-and-in (DI) binary options.

The value of the option depends on whether the settlement takes place immediately at knock-in or if it is deferred until the original expiry of the option. The value of immediate payment options is given by

$$\Pi_{\text{UI-binary}} = \left(\frac{X}{S}\right)^{(C_1+C_2)/\sigma^2} \Phi(-e_1) + \left(\frac{X}{S}\right)^{(C_1-C_2)/\sigma^2} \Phi(-e_2) \tag{52}$$

$$\Pi_{\text{DI-binary}} = \left(\frac{X}{S}\right)^{(C_2+C_2)/\sigma^2} \Phi(e_1) + \left(\frac{X}{S}\right)^{(C_1-C_2)/\sigma^2} \Phi(e_2) \tag{53}$$

where S, X, T, r, q, σ and Φ are defined in section 1.1 and

$$C_1 = r - q - \tfrac{1}{2}\sigma^2 \qquad C_2 = \sqrt{C_1^2 + 2r\sigma^2}$$
$$e_1 = \frac{\ln(X/S) + C_2 T}{\sigma\sqrt{T}} \qquad e_2 = \frac{\ln(X/S) - C_2 T}{\sigma\sqrt{T}}.$$

When payment is deferred to the expiry the value is given by

$$\Pi_{\text{UI-deferred}} = e^{-rT}\left(\Phi(d_2) + \left(\frac{X}{S}\right)^{2C_1/\sigma^2} \Phi(-d_3)\right) \tag{54}$$

$$\Pi_{\text{DI-deferred}} = e^{-rT}\left(\Phi(-d_2) + \left(\frac{X}{S}\right)^{2C_1/\sigma^2} \Phi(d_3)\right) \tag{55}$$

(56)

where S, X, T, r, q, σ and $d_2(S, X, T, r, q, \sigma)$ and Φ are defined in section 1.1 and

$$d_3 = \frac{\ln(X/S) + C_1 T}{\sigma\sqrt{T}}.$$

American binary knock-in options are not available for trading in Orc Trader™ . However, the value function of these options is used when calculating rebates of barrier options. Knock-out options pay a fixed amount of cash at expiry if the underlying is ITM during the whole life of the option. If the barrier is hit at any time before expiry the option becomes void. Binary up-and-out (UO) options are ITM when the underlying spot price lies below the strike, and down-and-out (DO) options are ITM when the spot lies above the strike. Their value is given by

$$\Pi_{\text{UO-binary}} = e^{-rT}\left(\Phi(-d_2) - \left(\frac{X}{S}\right)^{2C_1/\sigma^2} \Phi(-d_3)\right) \tag{57}$$

$$\Pi_{\text{DO-binary}} = e^{-rT}\left(\Phi(d_2) - \left(\frac{X}{S}\right)^{2C_1/\sigma^2} \Phi(d_3)\right) \tag{58}$$

with the same notation as above.

5.4. Barrier options

Barrier options are options which have a barrier H on the underlying S. The barrier determines whether the option is void or not. There are two types of barrier options:

- *In-options* (or *knock-in options*) are void until the underlying hits the barrier.
- *Out-options* (or *knock-out options*) become void when the underlying hits the barrier.

Barrier options are also grouped into up-options and down-options. For up-options the barrier lies above the current underlying spot price and for down-options the barrier lies below the current spot. Hence there are four kinds of barrier options: down-and-in (DI), down-and-out (DO), up-and-in (UI) and up-and-out (UO).

Orc Trader$^{\text{TM}}$ handles call and put barrier options of European type. The payoff is the same as the payoff of vanilla options given that the option is not void in accordance with the discussion above.

The following dynamical parameters are used to define barrier options:

- barrier hit
- barrier, H
- rebate, Π_R

Analytical solutions

The valuation of barrier options in Orc Trader$^{\text{TM}}$ is based on the *two-asset barrier options model* (see [**Ha**]). However, the Orc Trader$^{\text{TM}}$ model considers only the limit case as the correlation between the two assets goes to one. Hence it is a single asset model with two volatilities, the volatility of the strike σ_X and the volatility of the barrier σ_H. Here the volatility of the barrier, σ_H, is defined as the volatility read off from the barrier option contract if we would set the strike equal to the barrier level (assuming all other parameters, e.g. base contract price, stay the same).

(59)

$$\Pi_{\text{DIC}} = \begin{cases} \Pi_{\text{call}}, & S \le H \\ Se^{-qT}C_2\Phi(\min(d_3,-e_3)) - Xe^{-rT}C_1\Phi(\min(d_4,-e_4)), & S > H, d_1+e_1 < 0 \\ Se^{-qT}C_2\Phi(\min(d_3,-e_3)) - Xe^{-rT}C_1\Phi(\min(d_4,-e_4)) & \\ +Se^{-qT}(\Phi(d_1)-\Phi(-e_1)) - Xe^{-rT}(\Phi(d_2)-\Phi(-e_2)), & S > H, d_1+e_1 > 0 \end{cases}$$

(60)

$$\Pi_{\text{DOC}} = \Pi_{\text{call}} - \Pi_{\text{DIC}}$$

(61)

$$\Pi_{\mathsf{UIC}} = \begin{cases} \Pi_{\mathsf{call}}, & S \geq H \\ Se^{-qT}\Phi(\min(d_1, -e_1)) - Xe^{-rT}\Phi(\min(d_2, -e_2)), & S < H, d_1 + e_1 < 0 \\ Se^{-qT}\Phi(\min(d_1, -e_1)) - Xe^{-rT}\Phi(\min(d_2, -e_2)) \\ Se^{-qT}C_2(\Phi(d_3) - \Phi(-e_3)) - Xe^{-rT}C_1(\Phi(d_4) - \Phi(-e_4)), & S < H, d_1 + e_1 \geq 0 \end{cases}$$

(62)

$$\Pi_{\mathsf{UOC}} = \Pi_{\mathsf{call}} - \Pi_{\mathsf{UIC}}$$

(63)

$$\Pi_{\mathsf{DIP}} = \begin{cases} \Pi_{\mathsf{put}}, & S \leq H \\ -Se^{-qT}C_2(\Phi(-d_3) - \Phi(e_3)) + Xe^{-rT}C_1(\Phi(-d_4) - \Phi(e_4)) \\ -Se^{-qT}\Phi(\min(-d_1, e_1)) + Xe^{-rT}\Phi(\min(-d_2, e_2)), & S > H, d_1 + e_1 < 0 \\ -Se^{-qT}\Phi(\min(-d_1, e_1)) + Xe^{-rT}\Phi(\min(-d_2, e_2)), & S > H, d_1 + e_1 \geq 0 \end{cases}$$

(64)

$$\Pi_{\mathsf{DOP}} = \Pi_{\mathsf{put}} - \Pi_{\mathsf{DIP}}$$

(65)

$$\Pi_{\mathsf{UIP}} = \begin{cases} \Pi_{\mathsf{put}}, & S \geq H \\ -Se^{-qT}C_2\Phi(\min(-d_3, e_3)) + Xe^{-rT}C_1\Phi(\min(-d_4, e_4)) \\ -Se^{-qT}(\Phi(-d_1) - \Phi(e_1)) + Xe^{-rT}(\Phi(-d_2) - \Phi(e_2)), & S < H, d_1 + e_1 < 0 \\ -Se^{-qT}C_2\Phi(\min(-d_3, e_3)) + Xe^{-rT}C_1\Phi(\min(-d_4, e_4)), & S < H, d_1 + e_1 > 0 \end{cases}$$

(66)

$$\Pi_{\mathsf{UOP}} = \Pi_{\mathsf{put}} - \Pi_{\mathsf{UIP}}$$

where S, X, T, r, q and Φ are defined in section 1.1, Π_{call} and Π_{put} are given by (43) and (44) respectively and

$$\begin{aligned} C_1 &= (H/S)^{2(r-q)/\sigma_H^2 - 1} & C_2 &= C_1 \cdot (H/S)^{2\sigma_X/\sigma_H} \\ d_1 &= d_1(S, X, T, r, q, \sigma_X) & d_2 &= d_2(S, X, T, r, q, \sigma_X) \\ d_3 &= d_1 + 2\frac{\ln(H/S)}{\sigma_H\sqrt{T}} & d_4 &= d_2 + 2\frac{\ln(H/S)}{\sigma_H\sqrt{T}} \\ e_1 &= e_2 - \sigma_X\sqrt{T} & e_2 &= -d_2(S, H, T, r, q, \sigma_H) \\ e_3 &= e_4 - \sigma_X\sqrt{T} & e_4 &= -d_2(H, S, T, r, q, \sigma_H). \end{aligned}$$

Rebates

Barrier options can have a rebate Π_R. For knock-out barrier options rebates are payed out if the barrier is hit before expiry and for knock-in options rebates are payed out if the barrier is never hit. Hence, a barrier option with rebate is the sum of the corresponding barrier option without rebate plus Π_R times a binary option. The values of the eight cases available in Orc Trader™

are given by

$$
\begin{aligned}
\Pi_{\text{DIC with rebate}} &= \Pi_{\text{DIC}} + \Pi_R \cdot \Pi_{\text{DO-binary}} && (67)\\
\Pi_{\text{DOC with rebate}} &= \Pi_{\text{DOC}} + \Pi_R \cdot \Pi_{\text{DI-binary}} && (68)\\
\Pi_{\text{DIP with rebate}} &= \Pi_{\text{DIP}} + \Pi_R \cdot \Pi_{\text{DO-binary}} && (69)\\
\Pi_{\text{DOP with rebate}} &= \Pi_{\text{DOP}} + \Pi_R \cdot \Pi_{\text{DI-binary}} && (70)\\
\Pi_{\text{UIC with rebate}} &= \Pi_{\text{UIC}} + \Pi_R \cdot \Pi_{\text{UO-binary}} && (71)\\
\Pi_{\text{UOC with rebate}} &= \Pi_{\text{UOC}} + \Pi_R \cdot \Pi_{\text{UI-binary}} && (72)\\
\Pi_{\text{UIP with rebate}} &= \Pi_{\text{UIP}} + \Pi_R \cdot \Pi_{\text{UO-binary}} && (73)\\
\Pi_{\text{UOP with rebate}} &= \Pi_{\text{UOP}} + \Pi_R \cdot \Pi_{\text{UI-binary}} && (74)
\end{aligned}
$$

where $\Pi_{\text{UI-binary}}, \Pi_{\text{DI-binary}}, \Pi_{\text{UO-binary}}$ and $\Pi_{\text{DO-binary}}$ are given by (52), (53), (57), (58) and respectively.

5.5. Turbo warrants

A turbo warrant is a warrant with a knock-out barrier placed between the initial spot price of the underlying and the strike. If the barrier is hit a rebate is payed out.

The rebate depends on the value of the underlying during the rebate period. The rebate period starts when the barrier is hit, and the length of the period is specified by the dynamic parameter rebate period.

- For turbo call warrants the rebate equals the difference between the lowest recorded spot price during the rebate period and the strike.
- For turbo put warrants the rebate equals the difference between the strike and the highest recorded spot price during the rebate period.

The following dynamical parameters are used to define turbo warrants:

- barrier hit
- barrier, H
- rebate period

Analytical solutions

The value of a turbo warrant where the barrier has not been hit is fundamentally calculated as barrier options with a rebate. The value of turbo call warrants is given by (68) and turbo put warrants is given by (74) with some modifications. Only a single volatility is considered so σ_H is replaced by σ_X.

The rebate Π_R is calculated from the following function where the value of the variable Y depends on whether the barrier has been hit or not (see [E1]):

$$
\begin{aligned}
\Pi_{\text{R, call}}(S,X,Y) \quad = \quad & S\left(1+\frac{\sigma^2}{2(r-q)}\right)(\Phi(d_Y-d_+)-\Phi(d_X-d_+)) \\
& +e^{-(r-q)T_{\text{reb}}}\Bigg[\left(1-\frac{\sigma^2}{2(r-q)}\right)(k_Y\Phi(d_Y+d_-)-k_X\Phi(d_X+d_-)) \\
& -X(\Phi(d_Y-d_-)+k_Y\Phi(d_Y+d_-)-\Phi(d_X-d_-)-k_X\Phi(d_X+d_-)) \\
& +(Y-X)(\Phi(-d_-)+\Phi(d_-)-\Phi(d_Y-d_-)-k_Y\Phi(d_Y+d_-))\Bigg]
\end{aligned} \tag{75}
$$

$$
\begin{aligned}
\Pi_{\text{R, put}}(S,X,Y) \quad = \quad & S\left(1+\frac{\sigma^2}{2(r-q)}\right)(\Phi(d_Y-d_+)-\Phi(d_X-d_+)) \\
& +e^{-(r-q)T_{\text{reb}}}\Bigg[\left(1-\frac{\sigma^2}{2(r-q)}\right)(-k_Y\Phi(-d_Y-d_-)+k_X\Phi(-d_X-d_-)) \\
& +X(-\Phi(d_Y-d_-)+k_Y\Phi(d_Y+d_-)+\Phi(d_X-d_-)-k_X\Phi(d_X+d_-)) \\
& +(X-Y)(-\Phi(-d_-)+\Phi(d_-)+\Phi(d_Y-d_-)-k_Y\Phi(d_Y+d_-))\Bigg]
\end{aligned} \tag{76}
$$

or in case $r-q=0$

$$
\begin{aligned}
\Pi_{\text{R, call}}(S,X,Y) \quad = \quad & 2S\Big(\Phi(d_Y-d_+)-\Phi(d_X-d_+)\Big) \\
& -S\Bigg[\ln(Y/S)\Phi(d_Y-\sigma^2T_{reb})-\ln(X/S)\Phi(d_X-\sigma^2T_{reb}) \\
& -\sigma\sqrt{T_{reb}}\Big(f(d_Y-\sigma^2T_{reb})-f(d_x-\sigma^2T_{reb})\Big) \\
& -\frac{1}{2}\sigma^2T_{reb}\Big(\Phi(d_Y-\sigma^2T_{reb})-\Phi(d_X-\sigma^2T_{reb})\Big)\Bigg] \\
& -X\left[\Phi(d_Y-d_-)+\frac{k_Y}{Y}\Phi(d_Y-d_-)-\Phi(d_X-d_-)-\frac{k_X}{X}\Phi(d_X+d_-)\right] \\
& +(Y-X)\left[\Phi(-d_-)+\Phi(d_-)-\Phi(d_Y-d_-)-\frac{k_Y}{Y}\Phi(d_Y+d_-)\right]
\end{aligned}
$$

$$\begin{aligned}
\Pi_{\text{R, put}}(S,X,Y) \quad = \quad & 2S\Big(\Phi(d_X-d_+)-\Phi(d_Y-d_+)\Big) \\
& -S\Bigg[-\ln(Y/X)+\ln(Y/S)\Phi(d_Y-\sigma^2T_{reb})-\ln(X/S)\Phi(d_X-\sigma^2T_{reb}) \\
& -\sigma\sqrt{T_{reb}}\Big(f(d_Y-\sigma^2T_{reb})-f(d_x-\sigma^2T_{reb})\Big) \\
& -\frac{1}{2}\sigma^2T_{reb}\Big(\Phi(d_Y-\sigma^2T_{reb})-\Phi(d_X-\sigma^2T_{reb})\Big)\Bigg] \\
& +X\Bigg[-\Phi(d_Y-d_-)+\frac{k_Y}{Y}\Phi(d_Y-d_-)+\Phi(d_X-d_-)-\frac{k_X}{X}\Phi(d_X+d_-)\Bigg] \\
& +(X-Y)\Bigg[\Phi(d_Y-d_-)-\frac{k_Y}{Y}\Phi(-d_Y-d_-)\Bigg]
\end{aligned}$$

where T_{reb} is the length of the rebate period and S, X, T, r, q, σ and Φ are defined in section 1.1 and

$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}},$$

$$\begin{aligned}
k_X &= X\left(\tfrac{X}{S}\right)^{2(r-q)/\sigma^2-1} & k_Y &= Y\left(\tfrac{Y}{S}\right)^{2(r-q)/\sigma^2-1} \\
d_X &= \frac{\ln(X/S)}{\sigma\sqrt{T_{\text{reb}}}} & d_Y &= \frac{\ln(Y/S)}{\sigma\sqrt{T_{\text{reb}}}} \\
d_+ &= \left(\frac{1}{\sigma}(r-q)+\tfrac{1}{2}\sigma\right)\sqrt{T_{\text{reb}}} & d_- &= \left(\frac{1}{\sigma}(r-q)-\tfrac{1}{2}\sigma\right)\sqrt{T_{\text{reb}}}.
\end{aligned}$$

If the barrier has not been hit then the rebate is evaluated conditioned that $S = H$ and with $Y = H$, i.e. the rebate is given by $\Pi_{\text{R}}(H, X, H)$. If the barrier has been hit then the rebate is evaluated with $Y = S$, i.e. the rebate is given by $\Pi_{\text{R}}(S, X, S)$.

5.6. CBBC (Callable bull/bear contracts)

CBBC contracts are similar to turbo warrants. A knock-out barrier is placed between the initial spot price of the underlying and the strike. If the barrier is hit a rebate is payed out.

For CBBC contracts the barrier is hit when the trigger contract hits the barrier. The parameters of the trigger contract are however not used for valuation of CBBC contracts (see Approximation as turbo warrants for calculation details). If the dynamical parameter for trigger contracts is left blank the base contract is used as trigger contract.

Unlike for turbo warrants, the rebate period for CBBC contracts does not have fixed length. The rebate period starts when the barrier is hit. It then extends until the end of the *trading session* in which the barrier was hit, plus one additional trading session. Hence, $T_{\text{ts}} < T_{\text{reb}} < 2 \cdot T_{\text{ts}}$, where T_{reb} is the rebate period length and T_{ts} is the trading session length.

The rebate amount payed out equals the minimal absolute difference between the strike and the underlying spot price observed during the rebate period. Let m be the minimal and M the maximal value of the underlying observed during the rebate period. The dynamical parameter reference price must be manually updated with m respectively M for CBBC calls respectively puts, as new lows respectively highs are observed.

On some markets the rebate can be *floored*, i.e. a lower bound can be set on the rebate. The floor is a percentage α of the absolute difference between the barrier H and the strike X. Hence the rebate payoff function is given for put and call respectively by

$$P_{\text{rebate, call}} = \max\left(m - X, \tfrac{1}{100} \cdot \alpha(H - X)\right) \tag{77}$$

$$P_{\text{rebate, put}} = \max\left(X - M, \tfrac{1}{100} \cdot \alpha(X - H)\right) \tag{78}$$

where τ is the expiration time and τ_H is the barrier hit time. The percentage α is entered into the dynamical parameter rebate. The default value is $\alpha = 0$ which corresponds to no floor.

The following dynamical parameters are used to define CBBC contracts:

- start next trading session, $\tau_{\text{start next}}$
- end next trading session, $\tau_{\text{end next}}$
- end current trading session, $\tau_{\text{end cur}}$
- reference price, S_{ref}
- rebate - this parameter has a unique application for CBBC contracts.
- trigger contract
- barrier hit
- barrier, H

Approximation as turbo warrants

The value of the rebate of CBBC contracts depends on the probability distribution of the first time that the spot hits the barrier τ_H, which determines the length of the rebate period T_{reb}. In Orc Trader™ this value is approximated by

$$T_{\text{reb}} = \tau_{\text{end cur}} - t + \tau_{\text{end next}} - \tau_{\text{start next}} \tag{79}$$

where t is the current time and $\tau_{\text{end cur}}, \tau_{\text{end next}}$ and $\tau_{\text{start next}}$ are the end of the current, end of next and start of next trading sessions respectively. Conversion of T_{reb} from hours to years is done by division of $256 \cdot 8$, i.e. the number of trading days per year times number of trading hour per day. This approximation is only good when the underlying is close to the barrier. It is motivated by the contribution from the rebate being less significant in comparison with the barrier option part of the contract when the underlying is far from the barrier.

The barrier is hit when the trigger contract S_{trig} reaches the barrier. The parameters of S_{trig} are however not used for calculation. Instead a synthetic contract S_{synth} is constructed which inherits all parameters from the underlying contract S, except its value is shifted by the difference $S_{\text{trig}} - S$. Considering the spot values as a function of time, we have $S_{\text{synth}}(t) = S(t) + S_{\text{trig}}(0) - S(0)$, where $t = 0$ is present time.

The payoff from a floored CBBC contract equals the payoff from an un-floored CBBC contract with an additional constant contribution $\alpha|X - H|$ and where the strike has been shifted $X' = X + \alpha|H - X|$. Hence the payoffs (77) and (78) can be rewritten as

$$P_{\text{rebate, call}} = \tfrac{1}{100} \cdot \alpha(H - X) + \max\left(m - X', 0\right)$$

$$P_{\text{rebate, put}} = \tfrac{1}{100} \cdot \alpha(X - H) + \max\left(X' - M, 0\right)$$

The rebate part of a CBBC call is calculated using the turbo warrant call rebate function (75) where T_{reb} given by (79). The input parameters of the function depend on whether the barrier has been hit or not. If the barrier has not been hit then the rebate is calculated with the following input parameters

$$\Pi_{R,\text{call}} = \Pi_{R,\text{turbo call}}(H, X', H) + \alpha(H - X).$$

If the barrier has been hit then the rebate is calculated as

$$\Pi_{R,\text{call}} = \Pi_{R,\text{turbo call}}(S_{\text{synth}}, X', S_{\text{ref}}) + \alpha(H - X).$$

The value of the CBBC call is given by

$$\Pi_{\text{CBBC, call}} = \Pi_{\text{DOC}}(S, S_{\text{synth}}) + \Pi_{R,\text{call}} \cdot \Pi_{\text{DI-binary}}(S_{\text{synth}})$$

where $\Pi_{\text{DI-binary}}(S)$ is calculated from (53) and

$$\Pi_{\text{DOC}}(S_1, S_2) = \begin{cases} 0 & S_2 \leq H \\ \lim_{\rho \to 1} S_1 e^{-qT}\Big(\Phi_2(d_1, -e_1; \rho) - C_1 \Phi_2(d_3, -e_3; \rho)\Big) & S_2 > H \\ -Xe^{-rT}\Big(\Phi_2(d_2, -e_2; \rho) - C_2 \Phi_2(d_4, -e_4; \rho)\Big) & \end{cases} \tag{80}$$

where X, T, r, q, σ and Φ_2 are defined in section 1.1, $d_1(S_1, X, T, r, q, \sigma)$ and $d_2(S_1, X, T, r, q, \sigma)$ are given by (1) and (2) respectively and

$$\begin{aligned} C_1 &= \left(\frac{H}{S_2}\right)^{2(r-q)/\sigma^2+1} & C_2 &= \left(\frac{H}{S_2}\right)^{2(r-q)/\sigma^2-1} \\ d_3 &= d_1 + \frac{1}{\sigma\sqrt{T}} 2\rho \ln(H/S_2) & d_4 &= d_2 + \frac{1}{\sigma\sqrt{T}} 2\rho \ln(H/S_2) \\ e_1 &= \frac{1}{\sigma\sqrt{T}}(\ln(H/S_2) - (r - q - (\tfrac{1}{2} - \rho)\sigma^2)T) & e_2 &= e_1 + \rho\sigma\sqrt{T} \\ e_3 &= e_1 - \frac{1}{\sigma\sqrt{T}} 2\ln(H/S_2) & e_4 &= e_2 - \frac{1}{\sigma\sqrt{T}} 2\ln(H/S_2). \end{aligned}$$

The rebate part of a CBBC put is calculated using the turbo warrant put rebate function (76) where T_{reb} given by (79). The input parameters of the function depend on whether the barrier has been hit or not. If the barrier has not been hit then the rebate is calculated with the following input parameters

$$\Pi_{R,\text{put}} = \Pi_{R,\text{turbo put}}(H, X', H) + \alpha(X - H).$$

If the barrier has been hit then the rebate is calculated as

$$\Pi_{R,\text{put}} = \Pi_{R,\text{turbo put}}(S_{\text{synth}}, X', S_{\text{ref}}) + \alpha(X - H).$$

The value of the CBBC put is given by

$$\Pi_{\text{CBBC, put}} = \Pi_{\text{UOP}}(S, S_{\text{synth}}) + \Pi_{R,\text{put}} \cdot \Pi_{\text{UI-binary}}(S_{\text{synth}})$$

where $\Pi_{\text{UI-binary}}(S)$ is calculated from (52) and

$$\Pi_{\text{UOP}}(S_1, S_2) = \begin{cases} 0 & S_2 \geq H \\ \lim_{\rho \to 1} -S_1 e^{-qT}\Big(\Phi_2(-d_1, e_1; \rho) - C_1 \Phi_2(-d_3, e_3; \rho)\Big) & S_2 < H \\ +Xe^{-rT}\Big(\Phi_2(-d_2, e_2; \rho) - C_2 \Phi_2(-d_4, e_4; \rho)\Big) & \end{cases} \tag{81}$$

Note that if no trigger contract is set, then the underlying contract and synthetic contract are the same so $S_1 = S_2$ above. In this case (80) is the same as (60) and (81) is the same as (66) with $\sigma_H = \sigma_X$.

5.7. Quanto options

Quantos are fixed exchange-rate foreign-equity options. The underlying and strike are valued in a foreign currency. The payoff is payed in the domestic currency at a predetermined fixed exchange rate J_{fix}, with payoff functions given by

$$
\begin{aligned}
P_{\text{Call}} &= J_{\text{fix}} \cdot \max(S_f - X_f, 0) \\
P_{\text{Put}} &= J_{\text{fix}} \cdot \max(X_f - S_f, 0)
\end{aligned}
$$

The following dynamic parameters are used to define quanto options

- FX rate, J_{fix}
- FX volatility, σ_J
- FX/Spot correlation, ρ

Analytical solutions

The exchange rate is modeled by the following geometric Brownian motion

$$
\begin{aligned}
dJ &= (r_f - r_d)Jdt + \sigma_J dW_J && (82) \\
J(0) &= J_0, && (83)
\end{aligned}
$$

where r_f is the foreign interest rate, r_d is the domestic interest rate, σ_J is the FX volatility, J_0 is the spot exchange rate. The Brownian motions of the exchange rate W_J and of the underlying spot W from (13) are correlated by the FX/Spot correlation ρ.

The two-factor model can be transformed into a one factor model with a new yield

$$
q' = q + r_d - r_f + \rho \sigma_J \sigma \tag{84}
$$

where q is the underlying rate of the underlying from (13) and σ is the volatility from the same SDE (13), see [**KD**].

The value of the quanto call and put is given by

$$
\begin{aligned}
\Pi_{\text{call}} &= J_{\text{fix}} \cdot e^{-r_d T} \left(S_f e^{(r_f - q - \rho \cdot \sigma_J \cdot \sigma)T} \Phi(d_1) - X_f \Phi(d_2) \right) \\
&= J_{\text{fix}} \cdot \Pi_{\text{vanilla call}}(S_f, X_f, T, r_d, q', \sigma) \\
\Pi_{\text{put}} &= J_{\text{fix}} \cdot e^{-r_d T} \left(X_f \Phi(-d_2) - S_f e^{(r_f - q - \rho \cdot \sigma_J \cdot \sigma)T} \Phi(-d_1) \right) \\
&= J_{\text{fix}} \cdot \Pi_{\text{vanilla put}}(S_f, X_f, T, r_d, q', \sigma).
\end{aligned}
$$

where T is time to expiry and

$$
\begin{aligned}
d_1 &= \frac{\ln(S_f/X_f) + (r_f - q - \rho \sigma \sigma_J + \sigma^2/2)T}{\sigma \sqrt{T}} \\
d_2 &= d_1 - \sigma \sqrt{T}.
\end{aligned}
$$

Note that the option value is not affected by J_0. In Orc Trader™ the cross rate between the domestic and foreign currencies must be set for calculation of theoretical values. However, this rate does not affect the option value.

5.8. Quanto barrier options

The previous section shows that quanto options are scaled vanilla options with a modified yield. Analogously, quanto barrier options are scaled barrier options with the same modified yield. The value of a quanto barrier options is calculated from the corresponding barrier option formula (59)–(66) by

$$\Pi_{\text{quanto barrier}}(S_f, X_f, H_f, T, r_d, q, \sigma) = J_{\text{fix}} \cdot \Pi_{\text{barrier}}(S_f, X_f, H_f, T, r_d, q', \sigma)$$

where S_f, X_f and H_f are the spot, strike and barrier in the foreign currency, q' given by (84) and J_{fix} is the fixed foreign exchange rate.

5.9. Asian options

5.9.1. ARO options

An *average rate option* (ARO) is an option on the arithmetic average of the underlying $S(t)$. The arithmetic average of $S(t)$ is calculated over the *sampling period* on a number of *reset dates* $t_1, t_2, \ldots, t_N$

$$A = \frac{1}{N} \sum_{i=1}^{N} S(t_i).$$

For future reset dates the Orc Trader™ calculates the theoretical forward value of S. There is no way to retrieve the values of S for passed reset dates so these must be entered manually into the Orc Trader™ .

The payoff of the ARO call and put options are given by

$$\begin{aligned} P_{\text{call}}(A) &= \max(A - X, 0) \\ P_{\text{pus}}(A) &= \max(X - A, 0). \end{aligned}$$

The average A is not a geometric Brownian motion which makes valuation of ARO options hard and approximation methods are necessary.

Curran's approximation before entering the sampling period

The Orc Trader™ calculates theoretical prices of ARO options by applying the approximation method proposed by Curran in [**C1**], [**L**].

Let $F(t_i)$ be the theoretical futures values of the underlying that expire at time t_i, calculated from (10). The Curran approximation is based on conditioning on the geometric average of the underlying

$$G = \left(\prod_{i=1}^{N} S(t_i) \right)^{1/N}.$$

The mean μ_G and variance σ_G^2 of $\ln G$ and the covariance $\sigma_{iG} = \operatorname{cov}(\ln G, \ln S(t_i))$ are calculated by

$$\mu_G = \frac{1}{N}\sum_{i=1}^{N} \ln F(t_i) - \frac{1}{2}\sigma_i^2 t_i \tag{85}$$

$$\sigma_G^2 = \frac{1}{N^2}\sum_{i=1}^{N}(2N-2i+1)\sigma_i^2 t_i \tag{86}$$

$$\sigma_{iG} = \frac{1}{N}\left(\sum_{j=1}^{i-1}\sigma_j^2 t_j + (N-i+1)\sigma_i^2 t_i\right) \tag{87}$$

where σ_i is the volatility used for contracts with expiration date t_i.

Curran argues that for $\kappa \geq 0$ the following is a lower bound to the ARO call option

$$\Pi_{\text{low}}(\kappa) = e^{-rT}\left(\frac{1}{N}\sum_{i=1}^{N} F(t_i)\Phi(x_i) - X\Phi(y)\right). \tag{88}$$

where T is time to expiry of the option, Φ is given by (3) and

$$x_i = \frac{1}{\sigma_G}(\mu_G + \sigma_{iG} - \kappa) \quad \text{and} \quad y = \frac{1}{\sigma_G}(\mu_G - \kappa).$$

The value of the call option is obtained by maximizing this expression over κ

$$\Pi_{\text{ARO call}} = \Pi_{\text{low}}(\kappa^*) \tag{89}$$

where κ^* solves $\Pi'_{\text{low}}(\kappa^*) = 0$.

The value of the put option is obtained from

$$\Pi_{\text{ARO put}} = -e^{-rT}\frac{1}{N}\left(\sum_{i=1}^{N} F(t_i) - X\right) + \Pi_{\text{ARO call}}.$$

Curran's approximation after entering the sampling period

After the sampling period has started and $m < N$ reset dates have passed the recorded average is $A_m = \frac{1}{m}\sum_{i=1}^{m} S(t_i)$. The value of the ARO option is calculated using the function $\Pi_{\text{ARO call}}(t, X, [t_1, t_2, \ldots, t_N])$ obtained from (89), but with scaled strike

$$X_m = \frac{NX - mA_m}{N - m}$$

and only considering future reset dates, i.e. at time $t_m < t < t_{m+1}$ the value of the ARO option is calculated by

$$\Pi_{\text{ARO call}}(t, X, [t_1, t_2, \ldots, t_N]) = \frac{N-m}{N}\Pi_{\text{ARO call}}\left(t, X_m, [t_{m+1}, \ldots, t_N]\right).$$

If $X_m < 0$ the option is certain to end up ITM. Then the option value is obtained from

$$\Pi_{\text{ARO call}} = e^{-rT}\left(\frac{1}{N-m}\sum_{i=m+1}^{N} F(t_i) - X_m\right).$$

Implied volatility and actual volatility

See section 2.1.7 for a general discussion on implied volatilities. In Orc Trader™ the implied volatility of ARO options is the volatility which will produce the market value Π_m of the option when applied to all future reset dates.

As seen above when calculating (85)–(87), the value of ARO options depends on the volatility at each reset date, i.e. the value is given by $\Pi_{\text{ARO}}(\sigma_1, \sigma_2, \ldots, \sigma_N)$. The implied volatility σ_{imp} is obtained by replacing all σ_i by σ_{imp}, and looking for the value such that

$$\Pi_{ARO}(\sigma_{\text{imp}}, \sigma_{\text{imp}}, \ldots, \sigma_{\text{imp}}) = \Pi_m.$$

Actual volatility is calculated the same way with the market price Π_m exchanged by the theoretical price of the ARO option.

5.9.2. Forward start ARO options

Forward start ARO options are valued in the same way as regular ARO option with the difference that the strike is not fixed in advance. The following dynamical parameter determines the time of fixing of the strike

- start date, τ_{start}

Before the start date the expected strike X_{expec} by

$$X_{\text{expec}} = \frac{X}{100} \cdot F(T_{\text{start}})$$

where T_{start} is time to the start date and $F(T)$ is the future value of the underlying calculated from (10). The volatility σ_i in (85)–(87) is the volatility for the strike X_{expec}.

Before the start date the value of forward start ARO call options is given by (89) with X replaced by X_{expec} in (88).

After the start date the strike parameter is considered to be the strike of the option and the valuation is the same as for plain ARO options.

5.10. Variance swaps

A variance swap is a forward contract on annualized variance. The pay-off at expiration is the difference between the realized variance of the underlying contract and the variance delivery price.

Note that the variance delivery price is the price at which the variance swap is traded and cannot be set as a strike price. The pricing model depends on five parameters:

The following dynamic parameters affect the value of variance swaps

- realized volatility, σ_{real}
- start date, τ_{start}
- accuracy, N
- percentage of ATM, p_{ATM}
- use variance space
- use observations

Replicating variance swaps boils down to pricing the contract $\log(S/S^*)$. This can be done by linear combinations of regular European call and put options. The value S^* can be thought of as a liquidity boundary between puts and calls. For strikes above S^* calls are used and for strikes below S^* puts are used instead.

5.10.1. Theoretical value of a variance swap

Consider a variance swap with start date τ_{start} and expiry date τ. The theoretical value of a variance swap is composed of three terms

- The realized variance σ^2_{real} from τ_{start} until the day before the calculation date.
- The intraday variance σ^2_{intra} for the calculation date.
- The future expected variance σ^2_{expec} from the calculation date until τ. The future expected variance depends on the volatility model chosen for the underlying asset. In Orc Trader™ we use a log-normal assumption for the distribution of the underlying asset.

Let us define:

- T time to expiry (in years) of the variance swap
- T_{d} the number of remaining trading days, i.e. the number of trading days between the calculation date and τ
- $T_{\text{d,passed}}$ the number of passed trading days, i.e. the number of trading days between τ_{start} and the calculation date
- $T_{\text{d,tot}}$ the total lifetime (in trading days) of the variance swap, i.e. the number of trading days from τ_{start} to τ. We have $T_{\text{d,tot}} = T_{\text{d,passed}} + T_{\text{d}} + 1$.
- B the current base price
- B_{close} the closing price of the base contract the day before the calculation date
- S the market spot price of the underlying asset
- S_T the theoretical spot price at the expiry date T
- F the theoretical forward price w.r.t. the time left to expiry of the variance swap.

The total variance is given by

$$\sigma^2_{\text{tot}} = \frac{1}{T_{\text{d,tot}}} \left(T_{\text{d,passed}} \cdot \sigma^2_{\text{real}} + \sigma^2_{\text{intra}} + T_{\text{d}} \cdot \sigma^2_{\text{expec}} \right).$$

In Orc Trader™ the theoretical price of a variance swap, $\Pi_{\text{var swap}}$ is defined as the total variance. Note that the strike does not enter the valuation of the variance swap

$$\Pi_{\text{var swap}} = \sigma^2_{\text{tot}}. \tag{90}$$

Realized and intraday variances from observations

The variance swap model supports two methods for calculating realized and intraday variances. If the dynamic parameter use observations is non-zero, then the observations of the observation contract are used for calculating realized and intraday variances.

Consider the observations $S_1, S_2, \ldots S_N$ where S_0 is the observation at the start date (defined by the dynamic parameter start date) and S_N is the observation of the previous observation date, i.e. the last observation date not equal to todays date. If todays closing price has been entered into to observations table, we denote it by S_{N+1}.

The intraday variance is calculated as

$$\sigma^2_{\text{intra}} = 252 \cdot \left(\log\left(\frac{S_{\text{today}}}{S_N}\right)\right)^2$$

where S_{today} is the theoretical spot price if todays closing price is not present in the observation table, and $S_{\text{today}} = S_{N+1}$ otherwise.

The realized variance is given by

$$\sigma^2_{\text{real}} = \frac{252}{N-1}\sum_{i=2}^{N}\left(\log\left(\frac{S_i}{S_{i-1}}\right)\right)^2 \tag{91}$$

Note that dividends set on the observation contract are **not** taken into account when calculating realized and intraday variance. Hence, if a dividend occurs all previous values of the observation contract must be adjusted for correct realized variance calculation.

Realized and intraday variance not using observations

For backward compatibility reasons the realized and intraday variances can be calculated without using observations, by setting the dynamic parameter use observations to zero.

The realized variance needs to be computed using historical data and can be considered a constant (it is an input parameter to the model not computed by the Orc System).

The intraday variance is computed from the return of the underlying asset since the last trading day, i.e. relative yesterdays closing price B_{close}. Any dividends paid is taken into account explicitly (unless the underlying asset is a forward or a futures contract).

$$\sigma^2_{\text{intra}} = 252 \cdot \left(\log\left(\frac{B+D}{B_{\text{close}}}\right)\right)^2$$

where D denotes any dividends paid since the last trading day.

In case of a forward or a futures contract $D = 0$. If no closing price is available the settlement price is taken instead. If neither the closing price nor the settlement price are available, the return is considered to be zero.

Future expected variance

The future expected variance σ^2_{expec} is defined by

$$\sigma_{\text{expec}} = \frac{2}{T}\left(rT - \left(\frac{S}{F}e^{rT} - 1\right) - \log\left(\frac{F}{S}\right) + E\left(\frac{S_T - F}{F} - \log\left(\frac{S_T}{F}\right)\right)\right). \tag{92}$$

Algorithm for calculating the expected value of the log contract

To compute (92) we use an algorithm using vanilla put and call options with varying strikes for replicating the contract with payoff

$$P_{\text{log}}(S_T) = \frac{S_T - F}{F} - \log\left(\frac{S_T}{F}\right).$$

The strikes of the vanilla options are determined by the dynamic parameters accuracy, N, and percentage of ATM, p_{ATM}. The call strikes are $X_{c,i} = F + i\Delta X$ and put strikes $X_{p,i} = F - i\Delta X$ where $i = 0 \ldots N$ and $\Delta X = \frac{1}{N-1} \cdot p_{\text{ATM}} \cdot F$. The function P_{log} is replicated with $w_{c,i}$ vanilla call payoffs with strike $X_{c,i}$ and $w_{p,i}$ vanilla puts payoffs with strike $X_{p,i}$, where the call weight functions are calculated iteratively by

$$\begin{aligned} w_{c,0} &= \frac{P_{\text{log}}(X_{c,1}) - P_{\text{log}}(X_{c,0})}{X_{c,1} - X_{c,0}} \\ w_{c,i} &= \frac{P_{\text{log}}(X_{c,i+1}) - P_{\text{log}}(X_{c,i})}{X_{c,i+1} - X_{c,i}} - \sum_{j=0}^{i-1} w_{c,j} \qquad i = 1 \dots N-1 \end{aligned}$$

and the put weights are calculated analogously by replacing sub-index c by p.

This gives us the following approximation of the last term in (92) $E(P_{\text{log}}(S_T))$

$$e^{rT} \sum_{i=0}^{N-1} \left(w_{c,i} \Pi_{\text{call}}(S, X_{c,i}, T, r, q, \sigma_{\text{impl}}(X_{c,i}, T)) + w_{p,i} \Pi_{\text{put}}(S, X_{p,i}, T, r, q, \sigma_{\text{impl}}(X_{p,i}, T)) \right)$$

where Π_{call} and Π_{put} given by (43) and (44).

Since P_{log} is a convex function the approximation by the affine payoff functions of vanilla puts and calls will be greater than or equal to P_{log}. Hence, our approximation will overestimate the future expected variance. By increasing the accuracy the approximation is improved.

5.10.2. Calculated historic volatility

The calculated historic volatility column displays the variance (or volatility, depending on the use variance space setting) of the observations since the start date given by the dynamic parameter start date. Hence it is defined by σ_{real} as calculated in (91).

5.10.3. Delta and swimming skew delta for a variance swap

The definition of delta for a contract is the change in value when the base price changes, holding all other parameters fixed. For a variance swap, when changing the base price keeping all other parameters fixed, the only term changing its value is the intraday variance part. The expected future variance does not change since the options in the replicating portfolio are priced with the same volatilities independently of the base price, and strike price are all centered around the ATM forward. When it comes to swimming skew delta the expected future variance contribute to the delta since the volatilities are allowed to change for each strike if the variance swap is priced using implied volatilities.

In the trading window the following values are seen for Δ and SkewΔ:

(93) $$\Delta = \frac{\sigma^2_{\text{intra}}(B_t(1+0.00001)) - \sigma^2_{\text{intra}}(B_t(1-0.00001))}{2B_t 0.00001},$$

and

(94) $$\text{Skew}\Delta = \frac{\sigma^2_{\text{expec}}(S_t(1+0.00001)) - \sigma^2_{\text{expec}}(S_t(1-0.00001))}{2S_t 0.00001}.$$

Here $\sigma^2_{\text{intra}}(x)$ and $\sigma^2_{\text{expec}}(x)$ denotes the intraday and expected variance given that the base price is x.

In (94) the volatility is allowed to swim with the base price if a surface is used.

In the portfolio window the equations (93) and (94) multiplied by volume$\times$multiplier$\times$price multiplier are used for Δ and SkewΔ.

5.10.4. Vega of a variance swap

The vega of a variance swap is the sensitivity in the expected future variance to changed in the volatility of the underlying asset. It is computed as a finite difference quotient in which the volatility is shifted by 0.5%:

$$\mathcal{V} = \frac{\sigma^2_{\text{expec}}(\sigma + 0.005) - \sigma^2_{\text{expec}}(\sigma - 0.005)}{2 * 0.005}. \tag{95}$$

Here $\sigma^2_{\text{expec}}(v)$ denotes the expected variance given that the volatility is v.

In the portfolio window the value (95) is multiplied with volume $\times$ multiplier $\times$ price multiplier.

For a variance swap based on constant contract based volatility, vega in the trading window should be $(T - t)/T$.

5.10.5. Gamma and swimming skew gamma of a variance swap

The definition of gamma for a contract is the change in delta when the base price changes, holding all other parameters fixed. For a variance swap, when changing the base price keeping all other parameters fixed, the only term changing its value is the intraday variance part. The expected future variance does not change since the options in the replicating portfolio are priced with the same volatilities independently of the base price, and the strike prices are all centered around the ATM forward. When it comes to swimming skew gamma the expected future variance contribute to the gamma since the volatilities are allowed to change for each strike if the variance swap is priced using implied volatilities. In the trading window the following values are seen for Γ and SkewΓ:

$$\Gamma = \frac{\sigma^2_{\text{intra}}(B_t(1 + 0.001)) - 2\sigma^2_{\text{intra}}(B_t(1 - 0.001))}{2(B_t)^2 0.001^2}, \tag{96}$$

and

$$\text{Skew}\Gamma = \frac{\sigma^2_{\text{expec}}(S_t(1 + 0.001)) - 2\sigma^2_{\text{expec}}(S_t) + \sigma^2_{\text{expec}}(S_t(1 - 0.001))}{2(S_t)^2(0.001)^2}. \tag{97}$$

Here $\sigma^2_{\text{intra}}(x)$ and $\sigma^2_{\text{expec}}(x)$ denotes the intraday and expected variance given that the base price is x.

In (97) the volatility is allowed to swim with the base price if a volatility surface is used.

In the portfolio window Γ and SkewΓ are given by (96) and (97), respectively.

5.10.6. Theta of a variance swap

The theta of a variance swap is the change in theoretical value when one day passes; it is computed as the difference between the value at the next trading day less the value at the current trading day assuming no change in the price of the underlying instrument.

5.10.7. Expiry value of a variance swap

The expiry value of a variance swap is just the realized volatility at expiry as set in the dynamic model parameter.

5.11. Bond

The owner of a bond is entitled to coupons at prescribed dates. A coupon is defined by four parameters

- Ex-coup date
- Coupon date
- Coupon, c
- Kind: amount, maturity, %, % spot, yield

The kind parameter determines the amount C to be payed on the coupon date. For all kinds the amount to be payed is equal to the coupon value, $C = c$, except for kind % spot. For kind % spot the amount to be payed is given by $C = c \cdot B$ where B is the base price of the contract.

The theoretical value of a bond is the sum of the present value of the remaining coupons. Consider a bond with coupons (amounts to be payed) $C_1, \ldots C_N$ where coupons $C_k, \ldots C_N$ are future coupons. Let T_i be the time to the coupon date of coupon number i. From the yield curve entry r_i at T_i we get the present value factor pv_{T_i} by (8) (after conversion to continuous rate).

The accrued interest for the next coupon k is calculated from the coupon date of the previous coupon $k - 1$. This is done by subtracting the amount $C_k \cdot T_{k-1}/T_{k-1,k}$ where T_{k-1} is the time since previous coupon $k - 1$ until the current date and $T_{k-1,k}$ is the time between the previous coupon $k - 1$ and the next coupon k. If $k = 1$ then the accrued interest is zero.

The theoretical price of the bond is given by

$$\Pi_{\text{bond}} = -\frac{T_{k-1}}{T_{k-1,k}} C_k + \sum_{i=k}^{N} \text{pv}_{T_i} \cdot C_i. \tag{98}$$

5.12. Convertible bond

A convertible is a bond with an option to convert to an underlying (stock) up to expiry. The date from which the convertible can be converted can be the issuing date, the expiry date, or some date in between. Often, the convertible is callable, i.e. the writer has the opportunity to repurchase the convertible at a pre-decided price. Before the repurchase, the holder may convert the convertible. The convertible can also be putable, i.e. the owner has the right to sell the Convertible at a pre-decided price.

The following dynamic parameters are used to define convertible bonds

- Bond yield offset, r_{offset}
- Convertible from, τ_{from}

The coupons table in the contract inspector contains two types of values. Coupons of kind 'maturity' or '%' are associated with the bond part of the contract and are used for calculation as described in the bond section.

In addition to bond coupons, the callable price Π_{call} and putable price Π_{put} are also set in the coupons view (see Orc Trader Manual for details). The following parameters are associated with the callable resp. putable price:

Ex-coup date: The start date of the callable/putable condition, τ_{start}.
Coupon date: The end date of the callable/putable condition, τ_{end}.

Coupon: The callable price, Π_{call}.
The putable price, Π_{put}.
Kind: 'call' specifies that it is a callable condition.
'put' specifies that it is a putable condition.

The binomial method for convertible bonds

Spot tree The theoretical value of a convertible bond is calculated using the Cox-Ross-Rubinstein binomial method described in section 3.1.1. Dividends are treated as described in the section on dividends.

The following parameters are used for constructing the binomial spot tree

- contract financing rate r
- underlying rate q
- volatility σ
- number of time steps $N = 200$.

The result is a binomial spot tree with the spot price S_{ij} at node (i, j) given by (23) (or (32) when dividends are present) and parameters Δt and p as defined in section 3.1.

Bond part The bond price $\Pi_{\text{bond},i}$ at time step i is calculated from (98) with the yield curve shifted by the bond yield offset r_{offset}. Hence, the present value factor pv_{T_k} of the coupon k with time to coupon payment T_k is calculated from (8) with the financing rate $r_k + r_{\text{offset}}$ where r_k is the yield curve entry at T_k.

Value of convertion The convertion multiplier m_{convert} determines how many underlying (stocks) are obtained from converting one bond. It is defined by the strike price parameter X

$$m_{\text{convert}} = \frac{100}{X}$$

At the final time step N all nodes are set to the value of convertion to underlying

$$\pi_{Nj} = \max(m_{\text{convert}} \cdot S_{Nj} - \Pi_{\text{bond},N}, 0).$$

The value at node (i, j) is calculated by comparing the following values

$$\begin{aligned}
\pi_{\text{tree},ij} &= e^{-r\Delta t}(p\pi_{i+1,j} + (1-p)\pi_{i+1,j+1}) && \text{(value from time step } i+1\text{)} \\
\pi_{\text{convert},ij} &= m_{\text{convert}} \cdot S_{ij} - \Pi_{\text{bond},i} && \text{(value of convertion to underlying)} \\
\pi_{\text{call},ij} &= \Pi_{\text{call}} - \Pi_{\text{bond},i} && \text{(value of calling if callable at } i\text{)} \\
\pi_{\text{put},ij} &= \Pi_{\text{put}} - \Pi_{\text{bond},i} && \text{(value of puting if putable at } i\text{)}
\end{aligned}$$

where r is (non-offset adjusted) contract financing rate.

The convertion value at node (i, j) is initially set as $\pi_{ij} = \pi_{\text{tree},ij}$ and is then updated in accordance with the following conditions, where τ_i is the date at time step i

$$\begin{aligned}
\pi_{ij} &= \min(\pi_{ij}, \pi_{\text{call},ij}) && \text{if } \tau_{\text{start}} < \tau_i < \tau_{\text{end}} \text{ for a callable condition} \\
\pi_{ij} &= \max(\pi_{ij}, \pi_{\text{put},ij}) && \text{if } \tau_{\text{start}} < \tau_i < \tau_{\text{end}} \text{ for a putable condition} \\
\pi_{ij} &= \max(\pi_{ij}, \pi_{\text{convert},ij}) && \text{if } \tau_{\text{from}} < \tau_i
\end{aligned}$$

The theoretical value of the convertible bond is given by

$$\Pi_{\text{convertible}} = \pi_{0,0} + \Pi_{\text{bond},0}. \tag{99}$$

5.13. Interest rate swap

An interest rate swap has two legs each of which defines a series of future cash flows. Each leg can either be of floating or fixed type. One leg is short (paying) and has a negative value and the other is long (recieving) and has a positive value. In Orc the price of a swap is the sum of the present value of the legs based on a face value (notional) of 100 and where the present value of the short leg is multiplied by the FX factor set up on the contract.

5.13.1. Fixed leg

In Orc a fixed leg is set up as a series of cash flows. As an example consider the following setup.

Ex coupon date	Coupon date	Coupon
2014-03-19	2014-03-20	4
2014-06-19	2014-06-20	5
2014-09-19	2014-09-22	6

The Ex coupon date is the first date that the corresponding cash flow does not enter the swap valuation. The Coupon date is the date that the coupon is paid out. The Coupon is the size of the payment in percentage relative to the face value of the swap. Coupons with Ex coupon date after the expiry of the swap are disregarded. The present value factors used to value the leg are calculated using the yield curve set up for the leg when creating the swap. The theoretical price can in general be written as

$$\text{Swap price} = \sum \text{PV}(\text{calc date, coupon date})\text{coupon}. \tag{100}$$

The sum is over all coupons with ex coupon date after the calculation date and not after the expiry of the swap. If the calculation date is 2014-04-02 and the yield curve is set up as being of continuous type, flat at 5% and using Act/365 as day count convention then the price for the above example is 10.8057.

5.13.2. Floating leg

To describe how the floating leg works we can look at the following example. We assume that the calculation date is 2014-04-02.

Reset date	Pay date	Coupon(%)
2014-03-20	2014-06-24	4
2014-06-24	2014-09-26	0
2014-09-24	2014-12-24	0

The reset dates determine the floating rate periods. The Pay date for a reset determines the date when the payment correponding to the reset rate is due. In the example the three month rate for the first period has already reset and is known to be 4%. For the future reset dates the rates are unknown and any coupon values entered for them will not affect the valuation of the leg. The price contribution of each reset is given by

$$\text{Price of reset} = 100 \cdot \text{PV}_{y.c.}\,(\text{calc date, coupon date})\,(F(T_i, T_{i+1}) + \text{spread})\,\Delta_i.$$

The factors are defined in the table below.

$\mathrm{PV}_{y.c.}(d_1, d_2)$	The present value factor between d_1 and d_2 using the yield curve set up for the leg.
Δ_i	$T_{i+1} - T_i$, the accrual factor, the time in years between the reset dates T_i and T_{i+1}. This factor is calculated using the day count convention set on the swap.
$F(T_i, T_{i+1})$	If the rate has reset then it is the reset value, otherwise $\frac{1}{\Delta_i}\left(\frac{\mathrm{PV}_s(T_i)}{\mathrm{PV}_s(T_{i+1})} - 1\right)$.
$\mathrm{PV}_s(T)$	The present value factor corresponding to the period between the calculation date and T using the day convention set on the swap.
spread	The pread set on the leg.

Assume that the yield curve is set up as being of continuous type, flat at 5% and using Act/365 as day count convention. We also assume that the day count convention set on the swap is 30/360 and that the leg spread is 1%. Given this the price for the above example is calculated to be 3.9651.

5.14. Equity swaps

In Orc there are two types of equity swaps, fixed and floating notional equity swaps. The long leg of these swaps will periodically recieve an amount of money depending on the period return for the base contract. The short leg will periodically pay either a predetermined (fixed) amount or an amount depending on future rates. In the case of a Floating notional swap the interest rate leg will also depend on the future reset values of the base contract.

5.14.1. Fixed notional swap

The short fixed or floating leg for a Fixed notional swap is priced in the same way as the corresponding leg in an interest rate swap, see the section about Interest rate swaps. The long equity leg will, for each reset period, recieve

$$Notional \cdot \left(\frac{S_{T_{i+1}} - S_{T_i} + \frac{DP}{100} \cdot \sum d_j}{S_{T_i}}\right). \tag{101}$$

$Notional$ is an agreed size of the contract. S_T is the price of the base contract at time T. DP is the dividend participation factor, typically 0 or 100, and d_j is a dividend between reset times T_i and T_{i+1}. The price of the leg will be the value of the leg based on a $Notional$ of 100.

Equity leg setup

We use the following example to describe how the equity leg works. The calculation date is assumed to be 2014-04-02.

Reset date	Pay date	Coupon(%)
2014-03-20	2014-06-25	100
2014-06-24	2014-09-26	0
2014-09-24	2014-12-24	0

The reset dates determine the equity returns. The pay date for a reset determines the date when the payment corresponding to the reset date is due. In the example the value of the equity was at 2014-03-20 equal to 100. At 2014-06-24, the next reset, the equity value will be noted and entered in the Coupon column on row two. For example suppose that value is 110. Then, at 2014-06-25, the leg will recieve $(110/100-1)\cdot Notional$.

Dividends

The value of the swap will depend on the dividends of the base contract. Future dividends should be put on the base contract, the equity. Passed dividends can be put either on the base contract or the swap contract, not on both.

Equity leg pricing

The theoretical price of the equity leg is based on a notional of 100 and price contribution of each reset is given by

(102) $$\frac{F\left(R_{i+1}\right)-F\left(R_i\right)+\frac{DP}{100}\sum_{d_j\in(R_i,R_{i+1}]}d_j}{F\left(R_i\right)}\frac{PV\left(P_i\right)}{PV\left(T_{stl}\right)}\cdot 100.$$

P_i is the pay date for reset i. $PV(T)$ is the present value of one unit for time T. T_{stl} is the time to settlement of the contract, typically zero or a few days. The sum over d_j is the sum of all dividends between resets R_i and R_{i+1}. $F\left(R_i\right)$ is the forward value at reset R_i. For past resets it is the value noted on that day. The forward value is for future resets calculated as

(103) $$F(T)=\left(S-\sum PV(T_{d_j})d_j\right)/PV(T).$$

The formula is less accurate when the DP is less than 100. The reason for this goes back to the difficulty of exactly hedging $1/S_T$ where S_T is a future value of the underlying equity.

5.14.2. Floating notional swap

In a floating notional swap the long leg will for each reset period recieve

(104) $$Notional\cdot\left(S_{T_{i+1}}-S_{T_i}+DP\cdot\sum d_j\right).$$

$Notional$ is an agreed size of the contract. S_T is the price of the base contract at time T. DP is the dividend participation factor, typically 0 or 100, and d_j is a dividend between T_i and T_{i+1}. The price of the leg will be the value of the leg based on a $Notional$ of 1.

Equity leg setup

We use the following example to describe how the equity leg works. The calculation date is assumed to be 2014-04-02.

Reset date	Pay date	Coupon(%)
2014-03-20	2014-06-25	100
2014-06-24	2014-09-26	0
2014-09-24	2014-12-24	0

The reset dates determine the equity returns. The pay date for a reset determines the date when the payment corresponding to the reset date is due. In the example the value of the equity was at 2014-03-20 equal to 100. At 2014-06-24, the next reset, the equity value will be noted and entered in the Coupon column on row two. For example suppose that value is 110. Then, at 2014-06-25, the leg will recieve $(110/100 - 1) \cdot Notional$.

Dividends

The value of the swap will depend on the dividends of the base contract. Future dividends should be put on the base contract, the equity. Passed dividends can be put either on the base contract or the swap contract, not on both.

Equity leg pricing

The theoretical price of the equity leg is based on a notional of 1 and price contribution of each reset is given by

$$\left(F\left(R_{i+1}\right) - F\left(R_i\right) + \frac{DP}{100} \sum_{d_j \in (R_i, R_{i+1}]} d_j \right) \frac{PV\left(P_i\right)}{PV\left(T_{stl}\right)}. \tag{105}$$

P_i is the pay date for reset i. $PV(T)$ is the present value of one unit for time T. T_{stl} is the time to settlement of the contract, typically zero or a few days. The sum over d_j is the sum of all dividends between resets R_i and R_{i+1}. $F\left(R_i\right)$ is the forward value at reset R_i. For past resets it is the value noted on that day. The forward value is for future resets calculated as

$$F(T) = \left(S - \sum PV(T_{d_j}) d_j \right) / PV(T). \tag{106}$$

Interest rate leg pricing

The interest rate leg can either be of floating or fixed size.

For the floating case an example of a setup could be as in the table below. We assume that the calculation date is between 2014-03-20 and 2014-04-24.

Reset date	Pay date	Coupon(%)
2014-03-20	2014-06-25	3
2014-06-24	2014-09-26	0
2014-09-24	2014-12-24	0

At 2014-03-20 a rate of 3% was determined for the upcoming period 2014-03-20 to 2014-06-24. The rate for the future period 2014-06-24 to 2014-09-24 is yet unknown. The price swap contribution coming from the current reset period is

$$F(R_0)\left(T_1 - T_0\right) \frac{PV(P_0)}{PV(T_{stl})} \cdot 3/100. \tag{107}$$

Here we indicate the previous reset date by 0 and the next reset by 1. $F(R_0)$ is the equity value at the previous reset. $PV(P_0)$ is the present value factor for the current reset period. $T_1 - T_0$

is the time length in years for the first reset period using the 30/360 day convention. T_{stl} is the contract

The value of the future reset period is calculated as

$$F(R_1)\left(\frac{PV(R_1)}{PV(R_2)} - 1\right)\frac{PV(P_1)}{PV(T_{stl})}. \tag{108}$$

It there would be more future periods they would be valued correspondingly.

For the fixed rate case then, for all resets, the future rates are defined from the begining as in the following example.

Reset date	Pay date	Coupon(%)
2014-03-20	2014-06-25	r_1
2014-06-24	2014-09-26	r_2
2014-09-24	2014-12-24	0

The table is to be interpreted as before. Please note that the coupon for the last reset is a dummy value that will never be used in the calculations. The value of reset number i is calculated as

$$F(R_i)\,(T_{i+1} - T_i)\,\frac{PV(P_i)}{PV(T_{stl})} \cdot r_i/100. \tag{109}$$

5.15. Structured note

In Orc, two types of Structured Notes are considered: call and put. Structured Note calls are valued as a sum of a bond and a European call. Structured Note puts are valued as a sum of a bond and a (sold) European put. For both contracts the maturity date of the bond, the expiry date (which cannot be later than the maturity date), and the strike of the option must be specified.

The following dynamic parameter is used to define convertible bonds

- Option multiplier, m_{option}

The theoretical value of a structured note is calculated as the sum of the bond part and the multiplier adjusted option part. The value of the bond part Π_{bond} is calculated from (98). The option part is calculated with the Black-Scholes formula, Π_{call} from (43) if it is a call and Π_{put} from (44) if it is a put. The value of the structured note is given by

$$\begin{aligned}\Pi_{\text{structured note, call}} &= \Pi_{\text{bond}} + m_{\text{option}} \cdot \Pi_{\text{call}} \\ \Pi_{\text{structured note, put}} &= \Pi_{\text{bond}} + m_{\text{option}} \cdot \Pi_{\text{put}}\end{aligned}$$

6. Volatility models

6.1. Volatility surfaces in Orc Trader™

In section 2.1.7 the concept of volatility surfaces is covered briefly. The implied volatility is calculated for options with a range of strikes X and times to expiry T that share the same underlying. From the set of implied volatilities a volatility surface $\sigma(X, T)$ can be constructed.

6.1.1. Volatility skews

A volatility surface in Orc Trader™ is defined by a set of *volatility skews*. A volatility skew describes the volatility as a function of the strike price for a given expiration date. A surface is constructed by interpolation between neighboring skews and extrapolation of the skews with the earliest and latest available expiration date. The construction of the skew as well as the inter- and extrapolation is specific for respective volatility model.

A volatility skew is either defined by a set of points, by a set of parameters or a mixture of the both. The models provided in Orc Trader™ are classified as follows:

point based	**parameter based**	**mixed**
cubic spline (static)	wing	cubic spline (dynamic)
	time weighted wing	cubic spline (stddev)
	wing eurofuture	
	SABR	

6.1.2. Transformation of strike

Most volatility models do not use the strike X directly as a variable when calculating the volatility skew. Instead a transformed variant of the strike is introduced, which we denote x.

The transformed strike x is only used for calculation purpose. When plotting the volatility skew in the volatility manager the original strike X is used.

Let us denote the volatility skew as a function of the original strike $\sigma(X)$. To be precise, the transformed variable $x(X)$ introduces a new skew function $\hat{\sigma}(x)$ defined by $\hat{\sigma}(x(X)) = \sigma(X)$. We will use the sloppy notation $\sigma(x)$ instead of $\hat{\sigma}(x)$ as it is obvious from the context which function we mean.

Log-moneyness

The strike X is transformed to log-moneyness x according to

$$x = \log\left(\frac{X}{F_{\text{synth}}}\right) \tag{110}$$

where F_{synth} is given by (114). For such models all model parameters are expressed in terms of the transformation (110). For example, if a model uses a cutoff strike which equals $0.8 \cdot F_{\text{synth}}$ then the cutoff parameter value for that model is $e^{0.8}$ and not $0.8 \cdot F_{\text{synth}}$.

The parameter x is not used for display purpose, so before any graphs are plotted the volatility skew $\sigma(x)$ is transformed back to the original strike variable $\sigma(X)$ by the inverse relation

$$X = F_{\text{synth}} \cdot e^{x}. \tag{111}$$

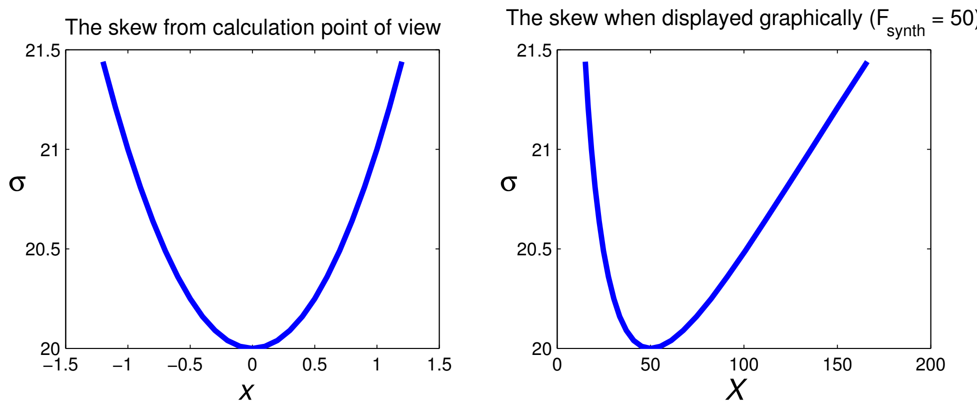

FIGURE 2. The skew plotted for the variables x and X respectively.

Consider for example a volatility skew modelled by $\sigma(x) = 20 + x^2$ and assume $F_{\text{synth}} = 50$. Figure 2 plots the skew for $-1.2 \leq x \leq 1.2$ which corresponds to $15 \leq X \leq 166$.

Time weighted log-moneyness

In order to add some dynamics to the volatility skew some models perform calculations with a time weighted strike parameter. These model provide the following model parameter

Alpha (α) The speed of the time-weighting contraction.

Time weighting will contract the volatility skew in strike-direction if the time to expiry is less than one year (and expand the skew if time to expiry is greater than one year). The center of the contraction is the strike which equals F_{synth} given by (114). In Figure 3 the parameter SSR=100 so F_{synth} equals the ATM forward.

The strike X is transformed to time weighted log-moneyness x according to

(112)
$$x = \frac{1}{T^\alpha} \log \left(\frac{X}{F_{\text{synth}}} \right)$$

where T is time to expiry. The parameter alpha, α, determines the impact of the time weighting. A large alpha gives a heavily contracted skew. Settig $\alpha = 0$ removes the time weighting effect so the expression (112) becomes equal to (110).

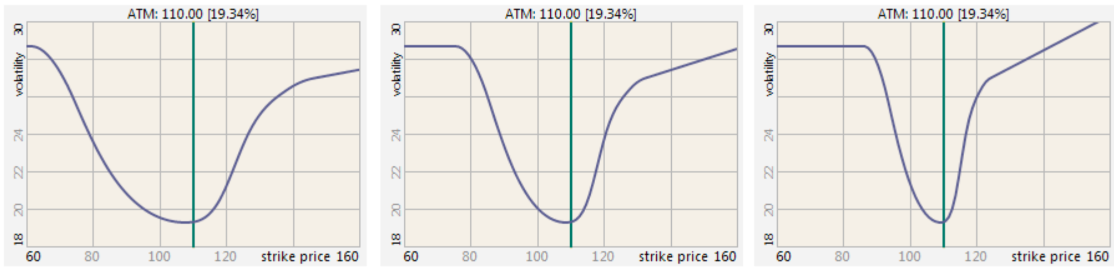

FIGURE 3. Dynamic spline skews from left to right: I. One year to expiry, II. six months to expiry and $\alpha = 0.5$, III. six months to expiry and $\alpha = 1.0$.

Stddev spline transformation

In the Cubic spline (stddev) model the strike X is transformed in the following way:

$$x = \frac{X - F_{\text{synth}}}{F_{\text{synth}} \cdot \sigma_{\text{ATM}} \cdot \sqrt{T_{\text{days}}/365}} \tag{113}$$

where we get F_{synth} from (114), σ_{ATM} is the volatility at $X = F_{\text{synth}}$ and T_{days} is days to expiry.

6.1.3. ATM forward, reference price and synthetic forward

The forward price of the underlying is called the *ATM forward* and we denote it F_{ATM}.

For some models the ATM forward is a key parameter when calculating the shape of the skew. This makes the model sensitive to movement in the underlying asset. In Orc Trader™ this sensitivity can be reduced or eliminated by constructing a synthetic forward.

As an alternative some models base their calculations on the *synthetic forward* F_{synth} rather than the ATM forward. Teh synthetic forward is calculated from the volatility model parameters

ATM forward (F_{ATM})	ATM forward
Ref. price (F_{ref})	Reference price
SSR	Swimming skewness rate

The synthetic forward is calculated as

$$F_{\text{synth}} = F_{\text{ATM}}^{SSR/100} \cdot F_{\text{ref}}^{1-SSR/100}. \tag{114}$$

The range of the skew swimmingness rate is given by $0 \le SSR \le 100$. For $SSR = 0$ the synthetic forward equals the reference price $F_{\text{synth}} = F_{\text{ref}}$ and the model is not affected by movement in the ATM forward. For $SSR = 100$ we have $F_{\text{synth}} = F_{\text{ATM}}$ and the reference price is not considered. For values in between the weight of the ATM forward increases and the weight of the reference price decreases as the skew swimmingness rate increases.

6.1.4. Current volatility and current slope

Consider a volatility skew $\sigma(x)$ with x given by (110). Several models make use of the *current volatility* $\sigma_{\text{cur}} = \sigma(0)$ and *current slope* $s_{\text{cur}} = \sigma'(0)$, i.e. the volatility and slope of the skew at $x = 0$. Such models provide some or all of the following parameters which are used to calculate σ_{cur} and s_{cur}

Vol. ref. (σ_{ref})	Volatility reference. The volatility at $x = 0$ for $VCR = 0$.
Slope ref. (s_{ref})	Slope reference. The slope at $x = 0$ for $SCR = 0$.
VCR	Volatility change rate
SCR	Slope change rate
SSR	Skew swimmingness rate
Ref. price (F_{ref})	Reference (forward) price

The calculations also require the ATM forward F_{ATM} which is calculated from market data.

The constant part of σ_{cur} and s_{cur} is given by the parameters σ_{ref} and s_{ref} respectively and the parameters VCR, SCR, SSR and F_{ref} determine the contribution from the change in the ATM forward. The parameters σ_{cur} and s_{cur} are calculated by

$$\begin{aligned}\sigma_{\text{cur}} &= \sigma_{\text{ref}} - VCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}} \\ s_{\text{cur}} &= s_{\text{ref}} - SCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}}.\end{aligned}$$

The impact of VCR on σ_{cur} can be explained as follows. When $SSR = 100$ an increase in the quotient $F_{\text{ATM}}/F_{\text{ref}}$ by 1% results in a decrease of σ_{cur} by VCR percentage units, and vice versa for a decrease in $F_{\text{ATM}}/F_{\text{ref}}$. As SSR decreases the impact from VCR decreases to reach zero for $SSR = 0$.

The analogous reasoning holds for the impact of SCR on s_{cur}.

6.1.5. Fitting volatility skews

There are several ways to fit implied volatilities in order to create a volatility skew and the method used depends on the volatility model.

Some models apply *least square minimization* of the parameters used to define the skew. Given a set of implied volatilities $\sigma_1, \sigma_2, \ldots \sigma_N$ and a set of parameters $p_1, p_2, \ldots, p_M$ which define a skew $\sigma(p_1, \ldots, p_M)$ the sum of the squared deviations is

$$D(p_1, \ldots, p_M) = \sum_{i=1}^{N} \left(\sigma_i - \sigma(p_1, \ldots, p_M)\right)^2. \tag{115}$$

The problem is to minimize D with respect to $p_1, \ldots, p_M$.

The implementation of the least square method is based on (but not identical with) the method presented in [**FPTV**].

Out-of-the-money weighted fitting

For some volatility models it is possible to increase the impact of implied volatilities from OTM options. This feature is possible for models which do fitting by solving a least square minimization problem with respect to the model parameters. ITM volatilities are given less weight when summing the deviation (115) to be minimized.

An option is considered to be ITM if the ATM forward F_{ATM} is ITM for that option, i.e. $X < F_{\text{ATM}}$ for calls and $X > F_{\text{ATM}}$ for puts. For a collection of strikes $X_1, \ldots, X_N$ the weights are calculated as

$$w_{\text{call},i} = \left(\frac{X_i}{F_{\text{ATM}}}\right)^{2\sqrt{T}} \qquad \text{if } X_i < F_{\text{ATM}}$$
$$w_{\text{put},i} = \left(\frac{F_{\text{ATM}}}{X_i}\right)^{2\sqrt{T}} \qquad \text{if } X_i > F_{\text{ATM}}.$$

where T is time to expiry. For OTM options $w_{\text{put}} = w_{\text{call}} = 1$.

In this case, the construction of the skew involves the problem to minimize a modified version of the least squared deviations (115), namely

$$D_w(p_1, \ldots, p_M) = \sum_{i=1}^{N} (\sigma_i - \sigma(p_1, \ldots, p_M))^2 \cdot w_i^2.$$

Outliers

For some volatility models it is possible to ignore strikes with implied volatilities that stick out to much when constructing the volatility skew. The process of choosing which volatilities to exclude is a matter of solving a least square minimization problem.

Given N call options with strikes X_i, $i = 1, \ldots N$ we get N implied volatilities σ_i from the bid prices of the options. Suppose we want to exclude at most k outliers before fitting the skew. We construct the skew $\sigma(X)$ for every possible setup of outliers (there are $\sum_{j=0}^{k} N!/(j!(N-j)!)$ possible setups) and calculate a *penalized absolute deviation* D for each skew

$$D = p \cdot \sum_{i=1}^{N} (\sigma(X_i) - \sigma_i)^2 \cdot w_i \tag{116}$$

where w_i is zero if σ_i is an outlier for that setup and one otherwise, and p is a penalty factor that depends on the number of outliers for the given skew. The problem of choosing outliers is solved by the setup which minimize the deviation D.

Outliers are chosen independently for bid, ask and average prices of put and call options. For example, the strikes with bid outliers generally do not coincide with the strikes with ask outliers for a series of call options.

The maximal number or outliers k to be excluded depends on the number of implied volatilities available and is specific for each model. The penalty factor p is one for the setup with no outliers and it grows with the number of outliers in the setup. For a setup with $\hat{k}$ outliers p is calculated as

$$p(\hat{k}) = \begin{cases} 1 & \text{for} \quad \hat{k} = 0 \\ p_1 & \text{for} \quad \hat{k} = 1 \\ p_2^{\hat{k} - \hat{k}_{\text{old}}} & \text{for} \quad \hat{k} \geq 2 \end{cases}$$

where p_1 and p_2 are specific for each model and $\hat{k}_{\text{old}}$ is the number of outliers in the solution when solving the problem with $k = \hat{k} - 1$.

6.1.6. Smoothing, affine and constant ranges

For some volatility models the domain of the volatility skew $\sigma(x)$ is split into an inner range, e.g. spline range or put and call wings, and outer ranges which we call smoothing, affine or constant ranges, see Figures 6.3. The outer ranges are constructed in such a way that the skew should be well behaved for all x. The smoothing, affine and constant regions are defined by some or all of the model parameters

Down sm. (dsm)	Down smoothing parameter. Defines the length of the down smoothing range.
Up sm. (usm)	Up smoothing parameter. Defines the length of the up smoothing range.
Down slope (k_{down})	Slope of the skew in the down affine region.
Up slope (k_{up})	Slope of the skew in the up affine region.

Let x_1 and x_2 be the end points of the inner region. Note that since x is typically log moneyness we typically have $x_1 < 0$ and $x_2 > 0$. The down smoothing region is defined as the interval $[(1 + dsm)x_1, x_1]$ and the up smoothing region is defined as $[x_2, (1 + usm)x_2]$. Denote $x_0 = (1 + dsm)x_1$ and $x_3 = (1 + usm)x_2$.

In the down smoothing region the skew is given by a second degree polynomial

$$P(x) = a + bx + cx^2 \tag{117}$$

which satisfies

$$\begin{aligned} P(x_1) &= \sigma(x_1) \\ P'(x_1) &= \sigma(x_1) \\ P'(x_0) &= k_{\text{down}}. \end{aligned}$$

The values $\sigma(x_1)$ and $\sigma'(x_1)$ are the end point values from the inner region. For models which do not offer the down slope or up slope parameters the slope is set to zero, $k_{\text{down}} = k_{\text{up}} = 0$. Solving this system gives

$$\begin{aligned} c &= \frac{\sigma'(x_1) - k_{\text{down}}}{2(x_1 - x_0)} \\ b &= k_{\text{down}} - \frac{\sigma'(x_1) - k_{\text{down}}}{x_1 - x_0} x_0 \\ a &= \sigma(x_1) - b \cdot x_1 - c \cdot x_1^2. \end{aligned}$$

By the same reasoning we have that the up smoothing range is given by (117) with

$$\begin{aligned} c &= \frac{\sigma'(x_2) - k_{\text{up}}}{2(x_2 - x_3)} \\ b &= k_{\text{up}} - \frac{\sigma'(x_2) - k_{\text{up}}}{x_2 - x_3} x_3 \\ a &= \sigma(x_2) - b \cdot x_2 - c \cdot x_2^2. \end{aligned}$$

In the affine or constant ranges the skew is given by the lines

down affine/constant range	$\sigma(x) = k_{\text{down}}(x - x_0) + \sigma(x_0)$
up affine/constant range	$\sigma(x) = k_{\text{up}}(x - x_3) + \sigma(x_3)$

6.2. Cubic spline (static)

No. of points (N)	The number of strikes used for interpolation.

Interpolation is done with cubic spline polynomials. Given the strike prices $X_1, X_2, \ldots, X_N$ and the volatilities $\sigma_1, \sigma_2, \ldots, \sigma_N$ at these strike prices, the curve is approximated by cubic polynomials on each interval $(X_i, X_{i+1}]$, i.e. for each strike price $X \in (X_i, X_{i+1}]$ the volatility at X is given by

$$P_i(X) = a_i + b_i(X - X_i) + c_i(X - X_i)^2 + d_i(X - X_i)^3.$$

The coefficients a_i, b_i, c_i and d_i are computed requiring that the resulting polynomial satisfies C^2-regularity at the internal nodes, i.e. that for each $i = 2, 3, \ldots, N - 1$

$$\begin{aligned} P_{i-1}(X_i) &= P_i(X_i) \\ P'_{i-1}(X_i) &= P'_i(X_i) \\ P''_{i-1}(X_i) &= P''_i(X_i). \end{aligned}$$

and that all implied volatilities lie on the curve

$$P_i(X_i) = \sigma_i, \qquad i = 1, 2 \ldots N. \tag{118}$$

At the endpoints we specify a so-called natural spline condition, $P''_1(X_1) = 0$ and $P''_{N-1}(X_N) = 0$. We get the following expression for the static spline volatility

$$\sigma_{\text{static}}(X) = \begin{cases} P_1(X_1) + P'_1(X_1) \cdot (X - X_1) & \text{if } X < X_1, \\ P_i(X) & \text{if } X_i \le X < X_{i+1} \\ P_{N-1}(X_N) + P'_{N-1}(X_N) \cdot (X - X_N) & \text{if } X_N \le X. \end{cases} \tag{119}$$

6.3. Cubic spline (dynamic)

ATM fwd. (F_{ATM})	ATM forward. Used to calculate the synthetic forward (see section 6.1.3)
Ref.price (F_{ref})	Reference (forward) price. Used to calculate the synthetic forward (see section 6.1.3).
Cur. vol (σ_{cur})	Current volatility. The volatility at the central skew point $x = 0$. Calculated at fitting.
Cur. slope (s_{cur})	Current slope. The slope at the central skew point $x = 0$. Calculated at fitting.
Down sm. (dsm)	Defines the length of the down smoothing range. Default value is 0.5.
Up sm. (usm)	Defines the length of the up smoothing range. Default value is 0.5.
Down slope (k_{down})	The slope of the line in the down affine range.
Up slope (k_{up})	The slope of the line in the up affine range.
VCR	Volatility change rate. Used to calculate current vol. σ_{cur} (see section 6.1.4). Determines the contribution to σ_{cur} from change in the ATM forward.
SCR	Slope change rate. Used to calculate current slope s_{cur} (see section 6.1.4). Determines the contribution to s_{cur} from change in the ATM forward.
SSR	Skew swimmingness rate. Used to calculate the synthetic forward (see section 6.1.3) and to calculate the σ_{cur} and s_{cur} (see section 6.1.4).
No. of points (N)	The number of strikes used for interpolation.
Rigidity (μ)	Increasing rigidity reduces the curvature of the spline at the cost of worse fitting to the data points. Defines the parameter λ by (124) (see cubic spline interpolation with smoothing)
Alpha (α)	Determines the amplitude of the impact of the time weighting (see the section Time weighted low-moneyness).

The dynamic spline model is based on the same principle as the static spline model; cubic spline interpolation for N strikes $X_1, X_2 \dots X_N$. The dynamic spline model has some additional features:

- (dynamic) The skews are contracted around the synthetic forwardas we get closer to expiry.
- (smoothing) A smoothing technique reduces the curvature of the volatility skews.
- (weighting) The implied volatilities are weighted by vega when smoothening the interpolation.
- (smooth extrapolation) The skews are customizable outside the spline range $[X_1, X_N]$.

Cubic spline interpolation with smoothing

Let us start by presenting the method used for spline interpolation by the dynamic spline model. Consider the set of points $x_1, x_2 \dots x_N$. A cubic spline is a union of cubic polynomials

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \qquad i = 1 \dots N - 1$$

which satisfy the following regularity conditions for $i = 2 \dots N-1$

$$P_{i-1}(x_i) = P_i(x_i) \tag{120}$$

$$P'_{i-1}(x_i) = P'_i(x_i) \tag{121}$$

$$P''_{i-1}(x_i) = P''_i(x_i). \tag{122}$$

For the purpose of our application we use so-called *natural cubic splines* which satisfy the additional boundary conditions $P''_1(x_1) = 0$ and $P''_{N-1}(x_N) = 0$. We denote the resulting cubic spline

$$P(x) = P_i(x), \qquad x \in [x_i, x_{i+1}].$$

Suppose that each point x_i is associated with a value y_i. We construct a curve which approximate the set of data points by calculating cubic spline $P(x)$ which minimizes the following functional

$$\lambda \sum_{i=1}^{N} \frac{(y_i - P(x_i))^2}{w_i^2} + (1-\lambda) \int_{x_1}^{x_N} (P''(x))^2 dz \tag{123}$$

where λ is calculated from rigidity parameter μ and the impact of the value y_i is weighted by a weight w_i. The relation between λ and rigidity is given by (see [P] for the background to this relation)

$$\lambda = \frac{1}{\frac{3}{2}\mu + 1}. \tag{124}$$

Note that if $\mu = 0$ then the minimization of (123) is reduced to the equality

$$P(x_i) = y_i. \tag{125}$$

For a large μ the emphasis is on minimizing the curvature and the solution of (123) approaches a straight line.

In Orc Trader™ the problem (123) with the constraints (120)–(122) is solved with the method presented in [P].

Dynamic spline volatility skews

For calculation purpose the skew $\sigma(x)$ is defined as a function of time weighted log-moneyness x given by (112). We denote the transformed strikes $x_1, x_2 \dots x_N$. The range of the skew is split into five regions defined as

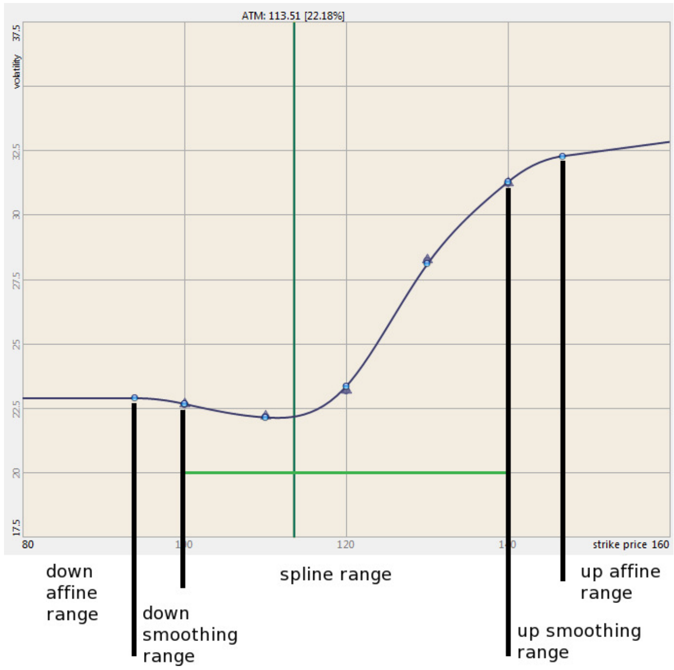

FIGURE 4. The regions of the dynamic spline model

down affine range		x	$< x_1(1+dsm)$
down smoothing range	$x_1(1+dsm) <$	x	$< x_1$
spline range	$x_1 <$	x	$< x_N$
up smoothing range	$x_N <$	x	$< x_N(1+usm)$
up affine range	$x_N(1+usm) <$	$x.$	

TABLE 1. Dynamic spline ranges

In the spline range the volatility skew is defined by

$$\sigma(x) = \sigma_{\text{cur}} + s_{\text{cur}} \cdot x + P(x) \tag{126}$$

where σ_{cur} and s_{cur} are given by (127) and (128) and $P(x)$ is the cubic spline given by (129). The parameters σ_{cur} and s_{cur} can also be set manually in the volatility manager (see Orc Trader Manual).

In the smoothing ranges the skew is given by second degree polynomials in such a way that it is continuous and differentiable in the points x_1 and x_N as described in section 6.1.6.

In the affine ranges the skew is given by an affine function in such a way that is is continuous in the end points $x_1(1+dsm)$ and $x_N(1+usm)$ as described in section 6.1.6.

Fitting algorithm

In the spline range the volatility skew (126) is fitted to the implied volatilities $\sigma_1 \ldots \sigma_N$ at the transformed strike points $x_1 \ldots x_N$ according to the following algorithm.

I. A non-smoothened cubic spline $Q(x)$ is calculated for the data points (x_i, σ_i) for $i = 1 \ldots N$. That is, $Q(x)$ satisfies (120)–(122) and (125) with $y_i = \sigma_i$.

II. Set

$$\begin{aligned}
\sigma_{\text{cur},1} &= Q(0) + VCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}} \\
s_{\text{cur},1} &= Q'(0) + SCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}}.
\end{aligned}$$

and subtract the tangent of $Q(x)$ at $x = 0$ from the implied volatilities

$$\varepsilon_{i,1} = \sigma_i - (\sigma_{\text{cur},1} + s_{\text{cur},1} \cdot x_i), \qquad i = 1 \ldots N.$$

III. Iterate this step 20 times, $j = 1 \ldots 20$:
Calculate a smoothened cubic spline polynomial $P_j(x)$ for the data points $(x_i, \varepsilon_{i,j})$. That is, $P_j(x)$ satisfies (120)–(123) where the weight w_i is set to vega for that strike, $w_i = \mathcal{V}(x_i)$. Set

$$\begin{aligned}
\sigma_{\text{cur},j+1} &= \sigma_{\text{cur},j} + P_j(0) - VCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}} \\
s_{\text{cur},j+1} &= s_{\text{cur},j} + P_j'(0) - SCR \cdot SSR \cdot \frac{F_{\text{ATM}} - F_{\text{ref}}}{F_{\text{ref}}} \\
\varepsilon_{i,j+1} &= \sigma_i - (\sigma_{\text{cur},j+1} + s_{\text{cur},j+1} \cdot x_i), \qquad i = 1 \ldots N.
\end{aligned}$$

IV. The dynamic spline polynomial which defines the skew in the spline range is obtained after 20 iterations

$$\sigma_{\text{cur}} = \sigma_{\text{cur},20} \tag{127}$$

$$s_{\text{cur}} = s_{\text{cur},20}. \tag{128}$$

$$P(x) = P_{20}(x). \tag{129}$$

6.4. Cubic spline (stddev)

ATM fwd. (F_{ATM})	ATM forward. Used to calculate the synthetic forward (see section 6.1.3)
Ref.price (F_{ref})	Reference (forward) price. Used to calculate the synthetic forward (see section 6.1.3).
ATM vol (σ_{ATM})	The volatility at the synthetic forward. Expressed in percentage.
SSR	Skew swimmingness rate. Used to calculate the synthetic forward (see section 6.1.3).
VCR	Volatility change rate. Used to calculate current vol. σ_{cur} (see section 6.1.4). Determines the contribution to σ_{cur} from change in the ATM forward.
SCR	Slope change rate. Used to calculate current slope s_{cur} (see section 6.1.4). Determines the contribution to s_{cur} from change in the ATM forward.
Up slope (k_{up})	The slope of the line describing the curve to the right of the uppermost strike.
Down slope (k_{down})	The slope of the line describing the curve to the left of the lowermost strike.
No. of points (N)	The number of strikes used for interpolation.

The stddev spline model is based on the same principle as the static spline model; cubic spline interpolation for N strikes $X_1, X_2 \ldots X_N$. The stddev spline model has some additional featuers

- (dynamic) The skews will contract around the synthetic forward as we get closer to expiry.
- (affine extrapolation) The skews can be set to behave well outside the spline range.

For calculation purpose the volatility skew $\sigma(x)$ is defined as a function of the transformed strike

$$x = \frac{X - F_{\text{synth}}}{F_{\text{synth}} \cdot \sigma_{\text{ATM}} \cdot \sqrt{T}},$$

where F_{synth} is the synthetic forward given by (114). The range of the skew is split into three regions defined by

down affine range		x	$< x_1$
spline range	$x_1 <$	x	$< x_N$
up affine range	$x_N <$	$x.$	

TABLE 2. stddev spline ranges

The strike transformation will make the spline range shrink as we get closer to expiration and in the limit as $T \to 0$ we have $x_1 = F_{\text{synth}} = x_N$.

In the spline range the volatility skew is calculated using the same fitting algorithm as dynamic spline model with rigidity $\mu = 0$ and all weights $w_i = 1$, see section (6.3). The skew is given by

$$\sigma_{\text{stddev}}(x) = \sigma_{\text{cur}} + s_{\text{cur}} \cdot x + P(x) \qquad \text{for } x_1 \leq x \leq x_N$$

where σ_{cur} and s_{cur} are given by (127) and (128) and $P(x)$ is the spline polynomial (129).

In the down and up affine ranges the skew is constructed so that it is continuous in the points x_1 and x_N and has the slope given by the model parameters k_{down} and k_{up}

$$\begin{aligned}
\sigma_{\text{stddev}}(x) &= \sigma_{\text{stddev}}(x_1) + k_{\text{down}}(x - x_1) \qquad \text{for } x < x_1 \\
\sigma_{\text{stddev}}(x) &= \sigma_{\text{stddev}}(x_N) + k_{\text{up}}(x - x_N) \qquad \text{for } x > x_N.
\end{aligned}$$

6.5. Wing model

The following parameters are available for the wing model.

Expiry	Expiry date.
Days	Days remaining until expiry.
ATM fwd. (F_{ATM})	ATM forward. Used to calculate the synthetic forward (see section 6.1.3)
Ref.price (F_{ref})	Reference (forward) price. Used to calculate the synthetic forward (see section 6.1.3) and to calculate the σ_{cur} and s_{cur} (see section 6.1.4).
Vol. ref (σ_{ref})	Volatility reference. Used to calculate current vol. σ_{cur} (see section 6.1.4).
Slope ref (s_{ref})	Slope reference. Used to calculate current slope s_{cur} (see section 6.1.4).
Cur. vol. (σ_{cur})	Current volatility. The volatility at the central skew point $x = 0$. Calculated from other parameters (see section 6.1.4).
Cur. slope (s_{cur})	Current slope. The slope at the central skew point $x = 0$. Calculated from other parameters (see section 6.1.4).
Put curv. (pc)	Put curvature. The curvature of the skew in the put wing.
Call curv. (cc)	Call curvature. The curvature of the skew in the call wing.
Down cutoff (x_{dc})	The down cutoff defines the transition point between the put wing and the down smoothing range. Corresponds to strike $X = F_{\text{synth}} \cdot \exp(x_{\text{dc}})$.
Up cutoff (x_{uc})	The up cutoff defines the transition point between the call wing and the up smoothing range. Corresponds to strike $X = F_{\text{synth}} \cdot \exp(x_{\text{uc}})$.
VCR	Volatility change rate. Used to calculate current vol. σ_{cur} (see section 6.1.4). Determines the contribution to σ_{cur} from change in the ATM forward.
SCR	Slope change rate. Used to calculate current slope s_{cur} (see section 6.1.4). Determines the contribution to s_{cur} from change in the ATM forward.
SSR	Skew swimmingness rate. Used to calculate the synthetic forward (see section 6.1.3) and to calculate the σ_{cur} and s_{cur} (see section 6.1.4).
Down sm. (dsm)	Defines the length of the down smoothing range. The length of the range is defined in relation to the length of the put wing. Default value is 0.5.
Up sm. (usm)	Defines the length of the up smoothing range. The length of the range is defined in relation to the length of the call wing. Default value is 0.5.

The volatility skew

The volatility skews of the wing model are split into six regions. On the inner regions the volatility is modeled by second degree polynomials and on the boundary regions it is constant. For calculation purpose the volatility skew $\sigma(x)$ is defined as a function of log-moneyness x defined by (110). The plotted figure displays the volatility $\sigma(X)$ as a function of strike X given by x from the relation (111). The value $x = 0$ corresponds to $X = F_{\text{synth}}$ where F_{synth} is given by (114).

The four parameters $x_{\text{dc}}, x_{\text{uc}}, dsm$ and usm define the six regions of the wing skew. They are defined as follows

FIGURE 5. The regions of the wing model

down constant range		x	$< x_{\text{dc}}(1 + dsm)$
down smoothing range	$x_{\text{dc}}(1 + dsm) <$	x	$< x_{\text{dc}}$
put wing	$x_{\text{dc}} <$	x	< 0
call wing	$0 <$	x	$< x_{\text{uc}}$
up smoothing range	$x_{\text{uc}} <$	x	$< x_{\text{uc}}(1 + usm)$
up constant range	$x_{\text{dc}}(1 + usm) <$	$x.$	

On the put and call wings the volatility skew $\sigma(x)$ follows second degree polynomials determined by the four parameters $\sigma_{\text{cur}}, s_{\text{cur}}, pc$ and cc

$$\sigma(x) = \begin{cases} \sigma_{\text{cur}} + s_{\text{cur}} \cdot x + pc \cdot x^2 & \text{for} \quad x_{\text{dc}} < x < 0 \\ \sigma_{\text{cur}} + s_{\text{cur}} \cdot x + cc \cdot x^2 & \text{for} \quad 0 < x < x_{\text{uc}}. \end{cases} \tag{130}$$

In the smoothing ranges the skew is given by second degree polynomials in such a way that it is continuous and differentiable in the points x_{dc} and x_{uc} as described in section 6.1.6.

In the constant ranges the skew assumes a constant in such a way that is is continous in the end points $x_{\text{dc}}(1 + dsm)$ and $x_{\text{uc}}(1 + usm)$ as described in section 6.1.6, i.e. $k_{\text{down}} = k_{\text{up}} = 0$.

Outliers

The process of choosing outlier implied volatilities to exclude when constructing the skew is discussed in section 6.1.5. The maximal number of outliers to pick k and the penalty factors p_1 and p_2 in section 6.1.5 are specific for each model. For the wing model $p_1 = 1.1$ and $p_2 = 1.2$.

The number k is determined from the number of available implied volatilities N. For the Wing model the relation is as follows

if $N < 8$ or $N > 200$	then	$k = 1$
if $8 \leq N < 11$ or $30 < N \leq 200$	then	$k = 2$
if $11 \leq N \leq 30$	then	$k = 3.$

Inter- and extrapolation of times to expiry

For an arbitrary time to expiry T a volatility skew is constructed by interpolation of neighboring skews or extrapolation of boundary skews. If skews exist for T_1 and T_2 where $T_1 < T < T_2$ then the parameters of the skew at T are obtained by linear interpolation. Set $u = (T - T_1)/(T_2 - T_1)$. The parameter $p(T)$ is calculated as

$$p(T) = (1 - u) \cdot p(T_1) + u \cdot p(T_2).$$

For example the volatility reference is calculated as $\sigma_{\text{ref}}(T) = (1 - u)\sigma_{\text{ref}}(T_1) + u\sigma_{\text{ref}}(T_2)$.

If T does not lie in between two existing skews, the parameter settings of the closes neighboring skew are used. Denote the shortest available time to expiry T_0 and the longest T_N. If $T < T_0$ then the parameter $p(T)$ is defined as $p(T) = p(T_0)$. If $T > T_N$ then $p(T) = p(T_N)$.

6.6. Wing power skew model

The following parameters are available for the wing power skew model:

Vol ref	Volatility reference.
Slope ref	$\text{Slope}_{\text{WPS}}$, Slope reference.
Put curv.	Put curv_{WPS}
Call curv.	Call curv_{WPS}
Cur. vol.	Current volatility.
Cur. slope	Current slope.
Down cutoff	Down $\text{cutoff}_{\text{WPS}}$
Up cutoff	Up $\text{cutoff}_{\text{WPS}}$
Slope Pwr	
Curva Pwr	
VCOR	Volatility Change Out-performance rate
SCOR	Slope Change Out-performance rate
VCR	VCR_{Wing}, Volatility change rate. Read-only parameter calculated according to formula (131) below.
SCR	SCR_{Wing}, Slope change rate. Read-only parameter calculated according to formula (131) below.
SSR	Skew swimmingness rate.
Down sm.	
Up sm.	
Time floor	Floor for parameter Time ref (default: 5 days)
Time ref	T_{ref}, reference duration (default: 60 days)
Ref.price	Reference (forward) price.

The idea with the Wing Power Skew model is to make the parameters less sensitive to time, i.e. to introduce better time weighting as compared to the regular Wing model. Please, note: when the model calculates the time to maturity, it uses a weekday calendar. It means that Saturdays and Sundays are excluded.

First the wing power skew parameters $\text{Slope}_{\text{WPS}}$, $\text{Put curv}_{\text{WPS}}$, $\text{Call curv}_{\text{WPS}}$, $\text{Down cutoff}_{\text{WPS}}$, $\text{Up cutoff}_{\text{WPS}}$, VCOR, SCOR, SSR, $\text{Down sm.}_{\text{WPS}}$ and $\text{Up sm.}_{\text{WPS}}$ are used to calculate the corresponding regular Wing model parameters $\text{Slope}_{\text{Wing}}$, $\text{Put curv}_{\text{Wing}}$, $\text{Call curv}_{\text{Wing}}$, $\text{Down cutoff}_{\text{Wing}}$, $\text{Up cutoff}_{\text{Wing}}$, VCR, SCOR, SSR, $\text{Down sm.}_{\text{Wing}}$ and $\text{Up sm.}_{\text{Wing}}$, using the following formulas:

$$
\begin{aligned}
\text{Slope}_{\text{Wing}} &= \text{Slope}_{\text{WPS}} \left(\frac{T}{T_{\text{ref}}}\right)^{-\text{Slope Power}} \\
\text{Call curv}_{\text{Wing}} &= \text{Call curv}_{\text{WPS}} \left(\frac{T}{T_{\text{ref}}}\right)^{-2(\text{Curva Power})} \\
\text{Put curv}_{\text{Wing}} &= \text{Put curv}_{\text{WPS}} \left(\frac{T}{T_{\text{ref}}}\right)^{-2(\text{Curva Power})} \\
\text{Up cutoff}_{\text{Wing}} &= \text{Up cutoff}_{\text{WPS}} \sqrt{\frac{T}{T_{\text{ref}}}} \\
\text{Down cutoff}_{\text{Wing}} &= \text{Down cutoff}_{\text{WPS}} \sqrt{\frac{T}{T_{\text{ref}}}} \\
\text{VCR}_{\text{Wing}} &= -\text{VCOR}_{\text{WPS}}\text{Slope}_{\text{WPS}} \left(\frac{T}{T_{\text{ref}}}\right)^{-\text{Slope Power}} \\
\text{SCR}_{\text{Wing}} &= -\text{SCOR}_{\text{WPS}}(\text{Call curv}_{\text{WPS}} + \text{Put curv}_{\text{WPS}}) \left(\frac{T}{T_{\text{ref}}}\right)^{-2(\text{Curva Power})} \\
\text{SSR}_{\text{Wing}} &= \text{SSR}_{\text{WPS}} \\
\text{Down sm.}_{\text{Wing}} &= \text{Down sm.}_{\text{WPS}} \\
\text{Up sm.}_{\text{Wing}} &= \text{Up sm.}_{\text{WPS}}
\end{aligned}
$$

(T is here the number of weekday calendar days to expiry.)

Second, the regular wing model parameters are used to calculate the volatility in the same way as in the regular wing model.

The time weighted wing differs from the regular wing model regarding the converted strike x. When using the Wing model, the converted strike is $x = ln(X/F)$. The transformed strike for the time weighted wing model is $x = \ln(X/F)/\sqrt{T/365.0}$ where T is the time in days to expiry, floored at one day. For Wing Power Skew model T is the number of *business* days to expiry, floored at volatility curve parameter *Time floor*, i.e. $T = \max(Days\_to\_expiry, Time\_floor)$.

6.7. SABR model

SABR volatility surfaces depend on the following parameters

Expiry	Expiry date.
Days	Days remaining until expiry.
ATM fwd. (F_{ATM})	ATM forward. Used to calculate the synthetic forward (see section 6.1.3)
Ref.price (F_{ref})	Reference (forward) price. Used to calculate the synthetic forward (see section 6.1.3).
Beta (β)	$0 \leq \beta \leq 1$, where $\beta = 0$ represents a stochastic normal model and $\beta = 1$ represents a stochastic lognormal model. Default value is zero.
ATM vol (σ_{ATM})	The volatility at the synthetic forward. The start volatility parameter α_0 is calculated implicitly from this value by (134). Expressed in percentage.
Volvol (ν)	The volatility of the volatility process (131)–(132). Expressed in percentage scaled by square root of time to expiry. For example, to get $\nu = 0.2$ with 70 days to expiry, set the value $20 \cdot \sqrt{70/365} = 8.76$
Corr. (ρ)	Correlation of the Brownian motion of the forward price and the Brownian motion of the stochastic volatility process. Expressed in percentage.
SSR	Skew swimmingness rate. Used to calculate the synthetic forward (see section 6.1.3).
Use as guess	The use as guess parameter is used for fitting. The curve fitting problem is non-linear and an iterative algorithm (the Levenberg-Marquardt algorithm) is used. Default value is 0 and means that the standard guess will be used. If set to 1, the current values are used as start guess.

The synthetic forward F_{synth} is calculated from the parameters ATM forward, reference price and skew swimmingness rate by (114).

Introduce a stochastic differential equation (SDE) for the forward price

$$\begin{aligned} dF(t) &= \alpha(t)F(t)^{\beta}dW_1(t) \\ F(0) &= F_{\text{synth}}. \end{aligned}$$

where the volatility $\alpha(t)$ solves the SDE

$$d\alpha(t) = \nu\alpha(t)dW_2(t) \tag{131}$$

$$\alpha(0) = \alpha_0 \tag{132}$$

where W_1 and W_2 are Brownian motions with correlation $dW_1 \cdot dW_2 = \rho dt$.

The SABR volatility skew $\sigma(X)$ is given by

$$\sigma(X) = \frac{\alpha_0 z}{\psi(X)} \cdot \varphi(X) \tag{133}$$

where

$$\begin{aligned}
\varphi(X) &= 1+\left(\frac{1}{24}\frac{(1-\beta)^2\alpha_0^2}{(F_{\text{synth}}\cdot X)^{1-\beta}}+\frac{1}{4}\frac{\rho\beta\alpha_0\nu}{(F_{\text{synth}}\cdot X)^{(1-\beta)/2}}+\frac{1}{24}(2-3\rho^2)\nu^2\right)T \\
\psi(X) &= x(z)(F_{\text{synth}}\cdot X)^{(1-\beta)/2}\left(1+\frac{1}{24}(1-\beta)^2\log^2\left(\frac{F_{\text{synth}}}{X}\right)+\frac{1}{1920}(1-\beta)^4\log^4\left(\frac{F_{\text{synth}}}{X}\right)\right) \\
z &= \frac{\nu}{\alpha_0}(F_{\text{synth}}\cdot X)^{(1-\beta)/2}\log\left(\frac{F_{\text{synth}}}{X}\right) \\
x(z) &= \log\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right)
\end{aligned}$$

where X, T are defined in section 1.1, β, ν and ρ are model parameters, α_0 is calculated from (134) and lower order terms are neglected.

For the special case of options where the strike equals the synthetic forward price, $X = F_{\text{synth}}$, this formula reduces to

$$\sigma_{\text{ATM}} = \sigma(F_{\text{synth}}) = \frac{\alpha_0}{F_{\text{synth}}^{1-\beta}}\varphi(F_{\text{synth}}) \tag{134}$$

neglecting lower order terms.

The SABR model implemented in Orc Trader™ is developed from the model presented in [**HKLW**].

6.8. Spline delta (dense)

6.8.1. Parameters

The Delta dense model has nineteen delta parameters and one rigidity parameter.

Rigidity	This is manually set and only impacts the calibration. The higher the value is the less curvature the calibrated curve will have. The default value is 10 and the value must not be negative.
x% put	The volatility for a put delta of -x%. The values of x are 0.5, 1, 2, 5, 10, 15, 25, 35 and 40.
ATM vol	The volatility for a put delta of -50% and call delta of 50%.
x% call	The volatility for a call delta of x%. The values of x are 0.5, 1, 2, 5, 10, 15, 25, 35 and 40.

6.8.2. The model

Given a volatility value for a delta parameter one can compute the corresponding (strike, volatility)-point using Black Scholes formula. Doing this for all the delta parameters hence yields nineteen (strike, volatility)-points. The final (strike, volatility)-curve curve is produced by spline interpolation between these points.

6.8.3. Calibration

The calibration is using the calibration routine for the Cubic spline (dynamic) model. The Cubic spline (dynamic) calibration is used to produce a (strike, volatility)-curve. After this one finds volatilities for the delta parameters such that the corresponding (strike, volatility)-points lie very close to the initially calibrated (strike, volatility)-curve. The obtained delta parameter volatilities are the result from the calibration.

6.9. SVI

6.9.1. Functional form

The SVI model is a parametrized volatility curve that, in a slightly different shape, appears as the limit of the Heston model as time to maturity goes to infinity , see [**GJ**]. The SVI parameterization of the total implied variance for a fixed time to maturity reads,

$$\omega_{imp}(x) = a + b\left(\rho(x-m) + \sqrt{(x-m)^2+\sigma^2}\right), \tag{135}$$

where x is the usual log-moneyness and $\{a, b, \sigma, \rho, m\}$ is the parameter set. The parameter sigma (σ) is not to be confused with the volatility of the underlying price process. Total implied variance equals implied volatility squared multiplied with the time to expiry and so the implied volatility curve as a function of x is,

$$\sigma_{imp}(x) = \sqrt{\frac{1}{T}\left(a + b\left(\rho(x-m) + \sqrt{(x-m)^2+\sigma^2}\right)\right)}. \tag{136}$$

The effects on the volatility smile of the raw parameters are listed below:

- Increasing a results is a vertical translation of the smile in the positive direction.
- Increasing b decreases the angle between th eput and call wing, i.e. tightens the smile.
- Increasing ρ results in a counter-clockwise rotation of the smile.
- Increasing m results in a horisontal translation of the smile in the positive direction.
- Increasing σ reduces the at-the-money curvature of the smile.

These properties of the parameters have been visualized in many articles, for example in [**JAG**].

6.9.2. Calibration

The calibration of the SVI model is mathematically involved. The main implementation in Orc is based on [**ZS**]. The resulting volatility curve may not be arbitrage-free and in [**GJ2**] a modification to the curve is described that can remedy this problem. In the Orc implementation there is a parameter called arb free fit that if set to 1 applies this modification. It should be said however that the fit of the resulting curve to market data may be less good than the original one.

Weights

When calibrating the model to market data each data point will enter the calibration together with a corresponding weight. This weight basically says how much influence the data point should have in the calibration. As for most other volatility models in Orc the weights can be influenced by the option vega and in-the-moneyness. In the case of the SVI model the OTM weighting will give zero weights to in the money options. For the SVI model there are also specific parameters in the model that will impact the data point weights. These parameters will make spread size affect the weights as follows. The spread size mean and standard deviation are first calculated. Some spreads may be very large and therefore only a percentage of the smallest spreads will be used when calculating the mean and standard deviations. The value of the parameter sp perc will determine how many spreads that will be included. The parameter n sp std will determine how many spread standard deviations away from the mean that will be given a non-zero weight factor by this weighting procedure. If the parameter sp w fun is zero then a zero-one cut-off will be applied; spreads larger than mean + n sp std standard deviations will be given zero weights, all others will be unaffected (weight factor 1). If the parameter sp w fun is 1 then a sliding weight factor will be applied; spreads larger than mean will get linearly smaller until they at mean + n sp std standard deviations will be given zero weights. *This weighting procedure only works if bid, ask and mid prices are selected to be used for the calibration (Fit all). Options with incomplete prices (bir or ask missing) will be disregarded in the calibration.*

6.9.3. Model parameters

The following parameters are used by the SVI model implemented in Orc.

a	The parameter a as described above.
b	The parameter b as described above.
sigma	The parameter σ as described above.
rho	The parameter ρ as described above.
m	The parameter m as described above.
arb free fit	If set to zero no modification will be done to the calibrated curve should it not be arbitrage free.
ssr	This defines the influence of the Reference price in the same way as for the Wing model. The default value is 1.
sp perc	The percentage of all options to be used when calculating mean and standard deviation in the weighting procedure described above. The default value is 85.
n sp std	The number of standard deviations away from the mean spread that will give zero weights according to the Weight section above. The deafult value is 2. If sliding weighting is used then this should probably be increased.
sp w fun	The kind of weighting prcedure applied; cut-off or linear as described in the Weight section above. The default value is 0 (cut-off).
nbr pts th	If there are less than this number of options with both bid and ask prices no calibration is done at all. The default value is 6.

7. Calculation parameters and preferences

7.0.1. Precision parameter

The precision parameter is used in Orc Trader™ to tune the speed vs. accuracy ratio when calculations are performed. The parameter can take values in the interval 0 to 5, where 0 is lowest precision. An example of application is the number of time steps used in binomial methods.

7.0.2. Day and hour count convention

Market conventions for the number of days per year differ for various situations. The notation typically used to denote a day count convention is Number of days in a month/Number of days in a year. In practice, there are six day count conventions:

- Actual/Actual (in period)
- Actual/365
- Actual/365 (366 in a leap year)
- Actual/360
- 30/360
- 30E/360

30/360

The number of days between two dates assumes 30-day months, according to the following rules for the number of days between the dates $d_1/m_1/y_1$ and $d_2/m_2/y_2$:

$$360 \cdot (y_2 - y_1) + 30 \cdot (m_2 - m_1) + d_2' - d_1' \tag{137}$$

where $d_1' = \min(30, d_1)$ and

$$d_2' = \begin{cases} 30 & \text{if } d_2 = 31 \text{ and } d_1' = 30 \\ d_2 & \text{otherwise} \end{cases}$$

30E/360

The number of days between two dates assumes 30-day months according to (137) where $d_1' = \min(30, d_1)$ and $d_2' = \min(30, d_2)$.

Volatility day count conventions

The following are the available conventions for calculating volatility days between two dates.

- Trading/245
- Trading/246
- Trading/247
- Trading/248
- Trading/249
- Trading/250
- Trading/251
- Trading/252
- Trading/253
- Trading/254
- Trading/255
- Trading/256
- Trading/257
- Actual/365

The number of trading days between to dates is obtained from the trading calendar.

7.0.3. Additional volatility time parameters

Volatility time mode

There are four volatility time modes:

- none
- quoting expiry
- quoting always
- always

When in mode 'none' additional volatility time is set then additional vol. time is calculated from the parameter additional volatility time in terms of one day. When mode is set to 'always' the additional volatility time is calculated from volatility time decrease parameters.

Additional volatility time in terms of one day

This parameter specifies how big portion of one day the additional volatility time consists of. When set to 0% additional volatility time is zero and when set to 100% additional volatility time is one full day, i.e. the parameter divided by number of days per year.

Volatility time decrease

The volatility time decrease parameters are:

- Expiry time, t_{expiry}
- Volatility decrease start, t_{start}
- Volatility decrease end, t_{end}

Assuming that $t_{\text{start}} < t_{\text{expiry}} < t_{\text{end}}$ additional volatility time is calculated as

$$t_{\text{additional}} = \frac{t_{\text{expiry}} - t}{t_{\text{end}} - t_{\text{start}}} \cdot \frac{1}{N_{\text{days}}}$$

where t is the calculation time and N_{days} is number of days per year. If $t \notin [t_{\text{start}}, t_{\text{end}}]$ or $t_{\text{expiry}} \notin [t_{\text{start}}, t_{\text{end}}]$ then it is moved to the closest boundary point, e.g. if $t > t_{\text{end}}$ then we set $t = t_{\text{end}}$.

The volatility time can decrease over midnight. Setting $t_{\text{end}} < t_{\text{start}}$ is interpreted as time decrease over midnight. E.g. $t_{\text{start}} =$ 14:00 and $t_{\text{end}} =$ 03:00 means that volatility time will decrease for 13 hours from 14:00 in the afternoon until 03:00 the coming night.

Setting $t_{\text{start}} = t_{\text{end}}$ then volatility time will decrease from 24 hours from t_{start} until t_{end} the following day.

7.0.4. Financing rate

The financing rate r is used to calculate present values of certain future payments and future values of fixed amounts (see section 2.1.4).

Contract financing rate

A contract that expires at time T is associated with a financing rate r given by the entry at time T of the contract yield curve. This financing rate r is used for theoretical calculations on that contract unless otherwise specified.

A contract that does not expire is associated with the financing rate r given by the entry at time zero of the contract yield curve.

Rate types

The following table lists available rate types in Orc Trader™ as well as present value factor pv_T used for calculating the present value of a certain payment with financing rate time to payment T.

TABLE 3. Present value factor for rate types

Rate type	Present value factor (pv_T)	Comment
Continuous	e^{-rT}	
Compound	$(1+r/n)^{-nT}$	annual: $n=1$ semi-annual: $n=2$ quarterly: $n=4$ monthly: $n=12$
Straight	$(1+rT)^{-1}$	
Discount	$1-rT$	
Present value factor	r	

A yield curve is basically defined by a set of (time, rate, rate type)-tuples where time is denoted in years and has been calculated using the user defined day count convention. Guided by table 3 above one can calculate a present value factor for each entry of a yield curve. To calculate a present value factor for a future time not readily availabe in the yield curve definition an interpolation procedure is used. First of all each present value factor corresponding to the yield curve can be transformed to a continuous rate by the formula

$$r_{\text{cont}}(T) = -\frac{1}{T}\ln(\text{pv}_T). \tag{138}$$

For any future time T between T_1 and T_2 an interpolated continuous rate is calculated as

$$r_{\text{inter. cont.}} = \frac{T_2 - T}{T_2 - T_1} r_{\text{cont}}(T_1) + \frac{T - T_1}{T_2 - T_1} r_{\text{cont}}(T_2). \tag{139}$$

Using this interpolated rate the present value factor for T is calculated as

$$e^{-r_{\text{inter. cont.}} T}. \tag{140}$$

The Orc Trader™ handles negative financing rates, as long as the present value factor pv_T is positive so that the expression (138) can be evaluated.

7.0.5. Underlying rate

The underlying rate q is a yield which is continuously payed out by the underlying asset. The cost of carry b is given by the continuously compound financing rate r and the underlying rate q by $b = r - q$.

If an underlying rate curve is used, the (interpolated) value from the curve is used as underlying rate.

7.0.6. Dividends

Discrete dividend payments in Orc Trader™ are defined by the parameters

- Ex-dividend date, τ
- Dividend date, $\hat{\tau}$
- Dividend, D
- Weight, w
- Kind

Kinds for equities

For equities there are two main choices for selecting dividend kind; amount and %spot.

Amount means that a fixed dividend amount will be paid out at the dividend date. If the dividend contract for an option has a different currency than the option itself the amount will be transformed to the option currency in all theoretical calculations.

%spot means that the dividend amount to be paid out at the dividend date is a percentage of the current spot price. This means that the dividend amount changes as the spot price changes. The calculation of the %spot dividend amount will not take the currency of the dividend contract into account but will always relate to the spot price.

Total present value of dividends

The present value of N dividends is calculated from the following parameters

- Time to expiry T
- Dividends $D_1, D_2, \dots D_N$
- Time to ex-dividend dates $T_1, \dots T_N$, where $T_N \leq T$.
- Interest rate r associated with T
- Underlying rate q

The total present value of the dividends is calculated as

$$D_{\text{tot},T} = \begin{cases} \sum_{i=1}^{N} \text{pv}_{T_i} D_i w_i \text{FX}_i & \text{for amount kind} \\ \sum_{i=1}^{N} \text{pv}_{T_i} D_i S_{\text{div}}/100 & \text{for \%spot kind} \end{cases} \tag{141}$$

where pv_T is given by (9), FX is the exchange rate between dividend currency and contract currency, S_{div} is the spot price of dividend contract.

Some models require higher accuracy on the value of $D_{\text{tot},T}$ and use different techniques to calculate this value (see section 3.1.2 for an example).

If base offsets are set on the contract then the present value factor pv_T is defined as

$$\text{pv}_T = \frac{F_T}{S}$$

where F_T is the forward value of the underlying and S is the spot price. The present value factors of the ex-dividend dates are given by $\text{pv}_{T_i} = \text{pv}_T{}^{T_i/T}$ which are used in expression (141) to calculate the total present value of the dividends.

Dividends on baskets

For derivative on baskets or basket indices, Orc Trader™ automatically calculates the dividends on the basket from the dividends set on the components of the basket.

7.0.7. Early exercise premium (EEP)

The EEP is an amount that is payed to the holder of the option if it is exercised immediately. The EEP is assumed to vanish as time approaches expiry.

For calculation of theoretical values of call options the EEP is added to the underlying spot price, S, and for put option EEP is subtracted from the spot.

$$S' = \begin{cases} S + \text{EEP} & \text{for call options} \\ S - \text{EEP} & \text{for put options} \end{cases}$$

The forward value of the underlying is not affected by the EEP. The difference between the spot and forward affects the cost of carry by modifying the dividend yield

$$q' = \begin{cases} q + \frac{1}{T}\log\left(1 + \frac{\text{EEP}}{S}\right) & \text{for call options} \\ q + \frac{1}{T}\log\left(1 - \frac{\text{EEP}}{S}\right) & \text{for put options} \end{cases}$$

Hence, for instance the Black-Scholes theoretical price of a contract is given by $\Pi_{BS}(S', X, T, r, q', \sigma)$ where X, T, r, σ are defined in section 1.1.

Note that EEP is *not* the same as the early exercise premium found in mathematical finance literature. Conventionally the early exercise premium is the difference between the value of an American option and value of the corresponding European option, i.e. the value of obtaining the right to exercise the option before maturity.

7.0.8. Position

The parameter *position* is the number of contracts in the portfolio.

7.0.9. Multiplier

On some markets the price refers to only one contract, but the buyer obtains a number of contracts. The *multiplier* is the number of contracts obtained when buying one position in the contract. It affects various portfolio window calculations.

7.0.10. Price multiplier

The *price multiplier* is used to adjust the theoretical price if the derivative contract gives the right to buy or sell more or less than one of the underlying asset. It can for instance be used to price a warrant which only gives the right to a fraction of a stock. The price multiplier is simply the factor with which the theoretical price is multiplied after the usual calculation.

7.0.11. Base price multiplier

The *base price multiplier* scales the value of the underlying for calculations on the derivative contract. The underlying S is multiplied by the base price multiplier m_{BP}, so the actual underlying value used for calculations is $S_{\text{BP}} = m_{\text{BP}} \cdot S$.

Example

If the value of the underlying is $S = 100$ and the base price multiplier is $m_{\text{BP}} = 0.1$ the value used for calculations is $S_{\text{BP}} = 10$. Hence, options with strike 10 are ATM.

7.0.12. Base offset

Base offsets determine the relation between the spot and forward value of the base contract. There are five different base offset modes available:

- Not used
- Simple
- Synthetic spot
- Forward adjustment
- No cost of carry

Not used

No offset is applied to the base contract. The spot and forward values are calculated as in section 2.1.5. The derivative of spot with respect to base is calculated as in section 2.1.5.

Simple

Let B be the base price of an option with expiry T, where the base contract is a spot, forward or future. The base offset o_{base} is applied so that the spot price of the base contract is B. The forward price at time T of the base contract is $B + o_{\text{base}}$, that is

$$\begin{aligned} S &= B \\ F_T &= B + o_{\text{base}}. \end{aligned}$$

For an arbitrary time t the forward value of the base contract is calculated with a synthetic yield $y = \frac{1}{T} \ln\left((B + o_{\text{base}})/B\right)$ by

$$F_t = Be^{yt}.$$

The derivative of spot with respect to base is calculated as in section 2.1.5.

Synthetic spot

Let B be the base price of an option where the base contract is a spot, forward or future. The base offset o_{base} is applied so that the spot price of the base contract is $B + o_{\text{base}}$

$$S = B + o_{\text{base}}. \tag{142}$$

The forward price of the base contract is calculated from (10) with S given by (142).

The derivative of spot with respect to base is one, $\frac{\partial S}{\partial B} = 1$.

Forward adjustment

The spot price of the base contract is calculated as in section 2.1.5 and is not affected by the base offset.

The forward price of the base contract is adjusted by the base offset o_{base}

$$F = F_{\text{theo}} + o_{\text{base}}$$

where F_{theo} is calculated as in section 2.1.5.

The derivative of spot with respect to base is calculated as in section 2.1.5.

No cost of carry

Both the spot and forward prices of the base contract are set to the offset adjusted base price

$$\begin{aligned} S &= B + o_{\text{base}} \\ F &= B + o_{\text{base}}. \end{aligned}$$

The derivative of spot with respect to base is one, $\frac{\partial S}{\partial B} = 1$.

7.0.13. Sqrt(t) normalized vega

Normalized vega term

The normalized vega term f_{sqrt} is used for calculating the weight when calculating sqrt(t) normalized vega. If the number of trading days to expiration is less than f_{sqrt} then the weight is greater than one. Else it is less than one. The default value is $f_{\text{sqrt}} = 45$.

Minimum days

The minimum days T_{min} makes sure that the weight does not go to infinity as the number of trading days goes to zero when calculating sqrt(t) normalized vega. Hence, the weight is bounded above by $\sqrt{f_{\text{sqrt}}/T_{\text{min}}}$. The default value is $T_{\text{min}} = 3$.

7.1. Dynamical parameters

7.1.1. Accuracy

Accuracy determines how many puts and calls will be used in the replication of the expected future variance. Default value is 10, which means that 10 put options and 10 call options will be used; in total 20 contracts. Finer accuracy means better replication but at the cost of computation time,

7.1.2. Barrier hit

Indicator which determines whether the barrier of the contract has been hit or not.

- Barrier hit = 0 means the barrier has not been hit
- Barrier hit = 1 means the barrier has been hit

This parameter must be updated manually when the barrier is hit.

7.1.3. Barrier

The value of the barrier of the contract. The barrier is *hit* when the value of the underlying equals or passes the barrier. Contracts with barriers include terms and conditions which are activated when the barrier is hit.

7.1.4. FX rate

The fixed foreign exchange rate.

7.1.5. FX volatility

The volatility σ_J of the foreign exchange rate, where the foreign exchange rate is modeled by geometric Brownian motion (82)–(83).

7.1.6. FX/Spot correlation

7.1.7. Percentage of ATM

Percentage of ATM, p_{ATM} determines how wide the strike interval should be around the ATM forward in the replication procedure. The min strike is $X_{\text{min}} = \frac{1}{100} \cdot F_{\text{atm}}(100 - p_{\text{ATM}})$ and the max strike is $X_{\text{max}} = \frac{1}{100} \cdot F_{\text{atm}}(100 + p_{\text{ATM}})$

7.1.8. Rebate

The amount payed out when rebates are payed out. For CBBC contracts the value of the rebate parameter has a slightly different meaning (see rebate of cbbc contracts). For knock-out options the rebate is payed out if the option is knocked out. For knock-in options the rebate is payed out if the option is not knocked in before maturity.

Rebate of CBBC contracts

For CBBC contract the value of the parameter rebate is the percentage α of the absolute difference between the barrier H and strike X which defines the floor of the rebate part of the contract. Hence the rebate floor is defined as $\frac{1}{100}\alpha \cdot |X - H|$.

Example: For a rebate floor $0.3 \cdot |X - H|$ set $\alpha = 30$.

7.1.9. Realized volatility

The annualized realized volatility of the underlying contract from the start date of the variance swap until the calculation date. This value must be calculated manually by the user and updated at the beginning of each trading day.

7.1.10. Rebate period

The length in hours of the period over which the rebate is calculated.

7.1.11. Reference price

The minimal respectively maximal value of the underlying spot during the rebate period, for call respectively put options.

This parameter must be updated manually when a new high or low is recorded.

7.1.12. Start date

Variance swaps

The date from which realized volatility is accounted for.

Forward start ARO options

The date at which the strike of a forward start ARO option is fixed.

7.1.13. Trigger contract

The contract used for monitoring the barrier and registering a knock-out event. It should be entered as the contract tag. If no value or zero is set, the base contract is taken as the trigger contract.

7.1.14. Trading session

The following parameters have self-explanatory names and are given on the format "hhmmss".

- start next trading session
- end next trading session
- end current trading session

7.1.15. Use variance space

Use variance space determines if prices and greeks should be displayed in terms of variance or volatility in the trading view for variance swaps. Variance swaps are booked in annualized variance so this parameter does not affect the portfolio window.

7.1.16. Use observations

Use observations enables the use of the observation values of the observation contract.

7.1.17. Bond yield offset

The offset applied to the yield curve for valuing the bond part of a convertible bond.

7.1.18. Convertible from

The date from which a convertible bond can be converted into the underlying (stock).

7.1.19. Option multiplier

8. Theoretical values

8.1. Risk measures

8.1.1. Risk measures, add-ons and document setup

In this section number of basic risk measures are listed, most of whom are partial derivative of first or higher order of the theoretical price of the contract.

The basic risk measures are:

- delta
- gamma
- theta
- rho
- vega

- speed
- charm
- color
- vanna
- weezu
- zomma
- duration
- convexity
- hedge volume
- dividend risk
- volatility surface parameters

Basic risk measures can be varied in several ways by adding risk measure add-ons. An add-on modifies the calculation of the risk measure in some way.

Available risk measure add-ons are:

- of position
- basket
- beta-adjusted
- cash
- PL
- day
- (derivative)
- perceived
- contract
- option
- option . . . of contract
- portfolio . . . result
- residual
- swimming skew
- (1%)
- (user %)
- (no basket split)
- (price display)

Some risk measures have a number of add-ons which are specific for that risk measure only.

- vega related risk measures

Document setup

A typical risk measure consists of a combination of basic risk measures and risk measure add-ons. For a full description on how a risk measure is calculated all components must be considered.

For example, to get the full description on how *swimming skew cash delta of position* is calculated consult the basic risk measure section delta and add-on sections swimming skew, cash and of position.

Some risk measures are not calculated by the default method for a particular add-on. In such cases the documentation contains a section on the combination of the basic risk measure and add-on. For example, *gamma of position* does not follow default the calculation for risk measures of position. The sections gamma and gamma of position must be consulted.

Basic risk measures include a list of add-ons which are available, by themselves or in combinations, for that risk measure. The number of combinations is limited, all possible combinations are not available in the Orc Trader™ .

8.1.2. Delta (Δ)

Delta is one of the Greeks. It measures how sensitive the theoretical price Π is towards movement in the underlying S. It is defined as the partial derivative of Π with respect to S

$$\Delta = \frac{\partial \Pi}{\partial S}.$$

The underlying asset is either the theoretical spot price of the underling or the base contract price depending on the context, see Trading window and Portfolio window sections.

Pricing models

The default implementation for calculating delta uses the symmetrized average rate of change formula (6) with $\varepsilon = 0.00001$.

The Black-Scholes model and the Black-76 model calculate delta analytically when applicable. For the European vanilla call options delta is

$$\Delta_{\text{call}} = e^{-qt}\Phi(d_1)$$

and for the European vanilla put option delta is

$$\Delta_{\text{put}} = e^{-qt}(\Phi(d_1) - 1)$$

where S, X, T, r, q, σ and Φ and $d_1(S, X, T, r, q, \sigma)$ are defined in section 1.1.

For the binomial model, binomial relative and binomial robust the partial derivative is calculated from the expression (41) of the derivative from binomial tree method.

The variance swap pricing model uses its own delta implementation, see section 5.10.3.

Trading window

The trading window delta is calculated by taking the partial derivative of the theoretical contract price with respect to the theoretical spot price of the underling. For partial derivative with respect to the base contract see perceived delta.

Trading window delta is affected by multipliers in accordance with the section default multipliers trading window.

Portfolio window

Portfolio window delta measures the delta of a whole position. See delta of position for details.

Related risk measures

Related risk measures
skew risk

Trading window add-ons

option	perceived	swimming skew	(1%)
(user%)	hedge volume		

Portfolio window add-ons

of position	contract	beta-adjusted	cash
day	option	option ... of contract	portfolio ... result
swimming skew			

Basket watch add-ons

of position	contract	beta-adjusted	cash
day	option	option ... of contract	portfolio ... result
swimming skew	residual	adjusted	basket

8.1.3. Gamma (Γ)

Gamma is one of the Greeks. It measures how sensitive delta Δ is towards movement in the underlying asset S. It is defined as the second partial derivative of the contract price Π with respect to S

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 \Pi}{\partial S^2}.$$

The underlying asset is either the theoretical spot price of the underling or the base contract price depending on the context, see Trading window and Portfolio window sections.

Pricing models

The default implementation for calculating gamma uses the symmetrized average rate of change formula (7) with $\varepsilon = 0.001$.

The Black-Scholes model and the Black-76 model calculate gamma analytically when applicable. For both European vanilla call and put options gamma is

$$\Gamma_{\text{call}} = \Gamma_{\text{put}} = e^{-qt} \frac{\Phi'(d_1)}{S\sigma\sqrt{t}}$$

where S, X, T, r, q, σ and Φ and $d_1(S, X, T, r, q, \sigma)$ are defined in section 1.1.

For the binomial model, binomial relative and binomial robust the partial derivative is calculated from the expression (41) of the derivative from binomial tree method.

The variance swap pricing model uses its own gamma implementation, see section 5.10.5.

Trading window

The trading window gamma is calculated by taking the partial derivative of the theoretical contract price with respect to the theoretical spot price of the underling. For partial derivative with respect to the base contract see perceived gamma.

Trading window gamma is affected by multipliers in accordance with default multipliers trading window.

Portfolio window

Portfolio window gamma measures the gamma of a whole position. See Gamma of position for details.

Related risk measures

Trading window add-ons

option	perceived	swimming skew	(1%)
(user%)			

Portfolio window add-ons

of position	beta-adjusted	cash	PL
day	option	option . . . of contract	portfolio . . . result
swimming skew	(1%)	(user%)	(no basket split)

Basket watch add-ons

of position	basket	beta-adjusted	cash
PL	day	option	option . . . of contract
portfolio . . . result	swimming skew	(1%)	(user %)
(no basket split)	residual		

8.1.4. Theta (Θ)

Theta is one of the Greeks. It measures how the derivative contract price Π changes as one trading day passes

$$\Theta = \Pi(T - 1_{\text{trading day}}) - \Pi(T). \tag{143}$$

where T is time to expiry and $1_{\text{trading day}}$ is one trading day.

For theta calculated as a partial derivative see risk measures derivative.

Related risk measures

Trading window add-ons

interest rate	rate	no dividend	option
volatility	(derivative)	(price display)	

Portfolio window add-ons

of position	option . . . of contract	day	(derivative)
interest rate	no dividend	volatility	portfolio theta result

Basket watch add-ons

of position	option . . . of contract	day	(derivative)
interest rate	no dividend	volatility	portfolio theta result

8.1.5. Rho (ρ)

Related risk measures

Trading window add-ons

day	+10 basis points	-1 basis point	(price display)

Portfolio window add-ons

of position	day	+10 basis points	-1 basis point
portfolio . . . result			

Basket watch add-ons

of position	day	+10 basis points	-1 basis point
portfolio . . . result			

8.1.6. Vega ($\mathcal{V}$)

Vega is one of the Greeks. It measures how sensitive the theoretical price Π is towards change in volatility σ. It is defined as the partial derivative of Π with respect to σ over one hundred

$$\mathcal{V} = \frac{1}{100} \cdot \frac{\partial \Pi}{\partial \sigma}.$$

It can be intrepreted as the change in theoretical price when the volatility changes by one percentage unit.

Pricing models

The default implementation for calculating vega uses the symmetrized finite difference formula (4) with $\varepsilon = 0.005$.

Related risk measures

Trading window add-ons

atm	vega per atm vega	option	(1%)
(user%)	weighted vega	v.c.r.-weighted vega	(price display)
exposure	hedge volume		

Portfolio window add-ons

of position	option . . . of contract	atm	day
(1%)	(user%)	(no basket split)	weighted vega
sqrt(t) normalized vega	risk vega	volatility weighted	v.c.r.-weighted vega
portfolio . . . result			

Basket watch add-ons

of position	option . . . of contract	atm	day
(1%)	(user%)	(no basket split)	weighted vega
sqrt(t) normalized vega	risk vega	volatility weighted	v.c.r.-weighted vega
portfolio . . . result			

8.1.7. Speed

Related risk measures

Portfolio window add-ons

of position

Basket watch add-ons

of position

8.1.8. Charm

Charm is defined as the effect on delta as one trading day passes

$$\text{charm} = \Delta(T - 1_{\text{trading day}}) - \Delta(T)$$

where Δ is delta, T is time to expiry and $1_{\text{trading day}}$ is one trading day. On expiration date, charm is zero for delivery settled derivatives.

Related risk measures

Trading window add-ons
swimming skew

Portfolio window add-ons
of position | cash | swimming skew

Basket watch add-ons
of position | cash | swimming skew

8.1.9. Color

Color is defined as the effect on gamma as one trading day passes

$$\text{color} = \Gamma(T - 1_{\text{trading day}}) - \Gamma(T)$$

where Γ is gamma, T is time to expiry and $1_{\text{trading day}}$ is one trading day. On expiration date, color is zero for delivery settled derivatives.

Related risk measures

Trading window add-ons
swimming skew

Portfolio window add-ons
of position | swimming skew

Basket watch add-ons
of position | swimming skew

8.1.10. Vanna

Vanna measures how sensitive delta is with respect to changes in volatility or equivalently how sensitive vega is with respect to changes in the underlying. It is defined as the second order cross partial derivative of the contract price Π with respect to underlying S and volatility σ, scaled by a factor $1/100$ in order to get the change as volatility moves by 1%

$$\text{vanna} = \frac{\partial^2 \Pi}{\partial S \partial \sigma} \cdot \frac{1}{100}$$

The default implementation calculates the cross derivative $\partial^2 \Pi / \partial S \partial \sigma$ in two steps. First delta Δ of the contract is calculated and then the derivative $\partial \Delta / \partial \sigma$ is calculated by applying the symmetrized finite difference method formula (4).

Related risk measures

Portfolio window add-ons
of position

Basket watch add-ons
of position

8.1.11. Weezu

Weezu measures how sensitive vega is towards changes in volatility. It is defined as the second derivative of the contract price Π with respect to volatility σ, scaled by a factor 100^{-2} in order to get the change as volatility moves by 1%

$$\text{weezu} = \frac{\partial^2 \Pi}{\partial \sigma^2} \cdot \frac{1}{100^2}$$

The default implementation calculates the second derivative $\partial^2\Pi/\partial\sigma^2$ by applying the symmetrized finite difference method formula (5) with $\varepsilon = 0.005$.

Related risk measures

Portfolio window add-ons
of position

Basket watch add-ons
of position

8.1.12. Zomma

Weezu measures how sensitive gamma is towards changes in volatility. It is defined as the derivative of gamma Γ with respect to volatility σ, scaled by a factor 100 in order to get the change as volatility moves by 1%

$$\text{zomma} = \frac{\partial \Gamma}{\partial \sigma} \cdot \frac{1}{100} = \frac{\partial^3 \Pi}{\partial \sigma \partial^2 S} \cdot \frac{1}{100}$$

The default implementation calculates the derivative $\partial\Gamma/\partial\sigma$ by applying the symmetrized finite difference method formula (4) with $\varepsilon = 0.005$.

Related risk measures

Portfolio window add-ons
of position swimming skew

Basket watch add-ons
of position swimming skew

8.1.13. Duration

Related risk measures

Related risk measures
modified duration

8.1.14. Convexity

8.1.15. Dividend risk

Dividend risk is only calculated for the add-ons mentioned below.

Related risk measures

Trading window add-ons

(1%)	(user%)	

Portfolio window add-ons

(1%)	(user%)	portfolio . . . result

Basket watch add-ons

(1%)	(user%)	portfolio . . . result

Related risk measures

Trading window add-ons

(delta based)	(delta ytm based)	(vega based)

8.1.16. Volatility surface parameters

These risk measures measure the effect on the theoretical price when changing the value of a parameter of the volatility surface.

ATM volatility risk

The change in theoretical price with respect to the volatility surface parameter ATM volatility σ_{ATM}. Defined as the partial derivative of the theoretical price with respect to σ_{ATM}

$$\mu_{\text{ATM vol}} = \frac{\partial \Pi}{\partial \sigma_{\text{ATM}}}.$$

For the SABR model it is calculated analytically by taking the partial derivative of (133) with respect to the parameter σ_{ATM}, where α_0 implicitly depends on σ_{ATM}.

8.2. Risk measure extensions and add-ons

8.2.1. Risk measures of position

In the portfolio window risk is measured on positions level rather than per contract level as in the trading window. Various multipliers affect the risk measures.

Multipliers

Risk measures *of position* are multiplied by the position p, the multiplier m and the price multiplier m_p. The underlying is multiplied by the base price multiplier when the risk measure is calculated. There are two exceptions to this rule: delta of position and gamma of position.

For a portfolio window risk measure that is not multiplied by any multipliers, see option risk measures of contract. For a risk measure μ, the relation between the risk measure *of position* $\mu_{\text{of position}}$ and the *option risk measure of contract* $\mu_{\text{option...of contract}}$ is

$$\mu_{\text{of position}} = p \cdot m \cdot m_p \cdot \mu_{\text{option...of contract}}. \tag{144}$$

For example the relation between *vega of position* and *option vega of contract* is $\mathcal{V}_{\text{of position}} = p \cdot m \cdot m_p \cdot \mathcal{V}_{\text{option...of contract}}$.

See section 8.2.2 for risk measures with alternative setup of multipliers.

Basket calculations

Basket split is applicable to some *risk measures of position* for basket derivatives. See section 8.2.15 for list of applicable risk measures and further details.

Partial derivatives with respect to the underlying

By default risk measures of position which are partial derivatives with respect to the underlying are calculated by using the base contract price as underlying. Theoretical spot price can be used as underlying by changing the preference setting Calculate Greeks using....

Delta of position

Delta of position measures how much the position moves when the base contract price changes. The value is an estimate on the number of base contracts needed to achieve the same price movement.

In addition to default risk measures of position multipliers, *delta of position* is divided by the multiplier of the base contract, m_B.

Delta of position is calculated as

$$\Delta_{\text{of position}} = \frac{p \cdot m \cdot m_p}{m_B} \cdot \frac{\partial \Pi}{\partial B} \tag{145}$$

where Π is the theoretical price of the contract, and B is the value of the base contract.

Gamma of position

Gamma of position measures how much the delta moves when the base contract price changes.

In addition to default risk measures of position multipliers, *gamma of position* is divided by the multiplier of the base contract, m_B.

Gamam of position is calculated as

$$\Gamma_{\text{of position}} = \frac{p \cdot m \cdot m_p}{m_B} \cdot \frac{\partial^2 \Pi}{\partial B^2} \tag{146}$$

where Π is the theoretical price of the contract, and B is the value of the base contract.

8.2.2. Multipliers and risk measures

Default multipliers, trading window

By default trading window risk measures are multiplied by the price multiplier m_p. The underlying is multiplied by the base price multiplier when the risk measure is calculated.

For risk measures which are not multiplied by any multipliers see option risk measures trading window. The relation between the risk measure μ and the option risk measure μ_{option} is

$$\mu = m_p \cdot \mu_{\text{option}}.$$

For example the relation between delta and option delta is $\Delta = m_p \cdot \Delta_{\text{option}}$.

Default multipliers, portfolio window

See section 8.2.1 for default risk measures of portfolio windows.

Option risk measures, trading window

In trading windows option risk measures are not multiplied by the price multiplier. Hence, option risk measures are not multiplied by any multipliers.

The underlying contract price is multiplied by the base price multiplier when calculating option risk measures.

Option risk measures, portfolio window

Observe that *option* risk measures and option risk measures of contract are two separate classes of risk measure add-ons.

In portfolio windows *option* risk measures are calculated in the same way as default risk measures of position, i.e. by formula (144). Hence, *option delta* and *option gamma* satisfy

$$\begin{aligned}\Delta_{\text{option...of position}} &= p \cdot m \cdot m_p \cdot \Delta_{\text{option...of contract}} \\ \Gamma_{\text{option...of position}} &= p \cdot m \cdot m_p \cdot \Gamma_{\text{option...of contract}}\end{aligned}$$

where p is position, m is multiplier and m_p is price multiplier.

The underlying contract price is multiplied by the base price multiplier when calculating option risk measures.

Contract risk measures

Contract risk measures are only multiplied by price multiplier m_p. For a risk measure μ the relation between *contract* risk measures and option risk measures of contract, which are not affected by any multipliers, is given by

$$\mu_{\text{contract}} = m_p \cdot \mu_{\text{option...of contract}}.$$

The underlying contract price is multiplied by the base price multiplier when calculating option risk measures.

Contract risk measures correspond to trading window basic risk measures, i.e. risk measures without any add-ons.

Option risk measures of contract

Option risk measures of contract are not multiplied nor divided by position, multiplier, price multiplier or multiplier of base contract. Hence, they are not multiplied nor divided by any multiplier.

The underlying contract price is multiplied by the base price multiplier when calculating option risk measures.

Option risk measures of contract correspond to the trading window option risk measures.

8.2.3. Portfolio ... result

The porfolio ... result risk measures show how much a particular risk measure contributes to the portfolio result since the last reset date.

Introduce the following

- the reset date (theoretical market value from), t_{reset}
- the calculation date (theoretical market value to), t_{calc}

- Theoretical portfolio value at the reset date, Π_{reset}
- Theoretical portfolio value at the calculation date, Π_{calc}

The portfolio ... result risk measures are defined as the terms in a Taylor expansion of the portfolio value. Therefore, the difference between the portfolio value on the reset date and the calculation date is approximately given by

$$\begin{aligned}\Pi_{\text{calc}} &= \Pi_{\text{reset}} + \Delta_{\text{portfolio result}} + \Gamma_{\text{portfolio result}} \\ &\quad + \Theta_{\text{portfolio result}} + \mathcal{V}_{\text{portfolio result}} + \rho_{\text{portfolio result}}\end{aligned}$$

The effect from the last two terms will most likely be neglectable since interest rate and volatility are unlikely to change much over a short time period.

To see the change due to the different greeks when starting Orc Trader the following day, you should use the following settings:

Theoretical mv from:	today
Theoretical mv to:	tomorrow
Delta:	today
Gamma:	today
Vega:	today
Rho:	today
Dividends from:	today
Dividends to:	tomorrow

Portfolio theta result

When performing a portfolio reset, dates are entered in the field "theoretical mv from" and "theoretical market value to". On the reset date portfolio theta result is zero. After the reset date it is defined as the difference in theoretical market value between the “Theoretical mv to date”, t_{calc}, and the “Theoretical mv from date”, t_{reset}:

$$\begin{aligned}\Theta_{\text{portfolio result}} &= \Pi_{\text{mv}}(t_{\text{calc}}, S(t_{\text{reset}}), \sigma(t_{\text{reset}}), r(t_{\text{reset}})) \\ &\quad - \Pi_{\text{mv}}(t_{\text{reset}}, S(t_{\text{reset}}), \sigma(t_{\text{reset}}), r(t_{\text{reset}}))\end{aligned}$$

Portfolio theta result is calculated with fixed base value date. This means that when calculating $\Pi_{\text{mv}}(t_{\text{calc}})$, the spot price is adjusted for any dividend payments. If a dividend occurs, the spot price is decreased with the same amount. (Technically, inside the calculator, this is achieved by setting ORC_PARAMETER_BASE_PRICE_VALID_DATE to the “Theoretical mv from” date).

Portfolio delta result

When performing a portfolio reset, a reset date is entered in the field "Delta:". The portfolio result column shows the difference in value of position due to delta between the calculation date and the entered reset date:

$$\Delta_{\text{portfolio result}} = \Delta_{\text{skew}}(t_{\text{theo mv from}}) \cdot (S(t_{\text{calc}}) - S(t_{\text{reset}})) \cdot m$$

where Δ_{skew} is skew delta and m is the contract multiplier.

Portfolio gamma result

When performing a portfolio reset, a reset date is entered in the field "Gamma:". The portfolio result column shows the difference in value of position due to gamma between the calculation date and the entered reset date:

$$\Gamma_{\text{portfolio result}} = \frac{1}{2} \cdot \Gamma_{\text{skew}}(t_{\text{theo mv from}}) \cdot (S(t_{\text{calc}}) - S(t_{\text{reset}}))^2 \cdot m$$

where Γ_{skew} is skew gamma and m is the contract multiplier.

Portfolio vega result

When performing a portfolio reset, a reset date is entered in the field "Vega:". The portfolio result column shows the difference in value of position due to vega:

$$\mathcal{V}_{\text{portfolio result}} = \text{Vega at reset} \cdot (\text{volatilityAtGivenBasePrice} - \text{Volatility at reset})),$$

where volatilityAtGivenBasePrice is the volatility calculated for the instrument (and e.g. strike taken from the instrument), with base price equal to the base prise valid when doing the portfolio reset.

Portfolio rho result

When performing a portfolio reset, a reset date is entered in the field "Rho:". The portfolio result column shows the difference in value of position due to rho between the calculation date and the entered reset date:

$$\rho_{\text{portfolio result}} = \rho(t_{\text{theo mv from}}) \cdot (r(t_{\text{calc}}) - r(t_{\text{reset}})).$$

Portfolio dividend result

When performing a portfolio reset, dates are entered in the field "Dividends from:" and "Dividends to:". On the reset date portfolio dividend result is zero. After the reset date it is defined as the present value of all dividends between the dates "Dividends from:" and "Dividends to:".

8.2.4. Beta-adjusted risk measures

The beta base allows for comparison of risk measures of contracts with different underlying. Consider two derivative contracts C and C_β with underlying contracts S and S_β respectively. Setting S_β as beta base will introduce a scaled version of S, $S' = S/q_\beta$ where

$$q_\beta = \frac{S}{S_\beta} \tag{147}$$

is a fixed ratio. For a risk measure μ that measures what happens when the underlying S increases by one unit, the corresponding beta-adjusted risk measure μ_β measures what happens when the scaled underlying S' increases by one unit, i.e. when the underlying S increases by q_β. Since the beta base S_β and S' are of equal magnitude, μ_β can be viewed as the risk measure of the contract C with respect to S_β, i.e. what happens with C as S_β increases by one unit.

Beta-adjusted delta

Beta-adjusted delta of position is calculated as

$$\Delta_{\beta,\text{of position}} = \beta \cdot \frac{\partial C}{\partial S'} = \beta \cdot q_\beta \cdot \Delta_{\text{of position}}$$

where β is the beta factor.

Beta-adjusted delta of position is divided by the multiplier of the beta base and not of the base contract, so m_B in formula (145) is the multiplier of the beta base.

Beta-adjusted gamma

Beta-adjusted gamma of position is calculated as

$$\Gamma_{\beta,\text{of position}} = \beta \cdot \frac{\partial^2 C}{\partial (S')^2} = \beta \cdot q_\beta^2 \cdot \Gamma_{\text{of position}}$$

where β is the beta factor.

Beta-adjusted gamma of position is divided by the multiplier of the beta base and not of the base contract, so m_B in formula (146) is the multiplier of the beta base.

8.2.5. Swimming skew risk measures

See section 2.3.3 for a short introduction on the concept of skew. The swimming skew risk measures take into account the effect from skew due to non-constant volatility.

Swimming skew delta

Measures the total effect on the theoretical price when the base contract price changes. It is defined as the total derivative of the contract price Π with respect to the base contract S.

$$\Delta_{\text{skew}} = \frac{\mathrm{d}\Pi(S, \sigma(S))}{\mathrm{d}S} = \Delta + \mathcal{V} \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}S} \tag{148}$$

where Δ is delta, $\mathcal{V}$ is vega and σ is volatility.

The default implementation calculates swimming skew delta as

$$\Delta_{\text{skew}} = \Delta_{\text{perceived}} \cdot \frac{\partial B}{\partial S}$$

where $\Delta_{\text{perceived}}$ is perceived delta and $\frac{\partial B}{\partial S}$ is the derivative of base with respect to spot.

Swimming skew gamma

Measures the total effect on swimming skew gamma when the base contract price changes. It is defined as the total derivative of the swimming skew delta Δ_{skew} with respect to the base contract S.

$$\Gamma_{\text{skew}} = \frac{\mathrm{d}\Delta_{\text{skew}}(S, \sigma(S))}{\mathrm{d}S^2} = \Gamma + 2\frac{\partial \Delta}{\partial \sigma} \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}S} + \mathcal{V}\frac{\mathrm{d}^2\sigma}{\mathrm{d}S^2} + \frac{\partial \mathcal{V}}{\partial \sigma}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}S}\right)^2 \tag{149}$$

where Γ is gamma, $\mathcal{V}$ is vega and σ is volatility.

The default implementation calculates swimming skew gamma as

$$\Gamma_{\text{skew}} = \Gamma_{\text{perceived}} \cdot \left(\frac{\partial B}{\partial S}\right)^2$$

where $\Gamma_{\text{perceived}}$ is perceived gamma and $\frac{\partial B}{\partial S}$ is the derivative of base with respect to spot.

The Black -76, binomial, binomial relative, binomial robust, Kim and euro option models apply the formula (149), where the components are calculated as follows: Γ and $\mathcal{V}$ use the model specific implementation, $\frac{\mathrm{d}\sigma}{\mathrm{d}S}$ use formula (6) with $\epsilon = 0.00001$, $\frac{\mathrm{d}^2\sigma}{\mathrm{d}S^2}$ use (7) with $\varepsilon = 0.001$, and derivatives with respect to σ use formula (4) with $\varepsilon = 0.005$.

Swimming skew charm

Swimming skew charm is defined as the effect on swimming skew delta when one trading day passes. It is calculated as

$$\text{charm}_{\text{skew}} = \Delta_{\text{skew}}(T - 1_{\text{trading day}}) - \Delta_{\text{skew}}(T)$$

where T is time to expiry and $1_{\text{trading day}}$ is one trading day.

Swimming skew color

Swimming skew color is defined as the effect on swimming skew gamma when one trading day passes. It is calculated as

$$\text{color}_{\text{skew}} = \Gamma_{\text{skew}}(T - 1_{\text{trading day}}) - \Gamma_{\text{skew}}(T)$$

where T is time to expiry and $1_{\text{trading day}}$ is one trading day.

Swimming skew zomma

Swimming skew zomma is defined as the effect on swimming skew gamma Γ_{skew} when volatility increases by 1%. It is calculated as $\partial\Gamma_{\text{skew}}/\partial\sigma$ by applying the symmetrized finite difference method formula (4) with $\varepsilon = 0.005$.

8.2.6. Time related risk measures

Day risk measures

Day risk measures calculate the risk measure of a portfolio position since reset of portfolio was last performed.

8.2.7. Cash risk measures

Cash risk measures of position

The product of the risk measure of position and the underlying spot price S. For a risk measure μ it is calculated as

$$\mu_{\text{cash, of position}} = S \cdot \mu_{\text{of position}}$$

Example. Cash delta of position is $\Delta_{\text{cash, of position}} = S \cdot \Delta_{\text{of position}}$. Cash delta of position corresponds to the cash amount the position (P%L) will change when the underlying moves by one unit.

Example. Cash gamma of position is $\Gamma_{\text{cash, of position}} = S \cdot \Gamma_{\text{of position}}$. Cash gamma of position corresponds to the cash amount the delta of the position will change when the underlying moves by one unit.

Cash risk measures (currency)

The currency cash delta for a currency, XYZ, shows the risk of a currency or currency derivative position with respect to movements in the currency XYZ.

Example: If you are long 1 USD/SEK which is traded at SEK 9, you will have a Currency cash delta (USD) of USD 1 and a Currency cash delta (SEK) of SEK -9. All other currency cash deltas are zero for this position.

8.2.8. PL risk measures

PL risk measures measure the monetary change in P&L due to that risk measure.

PL Gamma (1%) of position

PL Gamma measures the monetary change in P&L due to gamma when the underlying moves up by 1%. It is calculated as

$$\Gamma_{\text{PL, (1\%) of position}} = \frac{1}{2} \cdot \left(\frac{S}{100}\right)^2 \cdot \Gamma_{\text{of position}}.$$

where S is the underlying spot price.

Note that $\Gamma_{\text{PL, (1\%) of position}} = \frac{1}{200} \cdot \Gamma_{\text{cash, (1\%) of position}}$.

8.2.9. Perceived risk measures

Perceived delta and perceived gamma are modified variants of delta and gamma. They are calculated by taking the partial derivative with respect to the base contract. This is in contrast to trading window delta and gamma for which partial derivative is taken with respect to the theoretical spot price of the underling.

For all pricing models perceived risk measures are calculated by using the symmetrized average rate of change. Hence, perceived delta is calculated from (6) with $\varepsilon = 10^{-5}$ and perceived gamma is calculated from (7) with $\varepsilon = 10^{-3}$.

8.2.10. Risk measures (1%)

Risk measures (1%) are the special case of Risk measures (user%) when user% = 0.01.

Example. Delta (1%) is calculated as

$$\Delta_{1\%} = 0.01 \cdot S \cdot \Delta.$$

PL Gamma (1%)

PL Gamma (1%) of position is multiplied by 0.01^2 rather than by 0.01, see the section PL Gamma (1%) of position for details.

8.2.11. Risk measures (user %)

Risk measures (user %) are defined as the change in the measured quantity when the risk parameter is increased by a user defined percentage.

Delta (user %)

Delta (user %) measures the change in the derivative theoretical price Π as the underlying contract price S is increased by a user defined percentage u. It is calculated as

$$\Delta_{\text{user\%}} = u \cdot S \cdot \Delta,$$

where Δ is delta.

Gamma (user %)

Gamma (user %) measures the change in delta as the underlying contract price S is increased by a user defined percentage u. It is calculated as

$$\Gamma_{\text{user\%}} = u \cdot S \cdot \Gamma,$$

where Γ is gamma.

Vega (user %)

Vega (user %) measures the change in the derivative theoretical price as the volatility is increased by a user defined percentage u. It is calculated as

$$\mathcal{V}_{\text{user\%}} = u \cdot 100 \cdot \sigma \cdot \mathcal{V},$$

where $\mathcal{V}$ is vega.

Dividend risk (user %)

Dividend risk (user %) measures the change in the theoretical price Π as the dividends of type amount $D_{\text{amount},i}$, for $i = 1, \ldots, N$ of the dividend contract are increased by a user defined percentage u. Note that only dividends of type 'amount' are considered. It is calculated as

$$D\text{-risk}_{\text{user\%}} = \Pi((1+u)D_{\text{amount},1}, \ldots, (1+u)D_{\text{amount},N}) - \Pi(D_{\text{amount},1}, \ldots, D_{\text{amount},N}).$$

8.2.12. Delta related risk measures

Skew risk

See section 2.3.3 for a short introduction on the concept of skew.

Skew risk is the difference between delta and swimming skew delta

$$\text{skew risk} = \Delta - \Delta_{\text{skew}}.$$

In view of equation (148) skew risk $= -\mathcal{V} \cdot \mathrm{d}\sigma/\mathrm{d}S$.

8.2.13. Theta related risk measures

Volatility risk measures

Volatility risk measure risk over time but only account for value changes due to volatility and ignore the effects due to financing rate.

The change in value over time of an investment has two components: The financing rate contributes with a non-risky predictable change in value and the volatility contributes with a risky value change. Volatility risk measures do not take into account the non-risky part of the value change.

Volatility theta (Θ_σ)

Volatility theta is defined as theta when the financing rate r is zero. It is calculated from (143) with r set to zero.

Interest rate theta (Θ_r)

Interest rate theta measures the change in value of the theoretical price due to the financing rate. It is defined as the difference between theta Θ and volatility theta Θ_σ

$$\Theta_r = \Theta - \Theta_\sigma.$$

No dividend theta

No dividend theta is defined as theta when dividend is set to zero.

Risk measures (derivative)

Risk measures (derivative) are calculated by taking the partial derivative with respect to time and adjusting the result to give the effect as one trading day passes. This is in contrast to the default way to measure risk over time period, which is to calculate the difference between a future and the present value of the measured quantity.

Theta(derivative)

Theta(derivative) measures the change in the theoretical contract price Π when one trading day passes. It is the partial derivative of Π with respect to time to expiry T divided by the number of volatility days vd

$$\Theta = \frac{1}{vd} \cdot \frac{\partial \Pi}{\partial T}. \tag{150}$$

The default implementation approximates theta(derivative) as a finite difference. The calculation is done by simulating an additional 90 seconds to time to expiry $T_{\text{sim}} = T + 90$ seconds

$$\Theta = \frac{1}{vd} \cdot \frac{\Pi(T) - \Pi(T_{\text{sim}})}{\text{90 seconds}}. \tag{151}$$

Theta(derivative) measures the total change in the contract price as one trading day passes. For the change due to volatility as one trading day passes see volatility theta derivative.

The Black-Scholes model and the Black-76 model calculate theta(derivative) analytically when applicable. For the European vanilla call options it is

$$\Theta_{\text{call}} = \frac{1}{vd} \cdot \left(-\frac{Se^{-qT}\Phi(d_1)\sigma}{2\sqrt{T}} + qSe^{-qT}\Phi(d_1) - rXe^{-rT}\Phi(d_2) \right)$$

and for the European vanilla put

$$\Theta_{\text{put}} = \frac{1}{vd} \cdot \left(-\frac{Se^{-qT}\Phi(d_1)\sigma}{2\sqrt{T}} - qSe^{-qT}\Phi(-d_1) + rXe^{-rT}\Phi(-d_2) \right)$$

where notation is in accordance with section 1.1.

Volatility theta(derivative)

Volatility theta(derivative) measures the change in the theoretical contract price Π when one trading day passes not taking into account the value change due to the financing rate. It is defined by (150) where vd is the number of volatility days and $\Pi(T)$ has been redefined to exclude change in value due to financing rate.

For all pricing models volatility theta(derivative) is calculated as a finite difference. The calculation is done by simulating $T_{\text{add. vol. time}} = 90$ seconds of additional volatility time to the time to expiry, $T_{\text{sim}} = T + T_{\text{add. vol time}}$. Thus the present value factor e^{-rT} and forward price of underlying $F = Se^{(r-q)T}$ are kept fixed throughout the calculation

$$\Theta_\sigma = \frac{1}{vd} \cdot e^{-rT} \frac{\Pi_{\text{F}}(F, X, T, \sigma) - \Pi_{\text{F}}(F, X, T_{\text{sim}}, \sigma)}{T_{\text{add. vol. time}}}$$

where Π_{F} is given by (22).

8.2.14. Vega related risk measures

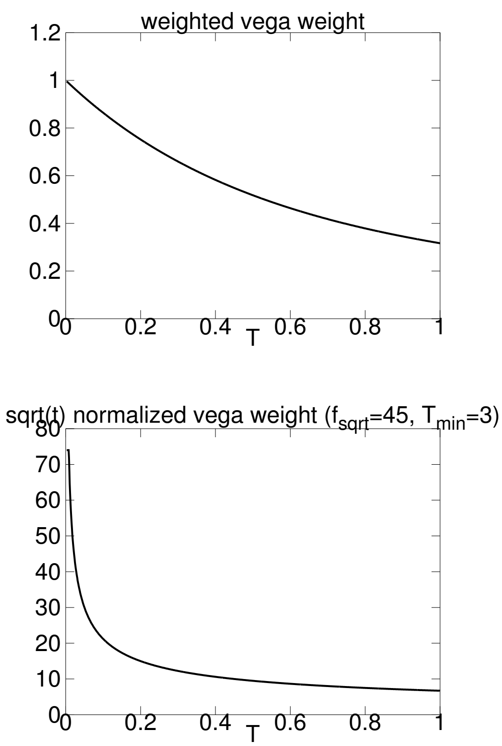

FIGURE 6. The weights of weighted vega, risk vega and sqrt(t) normalized vega as functions of time to expiry.

ATM vega

Vega of the option if it was the ATM forward option, i.e. let X_{ATM} be the strike which equals the forward price of the underling. Then

$$\mathcal{V}_{\text{ATM}} = \mathcal{V}(X_{\text{ATM}})$$

where $\mathcal{V}$ is vega.

Vega per ATM vega

Vega per ATM vega is defined as vega divided by atm vega

$$\mathcal{V}_{\text{per atm vega}} = \frac{\mathcal{V}}{\mathcal{V}_{\text{ATM}}}.$$

Weighted vega

Weighted vega has the following properties

- The weight is approximately inversely proportional to time to expiry.
- For contracts close to expiry the weight is approximately one.
- For contracts with one year to expiry it is approximately one third.

Figure 6 displays the weight as a function of time to expiry.

It is calculated as

$$\mathcal{V}_{\text{weight}} = \frac{1 - e^{-3T}}{3T} \cdot \mathcal{V}$$

where T is time to expiration and $\mathcal{V}$ is vega.

Sqrt(t) normalized vega

The weight of sqrt(t) normalized vega is determined by the number of trading days to expiry, T_{d}, and the preference settings normalized vega term, f_{sqrt}, and minimum days, T_{min}. The weight is a decreasing function of time to expiry (see Figure 6).

Sqrt(t) normalized vega is calculated as

$$\mathcal{V}_{\text{sqrt}} = \sqrt{\frac{f_{\text{sqrt}}}{\max(T_{\text{d}}, T_{\text{min}})}} \cdot \mathcal{V}$$

where $\mathcal{V}$ is vega.

Risk vega

Risk vega weighs vega by volatility and time to expiry. The weight (divided by volatility) is a decreasing function of time to expiry (see Figure 6). It is calculated as

$$\mathcal{V}_{\text{risk}} = \frac{25}{2\sqrt{T}}(1 - e^{-2\sqrt{T}}) \cdot \sigma \cdot \mathcal{V}$$

where T is time to expiration, σ is the volatility and $\mathcal{V}$ is vega (see Figure 6).

Volatility weighted vega

Volatility weighted vega is vega $\mathcal{V}$ weighted by the volatility σ

$$\mathcal{V}_{\text{vol}} = \sigma \cdot \mathcal{V}.$$

V.c.r.-weighted vega

The v.c.r.-weight is calculated as the quotient of the volatility surface VCR parameter (see section 6.1.4) for two expiry dates. Let T be the options expiry date and T_1 is the first expiry date of the volatility surface, i.e. the front month. V.c.r.-weighted vega is calculated as

$$\mathcal{V}_{\text{VCR}} = \frac{VCR(T)}{VCR(T_1)} \cdot \mathcal{V}$$

where $\mathcal{V}$ is vega.

If VCR of the front month is zero or if contract based volatility is used then $\mathcal{V}_{\text{VCR}} = \mathcal{V}$.

Vega exposure

Vega exposure is an input column from which hedge volume (vega based) is calculated. It is the vega exposure obtained when buying the hedge volume amount of the option.

8.2.15. Basket related risk measures

Basket split

Basket split is a way to display a risk measure for each component of the basket rather than displaying the total risk measure of the basket. Consider a risk measure μ and a basket derivative on a basket with N components. The total risk measure of the derivative equals the sum of the *basket split risk measures* $\mu_{\text{split},i}$ of its components

$$\mu = \sum_{i=1}^{N} \mu_{\text{split},i}.$$

Basket split is not a risk measure add-on. It automatically displayed in the portfolio window to applicable risk measures if the portfolio contains a basket and a basket derivative (to add the basket to the portfolio, (ctrl)-drag and drop the basket to the portfolio window).

The basket split risk measure for component i is calculated by multiplying the risk measure of the basket derivative with either the *total volume for component* i v_i, or the *weight of component* i w_i. The *total volume of component* i is the product of the *volume of component* i *in the basket* V_i, price multiplier of basket m_B and the multiplier of the contract m_i

$$v_i = V_i \cdot m_B \cdot m_i.$$

The *weight of component* i is the *total volume of the contract* times the theoretical price of the contract Π_i divided by the theoretical price of the basket Π_B

$$w_i = v_i \cdot \frac{\Pi_i}{\Pi_B}.$$

Basket split is calculated as follows for respective risk measure. Denote by $\mu_{\text{of position},B}$ the total risk measure of the basket derivative, e.g. $\Delta_{\text{of position},B}$ is the *delta of position* of the basket derivative.

delta of position	$\Delta_{\text{split},i} = v_i \cdot \Delta_{\text{ of position},B}$
swimming skew_delta of position	$\Delta_{\text{skew, split},i} = v_i \cdot \Delta_{\text{skew, of position},B}$
cash_delta of position	$\Delta_{\text{cash, split},i} = w_i \cdot \Delta_{\text{cash, of position},B}$
swimming skew_cash_ delta of position	$\Delta_{\text{skew, cash, split},i} = w_i \cdot \Delta_{\text{skew, cash, of position},B}$
gamma of position	$\Gamma_{\text{split},i} = v_i^2 \cdot \Gamma_{\text{ of position},B}$
vega_of position	$\mathcal{V}_{\text{split},i} = v_i \cdot \mathcal{V}_{\text{of position},B}$

8.2.16. Hedge volume

Entering a reference *hedge volume (risk measure)* for a reference contract will display the corresponding hedge volume (risk measure) for all other contracts in the same window. The hedge volume for a contract is the volume that will give the same total risk measure as buying the reference hedge volume of the reference contract. For the risk measure μ the hedge volume hv_μ of an option is calculated as

$$hv_\mu = \frac{\mu_{\text{ref}}}{\mu} hv_{\mu,\text{ref}}$$

where μ is the risk measure of the option, μ_{ref} is the risk measure of the reference contract and $hv_{\mu,\text{ref}}$ is reference hedge volume.

Example. Consider option A and option B in the same contract window. Delta of option A is 0.4 and delta of option B is 0.6. Entering hedge volume (delta based) 100 for option A will display hedge volume (delta base) $0.6 \cdot 100/0.4 = 150$ for option B.

Hedge volume (vega based)

The hedge volume (vega based) hv_{vega} is the volume which gives the vega $\mathcal{V}_{\text{expo}}$ that was entered in the vega exposure column. It is calculated as

$$hv_{\text{vega}} = \frac{1}{\mathcal{V}_{\text{expo}} \cdot m}$$

where m is the option multiplier.

8.2.17. Risk measures (price display)

Risk measures (price display) are displayed with the accuracy selected in the price display of the contract inspector. The custom number of decimals check box in the column picker must be unchecked for the price display to work.

8.3. Volatility manager API risk measures

The volatility manager API (VMAPI) provides a number of operations and corresponding risk columns.

8.3.1. Call/Put curvature risk

Some volatility models provide the parameters put curvature and call curvature. Call curvature risk μ_{cc} and put curvature risk μ_{pc} measure how much the theoretical price changes when the corresponding curvature parameter increases by one unit.

Wing model

On the put and call wing the wing model is given by a second degree polynomial (130) in the variable log-moneyness, x.

The call and put curvature risk measures are calculated as the partial derivative of the theoretical price with respect to the parameter. From the expression (130) we get on the put and call wings

$$\begin{aligned} \mu_{\text{cc}} &= \mathcal{V} \cdot x^2 \qquad \text{for } x > 0 \\ \mu_{\text{cp}} &= \mathcal{V} \cdot x^2 \qquad \text{for } x < 0 \end{aligned}$$

where $\mathcal{V}$ is vega. Put curvature risk is zero for $x > 0$ and call curvature risk is zero for $x < 0$.

Outside the put and call wings the risk measure is calculated analytically as $\mathcal{V} \cdot \dfrac{\partial \sigma}{\partial \text{cc}}$ and $\mathcal{V} \cdot \dfrac{\partial \sigma}{\partial \text{pc}}$ in accordance with the expressions for the volatility as a function of log moneyness in these regions, see section 6.1.6.

Bibliography

[C1] M. Curran, *Valuing Asian and Portfolio Options by Conditioning on the Geometric Mean Price*, Management Science **40** (1994), no. 12, 1705–1711.

[DDKZ] K. Demterfi, E. Derman, M. Kamal, and J. Zou, *More than you ever wanted to know about variance swaps*, Goldman Sachs Quantitative Strategies Research Notes. (1999).

[E1] J. Eriksson, *"Explicit pricing formulas for turbo warrants" in On the pricing equations of some path-dependent options*, Thesis (PhD). University of Uppsala., 2006.

[FPTV] W. Press, S. Teukolsky, W. Vetterling, and B. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd ed., Cambridge University Press, 1992.

[HKLW] P. Hagan, D. Kumar, A. Lesniewski, and D. Woodward, *Managing smile risk*, WILMOTT Magazine September (2002), 84–108.

[Ha] E. Haug, *The complete guide to option pricing formula*, 2nd ed., McGraw-Hill, 2007.

[Hu] J. Hull, *Options, futures, and other derivatives*, 5th ed., Prentice-Hall, 2003.

[KD] A. Konishi and R.E.Dattatreya, *Frontiers in Derivatives*, Irwin Professional Pub.r, 1997.

[L] E. Levy, *Exotic options, the state of the art*, 1st ed., International Thomson Business Press, 1997.

[PV] A. Pelsser and T. Vorst, *The binomial model and the Greeks*, The journal of derivatives **1** (1994), no. 3, 45–49.

[P] D.S.G. Pollock, *Smoothing with cubic splines*, University of London, Queen mary and Westfield College (1993).

[VN] M.H. Vellekoop and J.W. Nieuwenhuis, *Efficient pricing of derivatives on assets with discrete dividends*, Applied Mathematical Finance **13** (2006), no. 3, 265–284.

[JAG] J. Gatheral and J.W. Nieuwenhuis, *A parsimonous arbitrage-free implied volatiltiy parameterization with application to the valuation of volatility derivatives.*, Presentation at Global Derivatives and Risk Management, Madrid (2004).

[ZS] *Quasi-explicit calibration of Gatheral's SVI model.*, Zeliade White Papers, 2009.

[GJ] J. Gatheral and A. Jacquier, *Convergence of Heston to SVI*, Quantitative Finance **11** (2011), no. 8, 1129–1132.

[GJ2] ______, *Arbitrage-free SVI volatility surfaces.*, Quantitative Finance **14** (2014), no. 1, 59–71.

Appendix Contents

APPENDIX

EXOTIC OPTIONS AND PRICING MODELS IN ORC

Range options

Range, double barrier, or corridor options are barrier options with two barriers. There are two types: knock-in and knock-out. As with barrier options the knock-ins become valid only after one of the barriers is hit while the knock-outs become worthless (or pay a rebate) when one of the barriers is hit.

A combination of a knock-in call and a knock-out call with the same barriers and strike is equal to a vanilla call (if there are no rebates), since either the knock-in or the knock-out will be valid on the expiration day.

The two barriers are supposed to lie in either side of the underlying price when the option starts (otherwise the option is just a single barrier option). Also the strike is supposed to be between the barriers.

Since there is an additional barrier, knock-out corridor options are cheaper than each of the corresponding knock-out single barrier options. Similarly, knock-in options are more expensive than each of the corresponding knock-in single barrier options.

Analytical models

The value of an out-corridor call, occ, with barriers $U > L$ is

$$occ = Se^{-qt}f(b + \sigma^2/2) - Ke^{-rt}f(b - \sigma^2/2) \ ,$$

where

$$f(\upsilon) = \sum_{n=-N}^{N} \Gamma(\phi(a_1) - \phi(a_2)) - \left(\frac{U}{S}\right)^{(2\upsilon)/\sigma^2} \Gamma^{-1}(\phi(a_3) - \phi(a_4)), \ \Gamma = (U/L)^{(2n\upsilon)/\sigma^2},$$

$$a_1 = \frac{ln((SU^n)/(KL^n)) + \upsilon t}{\sigma\sqrt{t}} \ , \ a_2 = \frac{ln((SU^{2n-1})/(L^{2n})) + \upsilon t}{\sigma\sqrt{t}} \ ,$$

$$a_3 = \frac{ln((SU^{2n-1})/(L^{2n})) + \upsilon t}{\sigma\sqrt{t}} \ \text{and} \ a_4 = \frac{ln((SU^{2n-1})/(L^{2n})) + \upsilon t}{\sigma\sqrt{t}}$$

The value of the put, ocp is given by

$$ocp = Ke^{-rt}g(b - \sigma^2/2) - Se^{-qt}g(b + \sigma^2/2),$$

where

$$g(\upsilon) = \sum_{n=-N}^{N} \Gamma(\phi(a_2 + ln(U/L) + \phi(a_1))) - \left(\frac{U}{S}\right)^{(2\upsilon)/\sigma^2} \Gamma^{-1}(\phi(a_4) + ln(U/L) - \phi(a_3)).$$

The corresponding knock-in options are valued by using the fact that the sum of the values of a knock-in option and the corresponding knock-out option equals a plain vanilla (call or put).

For a calculated example of a Range option, see “Table 5: Theoretical values for different Barrier contracts and models.” on page 52.

The Orc also handles rebates for corridor options. As in the single barrier case, the rebate is assumed to be deferred for knock-in options and non-deferred for knock-out options. For the theoretical values, see reference [L] in the Bibliography section.

Foreign Equity options

Foreign equity options are options on a foreign equity with strike in the foreign currency. The contract is valued in the domestic currency.

Foreign Equity options in Orc

Because of the similarity with vanilla options, Foreign equity options are entered into the Orc as a (vanilla) option. The strike currency is thus the foreign currency and the contract currency is the domestic currency.

Theoretical values

The theoretical value expressed in the foreign currency is simply that of a vanilla option. The value is then transformed into domestic currency with the prevailing FX rate.

Composite options

Composite options are options where the strike is in another currency than the underlying spot. The same assumptions are used for these contracts as for Quantos.

Composite options in Orc

Composite options are valued in Orc either as a separate contract or as a vanilla option. In the latter case the user must adjust the volatility to take into account the volatility of the FX-rate and the correlation, see Eq. (1.1). If it is valued as a separate contract you enter the correlation ρ and exchange rate volatility, σ_j, as dynamic parameters. The strike currency should be set to the currency of the contract and you can only set the yield curve of the contract. The value of the base contract will be transformed into the strike currency with the prevailing FX-rate.

The same theoretical values will be obtained for the two alternatives. The vega, however, will be different, since only the spot volatility is assumed to vary in the first case. In the vanilla case the total volatility is altered.

Theoretical values

We assume the FX-rate and spot price varies according to Equations. The two-factor model can be transformed into a one factor model with a new volatility. The value is

$$\sigma' = \sqrt{\sigma_S^2 + \rho\sigma_S\sigma_J + \sigma_J^2} \tag{1.1}$$

The valuation is then done with the adjusted volatility and the Black & Scholes formula for European Composites and with a Binomial model for American Composites. For a calculated example, reference [KD] in the Bibliography section.

Compound options

Compound options are options on options. There are four different kinds: Call on call (cc), call on put (cp), put on call (pc), and put on put(pp). We assume that all involved options are European. For the compound, we denote the time to expiry by t, and the strike by X. The underlying option has time to expiry t_u and strike X_u. We assume that the underlying is still alive at the time of expiration of the contract, i.e. $t_u \geq t$. The theoretical values are then given by

$$cc = Se^{-q_u t_u}\Phi_2(z_1, y_1;\rho) - X_u e^{rt_u}\Phi_2(z_2, y_2;\rho) - Xe^{-rt}\Phi(y_2) \;,$$

$$pc = X_u e^{rt_u}\Phi_2(z_2, -y_2;-\rho) - Se^{-q_u t_u}\Phi_2(z_1, -y_1;-\rho) + Xe^{-rt}\Phi(-y_2) \;,$$

$$cp = X_u e^{rt_u}\Phi_2(-z_2, -y_2;\rho) - Se^{-q_u t_u}\Phi_2(-z_1, -y_1;\rho) - Xe^{-rt}\Phi(-y_2) \;,\; X \leq X_u \;, \tag{1.2}$$

$$pp = Se^{-q_u t_u}\Phi_2(-z_1, y_1; -\rho) - X_u e^{r t_u}\Phi_2(-z_2, y_2; -\rho) - Xe^{-rt}\Phi(y_2) \ , \ X \le X_u$$

where q_u is the dividend yield during the life of the underlying and

$$z_1 = d_1(S, X_u, t_u, q_u, \sigma) \ , \ z_2 = d_2(S, X_u, t_u, q_u, \sigma) \ ,$$

$$y_1 = d_1(S, S', t, q, \sigma), \text{ and } y_2 = d_2(S, S', t, q, \sigma).$$

q is the dividend yield during the time of the contract and S' is the stock price at expiry for which the option is at-the-money. Thus for options on calls it is the stock price which solves

$$c = c(S, X_u, t_u - t, r, q', \sigma) = X \ ,$$

and for options on puts S' solves

$$p = p(S, X_u, t_u - t, r, q', \sigma) = X \tag{1.3}$$

where $q' = (q_u t_u - qt)/(t_u - t)$ is the dividend yield during the time when only the underlying is alive. Note that the condition $X \le X_u$ in Eq. (1.2) implies that Eq. (1.3) has a solution. If $X > X_u$ then options on puts are somewhat degenerate since it is known beforehand whether the compound will end up in-the-money. The theoreticals in the Orc are

$$cp = 0 \text{ and } pp = Xe^{-rt} - p \text{ , when } X > X_u \ ,$$

where p is the value of the underlying put option.

Asian Quanto options

Asian Quantos are fixed exchange-rate foreign-equity options. The underlying is valued in foreign currency, and is the strike. The payoff, however, of these cash-settled options are in the domestic currency with a fixed exchange rate, $\bar{J}$. As for other Asian options, the payoff is also determined from an average of the price of the underlying on certain pre-specified dates, the sampling dates. Thus for ARO options, $aroqc$ and $aroqp$, the pay-offs are

$$payoff(aroc) = \bar{J}max(A - X, 0) = \bar{J}max\left(\frac{1}{N}\sum_{i=1}^{N} S(t_i) - X, 0\right) \text{ and}$$

$$payoff(arop) = \bar{J}max(X - A, 0) \ ,$$

respectively. The value of the option at expiry depends not only on the underlying spot price but the exchange rate. We model the exchange rate, J, with a geometric Brownian motion with a drift equal to the difference in interest rates, $r_f - r$ (cf. the interest rate parity theorem)

$$dJ = (r_f - r)Jdt + \sigma_J J dW_J \ , \tag{1.4}$$

according to risk neutral valuation. In many situations - such as when the underlying is a company that charge in foreign currency - it is not reasonable to assume that the exchange rate and underlying vary independently. This is modelled with a correlation, ρ, between the noise of the underlying, dW_S, and the noise of the exchange rate, dW_J.

Quantos in Orc

Quantos are made in Orc as a separate contract. You enter the correlation ρ and exchange rate volatility, σ_j, as dynamic parameters. The strike currency should be set to the foreign currency and you set the foreign yield curve as the strike yield curve. The domestic currency is set as the contract currency.

Theoretical values

The valuation is performed with two adjustments to the normal Asian valuation. First the fixed fx-rate, $\bar{J}$, is taken into account. Second, the forward values are adjusted for interest rate difference correlation and fx volatility according to

$$q' = q - r + r_f + \rho\sigma_j .$$

With additional notations

$$aroqc = \bar{J}e^{-rt}\left(\frac{1}{N}\sum_{i=1}^{N}\tilde{F}(t_i)\Phi_1(x_i) - X\Phi_1(y)\right) .$$

$$aroqc = \bar{J}e^{-rt}\left(\frac{1}{N}\sum_{i=1}^{N}\tilde{F}(t_i)\Phi_1(x_i) - X\Phi_1(y) - \frac{1}{N}\sum_{i=1}^{N}\tilde{F}(t_i) + X\right)$$

Installment warrants

Installment warrants are priced like European options with the Black&Scholes model. Because the holder of the warrant is entitled to dividends, dividends will add value to the theoretical price in the following way:

D_p = Sum of passed dividends set on the warrant.

D_f = Sum of future (but prior to expiration of warrant) dividends on the dividend contract.

The D_p and D_f are added to the theoretical Black&Scholes value to become the theoretical price of the Installment warrant.

Risk

Apart from Dividend risk, the risk is the same as for European options. The dividend risk differs in accordance with the specific dividend treatment for the installment warrants.

Forward Start options

Forwards start options are call or put options where the strike is determined at a later date, the start date. Thus in the time up to the start date the option is only sensible to changes in the volatility and the interest rate. This is thus a natural instrument for hedging volatility exposure. The strike is determined as a percentage, α, of the underlying price at the start date.

Forward Start options in Orc

The strike-percentage, α, is entered as the strike. Typically this number will be 100. The start date is entered as a dynamic parameter. It is only possible to value European Forward start options.

At start date, when the (vanilla) option starts, the holder can either close the position or enter a (vanilla) options position.

In the **Contract Inspector>Dynamic Parameters** view for the forward start option you can change the type of the option. When the strike has been set, the option should be changed from a forward start option to a standard option and the correct strike should be set for the contract. If this is done, then the position in the forward start option is kept as a position in a standard option.

Theoretical values

At start date, the option is easy to value with the Black & Scholes formula. Up to the start date, the option has no risk with respect to price movements in the underlying and is therefore valued with the appropriate pv-factor. The formulas are a simple adaption of Black & Scholes formulas Eq. (6.3) and Eq. (6.4).

$$c = \frac{100}{X'} e^{-rt}(\tilde{F}\Phi_1(d_1) - X'\Phi_1(d_2)),$$
$$p = \frac{100}{X'} e^{-rt}(X'\Phi_1(-d_2) - \tilde{F}\Phi_1(-d_1)),$$

where X' is the expected strike, $X' = \alpha(\tilde{F}'/100)$, $\tilde{F}'$, and $\tilde{F}$ are the theoretical forward prices at start date and expiry respectively, $d_1 = d_1(\tilde{F}, X', t-t', 0, \sigma)$ and $d_2 = d_{1-\sigma\sqrt{t-t'}}$. Note that if there is a term structure on the volatility surface, the forward volatility is used. The factor $100/X'$ is due to the fact that these contracts are traded in percentage of nominal. See also reference [Ha] in the Bibliography section.

Open ended options

An open ended option is an option without expiry date. For such a contract to make sense it has to be of American type. The holder of an open ended call/put with strike price X has the right to buy/sell the underlying asset at any time to the price X.

Pricing and modelling - No discrete dividends

In Orc the underlying asset is modelled by a geometric Brownian motion

$$S_t = se^{(r-q-\frac{1}{2}\sigma^2)+\sigma W_t}$$

where r, q and σ is the short rate, dividend yield and volatility. In the modelling of the option price both discrete dividends and dividend yield are allowed.

Open ended put options without discrete dividends

The basic pricing equation for the put option price $P(S)$ is the time-independent Black-Scholes equation

$$\begin{aligned}(r-q)s\frac{dP}{ds}(s) + \frac{1}{2}\sigma^2 s^2 \frac{d^2P}{ds^2}(s) - rP(s) &= 0 \text{ for } s > s^*,\\ P(s^*) &= (X - s^*),\\ \frac{dP}{dS}(s^*) &= -1.\end{aligned}$$

Here the number s* is the so-called optimal exercise boundary dividing the continuation region $(s > s)$ and the stopping region ($s \le s^*$). The option should be held whenever $S_t > s^*$ and exercised at the first instant $S_t \le s^*$. The above equation can be solved explicitly and the solution is given by

$$P(s) = \begin{cases} (X - s) & \text{if } s \le s^*;\\ (X - s^*)\left(\frac{s}{s^*}\right)^\gamma & \text{if } s > s^*.\end{cases}$$

The numbers $s^*$and γ are given by

$$s^* = \frac{X\gamma}{\gamma - 1}$$

$$\gamma = A - \sqrt{A^2 + 2r/\sigma^2}$$

where

$$A = -(r - q - \tfrac{1}{2}\sigma^2)/\sigma^2.$$

Open ended call options without discrete dividends

The basic equation for the call option is given by

$$\begin{aligned}(r-q)s\frac{dC}{ds}(s) + \frac{1}{2}\sigma^2 s^2 \frac{d^2C}{ds^2}(s) - rC(s) &= 0 \text{ for } 0 < s < s^*,\\ C(s^*) &= (s^* - X),\\ \frac{dC}{dS}(s) &= 1.\end{aligned}$$

In this case the solution will be

$$C(s) = \begin{cases} (s - X) & \text{if } s \geq s^*;\\ (s^* - X)\left(\frac{s}{s^*}\right)^\gamma & \text{if } 0 < s < s^*.\end{cases}$$

The numbers s and are in the call case given by

$$s^* = \frac{X\gamma}{\gamma - 1}$$

$$\gamma = A + \sqrt{A^2 + 2r/\sigma^2}$$

where

$$A = -(r - q - \tfrac{1}{2}\sigma^2)/\sigma^2.$$

A note about γ, $s^*$and χ

In the put-case $\gamma < 0$ to insure a bounded option price as $s \to \infty$. This implies that $s^* < \chi$ and the price as described above is well-defined.

In the call case we need to consider a special case separately, namely the case $q = 0$. For $q = 0$ we have $\gamma = 1$ and the above expression for the call-price breaks down. A separate analysis of the case $q = 0$ reveals that the only reasonable price is

$$C(s) = s \text{ when } q = 0.$$

Note that the price in this case does not completely solve the pricing equation since no optimal exercise boundary s^* exists. This is consistent with the theory saying that an American call option without dividends should be exercised as late as possible (and in this case never). Intuitively one can understand the price from the observation that in the long run the strike price will be negligible compared to the stock price and the option has to price at the current underlying level.

In the case of $q = 0$ we have $\gamma > 1$ and $s^* > \chi$ and the price given by the formula is well-defined.

Pricing and modelling in the presence of discrete dividends

When there are discrete dividends present no explicit pricing formula is known. Instead the pricing is done by combining the above formulas with a binomial tree of Cox-Ross-Rubinstein type. After the final ex-dividend date the option is just an ordinary open ended put/call without discrete dividends and can be priced as such. Thus by generating a binomial spot tree we can compute the values of the options immediately after the final ex-dividend date (i.e. on the next day) and use these values as input into a binomial option tree valuing the option at present date. The option tree has final date = final ex-dividend date + 1 day.

Example Call option, Strike $X = 91$, Spot $S = 100$, 1 discrete dividend, size $D = 10$, ex-dividend date + 1 day, short rate $r = 0$, dividend yield $q = 0$, volatility = 20%. The value of exercising the option today is 100 - 91 = 11. The value in waiting until after the dividend (e.g. until tomorrow) is 100-10 = 90 which is the theoretical value of the option tomorrow (considering that S does not move). The rational thing to do is to not exercise the option now but keeping it until after the dividend and the value of the option today is 90.

Call options does in general theoretically price close to S - PV(future dividends) unless the present value of the future dividends is larger than the strike.

APPENDIX

CERTIFICATES

A certificate (or zero-strike call) is a cash-settled contract where the holder will receive the equivalent of a percentage of an underlying (often an equity basket) at a future date. The holder thus benefits from the performance of the basket without actually owning it. The buyer pays at the purchase, so it differs from forwards/futures. The holder can get some part, α of the dividends during the lifetime of the derivative.

Certificates in Orc

In the OTC creator for certificates, you specify the redemption, R, which is the percentage of the basket value the holder obtains. The estimated net dividends, $1-\alpha$, is the fraction of the future dividends that you estimate the owner of the basket (and writer of the Certificate) will obtain. The dividends paid to the holder, β, is the percentage of the past dividends that the holder of the certificate is entitled to. In the cash payback entry, you can specify a value that has already been paid out to the holder of the certificate due to some extraordinary circumstance, such as if a spot has disappeared from the basket due to a merger.

Note that past dividends are taken from the contract itself, whereas future dividends are taken from the dividend contract.

Valuation of Certificates

The valuation is done according to formula Eq. , with correction for dividends already paid out to the holder of the certificate.

$$Cer = e^{-rt}FR + \alpha\sum_{i=1}^{n} D_i e^{-rb_i} + \beta D + P,$$

where D is the sum of the dividends already paid and P, an additional value due to corporate actions that will be paid to the holder.

INTEREST RATE DERIVATIVES

Bond futures

A Bond future contract is defined as follows in the Orc:

- Years = 3 or 10
- Interest rate = This is different for different futures, but it is important to set. A typical value is 6.
- Price display = 6 decimals
- Ytm display = 2 decimals
- IsYTMQuoted = YES
- IsMatchingDescending = NO
- YTMDayConvention = Actual/365.
- Multiplier = 10.000.
- Base contract = The contract itself.

These contracts are quoted in yield but margined in price. The formula for translating yield to price is done with a specified synthetic bond:

$$price(yield) = r\frac{(1-v)}{1-q} + v$$

where

$$v = \left(\frac{n}{n+1+q}\right)^{tn}$$

The interest rate (r) is typically 6 and set on the bond future.

t is 3 or 10 (years) and also set on the future.

n is the number of coupons per year and is set via the ytm rate convention.

OTHER MODELS

Mirror own model

The Mirror own model, is a model specially designed for generating quotes that are the same as another contract. The theoretical bid price for a contract with the Mirror own model is simply the quote bid of the base contract. The theoretical ask price for a contract with the Mirror own model is simply the quote ask of the base contract.

Mirror market model

The Mirror market model, is a model specially designed for generating theoretical prices that are the same as another contract. The theoretical bid price for a contract with the Mirror market model is the bid of the base contract (adjusted for differences in price multiplier and currency) + the bid offset of the contract. The theoretical ask price for a contract with the Mirror market model is the ask of the base contract (adjusted for differences in price multiplier and currency) + the ask offset of the contract.

External model

The model External is designed to make it possible to feed external prices and risk values into Orc Trader, for instance by using EXCEL or the Orc Protocol. The model has 6 dynamical parameters:

- External theoretical price
- External delta
- External gamma
- External vega
- External theta
- External rho

These are the values that will be displayed in the columns Theoretical price, Delta, Gamma. Vega, Theta and Rho in the Orc Trader.

Note The model does not support sanity checks on values, i.e. it is possible to feed the model with Delta = 2.0 , Gamma = 3 .0 and Theoretical price of a call = -1.23 and these values will be used in risk calculations.

Fed fund binary

The Fed binary model is designed for binary options of put and call kind, and is applicable for European executions of the asset types Interest Rate futures/Bonds. The base contract should be a future (quoted 0-100) and the strike should be specified in 100-rate terms.

Volatility markup

The model Volatility markup allows for a user to specify a volatility offset that will be used in the theoretical calculations. Theoretical values are calculated based on the volatility used for the contract + offset value. The implied volatility is calculated based on offset value = 0. Thus pricing with implied volatility would result in pricing with an actual volatility implied vol. + offset.

Example If implied volatility is 17% and the offset is 3% then the theoretical value of the option will be caluclated using 20%. The price will thus not be equal to the market price.

If the price is calculated using contract based volatility and the volatility is set to 17% and the offset is set to 3% the contract will be priced using 20% volatility. To obtain the market price the volatility have to be set to 14%, then the contract will be priced using 14% + 3% = 17%.

Volatility quoted

The model Volatility quoted is for options quoted in volatility.

The theoretical valuation, i.e. conversion between volatility and price is done using the Black-Scholes model in the European case and the Barone-Adesi, Whaley and Macmillan model (*Journal of Finance, Vol XLII, No 2, June 1997*) in the American case.

The pricing model in Orc has three parameters:

- **Tolerance** - Specifies the tolerance for ending the iterations in the Barone-Adesi, Whaley and Macmillan model. The stopping criteria is *abs(old value - new value) < tolerance*.
  Default value 0.000001.
- **Accuracy** - Specifies how many iterations the model should allow before forcing termination. If the tolerance is not reached within the number of iterations specified, the model falls back on the Black-Scholes pricer instead. Default value 10000.
- **Premium quoted** - Specifies if the value should be displayed in volatility terms or premium terms in the trading window. Default is in volatility terms.

When trading a contract using this model, all theoretical values, invested and market value are displayed in premium in the portfolio.

Dividend future

The Dividend future model calculates the theoretical price for dividend futures, and sums all the dividends for a contract, from the dynamic parameter *start date* to the expiration date, to calculate the final settlement price.

The final settlement price is then discounted to the present value. If the currency of the dividend contract differs from the currency of the dividend future, the yield curve corresponding to the currency of the dividend contract is used.

APPENDIX

PARAMETERS IN ORC AND THEIR USE IN MODELS

The base contract

In the Orc, the user has the opportunity to choose on which contract the calculations should be based, the base contract. In each option pricing model, the value of an underlying appears as a parameter, denoted S or F. The value of the base contract is the raw input for the underlying value in the pricing model. Occasionally, it is not used directly, but first transformed, in order to make sense. This is different for different models and different types of base contracts. We distinguish two types of models: Black & Scholes type and Black -76 type. In the first group, it is assumed that the price of the underlying is for immediate delivery (spot price). For Black -76 type of models it is assumed that the price of the underlying is a future price.

The Black & Scholes type models are Black & Scholes, Bjerksund, Barone-Adesi, Geske, Binomial, Binomial (relative), and all the exotic models. Before the asset spot price parameter is plugged into the theoretical models, we need to calculate it theoretically from the base contract price. We consider two cases:

- The base contract is the underlying spot, index, or basket contract. This is the default base contract for Black & Scholes type models, so if no base contract has been specifically defined or set, this is the contract that will be used.
- The base contract is a forward or a futures contract. Assume also that only the (base) spot contract pays (discrete) dividends, D_i. Then the spot price, S, used is calculated from the forward market price F (the base contract) as in Eq. .

where the sum is taken over all dividends between the calculation date and the expiry of the base contract.

The Black -76 type models are Black -76, Black -76 (Clean), and Eurooption. The price of the base contract B is used directly without any calculations. This model is only applicable when the base contract is a futures or forward contract.

When to use which type of model? The idea is to use the Black & Scholes type models to price options with spot delivery or which are cash-settled. The Black -76 type models should be used when the underlying asset of the options is a forward or a futures contract with the same, or with longer time to expiry than the options, and where the options give physical delivery of the forward or futures contract.

If (none) default is selected, the spot contract for the underlying is used as base contract if the model is of Black & Scholes type and a future or forward with the same expiry date if the model is Black -76 type.

The Dividend contract

The user can define from which contract the dividends are to be taken. If (none) default is selected, the spot contract for the underlying of the base contract is selected.

The Strike price and strike currency

The strike price and strike currency can be entered for options and convertibles. The strike currency shows which currency the strike is in.

The Expiration date

The expiration date is a static component of a forward, futures, option, or convertible. The time to expiry t is calculated as the number of days between the day of calculation (which is variable) and the expiry date, divided by 365 (or as #trading/256).

The Calculation date

The calculation date is the date when all calculations are supposed to take place. As default it is the date of today (the date set on the client), but it can be changed in simulations.

Dividends

In all models except Binomial (relative), dividends are treated as fixed. for details on how dividends are treated in the Binomial (relative) model.

For each dividend an ex-dividend date, a dividend date, a dividend, and a weight can be entered. The ex-dividend date is the first date the underlying asset is traded without the right to the dividend. The dividend date is the date the dividend is paid. The dividend is simply the value in the currency of the dividend contract. The weight is a multiplicative factor, normally 100%, but can be set to something else if the actual money obtained differs from the value of the dividend.

For derivative on baskets or basket indices, note that the Orc automatically calculates the dividends on the basket from the dividends set on the components of the basket.

Underlying rate

The underlying rate q normally means a yield which is continuously paid out by the underlying asset. It should be entered as a continuously compounded interest rate. The underlying rate can either be regarded as a dividend yield, which is paid during the lifetime of the derivative contract, or it can be used to define a cost-of-carry denoted by b. The relation between continuously compounded risk free rate, r, underlying rate q, and b is $b = r - q$. Normally, $b = r$ which is the same as $q = 0$.

If an underlying rate curve is used, the (interpolated) value from the curve is used as underlying rate.

Volatility

The standard deviation of the asset's returns, or the volatility, as it is called when annualized, is a measure of possible changes in the market price of the asset in the future. An asset with a high volatility can be expected to have large changes in its future price. An asset with low volatility is less risky, since its future price will be more determined by the present spot price. If the volatility rises, prices of options on the asset will also go up.

Most parameters in the Black & Scholes framework are easily estimated. The underlying asset price, the exercise price, and the time to expiry are observable parameters. The interest rate is somewhat more involved. Usually the yield to maturity for a zero-coupon treasury bill with a time to maturity equal to the time to expiry of the option can be used. Since the financing rate does not have such a strong impact on the option price in the Black & Scholes framework - at least not for options with relatively short time to expiry and in countries where financing rates are relatively low - the most involved parameter in the model is the volatility of the asset price.

For the Black & Scholes framework to give any reasonable result, the volatility has to be estimated. The main problem is that we have to estimate a future value, i.e. the standard deviation from the day of the calculation until the expiry date. Note that the estimated volatility is assumed to be constant during time to expiry.

The volatility can also be seen as a price indicator of an option. This interpretation is natural also in the light of that the volatility seen in the market almost always varies for different strikes and different time to maturities (for the same underlying). In Orc, the Volatility manager is used for analyzing this skew and term structure of the volatility surface.

Volatility surface

Refer to “Volatility models specifics” in the Volatility management chapter of the Orc Trader user manual.

Volatility days

It has been claimed that market volatility only occurs on trading days. If this is true, a 256 day year - the convention for the amount of trading days in a year - must be used.

In Orc you can choose which way of calculating volatility you would like to use by either selecting Actual or Trading in the Volatility days pop-up menu in the inspector.

If you have chosen Trading, then the volatility that will be used in all models is $\tilde{\sigma} = \sigma\sqrt{t_{\sigma}/t}$, where t_{σ} is defined as the number of trading days in the period divided by 256 and σ is the volatility entered in the Contract Inspector in the Orc client.

The difference can be significant close to expiry. If expiry is on a Monday, then on the Friday before there will be three actual days to expiry, but only one trading day to expiry. In this case a volatility of 40 with trading days convention corresponds to a volatility of $40 \cdot \sqrt{\left(\frac{1}{256}\right)/\left(\frac{3}{365}\right)} = 27{,}58$ with actual days.

Rate type	Present value
Continuous	e^{-rt}
Annual compounding	$1/(1+r)_t$
Semiannual compounding	$1/(1+r/2)_{2t}$
Quarterly compounding	$1/(1+r/4)_{4t}$
Monthly compounding	$1(1+r/12)_{12t}$
Straight	1/(1+rt)
Discount	1 - rt
Present value factor	r

Table 1: Present value of 1 paid out after t years at the rate for different rate types.

Financing rate

The financing rate has to be guaranteed to be risk-free, i.e. and investor has to know for sure how big the return from the investment will be at the end of the investment period. Usually a treasury bill interest rate is used, the time to maturity of which more or less coincides with that of the time to expiry of the option. It has to be understood, however, that yield to maturities for T-bills vary, which violates the assumption in the basic Black & Scholes model.

Before the financing rate is used in Orc calculation, it is converted into a continuously compounded interest rate, see below, and then adjusted by the formula $\tilde{r} = (rt_r)/t$, where t_r is defined as the number of days in the period divided by the number of days in a year, according to the day count convention chosen, when defining the yield curve.

Rate model

The yield curve is defined as a number of ordered pairs $\{(d_i, r_i)\}^n_{i=1}$, where d_i denotes a number of days $(d_1 < d_2 < \ldots < d_n)$ (from the calculation date), r_i the corresponding interest rate and n is the number of points on the yield curve defined. Assume that d is the actual number of days to expiry. The financing rate is picked from the yield curve by a linear interpolation. First, if $n = 0$, i.e. no yield curve has been defined, we set $r = 7$%. If $d < d_1$ then we set $r = r_1$ and if $d > d_n$ we set $r = r_n$. Now, if $d_i \le d \le d_{i+1}$ for some $1 \le i \le n$, then we linearly interpolate r from the yield curve in the following way:

$$r = r_i + \frac{r_{i+1} - r_i}{d_{i+1} - d_i} \cdot (d - d_i) \quad (5.5)$$

Note that this is for the “actual financing rate used” as seen in the corresponding column in the Orc Trading Window and in the Contract Inspector and is not necessary valid when computing interest rates used for PV calculations.

Calculation of present value factors and interest rates

Given the interest curve structure above the interest rate used for PV calculations and option valuations are computed by first converting the interest rate entries to continuous rates. Hence if $d_i \le d \le d_{i+1}$ we first convert r_i and r_{i+1} to continuous rates r'_i and r'_{i+1} and then compute the interest rate r as

$$r = r'_i + \frac{r'_{i+1} - r'_i}{t_{i+1} - t_i}(t - t_i)$$

Here t_{i+1} and t_i are the times in years corresponding to the number of days d_{i+1} and d_i with the rate day conventions set on the corresponding yield curve entries. The time t is the time in years for the periond of time d, computed using the day convention for the yield curve entry closest before d days.

Before r_i and r_{i+1} are converted to continuous rates any yield offset is added.

The PV corresponding to t and the interpolated continuous interest rate is computed as

$$PV = e^{-rt}$$

In the computation of PV for a derivative contract, the time period d is typically the number of days between the settlement day of the contract and and the final settlement day of the contract, except for forward contracts and future settled contracts, where the start date and end date are both set to the final settlement date.

Extrapolation of interest rates

Extrapolation of interest rates is handled as follows: The (start or end) entry is converted to a continuous rate. This rate is applied together with the (extrapolating) time to get the present value factor. For example, say it is 1 day to expiry and the first yield curve entry has 5% straight act/360 for 30 days.

The extrapolated present value factor (pv) is calculated in the following way:

$$pv = \left(\frac{1}{\left(1 + 0{,}05 \times \frac{30}{360}\right)}\right)^{\left(\frac{1}{30}\right)} = 0{,}99986141....$$

Rate type

All calculations in Orc are internally done by using a so called continuously compounded interest rate model. However, there are several other rate types in the Orc. A list including how present values are calculated in the Orc is found in Table 1.

Day count convention

Market conventions for the number of days in a period and the number of days in a year differ by type of market and country. The notation typically used to denote a day count convention is Number of days in a month/Number of days in a year. In practice, there are six day count conventions:

1 Actual/Actual (in period)

2 Actual/365

3 Actual/365 (366 in a leap year)

4 Actual/360

5 30/360

6 30E/360

In calculations of the actual number of days, only one of the two bracketing dates in question is included. For example, the actual number of days between August 20 and August 24 is four days. The last two day conventions, 30/360 and 30E/360, require some explanation.

30/360 day count: the number of days between two dates assumes 30-day months, according to the following rules for the number of days between the dates $d_1/m_1/y_1$ and $d_2/m_2/y_2$:

$$d_c = 360 \cdot (y_2 - y_1) + 30 \cdot (m_2 - m_1) + d_2^* - d_1^*,$$

where $d_1^* = min(30, d_1)$ and

$$d_2 = \begin{cases} 30, \text{ if } d_2=31 \text{ and } (d_1=30 \text{ or } d_1=31) \\ d_2 \text{ otherwise} \end{cases}$$

For example, there are 29 days between May 1 and May 30 and 30 days between May 1 and May 31.

30E/360 day count: The number of days between two dates assumes 30-day month, according to the following rules for the number of days between the dates $d_1/m_1/y_1$ and $d_2/m_2/y_2$: if d_1 is 31, change d_1 to 30, if d_2 is 31, change d_2 to 30, then the number of days between the two dates is:

$$(y_2 - y_1) \cdot 360 + (m_2 - m_1) \cdot 30 + d_2 - d_1.$$

For example, there are 29 days between May 1 and May 30 as well as 29 days between May 1 and May 31.

Yield offset

The user can specify an offset to the yield curve that the contract should be valued with.

Calendars, Settlement days, and Final settlement days

In Orc, settlement dates are calculated from the date of the calculation and the settlement days entered into the system. Final settlement dates are calculated for contracts with an expiry date, and use the final settlement days entered for that contract. The settlement dates are calculated by using the trading and settlement calendars. If there are three final settlement days, the final settlement date is the first settlement date (according to the settlement calendar) after or coinciding with three days (according to the trading calendar) after the expiration date.

Financial costs in Orc are always calculated by using the time between settlement dates.

EEP

EEP is short for Early Exercise Premium. It is added to the theoretical spot price when pricing a call option, and subtracted from theoretical spot price when pricing a put option.

Multiplier and face value

The multiplier adjusts for the fact that although the market price for the contract can refer to only one contract, the buyer may actually obtain another number. Thus the multiplier affects the value in portfolios but not in trading pages.

Bond products are frequently quoted in percentage. The buyer, however obtains the face value. The relation is

$$Face\ value = 100 \cdot multiplier.$$

Price multiplier

The price multiplier is used to adjust the theoretical price if the derivative contract gives the right to buy or sell more or less than one of the underlying asset. It can for instance be used to price a warrant which only gives the right to a fraction of a stock. The price multiplier is simply the factor with which the theoretical price is multiplied after the usual calculation.

Settlement

The user can set whether the contract is cash-settled or delivery-settled. This affects the behavior at expiration. Also, the theoretical forward prices are calculated a little differently. In addition, expired cash-settled contracts have no risk, while delivered contracts are assumed to have a delta, if there is a simulation.

Premium Payment

This seeting allows to set for a contract as whether the premium is paid at trade adjusted for settlement days or at expiration. In calculations, the effect will be different present value adjustment of the premium.

Offset

For non-interest rate contracts, the Orc supports five different offset techniques.

- No offset
- Simple
- Synthetic spot
- Forward adjustment
- No cost of carry

The first alternative makes no offset adjustments at all.

When using Simple offset to spots, forwards, or futures, the theoretical price is simply the sum of the base price and the offset. For options, the Orc calculates a synthetic dividend yield, which when combined with the financing rate results in a theoretical forward price which is the sum of the base price and the offset. The option is then valued according to the base price and the synthetic yield. No dividends are used, $S = e^{-bt}F$ and $F = B + offset$.

Depending on if the base contract is the spot or the forward, different yields are used to relate the spot and the forward.

If $B = F$ then the spot and the forward are related by $S = Fe^{-(r-y)t}$.

If on the other hand $B = S$ then a synthetic yield y^* is used to make sure that the relation $S = Fe^{-(r-y)t}$ still holds with $F = B + offset$, i.e. that

$y^* = ln\left(\frac{B}{B + offset}\right) / t + r$, which is the same as $y^* = r - \left(ln\left(\frac{F}{S}\right)\right) / t$.

The synthetic spot offset alternative, a theoretical spot price is calculated as the sum of the base price and the base offset of the contract. The theoretical forward price is then calculated using this theoretical spot price, the financing rate, underlying rate and dividends (unless e.g. underlying rate mode is set to use no dividends).

The forward adjustment is a correction to the theoretical forward price, without changing the theoretical spot price. When the offset is non-zero, a synthetic yield is calculated to compensate for the changed theoretical forward price with the unchanged theoretical spot price. This yield as well as dividends are then used in future/forward/option calculations.

Note that if the offset is zero, forward adjustment offset mode results in the value as if offset mode were 'Not used'.

$$S = e^{-bt_B}B + \sum_{i=1}^{n_B} D_i e^{-b_i t_i}$$

$$F = e^{bt}\left(S - \sum_{i=1}^{n} D_i e^{-b_i t_i}\right) + offset$$

The offset logic "No cost of carry" can be used when the underlying is a future. In valuation q will be set to be equal to r. $S = B + offset$, $F = B + offset$ will be used as spot price in option / future valuation formulas. No dividends are applied.

Example We are trading 990423 OMX-futures on 990219. We base the futures on the 990326 future. The interest rate is 3%, and dividend yield is 5%. The March future is traded at 701, which results (with no offsets) in a theoretical value of 699.90.

If you believe that the April future should be valued 120 points below the March future, you apply a simple offset of -1.20. As a result the theoretical value of the April futures is 699.80.

With the simple offset, the theoretical price will move in parallel with the base contract. This is reflected in that the delta in a trading page for the April future will be:

$$(1 + 0{,}03(35/365))e^{-0{,}05 \cdot (35/365)} = 0{,}9981$$

Alternatively you might want to value the future from a (synthetic) spot price and the spot value should be 40 points below March future. Choose synthetic spot offset for the April future, and enter a offset -0.40. Then the theoretical value of the April future, Π is

$$\Pi = (701 - 0{,}40) \cdot (1 + 0{,}03 \cdot 63/365)e^{-(0{,}05 \cdot 63/365)} = 698{,}18 .$$

With the synthetic spot offset, the delta in the trading page will be:

$$(1 + 0{,}03(63/365))e^{-0{,}05 \cdot (63/365)} = 0{,}9965$$

If the non offset theoretical value of 699.90 is not correct, perhaps because of a missing dividend, you can adjust this with a forward adjustment offset. If you enter -0.60, the theoretical value becomes 699.30.

With the forward adjustment offset logic the delta will be the same as for "no offset":

$$(1 + 0{,}03(63/365))e^{-0{,}05 \cdot (63/365)} = 0{,}9965$$

For interest rate contracts the Orc supports three different offset types

- No offset
- Price offset
- YTM offset

Price offset allows for the user to add an offset to the clean base price to obtain a clean price for the contract. This is similar to simple offset for other asset types.

YTM offset allows for the user to add an offset to the (market or implied) YTM of the base contract, and then value the contract from the adjusted YTM.

YTM

The *yield to maturity* is a market convention for quoting bonds. It can also be used as a price indicator of the risk rate for a convertible.

Accrued days

Bonds are usually quoted in *clean price*, which means that the price you see in the market does not include accrued interest on the next coupon to be paid out to the holder of the bond. This means that if you are buying a bond, you will actually pay the quoted price plus the accrued interest on the coupon. There are different day conventions for calculating accrued interest rate. See "Day count convention"and "Rate type".

Spread

If you would like to send prices with a wider spread than that obtained by the difference between the theoretical bid and ask prices, it is possible to set a spread for a contract, either by using a spread table or manually enter a spread. A manually entered spread forces the Orc to never suggest prices with a difference between ask and bid prices less than the spread.

See also spread tables and spread bias.

Spread bias

In addition the spread bias governs how the spread should be fit in relation to the average of the theoretical bid and ask prices.

Example If the theoretical bid and ask are 210 and 220, the spread is 40, and the market accepts integer prices (at this price range), the following prices are suggested for different spread bias.

Spread bias	Suggested bid	Suggested ask
0	180	220
42	187	227
70	195	235
100	210	250

Note that the Orc does not suggest prices better than theoretical bid and ask.

Spread tables

A spread table allows for different spreads at different price levels.

Strike conversion

Strike conversion is set in the volatility model . Using the Wing model means that the there should be no conversion of the strike values in the volatility calculation. The Wing Eurofuture model will will make a conversion from X to 100 - X (and a similar conversion for forward values) in the volatility calculation (see “Volatility surface” on page 14). This is to make it more consistent with the Euro option model which assumes that 100-F is lognormal.

THEORETICAL VALUES FOR THE TRADING PAGE

Theoretical prices

Theoretical price

The theoretical price column displays the theoretical price of the contract which is determined by the parameters for the contract in the database. In the theoretical price column the theoretical price Π of the contract times the price multiplier is displayed. It is presented in contract currency.

Theoretical bid and ask

There is a special functionality for generating quotes in Orc, which use the so called Theoretical bid and ask prices. Normally (when the model is not Mirror own or market), they are generated as the theoretical price with two exceptions

- Different base prices are used.
- Theoretical bid/ask offsets are added.

A bought call is hedged on the bid side of the base contract, but a bought put hedged on the ask side of the base contract. For this reason the following logic is used

- Theoretical bid for Call options is based on the bid price of the theoretical bid/ask basis
- Theoretical ask for Call options is based on the ask price of the theoretical bid/ask basis
- Theoretical bid for Put options is based on the ask price of the theoretical bid/ask basis
- Theoretical ask for Put options is based on the bid price of the theoretical bid/ask basis

Other (non-composite) contracts, such as spots, forwards, and futures have a positive delta and are treated as calls.

The offsets that are added to the Theoretical bid/ask prices are the Bid/Ask vol. offset, the Bid/Ask rate offset, and the Bid/Ask price offset.

The Mirror market model, is a model specially designed for generating theoretical prices that are the same as another contract. The theoretical bid price for a contract with the Mirror market model is the bid of the base contract (adjusted for differences in price multiplier and currency) + the bid offset of the contract. The theoretical ask price for a contract with the Mirror market model is the ask of the base contract (adjusted for differences in price multiplier and currency) + the ask offset of the contract.

The Mirror own model, is a model specially designed for generating quotes that are the same as another contract. The theoretical bid price for a contract with the Mirror own model is simply the quote bid of the base contract. The theoretical ask price for a contract with the Mirror own model is simply the quote ask of the base contract.

For combination and composite contracts the situation is a little bit more complicated since one does not know a priori on which side the combination is to be hedged. In Orc, there are two different models which differs only in the way theoretical bid/ask prices are generated, Wide spread and Tight spread. Note that for combinations, the offsets of the combinations are used. The component offsets are not used. This allows for using different offsets for the combination and the individual components.

The Wide spread model uses the above logic for each component individually, with the logic reversed if the component is sold.

The Tight spread model first calculates the delta, vega, and rho for the entire composite. Then all components are treated the same. If delta is positive, each component is treated as a call (with respect to which base price that is to be used), and if the delta is negative each component is treated as a put (with respect to which base price that is to be used). A similar logic is used with respect to vega and rho.

The Tight spread model is designed for use when calculating theoretical bid and theoretical ask prices for options combinations when:

1 Both sides of the underlying price are being considered in the calculation (for example, Market Bid/Ask).

2 All the components are being hedged in exactly the same underlying (it should generally not be used for calendar spreads).

The reason is that the Wide spread model has some limitations as shown in the following example scenario:

Example:

100 / 101 Call Spread

Underlying Price: 100.20 - 100.21

100 / 101 call spread theoretical bid price: 0.30 (Wide spread)

Component Price for 100C: 0.45 (based on underlying bid)

Component Price for 101C: 0.15 (based on underlying ask)

100 / 101 Call Spread

Underlying Price: 100.20 - 100.24

100 / 101 Call Spread theoretical bid price: 0.29 (Wide spread)

Component Price for 100C: 0.45 (based on underlying bid)

Component Price for 101C: 0.16 (based on underlying ask)

In this example, the theoretical bid of the call spread is lowered when only underlying ask has moved. This is because Wide spread model considers all the components separately. The Tight spread model uses the net delta of the combination and therefore there would be no change in the theoretical bid of this combination with a move up in the underlying ask.

The Arbitrage model has a constant theoretical bid and ask. These can be entered as parameters “Bid limit” and “Ask limit” respectively.

Banking

The stock dividend is an integral factor in the cost of carry equation for conversions and reversals. The long conversion has a negative interest carry but a positive dividend flow. The opposite is true for the reversal. Therefore, the conversion and reversal formulas and values must be adjusted as dividends change. An increase or decrease in the dividend as well as a change in the ex-dividend date has a major impact on the valuation of a conversion or reversal. This can also have dramatic impact on box pricing.

When we do the reversal, we are buying the right to receive interest. However, we are also committing to the obligation to pay the dividend. When we do the long conversion, we are committing to pay interest but gain the dividend.

The conversion value or Banking is for equity contracts defined as $b = X + c - p - S$. I.e. put-call parity is assumed to hold. Using Eqs. Eq. and Eq. (5.3), we get

$$b = X + c - p - \left(e^{-rt}F + \sum_{i=1}^{n} D_i e^{-rb_i}\right) = X + e^{-rt}(F - X) - e^{-rt}F - \sum_{i=1}^{n} D_i e^{-rb_i} \qquad (6.6)$$

$$= X(1 - e^{-rt}) - \sum_{i=1}^{n} D_i e^{-rb_i}.$$

A well known rule of thumb states that an in-the-money option becomes a candidate for early exercise when the price of the corresponding out-of-the-money option (with the same strike price) is less than the banking cost, b.

Theoretical Intrinsic Value

The theoretical intrinsic value for an option is calculated as:

$$max(S-X,\ 0)$$

for puts and

$$max(X-S,\ 0)$$

for calls.

For the Euro option model, the theoretical intrinsic value is

$$max(\textit{offset adjusted base price - X, 0})$$

for puts and

$$max(X-\textit{offset adjusted base price, 0})$$

for calls.

Risk measures

Delta and option delta

The Delta (Δ) is the partial derivative of the theoretical price of the option with respect to the price of the underlying asset (or the base contract, to be exact), i.e. $\Delta = (\partial\Pi)/(\partial S)$. The risk measure tells you how sensitive the price of the contract is with respect to changes in the price of the underlying asset. This can be interpreted as the approximate effect on the theoretical price when the price of the underlying base contract increases, or decreases by one price unit. If the underlying asset price moves one price unit up then the new theoretical price roughly equals the old theoretical price plus the Delta. If the underlying asset price moves one price unit down, then the new theoretical roughly equals the old theoretical price less the Delta. The following holds for the delta of a call and put respectively

$$0 \le \Delta_c \le e^{-qt} \text{ and } -e^{-qt} \le \Delta_p \le 0 .$$

When using the Black & Scholes model the equation for the delta of the European call is

$\Delta_c = (\partial c)/(\partial S) = e^{-qt} \cdot \Phi_1(d_1)$

and for the put option

$\Delta_p = (\partial p)/(\partial S) = e^{-qt}(\Phi_1(d_1) - 1)$,

so that $\Delta_c - \Delta_p = e^{-qt}$. This is an important property of European options.

For other models than Black & Scholes and Black -76, the delta is calculated by using the symmetrized form of the average rate of change, which is

$$\Delta \sim \frac{\Pi(1{,}00001 \cdot S) - \Pi(0{,}99999 \cdot S)}{2 \cdot 0{,}00001 \cdot S} \tag{6.7}$$

It is well known that the approximation (6.7) tends to give inaccurate estimates for the binomial model, and that another approach for calculating the delta must be used. In Orc, for the binomial model, the delta of the option price at $S = S_{00} = S_{21}$ is approximated to be the slope (i.e. derivative or delta) at $S = S_{00} = S_{21}$ of the parabola going through (S_{20}, C_{20}), (S_{21}, C_{21}), and (S_{22}, C_{22}). This yields the formula

$$\Delta \sim \frac{S_{20}(2S_{21}-S_{20})(C_{21}-C_{22}) + S_{21}^2(C_{22}-C_{20}) + S_{22}(2S_{21}-S_{22})(C_{20}-C_{21})}{(S_{21}-S_{20})(S_{22}-S_{20})(S_{22}-S_{21})} . \tag{6.8}$$

To make sure that the option price C_{21} at S_{21} has the right time to expiry, t has to be adjusted by a factor which depends on the amount of the steps used in the binomial tree.

In addition, the delta is multiplied with the price multiplier, but option delta is not. This is the only difference between these two columns.

Swimming skew delta and option swimming skew delta

The swimming skew delta Δ_{skew} measures the total effect of base price changes on the theoretical value. The difference to delta is that the effect the volatility may change when the base price changes is included. The calculation is

$$\Delta_{skew} = \Delta + \Lambda\frac{d\sigma}{dS}. \tag{6.9}$$

In addition, the swimming skew delta is multiplied with the price multiplier, but option swimming skew delta is not. This is the only difference between these two columns.

Skew risk

The skew risk measures the difference between the swimming skew delta and the delta. Thus in view of Eq. (6.9), this is the product of the vega and the derivative of volatility with respect to the spot price.

Swimming skew color

The definition of swimming skew color is :

The effect on swimming skew gamma when one trading day passes.

Swimming skew gamma

The swimming skew gamma measures the total effect on swimming skew delta, when the base price changes. It is calculated as the total derivative of the swimming skew delta with respect to s:

$$\frac{d\Delta_{skew}}{ds} = \Gamma + \frac{\partial\Lambda}{\partial\sigma}skew^2 + 2\frac{\partial\Lambda}{\partial s}skew + \Lambda\frac{\partial}{\partial s}skew \tag{6.10}$$

Gamma and option gamma

The Gamma (Γ) is the second partial derivative of the price of the option with respect to the price of the underlying asset, i.e. $\Gamma = \partial\Delta/\partial S$. The risk measure tells you how sensitive the delta of the contract is with respect to changes in the price of the underlying asset. This can be interpreted as the approximate effect on the delta when the price of the underlying base contact increases or decreases by one unit. If the underlying asset price moves one price unit up, the new delta equals the old delta plus the gamma and if the underlying asset price moves one price unit down, then the new delta equals the old delta less the gamma. The convexity of options prices means that $\Gamma \geq 0$.

The analytic expression for the gamma of European options when using the Black & Scholes model is
$\Gamma_c = \frac{\partial\Delta_c}{\partial S} = \frac{\Phi'_1(d_1)}{S\sigma\sqrt{t}}e^{-qt}$, and for the put option it is $\Gamma_p = \frac{\partial\Delta_p}{\partial S} = \frac{\Phi'_1(d_1)}{S\sigma\sqrt{t}}e^{-qt}$
For other models than Black & Scholes and Black -76, the gamma is calculated by using the symmetrized average rate of change, namely

$$\Gamma \sim \frac{\Pi(1{,}001 \cdot S) - 2\Pi(S) + \Pi(0{,}999 \cdot S)}{(0{,}001 \cdot S)^2} \tag{6.11}$$

It is well known that the approximation (Eq. (6.11)) tends to give inaccurate estimates for the binomial model, so that another approach for calculating the gamma must be used. In Orc, for binomial trees, the gamma of the option price at $S = S_{00} = S_{21}$ is approximated to be the convexity (i.e. gamma) of the parabola going through the points (S_{20}, C_{20}), (S_{21}, C_{21}), and (S_{22}, C_{22}). This yields the formula

$$\Gamma \sim 2\frac{S_{20}(C_{21} - C_{22}) + S_{21}(C_{22} - C_{20}) + S_{22}(C_{20} - C_{21})}{(S_{21} - S_{20})(S_{22} - S_{20})(S_{22} - S_{21})}. \tag{6.12}$$

Here, t has to be adjusted in the same way as for the delta case.

In addition, the gamma is multiplier with the price multiplier, but option gamma is not. This is the only difference between these columns.

Theta and option theta

The theta (Θ) of a derivative contract is defined as the difference between the theoretical price at the present calculation date and its theoretical price at the next trading day, on the assumption that all other parameters remain unchanged. The user may specify as whether the base price or the implied spot price should remain constant. The Theta is thus not defined as a derivative. The theta reflects the time decay of an option and is a risk measure of the options time sensitivity.

The following relation holds: the theoretical option price the next trading day is equal to the theoretical price today plus the Theta, given that the relevant settings and price are constant.

Option theta shows the theta effect not adjusted for the price multiplier.

The time used for the theta calculation changes every 30 minutes if “vol. time mode” is set to “always” in the Contract Inspector. Otherwise, theta is only changing when the calculation date changes.

Volatility theta and Interest rate theta

The Volatility theta Θ_σ is defined as the theta value of the contract when the financing rate is zero. It thus isolates time decay of an option which is only due to less exercise opportunities, and not to carry cost. Hence $\Theta_\sigma = \Theta|_{r=0}$. The interest rate theta Θ_r is defined as the theta value of the contract less the Volatility theta: $\Theta_r = \Theta - \Theta_\sigma$.

Vega and option vega

The vega (Λ) of the contract is defined as $0{,}01 \cdot (\partial\Pi)/(\partial\sigma)$. This can be interpreted as the approximate effect on the theoretical price when the volatility of the contract increases by one percentage unit.

The following relation holds: if the volatility goes up by one percentage unit, the new theoretical price is approximately equal to the old theoretical price plus Vega and if the volatility goes down by one percentage unit the new theoretical price is approximately equal to the old theoretical price less Vega.

For the Black & Scholes model vega is calculated analytically by the formula

$\Lambda = S\sqrt{t}e^{-qt}\varphi(d_1)$.

For other models, Vega is calculated by using the symmetrized form of the average rate of change, namely

$$\Lambda = 0{,}01 \cdot \frac{\Pi(1{,}01 \cdot \sigma) - \Pi(0{,}99 \cdot \sigma)}{2 \cdot 0{,}01 \cdot \sigma}.$$

Option vega shows the vega effect not adjusted for the price multiplier.

Weighted Vega

The weighted vega is the usual vega multiplied with a weight w_t that decreases from one to zero as time t to expiry increases. The weight is define as

$w_t = \frac{1 - e^{-3t}}{3t}$

and was chosen because of three properties:

- w_t is (approximately) inversely proportional to the time to expiry,
- for contracts very close to expiry w_t is close to one
- contracts with one year to expiry has approximately a third of the weight of contracts close to expiry.

Rho

The rho ρ of the contract is defined as the effect on the contract theoretical price when every financing rate is increased by one percentage unit and all other parameters remain unchanged.

The user may specify as whether the base price or the implied spot price should remain constant. In the case that several yield curves are involved all financing rates are moved up by one percentage unit.

The rho is thus not defined as a derivative. the rho is a risk measure of the option price sensitivity to changes in the financing rate.

Rho -1 basis point is the rho value define as the effect on the contract price when the interest rate decreases by one point, and Rho +10 basis points is the rho value defined as the effect on the contract price when the interest rate increases by 10 points.

Speed

The speed of the contract in defined as $\partial^3\Pi/\partial^3 S$. This can be interpreted as the approximate effect on the gamma when the price of the underlying base contract increases by one price unit.

Charm

The charm value of the contract is define as the effect on the delta when one trading day passes. On expiration date, charm is 0 for delivery settled derivatives.

Color

The color value is defined as the effect on the gamma when on trading day passes. On expiration date, color is 0 for delivery settled derivatives.

Model	ý	Γ	θ	Vega	ρ	Speed	Charm	Color
Black&Scholes	0.37	0.0273	-0.033	0.17	0.08	0.0004	-0.0015	0.0001
Roll-Geske	0.46	0.0438	-0.052	0.16	0.06	0.00021	-0.0038	0.0003
Binomial (European Execution)	0.37	0.0272	-0.033	0.17	0.08	0.0004	-0.0013	0.0001
Binomial (American Execution)	0.46	0.0438	-0.045	0.16	0.05	0.00021	-0.0002	0.0007

Table 2: Theoretical values of Greeks for different models. It is assumed to be a call option with parameters as in Table 4.

Delta(1%), Gamma(1%) and Vega(1%)

Delta (1%) is defined as the change in the theoretical value when the price of the underlying base contract increases by 1%. It is calculated by $\Delta_{1(\circ/\circ)} = 0{,}01 \cdot S \cdot \Delta$. Gamma is defined as the change in the contract delta when the price of the underlying base contract increases by 1%. It is calculated by $\Gamma_{1(\circ/\circ)} = 0{,}01 \cdot S \cdot \Gamma$. Vega(1%) is defined as the change in the theoretical value of the contract if the volatility increases by 1% in relative terms.

Delta(user%), Gamma(user%) and Vega(user%)

The delta(user%) value is defined as the change in the theoretical price if the price of the underlying base contract increases by a user defined percentage.

The gamma(user%) value is defined as the change in the delta value if the price of the underlying base contract increases by a user defined percentage.

The vega(user%) value is defined as the change in the theoretical price if the price of the volatility increases by a user defined percentage.

Delta(ytm)

The Delta (ytm) is the delta in terms of yield to maturity. The value shows the effect on the contract price when the yield to maturity of the base contract decreases by one point. This risk measure is for interest rate instruments. For combined instruments such as convertibles, only the effect on the bond part is accounted for.

To obtain numbers representing money, the value is multiplied with the multiplier.

Gamma(ytm)

The Gamma (ytm) is the gamma value in terms of yield to maturity. The value is defined as the effect on the contract delta rate when the yield to maturity of the base contract decreases by one point. This risk measure is for interest rate instrument. For combined instruments such as convertibles, only the effect on the bond part is accounted for.

To obtain numbers representing money, the value is multiplied with the multiplier.

Hedge volume(delta based)

This column displays the factor between delta for different contract on the same trading page.

Example A call has delta 0.50 and the multiplier is 100. A future in the same window has delta 1.02 and multiplier 1. If you enter 1 in the hedge column field for the call, the Orc displays 49 in the hedge volume field for the future, which is the delta equivalent number of futures. Similarly, if you enter 100 in the hedge volume field for the future, the Orc displays 2 in the hedge volume field for the call.

Hedge volume (delta(ytm) based)

This column displays the factor between delta(ytm) for different contract on the same trading page.

Example A bond has delta(ytm) 0.020 and the multiplier is 10,000. Another bond in the same window has delta(ytm) 0.010 and multiplier 10,000. If you enter 1 in the hedge volume field for the first bond, the Orc displays 2 in the hedge volume field for the other, which is the delta(ytm) equivalent number of this bond.

Duration, Duration (bid) and Duration (ask)

The duration measures the average time to cash flow of an interest rate instrument. For the Duration column, future cash flows are valued with a yield corresponding to theoretical price, for the Duration (bid) and Duration (ask), the bid and ask yield respectively are used.

Modified duration, Modified duration (bid) and Modified duration (ask)

The modified duration, D_m, measures how yield changes affects the theoretical price of the interest rate instruments. It is calculated as

$$D_m = \frac{\Pi(r - 0{,}01r) - \Pi(r + 0{,}01r)}{0{,}02r\Pi(r)}$$

where $\Pi(r)$ is the theoretical dirty price at rate r. For the Modified duration column, r is the yield corresponding to theoretical price, for the Modified duration (bid) and Modified duration (ask), the bid and ask yield respectively are used.

Convexity, Convexity (bid) and Convexity (ask)

The convexity, Con, measures how the yield changes affects the modified duration for an interest rate instrument. It is calculated as

$$Con = \frac{\Pi(r + 0{,}01r) - 2\Pi(r) + \Pi(r + 0{,}01r)}{(0{,}01r)^2\Pi(r)}$$

where $\Pi(r)$ is the theoretical dirty price at rate r. For the Convexity column, r is the yield corresponding to theoretical price, for the Convexity (bid) and Convexity (ask), the bid and ask yield respectively are used.

Multi-currency setup

When a contract's price currency and the currency of the strike, spot and underlying differ, then this affects the risk value calculations.

Assume that price currency is EUR and the strike, spot and underlying are in SEK, then risk values are affected in the following way:

- Delta, skew delta, gamma(%) and skew gamma(%) are unaffected by changes in the EUR/SEK spot rate,
- Gamma, skew gamma, color, skew color are linear with respect to the EUR/SEK spot rate,
- Skew delta(%), vega, vega(%) and theta are inversely proportional to the EUR/SEK spot rate.

Implied values

Implied volatility

Implied volatilities are presented in four columns, one for the ask price and one for the bid price for both calls and puts. Different colors denote the highest implied volatility for the bid side and the lowest volatility for the ask side for all calls and puts in one expiry (given that they have market prices different from zero).

The volatility is the momentum standard deviation for the underlying price. The price of a vanilla call or a vanilla put can be described as $\Pi = \Pi(S, X, t, r, \sigma)$. From this formula the implied volatility σ_i can be calculated from a given market price Π_m, since the function $\Pi = \Pi(S, X, t, r, \circ)$ is a strictly increasing function with respect to σ, and hence injective for positive σ. Thus $\sigma_i = \sigma_i(\Pi_m)$ is the volatility that solves

$$\Pi(S, X, t, r, \sigma) = \Pi_m, \tag{6.13}$$

where S, X, t, r are held constant. A necessary requirement is, of course, that Π_m is in the value set of $\Pi(S, X, t, r, \sigma)$ as σ varies. By the Black & Scholes formula, σ_i cannot be given as a closed-form formula, and numerical methods have to be used to calculate σ_i from Π_m. In Orc, an algoritm is used which combines root bracketing, bisection, and inverse quadratic interpolation. Observe that it would make no numerical sense to use classical Newton-Raphson, since the derivative of the theoretical value with respect to the volatility $\partial\Pi/\partial\sigma$ is calculated by using a numerical approximation.

It is easy to show that Π grows with σ, and there is hence a correspondence between high prices and high volatilities. By weighting different implied volatilities for different strike prices, it is possible to get a picture of how the market estimates future volatility of the underlying. Often at-the-money options are given a high weight in such calculations, since these options are more liquid and also the most sensitive to changes in volatility. For a quick estimate of market volatility, it is often enough to look at the at-the-money options.

For an example with some different implied values, see Table 3.

Most contracts with volatility in Orc are monotone with respect to volatility and the implied volatility calculation is performed in the same way for these. But some combinations such as call spreads and some contracts such as Binaries and Caps are not monotone with respect to volatility. As a consequence the concept of implied volatility is not well defined and the implied volatility calculation is not available for these contracts.

Implied base price

Implied base prices are presented i four columns, one for the ask price and one for the bid price for both calls and puts. Different colors denote the highest implied base value of rate bid side and the lowest for the ask side for all calls and puts in one expiry (given that they have market prices different from zero).

Model	Implied volatility	Implied baseprice	Implied Strike price	Implied Yield offset	Implied Underlying rate
Black&Scholes	28.51	99.28	100.86	-352.35	3.15
Roll-Geske	28.83	99.58	100.48	-356.79	3.28
Binomial (European Execution)	28.51	99.28	100.86	-351.78	3.14
Binomial (American Execution)	28.82	99.60	100.45	-362.78	3.35

Table 3: Different implied values. The market value of the call option defined by Table 4 is supposed to be 3.00 for the European option an 3.50 for the American option.

Similar calculations are made for the implied base price, S_i as for the implied volatility. It is then given by the S (more precisely, the value of base contract) that solves , where everything but the spot price is held constant.

For an example with some different implied values, see Table 3.

Implied strike price

Implied strikes are presented in four columns, one for the ask price and one for the bid price for both calls and puts.

Similar calculations are made for the implied strike price, as for the implied volatility. The implied strike for a given market value Π_m, is the strike that solves Eq. (6.13), where everything but the strike is held constant.

For an example with some different implied values, see Table 3.

Implied yield-curve offset

Implied yield-curve offsets are presented in four columns, one for the ask price and one for the bid price for both calls and puts. Different colors denote the highest implied yield-curve offset for the bid side and the lowest for the ask side for all calls and puts in one expiry (given that they have market prices different from zero). The implied yield-curve offset is given by the difference between the implied financing rate and the financing rate.

The implied financing rate for a given market value Π_m, is the financing rate that solves Eq. (6.13), where everything but the financing rate is held constant.

For r the situation is more intricate, since it is not true that the option price as a function of r is injective, but if a solution is found, the corresponding implied yield-curve offset is calculated.

For an example with some different implied values, see Table 3.

Implied E.E.P.

Implied early exercise premiums (E.E.P.) are presented in four columns, one for the ask price and one for the bid price for both calls and puts.

Similar calculations are made for the implied eep, as for the implied volatility.

Synthetic values

Synthetic futures, calls, and puts

The synthetical futures theoretical prices are calculated for a strike if there are market prices in the corresponding calls and puts. The synthetic future is obtained by creating a combination of calls and puts with the same strike prices and expiry date. By buying a call and selling a put, you are effectively buying a future at a certain price. A sold future is created by selling a Call and buying a Put option with the same strike and expiry date. The analysis is done in bid and ask columns. The best offered price for a certain expiration month is highlighted as well as the best asked price.

To price the synthetic future put-call parity, Eq. (5.3) is used. The formula used for the offered price of the synthetic future is

$$F_b = (c_b - p_a)e^{rt} + X, \tag{6.14}$$

where F_b is the synthetic future bid price, c_b the market call bid price and p_a the market put ask price.

The asked price of the synthetic future is

$$F_a = (c_a - p_b)e^{rt} + X, \tag{6.15}$$

where F_a is the synthetic future ask price, c_a the market call ask price and p_b the market put bid price.

The bid and ask prices of the synthetic call are with the same notation

$$c_b = (F_b - X)e^{-rt} + p_a \text{ and } c_a = (F_a - X)e^{-rt} + p_b. \tag{6.16}$$

The bid and ask prices of the synthetic put are

$$p_b = (X - F_a)e^{-rt} + c_a \text{ and } p_a = (X - F_b)e^{-rt} + c_b. \tag{6.17}$$

Example The 100 call with time to expiry 90 days is traded at 5.50-5.80. The 100 put with the same expiry is traded at 3.40-3.70. Let $r = 0{,}05$, then a synthetic future can be bought at $2{,}40 \cdot 1{,}0126 + 100 = 102{,}40$. Similarly a synthetic future can be sold at $1{,}80 \cdot 1{,}0126 + 100 = 101{,}80$.

Synthetic bills

The synthetic bill columns show the return given if you buy the spot and sell it on the forward market (directly or synthetically). In the spot position you get dividends paid during the time to expiry.

The return on a long underlying and a short future or a short underlying and a long future over the contract period is calculated as follows. Let S_b and S_a denote the offered and asked asset spot prices in the market and F_b and F_a the offered and asked futures price of the same asset in the market.

Then the offered return $(R_b)_p$ for the synthetic bill is

$$(R_b)_p = 100\left(\frac{F_a}{S_b - \sum_{i=1}^{n} D_i e^{-rt_i}} - 1\right)\%,$$

and the asked return is

$$(R_a)_p = 100\left(\frac{F_b}{S_a - \sum_{i=1}^{n} D_i e^{-rt_i}} - 1\right)\%.$$

Example The underlying is traded at 100-101. The future at 102-102.50. The underlying have no dividends during the period. Then the offered return for the period is 2.5% and the asked is 1%.

The yearly return of a synthetic bill is calculated exactly as the return over a period, except that the return is given on a yearly basis. Then the offered yearly return $(R_a)_y$ for the synthetic bill above is

$$(R_a)_y = \frac{100}{t}\left(ln\left(\frac{F_b}{S_a - \sum_{i=1}^{n} D_i e^{-rt_i}}\right) \%, \right)$$

and the asked yearly return is

$$(R_b)_y = \frac{100}{t}\left(ln\left(\frac{F_a}{S_b - \sum_{i=1}^{n} D_i e^{-rt_i}}\right) \%, \right)$$

These continuously compounded interest rates can, if necessary, be converted into straight or effective yearly compounded interest rates.

Example If the period in the previous example is 180 days, then the offered yearly return is $(100/2) \cdot ln(1{,}025) = 4{,}94$ % (continuously compounded). The yearly return is $(100/2) \cdot ln(102/101) = 1{,}97$ % (continuously compounded).

Covered calls

A basic option strategy is the so called covered call strategy, which consists of buying the underlying asset and writing a call option against it. Writing covered calls is probably the singe most common option strategy used by institutional investors. The call writer receives the option premium at the outset in return for standing ready to sell the asset fort the exercise price at a later date. Writing the call limits the maximum profit that an be earned on the position, since the stock will be called away if its price ends up above the strike price. The maximum return is therefore what is often called the "If called" return, since this is the return that will be earned if the call is exercised.

The objective of writing the call is to generate extra income in a flat or down market, because the time value dissipates. The standstill return is positive and equals the option's initial time value. The call provides a cushion if the stock price falls: the position breaks even when the stock has fallen by the amount of the premium received.

Covered call writing is a risk-reducing strategy. It is designed to do precisely what investors do when they choose stocks that pay high dividends over stocks that have greater growth prospects. Covered call writing converts the prospects for uncertain future capital gains into immediate cash flows that resemble dividends. There may be good reasons for an occasional covered call, but a formal program of covered call writing might well be no better than simply buying high dividend stocks instead.

Another misunderstanding about covered calls arises from the way in which the profit is calculated. Let S be the stock price when the call is written, X the strike, C be the call premium, and $\hat{S}$ be the stock price when the call expires. If the call expires in the money, the profit is $X - S + C$. When you attached the call to the stock, you made a conscious decision to hold the stock. You could have sold it, liberating the S dollar for use elsewhere. thus, you have to count the S dollars even though you may have bought the stock much earlier when calculating the invested amount in your strategy.

In Orc two cases are considered. The first case is when the spot price is assumed to remain unchanged during the life of the call, and the second case is where we assume that exercise will take place. Also, the result is presented in two ways. Either, the return (in%) for the period, or the corresponding yearly return.

Case 1: Covered call period return if the spot remains constant

The return R^p on a long underlying and short call over the contract period is given by

$$\begin{cases} R^p = 100\left(\frac{S + \sum_{i=1}^{n} e^{-rb_i} D_i}{S - C} - 1\right) \% \text{ when } S < X; \\ R^p = 100\left(\frac{X}{S - C} - 1\right) \% \text{ if } S \geq X \end{cases}$$

The yearly return on the covered call is

$$\begin{cases} R^y = \dfrac{100}{t} \cdot ln\left(\dfrac{S + \sum_{i=1}^{n} e^{-rb_i} D_i}{S - C}\right) \%\ when\ S < X; \\ R^y = \dfrac{100}{t} \cdot ln\left(\dfrac{X}{S - C}\right) \%\ when\ S \geq X. \end{cases}$$

The spot price S and the price of the call C are averages of the bid/ask market prices.

Case 2: covered call period return if the call is assumed to end up in-the-money

If there is a dividend during the life of the option, it is assumed that the call is exercised at the dividend date. Otherwise the call is assumed not to be exercised until expiration.

The period return R^p is given by

$$\begin{cases} R^p = 100 \cdot \left(\dfrac{X + \sum_{i=1}^{n} e^{-rb_i} D_i}{S - C} - 1\right) \%\ when\ S < X; \\ R^p = 100\left(\dfrac{X}{S - C} - 1\right) \%\ if\ S \geq X. \end{cases}$$

The yearly return R^y on a covered call this is assumed to be exercised is

$$\begin{cases} R^y = \dfrac{100}{t} \cdot ln\left(\dfrac{X + \sum_{i=1}^{n} e^{-rb_i} D_i}{S - C}\right) \%\ when\ S < X; \\ R^y = \dfrac{100}{t} \cdot ln\left(\dfrac{X}{S - C}\right) \%\ when\ S \geq X. \end{cases}$$

Deposits

These are the basic ingredients in deposits calculations and constructions.

- Start date = the date on which the deposit is issued
- end date = the date on which the deposit ends and the end amount is paid
- settlement date = the date on which the payment of the deposit takes place, it is usually onor a few days after the start date
- start amount = the amount paid on the settlement date
- end amount = the amount paid back on the end date
- rate amount = end amount - start amount
- ytm = the yield to maturity seen in the New contract window and is calculated as 100*((rate amount)/(start amount))/(end date - start date).
  The difference “end date - start date” should be expressed in years.
- YTM day conventions, rate calendars and how the rate should be compounded (i.e straight,continuously, effective...).

All theoretical calculations are based on a present value factor for the deposit.

How to calculate present value factor for deposit

Given the calculation date T_c, the settlement date T_s and the end date T_e and the current yield curve chosen, a present value factor is calculated. It is done in the following way.

- Let $PV_1 = PV(T_c, T_s)$ be the present value factor between T_c and T_s based on the chosen yield curve.
- Let $PV_2 = PV(T_c, T_e)$ between T_c and T_e based on the chosen yield curve.
- The present value factor for the deposit is then given by PV = PV_2/PV_1. I.e. if the yield curve is flat the present value factor is the factor for the dates T_c and T_e only. If T_c # T_s then PV_1 = 1, if T_c # T_e then PV_2 = 1 also.

Basic calculations in the New contracts window

A deposit can be thought of as a zero coupon bond with face value equal to the end amount. In the New contracts window for deposits the following basic calculations are performed,

- rate amount = end amount - start amount
- ytm = 100*((rate amount)/(start amount))/(end date - start date)

The only variable held fixed throughout this calculation when both equations are involved is the start amount. By varying for instance the end amount both the rate amount will change and the ytm will change, and varying the end date will change the rate amount and the and amount by holding the start date and ytm fixed. Lets consider an example:

If start amount = 1000, end amount = 1003, start date = 16th of june 2006 and end date =16th of june 2006 and the convention for ytm days are 30/360, then ytm = 100*(1003-1000)/1000*(1 year) = 0.3 per cent. By changing the end amount to 1004 will change the rate amount to 4 and the ytm to 0.4 per cent, or by changing the end date to 6 months earlier so that end date - start date = 0.5 years, then the ytm is held fixed, the rate amount will be recalculated to 2 and the end amount will be changed according to the first equation to be 1002.

Theoretical price and YTM calculations for already existing contracts

The theoretical price = the value of the deposit today. Let PV be the present value factor for the deposit calculated as described above. The theoretical price is then calculated as th. price = PV*end amount.

The theoretical YTM as displayed in the options window or trading window is calculated a bit differently depending on the YTM rate type chosen for the deposit in the contracts inspector, i.e if it is straight, continuous, compound or annual compounding.

Let T = time between settlement date and end date. Then for instance

- $th.ytm = -ln(PV)/T$ if it is continuous rate type
- $th.ytm = (1-PV)/(PV \cdot T)$ if it is straight rate type
- $th.ytm = (1-PV)/T$ if it is discount rate type
- $th.ytm = (PV^{-1/(nT_n)} - 1) \cdot n$ if it is compound. Here, n is the number of compoundings and T_n is the length if interval for compounding n.

Implied YTM from market prices can also be calculated. The calculations are basically the same as for the theoretical value, except that the present value is calculated implicitly from the market price according to $PV_{impl} = (marketprice)/(endamount)$.

APPENDIX A-33 Theoretical values for the trading page

THEORETICAL VALUES FOR THE CONTENTS WINDOW

Position

Average paid for position

The average premium paid for the contract defined as the Invested amount divided by the Volume. Average paid is undefined for a zero volume position.

Call count and put count

The number of call contracts $\#C$ is calculated for each contract with positive delta as

$$\#C = v \cdot m \cdot (pm) / m_b,$$

where v is the volume, m the multiplier, pm is the price multiplier, and m_b the multiplier of the base contract.

The number of put contracts $\#P$ is calculated for each contract with negative delta as

$$\#P = v \cdot m \cdot (pm) / m_b$$

with the same notations as earlier. For contracts with positive delta, except calls, the contribution is $\#P = -v \cdot m \cdot (pm) / m_b$.

Result and Valuation

Invested in position

The invested amount is calculated as the sum of the invested for each trade in the contract since the latest Mark-to-market added to the invested amount set in the latest Mark-to-market. The value is present value adjusted for forwards.

Example If 5000 XYZ shares were bought at 300 and 4000 XYZ shares were sold at 302, the invested in position is calculated as follows:

Transaction	Invested
5000 bought XYZ shares at 300	+1500000
4000 sold XYZ shares at 302	-1208000

A total amount of 292 000 were thus invested (1 500 000-1 208 000).

Invested accrued interest of position

The amount accrued interest is calculated as the sum of the accrued for each trade in the contract since latest Mark-to-market added to the accrued amount set in the latest Mark-to-market. The value is present value adjusted for forwards.

Market value of position

The market value of a position is calculated as

$$\textit{Market value of contract} \cdot \textit{volume} \cdot \textit{multiplier} \; .$$

The market value is present value adjusted for forwards.

Result of position (market)

The market value of position less the invested amount.

Example If 5000 XYZ shares were bought at 300 and 4000 shares were sold at 302. A total amount of 292000 were thus invested. The day result shows what you have received for this money, namely 1000 XYZ shares. If the share now is trading at 310, the result is:

Present market value of position	+310 000
- Invested amount	-292 000
= Result of position (market)	+18 000

Day result of position (market)

The market value less the invested amount of the trades performed since the last reset. The value is present value adjusted for forwards.

Plain invested of position

The invested amount not present value adjusted for forwards.

Plain market value of position

Plain market value of position, not present value adjust for forwards.

Plain result of position

Plain market value of position less the plain invested amount.

Theoretical market value of position

The value is calculated as

$$\textit{Theoretical value of contract} \cdot \textit{volume} \cdot \textit{multiplier} \, .$$

The value is present value adjusted for forwards.

Result of position (theoretical)

The theoretical market value less the invested amount.

Day result of position (theoretical)

The theoretical market value less the invested amount of the trades performed since the latest reset.

Theoretical edge

The theoretical value of position less the market value of position.

Theoretical value of contract

The theoretical value of the contract (expressed in the contract currency).

Value of position (market or theoretical)

The market value of the position and if there is no market price in the contract the theoretical market value of position.

Result of position (market or theoretical)

The value of position (market or theoretical) less the invested amount.

Accrued interest rate of position

The current accrued interest rate of position.

Accrued interest result

The accrued interest rate of position less the Accrued interest paid.

Theoretical value of contract (in base currency)

The theoretical value of the position expressed in the base currency.

Account balance

The result of the position unless it is a forward or future. Forwards and futures have account balance zero.

Theoretical forward price of contract

The implied forward price (at the expiry of the contract) of the base contract.

Theoretical forward ytm of contract

The implied forward ytm (at the expiry of the contract) of the base contract.

Plain closing value of position

The market value of the position based on the close price.

Plain result of position (close)

The market value of the position based on the close price less the invested.

Total result of position (market)

The sum of Result of position (market) and Accrued interest result.

Total result of position (theoretical)

The sum of Result of position (theoretical) and Accrued interest result.

Total result of position (market or theoretical)

The sum of Result of position (market or theoretical) and Accrued interest result.

Fees paid for position

The accumulated fees paid for the position.

Exercise value

the value of the strike for the position.

Short exercise value

The value of the strike for short positions.

Long exercise value

The value of the strike for long positions.

Value of position (plain spread or theoretical)

The plain spread market value of position and if there is no market price the theoretical market value (but present value adjusted for forwards) is used.

Short value of position (plain spread or theoretical)

The Value of position (plain spread or theoretical) if the position is short, otherwise zero.

Long value of position (plain spread or theoretical)

The Value of position (plain spread or theoretical) if the position is long, otherwise zero.

Theoretical value of contract (in position currency)

The theoretical value of the contract expressed in the currency of the position.

Total invested amount in position

The sum of the invested in position and the invested accrued in position.

Total result of position including fees (market)

The sum of total result of position (market) and Fees paid for position.

total result of position including fees (theoretical)

The sum of Total result of position (theoretical) and Fees paid for position.

Change in invested in position

The change in invested in position since the latest reset.

Change in invested accrued interest in position

The change in invested accrued in position since the latest reset.

Theoretical spot price for contract

The implied spot (cash) price of the base contract.

Financing

Financing cost of position

the financing cost of position is defined as the cost of keeping the position another trading day. The interest rate used is taken from the default yield curve for the currency that the position is invested.

Theoretical financing cost of position

Orc calculates the financing cost using the theoretical value of the position. The interest rate used is taken from the default yield curve for the currency that the position is invested in.

Overnight financing

The overnight financing cost determined by the Invested amount and the one-day interest rate taken from the default yield curve for the currency that the position is invested in.

There is another portfolio-based functionality for overnight financing. The user may set an overnight rate and a number of days and perform overnight financing for every position in the portfolio.

Accrued change

The change in accrued interest to the next trading day.

Cost of carry

The sum of the financing cost of position and accrued change.

Net cost of carry

The sum of the financing cost of position, accrued change, and theta.

Theoretical cost of carry

The sum of the theoretical financing cost of position and accrued change.

Theoretical net cost of carry

The sum of the theoretical financing cost of position, accrued change, and theta.

Risk

Beta adjusted delta of position

This shows the equivalent number of beta base contracts.

$$Delta\ of\ position \cdot Beta\ value \cdot Base\ price \cdot \frac{(Base\ contract\ multiplier)}{Price\ of\ Beta\ base \cdot Multiplier\ of\ beta\ base\ contract}$$

This concept allows to sum and compare delta from different underlyings but with the same beta base.

Beta adjusted swimming skew delta of position

This shows the equivalent number of beta base contracts.

$$Swimming\ skew\ delta\ of\ position \cdot Beta\ value \cdot Base\ price \cdot \frac{(Base\ contract\ multiplier)}{Price\ of\ Beta\ base \cdot Multiplier\ of\ beta\ base\ contract}$$

This concept allows to sum and compare swimming skew delta from different underlyings but with the same beta base.

Beta adjusted gamma of position

This shows gamma translated to beta base contracts.

$$Gamma\ of\ position \cdot (Beta\ value \cdot Base\ price)^2 \cdot \frac{(Base\ contract\ multiplier)}{Price\ of\ Beta\ base \cdot Multiplier\ of\ beta\ base\ contract}$$

This concept allows to sum and compare gamma from different underlyings but with the same beta base.

Beta adjusted swimming skew gamma of position

This shows swimming skew gamma translated to beta base contracts.

$$Swimming\ skew\ gamma\ of\ position \cdot (Beta\ value \cdot Base\ price)^2 \cdot \frac{(Base\ contract\ multiplier)}{Price\ of\ Beta\ base \cdot Multiplier\ of\ beta\ base\ contract}$$

This concept allows to sum and compare swimming skew gamma from different underlyings, but with the same beta base.

Delta 1% of position

$$0.01 \cdot Delta\ of\ contract \cdot Base\ price \cdot Position\ volume \cdot Multiplier$$

Gamma 1% of position

$$0.01 \cdot Gamma\ of\ contract \cdot Base\ price \cdot Position\ volume \cdot Multiplier$$

In beta base summation row, the contribution from each position is instead a beta adjusted version:

$$0.01 \cdot Gamma\ of\ contract \cdot Base\ price \cdot Base\ price \cdot Position\ volume \cdot Multiplier \cdot beta \cdot beta\ /\ Price\ of\ beta\ base$$

Delta of position

An estimate of the number of base contract that your position equals with respect to price movements in the base contract. It is calculated as

$$Delta\ of\ contract \cdot volume \cdot (multiplier)/(multiplier\ of\ base\ contract)\ .$$

For forwards this value is multiplied by e^{-rt} (present value adjustment), i.e. forwards have $\Delta = 1$. This ensures that $c + p - f$ have zero risk in the portfolio. As delta represents a number of contracts, it is displayed as an integer. In addition, depending on the preference setting, if the base contract is a future the delta is transformed to future equivalents by dividing with the delta of the future.

In summation rows, the sum consists of beta adjusted delta values.

Swimming skew delta of position

Another estimate of the number of base contracts that your position equals with respect to price movements in the base contract. It is calculated as

$$Swimming\ skew\ \Delta\ of\ contract \cdot volume \cdot (multiplier)/(multiplier\ of\ base\ contract)$$

For forwards this value is present value adjusted. As swimming skew delta represents a number of contracts, it is displayed as an integer. In addition, depending on preference setting, if the base contract is a future the swimming skew delta is transformed to future equivalents by dividing with the delta of the future.

In summation rows the sum consists of beta adjusted (swimming skew) delta values.

Gamma of position

An estimate of how Delta of position changes with respect to price movements in the base contract. It is calculated as

$$Gamma\ of\ contract \cdot volume \cdot (multiplier)/(multiplier\ of\ base\ contract)\ .$$

For forwards this value is multiplied by e^{-rt} (present value adjustment). Gamma of position is displayed as an integer.

In summation rows the sum consists of beta adjusted gamma values.

Theta and Rho

These greeks represent money and are therefore summed directly in the contents window. The values are calculated as the effect of a simulation on the Theoretical result. For theta, the simulation is one trading day, for volatility theta, all interest rates are simultaneously simulated to be zero. Interest rate theta is the difference between theta and volatility theta.

For rho it is a simulation on all yield curves up by one percentage unit.

The same formula applies to volatility theta, interest rate theta, Rho +10bp, and Rho -1bp.

Vega of position

This greek represents money and is therefore summed directly in the contents window. the value is calculated as

$$Greek\ of\ contract \cdot volume \cdot multiplier\ .$$

For forwards this value is multiplier by e^{-rt} (present value adjustment). The same formula applies to Weighted vega, Delta (1%), Gamma (1%), Vega (1%), Delta (user%), Gamma (user%), and Vega (user%) of position.

Speed, Charm, and Color of position

These greeks are calculated analogously to delta and gamma.

$$Greek\ of\ contract \cdot volume \cdot (multiplier)/(multiplier\ of\ base\ contract)\ .$$

For forwards this value is multiplied by e^{-rt} (present value adjustment).

Cash delta of position

the net exposure of the position. It is defined as the delta of the position times the price of the underlying base contract, converted into the base currency.

Beta-adjusted delta

An estimate of the number of beta-base contracts that your position equals with respect to price movements in the beta-base. It is calculated as

$$Betabase\ adjusted\ delta \cdot volume \cdot (multiplier)/(multiplier\ of\ betabase\ contract)\ ,$$

For forwards this value is multiplied by e^{-rt} (present value adjustment).

Beta-adjusted gamma

An estimate of how the beta-adjusted delta changes with respect to price movements in the beta-base. It is calculated as

$$Betabase\ adjusted\ gamma \cdot volume \cdot (multiplier)/(multiplier\ of\ betabase\ contract)$$

For forwards this value is multiplied by e^{-rt} (present value adjustment).

Delta (ytm) of position

The delta value in terms of yield to maturity. The Delta (ytm) shows the effect on the contract price when the yield to maturity of the base contract decreases by one point. This risk measure is for interest rate instruments. For combined instruments, such as convertibles, only the effect on the bond part is accounted for.

Contract delta, Duration, Modified duration and Convexity

These contract parameters return the same values in the contents window as on the trading page.

Market move

Market move columns provide a quick way to see several risk measures under a standard simulation. the Up columns return the risk measure value if the base price were to move up by a user-defined percentage. Similarly, the Down columns return the risk measure value if the base price were to move down by a user-defined percentage.

BASKET CALCULATIONS AND THE BASKET WATCH WINDOW

Basket bid and ask

The basket bid B_b and ask B_a are calculated from the components bid and ask prices respectively as

$$B_a = Pricemultiplier\ of\ basket \cdot \sum_{i=1}^{n} v_i m_i (S_a)_i$$

and

$$B_b = Pricemultiplier\ of\ basket \cdot \sum_{i=1}^{n} v_i m_i (S_b)_i ,$$

where m_i is the multiplier, v_i the volume, $(S_a)_i$ and $(S_b)_i$ the ask and bid prices of component i. If there is no ask or bid prices the last paid or close price is used.

Basket split calculations

In portfolios there is functionality for moving some greeks from positions in basket derivatives to virtual position in the components. Thus, when entering an OMX forward position into a basket and then split over the OMX Basket, the resulting risk appears for each of the components. As a consequence, it is immediately seen the number of, say LME b, shares you must hedge with to be delta neutral in the LME b underlying. The functionality applies to the following columns: delta of position, gamma of position, vega of position, rho of position, swimming skew delta of position, cash delta of position, and swimming skew cash delta of position.

Calculation of virtual positions

For the greeks that are shown in money, i.e. vega of position, rho of position, cash delta of position, and swimming skew delta of position, the greek value from the derivative is distributed according to the market value weight of the respective components.

For delta and swimming skew delta of position, the greek value is distributed according to the total volume of the component. The total volume is

$$Total\ volume\ for\ component\ i = Pricemultiplier\ of\ basket \cdot volume \cdot m_i .$$

In addition, since the delta and swimming skew delta of position represent a number of base contracts, there could be additional adjustments since the base contract (and thus possibly the multiplier and time to expiry of the base contract) has changed.

For gamma of position, the greek value is distributed according to the square of total volume of the component. In addition, since the gamma of position represents a derivative of a number of base contracts, there could be additional adjustments since the base contract (and thus possibly the multiplier and time to expiry of the base contract) has changed.

Basket watch columns

Most of the columns available for a Basket Watch window are identical to the Contents widow columns. Columns that are specific for Basket Watch windows are described below

Component volume

The volume of the component in the basket

Component weight

The number of shares in the component divided by the total number of shares in the basket.

Next dividend of component

The next dividend of the component

Date of next dividend

The date of the next dividend.

Weight of next dividends of component

The weight of the next dividend.

Weighted next dividend of component

The effect that the dividend will have on the price of the basket.

Component market value weight

The total market value of the component divided by the total market value of the basket.

Index split in points

The Component market value weight multiplied by the price of the basket.

Performance

The performance of the component since closing. Where c indicates the price of the component and b indicates the price of the basket.

Basket delta residual

This column shows how much delta you have in this component. It is calculated as a sum of contributions (to delta) from component position and from basket derivative positions. The contribution from the basket positions to component i is calculated as

$$Contract\ delta \cdot volume \cdot m_i \cdot price\ multiplier \cdot v_i \cdot price\ multiplier\ of\ basket\ .$$

Basket gamma residual

This shows an estimate of the effect on Basket delta residual when the component price moves by one unit. It is calculated as a sum of contributions (to gamma) from component position and from basket derivative positions. The contribution from the basket positions is calculated as

$$Contract\ gamma \cdot volume \cdot m_i \cdot price\ multiplier \cdot (v_i \cdot price\ multiplier\ of\ basket)^2$$

Basket delta

The value shows the number of baskets that corresponds to the Basket delta residual for this component. It is a sum of the delta for the basket positions and a contribution for each position in the component as

$$\frac{Contract\ delta \cdot volume \cdot multiplier \cdot price\ multiplier}{price\ multiplier\ of\ basket \cdot v_i \cdot Basket\ multiplier}\ .$$

Basket gamma

This shows and estimate of the effect on Basket delta when the component price moves by one unit. It is a sum of the gamma for the basket positions and a contribution for each position in the component as

$$\frac{Contract\ gamma \cdot volume \cdot multiplier \cdot price\ multiplier}{(price\ multiplier\ of\ basket \cdot v_i)^2 \cdot Basket\ multiplier}.$$

Adjusted basket delta residual

This is the Basket delta residual measure in terms of the basket. The value is

$$Basket\ delta\ residual \cdot \frac{multiplier \cdot component\ value}{Basket\ multiplier \cdot (Basket\ value)}\quad .$$

PORTFOLIO MANAGEMENT

This section gives a short description of how result management is done in Orc and how it enables control of portfolios with high transaction frequency, e.g. market marker portfolios. The goal with the principles for result management given is to avoid psychological barriers and irrational behavior from traders, and also to give an opportunity to have an instant overview of results and risk exposure according to the so called mark-to-market principle.

Back-office labour is also greatly simplified if you concentrate on the two most essential pieces of information, payment cash flows and position reconciliation.

Risk management

Rule 1

All risk is measured from the actual market values. How the price has moved earlier is of no relevance. Historical price movements only decide the result of today and cannot be influenced.

Example If you have bought 10000 XYZ share at 400 and the market value is 200, the risk is measured from 200. If you want to protect yourself from a decline in the market price, the hedge should be decided on bases of a market price of 200 and not from a market price of 400.

Rule 2

Profits and losses are “realized” by diminishing the risk, not by selling or buying individual holdings. The risk should always be diminished by choosing the best alternative, i.e. the cheapest and most efficient alternative to achieve what you *really* want.

Example Assume you have a portfolio containing 10000 XYZ share with a market value of 2000000. You want to diminish the risk in this portfolio and you have the following means of doing it

- Sell some of your holdings
- Buy put options on XYZ
- Sell some XYZ futures or forwards
- Use all of the above alternatives on the market index

Which is the best alternative? To answer that question you have to ask yourself

- Which part of risk do I wan to get rid of?
- the total risk?
- The market risk?
- The company specific risk?
- How large part of the risk do I want to get rid of?
- Which of the alternatives gives the best return given the chosen level of risk?

When the questions above have been answered, you can make a rational decision of what is “best”. The decision is of course to a certain degree subjective, but *the subjective decision is based on a rational analysis of the alternatives available.*

Advantages

There are many advantages in measuring your results as above:

- the price at which the position was taken is irrelevant. You do not have to spend time on complex models like “First in, first out”, “Last in, first out”, average paid and so on to be able to calculate unrealized results. This saves a lot of energy.

- It is sufficient to know the position and the invested amount. The accumulated result can always be calculated by looking at how much have been invested and relate to the actual position. The difference between the invested amount and the market value is the accumulated result for the position.
- Realized and unrealized profits are put on an equal basis. The trader does not have to make the always difficult decision to unwind a loosing position. The loss is taken instantly as the market value is changing. Performance during the day isn't affected by an unpleasant position somewhere which the trader does not want to unwind because of unwillingness to take a loss.
- Easier administration. The administration can concentrate on controlling what is important, namely to reconcile the invested amounts and positions with the true ones instead of checking each separate transaction (which is necessary if average prices are measured). It is enough to look at separate transactions if there are difference in invested amounts or positions.
- More rational investment decisions. Realized profits and losses will no longer govern the trading activity. Each day is a new day, where you start from zero, in spite of earlier results.
- Risk management is made easier. By always taking the invested amount as a starting point you can strive for optimizing risk and return from the actual market price. There are no ties to historical performance in the portfolio. If you think that an option or a stock is worth selling, you should sell it without bothering about whether losses are realized or not. This is easier if the mark-to-market principle is followed.

Position currency

For each trade the user can specify which currency to invest in. This enables the possibility to trade the same asset in different currencies with a result that does not depend on exchange rates.

Example Buy 1000 Nokia shares at EUR 50 and sell 1000 Nokia shares at SEK 459. If you invest both positions in EUR (at a cross rate of 9), the result is EUR 1000. The result will not be changed if the EUR/SEK exchange rate changes.

Result management

The main principle behind the result management in Orc is that the result is equal to the market value less the invested amount.

Book-keeping

If the invested amount is incorrect it can be adjusted by a cash transaction, like carry, correction, cash, balance, fee, or lending. Note that cash transactions only affects the invested amount for the underlying. If the invested is to be change by 50000 a cash transaction of 50000 must be performed. To do this, enter the Book-keeping form as below. the sign in the Amount field determines if the invested amount should be increased or decreased, +50,000 means that the invested amount will be increased and -50,000 means that the invested will be decreased. Note that an increase of the invested amount will decrease the result, since the result equals the market value less the invested amount.

Overnight financing

This functionality allows the user to book overnight financing for all positions in a portfolio. The user can enter a financing rate and a number of days.

Expiration

Suppose the following position:

Underl	Vol	Inv	Mv	Res
XYZ Spot	1000	150000	172500	22500
XYZ K150	10	5000	25000	20000

If the Expiration command is selected, where Perform closing transactions is selected, Orc will generate a transaction and update the portfolio as below:

Underl	Vol	Inv	Mv	Res
XYZ Spot	2000	300000	345000	45000

The 150 calls are sold and stocks bought at 150. The invested amount for spot is updated with the premium, 150000. The invested amount for the calls can be displayed if Show zero volume is selected in the What to list dialog.

Underl	Vol	Inv	Mv	Res
XYZ Spot	1000	150000	172500	22500
XYZ K150	10	5000	25000	20000

Mark-to-market

The invested amount will be set equal to the current market value of position. Balance will be updated accordingly.

APPENDIX A-48 Portfolio management

APPENDIX

CALCULATED EXAMPLES

Pricing a forward

Example Assume that the underlying asset is trading at 760 and that the forward expires in 30 days. The present value of 1 paid out in thirty days is assumed to be $1/1{,}009 \approx 0{,}991.$ We also assume that no dividends are paid during the life time of the forward. The theoretical price of the forward then becomes

$$F = 1{,}009 \cdot 760 = 766{,}84$$

If we assume that some dividends are paid out, and that the present value of these divdends is equal to 14.1 we get

$$F = 1{,}009 \cdot (760 - 14{,}1) \approx 752{,}61$$

In this last case, the theoretical price will be below the spot price, since the present value of the dividends exceeds the interest rate effect.

Example The pricing of bond forwards are made in slightly different ways in Orc and Bloomberg. This can best be seen by an example accompanied by all necessary calculations. In our example, we will consider the Swedish benchmark bond 1028.

As the trade date we take Wednesday 961023. This gives a settlement date of Monday 961028. Let us also assume that the 1028 spot contract is trading at 5.35 ytm, which gives a clean price of 111.539 and a dirty price of 120.003.

We will price a forward with expiration at 970423, assuming a repo rate of 4.6%. The final settlement date is 970428. The rate day convention used is Actual/360.

Orc calculations

The number of interest rate days between 961028 and 970121 is 85. The number of interest rate days between 961028 and 970428 is 182. Calculation of theoretical dirty forward price:

$$F = \left(120{,}003 - \frac{11}{1 + 0{,}046 \cdot 85/360}\right) \cdot \left(1 + 0{,}046 \cdot \frac{182}{360}\right) = (120{,}003 - 10{,}882) \cdot 1{,}2325\ldots = 111.659$$

This gives a theoretical clean price of 111.657 - 2.964 = 108.693395. The difference between the two values is explained by the fact that

$$\frac{1 + rt_1}{1 + rt_2} \neq 1 + r(t_1 - t_2),$$

if r>0 and t_1¦t_2¡0.

In words this means that for straight interest rates, there is a difference between taking the future value of a future cash flow and taking the same future value, from today, of the present value of the cash flow.

Bond Options

There are two reasons for the difference between a theoretical option price in Orc and Bloomberg. First, and most important, there is a difference in the calculation of the underlying synthetic forward price in Orc and Bloomberg. This means that there will be a slight difference in the forward price used in the Black & Scholes formula. But it also means that the conversion between yield volatility and clean price volatility is not exactly the same in Orc and Bloomberg. Secondly, Orc has two conventions for volatility time. If Actual is used, we are using the same convention as in Bloomberg.

Example To illustrate this we use the same example as above for a call option with strike 5.35 ytm, corresponding to a clean strike price of 109.052. We assume a yield volatility of 17.2:

Orc calculations

the theoretical clean price is F = 108.695, which is equivalent to 5.556 yield. Assuming an Actual over Actual volatility time convention, the volatility time t_σ =182/365=.4986... The conversion to the price volatility is given by

$$\sigma_p = \frac{\sigma_y}{F_p} \cdot \Delta_y \cdot F_y = \frac{0,172}{108,695} \cdot 1,73045 \cdot 5,556 = 0,01521$$

Black 76 (Clean) gives:

$$c = \frac{1}{0,046 \cdot 182/360}(FN(d_1) - XN(d_2)),$$

where

$$d_1 = \frac{\ln(F/X) + \sigma_p^2 t_\sigma/2}{\sigma_p\sqrt{t_\sigma}} = \frac{\ln(108,695/109,052) + 0,0152^2 \cdot 0,4986/2}{0,0152 \cdot \sqrt{0,4986}} = -0,297696$$

and

$$d_2 = d_1 - \sigma_p\sqrt{t_\sigma} = -0,297696\ldots - 0,010742\ldots = -0,310468\ldots$$

The theoretical price given by Orc is

$$\begin{aligned} c &= 0,97727 \cdot ((108,695)N(-0,2997) - 109,052N(-0,3104)) \\ &= 0,97727 \cdot (108,695 \cdot 0,38219 - 109,052 \cdot 0,378102) = 0,3027\ldots \end{aligned}$$

Bloomberg Calculations

The conversion to the price volatility is given by

$$\sigma_p = \frac{\sigma_y}{F_p} \cdot \Delta_y \cdot F_y = \frac{0,172}{108,693} \cdot 1,730 \cdot 5,557 = 0,01511$$

After this, the calculation is done in the following way, all according to page OVY 27 in Bloomberg. $F=108.693, X=109.052, \sigma_p=0.01511, t=182/365$ and $r=0.46$ (this is 0.46 actual/360 converted into a continuously compounded interest rate with exp(r182/365)=1+0.046·182/360.)

$$d_1 = \frac{\ln(F/X) + \sigma_p^2 t_\sigma/2}{\sigma_p\sqrt{t_\sigma}} = \frac{\ln((108,693/109,052) + 0,01511^2 \cdot 0,4986/2)}{0,01511 \cdot \sqrt{0,4986}} = -0,301491\ldots$$

and

$$d_2 = d_1 - \sigma_p\sqrt{t_\sigma} = -0,301491\ldots - 0,010669\ldots = -0,31438\ldots$$

The theoretical price given by Bloomberg is

$$\begin{aligned} c &= 0,97727 \cdot (108,693N(-0,30372) - 109,052N(-0,31438)) \\ &= 0,97727 \cdot (108,693 \cdot 0,3806706 - 109,052 \cdot 0,3766162) = 0,2985\ldots \end{aligned}$$

Options on stocks and baskets

Example We calculate theoretical prices for all models that can be based on stocks or baskets in the Orc. The relevant parameters are as follows:

Base contract	Spot	Base price	100
Trade date	980630		
Strike	100		
Exercise date	980928		
Settlement days: Spot:	5 days	option:	3 days

Final Settlement days: Spot:	5 days	option:	3 days		
Dividend 1 (of Spot): Ex-div date	980713	Div date	980714	Dividend	3
Dividend 2 (of Spot): Ex-div date	980813	Div date	980814	Dividend	4
Volatility (actual)	30				
Financing rate (simple)	5				
Day count conversion	Actual/ 360				

The theoretical values of the options are then as in Table 4:

Model	Call	Put
Default (European execution)	3.25962	9.04543
Black&Scholes	3.25962	9.04543
Binomial (European execution)	3.25875	9.04754
Trinomial (European execution)	3.25829	9.04551
Bjerksund	3.84554	9.04543
Barone-Adesi		9.20099
Geske	3.69121	
Binomial (American execution)	3.68919	9.23324

Table 4: Theoretical values for different models.

For European options, all models are in good agreement. Black&Scholes is the fastest and is therefore recommended. Bjerksund is not accurate, since there are large (discrete) dividends.

Contract	Model	Call	Put	Barrier
Binary	Cash or Nothing	0.44	0.55	
	One-Touch deferred	0.99	0.83	
	One-Touch immediate	1.00	0.84	
Barrier	Down and In	0.77	5.90	L=90

Contract	Model	Call	Put	Barrier
	Up and In	4.45	0.45	U=110
Range	Range In	4.83	5.93	L=90
				U=110

Table 5: Theoretical values for different Barrier contracts and models.

Example Consider a Quanto call with 180 days to expiry. The underlying price is 100, the strike is 105, the predetermined exchange rate is 1.5, the domestic interest rate 8% continuously compounding, the foreign interest rate 5% continuously compounding. The dividends of the underlying corresponds to a continuous dividend yield of 4%. The yearly volatility of the underlying is 20%, the yearly volatility of the exchange rate is 10% and the correlation is 0.30. The calculations for the theoretical value

$$d_1 = \frac{ln(100/105) + (0{,}05 - 0{,}04 - 0{,}3 \cdot 0{,}2 \cdot 0{,}1 + 0{,}2^2/2)0{,}5}{0{,}2\sqrt{0{,}5}} = -0{,}2601457,$$

$$d_2 = d_1 - 0{,}2\sqrt{0{,}5} = -0{,}401567,\ N(d_1) = 0{,}3974,\ N(d_2) = 0{,}3440,$$

$$c = 1{,}5e^{-0{,}08 \cdot 0{,}5}(100e^{(0{,}05 - 0{,}04 - 0{,}3 \cdot 0{,}2 \cdot 0{,}1)0{,}5}0{,}3974 - 105 \cdot 0{,}3440) = 5{,}3280.$$

Financing rate from a yield curve

1. Convert the yields to continuous interest rates.
2. Interpolate according to the formula (15.1) in "Rate model" on page 69. The day convention for the contract is the same day convention as for the entry i in the yield curve.
3. After the interpolation, a continuous interest rate is received. Adjust this rate with the term t_r/t_i where t_r is calculated using the same day convention as for the entry i in the yield curve. This adjusted rate is then used in the calculations.

Example1 Yield curve:

Days	Rate (%)	Rate type	Rate days	Entry
365	5	Annual comp.	30/360	i
1095	6	Annual comp.	30/360	i+1

Calculation date: 2001-08-24

Contract = Forward

Expiry date = 2003-08-24

Spot price = 1000

No dividends or settlement days.

Actual number of days between expiry and calculation date is 730.

Since 2003-08-24 is a Sunday, 731 days are used.

Number of days with 30/360 convention is 721 (adjusted with one day due to Sunday at expiry).

The interpolated continous rate, r, is calculated as follows:

$$r = ln(1{,}05) + \left(ln(1{,}06) - \frac{ln(1{,}05)}{\frac{1095}{360} - \frac{365}{360}} \right)\left(\frac{721}{360} - \frac{365}{360}\right) = 0{,}05341267$$

r is adjusted by $\frac{t_r}{t}$ to get

$$r' = r\frac{721/360}{731/365} = 0{,}53413690$$

This rate is used to calculate the theoretical forward price as

$$F = Se^{r't} = 1000e^{\left(0,5341369\frac{731}{365}\right)} = 1112,905$$

Example2 Same as example1, but with a slightly different yield curve.

Yield curve:

Days	Rate (%)	Rate type	Rate days	Entry
365	5	Annual comp.	**Act**/360	i
1095	6	Annual comp.	30/360	i+1

Since the entry i has a different day convention, the calculations will differ a little bit:

$$r = ln(1,05) + \left(ln(1,06) - \frac{ln(1,05)}{\frac{1095}{360} - \frac{\mathbf{365}}{\mathbf{365}}}\right)\left(\frac{\mathbf{731}}{\mathbf{365}} - \frac{365}{\mathbf{365}}\right) = 0,05344553$$

Since the day convention for the entry is act/365, no adjustment has to be done in this case and this rate is used to calculate the theoretical forward price as:

$$F = Se^{rt} = 1000e^{\left(0,53445533\frac{731}{365}\right)} = 1112,976$$

Example3
- Contract: Put
- Expiry date: 050916
- Expiry type: European
- Strike: 11.00
- Underlying price: 11.13
- Volatility: 15.40860%
- Settlement days: 0
- Final settlement days: 3
- Underlying spot settlement days: 0
- Settlement type: Delivery
- Calculation date: 041217

Calculation of interest rate to use:
- Entry prior to expiry date, 050913.
- Rate 2.28.
- Rate type Straight 1/(1+rt).
- Rate days Actual/360.
- Entry past expiry date, 051214
- Rate 2.3
- Rate type Annual compounding 1/((1+r)^t)
- Rate days 30/360

1) Conversion of yields to continuously compounded.

Entry prior to expiry date

$$PV = \frac{1}{1 + r_s t} = e^{-r_{cc}t},$$

where r_s is the rate expressed as straight, r_{cc} is the rate expressed as continuously compounding and t is the time to entry expressed in years using Actual/360 as day convention.

This gives us $r_{cc} = \frac{ln(1 + r_s t)}{t}$.

Days is Actual number of days between 041216 and 050913 = 270 and t will be 270/360.

$$r_{ccprior} = \frac{ln(1 + 0,0228(270/360))}{270/360} = 2,26072542\ \%$$

Entry past expiry date is $PV = (1 + r_{ac})^{-t} = e^{-r_{cc}t}$, where r_{ac} is the rate expressed as Annual compounded, r_{cc} as above and t is t he time to entry expressed in years using 30/360 as day convention.

This gives us

$$r_{cc} = ln(1 + r_{ac})$$

$$r_{ccpast} = ln(1 + 0,023) = 2,2739487\ \%$$

2) Interpolation of yields

The interpolation is done linearly with respect to time. The day convention used for t_{ccpast} and $t_{ccprior}$ is the convention set for the entry and the day convention used for $t_{expiration}$ is the day convention from the entry prior to expiry date.

$$r = r_{ccprior} + \frac{r_{ccpast} - r_{ccprior}}{t_{past} - t_{prior}}(t_{expiration} - t_{prior})$$

t_{prior} is Actual days to prior entry/360 = 270/360

$t_{expiration}$ is Actual days to expiry(including settlement days)/360 = 278/360

t_{past} is Days to past entry (30/360 convention) / 360 = /See below for full calculation/ = 357/360

This gives us

$$r = 2,26072542 + \frac{2,2739487 - 2,26072542}{\frac{357}{360} - \frac{270}{360}}\left(\frac{278}{360} - \frac{270}{360}\right) = 2,2619413\ \%$$

Days is Number of days between 041217 and 051214 using 30/360 day convention.

d1/m1/y1 = 17/12/04

d2/m2/y2 = 14/12/05

Days = 360 * (y2 - y1) + 30 * (m2 - m1) + d2° - d1° = 360*1 + 30*0 + 14 - 17 = 357

3) Adjustment to Actual/365

The calculated rate is adjusted by a factor to get the rate $r°$ to use in forward calculation according to $r° = r(t_{priorexpiry}/t_{act}/365)$, where $t_{priorexpiry}$ is the time to expiration using day convention from prior rate entry and $t_{act}/365$ is the time to expiration using Actual/365 as day convention (including settlement days).

$$r° = 2,260476\frac{278/360}{278/365} = 2,2933572\%$$

Calculation of theoretical forward:

The time for theoretical forward calculation always starts on Calculation date + Underlying base contract settlement days. The end date is determined by the final settlement days on the option if Settlement type is Delivery and by Underlying base contract settlement days if it is Cash. In this case the end date will be Expiry date + Settlement days.

Days = (Expiry date + Settlement days) - (Calculation date + Underlying base contract settlement days) = 050921 - 041217 = 278 t° = 278/365

This gives us

$$Theoforw = 11{,}13e^{r^\circ t^\circ} = 11{,}32611792$$

$$Theoforw_{Orc} = 11{,}326112$$

Calculation of theoretical put price:

Vol time to expiration is 273

actual days to expiry date/365

$$d_1 = \frac{ln\left(\frac{F}{X}\right) + \frac{v^2}{2t}}{vt^{0,5}} = \frac{0{,}02921611 + 0{,}00887904}{0{,}13325947} = 0{,}28587199$$

$$d_2 = d_1 - vt^{0,5} = 0{,}28587199 - 0{,}13325947 = 0{,}15261252$$

$$CDF(-d_1) = CDF(-0{,}28587199) = 0{,}387488$$

$$CDF(-d_2) = CDF(-0{,}15261252) = 0{,}439352$$

e^{-rt} is the PV factor for the time calculation date + option settlement days to expiry date + option final settlement days for delivery settled options.

For cash settled the final date is instead expiry date + underlying base contract settlement days (same interpolation logic for r as when calculating theoretical forward).

$$p = e^{-rt}(CDF(-d_2)X - CDF(-d_1)F) = ""$$

$$e^{-0{,}022933\frac{278}{365}} 11{,}00(0{,}439) - 11{,}326(0{,}387) = 0{,}43644674$$

In Orc the theoretical price returned is 0.436447

If instead the options were priced using simple base offset mode, the pricing would be done in the same way as written earlier but with theoretical forward instead calculated as:

F = Base price + base price offset.