Tbricks 2.13. Pricing Models

Contents

1 Instrument Styles

- FXXXXX – Future instrument
- OCEXXX/RWXXCE – European call options
- OPEXXX/RWXXPE – European put options
- OCAXXX/RWXXCA – American call options
- OPAXXX/RWXXPA – American put options
- OCEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In – European call option with barrier
- OPEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In – European put option with barrier
- RWXXCE, Variant Turbo; OCXXXX, Variant Turbo – Turbo call option
- RWXXPE, Variant Turbo; OPXXXX, Variant Turbo – Turbo put option
- RWXXCE, Variant CBBC; OCXXXX, Variant CBBC – CBBC call instrument
- RWXXPE, Variant CBBC; OPXXXX, Variant CBBC – CBBC put instrument
- RWXXCE, Variant Average price option; OCXXXX, Variant Average price option – European exercise arithmetic averaging Asian call option with discrete monitoring
- RWXXPE, Variant Average price option; OPXXXX, Variant Average price option – European exercise arithmetic averaging Asian put option with discrete monitoring
- OCEXXX, Variant Binary Cash – European binary cash call options
- OPEXXX, Variant Binary Cash – European binary cash put options

- OCAXXX, Variant Binary Cash – American binary cash “one-touch” call options
- OPAXXX, Variant Binary Cash – American binary cash “one-touch” put options
- EUXXXX – Units and funds
- RWXXCE, Variant EUSIPA 2300; OCXXXX, Variant EUSIPA 2300 – Leverage certificate call instrument
- RWXXPE, Variant EUSIPA 2300; OPXXXX, Variant EUSIPA 2300 – Leverage certificate put instrument
- OCEDXX – European call options on interest rates/notional debt securities
- OPEDXX – European put options on interest rates/notional debt securities
- OCADXX – American call options on interest rates/notional debt securities
- OPADXX – American put options on interest rates/notional debt securities
- XXXXXX – Any instrument in the system
- XXXXXX – Any option with Quanto instrument group set to target group with quanto correlation parameters, except debt instruments and options with physical delivery.
- RWXXCX, Variant Mini Futures; RWXXXX (is treated as RWXXCX), Variant Mini Futures; OCXXXX, Variant Mini Futures; OXXXXX (is treated as OCXXXX), Variant Mini Futures – Mini Futures call
- RWXXPX, Variant Mini Futures; OPXXXX, Variant Mini Futures – Mini Futures put
- RWXXCE, Variant Forward Start; OCEXXX, Variant Forward Start – Forward Start call
- RWXXPE, Variant Forward Start; OPEXXX, Variant Forward Start – Forward Start put
- RMXXXX, Variant EUSIPA 1300 – Tracker Certificate
- SVXXXX – Variance Swap
- DBFXXX – Bond
- DXXXXA – Deposit
- DTVXXX – Floating Rate Note
- EXXXXX, Variant Equity return – Equity return instrument – equity leg of Equity swap

- RMXXXX, Variant Capped Call – Capped call options, call spreads
- RMXXXX, Variant Capped Put – Capped put options, put spreads
- RMXXXX, Variant EUSIPA 1200 – Max Certificate
- RMXXXX, Variant EUSIPA 1250 – Capped bonus certificate
- RMXXXX, Variant EUSIPA 1320 – Uncapped bonus certificate
- FXXXXX, Variant Dividend futures – Dividend futures instrument

2 Settings

This section contains the set of different instrument parameters, attributes, strategy parameters and other data provided by the Tbricks system. Settings of the type Instrument parameter can naturally be specified for any instrument or group, but Pricing app will use only those sources (i.e. instruments and groups) that are listed in the "Sources" section of the respective setting. If the setting of the type Instrument parameter is marked as "Aggregated: ✓", it means that Pricing app will open an aggregated parameters stream in order to receive respective setting, in which case the value for such setting can also be specified for any parental group of the given source. Settings marked as "Supports overriding: ✓" can be overridden in the calculated values request via (instrument) override parameters. Settings of the type Strategy parameter sometimes can define an offset or a factor for some entity, in which case they will be marked as "Supports offsetting: ✓". Last, in the current notation we refer to the instrument for which calculated values are requested as Valuation instrument.

- Underlying price scale factor Instrument parameter
  Default: 1.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Underlying price translation Instrument parameter
  Default: 0.0
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Underlying forward price offset Instrument parameter
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Underlying forward price adjustment Instrument parameter
  Default: 0.0
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Rate Strategy parameter
  Unit: %
  Supports offsetting: ✓

- Rate Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- ZCYC override Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- ZCYC table Instrument parameter
  Sources: Underlying instrument, ZCYC override group
  Aggregated: ✓
  Usually specified for: Currency group, ZCYC override group
  See also: Setting up theoretical pricing, ZCYC provider

- Financing rate spread Instrument parameter
  Default: 0.0
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Rate offset Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Convenience yield Strategy parameter
  Unit: %
  Supports offsetting: ✓

- Convenience yield Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Convenience yield curve table Instrument parameter
  Sources: Underlying instrument
  Aggregated: ✓
  Usually specified for: Underlying instrument
  See also: Setting up theoretical pricing, Time variables used in Tbricks

- Use CCY1 ZCYC Instrument parameter
  Default: false
  Sources: Underlying instrument
  Aggregated: ✓
  Usually specified for: Underlying instrument

- Maturity date Instrument attribute
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
  See also: Time variables used in Tbricks

- Maturity time Instrument attribute
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
  See also: Time variables used in Tbricks

- Maturity time fallback Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: Time variables used in Tbricks

- Dividends table Instrument parameter
  Sources: Dividend instrument
  Aggregated: ✓
  Usually specified for: Dividend instrument
  Supports overriding: ✓
  See also: Setting up theoretical pricing, Basket dividend provider

- Dividend instrument Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Underlying instrument Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Dividend adjustment bid Instrument parameter
  Default: 100.0
  Unit: % (100.0 means no change)
  Sources: Valuation instrument

Aggregated: ✓
Usually specified for: Valuation instrument

- Dividend adjustment ask Instrument parameter
  Default: 100.0
  Unit: % (100.0 means no change)
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Dividend adjustment Strategy parameter
  Supports offsetting: ✓

- Strike price Strategy parameter
  Unit: Strike currency or Underlying currency

- Strike price Instrument attribute
  Unit: Strike currency or Underlying currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Instrument volatility Strategy parameter
  Unit: %
  Supports offsetting: ✓
  See also: Volatility models, Setting up theoretical pricing, How to check implied volatility calculation

- Instrument volatility Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: Volatility models, Setting up theoretical pricing, How to check implied volatility calculation

- Volatility model Instrument parameter
  Default: SVI
  Sources: Valuation instrument, Volatility group, Volatility surface group
  Aggregated: ✓
  Usually specified for: Product group, Volatility surface group, Volatility group
  Supports overriding: ✓
  See also: Volatility models, Volatility managers, How to check implied volatility calculation, Setting up theoretical pricing

- Volatility offset Instrument parameter
  Unit: %
  Sources: Valuation instrument, Volatility group, Volatility surface group
  Aggregated: ✓
  Usually specified for: Valuation instrument, Product group, Maturity group, Volatility surface group, Volatility group
  Supports overriding: ✓

See also: Volatility models, Volatility managers, How to check implied volatility calculation, Setting up theoretical pricing

- Volatility group Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Volatility models, Volatility managers, Setting up theoretical pricing, How to check implied volatility calculation

- Volatility surface group Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Product group, Maturity group
  See also: Volatility models, Volatility managers, Setting up theoretical pricing, How to check implied volatility calculation

- Fair bid volatility adjustment Instrument parameter
  Default: 0.0
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Fair ask volatility adjustment Instrument parameter
  Default: 0.0
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Barrier level Instrument attribute
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Barrier rebate Instrument attribute
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Barrier observation frequency Instrument parameter
  Default: 0 (continuous monitoring)
  Unit: observations/day
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Barrier period Instrument attribute
  Sources: Valuation instrument

Usually specified for: Valuation instrument
See also: Barrier watch app

- Rebate reference price Instrument parameter
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Barrier instrument Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Barrier crossed Instrument parameter
  Default: false
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Barrier watch app

- Barrier crossed time Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Barrier watch app

- Rebate min percentage Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Fixing prices table Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Setting up theoretical pricing

- Best price Market data
  Sources: Valuation instrument, Underlying instrument, Beta instrument
  See also: Market data subscription and processing

- Statistics Market data
  Sources: Valuation instrument, Underlying instrument, Beta instrument
  See also: Market data subscription and processing

- Bid/Ask override price Instrument parameter
  Unit: Currency of the respective source instrument
  Sources: Valuation instrument, Underlying instrument, Barrier instrument, Beta instrument
  Aggregated: ✓

Usually specified for: Respective source
Supports overriding: ✓
See also: Market data subscription and processing

- Bid/Ask fallback price Instrument parameter
  Unit: Currency of the respective source instrument
  Sources: Valuation instrument, Underlying instrument, Barrier instrument, Beta instrument
  Aggregated: ✓
  Usually specified for: Respective source
  Supports overriding: ✓
  See also: Market data subscription and processing

- Manual reference price Instrument parameter
  Unit: Currency of the respective source instrument
  Sources: Valuation instrument, Underlying instrument, Beta instrument
  Aggregated: ✓
  Usually specified for: Respective source
  Supports overriding: ✓

- Underlying price Strategy parameter
  Unit: Underlying instrument currency
  Supports offsetting: ✓
  See also: Market data subscription and processing

- Underlying price source Strategy parameter
  See also: Market data subscription and processing

- Underlying price source ranking Strategy parameter
  See also: Market data subscription and processing

- Underlying MIC Strategy parameter
  See also: Market data subscription and processing

- Underlying volume Strategy parameter
  See also: Market data subscription and processing

- Underlying use primary market Strategy parameter
  See also: Market data subscription and processing

- Reference price date Strategy parameter
  See also: Market data subscription and processing

- Reference price
  See also: Market data subscription and processing

- Underlying price source ranking Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Market data subscription and processing

- Default underlying price source ranking User preference
  See also: Market data subscription and processing

- Underlying use primary market User preference
  See also: Market data subscription and processing

- Underlying instrument Strategy parameter

- Net asset value Instrument parameter
  Unit: Instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: ETF parameters importer

- ETF cash Instrument parameter
  Unit: ETF cash CCY
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- ETF cash currency Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- ETF reference price Instrument parameter
  Default: 0.0
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- Units per block Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- Underlyings pre block Instrument parameter
  Default: 1.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- ETF FX reference price Instrument parameter
  Default: 1.0

Sources: Valuation instrument
Aggregated: ✓
Usually specified for: Valuation instrument
Supports overriding: ✓
See also: ETF parameters importer

- Use snap ETF pricing Instrument parameter
  Default: false
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- ETF adjustment factor Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- ETF tracking factor Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: ETF parameters importer

- Instrument fair market price Instrument parameter
  Unit: Instrument currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Taylor reference values table Instrument parameter
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Taylor derivatives 1 table Instrument parameter
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Instrument theta Instrument parameter
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Taylor derivatives 2 table Instrument parameter
  Sources: Valuation instrument
  Usually specified for: Valuation instrument

- Ratio of the Instrument component
  Sources: Valuation instrument, Underlying instrument

- Volatility relations table Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
- Expand underlying legs Instrument parameter
  Default: false
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
- External values are valid Instrument parameter
  Default: false
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
- External model valid until Instrument parameter
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- External model valid range Instrument parameter
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- Respective Instrument parameter/ Strategy parameter/ Instrument attribute/ etc
- Grant date Instrument attribute
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- Forward start ratio Instrument attribute
  Default: 1.0
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- Certificate dividend multiplier Instrument parameter
  Default: 1.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
- Issue date Instrument attribute
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- Last reference price Instrument parameter
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Realized variance Instrument parameter
  Unit: %
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Variance swap replication accuracy Instrument parameter
  Default: 500
  Unit: Number of strikes
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Variance swap percentage of ATM Instrument parameter
  Default: 99.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Time decay granularity Instrument parameter
  Default: 24
  Unit: hours
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Product group
  See also: Time variables used in Tbricks

- Time decay start Instrument parameter
  Default: 00:00:00
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Product group
  See also: Time variables used in Tbricks

- Time decay spread Instrument parameter
  Default: 5
  Unit: minutes
  Sources: Underlying instrument
  Aggregated: ✓
  Usually specified for: Underlying instrument
  See also: Time variables used in Tbricks

- Valuation datetime Strategy parameter
  See also: Time variables used in Tbricks

- Payment instrument Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Payments table Instrument parameter
  Sources: Payment instrument

Aggregated: ✓
Usually specified for: Payment instrument

- Dividend time fallback Instrument parameter
  Sources: Valuation instrument, Dividend instrument, Payment instrument
  Aggregated: ✓
  Usually specified for: Dividend instrument
  Supports overriding: ✓
- Start date Instrument parameter
  Sources: Payment instrument
  Aggregated: ✓
  Usually specified for: Payment instrument
- End date Instrument parameter
  Sources: Payment instrument
  Aggregated: ✓
  Usually specified for: Payment instrument
- Start amount Instrument parameter
  Unit: Payment instrument currency
  Sources: Payment instrument
  Aggregated: ✓
  Usually specified for: Payment instrument
- End amount Instrument parameter
  Unit: Payment instrument currency
  Sources: Payment instrument
  Aggregated: ✓
  Usually specified for: Payment instrument
- Cap price Instrument attribute
  Unit: Underlying instrument currency
  Sources: Valuation instrument
  Usually specified for: Valuation instrument
- Barrier window start Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
- Settlement Calendar
  Default: XXXX
  Sources: Underlying instrument
  See also: Calendar resource, Time variables used in Tbricks
- Settlement days Instrument parameter
  Default: 0
  Unit: days
  Sources: Underlying instrument, Valuation instrument
  Aggregated: ✓
  Usually specified for: Underlying instrument

Supports overriding: ✓
See also: Time variables used in Tbricks

- Final settlement days Instrument parameter
  Default: 0
  Unit: days
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: Time variables used in Tbricks

- Trading Calendar
  Default: XXXX
  Sources: Underlying instrument
  See also: Calendar resource, Time variables used in Tbricks

- Volatility day convention Instrument parameter
  Default: Actual/365
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓
  See also: Time variables used in Tbricks

- Volatility weight of weekend Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Time variables used in Tbricks

- Volatility weight of holiday Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Time variables used in Tbricks

- Volatility weight exceptions table Instrument parameter
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  See also: Time variables used in Tbricks

- Fair price offset Instrument parameter
  Default: 0.0
  Unit: Instrument currency
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Fair price multiplier Instrument parameter
  Default: 1.0

Sources: Valuation instrument
Aggregated: ✓
Usually specified for: Valuation instrument
Supports overriding: ✓

- Fair bid price adjustment Instrument parameter
  Default: 0.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Fair ask price adjustment Instrument parameter
  Default: 0.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

- Fair bid/ask decay start adjustment Instrument parameter
  Default: 0.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Fair bid/ask decay end adjustment Instrument parameter
  Default: 0.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Fair bid/ask decay start factor Instrument parameter
  Default: 1.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Fair bid/ask decay end factor Instrument parameter
  Default: 1.0
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- YTM day count convention Instrument parameter
  Default: Actual/Actual
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- YTM compounding basis Instrument parameter
  Default: Continuous
  Sources: Valuation instrument

Aggregated: ✓
Usually specified for: Valuation instrument

- Accrued interest day count convention Instrument parameter
  Default: Actual/Actual
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument

- Bond quoted Instrument parameter
  Default: Clean
  Sources: Valuation instrument
  Aggregated: ✓
  Usually specified for: Valuation instrument
  Supports overriding: ✓

3 Model Parameters

- S – The spot price of the underlying instrument.
  Dependencies: U , r, q, t_e, t_r, $D_i, 1 \leq i \leq N_D$, T_i^d
  Settings:
  - Underlying price scale factor Instrument parameter
  - Underlying price translation Instrument parameter
  - Underlying forward price offset Instrument parameter
  - Underlying forward price adjustment Instrument parameter

- r – The risk-free interest rate.
  Settings:
  - Rate Strategy parameter
  - Rate Instrument parameter
  - ZCYC override Instrument parameter
  - ZCYC table Instrument parameter
  - Financing rate spread Instrument parameter
  - Rate offset Instrument parameter

- q – The continuous convenience yield.
  Settings:
  - Convenience yield Strategy parameter
  - Convenience yield Instrument parameter
  - Convenience yield curve table Instrument parameter
  - Use CCY1 ZCYC Instrument parameter
  - Underlying forward price offset Instrument parameter

- T – The date and time of maturity for the instrument.
  Settings:
  - Maturity date Instrument attribute

- Maturity time Instrument attribute
- Maturity time fallback Instrument parameter

- $D_i, 1 \leq i \leq N_D$ – The discrete dividends paid by the underlying instrument.
  Settings:
  - Dividends table Instrument parameter
  - Dividend instrument Instrument parameter
  - Underlying instrument Instrument parameter
  - Dividend adjustment bid Instrument parameter
  - Dividend adjustment ask Instrument parameter
  - Dividend adjustment Strategy parameter

- t_e^U – Time to expiration of the underlying instrument expressed in years.
  Dependencies: T_0, T
  Settings:
  - Underlying instrument Instrument parameter

- K – The instrument strike price.
  Settings:
  - Strike price Strategy parameter
  - Strike price Instrument attribute

- σ – The volatility of the underlying instrument.
  Dependencies: F, K, t_σ
  Settings:
  - Instrument volatility Strategy parameter
  - Instrument volatility Instrument parameter
  - Volatility model Instrument parameter
  - Volatility offset Instrument parameter
  - Volatility group Instrument parameter
  - Volatility surface group Instrument parameter
  - Fair bid volatility adjustment Instrument parameter
  - Fair ask volatility adjustment Instrument parameter

- H – The barrier level for the instrument.
  Settings:
  - Barrier level Instrument attribute

- R – The cash rebate paid out if the instrument is rendered worthless by the barrier condition.
  Settings:
  - Barrier rebate Instrument attribute

- σ_H – The volatility of the underlying instrument at the barrier level.
  Dependencies: σ, H

- f_{obs} – The frequency with which the barrier condition is observed.
  Settings:
  - Barrier observation frequency Instrument parameter
- Δ_{t_R} – The length of the period of time after crossing the barrier that the value of the underlying is measured to determine the rebate.
  Settings:
  - Barrier period Instrument attribute
- S_{RR} – Maximal (for puts) or minimal (for calls) recorded price of underlying during rebate period after knockout event.
  Settings:
  - Rebate reference price Instrument parameter
- BC – Boolean value, determines whether the barrier was hit in the past.
  Settings:
  - Barrier instrument Instrument parameter
  - Barrier crossed Instrument parameter
- t_{BC} – Time of barrier knockout event.
  Settings:
  - Barrier instrument Instrument parameter
  - Barrier crossed time Instrument parameter
- R_{min} – Minimal rebate payment given as percentage of absolute difference between strike and barrier.
  Settings:
  - Rebate min percentage Instrument parameter
- $t_i, 1 \leq i \leq N_f$, $S_{t_i}, t_i < t$ – The discrete monitoring dates, and already fixed prices.
  Dependencies: t_e
  Settings:
  - Fixing prices table Instrument parameter
- U – The price of the underlying instrument.
  Settings:
  - Underlying instrument Instrument parameter
  - Best price Market data
  - Statistics Market data
  - Bid/Ask override price Instrument parameter
  - Bid/Ask fallback price Instrument parameter
  - Manual reference price Instrument parameter
  - Underlying price Strategy parameter
  - Underlying price source Strategy parameter
  - Underlying price source ranking Strategy parameter

- Underlying MIC Strategy parameter
- Underlying volume Strategy parameter
- Underlying use primary market Strategy parameter
- Reference price date Strategy parameter
- Reference price
- Underlying price source ranking Instrument parameter
- Default underlying price source ranking User preference
- Underlying use primary market User preference
- Underlying instrument Strategy parameter

- NAV – Net asset value.
  Settings:
  - Net asset value Instrument parameter

- Cash – Cash.
  Settings:
  - ETF cash Instrument parameter

- ETF Cash CCY – Currency of ETF cash component.
  Settings:
  - ETF cash currency Instrument parameter

- U_{Ref} – The reference price of ETF underlying instrument.
  Settings:
  - ETF reference price Instrument parameter

- Units – Units per block.
  Settings:
  - Units per block Instrument parameter

- Und – Underlyings per block.
  Settings:
  - Underlyings pre block Instrument parameter

- $\mathrm{FX}_{\mathrm{U_{Ref}}}$ – Reference exchange rate for the underlying vs ETF.
  Settings:
  - ETF FX reference price Instrument parameter

- Use snap ETF pricing – Use snap ETF pricing.
  Settings:
  - Use snap ETF pricing Instrument parameter

- Af – ETF adjustment factor.
  Settings:
  - ETF adjustment factor Instrument parameter

- Tf – ETF tracking factor.
  Settings:

- ETF tracking factor Instrument parameter

- F – The forward price of the underlying instrument.
  Dependencies: U , r, q, t_e, t_r, $D_i, 1 \leq i \leq N_D$, T_i^d
  Settings:
  - Underlying price scale factor Instrument parameter
  - Underlying price translation Instrument parameter
  - Underlying forward price offset Instrument parameter
  - Underlying forward price adjustment Instrument parameter

- V_0 – Value of the fair market price at the reference point provided by the external system.
  Settings:
  - Instrument fair market price Instrument parameter

- U_0^i – $1 \leq i \leq N, N$ - number of underlyings. Price of the i-th underlying instrument at the reference point provided by the external system.
  Settings:
  - Taylor reference values table Instrument parameter

- r_0^j – $1 \leq j \leq M \leq N, N$ - number of underlyings, M - number of unique underlying currencies. Rate of the j-th underlying instrument at the reference point provided by the external system.
  Settings:
  - Taylor reference values table Instrument parameter

- σ_0^i – $1 \leq i \leq N, N$ - number of underlyings. Volatility of the i-th underlying instrument at the reference point provided by the external system.
  Settings:
  - Taylor reference values table Instrument parameter

- Δ_0^i – $1 \leq i \leq N, N$ - number of underlyings. Value of $\Delta^i = \dfrac{\partial V}{\partial S^i}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter

- $\Gamma_0^{i,i}$ – $1 \leq i \leq N, N$ - number of underlyings. Value of $\Gamma^{i,i} = \dfrac{\partial^2 V}{\partial S_i^2}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter

- ρ_0^j – $1 \leq j \leq M \leq N, N$ - number of underlyings, M - number of unique underlying currencies. Value of $\rho^j = \dfrac{\partial V}{\partial r^j}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter

- ν_0^i – $1 \le i \le N, N$ - number of underlyings. Value of $\nu^i = \dfrac{\partial V}{\partial \sigma^i}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter
- ν_0^{Basket} – Reference value of the derivative of the fair value with respect to the volatility of the whole basket.
  Settings:
  - Taylor derivatives 1 table Instrument parameter
- $Vomma_0^{i,i}$ – $1 \le i \le N, N$ - number of underlyings. Value of $Vomma^{i,i} = \dfrac{\partial^2 V}{\partial \sigma_i^2}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter
- Θ_0 – Value of $\Theta = V(T + 1\ day) - V(T)$ at the reference point provided by the external system.
  Settings:
  - Instrument theta Instrument parameter
- $Charm\ C_0^i$ – $1 \le i \le N, N$ - number of underlyings. Value of $Charm\ C^i = \Delta^i(T + 1\ day) - \Delta^i(T)$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter
- $Charm\ C_0^{Basket}$ – Value of $Charm\ C^{Basket} = \Delta^{Basket}(T + 1\ day) - \Delta^{Basket}(T)$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 1 table Instrument parameter
- $\Gamma_0^{i,j}$ – $1 \le i \le N, 1 \le j \le N, i \ne j, N$ - number of underlyings. Value of $\Gamma^{i,j} = \dfrac{\partial^2 V}{\partial S^i \partial S^j}, i \ne j$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 2 table Instrument parameter
- $Vanna_0^{i,j}$ – $1 \le i \le N, 1 \le j \le N, N$ - number of underlyings. Value of $Vanna^{i,j} = \dfrac{\partial^2 V}{\partial S^i \partial \sigma^j}$ at the reference point provided by the external system.
  Settings:
  - Taylor derivatives 2 table Instrument parameter
- $Vomma_0^{i,j}$ – $1 \le i \le N, 1 \le j \le N, i \ne j, N$ - number of underlyings. Value of $Vomma^{i,j} = \dfrac{\partial^2 V}{\partial \sigma^i \partial \sigma^j}$ at the reference point provided by the external system.
  Settings:

- Taylor derivatives 2 table Instrument parameter

- w_i – $1 \leq i \leq N, N$ - number of underlyings, weight of the i-th leg of the underlying basket.
  Dependencies: Expand underlying legs
  Settings:
  - Ratio of the Instrument component
  - Underlying instrument Instrument parameter

- Volatility relations – Table specifying volatility group for each of the instrument's underlyings.
  Settings:
  - Volatility relations table Instrument parameter

- Expand underlying legs – Checkbox enabling subscription for the legs of the underlying instrument if the underlying is a basket.
  Settings:
  - Expand underlying legs Instrument parameter

- External values are valid – Checkbox enabling calculation of values based on the reference values for the external models.
  Settings:
  - External values are valid Instrument parameter

- Valid until – Date and time of expiration of the reference values for the external models.
  Settings:
  - External model valid until Instrument parameter

- Valid range – Range of the underlying prices within which reference values are considered valid.
  Settings:
  - External model valid range Instrument parameter

- Whichever parameters are relevant for the base model – .
  Settings:
  - Respective Instrument parameter/ Strategy parameter/ Instrument attribute/ etc

- t_g – The time until grant date for the instrument with forward start.
  Settings:
  - Grant date Instrument attribute

- α_{fs} – Spot price is multiplied by this value to get forward start contract strike price.
  Settings:
  - Forward start ratio Instrument attribute

- D_p – The discrete dividends already paid by the underlying instrument.
  Dependencies: T_0, $D_i, 1 \leq i \leq N_D$, T_i^d

- α_c – Fraction of all dividends received by the writer of a tracker certificate.
  Settings:
  - Certificate dividend multiplier Instrument parameter
- T_{issue} – Date on which an instrument is issued.
  Settings:
  - Issue date Instrument attribute
- Last Reference Price – price of the underlying asset on the last date of reference.
  Settings:
  - Last reference price Instrument parameter
- Realized Variance – realized variance of the underlying asset over the life of contract.
  Settings:
  - Realized variance Instrument parameter
- Variance swap replication accuracy – amount of put and call options used to replicate variance swap.
  Settings:
  - Variance swap replication accuracy Instrument parameter
- Variance swap percentage of ATM – width of replicating grid of variance swaps.
  Settings:
  - Variance swap percentage of ATM Instrument parameter
- T_0 – Date and time that pricing considers as the current ones.
  Settings:
  - Time decay granularity Instrument parameter
  - Time decay start Instrument parameter
  - Time decay spread Instrument parameter
  - Valuation datetime Strategy parameter
- T_i^f – $1 \leq i \leq N_n$ - pre-specified dates and times when rate is fixed.
  Settings:
  - Payment instrument Instrument parameter
  - Payments table Instrument parameter
  - Dividend time fallback Instrument parameter
- T_i^{st} – $1 \leq i \leq N_n$ - pre-specified dates and times when Pricing starts accruing the coupon.
  Settings:
  - Payment instrument Instrument parameter
  - Payments table Instrument parameter
  - Dividend time fallback Instrument parameter

- T_i^{ex} $- 1 \leq i \leq N_n$ - pre-specified dates and times of coupon payments.
  Settings:
  - Payment instrument Instrument parameter
  - Payments table Instrument parameter
  - Dividend time fallback Instrument parameter
- T_i^p $- 1 \leq i \leq N_n$ - pre-specified dates and times when coupon is actualy paid and registered in portfolio.
  Settings:
  - Payment instrument Instrument parameter
  - Payments table Instrument parameter
  - Dividend time fallback Instrument parameter
- c_i $- 1 \leq i \leq N_n$ - coupons/payments amounts for debt instruments.
  Settings:
  - Payment instrument Instrument parameter
  - Payments table Instrument parameter
- c_N – cash amount that is paid at maturity.
  Settings:
  - Payments table Instrument parameter
- T_{start} – start time of some period related to the instrument.
  Settings:
  - Start date Instrument parameter
- T_{end} – end time for deposit.
  Settings:
  - End date Instrument parameter
- B_{start} – amount paid at the start time.
  Settings:
  - Start amount Instrument parameter
- B_{end} – amount paid at the end time.
  Settings:
  - End amount Instrument parameter
- Cap Price – Cap price of spreads / capped instruments.
  Settings:
  - Cap price Instrument attribute
- t_{Window} – Start of barrier observation period.
  Settings:
  - Barrier window start Instrument parameter
- σ_{cap} – The volatility of the underlying instrument at the cap price.
  Dependencies: σ, Cap Price

- t_e – Time to expiration expressed in years.
  Dependencies: T_0, T

- t_r – Rate (settlement) time to expiration expressed in years.
  Dependencies: T_0, T
  Settings:
  - Settlement Calendar
  - Settlement days Instrument parameter
  - Final settlement days Instrument parameter

- T_i^d – $1 \le i \le N_n$ - pre-specified dates and times of dividend payments.
  Settings:
  - Dividends table Instrument parameter
  - Dividend time fallback Instrument parameter

- t_σ – Volatility (trading) time to expiration expressed in years.
  Dependencies: T_0, T
  Settings:
  - Trading Calendar
  - Volatility day convention Instrument parameter
  - Volatility weight of weekend Instrument parameter
  - Volatility weight of holiday Instrument parameter
  - Volatility weight exceptions table Instrument parameter

4 Calculated values

- V – Fair values (Fair market price of the instrument, fair bid and fair ask)
  Settings:
  - Fair price offset Instrument parameter
  - Fair price multiplier Instrument parameter
  - Fair bid price adjustment Instrument parameter
  - Fair ask price adjustment Instrument parameter
  - Fair bid/ask decay start adjustment Instrument parameter
  - Fair bid/ask decay end adjustment Instrument parameter
  - Fair bid/ask decay start factor Instrument parameter
  - Fair bid/ask decay end factor Instrument parameter

- $\Delta = \frac{\partial V}{\partial S}$, is calculated either analytically or numerically, depending on the model type

- $\Gamma = \frac{\partial^2 V}{\partial S^2}$, is calculated either analytically or numerically, depending on the model type

- $\nu = 0.01 \cdot \frac{\partial V}{\partial \sigma}$, is calculated either analytically or numerically, depending on the model type. Multiplication by 0.01 can be interpreted as the approximate effect on the theoretical price when the volatility of the contract increases by one percentage unit.

- $\rho = 0.0001 \cdot \frac{\partial V}{\partial r}$, is calculated either analytically or numerically, depending on the model type. Multiplication by 0.0001 can be interpreted as the effect on the contract price when the interest rate increases by one basis point.

- Vanna $= 0.01 \cdot \frac{\partial^2 V}{\partial S \partial \sigma} = 0.01 \cdot \frac{\partial \Delta}{\partial \sigma}$, is calculated by finite-differences. Multiplication by 0.01 can be interpreted as the approximate effect on the Δ when the volatility of the contract increases by one percentage unit.

- Vomma $= 0.0001 \cdot \frac{\partial^2 V}{\partial \sigma^2}$, is calculated by finite-differences. Multiplication by 0.0001 can be interpreted as the approximate effect on the theoretical price when the volatility of the contract increases by one basis point.

- Skew Greeks : Δ, Γ, Speed

- Calendar/Opening/Overnight Greeks
  Calendar:
  θ - the difference between instrument fair prices the next business day and today;
  charm - the difference between Δ-s the next business day and today;
  color - the difference between Γ-s the next business day and today;
  skew charm;
  Opening:
  θ - the difference between instrument fair prices the next trading day and today;
  charm - the difference between Δ-s the next trading day and today;
  color - the difference between Γ-s the next trading day and today;
  skew charm;
  Overnight:
  θ - the difference between instrument fair prices the next trading day market open and today market close;
  charm - the difference between Δ-s the next trading day market open and today market close;
  color - the difference between Γ-s the next trading day market open and today market close;
  skew charm;
  Volatility theta θ_σ - is θ calculated with the assumption of zero interest rate;
  Interest rate theta θ_r - the difference between θ and θ_σ

- Zomma $= \frac{\partial^3 V}{\partial S^2 \partial \sigma} = \frac{\partial \Gamma}{\partial \sigma}$, is calculated by finite-differences

- Speed $= \frac{\partial^3 V}{\partial S^3} = \frac{\partial \Gamma}{\partial S}$, is calculated by finite-differences

- OEG option equivalent gamma $\frac{\Gamma}{\Gamma_{\mathrm{ATM}}}$. Γ_{ATM} is at the strike price being equal to the forward price

- OEV option equivalent vega $\frac{\nu}{\nu_{\text{ATM}}}$. ν_{ATM} is at the strike price being equal to the forward price
- σ_{impl} – implied volatility (including bid and ask values)
- Exercise boundary indicating the market price threshold for early exercise
- Exercise boundary reached – identify positions which should be exercised early if possible
- Exercise boundary div indicating the early exercise boundary at the moment of the next dividend
- Exercise boundary div reached – identify positions which should be exercised early at the moment of the next dividend if possible
- Calculated values mentioned in the description of the model
- YTM Yield to maturity
  Settings:
  - YTM day count convention Instrument parameter
  - YTM compounding basis Instrument parameter
- R The rate amount for deposit
- bid implied YTM Yield to maturity corresponding to a bid price
- ask implied YTM Yield to maturity corresponding to an ask price
- fair YTM Yield to maturity corresponding to a fair market price
- Δ_{YTM} YTM-based Greek for bonds
- Γ_{YTM} YTM-based Greek for bonds
- a The fraction of coupon that belongs to the seller
  Settings:
  - Accrued interest day count convention Instrument parameter
  - Bond quoted Instrument parameter

5 Future / Forward Pricing Model

Front-end name: Future / Forward

Type: Analytical

Target Instruments: FXXXXX

Relevant Parameters: S , r, q, T, $D_i, 1 \leq i \leq N_D$, t_e^U

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

The mathematical concepts of forward price calculation are described in the "Financial mathematics" section.

For future instruments, forward and fair market prices are always equal.

6 Black-Scholes with Analytical Dividend Corrections

Front-end name: European / European Escrowed Dividend

Type: Analytical/Approximative Analytical

Target Instruments: OCEXXX/RWXXCE; OPEXXX/RWXXPE

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

For options simple enough to fulfill its rather stringent requirements, the celebrated Black-Scholes formula is an incredibly efficient way of obtaining a theoretical valuation. Relying on the fact that the probability distribution of the stock process at maturity is readily available as the well-studied log-normal distribution, the basic version of the formula does not handle any complications beyond the most fundamental.

A particular complication which is not naturally handled very well by the formula is that of discrete dividend payments. Due to its importance in concrete usage, this case has garnered a certain amount of attention. Two observations lead to two different approaches:

1. A dividend paid just after current time is equivalent to reducing the current spot price by the dividend amount
   - Discount each dividend back to current time, including the effect of any continuous yields. Reduce the current spot price by the sum of the results
2. A dividend paid just before maturity is equivalent to increasing the strike price by the dividend amount
   - Compound each dividend to maturity, including the effect of any continuous yields. Increase the strike price by the sum of the results

The former approach enjoys rather widespread usage, and is generally termed the *escrowed dividend model*.

Another approach is suggested by Bos and Vandermark [4], where the two above observations are combined. Each dividend is split into "near" and "far" fractions, and these are used to reduce the current stock value and raise the strike price respectively.

Formulae

The base formulae for the price of European call and put options are

$$V_C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$V_P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r - q + \sigma^2/2\right)}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r - q - \sigma^2/2\right)}{\sigma\sqrt{T}},$$

and $N(x)$ is the standard normal cumulative distribution function. In the case of discrete dividend payments being present, the two pricing models diverge. For the `European Escrowed Dividends` pricing model, the escrowed dividend model applies fully, replacing the current spot S by S', where

$$S' = S - \sum_{i=1}^{N_D} D_i e^{t_i(q-r)}.$$

and t_i, $i \in [1, N_D] \cap \mathbb{Z}$ are the dividend times.

For the `European` pricing model, meanwhile, the correction suggested by Bos and Vandermark applies and modifies S and K to S' and K', where

$$S' = S - \sum_{i=1}^{N_D} \frac{T - t_i}{T} D_i e^{t_i(q-r)},$$

$$K' = K + \sum_{i=1}^{N_D} \frac{t_i}{T} D_i e^{(T-t_i)(r-q)},$$

and the definitions are otherwise as above.

7 Binomial Pricing Model

Front-end name: American/American escrowed dividends

Type: Numerical – Lattice / Approximative Numerical – Lattice

Target Instruments: OCAXXX/RWXXCA; OPAXXX/RWXXPA

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}, Exercise boundary, Exercise boundary reached, Exercise boundary div, Exercise boundary div reached

Description

The binomial lattice model is a highly popular numerical method, owing greatly to its intuitive nature and how straightforward it is to implement. The basic idea is that an approximation of the stock process can be obtained by restricting its movement to a progressively expanding lattice of nodes. As the name implies, the process is allowed to move to one of two alternatives in each time step. The probabilities of each destination – as well as the values they represent – are degrees of freedom used to match characteristics of the process.

The degrees of freedom are spent to obtain the following behaviors

1. Probabilities summing to 1
2. Multiplying the upward and downward movement multipliers gives 1
3. First moment matching that of the stock process
4. Second moment approximately matching that of the stock process

A lattice constructed according to the above will have the important property that it is *recombining*. This means that a given combination of steps up and down will lead to the same point irrespective of the order in which they are taken, which is imperative in limiting the growth of the number of nodes in the lattice.

Once a lattice has been established with probabilities of progressing between the different states, the fact that the payoff function is known can be used to find option values in the nodes at maturity. Each previous node – being the discounted expectation of its successors – can then be calculated using the already estimated values and the probability of reaching them.

Handling the complication of early exercise is straightforward – the value estimation obtained from a node's successors is compared to the value of immediate exercise, and the maximum is kept.

Less straightforward is the correction for discrete dividend payments. Two approaches are available in Tbricks:

Vellekoop-Nieuwenhuis

The preferred method (front-end model name "American") follows a paper by Vellekoop and Nieuwenhuis [15]. Interpolation is used to recreate the truncated data that is required for the absence of arbitrage jump conditions. Should a discrete dividend payment fall close to the current time, the data available for interpolation might be insufficient to yield accurate results. In such cases the lattice is expanded to handle the problem at the cost of additional node calculations.

Escrowed dividends

Use not recommended: This model solves a rather different mathematical problem, one that is not consistent with most other models. Use only if there is a significant and clear argument in its favor.

For legacy purposes, an approach using escrowed dividends is also available (front-end model name "American escrowed dividends"). For this method, the total discounted dividend amount is calculated (see Black-Scholes with Analytical Dividend Corrections), and a lattice is built from the spot price minus this amount. Throughout the solution process, the discounted amount of dividends paid out between current time and maturity is tracked. This number is then added to the spot price of the lattice nodes when calculating the intrinsic option price to determine whether early exercise is optimal. Due to its construction, this approach only supports absolute-sized discrete dividends.

8 Trinomial Pricing Model

Front-end name: Trinomial

Type: Numerical – Lattice

Target Instruments: OCEXXX/RWXXCE; OPEXXX/RWXXPE; OCAXXX/RWXXCA; OPAXXX/RWXXPA

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}, Exercise boundary reached

Description

The trinomial lattice model is an alternative to its popular binomial sibling. They share the advantages of generally being considered highly intuitive, and simple to implement. Both exhibit first order convergence in number of time steps, but due to an added degree of freedom, the trinomial model is able to more accurately match the distribution characteristics of the simulated geometric Brownian motion.

The basic principle is similar to the binomial model — the (known) price at maturity is propagated backward in time until the present, using an approximation of the dynamics of the underlying process. At maturity the payoff condition can be immediately evaluated for any given set of stock values. Taking a step backward in time, this is no longer true, and so for any given point the value becomes an expression of the values in preceding step and the probabilities of reaching them. Selecting transition probabilities such that aggregate characteristics of the geometric Brownian motion are retained in the approximating lattice, each evaluation point is then tasked with representing the value and probability of its immediate vicinity.

Structure

The geometric Brownian motion process is transformed into a standard Brownian motion with drift by a logarithm transformation. Further, the deterministic drift component is stripped, leaving a scaled Brownian motion as the target for simulation. For the simulation, the evolution from each point of evaluation is assumed to follow one of three paths, i.e.

$$X(t+\Delta t) = X(t) + \begin{cases} u & \text{with probability } p_u \\ m & \text{with probability } p_m \\ d & \text{with probability } p_d, \end{cases}$$

with $X(t)$ being the process approximating the distribution $\mathcal{N}(0, \sigma\sqrt{t})$. There are thus six degrees of freedom: three representing the change for each path, and three representing their probabilities. In order to approximate the normal distribution these can be spent [9] to

1. Have the probabilities sum to one
2. Set the middle path to represent no change
3. Set the changes in up and down directions to be equal
4. Match the first moment of the normal distribution
5. Match the second moment of the normal distribution
6. Match the fourth moment of the normal distribution

The combination of 2, 3 and 4 leads to all odd moments of the normal distribution being properly matched as zero. Solving the system of equations resulting from this set of decisions gives the parameters as

$$p_u = p_d = 1/6$$

$$p_m = 2/3$$

$$u = \sigma\sqrt{3\Delta t} = -d$$

$$m = 0.$$

The described lattice structure is referred to as the Analytical High Order Trinomial model [1] and is visualized in Figure 1.

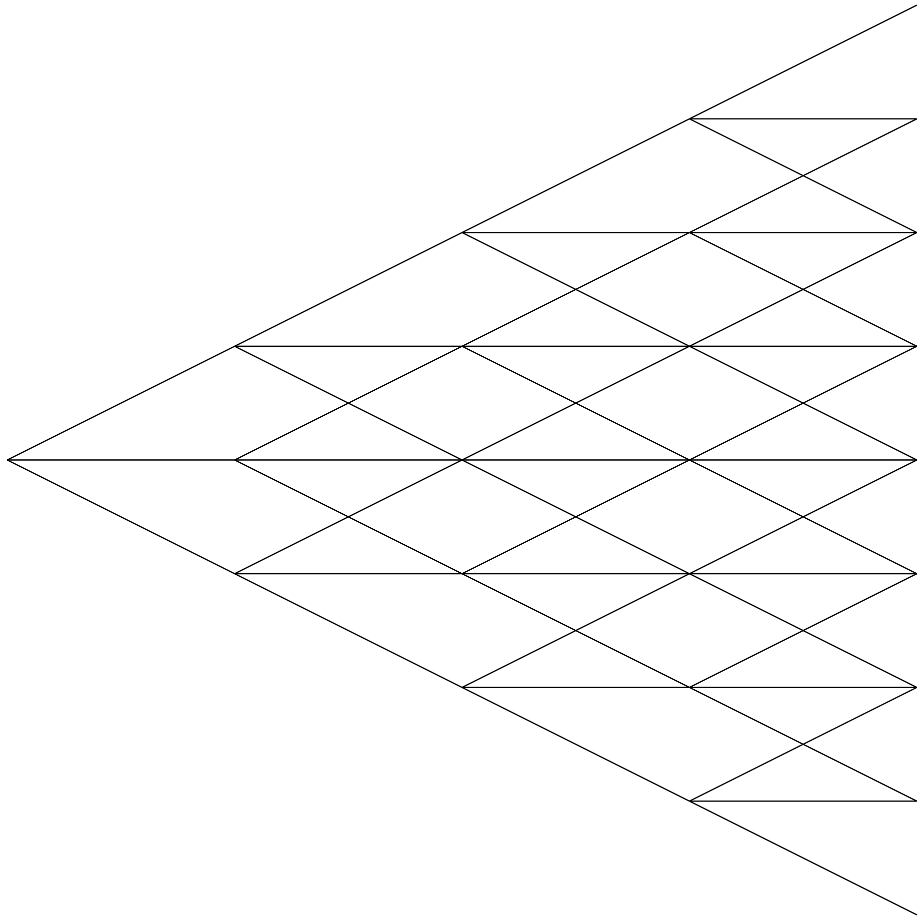

Figure 1: Basic trinomial lattice structure

Solution

Once a lattice has been established according to the above principle, a function is obtained for retrieving the corresponding stock value. The transformations are rolled back by adding the time-dependent, deterministic drift component back in, and transforming back to linear space. This gives a lattice

$$S = \left\{ S_j^i \in (0, \infty),\ i \in [0, N_t],\ j \in [-i, i] \right\}$$

where i is the time step number of the lattice point, N_t is the maximum number of time steps, and j is the direction and number of steps taken away from the middle point. Using the option payoff at maturity, the corresponding option values in the final time step $\{V_j^{N_t}\}$ can then immediately be calculated. This solution is used to progressively move backward in the lattice through the formula

$$V_j^i = e^{-r_i \Delta t} \left(p_u V_{j+1}^{i+1} + p_m V_j^{i+1} + p_d V_{j-1}^{i+1} \right),$$

where r_i is the prevailing interest-free rate between times $i\Delta t$ and $(i+1)\Delta t$. Finally, V_0^0 is identified as the price at current time.

Complications

From its structure, the trinomial model is set up to handle American exercise with relative ease. The standard stepping algorithm is augmented with a check against intrinsic value and choosing the maximum. A decidedly more delicate issue is that of discrete, absolute dividend payments, where the truncation of information outside the lattice becomes a noticeable issue. Vellekoop and Nieuwenhuis [15] suggest an approach of using interpolation to recreate the information that has been lost. The issue that then arises is that the interpolation base might be small, particularly close to current time. In order to avoid having to refine the entire lattice, a degree of adaptivity is introduced in the form of the *Adaptive Mesh Model* (AMM) of Figlewski and Gao [9]. The combination of the concepts of the AMM and Vellekoop-Nieuwenhuis' interpolation approach has been described and examined in detail by Nordström [14].

Adaptivity

The idea behind the AMM is to identify time periods which contribute disproportionately to the error in the solution, and locally increase the number of nodes in the lattice to compensate. This is done by creating a new stepping pattern that leads to a near doubling in number of nodes, together with a pattern to return to the original structure. Figure 2 shows the resulting lattice structure. An advantage of the particular solution suggested by Figlewski and Gao is that the refined lattice retains the structure of the coarse lattice. Not only does this simplify the algorithm implementation itself, but it also allows the refinement to be performed repeatedly, until the desired number of nodes is available. Figure 3 shows an example of this property being utilized.

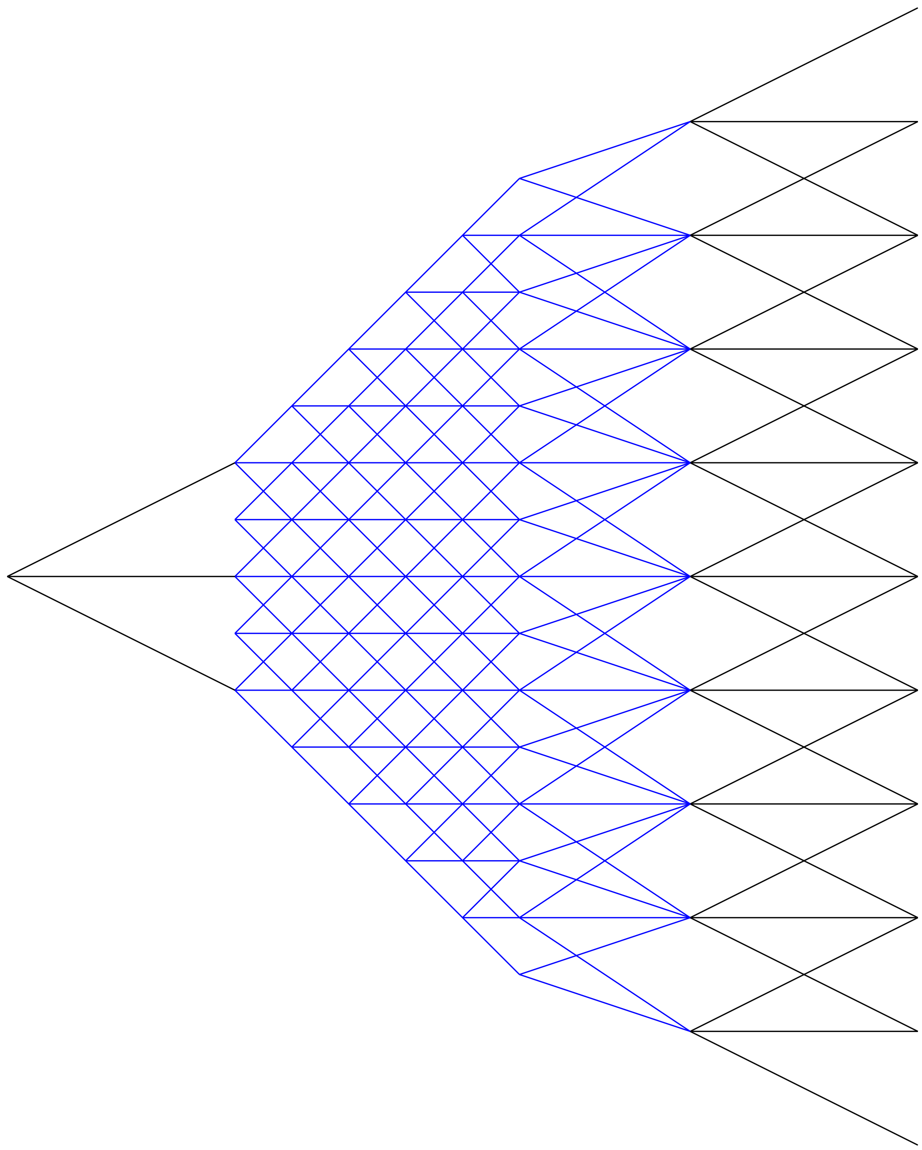

Figure 2: AMM using one level of refinement

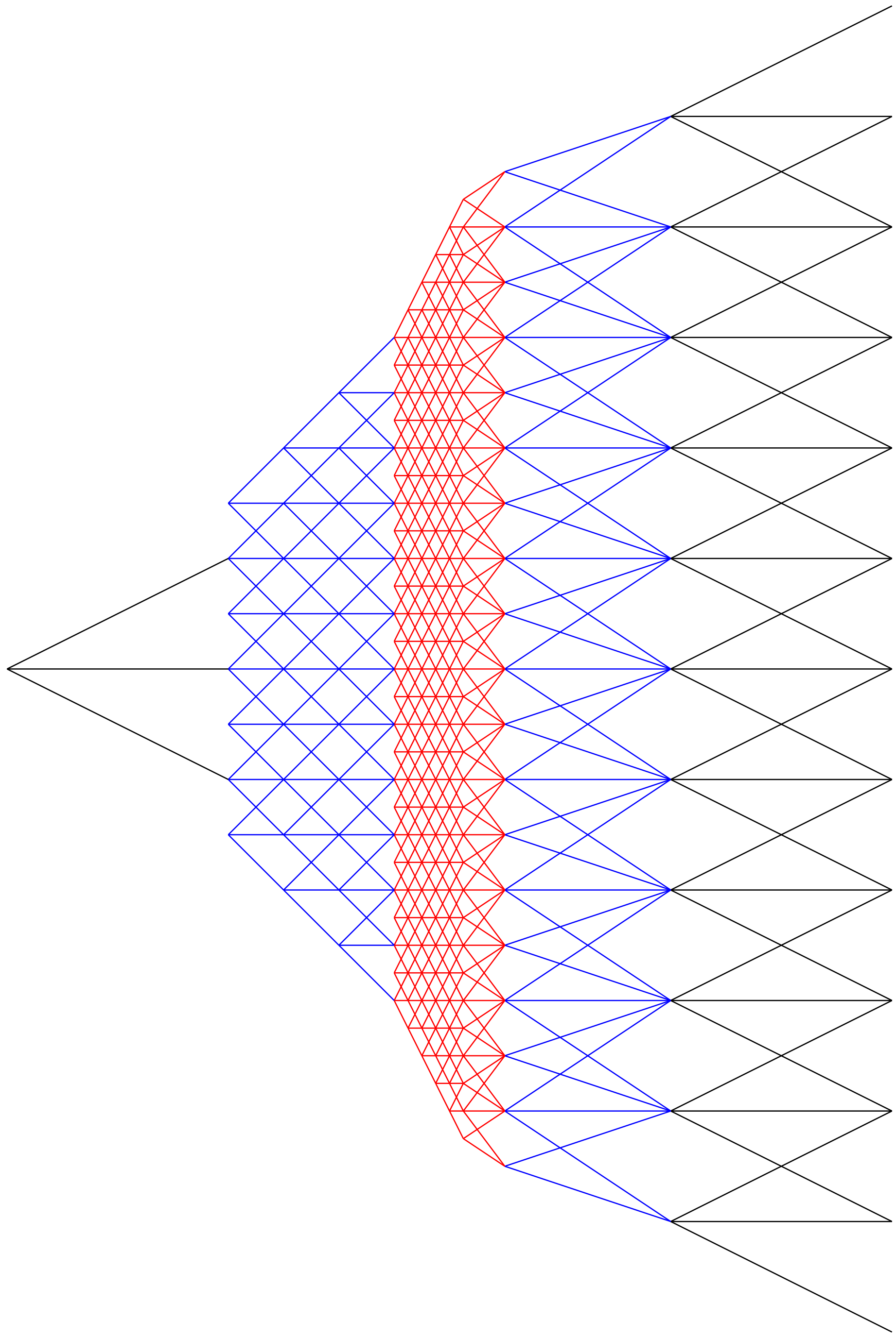

Figure 3: AMM using two levels of refinement

9 Finite Difference Vanilla Models

Front-end name: European FDM/American FDM

Type: Numerical – Finite Differences

Target Instruments: OCEXXX/RWXXCE; OPEXXX/RWXXPE; OCAXXX/RWXXCA; OPAXXX/RWXXPA

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}, Exercise boundary, Exercise boundary reached, Exercise boundary div, Exercise boundary div reached

Description

The partial differential equation formulation of the option pricing problem in the Black-Scholes framework is well suited for solution by the use of finite difference methods. The standard setup of finite difference schemes in the system contains an initial period of Rannacher timestepping [11] to handle the unsmoothness in the solution at maturity. This initial period is solved using an implicit Euler scheme, while the main part of the solution is done with a Crank-Nicolson scheme.

The issue of discrete dividend payments is handled rather straightforwardly by enforcing jump conditions, using interpolation to recreate any information that has been truncated by restricting calculations to the finite difference grid. The selection of method of compensation for American exercise is a less clear-cut issue, with several approaches being available. The method used is the one suggested by Brennan and Schwartz [5], which consists in performing the correction as part of the solution of the system of equations generated by the implicit solvers.

Target PDE

The solved PDE is the standard Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S_t^2}{2}\frac{\partial^2 V}{\partial S^2} + S_t(r(t) - q(t))\frac{\partial V}{\partial S} - r(t)V = 0,$$

with the terminal condition

$$V(S_T, T) = \begin{cases} \max(S_T - K, 0) & \text{Call} \\ \max(K - S_T, 0) & \text{Put,} \end{cases}$$

and boundary conditions

$$\frac{\partial V}{\partial t}(0,t) = r(t)V(0,t)$$

$$\frac{\partial^2 V}{\partial S^2}(S_{\max}, t) = 0.$$

Any discrete dividends are handled by the jump condition

$$V(S_{t_D^+}, t_D^+) = V(S_{t_D^-} + D, t_D^-),$$

where t_D^- and t_D^+ are the times just before and just after a dividend payment of D arbitrary units, respectively.

10 Barone-Adesi & Whaley Pricing Model

Front-end name: Barone-Adesi & Whaley

Type: Approximative Analytical

Target Instruments: OCAXXX/RWXXCA; OPAXXX/RWXXPA

Relevant Parameters: S, K , r, q, σ, T

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, σ_{impl}, Exercise boundary, Exercise boundary reached

Description

The option pricing problem for American options is rather complex because of the early exercise opportunity. This problem is equivalent to a partial differential equation with a free-boundary condition. While problems of this class cannot be solved analytically, a wide variety of different approaches have been investigated.

It is well-known (e.g. [12]) that in the case of $q \leq 0$ early exercise for the American call option is unprofitable — prices of American and European call options are equal in this case. For the American put option some possibility of early exercise always exists.

Giovanni Barone-Adesi and Robert Whaley [2] proposed an analytical approximation for the price of an American option without dividends. This approach is based on an approximation of the early exercise premium, i.e. the difference between the European and American option prices:

$$\varepsilon_C(S,t) = C(S,t) - c(S,t),$$
$$\varepsilon_P(S,t) = P(S,t) - p(S,t).$$

Both option prices satisfy the Black–Scholes equation, and thus the premiums $\varepsilon_C(S,t)$ and $\varepsilon_P(S,t)$ are also solutions of the Black–Scholes equation. Ignoring the derivative with respect to time results in an explicit formula for these premiums, for more details see [2].

Formulae

$\varepsilon_C(S,t)$ and $\varepsilon_P(S,t)$ should satisfy the following equation:

$$S^2 f_{SS} + NSf_S - \tfrac{M}{B(t)} f - (1-B(t))Mf_B = 0$$

with

$$M = \frac{2r}{\sigma^2}, \quad N = \frac{2(r-q)}{\sigma^2}, \quad B(t) = 1 - e^{-r(T-t)} \quad \text{and} \quad f(S,B) = \tfrac{\varepsilon(S,t)}{B(t)}.$$

The main approximation used ignores the $(1-B(t))Mf_B$ term.

The American call option price is expressed in terms of the critical commodity price S^*:

$$C(S,t) = \begin{cases} c(S,t) + A_2\left(\frac{S}{S^*}\right)^{q_2} & \text{when} \quad S < S^*, \quad \text{and} \\ S - K \quad \text{when} \quad S \geq S^*, \end{cases}$$

where

$$A_2 = \tfrac{S^*}{q_2}\left(1 - e^{-q(T-t)} N(d_1(S^*))\right),$$

$$q_2 = \frac{-N + 1 + \sqrt{(N-1)^2 + \frac{4M}{B(t)}}}{2},$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r - q + \sigma^2/2\right)}{\sigma\sqrt{T}}.$$

The critical commodity price S^* can be found from the equation

$$S^* - K = c(S^*, t) + A_2,$$

which in Tbricks is solved using Newton's method.

Similarly for American put options, the price is expressed in terms of the equivalent critical commodity price S^{**}:

$$P(S,t) = \begin{cases} p(S,t) + A_1\left(\frac{S}{S^{**}}\right)^{q_1} & \text{when} \quad S > S^{**}, \quad \text{and} \\ K - S \quad \text{when} \quad S \leq S^{**}, \end{cases}$$

where

$$q_1 = \frac{-N + 1 - \sqrt{(N-1)^2 + \frac{4M}{B(t)}}}{2}$$

$$A_1 = -\tfrac{S^{**}}{q_1}\left(1 - e^{-q(T-t)} N(-d_1(S^{**}))\right).$$

The equation for this critical commodity price S^{**} is

$$K - S^{**} = p(S^{**}, t) + A_1$$

11 Barrier Option Model

Front-end name: Barrier w/o dividends

Type: Analytical

Target Instruments: OCEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In; OPEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In

Relevant Parameters: S, K , r, q, σ, T, H, R

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

The expression for the value of a European barrier option is possible to derive directly from the basic assumptions of the Black-Scholes model. The basic model currently allows for neither American exercise nor discrete dividend payments.

Formulae

Let V_{XYZ}, $X \in \{D, U\}$, $Y \in \{I, O\}$, $Z \in \{C, P\}$ denote the value of a European barrier option, with the indices representing "down"/"up", "in"/"out" styles and call or put payoff types. If the barrier condition is already fulfilled the values of "in"-type options are equal to their vanilla variants. Similarly, the values of "out"-type options are equal to the present value of their rebate payments, if any. Assume thus in the following that the barrier conditions are not yet fulfilled. Following the brilliant exposition in Haug [12], the formulae for the value of the various barrier option types are then

$$V_{DIC} = \begin{cases} C+E & K>H \\ A-B+D+E & K<H \end{cases} \qquad \eta = 1,\ \phi = 1$$

$$V_{UIC} = \begin{cases} A+E & K>H \\ B-C+D+E & K<H \end{cases} \qquad \eta = -1,\ \phi = 1$$

$$V_{DIP} = \begin{cases} B-C+D+E & K>H \\ A+E & K<H \end{cases} \qquad \eta = 1,\ \phi = -1$$

$$V_{UIP} = \begin{cases} A-B+D+E & K>H \\ C+E & K<H \end{cases} \qquad \eta = -1,\ \phi = -1$$

$$V_{DOC} = \begin{cases} A-C+F & K>H \\ B-D+F & K<H \end{cases} \qquad \eta = 1,\ \phi = 1$$

$$V_{UOC} = \begin{cases} F & K>H \\ A-B+C-D+F & K<H \end{cases} \qquad \eta = -1,\ \phi = 1$$

$$V_{DOP} = \begin{cases} A-B+C-D+F & K>H \\ F & K<H \end{cases} \qquad \eta = 1,\ \phi = -1$$

$$V_{UOP} = \begin{cases} B-D+F & K>H \\ A-C+F & K<H \end{cases} \qquad \eta = -1,\ \phi = -1$$

where

$$A = \phi S e^{-qT} N\left(\phi x_1\right) - \phi K e^{-rT} N\left(\phi x_1 - \phi\sigma\sqrt{T}\right),$$

$$B = \phi S e^{-qT} N\left(\phi x_2\right) - \phi K e^{-rT} N\left(\phi x_2 - \phi\sigma\sqrt{T}\right),$$

$$C = \phi S e^{-qT}\left(H/S\right)^{2(\mu+1)} N\left(\eta y_1\right) - \phi K e^{-rT}\left(H/S\right)^{2\mu} N\left(\eta y_1 - \eta\sigma\sqrt{T}\right),$$

$$D = \phi S e^{-qT}\left(H/S\right)^{2(\mu+1)} N\left(\eta y_2\right) - \phi K e^{-rT}\left(H/S\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right),$$

$$E = R e^{-rT}\left(N\left(\eta x_2 - \eta\sigma\sqrt{T}\right) - (H/S)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right)\right),$$

$$F = R\left(\left(H/S\right)^{\mu+\lambda} N(\eta z) + \left(H/S\right)^{\mu-\lambda} N\left(\eta z - 2\eta\lambda\sigma\sqrt{T}\right)\right),$$

$$x_1 = \frac{\ln(S/K)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T},$$

$$x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T},$$

$$y_1 = \frac{\ln\left(H^2/(SK)\right)}{\sigma\sqrt{T}} + \left(1+\mu\right)\sigma\sqrt{T},$$

$$y_2 = \frac{\ln\left(H/S\right)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T},$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T},$$

$$\mu = \frac{r - q - \sigma^2/2}{\sigma^2},$$

$$\lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}.$$

12 Finite Difference Barrier Model

Front-end name: Barrier FDM

Type: Numerical – Finite Differences

Target Instruments: OCEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In; OPEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$, H, R

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

The finite difference setup for a barrier option differs comparatively little from that of vanilla options. In general, the solution process is identical, but the terminal condition and boundary conditions are changed to reflect the specifics of the option. The discussion in this document will assume that the barrier has neither already been, nor is currently, breached.

For "up"-style options, the upper boundary condition is put at the barrier level, while "down"-style options instead have their lower boundary condition moved to the barrier level. For "out"-style options the barrier boundary condition is a potentially time-dependent Dirichlet condition set to the present value of any rebate payments. "In"-style options instead have this Dirichlet condition set to the current value of the corresponding vanilla option, as they are knocked in. The terminal condition of an "out"-style option will be the same as its vanilla equivalent except still being subject to the barrier boundary condition. Meanwhile, "in"-style options at maturity have the value of their rebate payments, except the aforementioned boundary condition still holds.

Target PDE

Let $V_{van}(S, t)$ denote the value at stock S and time t of the barrierless vanilla equivalent of the barrier contract. Further, let $C_{PV}(t) = \exp\left(-\int_t^T r(s)ds\right)$, the present value factor at time t.

The solved PDE is the standard Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S_t^2}{2}\frac{\partial^2 V}{\partial S^2} + S_t(r(t) - q(t))\frac{\partial V}{\partial S} - r(t)V = 0,$$

on the stock interval

$$S \in \begin{cases} [0, H] & \text{“up”-style options} \\ [H, S_{max}] & \text{“down”-style options} \end{cases}$$

with lower boundary condition

$$\begin{cases} V(0,t) = C_{PV}(t)R & \text{“up-and-in” options} \\ V(0,t) = 0 & \text{“up-and-out” call options} \\ V(0,t) = C_{PV}(t)K & \text{“up-and-out” put options} \\ V(H,t) = V_{van}(H,t) & \text{“down-and-in” options} \\ V(H,t) = R & \text{“down-and-out” options,} \end{cases}$$

and upper boundary condition

$$\begin{cases} V(H,t) = V_{van}(H,t) & \text{“up-and-in” options} \\ V(H,t) = R & \text{“up-and-out” options} \\ \dfrac{\partial^2 V}{\partial S^2}(S_{max}, t) = 0 & \text{“down”-style options.} \end{cases}$$

Denoting by $\Phi(S)$ the payoff function at maturity of the barrierless vanilla equivalent of the barrier option, the terminal condition in the segment between boundary condition levels is

$$\begin{cases} V(S,T) = \Phi(S) & \text{“knock-out” options} \\ V(S,T) = R & \text{“knock-in” options} \end{cases}$$

Any discrete dividends are handled by the jump condition

$$V(S_{t_D^+}, t_D^+) = V(S_{t_D^-} + D, t_D^-),$$

where t_D^- and t_D^+ are the times just before and just after a dividend payment of D arbitrary units, respectively.

13 Two-Volatility Barrier Option Pricing Model

Front-end name: Two-volatility barrier

Type: Analytical

Target Instruments: OCEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In; OPEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In

Relevant Parameters: S, K , r, q, σ, T, σ_H, H, R, f_{obs}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

The two-volatility barrier option pricing model allows for limited use of multiple volatilities in the pricing of barrier options while retaining an analytical formula. Two volatilities are identified: the standard volatility taken at the forward price of the underlying, and in addition the volatility in effect at the barrier level. Subsequently, the option value calculation is separated into distinct components representing the payoff at maturity, and the probability of crossing the barrier, respectively. For the former component, the standard volatility is used, but the latter makes use of the barrier volatility. Assuming that the volatility smile will be reasonably centered around the area of strike and current stock value, this will result in an elevated value of volatility being used for the calculation regarding the barrier condition. As a result, knock-in style options will be optimistically evaluated, and knock-out style options in opposite fashion pessimistically evaluated.

The model has support for approximating discrete barrier observations. Setting a barrier observation frequency results in a change in barrier level to compensate for the lower probability of the barrier being registered as crossed. This does not, however, affect the stock level used to fetch the barrier volatility.

The additional degree of freedom leads to the implied volatility problem being naturally underdetermined for this model. As a result, they are currently disabled. Another important peculiarity affects the sensitivities related to volatility. Due to it being unclear what effects a change in standard volatility has on the barrier volatility, the sensitivities are calculated with respect only to this standard volatility and do not include changes to the probability of crossing the barrier.

This model uses dividend equivalent yield approximation in order to handle discrete and / or percentage dividends.

Formulae

The formulae are based on a two-asset model given in Haug [12], with the two assets set as identical (including using the same Wiener process to drive both) with the exception of volatility which is set as described above. This leads to the possibility of a set of simplifications (the derivation is available as a separate document) resulting in the formulas below.

Let $V_{\text{<D/U><I/O><C/P>}}$ denote the value of a barrier option. The first subscript letter designates whether the barrier is considered breached when the stock price falls below the barrier level (D – Down-and-x option) or when the stock price exceeds the barrier level (U – Up-and-x option). The second signifies whether the option becomes alive upon fulfilling the barrier condition (I – knock-In option) or whether it is rendered useless upon fulfilling the barrier condition (O – knock-Out option). Finally, the last subscript letter defines the payoff of the option as either call-type or put-type.

The values of knock-out type options can be found from the value of their associated knock-in contract, the barrierless version of the contract, and the in-out parity relationship

$$V_{xOy} = V_y - V_{xIy},$$

where V_y is the equivalent barrierless version of V_{xOy}. Thus, only the knock-in formulae will be given explicitly, and they are:

$$V_{\mathrm{DIC}} = \begin{cases} A_2 S e^{-qT} N(d_3) - A_1 K e^{-rT} N(d_4) & d_1 \le -e_1 \\ \\ \begin{array}{l} S e^{-qT}(N(d_1) - N(-e_1) + A_2 N(-e_3)) - \\ K e^{-rT}(N(d_2) - N(-e_2) + A_1 N(-e_4)) \end{array} & d_1 > -e_1 \end{cases}$$

$$V_{\mathrm{UIC}} = \begin{cases} S e^{-qT} N(d_1) - K e^{-rT} N(d_2) = V_C & d_1 \le -e_1 \\ \\ \begin{array}{l} S e^{-qT}(N(-e_1) + A_2(N(d_3) - N(-e_3))) - \\ K e^{-rT}(N(-e_2) + A_1(N(d_4) - N(-e_4))) \end{array} & d_1 > -e_1 \end{cases}$$

$$V_{\mathrm{DIP}} = \begin{cases} -S e^{-qT} N(-d_1) + K e^{-rT} N(-d_2) = V_{\mathrm{P}} & d_1 \ge -e_1 \\ \\ \begin{array}{l} S e^{-qT}(-N(e_1) - A_2(N(-d_3) - N(e_3))) - \\ K e^{-rT}(-N(e_2) - A_1(N(-d_4) - N(e_4))) \end{array} & d_1 < -e_1 \end{cases}$$

$$V_{\mathrm{UIP}} = \begin{cases} -A_2 e^{-qT} N(-d_3) + A_1 K e^{-rT} N(-d_4) & d_1 \ge -e_1 \\ \\ \begin{array}{l} S e^{-qT}(-N(-d_1) + N(e_1) - A_2 N(e_3)) - \\ K e^{-rT}(-N(-d_2) + N(e_2) - A_1 N(e_4)) \end{array} & d_1 < -e_1 \end{cases}$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln(S/K) + (r - q + \sigma^2/2)T\right)$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{1}{\sigma\sqrt{T}}\left(\ln(S/K) + (r - q - \sigma^2/2)T\right)$$

$$d_3 = d_1 + \frac{2\ln(H/S)}{\sigma_H\sqrt{T}}$$

$$d_4 = d_2 + \frac{2\ln(H/S)}{\sigma_H\sqrt{T}}$$

$$e_1 = \frac{1}{\sigma_H\sqrt{T}}\left(\ln(H/S) - (r - q - \sigma_H^2/2 + \sigma\sigma_H)T\right)$$

$$e_2 = e_1 + \sigma\sqrt{T} = \frac{1}{\sigma_H\sqrt{T}}\left(\ln(H/S) - (r - q - \sigma_H^2/2)T\right)$$

$$e_3 = e_1 - \frac{2\ln(H/S)}{\sigma_H\sqrt{T}}$$

$$e_4 = e_2 - \frac{2\ln(H/S)}{\sigma_H\sqrt{T}},$$

and $N(x)$ is the standard normal cumulative distribution function.

14 Turbo Option Model

Front-end name: Turbo w/o divs

Type: Approximative Analytical

Target Instruments: RWXXCE, Variant Turbo; OCXXXX, Variant Turbo; RWXXPE, Variant Turbo; OPXXXX, Variant Turbo

Relevant Parameters: S, K , r, q, σ, T, H, Δ_{t_R}, S_{RR}, BC, t_{BC}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, σ_impl

Description

Turbo options are conceptually similar to knock-out barrier options, but their rebates take the rather complex form of look-back contracts. If a turbo contract is knocked out, the holder receives an instrument of the same payoff type (call/put) and with the same strike price as the turbo. The underlying for such a call (put) contract is the minimum (maximum) process based on the underlying of the turbo instrument from the point of barrier crossing. This contract matures Δ_{t_R} years after the crossing.

A replicating portfolio for this option can be constructed from a standard knock-out option with a zero rebate, together with a knock-in lookback option with expiry dependent on the knock-in point. Assuming constant rate, convenience yield and volatility, the latter term can be further decomposed [8] into the product of an American binary option and a function related to the probability density of the extremum of the stock process within the lifespan of the lookback option.

Formulae

The value of the call and put variants can, in accordance with the above, be expressed as

$$V_{TC}(S,t) = V_{DOC}(S,t) + A_C V_{ABP}\,(S,t)$$

and

$$V_{TP}(S,t) = V_{UOP}(S,t) + A_P V_{ABC}\,(S,t)$$

respectively, where

- $V_{DOC/UOP}$ are the values of the knock-out barrier options. Their parameters are identical to that of the turbo option, except they pay no rebate in the case of a knock-out.
- $A_{C/P}$ are the functions involving the extremum process probability density, to be defined.
- $V_{ABP/ABC}$ are the values of the American binary options. Their strike level is the barrier level of the turbo option. All other parameters are identical to that of the turbo option.

The first and the second summands will be referred in future chapters as "unknocked" and "rebate" parts of the turbo, respectively. The expressions for the values of the barrier options and the American binary options are found in the documentation for the barrier option model and the binary option model respectively. The $A_{C/P}$ terms are given by the following expressions

$$\begin{aligned} A_C = He^{-q\Delta_{t_\mathrm{R}}} &\left(1+\frac{\sigma^2}{2(r-q)}\right)\left[N(-\phi_{2+})-N(\phi_1-\phi_{2+})\right]+ \\ He^{-r\Delta_{t_\mathrm{R}}} &\left(1-\frac{\sigma^2}{2(r-q)}\right)\left[N(\phi_{2-})-\phi_3 N(\phi_1+\phi_{2-})\right]- \\ Ke^{-r\Delta_{t_\mathrm{R}}} &\left[N(-\phi_{2-})-N(\phi_1-\phi_{2-})\right]- \\ Ke^{-r\Delta_{t_\mathrm{R}}} &\left[N(\phi_{2-})-\hat{\phi}_3 N(\phi_1+\phi_{2-})\right], \end{aligned}$$

$$\begin{aligned} A_P = He^{-q\Delta_{t_\mathrm{R}}} &\left(1+\frac{\sigma^2}{2(r-q)}\right)\left[N(-\phi_{2+})-N(\phi_1-\phi_{2+})\right]+ \\ He^{-r\Delta_{t_\mathrm{R}}} &\left(1-\frac{\sigma^2}{2(r-q)}\right)\left[N(\phi_{2-})-\phi_3 N(\phi_1+\phi_{2-})-(1-\phi_3)\right]+ \\ Ke^{-r\Delta_{t_\mathrm{R}}} &\left[N(\phi_1-\phi_{2-})-\hat{\phi}_3 N(-\phi_1-\phi_{2-})\right], \end{aligned}$$

where

$$\begin{aligned} \phi_1 &= \frac{\ln\left(\frac{K}{H}\right)}{\sigma\sqrt{\Delta_{t_\mathrm{R}}}}, \\ \phi_{2\pm} &= \frac{\Delta_{t_\mathrm{R}}\left(r-q\pm\frac{1}{2}\sigma^2\right)}{\sigma\sqrt{\Delta_{t_\mathrm{R}}}}, \\ \phi_3 &= \left(\frac{K}{H}\right)^{\frac{2(r-q)}{\sigma^2}}, \\ \hat{\phi}_3 &= \left(\frac{K}{H}\right)^{\frac{2(r-q)}{\sigma^2}-1}, \end{aligned}$$

and $N(x)$ is the cumulative distribution function of the standard normal distribution.

Valuation after knockout

Rebate payment of turbo instruments is equal to the minimal absolute difference between strike and underlying spot price observed during the rebate period. Given this, rebate payoff functions for turbo puts and calls look like:

$$\begin{aligned} R_{call} &= \max\left(S_{RR}-K,0\right)) \\ R_{put} &= \max\left(K-S_{RR},0\right)), \end{aligned}$$

where S_{RR} is the minimal (maximal) recorded value of the underlying price during the rebate period for calls (puts) respectively.

Suppose a knockout event happened for a turbo option at a known time t_{BC} and we are in some point t_0 between t_{BC} and the end of the rebate period $t_{BC} < t_0 < t_{BC} + \Delta_{t_\mathrm{R}}$. Further denote remaining lifetime of the contract by $\Delta^{'}_{t_\mathrm{R}} = t_{BC} + \Delta_{t_\mathrm{R}} - t_0$. The un-knocked part of turbo price is now gone, only the rebate part matters.

Only call instruments are considered for the remainder of the section, exactly the same argument holds for puts.

Denote by P probability of not going below recorded reference price during rebate period:

$$P = P(S_t \geq S_{RR};\ \forall t \in [t_0; t_{BC} + \Delta_{t_\mathrm{R}}]).$$

Further define fair price of the rebate part as:

$$V_{TR}(H, S, t, T, \Delta_{t_\mathrm{R}}) = A_C(H, \Delta_{t_\mathrm{R}})\, V_{ABP}\,(S, t, T)\,.$$

Fair price of a turbo instrument after a knockout event is then given as:

$$V = P\ \max\left(S_{RR} - K, 0\right) + (1 - P)\ V_{TR}(S_{RR}, S, t, \Delta^{'}_{t_\mathrm{R}}, t_{BC} + \Delta_{t_\mathrm{R}})$$

where $1-P$ is equal to un-discounted fair value of a one-touch binary call option with payment at maturity, maturity = T and strike = S_{RR}. binary-one-touch)

The logic behind is:

- With probability $P = P(S_t \geq S_{RR};\ \forall t \in [t_0; t_{BC} + \Delta_{t_\mathrm{R}}])$ stock price does not drop below the recorded minimal price until the end of rebate period and the final payment is $\max\left(S_{RR} - K, 0\right)$.
- With probability $1 - P$ stock price actually drops below the recorded minimal price until the end of rebate period and the (expected) rebate payment is given by $V_{TR}(S_{RR}, S, t, \Delta^{'}_{t_\mathrm{R}}, t_{BC} + \Delta_{t_\mathrm{R}})$ with minimal recorded price acting as a barrier.

15 Turbo Option Finite Difference Model

Front-end name: Turbo FDM

Type: Approximative Numerical – Finite Differences

Target Instruments: RWXXCE, Variant Turbo; OCXXXX, Variant Turbo; RWXXPE, Variant Turbo; OPXXXX, Variant Turbo

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$, H, Δ_{t_R}, BC

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

Compensating for discrete dividend payments in the turbo model is associated with the same set of complications seen for other analytical models. A numerical method is thus necessitated, and the one used in this case is the finite difference method.

As with the analytical model, a replicating portfolio can be set up consisting of a knock-out contract added to the product of an American binary option and a term representing the distribution of the extremum process under the lifespan of the rebate option. Due to the lifetime of this rebate option generally being short, it is assumed that the dividend payment will not fall within it. Thus, the calculation of the distribution-related term can be done as in the analytical case. The remaining two terms are calculated using standard finite differences.

Target PDE

The calculations for the knock-out barrier options are performed as described in the barrier option finite difference model documentation. For the American binary option, the solved PDE is the standard Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S_t^2}{2}\frac{\partial^2 V}{\partial S^2} + S_t(r(t) - q(t))\frac{\partial V}{\partial S} - r(t)V = 0,$$

with the terminal condition

$$V(S_T, T) = \begin{cases} \mathbf{1}_{[K,\infty)}(S_T) & \text{Call} \\ \mathbf{1}_{[0,K]}(S_T) & \text{Put,} \end{cases}$$

where K is the strike price of the binary option, which is set to the barrier level of the turbo option. The boundary conditions for call options are

$$V(K, t) = 1, \qquad V(\infty, t) = 0,$$

while for put options they are

$$V(0, t) = 0, \qquad V(K, t) = 1.$$

Since the early exercise decision will always be made if the contract is in the money, and never otherwise, these boundary conditions also fully correct for American exercise.

Any discrete dividends are handled by the jump condition

$$V(S_{t_D^+}, t_D^+) = V(S_{t_D^-} + D, t_D^-),$$

where t_D^- and t_D^+ are the times just before and just after a dividend payment of D arbitrary units, respectively.

If the barrier is hit, the model passes calculation to the analytical one, ignoring all dividends within rebate period.

16 Callable bull/bear contracts (CBBC)

Front-end name: CBBC

Type: Approximative Analytical

Target Instruments: RWXXCE, Variant CBBC; OCXXXX, Variant CBBC; RWXXPE, Variant CBBC; OPXXXX, Variant CBBC

Relevant Parameters: S, K , r, q, σ, $D_i, 1 \leq i \leq N_D$, T, H, R_{min}, S_{RR}, BC, t_{BC}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, σ_{impl}

Description

Callable bull/bear contracts or simply CBBCs are very similar to Turbo options. Given this, the reader is highly advised to familiarize himself with chapter about Turbo instruments prior to reading this one. (see Turbo Option Model)

The main difference between Turbos and CBBCs lies in determination of *rebate period length*. While for the former type of instruments rebate period length is fixed to some value (e.g. 3 hours) at contract initiation, for the latter type situation is conceptually different. Rebate period for CBBC does not have a predefined length. Instead, it starts when the barrier is hit and then extends until the end of the trading session in which the barrier was hit, plus one additional trading session.

Note: For a proper operation of CBBC one has to configure trading sessions via trading calendar resource, see this page for more details.

Let $\tau_{end\,cur}$ be the end of current trading session, $\tau_{start\,next}$, $\tau_{end\,next}$ - start/end of the next trading session respectively.

Rebate time for CBBCs is then fixed at the moment of barrier breach and given as

$$\Delta_{t_{\mathrm{R}}} = (\tau_{end\,cur} - t_{BC}) + (\tau_{end\,next} - \tau_{start\,next}),$$

where t_{BC} - barrier crossing time.

Floored rebate payment

As in case of turbo instruments, the rebate payment is equal to the minimal absolute difference between the strike and the underlying spot price observed during the rebate period. Recall S_{RR} is the minimal (maximal) recorded value of the underlying price during the rebate period for calls (puts) respectively.

It is possible to floor rebate payment for CBBCs via setting *Rebate min percentage* instrument parameter. The rebate is then bounded from below by

$$\frac{R_{min}}{100} \times |H - K|.$$

By default, rebate payments are not floored, i.e. $R_{min} = 0$.

Rebate payoff functions for CBBC puts and calls then look like:

$$R_{call} = \max\left(S_{RR} - K, \frac{R_{min}}{100}(H - K)\right)$$
$$R_{put} = \max\left(K - S_{RR}, \frac{R_{min}}{100}(K - H)\right).$$

In order to unify this with the un-floored approach, the latter functions can be formulated as a non-floored rebate function with a modified strike and an offset:

$$R_{call} = C_{off} + \max\left(0, S_{RR} - K'\right)$$
$$R_{put} = C_{off} + \max\left(0, K' - S_{RR}\right)$$

where

$$C_{off} = \frac{R_{min}}{100} \times |H - K|$$

and

$$K' = K + \frac{R_{min}}{100}(H - K)$$

Approximation of rebate period

If the barrier is not crossed, rebate part of CBBC is calculated as if the barrier was "just hit", i.e. rebate period is approximated by

$$\Delta_{t_{\mathrm{R}}} = (\tau_{end\,cur} - t) + (\tau_{end\,next} - \tau_{start\,next}),$$

where t corresponds to the present moment.

The logic behind this approach is:

1. If we are close to the barrier, knockout is likely to happen in the nearest future and the approximation of rebate length is consistent
2. If we are far from the barrier, the approximation is not very precise, however, the rebate part is relatively small in comparison to the un-knocked part

This rebate time is simply plugged into the formula for Turbo contacts (both before and after hitting the barrier). (see Turbo Option Model)

The model uses dividend equivalent yield approximation in order to handle discrete dividends, see "Financial mathematics" section for more details. Such approach tends to smooth out the impact of the discrete dividends, which can adversely affect the accuracy of the estimated probability of crossing the barrier. This effect becomes more influential as underlying price approaches the barrier - in such cases one could consider using the finite difference solver (namely, CBBC FDM).

17 Callable bull/bear contracts Finite Difference Model

Front-end name: CBBC FDM

Type: Approximative Numerical – Finite Differences

Target Instruments: RWXXCE, Variant CBBC; OCXXXX, Variant CBBC; RWXXPE, Variant CBBC; OPXXXX, Variant CBBC

Relevant Parameters: S, K , r, q, σ, $D_i, 1 \leq i \leq N_D$, T, H, R_{min}, BC

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, σ_{impl}

Description

- For the structure of the finite difference solver refer to the chapter about Turbo FDM model. (see Turbo Option Model)
- Calculation of rebate part is described in the chapter about analytical CBBC model. (see Callable bull/bear contracts (CBBC))
- If the barrier is hit, the model passes calculation to the analytical one, ignoring all dividends within rebate period.

18 Curran's Model

Front-end name: Asian w/o divs / Asian escrowed dividends

Type: Approximative Analytical

Target Instruments: RWXXCE, Variant Average price option; OCXXXX, Variant Average price option; RWXXPE, Variant Average price option; OPXXXX, Variant Average price option

Relevant Parameters: S, K , r, q, σ, T, $t_i, 1 \leq i \leq N_f$, $S_{t_i}, t_i < t$, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

Michael Curran's approximation [6, 7] for the value of discretely monitored arithmetic Asian options is based in the fact that option pricing where exercise is guaranteed is a comparatively simple problem which can often be analytically evaluated. Further, it relies on an important result saying that for any set of non-negative reals, the arithmetic average is greater than or equal to the geometric average. This allows a separation of the call option valuation formula into two distinct terms representing the following cases:

1. The geometric average is greater than or equal to the strike, and thus so is the arithmetic average.
2. The geometric average is less than the strike, and the arithmetic average may or may not be.

For case 1, exercise is guaranteed and an analytical expression of the value contribution can be found. The contribution of case 2 can not be calculated analytically, and must instead be approximated. In the Tbricks implementation, this approximation is done by swapping the order of the expectation operator and the maximum function, which leads to an underestimation of the option value.

In the case of discrete dividend payments being paid by the underlying instrument, models such as the escrowed dividend model (see Black-Scholes with Analytical Dividend Corrections) can be used to calculate the forward values used in the model caculations.

There are two main issues to consider with regards to accuracy of the Curran approximation. Firstly, having monitoring dates focused near maturity increases the correlation between the monitored stock values. This has the twin effects of increasing the relative contribution of the analytically available term from case 1, and improving the accuracy of the approximation of case 2. Secondly, the `Asian w/o divs` pricing model assumes that the monitoring dates are equidistantly placed between the time of the first monitoring date and maturity. Deviations from this pattern can impair precision, and might necessitate the use of the `Asian escrowed dividends` model, even without the presence of discrete dividend payments.

Formulae

Denote by A_t and G_t the arithmetic average process and the geometric average process respectively at time t. Let further

$$S' = S - \sum_{i=1}^{N_D} D_i e^{t_{D,i}(q-r)},$$

$$F_i = S' e^{(r-q)t_i},$$

$$\mu_i = \ln(F_i) - \frac{1}{2}\sigma^2 t_i,$$

$$\mu = \frac{1}{N_f}\sum_{i=1}^{N_f} \mu_i,$$

$$\sigma_i = \sqrt{\sigma^2 t_i},$$

$$\sigma_{xi} = \frac{\sigma_i}{N_f}\sum_{j=1}^{N_f} \sigma_j(\sqrt{\min(t_i, t_j)}),$$

$$\sigma_x = \frac{1}{N_f^2} \sum_{i=1}^{N_f} \sum_{j=1}^{N_f} \sigma_i \sigma_j \sqrt{\min(t_i, t_j)}.$$

Curran's approximation of the value of an Asian call option is then

$$V_C = e^{-rT} \int_{\hat{K}}^{\infty} (\mathbb{E}\left[A_T | G_T = G\right] - K) g(G) dG$$

where $g(x)$ is the cumulative density function of the geometric average at maturity. The lower integration boundary would optimally be chosen as $\hat{K}^*$ such that $\mathbb{E}\left[A_T | G_T = \hat{K}^*\right] = K$, but since this $\hat{K}^*$ is not analytically available an approximation is made. For the function

$$f(y) = \mathbb{E}\left[A_T | G_T = y\right] = \sum_{i=1}^{N_f} \exp\left[\mu_i + \left(\sigma_{xi}/\sigma^2\right)(y - \mu) + \left(\sigma_i^2 - \sigma_{xi}^2/\sigma^2\right)/2\right]$$

$\frac{\partial f}{\partial y}$ is naïvely approximated as 1 between K and $\hat{K}^*$. Thus, the estimate $\hat{K} = 2K - f(K)$ is used, resulting in the overall expression as shown in Haug [12]

$$V_C = e^{-rT} \left[\frac{1}{N_f} \sum_{i=1}^{N_f} F_i \, N\left(\frac{\mu - \ln \hat{K}}{\sigma_x} + \frac{\sigma_{xi}}{\sigma_x} \right) - KN\left(\frac{\mu - \ln \hat{K}}{\sigma_x} \right) \right]$$

where $N(x)$ is the standard normal cumulative distribution function. Put-call parity yields the value of a put option as

$$V_P = e^{-rT} \left[KN\left(-\frac{\mu - \ln \hat{K}}{\sigma_x} \right) - \frac{1}{N_f} \sum_{i=1}^{N_f} F_i \, N\left(-\left(\frac{\mu - \ln \hat{K}}{\sigma_x} + \frac{\sigma_{xi}}{\sigma_x} \right) \right) \right]$$

19 Hybrid Curran Model

Front-end name: Asian FDM

Type: Approximative Numerical – Finite Differences

Target Instruments: RWXXCE, Variant Average price option; OCXXXX, Variant Average price option; RWXXPE, Variant Average price option; OPXXXX, Variant Average price option

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \le i \le N_D$, $t_i, 1 \le i \le N_f$, $S_{t_i}, t_i < t$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

A significant flaw in Curran's approximation is that it is incapable of handling discrete dividend payments. To handle this issue, dividends are separated into two categories – those before the monitoring interval starts and those inside it.

Handling dividends outside the monitoring interval is reasonably straightforward. Curran's approximation can be used without modification to obtain the price at any point after the dividend payment. The values thus found can be used to establish a terminal condition for use in a standard finite difference solver for the Black-Scholes PDE. This solver can then use interpolation and jump conditions to compensate correctly for the dividends. This means that the rather common use case of Asian tail options with monitoring isolated to the last few days of the option lifespan can be handled without any major complexities.

The case of dividends inside the monitoring interval is significantly less straightforward, and is handled through an approximation. For each dividend in the monitoring interval, its effect on any remaining fixings is estimated. The sum of this impact over all dividends is then used to increase the strike price. The main approximation involved is that this method ignores the reduction in the diffusion coefficient in which the lower stock price after dividend would result.

Combining these measures, a hybrid method of Curran's approximation and finite differences is obtained which is capable of handling arbitrarily placed dividends.

Formulae

Letting $\tilde{t}_i$, $1 \leq i \leq N_D$ denote the times for the discrete dividend payments, the amount that the strike price is changed to compensate for dividends inside the monitoring interval is determined by the formula

$$K_{\text{shift}} = \frac{D_i}{N_f} \sum_{t_j > \tilde{t}_i} e^{(r-q)\left(t_j - \tilde{t}_i\right)}$$

20 Full PDE Asian Option Pricing Model

Front-end name: Asian Full PDE

Type: Numerical – Finite Differences

Target Instruments: RWXXCE, Variant Average price option; OCXXXX, Variant Average price option; RWXXPE, Variant Average price option; OPXXXX, Variant Average price option

Relevant Parameters: S, K , r, q, σ, T, $D_i, 1 \leq i \leq N_D$, $t_i, 1 \leq i \leq N_f$, $S_{t_i}, t_i < t$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

The full PDE finite difference pricing model for arithmetic averaging Asian options is the most straightforward application of finite differences to the problem.

It is observed that information can only flow between different averaging levels at monitoring dates, and thus the Black-Scholes equation holds unchanged at other times. A standard finite difference grid is set up in stock and time dimensions, but is also extended in the third dimension of current running average. Between monitoring dates, the problem for each averaging level is a fully independent instance of the standard Black-Scholes equation, with the interaction between averaging levels handled through absence-of-arbitrage jump conditions. This pattern is described in more detail by Zvan et al. [16].

An important observation is that the current running average is known. Thus, the averaging dimension grid collapses to a single level in the time span between current time and the first monitoring date. Performance of the solver is thus highly dependent on the size of the monitoring interval. For options with an Asian tail, where all the monitoring dates are concentrated around maturity, the performance of the method improves dramatically.

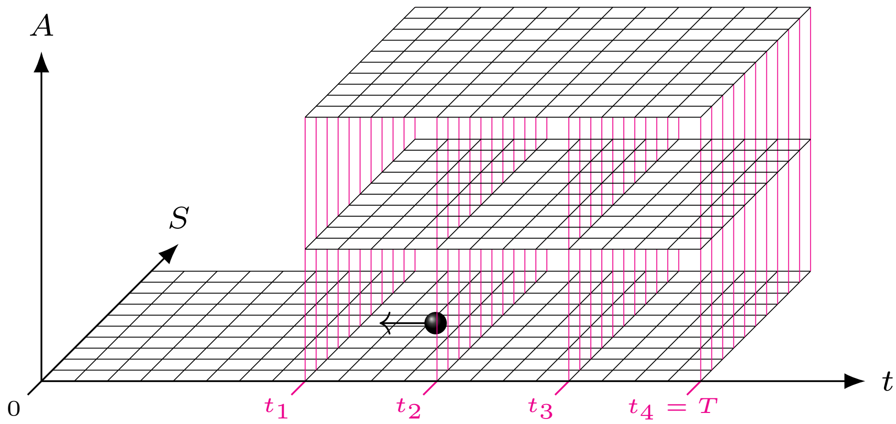

Figure 4: Structure of the full PDE FD solver

Target PDE

The target PDE for each averaging dimension level is

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S_t^2}{2}\frac{\partial^2 V}{\partial S^2} + S_t(r-q)\frac{\partial V}{\partial S} - rV = 0,$$

with the terminal condition

$$V(S_T, A_T, T) = \begin{cases} \max(A_T - K, 0) & \text{Call} \\ \max(K - A_T, 0) & \text{Put,} \end{cases}$$

and boundary conditions

$$\frac{\partial V}{\partial t}(0, A_t, t) = rV(0, A_t, t)$$

$$\frac{\partial^2 V}{\partial S^2}(S_{\max}, A_t, t) = 0.$$

Any discrete dividends are handled by the jump condition

$$V(S_{t_D^+}, A_{t_D^+}, t_D^+) = V(S_{t_D^-} + D, A_{t_D^-}, t_D^-),$$

where t_D^- and t_D^+ are the times just before and just after a dividend payment of D arbitrary units, respectively. Similarly, monitoring is handled by

$$V(S_{t_f^+}, A_{t_f^+}, t_f^+) = V(S_{t_f^\pm}, A_{t_f^-} + \frac{S_{t_f^\pm} - A_{t_f^-}}{k}, t_f^-),$$

where t_f^- and t_f^+ are the times just before and just after the k^{th} fixing. Due to the structure of the finite difference grid, the information required for these jump conditions will generally not be available, and is reconstructed using four-point Lagrange interpolation.

Discretization Schemes

The main finite difference scheme used for solution is the Crank-Nicolson scheme, but after each enforcement of a jump condition and at maturity, four higher accuracy steps of the implicit Euler scheme are taken. This is done to improve the handling of the non-smooth solution which arises in these situations. The result is a composite scheme with second order convergence in all dimensions.

21 Binary Option Pricing Model

Front-end name: Binary

Type: Analytical

Target Instruments: OCEXXX, Variant Binary Cash; OPEXXX, Variant Binary Cash; OCAXXX, Variant Binary Cash; OPAXXX, Variant Binary Cash

Relevant Parameters: S, K , r, q, σ, T

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

Binary cash options are comparatively simple constructs which yield a payoff of 1 unit of currency upon exercise under the condition that the price of the underlying is greater (smaller) than or equal to the strike price in the case of the call (put) version of the contract.

The pricing formula for European style binary cash options features a highly significant dependency on the *skew*, the derivative of volatility with respect to strike. This skew is obtained by a finite difference approximation using data from the volatility model.

In the case of American exercise being allowed, the assumption is made that exercise is immediate upon reaching the strike price. This assumption allows an analytical formula to be obtained despite the complication of the exercise type. Generally this assumption holds true, but for negative interest rates no such guarantee can be made, and the model is thus currently unavailable for instruments with negative interest rates. In this case, "Binary one-touch" model can be used.

Currently, implied volatility calculations are disabled for the model, in part due to the dependency of the European options on the skew. This dependency means that the implied volatility problem has an extra degree of freedom and thus that multiple solutions are available. In addition, the value of binary options as a function of volatility is not always monotonic (out-of-the-money calls, in-the-money puts are not), and thus it is again impossible to invert the function without additional information.

This model uses dividend equivalent yield approximation in order to handle discrete and / or percentage dividends, see "Financial mathematics" section for more details.

Formulae

The formulae for the European style contracts can be found in e.g. Gatheral [10], and are

$$V(S,t) = \begin{cases} e^{-rT}N(d_2) - \sqrt{T}Se^{-qT}N'(d_1)\dfrac{\partial\sigma}{\partial K} & \text{Binary Call} \\ e^{-rT}N(-d_2) + \sqrt{T}Se^{-qT}N'(d_1)\dfrac{\partial\sigma}{\partial K} & \text{Binary Put,} \end{cases}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r - q + \sigma^2/2\right)}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r - q - \sigma^2/2\right)}{\sigma\sqrt{T}},$$

$N(x)$ is the standard normal distribution cumulative distribution function and $N'(x)$ is the standard normal distribution probability density function.

The formulae for the American "one-touch" contracts can be found in Haug [12], but are only identified for their role as correction terms for rebates in knock-out barrier contracts. They are

$$V_C(S,t) = \left(\frac{S}{K}\right)^{\lambda_+} N(d_+) + \left(\frac{S}{K}\right)^{\lambda_-} N(d_-),$$

$$V_P(S,t) = \left(\frac{S}{K}\right)^{\lambda_+} N(-d_+) + \left(\frac{S}{K}\right)^{\lambda_-} N(-d_-),$$

where

$$\lambda_\pm = \alpha \pm \sqrt{-\beta}$$

$$\alpha = \frac{1}{2} - \frac{r-q}{\sigma^2}$$

$$\beta = -\frac{1}{\sigma^4}\left(\left(r - q - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2\right)$$

$$d_+ = \frac{\ln\left(\frac{S}{K}\right) + \sigma^2(T)\sqrt{-\beta}}{\sigma\sqrt{T}}$$

$$d_- = d_+ - 2\sigma\sqrt{T}\sqrt{-\beta} = \frac{\ln\left(\frac{S}{K}\right) - \sigma^2(T)\sqrt{-\beta}}{\sigma\sqrt{T}},$$

22 Binary One-touch Option Pricing Model

Front-end name: Binary one-touch

Type: Analytical

Target Instruments: OCAXXX, Variant Binary Cash; OPAXXX, Variant Binary Cash

Relevant Parameters: S, K , r, q, σ, T

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

Readers of this paragraph are advised to familiarize themselves with the documentation on the original binary pricing model before going further.

One of the shortcomings of the original model was its inability to price American Binary options when interest rates are negative. This model drops this limitation by assuming immediate exercise for in-the-money options, which is where "one-touch" naming comes from.

For standard American binary options in a non-negative interest rate setting, all calculated values obtained from this model exactly match ones from the "Binary" model, since with rates being non-negative, it is always optimal to exercise when the strike price is reached.

23 Exchange Trade Fund (ETF) pricing model

Front-end name: ETF

Type: Analytical

Target Instruments: EUXXXX; RWXXCE, Variant EUSIPA 2300; OCXXXX, Variant EUSIPA 2300; RWXXPE, Variant EUSIPA 2300; OPXXXX, Variant EUSIPA 2300

Relevant Parameters: U, NAV, Cash, ETF Cash CCY, U_{Ref}, Units, Und, $\mathrm{FX}_{\mathrm{U}_{\mathrm{Ref}}}$, Use snap ETF pricing, Af, Tf

Calculated values: V, Δ

Description

An ETF, or exchange traded fund, is a marketable security that tracks an index, a commodity, bonds, or a basket of assets like an index fund. A simple analytical formula is used to price such instruments. All relevant parameters besides Underlying price are manually input or imported from an XML file by the ETF parameters app.

Formulae

The following generic expression is used to calculate the theoretical value of an ETF:

$$V = \left(\mathrm{NAV} + \left(\mathrm{U} - \mathrm{U}_{\mathrm{Ref}}\right) * \mathrm{M} * \mathrm{Und}/\mathrm{Units} * \mathrm{FX}_{\mathrm{U}}\right)/\mathrm{FX}_{\mathrm{U}_{\mathrm{Ref}}}$$

where NAV is the net asset value, the value of the total equity minus fund liabilities divided by the number of ETF shares outstanding. If the instrument parameter NAV is empty a modified expression is applied:

$$V = \left(\mathrm{Cash} * \mathrm{FX}_{\mathrm{Cash}} + \left(\mathrm{U} - \mathrm{U}_{\mathrm{Ref}}\right) * \mathrm{M} * \mathrm{Und}/\mathrm{Units} * \mathrm{FX}_{\mathrm{U}}\right)/\mathrm{FX}_{\mathrm{U}_{\mathrm{Ref}}}$$

where:
$\mathrm{FX}_{\mathrm{Cash}}$ is the exchange rate between cash and ETF currencies, thus the currency in which ETF cash is presented has to be set.

Cash represents the cash component of the ETF.

U is the price of the underlying instrument.

U_{Ref} is the reference price of the ETF underlying instrument, for example the settlement or closing price of futures. In many setups this parameter should be set to zero.

Units is the number of units (ETF shares) per block, i.e. the amount of units that are redeemed for a block of the underlying security, which could be for example a basket, future, bond, etc. Units is the same as the prescribed number of units or the fund divisor.

Und is the number of underlying instruments in one block.

M is the multiplier of the underlying and describes how much/many of the underlying a derivative contract represent. It is the standard multiplier used in Tbricks and normally there is no need to modify it for ETF setup, but it is always useful to check its value in order to properly adjust ETF pricing.

FX_U is the exchange rate between underlying currency and ETF currency.

$FX_{U_{Ref}}$ is the reference exchange rate for the underlying vs ETF and is set to 1.0 by default. It differs from unity if the ETF and Underlying Instrument are in different currencies and FX risk should be accounted for, $FX_{U_{Ref}}$ = $FX_{U_{t-1}}$, where $t-1$ denotes the previous business day.

If "Use snap ETF pricing" is enabled the following formula is used:

$$V = \mathrm{Cash} * \mathrm{FX_{Cash}} + (\mathrm{U} * \mathrm{Af} * \mathrm{Tf}) * \mathrm{FX_U}$$

Af and Tf are the "ETF adjustment factor" and the "ETF tracking factor", respectively. These are mandatory parameters used to adjust the ETF price. This functionality allows one to configure alternative price sources for certain stocks to be used at certain times, for example when the real stock is not actively traded.

Pricing of leverage certificates via the ETF model

Leverage certificates have one additional relevant instrument attribute - namely, the barrier level. Such instruments are priced in exactly the same fashion as usual ETF's, except that they account for barrier being crossed.

- If the barrier **is not** breached, nothing is changed in the above formulas.
- If the barrier **is** breached, a leverage certificate does not track its underlying instrument any more. This results in delta being set to 0 and the model assumes that underlying price is equal to the reference price U_{Ref} (if snap pricing is not enabled). Thus the formula for fair market price simplifies to
  $$V = \mathrm{NAV},$$
  or, if NAV happens to be empty,
  $$V = \mathrm{Cash} * \mathrm{FX_{Cash}}.$$
  Similarly, if snap pricing is enabled, tracking factor becomes 0 and the formula once again shrinks to just
  $$V = \mathrm{Cash} * \mathrm{FX_{Cash}}.$$

24 Pricing of Short Term Interest Rate options

Front-end name: European, Barone-Adesi & Whaley, European FDM/American FDM, American, Trinomial

Type: Analytical/Approximative Analytical/Numerical – Finite Differences/Numerical – Lattice

Target Instruments: OCEDXX; OPEDXX; OCADXX; OPADXX

Relevant Parameters: S, K , r, q, σ, T

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}, Exercise boundary, Exercise boundary reached

Description

There are two types of Short Term Interest Rate (STIR) derivatives: STIR futures and STIR options. The CFI code of STIR futures is FXDXXX.

STIR futures are based on time limited deposits linked to the respective interest rate. In other words, buying / selling such a future contract is equivalent to lending / borrowing a given amount of a specified currency at an agreed interest rate r on a specific date in the future for a specified period. The quoted price of a STIR future is $100(1-r)$. STIR futures are underlying instruments to STIR options and strike prices of the latter are also expressed in terms of $100(1-r)$ and usually vary in the range of 90 to 100.

Examples of STIR options are Euribor, Eurodollar and Euroyen options, based on the respective future contracts.

One of the standard approaches to pricing STIR options is to use the transformations $F = 100 - U$ and $K = 100 - K$ and substitute these values into models for vanilla options on futures, with F as the forward price, U as the underlying price and K as the strike price. This approach is equivalent to using a lognormal distribution to model the interest rate associated with the underlying future price. It works for all the standard models Bos-Vandermark (European options), Barone-Adesi-Whaley (American options), Binomial and Trinomial (American options) and Finite difference (European and American options). For details, please refer to the individual documentation for each model.

Due to the lognormal model, negative interest rates — or equivalently, quoted prices above 100 — are not possible using this approach.

25 European Bachelier Pricing Model

Front-end name: European Bachelier

Type: Analytical

Target Instruments: OCEDXX; OPEDXX

Relevant Parameters: F, K , r, σ, T

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}

Description

The Bachelier model is a useful approach to the problem of pricing short term interest rate (STIR) options. These rates can be negative, and since modeling the underlying asset using a lognormal distribution only allows for positive rates, the Black–Scholes model is not applicable. The premium payment for STIR options takes one of two forms. The first alternative is *up-front payment*: you should immediately pay some money to buy an option. The second is the *margin payment* style which calculates cash flows at expiry. Mathematically speaking, the present value factor $e^{-r(T-t)}$ should be introduced to discount the expected value of the option for the up-front payment.

The option price in the Bachelier model with up-front payment can be obtained as the solution of the following linear parabolic equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial F^2} - rV = 0,$$

with corresponding terminal condition:

$$V(T,F) = \begin{cases} (F-K)^+ & \text{for Call} \\ (K-F)^+ & \text{for Put.} \end{cases}$$

For margin payment style options, the Bachelier model gives the option price as the solution for the next linear parabolic equation:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial F^2} = 0,$$

with the same terminal condition:

$$V(T,F) = \begin{cases} (F-K)^+ & \text{for Call} \\ (K-F)^+ & \text{for Put.} \end{cases}$$

For more information see, e.g. [12].

Formulae

Both of the equations above can be explicitly solved. The formulae for the case of up-front payment:

$$c(t,F) = e^{-r(T-t)}\left[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right]$$

$$p(t,F) = e^{-r(T-t)}\left[(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right]$$

where

$$d_1 = \tfrac{F-K}{\sigma\sqrt{T-t}}.$$

For the margin payment style the present value factor $e^{-r(T-t)}$ is dropped

$$c(t,F) = (F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$
$$p(t,F) = (K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.$$

26 American Bachelier Pricing Model

Front-end name: American Bachelier

Type: Approximative Analytical

Target Instruments: OCADXX; OPADXX

Relevant Parameters: F, K , r, σ, T

Calculated values:

Description

The Bachelier model is a useful approach to the problem of pricing short term interest rate (STIR) options. These rates can be negative, and since modeling the underlying asset using a lognormal distribution only allows for positive rates, the Black–Scholes model is not applicable. The premium payment for STIR options takes one of two forms. The first alternative is *up-front payment*: you should immediately pay some money to buy an option. The second is the *margin payment* style which calculates cash flows at expiry. Mathematically speaking, the present value factor $e^{-r(T-t)}$ should be introduced to discount the expected value of the option for the up-front payment.

The option price in the Bachelier model with up-front payment can be obtained as the solution of the following linear parabolic equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial F^2} - rV = 0,$$

with corresponding terminal condition:

$$V(T,F) = \begin{cases} (F-K)^+ & \text{for Call} \\ (K-F)^+ & \text{for Put.} \end{cases}$$

For margin payment style options, the Bachelier model gives the option price as the solution for the next linear parabolic equation:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial F^2} = 0,$$

with the same terminal condition:

$$V(T,F) = \begin{cases} (F-K)^+ & \text{for Call} \\ (K-F)^+ & \text{for Put.} \end{cases}$$

Straightforward calculations show that for margin payment style options early exercise is unprofitable, and thus prices of American and European options are equal in this case. Moreover, American options with up-front payment and non-positive interest rate r should be priced the same way as European options — early exercise is unprofitable for them too.

Giovanni Barone-Adesi and Robert Whaley [2] have proposed an analytical approximation for the price of American options without dividends in the Black–Scholes model. This approach is based on approximating the early exercise premium, i.e. the difference between European and American option prices. In the Tbricks system a similar approach is provided for the Bachelier model

$$\varepsilon_C(F,t) = C(F,t) - c(F,t),$$
$$\varepsilon_P(F,t) = P(F,t) - p(F,t).$$

Both option prices satisfy the Bachelier equation, therefore the differences $\varepsilon_C(F,t)$ and $\varepsilon_P(F,t)$ are also solutions of the Bachelier equation. Ignoring the derivative with respect to time results in an explicit formula for these premiums.

Formulae

$\varepsilon_C(F,t)$ and $\varepsilon_P(F,t)$ should satisfy the following equation:

$$f_{FF} + \tfrac{2r}{\sigma^2 B(t)} f_F - \tfrac{2r}{\sigma^2}(1 - B(t)) f_B = 0$$

with

$$B(t) = 1 - e^{-r(T-t)} \quad \text{and} \quad f(F,B) = \tfrac{\varepsilon(F,t)}{B(t)}.$$

The main approximation used ignores the $\frac{2r}{\sigma^2}(1 - B(t)) f_B$ term.

The American call option price in the Bachelier model is expressed in terms of the critical forward price F^*:

$$C(F,t) = \begin{cases} c(F,t) + (1 - e^{-r(T-t)} N(d(F^*)) M e^{\frac{F-F^*}{M}} & \text{when} \quad F < F^*, \quad \text{and} \\ F - K \quad \text{when} \quad F \geq F^*, \end{cases}$$

where

$$d(F) = \frac{F-K}{\sigma\sqrt{T-t}}$$
$$M = \sqrt{\tfrac{\sigma^2(1-e^{-r(T-t)})}{2r}}.$$

The critical forward price F^* can be found from the equation

$$F^* - K = c(F^*, T-t) + \sqrt{\tfrac{\sigma^2 B}{2r}}(1 - e^{-r(T-t)} N(d(F^*))),$$

which in Tbricks is solved using Newton's method.

Similarly for American put options, the price is expressed in terms of the equivalent critical forward price F^{**}:

$$P(S,t) = \begin{cases} p(S,t) + (1 - e^{-r(T-t)} N(-d(F^{**})) M e^{\frac{F^{**}-F}{M}} & \text{when} \quad F > F^{**}, \quad \text{and} \\ K - F \quad \text{when} \quad F \leq F^{**}, \end{cases}$$

The equation for this critical forward price F^{**} is

$$K - F^{**} = p(t, F^{**}) + \sqrt{\tfrac{\sigma^2 B}{2r}}(1 - e^{-r(T-t)}N(-d(F^{**})).$$

27 Finite Difference Bachelier Pricing Model

Front-end name: Bachelier FDM

Type: Numerical – Finite Differences

Target Instruments: OCEDXX; OPEDXX; OCADXX; OPADXX

Relevant Parameters: F, K , r, σ, T

Calculated values:

Description

The Bachelier model is a useful approach to the problem of pricing short term interest rate (STIR) options. These rates can be negative, and since modeling the underlying asset using a lognormal distribution only allows for positive rates, the Black–Scholes model is not applicable. The premium payment for STIR options takes one of two forms. The first alternative is *up-front payment*: you should immediately pay some money to buy an option. The second is the *margin payment* style which calculates cash flows at expiry. Mathematically speaking, the present value factor $e^{-r(T-t)}$ should be introduced to discount the expected value of the option for the up-front payment.

Formulae

Under assumption that the underlying price (equal to forward price F) is normally distributed, the option price V satisfies the Bachelier PDE [13]:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial F^2} + (r-q)\frac{\partial V}{\partial F} - rV = 0 \tag{1}$$

where F is the forward price of underlying asset, r is the interest rate, q is the constant dividend yield and σ is the volatility. The first order derivative with respect to F vanishes since for STIR options $r = q$. The corresponding finite-difference scheme has the coefficients:

$$A_i^n = 0.25 \cdot \nu_2 \cdot b_i^n - 0.5 \cdot \nu_1 \cdot a_i^n;$$
$$B_i^n = \nu_1 \cdot a_i^n - 0.5 \cdot dt \cdot r;$$
$$C_i^n = -(0.5 \cdot \nu_1 \cdot a_i^n + 0.25 \cdot \nu_2 \cdot b_i^n);$$

where $\nu_2 = dt/dF, \nu_2 = dt/(dF^2), a_i^n = 0.5 \cdot \sigma^2, b_i^n = F_i^n \cdot (r-q) = 0$

A uniform grid in the dimension of the underlying is introduced as:

$$F_i = F_{\min} + i\Delta F, \Delta F = (F_{\max} - F_{\min})/I,$$

where $F_{\min} = 0$, $F_{\max} = mK$, the multiplier m from range $[1.2; 12]$ is an input parameter, I is the total number of steps, K is strike price. The time discretization is defined between the final time moment $T_{\max}$ coincident with the maturity date T, where the final condition is applied, and the evaluation date $t = 0$:

$$t^{n+1} = t^n - \Delta t^{n+1}, t^0 = T_{\max}, t^N = 0,$$

where N is the number of time steps and Δt^n is the constant time step.

Boundary conditions are similar to the Black-Scholes model.

28 Multidimensional Taylor series

Front-end name: External multidimensional Taylor

Type: External

Target Instruments: XXXXXX

Relevant Parameters: S , r, σ, T, V_0, U_0^i, r_0^j, σ_0^i, Δ_0^i, $\Gamma_0^{i,i}$, ρ_0^j, ν_0^i, ν_0^{Basket}, $Vomma_0^{i,i}$, Θ_0, $Charm\ C_0^i$, $Charm\ C_0^{Basket}$, $\Gamma_0^{i,j}$, $Vanna_0^{i,j}$, $Vomma_0^{i,j}$, w_i, Volatility relations, Expand underlying legs, External values are valid, Valid until, Valid range

Calculated values: Calculated values mentioned in the description of the model

Description

External multidimensional Taylor model can be used to price instruments based on several underlyings simultaneously. Being an external model it depends on the values of the fair market price and its derivatives provided at some reference point. If the reference values of a derivative is not provided, its value is considered to be 0. The multidimensional Taylor series used can be presented in the following form:

$$\begin{aligned}
V = V_0 &+ \sum_{i=1}^{N} \Delta_0^i \left(S^i - S_0^i\right) + \rho_0^d \left(r^d(T) - r_0^d\right) + \sum_{i=1}^{M} \rho_0^i \left(r^i(T) - r_0^i\right) + \\
&+ \sum_{i=1}^{N} \sum_{j>i}^{N} \Gamma_0^{i,j} \left(S^i - S_0^i\right) \left(S^j - S_0^j\right) + \frac{1}{2} \sum_{i=1}^{N} \Gamma_0^{i,i} \left(S^i - S_0^i\right)^2 + \\
&+ \sum_{i=1}^{N} \sum_{j>i}^{N} Vomma_0^{i,j} \left(\sigma^i(F(S^{Basket}, T), T) - \sigma_0^i\right) \left(\sigma^j(F(S^{Basket}, T), T) - \sigma_0^j\right) + \\
&+ \frac{1}{2} \sum_{i=1}^{N} Vomma_0^{i,i} \left(\sigma^i(F(S^{Basket}, T), T) - \sigma_0^i\right)^2 + \sum_{i=1}^{N} \nu_0^i \left(\sigma^i(F(S^{Basket}, T), T) - \sigma_0^i\right) + \\
&+ \sum_{i=1}^{N} \sum_{j=1}^{N} Vanna_0^{i,j} \left(S^i - S_0^i\right) \left(\sigma^j(F(S^{Basket}, T), T) - \sigma_0^j\right)
\end{aligned} \tag{2}$$

where N is the number of the underlying instruments, M is the number of unique underlying currencies, r^d is the risk-free rate curve of the domestic currency for the derivative instrument itself and ρ_0^d is the derivative of the fair price with respect to that curve calculated at the reference point r_0^d. It is important to mention that volatility of each leg is calculated using forward price of the whole basket.

Spot price in this model is calculated as market price of the underlying basket. Forward price is calculated from the spot price as usual. Note that these two values are calculated independently from the parameters described below.

Here is a list of mandatory parameters for proper operation of the model:

- "Pricing model" should be set to "External multidimensional Taylor"
- "External values are valid" should be set to True on the instrument level (the only values calculated for this model when the latter parameter is set to False are Spot and Forward prices)
- "External model valid range" should be the fraction of the reference underlying price with which the current values are allowed to deviate before being considered invalid
- "External model valid until" should specify date and time of invalidation of the reference values
- "Instrument fair market price" should be specified on the instrument level, it will be used as the reference fair market price

There is a possibility to dismiss all errors caused by violation of time and/or price range by checking "Disable external out-of-range error". Note that even if external errors are ignored, it is necessary to have "External model valid range" & "External model valid until" set.

If usage of parameter context rankings is desired, one should specify "Theoretical values parameters context" instrument parameter in each context. E. g. if "Risk" and "Global" parameter contexts should be used within the external model setup, one should set "Theoretical values parameters context" to "Global" in "Global" context and to "Risk" in the "Risk" context.

Due to calculation specifics, it is important to make sure that the symmetric second derivatives are included only once in the table, while $Vanna$ is specified for every possible combination of the underlying instruments. E.g. if there is an instrument based on the underlying instrument $U1$ and $U2$, the table "Taylor derivatives 2" should contain exactly two rows and look like this:

$\boldsymbol{\partial U_i}$	$\boldsymbol{\partial U_j}$	$\boldsymbol{\Gamma}$	$\boldsymbol{Vanna}$	$\boldsymbol{Vomma}$
$U1$	$U2$	$\Gamma_0^{(1,2)}$	$Vanna_0^{(1,2)}$	$Vomma_0^{(1,2)}$
$U2$	$U1$		$Vanna_0^{(2,1)}$	

Similarly, since the rho compensation is on a per-currency basis, only one instrument per currency should have its rho column filled out in the "Taylor derivatives 1" table, and the value should be the total rho for the target option with respect to that currency. For example consider an option based on the three underlying instruments $U1$, $U2$ and $U3$. $U1$ and $U2$ are quoted in USD, while $U3$ is quoted in EUR. The "Taylor derivatives 1" tables in this case should be as follows:

$\boldsymbol{\partial U_i}$	$\boldsymbol{\Delta}$	$\boldsymbol{\Gamma}$	$\boldsymbol{\rho}$	$\boldsymbol{\nu}$	$\boldsymbol{Vomma}$	$\boldsymbol{Instrument\ charm}$
$U1$	$\Delta_0^{(1)}$	$\Gamma_0^{(1,1)}$	ρ_0^{USD}	$\nu_0^{(1)}$	$Vomma_0^{(1,1)}$	$Instrument\ charm_0^{(1)}$
$U2$	$\Delta_0^{(2)}$	$\Gamma_0^{(2,2)}$		$\nu_0^{(2)}$	$Vomma_0^{(2,2)}$	$Instrument\ charm_0^{(2)}$
$U3$	$\Delta_0^{(3)}$	$\Gamma_0^{(3,3)}$	ρ_0^{EUR}	$\nu_0^{(3)}$	$Vomma_0^{(3,3)}$	$Instrument\ charm_0^{(3)}$

Greeks

The following Greeks are provided by the external multidimensional Taylor model:

Delta w.r.t. leg	$\Delta^k = \frac{\partial V}{\partial S^k} =$ $= \Delta_0^k + \sum_{j=1}^{N} \Gamma_0^{k,j} \left(S^j - S_0^j \right) + \sum_{j=1}^{N} Vanna_0^{k,j} \left(\sigma^j - \sigma_0^j \right)$
Delta w.r.t. basket	$\Delta^{Basket} = \frac{\partial V}{\partial S^{Basket}} \approx$ $\approx \frac{1}{2\delta \sum_{i=1}^{N} S^i w^i} \left[V \left((1+\delta)\{S^i\}, \{\sigma^i(S^i)\} \right) - \right.$ $\left. - V \left((1-\delta)\{S^i\}, \{\sigma^i(S^i)\} \right) \right]$
Skew delta w.r.t. leg	$\Delta \ skew^k = \frac{dV}{dS^k} =$ $= \Delta_0^k + \sum_{i=1}^{N} \Gamma_0^{i,k} \left(S^i - S_0^i \right) + \nu_0^k \frac{d\sigma^k(F(S^{Basket}, T), T)}{dS^k} +$ $+ \sum_{i=1}^{N} Vomma_0^{i,k} \left(\sigma^i(F(S^{Basket}, T), T) - \sigma_0^i \right) \frac{d\sigma^k(F(S^{Basket}, T), T)}{dS^k} +$ $+ \sum_{i=1}^{N} Vanna_0^{i,k} \left(S^i - S_0^i \right) \frac{d\sigma^k(F(S^{Basket}, T), T)}{dS^k} +$ $+ \sum_{j=1}^{N} Vanna_0^{k,j} \left(\sigma^j(F(S^{Basket}, T), T) - \sigma_0^j \right)$
Skew delta w.r.t. basket	$\Delta \ skew^{Basket} = \frac{dV}{dS^{Basket}} \approx$ $\approx \frac{1}{2\delta \sum_{i=1}^{N} S^i w^i} \left[V \left((1+\delta)\{S^i\}, \{\sigma^i((1+\delta)S^i)\} \right) - \right.$ $\left. - V \left((1-\delta)\{S^i\}, \{\sigma^i((1-\delta)S^i)\} \right) \right]$
Gamma w.r.t. leg	$\Gamma^k = \frac{\partial^2 V}{\left(\partial S^k\right)^2} = \Gamma_0^{k,k}$

Gamma w.r.t. basket	$\Gamma^{Basket} = \dfrac{\partial^2 V}{(\partial S^{Basket})^2} \approx$ $\approx \dfrac{1}{\delta^2 \left(\sum_{i=1}^{N} S^i w^i\right)^2} \left[V\left((1+\delta)\{S^i\}, \{\sigma^i(S^i)\}\right) - \right.$ $\left. - 2V\left(\{S^i\}, \{\sigma^i(S^i)\}\right) + V\left((1-\delta)\{S^i\}, \{\sigma^i(S^i)\}\right) \right]$
Skew gamma w.r.t. leg	$\Gamma\ skew^k = \dfrac{d^2 V}{(dS^k)^2} = \Gamma_0^{k,k} + \nu_0^k \dfrac{d^2\sigma^k(F(S^{Basket},T),T)}{(dS^k)^2}$ $+ Vomma_0^{k,k} \left(\dfrac{d\sigma^k(F(S^{Basket},T),T)}{dS^k}\right)^2$ $+ \sum_{i=1}^{N} Vomma_0^{i,k} \left(\sigma^i(F(S^{Basket},T),T) - \sigma_0^i\right) \dfrac{d^2\sigma^k(F(S^{Basket},T),T)}{(dS^k)^2} +$ $+ 2Vanna_0^{k,k} \dfrac{d\sigma^k(F(S^{Basket},T),T)}{dS^k}$ $+ \sum_{i=1}^{N} Vanna_0^{i,k} \left(S^i - S_0^i\right) \dfrac{d^2\sigma^k(F(S^{Basket},T),T)}{(dS^k)^2}$
Skew gamma w.r.t. basket	$\Gamma\ skew^{Basket} = \dfrac{d^2 V}{(dS^{Basket})^2} \approx$ $\approx \dfrac{1}{\delta^2 \left(\sum_{i=1}^{N} S^i w^i\right)^2} \left[V\left((1+\delta)\{S^i\}, \{\sigma^i((1+\delta)S^i)\}\right) - \right.$ $\left. - 2V\left(\{S^i\}, \{\sigma^i(S^i)\}\right) + V\left((1-\delta)\{S^i\}, \{\sigma^i((1-\delta)S^i)\}\right) \right]$
Rho w.r.t. leg	$\rho^k = \dfrac{\partial V}{\partial r} = \rho_0^k$
Vega w.r.t. leg	$\nu^k = \dfrac{\partial V}{\partial \sigma^k} = \nu_0^k$
Vega w.r.t. basket	$\nu^{Basket} = \nu_0^{Basket}$
Calendar theta	$\Theta = V(T + 1\ day) - V(T) = \Theta_0$
Calendar charm w.r.t. leg	$Charm\ C^k = \Delta^k(T + 1\ day) - \Delta^k(T) = Charm\ C_0^k$
Calendar charm w.r.t. basket	$Charm\ C^{Basket} =$ $= \Delta^{Basket}(T + 1\ day) - \Delta^{Basket}(T) = Charm\ C_0^{Basket}$

Derivatives calculated with respect to one of the legs of the underlying basket are available in the “Portfolios” and “Portfolio overview” types of windows. They are shown in the respective columns when the “Expand underlying baskets” item of the context menu is chosen in the grid.

Note, that the resulting values of the ν and ρ are divided by the factor of 100 and 10000 respectively to be in-line with other models.

Disregard inner structure of the underlying basket

When the "Expand underlying legs" instrument parameter is set to false (or is empty), multidimensional Taylor model uses reference values of the Greeks w.r.t. the underlying basket itself. In this case reference values should be specified in the row with id corresponding to the underlying basket for "Taylor reference values", "Taylor derivatives 1" and "Taylor derivatives 2" tables.

Fair market price is calculated as follows:

$$\begin{aligned}
&V = V_0 + \Delta_0^{Basket}\left(S^{Basket} - S_0^{Basket}\right) + \rho_0^d\left(r^d(T) - r_0^d\right) + \\
&+ \rho_0^{Basket}\left(r^{Basket}(T) - r_0^{Basket}\right) + \frac{1}{2}\Gamma_0^{Basket}\left(S^{Basket} - S_0^{Basket}\right)^2 + \\
&+ \frac{1}{2}Vomma_0^{Basket}\left(\sigma^{Basket}(F(S^{Basket}, T), T) - \sigma_0^{Basket}\right)^2 + \\
&+ \nu_0^{Basket}\left(\sigma^{Basket}(F(S^{Basket}, T), T) - \sigma_0^{Basket}\right) + \\
&+ Vanna_0^{Basket}\left(S^{Basket} - S_0^{Basket}\right)\left(\sigma^{Basket}(F(S^{Basket}, T), T) - \sigma_0^{Basket}\right)
\end{aligned} \tag{3}$$

Δ, Δ *Skew*, Γ and Γ *Skew* are calculated with finite differences the same way as described in the previous section.

29 Pricing of Quanto options

Front-end name: All models for option pricing

Type: Analytical/Approximative Analytical/Numerical – Finite Differences/Numerical – Lattice

Target Instruments: XXXXXX

Relevant Parameters: Whichever parameters are relevant for the base model

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV, σ_{impl}, Exercise boundary, Exercise boundary reached

Description

Quanto options supported in Tbricks are options based on foreign equity where strike is also denominated in foreign currency, while option is priced in domestic one. Spot and strike prices in foreign currency are passed into respective model and at expiry the option value is translated into domestic currency. Convenience yield q, that particularly affects calculation of the forward price, is replaced by

$$r_d - r_f + q + \rho * \sigma_J * \sigma_f.$$

Here r_d is the domestic interest rate, r_f is the foreign interest rate, σ_f is the FX volatility, σ_J is the volatility, ρ is the FX/spot correlation.

Calculated values are converted according to the following rules

1. "Quanto fx rate" affects all of the calculated values, excluding Option Equivalent Vega/Gamma as well as CV's related to sensitivities to parameters of volatility models.
2. Current market exchange rate between strike currency and instrument's currency now affects all Greeks, excluding Vega/Vomma and Calendar/Opening/Overnight Greeks

Quanto options strike currency can be set to instrument currency and the strike price is then converted ro underlying currency using FX rate.

30 Mini Futures model

Front-end name: Mini-Futures

Type: Analytical

Target Instruments: RWXXCX, Variant Mini Futures; RWXXXX (is treated as RWXXCX), Variant Mini Futures; OCXXXX, Variant Mini Futures; OXXXXX (is treated as OCXXXX), Variant Mini Futures ; RWXXPX, Variant Mini Futures; OPXXXX, Variant Mini Futures

Relevant Parameters: S, K, H, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

A mini future is essentially an open ended deep in the money knock out option. Mini long is a down and out call and Mini short is a up and out put. Its strike price is also referred as financing level, the barrier level is also known as the stop loss level. Barrier rebate is taken as a difference between these two values. Pricing of such instruments is performed via "Turbo without dividends model". Any dividends, if present, are ignored in this model.

Formulae

Theoretically, if the barrier has not been crossed yet, a mini-futures value is calculated on the basis of present value of underlying instrument's future price and financing level:

$$V = \text{PV(future price of underlying instrument)} - \text{financing level} = S - K.$$

The logic behind is: consider a down-and-out call option with infinite maturity, initially specified cash rebate of H-K is paid at barrier level H, interest rate =

0. With such boundary conditions BS equation gives $S - K$ for $S \geq H$ at any time moment. As always with barrier instruments, the rebate is paid on barrier crossing and fair price in general case is the given as:

$$V_{call}(S) = max(S - K, H - K), \qquad V_{put}(S) = max(K - S, K - H).$$

Note that fair price is independent of volatility since the function is linear and the second derivative with respect to S is zero. Simplicity of this model leads to all Greeks, except Δ and Skew Δ being zero-valued. Δ and Skew Δ are both equal to 1 in case of call option and -1 in case of put (such instruments are deep in the money). Since mini-future is an open end instrument (does not have maturity), forward price calculation does not make much sense and its value is set to the price of underlying asset.

31 Forward Start Option model

Front-end name: Forward Start

Type: Analytical

Target Instruments: RWXXCE, Variant Forward Start; OCEXXX, Variant Forward Start; RWXXPE, Variant Forward Start; OPEXXX, Variant Forward Start

Relevant Parameters: S , r, q, σ, T, $D_i, 1 \leq i \leq N_D$, t_g, α_{fs}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, σ_{impl}

Description

Forward start options are essentially European options with strike price determined at grant date as:

$$K = \alpha_{fs} S_g, \text{where}$$

S_g - price of the underlying asset at the grant date.

For example, setting $\alpha_{fs} = 1$ leads to an at-the-money strike. As in the case of vanilla European options, the payoff of such contracts is given as $(S_T - K)^+$ for call options and $(K - S_T)^+$ for put options.

Formulae

After grant date, since strike price has already been fixed, a forward start option becomes Vanilla European and Black-Scholes model with Bos-Vandermark correction is used in valuation. (see Black-Scholes with Analytical Dividend Corrections)

Before grant date, according to Benson[3] fair price of a forward start call option is given as:

$$V_c = e^{-rt_g} F_g BV_c(1, \alpha_{fs}, \tau),$$

where

- F_g - forward price of the underlying asset at grant date,
- $\tau = T - t_g$ - time from grant date until maturity,
- $BV_c(S, K, T)$ - fair price of a European call option with strike K, underlying price S and time to maturity T given by Black-Scholes model with Bos-Vandermark correction.

Fair price of a put option is analogously calculated as:

$$V_p = e^{-rt_g} F_g BV_p(1, \alpha_{fs}, \tau).$$

It is important to note that is case of a dividend-free underlying there is no need to use Bos-Vandermark correction and the latter formula simplifies into the following form:

$$V_c = e^{-qt_g} S \cdot BS_c(1, \alpha_{fs}, \tau),$$

where

- $BS_c(S, K, T)$ - fair price of a European call option given by Black-Scholes model.

32 Tracker Certificate model

Front-end name: Tracker Certificate

Type: Analytical

Target Instruments: RMXXXX, Variant EUSIPA 1300

Relevant Parameters: S , r, q, σ, T, $D_i, 1 \le i \le N_D$, D_p, α_c, T_{issue}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

Tracker certificates (also known as participation certificates) are used to replicate ownership of a specific index. One of the major advantages of such contracts is their cost-efficiency: typically, no load or management fees are charged. Owner of the certificate usually receives some part of the dividends during lifetime of a contract. Tracker certificates also allow to buy "a certain part" of an index, most commonly 1/10 or 1/100.

One of the complications of this model is necessity to account for dividends paid in the past. Reasoning behind is: the tracker certificate is replicating ownership of a stock, or a basket of stocks. If we have a dividend on a stock present, the stock price drops on ex-dividend date, hence, price of a tracker certificate moves down as well. However, certificate's value also gains the cash value of the dividend (minus correction for taxes on the dividend).

For both past and future payments only discrete dividends are supported by this model.

Formulae

Fair price of a tracker certificate is given as:

$$V = e^{-rT} FR + (1 - \alpha_c) R \sum_{i=1}^{N} D_i e^{-rt_i} + (1 - \alpha_c) R D_p + P,$$

where

- D_i - future dividends,
- D_p - sum of all past dividends, starting from issue date until today,
- R - redemption, i.e. part of the basket holder of certificate obtains, specified via *fair price multiplier* instrument parameter,
- $1 - \alpha_c$ - percentage of dividends paid to the holder of certificate,
- P - an additional value due to corporate actions that will be paid to the holder, specified via *fair price offset* instrument parameter.

Things to note about dividend treatment:

1. Both past and future dividends are taken from *Dividends* table of the dividend instrument
2. Model currently supports discrete dividends only
3. If issue date is not specified, all past dividends are included in D_p

33 Variance Swap Pricing Model

Front-end name: Variance Swap

Type: Numerical

Target Instruments: SVXXXX

Relevant Parameters: S , r, q, σ, T, T_{issue}, Last Reference Price, Realized Variance, Variance swap replication accuracy, Variance swap percentage of ATM

Calculated values: V, Δ, Γ, ν, Skew Greeks

Description

Historical data convention

In this paragraph, we assume that historical price data is adjusted to remove gaps caused by all corporate actions such as:

- stock splits
- dividends/distributions
- rights offerings

The are two reasons behind this:

- We need to compare the security's current price to its historical price consistently (for example, stocks splitting itself causes some variance in underlying, which we want to sort out).
- In Tbricks, replication method is used in order to make some expectations about future variance, one of this method's limitations is assumption of underlying to follow general continuous diffusion (i.e. no jumps are allowed).

Realized variance

Let $S_0, S_1, \ldots S_N$ denote prices of the underlying asset at arbitrary equidistant time points $0 = t_0 < t_1 < \ldots < t_N = T$. From now on we assume that the time period between observations $k = t_i - t_{i-1}$ is an integer number of days. Define (natural) log-returns by

$$R_i = ln(\frac{S_i}{S_{i-1}}),\ i = 1 \ldots N.$$

The discrete realized variance, measured from time zero to maturity at time T with N interim samplings is given as:

$$\sigma_R^2(0;T) = \frac{1}{N}\sum_{i=1}^{N} ln^2(\frac{S_i}{S_{i-1}}) = \frac{1}{N}\sum_{i=1}^{N} R_i^2.$$

In practice, one is often interested in annualizing realized variance - this is done via multiplying by so called annualizing factor $AF = \frac{N}{T}$. This can be viewed as number of observations during a year, where AF = 252 corresponds to a daily sampling frequency and 252 trading days in a year. Factors taking on values such as 52 and 12 correspond to weekly and monthly samplings respectively.

$$\sigma_R^2(0;T) = \frac{AF}{N}\sum_{i=1}^{N} ln^2(\frac{S_i}{S_{i-1}}) = \frac{AF}{N}\sum_{i=1}^{N} R_i^2 = \frac{\frac{N}{T}}{N}\sum_{i=1}^{N} R_i^2 = \frac{1}{T}\sum_{i=1}^{N} R_i^2.$$

Note that time T as well as all t_i are measured in years.

Formulae

A variance swap (VAS) is essentially a forward contract on annualized variance, its payoff of is realized at maturity and given by:

$$\text{Payoff} = (\sigma_R^2(0;T) - K_{var})N_{var}, \text{where} \tag{4}$$

σ_R^2 - annualized realized volatility
K_{var} - annualized variance delivery price (sometimes referred as strike)
N_{var} - notional amount

Notional amount is typically quoted in dollars per volatility point, for example, N_{var} = 250,000USD/(volatility point). In TBricks, notional amount is expressed using "Fair price multiplier" instrument parameter. Delivery price is usually set to annualized risk neutral expected future variance over the maturity of the swap, hence, variance swaps have zero value at initiation, just like any other forward contract.

$$K_{VAR} = E_0[\sigma_R^2(0;T)].$$

By convention, no cash flows are exchanged when a swap is initiated. As all forward contracts, variance swaps are quoted in terms of delivery price, i.e. K_{VAR} is the price at which the variance swap is traded. At some time point t between initiation and maturity, delivery price is given as:

$$K_{VAR}(t) = \frac{\sigma_R^2 * (d_p - k) + \sigma_{intra}^2 * k + E_t[\sigma_T^2] * d_{TTM}}{d_p + d_{TTM}}$$

d_p - number of trading days passed since contract initiation
d_{TTM} - number of trading days until expiration
k - $1\{d_p > 0\}$
σ_R^2 - realized underlying variance over the life of the contract
σ_{intra}^2 - intraday variance for the calculation date
$E_t[\sigma_T^2]$ - expected future variance until expiration

Calculation of realized variance is currently not supported in TBricks and has to be set using "Realized variance" instrument parameter.

The intraday variance σ_{intra}^2 is derived from the logarithmic return of the underlying asset since the last recorded reference price. Any dividends paid are taken into account explicitly.

$$\sigma_{intra}^2 = 252 \log^2 \frac{S_t}{S_{t-1}}$$

S_t - underlying price on day t
S_{t-1} - adjusted reference price recorded on market open

The latter value is taken from the "Last reference price" instrument parameter.

The future expected variance $E_t[\sigma_T^2] = E[\sigma_T^2]$ depends on the volatility model of the underlying asset. We use the replication method to compute it:

$$E_t[\sigma_T^2] = \frac{2}{(T-t)}\left(r(T-t) - (\frac{S_t}{F}\exp^{r(T-t)} - 1) - \log\frac{F}{S_t} + E_t[\frac{S_t - F}{F} - \log\frac{S_t}{F}]\right)$$

$$\begin{aligned}
E_t[\frac{S_t - F}{F} - \log\frac{S_t}{F}] &= \exp^{r(T-t)}\left(\sum^{K>F}\frac{\Delta K}{K^2}C(K,\sigma(K,T)) + \sum^{K<F}\frac{\Delta K}{K^2}P(K,\sigma(K,T))\right)\\
&= \exp^{r(T-t)}\left(\sum_{i=0}^{N-1}\frac{\Delta K}{K_{c,i}^2}C(K_{c,i},\sigma(K_{c,i},T)) + \sum_{i=0}^{N-1}\frac{\Delta K}{K_{p,i}^2}P(K_{p,i},\sigma(K_{p,i},T))\right)
\end{aligned}$$

$C(K,\sigma(K,T))$ - BS price of a vanilla call with strike K and volatility $\sigma(K,T)$
$P(K,\sigma(K,T))$ - BS price of a vanilla put with strike K and volatility $\sigma(K,T)$
F - current theoretical forward price of the underlying asset
T - time to contract expiration

The grid of strikes is determined by two parameters: accuracy (N) and percentage of ATM (p_{ATM}). Note that only out-of-the-money options are used in order to replicate a variance swap.

$$K_{c,i} = F + i\Delta K, \qquad i = 0...N-1$$

$$K_{p,i} = F - i\Delta K, \qquad i = 0...N-1$$

$$\Delta K = p_{ATM} * F/(N-1)$$

N and (p_{ATM}) are taken from "Variance swap replication accuracy" and "Variance swap percentage of ATM" instrument parameters, respectively.

34 Bond Model

Front-end name: Bond

Type: Analytical

Target Instruments: DBFXXX; DXXXXA

Relevant Parameters: S, F , r, T_0, T_i^f, T_i^{st}, T_i^{ex}, T_i^p, c_i, c_N, T_{start}, T_{end}, B_{start}, B_{end}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, YTM, R, bid implied YTM, ask implied YTM, fair YTM, Δ_{YTM}, Γ_{YTM}, a

Description

Bonds are debt securities under which the issuer is obliged to make payments at pre-specified times. The amount of payments is pre-defined at the beginning: the payments c_i, $i = 1, \ldots, N-1$ are called coupons and represent the interest paid to the holder of the bond, the payment c_N is called notional and corresponds to the main amount of the debt that is paid at the bond expiration. The ZCYC override group and the rate offset are taken into account.

The payments are represented as a table parameter with following columns:

column name	**notation**	**description**
coupon′s fixing date	T_i^f	Rate is fixed
period start date	T_i^{st}	Pricing starts accruing the coupon
ex − coupon date	T_i^{ex}	Accruing of the "current" coupon ends
payable date	T_i^p	Cash is registered
cash amount	c_i	Cash amount of the coupon
coupon rate		Coupon rate
payment currency		Payment currency
payment type		Coupon or notional

In the usual setup period start date is equal to the previous ex-coupon date, fixing date can be equal to period start and ex-coupon to payable.

The fair price of the bond is computed as

$$V = \sum_{i=1}^{N} c_i e^{-rt_i}$$

where $t_i = t_i^{ex} + \Delta t_i^p$, $t_i^{ex} = T_i^{ex} - T_0$ is time to ex-coupon date for each payment, $\Delta t_i^p = T_i^p - T_i^{ex}$, N herein is number of coupons with $T^{ex} > T_0$, coupons with T^{ex} in the past are ignored.

To compare different bonds between each other, it is common to look at the bond *yield to maturity* (YTM) rather than the price. The yield corresponding to a market price V_{market} is a value y such that

$$V_{\text{market}} = \sum_{i=1}^{N} c_i e^{-yt_i}$$

We have bid implied YTM, ask implied YTM and fair YTM that correspond to the bid/ask market prices and to the fair price.

There are two additional YTM-based Greeks for bonds: Delta YTM and Gamma YTM given by

$$\Delta_{YTM} = \frac{\partial V}{\partial y} = -\sum_{i=1}^{N} c_i t_i e^{-yt_i}$$

and

$$\Gamma_{YTM} = \frac{\partial^2 V}{\partial y^2} = \sum_{i=1}^{N} c_i t_i^2 F e^{-y t_i}.$$

All the usual Greeks, besides ρ are equal to zero.

If the bond is sold at time T between the coupon payments $T_i < T < T_{i+1}$, the fraction a of the coupon c_{i+1} that corresponds to the period (T_i, T) is considered to belong to the seller. This part is called *accrued interest*; to compute it, the time period (T_i, T) is taken according to the accrued interest day count convention. Note that T is the settlement date of the trade; for example, if the trade is made on a Friday and the "Settlement Days" parameter is set to 1, the days will be counted from the following Monday.

Note: Accruing of the "current" coupon starts on the period start (effective) date and ends on the ex-coupon date. For the usual setup end date of the "current" coupon coincide with start date of the "next" one. If the periods overlap then total accrued interest is given by the sum of the positive parts related to both coupons.

Bonds can be quoted in *clean* (by default) or *dirty* prices, where

$$V_{\text{dirty}} = V_{\text{clean}} + a.$$

The switch between clean and dirty pricing is controlled by an instrument parameter "Bond quoted". Note that there is no accrued interest on the notional payment. If bond is quoted clean accrued interest will be subtracted from the bid/ask and fair market prices $V_{\text{market}} - a$, and then YTM is calculated based on the "clean" price.

Bond instrument can act as a fixed leg of Interest Rate swap.

Bond model is also used for pricing of deposit instruments. A **deposit** is a debt instrument where the amount B_{start} is paid at the time T_{start} and the amount B_{end} is received at the time T_{end}. The *rate amount* for the deposit is computed as

$$R = B_{\text{end}} - B_{\text{start}}$$

and the YTM is computed following

$$\frac{R}{B_{\text{start}} \cdot (T_{\text{end}} - T_{\text{start}})},$$

where the time difference in the denominator is computed according to the YTM day count convention.

The fair price of the deposit is computed as:

$$V = B_{\text{end}} e^{-r t_{\text{end}}}$$

where $t_{\text{end}} = T_{\text{end}} - T_0$. If either B_{end} or T_{end} is not defined the Payments table must be setup with just one row with notional payment and identical dates. Then fair price of deposit is computed identical to bond with just one coupon:

$$V = c_N e^{-rt}$$

where $t = t^{ex} + \Delta t^p$, $t^{ex} = T^{ex} - T_0$ is time to ex-coupon date for notional payment, $\Delta t^p = T^p - T^{ex}$.

35 Floating Rate Note Model

Front-end name: Bond

Type: Analytical

Target Instruments: DTVXXX

Relevant Parameters: S , r, T_0, T_i^f, T_i^{st}, T_i^{ex}, T_i^p, c_i, c_N

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, YTM, bid implied YTM, ask implied YTM, fair YTM, Δ_{YTM}, Γ_{YTM}

Description

Floating Rate Notes (FRNs) are debt securities under which the issuer is obliged to make payments at pre-specified times t_i, $i = 1, \ldots, N$. The amount that is paid at times t_i is unknown at the start and is fixed at pre-specified future times to some interest rate on the Notional c_N. The fair price is computed as sum of estimated coupons (with fixing dates T_i^f in the future) and known coupons (with Ex-coupon dates T_i^{ex} in the future and fixing dates in the past):

$$V = \sum_{T_i^f > T_0} \left[c_N e^{-rt_i} \left(\frac{e^{-rt_i^{st}}}{e^{-rt_i^{ex}}} - 1 \right) \right] + \sum_{T_i^{ex} > T_0, T_i^f < T_0} c_i e^{-rt_i}$$

where $t_i = t_i^{ex} + \Delta t_i^p$, $t_i^{ex} = T_i^{ex} - T_0$ is time to ex-coupon date for each payment, $t_i^{st} = T_i^{st} - T_0$ is time to period start date for each payment, $\Delta t_i^p = T_i^p - T_i^{ex}$, T_i^p is payable date, T_i^{ex} - ex-coupon date.

The payments are specified using the same table parameter as for the bond model: on effective date Pricing starts accruing the coupon, on ex-coupon date cash amount is calculated using the rate corresponding to the fixing date and accruing stops, and on Payable date coupon amount is paid out. Rate curve is used to estimate future payments. The ZCYC override group and the rate offset are taken into account in all computations.

FRN instrument can also act as a floating leg of Interest Rate swap or Rate leg of Equity swap.

36 Equity Return Model

Front-end name: Equity return

Type: Analytical

Target Instruments: EXXXXX, Variant Equity return

Relevant Parameters: S, F, U , r, T_0, T_i^f, T_i^{st}, T_i^{ex}, T_i^p, c_i, c_N

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

Equity return model is used to price instruments representing either total, price or dividend return from its underlying given the fixing dates, the initial price U_{initial} and the initial quantity Q_{initial} (notional amount $c_N = U_{\text{initial}} Q_{\text{initial}}$). Fixing, ex-coupon, and payable dates are stored in the Payments table associated with the instrument. This instrument fair price is the total present value of its cash flows with dividends d taken into consideration for total and dividend return instruments. For instruments with *Pricing scheme* parameter set to "Notional reset" Notional is recalculated for each period as $c_N = UQ_{\text{initial}}$ by Structured product maintenance app.

Fair price of total return instrument is:

$$V_{tr} = \sum_{i=1}^{N-1} \frac{F_{i+1} - F_i + \sum d_i}{F_i} c_N e^{-rt_i} + c_N e^{-rt_N}$$

where $\sum d_i = \sum_{T_{i-1}^{ex} < T_e < T_i^{ex}} d_i e^{-r(T_{divp} - T_e)}$, T_i^{ex} is ex-coupon date, T_e - ex-dividend date, T_{divp} - dividend payable date, $t_i = t_i^{ex} + \Delta t_i^p$, $t_i^{ex} = T_i^{ex} - T_0$ is time to ex-coupon date for each payment, $\Delta t_i^p = T_i^p - T_i^{ex}$, where T_i^p is payable date, T_i^{ex} - ex-coupon date, t_n is time to notional payment.

in case of price return:

$$V_{pr} = \sum_{i=1}^{N-1} \frac{F_{i+1} - F_i}{F_i} c_N e^{-rt_i} + c_N e^{-rt_N}$$

and dividend return:

$$V_{dr} = V_{tr} - V_{pr}$$

Delta is set to 1.0, Greeks are set to 0.0.

Equity return instrument represents equity leg of an equity swap.

37 Spreads / Capped Options

Front-end name: Spread

Type: Analytical

Target Instruments: RMXXXX, Variant Capped Call; RMXXXX, Variant Capped Put; RMXXXX, Variant EUSIPA 1200

Relevant Parameters: S, K, Cap Price , r, q, σ, T, $D_i, 1 \leq i \leq N_D$

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

Please note that this is a pricing model for instruments with CFI codes mentioned above, not to be confused with **spreads** as combinations consisting of two option legs and priced via the regular combination model.

As European options are sometimes too expensive, cheaper capped instruments can be used instead. A capped call instrument can be replicated as a long position in a call instrument together with a short position in another call with typically higher strike price. Such instrument yield payoffs that are bounded from above by $(CapPrice - K)$ and that is where "capped" terminology comes from. All the above holds for capped puts, except that strike price of the short position (i.e. cap price) is almost always lower than strike price of the long position.

From Tbricks point of view, a capped instrument (or a spread) has the same instrument attributes and parameters as a Vanilla European option with only one more parameter being necessary for pricing - CapPrice.

Pricing of such options proceeds relatively easy and reflects their "Buy option 1, sell option 2" nature: fair price and all Greeks is given as a difference between respectful values of two European options with all parameters being equal to ones of the capped instrument, except strike price. Strike of the first option is set equal to strike price of the capped instrument, of the second - to cap price.

A max certificate is essentially a capped call with strike price being equal to zero. Strike price instrument attribute is ignored for max certificates and can be left empty.

Implied volatility calculation for both capped calls and puts is disabled, since fair price of such instruments is not monotone with respect to volatility and the solution is undefined (fair price of both "bought" and "sold" "components" rises with volatility). Alternatively, one can state that there are multiple solutions of the implied volatility problem. Implied volatility is, however, supported for max certificates, due to their specific nature: fair price of a call option with zero strike price does not depend on volatility (vega is equal to zero) and fair value of the whole certificate is than monotone w.r.t. volatility.

Formulae

The value of capped calls and puts can be expressed as:

$$V_{CC}(S, K, t, C) = V_{EC}(S, K, t) - V_{EC}(S, C, t)$$

and

$$V_{CP}(S, K, t, C) = V_{EP}(S, K, t) - V_{EP}(S, C, t)$$

respectively, where

- C is the cap price.
- $V_{EP/EC}$ are the values of European options given by Bos-Vandermark model (see Black-Scholes with Analytical Dividend Corrections).

38 At-expiry barrier

Front-end name: At-expiry barrier

Type: Approximative Analytical

Target Instruments: OCEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In; OPEXXX, Variant Up-and-Out/Up-and-In/Down-and-Out/Down-and-In

Relevant Parameters: S, K , r, q, σ, T, H, R, t_{Window}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks, OEG, OEV

Description

In contrast to the usual Barrier options, knockout event of an At-Expiry option can only happen at maturity. This contraction of barrier observation period makes it possible to replicate an At-Expiry Barrier instrument with a portfolio of European Vanilla and / or European Binary options. Vanilla options are priced via Bos-Vandermark model (see Black-Scholes with Analytical Dividend Corrections), while Binary options are priced via Binary model (see Binary Option Pricing Model).

In Tbricks, the At-Expiry instruments have exactly the same CFI + CFI variant combination as the regular Barrier options. To specify that the instrument belongs to the former type, one needs to set instrument parameter "Barrier window start" to maturity datetime of the option.

As always in case of barrier options, there are eight possible combinations of payoff type (put or call) and barrier type (Down-and-In etc.). The replication process for each variant will be demonstrated graphically in the next two sections.

Due to symmetry of barrier conditions, replicating portfolios of the following pairs:

1. Down-and-In and Up-and-Out
2. Down-and-Out and Up-and-In

are exactly the same, which leads to equal fair price and Greeks.

Replicating portfolios for Call options

- **Down-and-In** Call and **Up-and-Out** Call
  - When $H < K$ the option always expires worthless
  - When $H > K$ the option can be replicated as a sum of
    1. Long position in one European put with strike K
    2. Short position in one European put with strike H
    3. Long position in $H - K$ European Binary puts with strike H

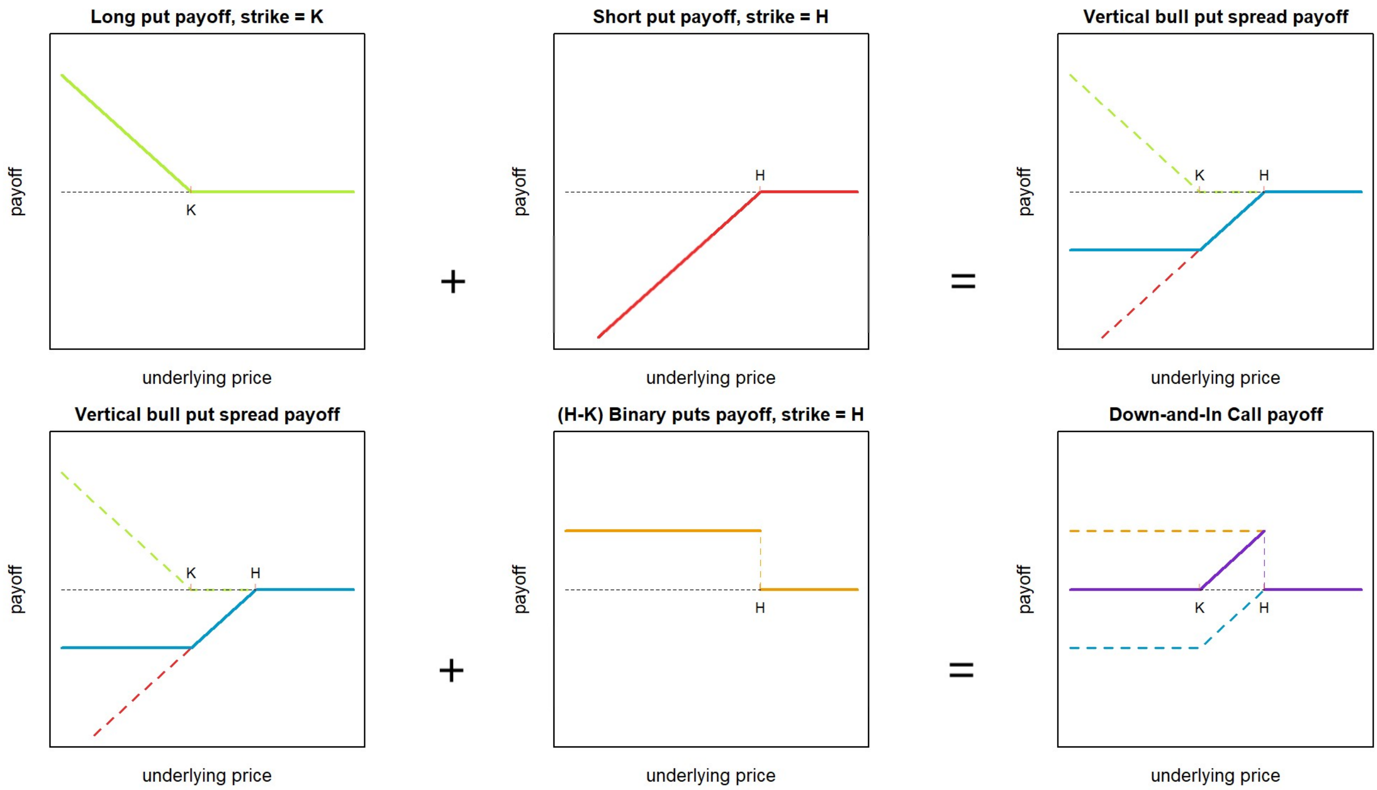

Figure 5: Replicating process for a Down-and-In Call

  - Same argument holds for the Up-and-Out variant

- **Down-and-Out** Call and **Up-and-In** Call
  - When $H < K$ the option is equal to a European Vanilla call with the same strike price
  - When $H > K$ the option can be replicated as a sum of
    1. Long position in one European call with strike H
    2. Long position in $H - K$ European Binary calls with strike H

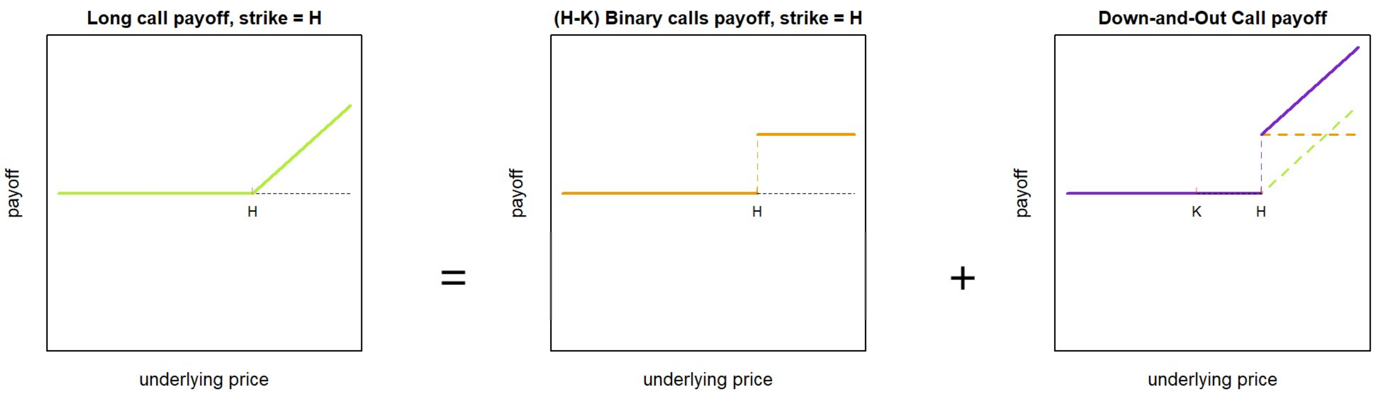

Figure 6: Replicating process for a Down-and-Out Call

  – Same argument holds for the Up-and-In variant

Replicating portfolios for Put options

- **Down-and-In** Put and **Up-and-Out** Put
  – When $H > K$ the option is equal to a European Vanilla put with the same strike price
  – When $H < K$ the option can be replicated as a sum of
    1. Long position in one European put with strike H
    2. Long position in $K - H$ European Binary puts with strike H

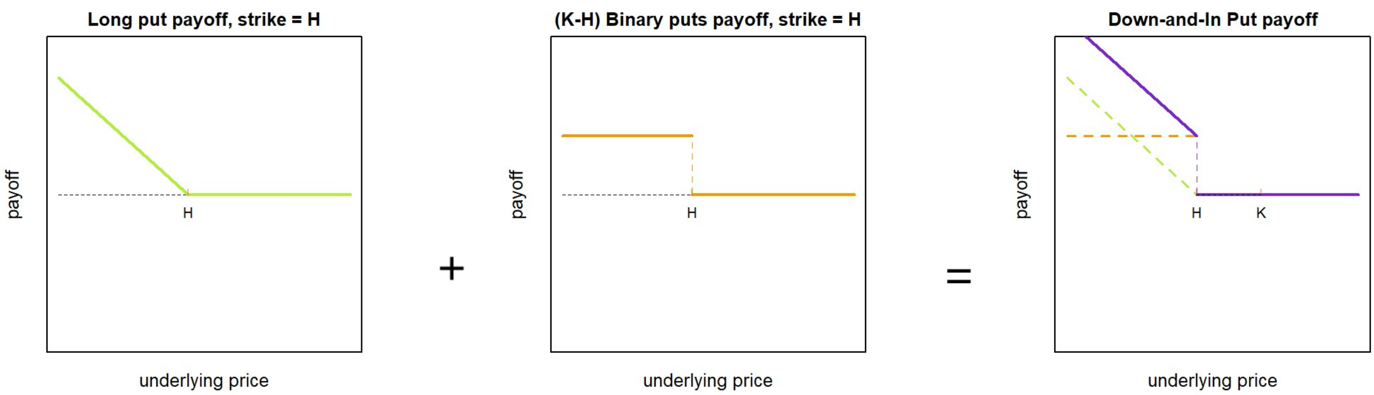

Figure 7: Replicating process for a Down-and-In Put

  – Same argument holds for the Up-and-Out variant

- **Down-and-Out** Put and **Up-and-In** Put

- When $H > K$ the option always expires worthless
- When $H < K$ the option can be replicated as a sum of
  1. Long position in one European call with strike K
  2. Short position in one European call with strike H
  3. Long position in $K - H$ European Binary calls with strike H

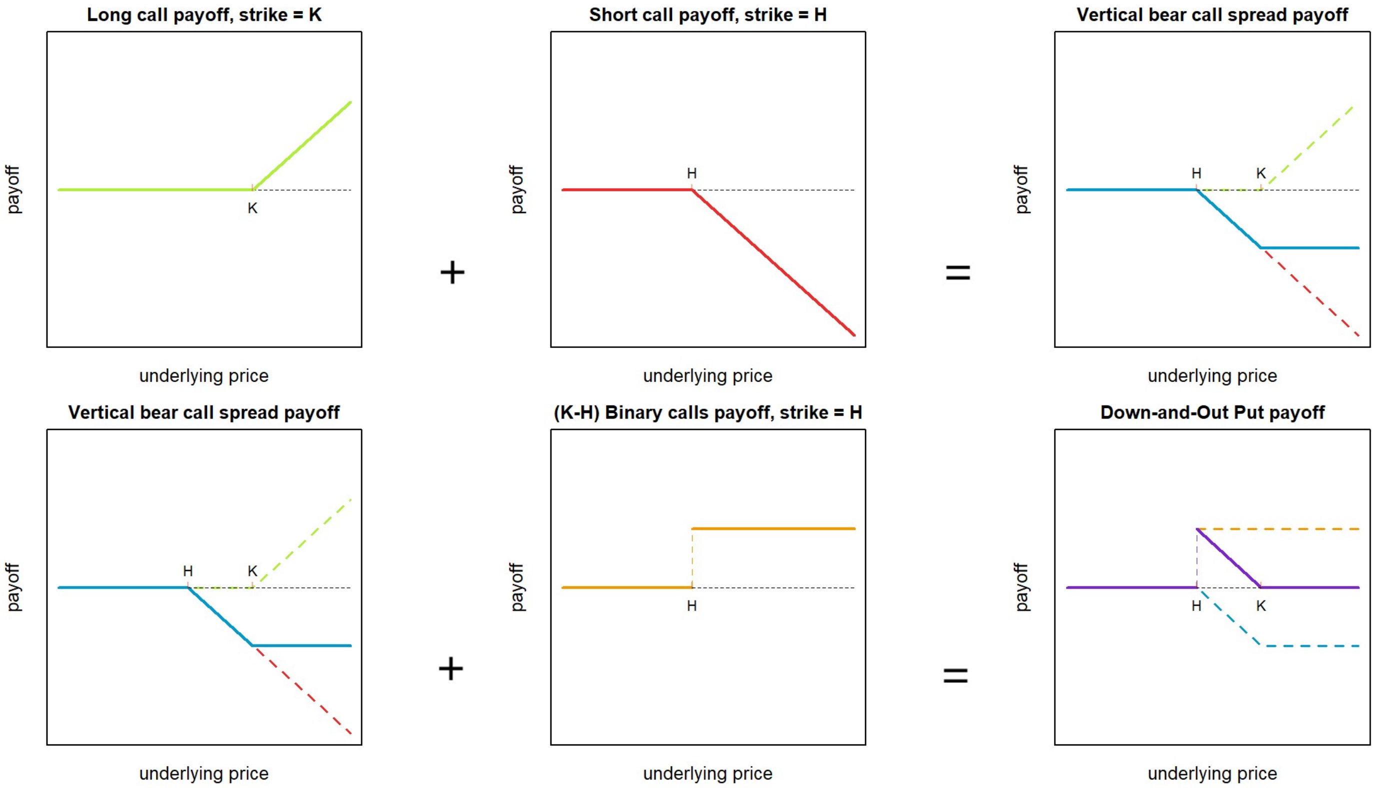

Figure 8: Replicating process for a Down-and-Out Put

- Same argument holds for the Up-and-In variant

This model does not support mixed discrete dividends.

39 Bonus Certificate

Front-end name: Bonus certificate

Type: Approximative Analytical

Target Instruments: RMXXXX, Variant EUSIPA 1250; RMXXXX, Variant EUSIPA 1320

Relevant Parameters: S, K , r, q, σ, $D_i, 1 \leq i \leq N_D$, T, H, BC, σ_H, f_{obs}, Cap Price, σ_{cap}

Calculated values: V, Δ, Γ, ν, ρ, Vanna, Vomma, Zomma, Speed, Skew Greeks, Calendar/Opening/Overnight Greeks

Description

A **bonus certificate** is a structured product that gives the holder an opportunity to avail sideway trends of the market over the short run. At contract initiation the barrier level is set below the underlying price, in contrast to the strike price which is set above. Optionally, the payoff of a bonus certificate may be capped. The subtype of the instrument is determined based on CFI variant: **EUSIPA 1250** defines capped certificates, **EUSIPA 1320** - uncapped ones. Barrier, strike, and cap (if any) prices must satisfy the following double inequality:

$$H \leq K \leq C,$$

where C denotes the cap price.

The payoff at maturity depends on whether the barrier level has been crossed during the lifetime of the contract:

1. If the barrier was never breached, the holder receives a cash payoff of

$$max(S, X),$$

which may then be capped like

$$min(max(S, X), C).$$

2. If the barrier was breached, the holder does not receive a bonus payment and the payoff is simply equal to price of the underlying instrument at maturity. It may still be capped like

$$min(S, C).$$

In Tbricks, the "Barrier crossed" instrument parameter defines whether an instrument has been knocked out.

Both capped and uncapped Bonus certificates are replicated by portfolios of vanilla European call options and European Down-and-out barrier put options.

Replicating portfolios

- An uncapped bonus certificate can be decomposed into
  1. Long position in a zero-strike European call
  2. Long position in a Down-and-Out put with barrier and strike equal to those of the bonus certificate
- A capped bonus certificate can be decomposed into

1. Long position in a zero-strike European call
2. Long position in a Down-and-Out put with barrier and strike equal to those of the bonus certificate
3. Short position in a European call with strike price equal to the cap price of the bonus certificate

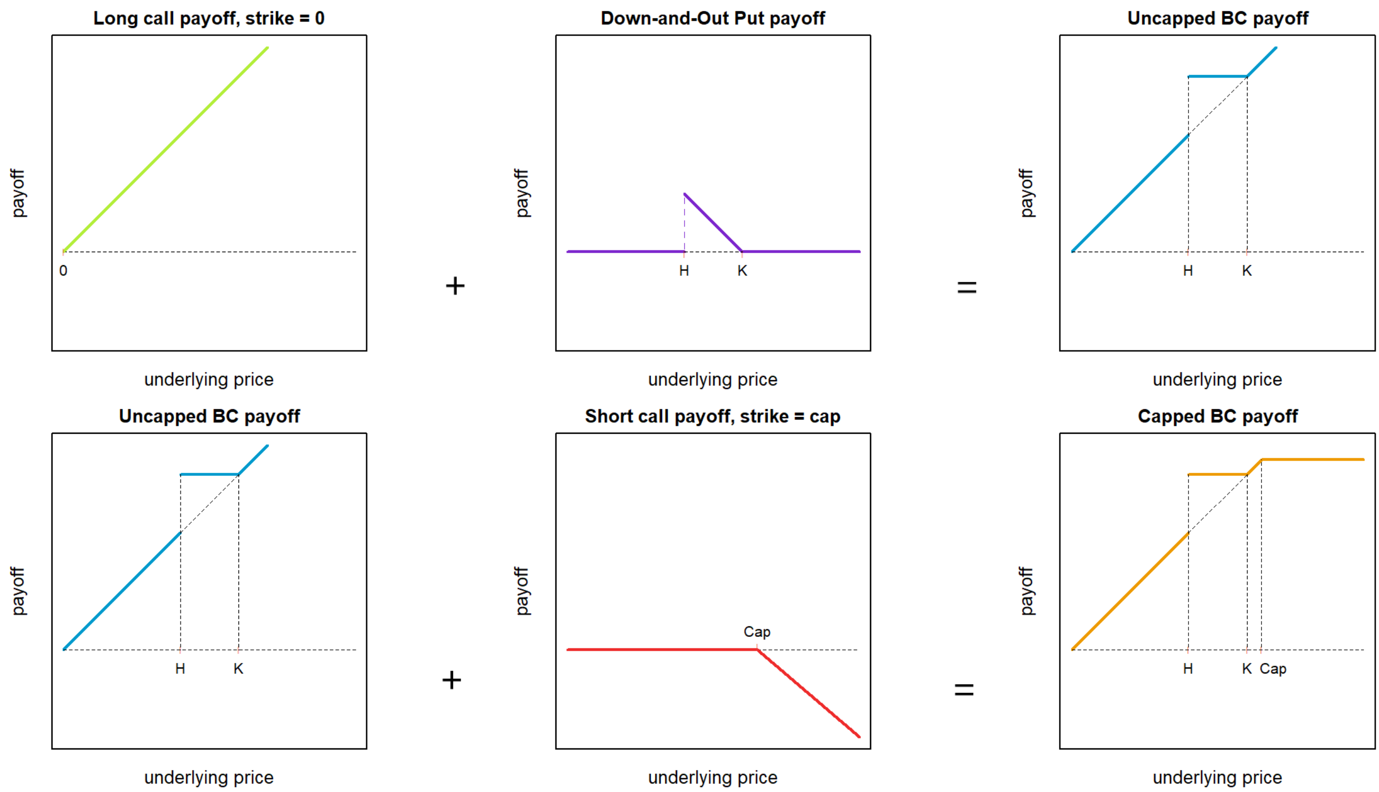

Figure 9: Replicating process for a Down-and-In Call

If the barrier is crossed, the Barrier part vanishes and the instrument transforms into a simple European call (possibly capped).

The European options are priced via the "Bos-Vandermark" model (see Black-Scholes with Analytical Dividend Corrections), while the "Two volatility barrier" (see Two-Volatility Barrier Option Pricing Model) model is applied for the barrier option. Usage of the latter model leads to the implied volatility problem being undefined for Bonus certificates.

40 Dividend Futures Pricing Model

Front-end name: Future / Forward

Type: Analytical

Target Instruments: FXXXXX, Variant Dividend futures

Relevant Parameters: T , r, $D_i, 1 \leq i \leq N_D$, T_{start}

Calculated values: V, Calendar/Opening/Overnight Greeks

Description

Fair market price of a dividend futures is given as a present value of the sum of all (both past and future) discrete dividend payments between Start and Maturity dates. If Start date is left empty, the summation starts from valuation date.

$$D = \{D_i : T_{start} \leq T_i \leq T\}$$
$$V = e^{-rT} \sum_{d \in D} d,$$

where T_i denotes datetime of i-th dividend payment.

41 Financial Mathematics

1.1 Forward price calculation

In the following paragraph, "forward price" (or simply $F(t)$) refers to the *expected spot price* at some future time moment t rather than delivery price of a forward contract. For derivative instruments, the "Forward price" calculated value shows $F(T)$ or expected price of the underlying instrument at maturity date of the derivative.

If the underlying asset is dividend free or all of discrete dividend payments are either absolute sized or percentage, forward price can be computed analytically. However, when dividend payments of both types are present, an explicit formula no longer exists and a finite-difference scheme is required. Moreover, in the latter setting $F(t)$ starts to show dependence on volatility.

Forward price calculation also depends on whether the underlying instrument is an equity or a future (with maturity date T^U).

1.1.1 Underlying instrument is an equity

As always, $D_i, 1 \leq i \leq N_D$ denotes all dividends paid until maturity date of the derivative.

$$F(T) = \begin{cases} S \cdot e^{(r-q)T} & \text{no dividends} \\ S \cdot e^{(r-q)T} - \sum\limits_{i=1}^{N_D} D_i e^{(r-q)(T-t_i)} & \text{only absolute-sized dividends} \\ S \cdot (1-D_1) \cdot \ldots \cdot (1-D_{N_D}) \cdot e^{(r-q)T} & \text{only percentage dividends} \end{cases}$$

1.1.2 Underlying instrument is a future, $T^U < T$

If underlying is a future, its price already contains the information of all dividends paid before its maturity.
Let $D_i, n^* \leq i \leq N$ denote all dividends between T^U and T.

$$F(T) = \begin{cases} S \cdot e^{(r-q)(T-T^U)} & \text{no dividends} \\ S \cdot e^{(r-q)(T-T^U)} - \sum\limits_{i=n^*}^{N_D} D_i e^{(r-q)(T-t_i)} & \text{only absolute-sized dividends} \\ S \cdot (1-D_{n^*}) \cdot \ldots \cdot (1-D_{N_D}) \cdot e^{(r-q)(T-T^U)} & \text{only percentage dividends} \end{cases}$$

1.1.3 Underlying instrument is a future, $T^U > T$

If the underlying future expires later than the derivative contract itself, calculating the forward price at T consists in discounting future price back from T_f to the maturity date of the option. This means that all dividends paid prior to the expiration of the derivative have no effect on the forward price.

Let $D_i, N_D + 1 \leq i \leq N^*$ denote all dividends between T and T^U.

$$F(T) = \begin{cases} \dfrac{S}{e^{(r-q)(T^U-T)}} & \text{no dividends} \\ \dfrac{S}{e^{(r-q)(T^U-T)}} + \sum\limits_{i=N_D+1}^{N^*} \dfrac{D_i}{e^{(r-q)(T_i-T)}} & \text{only absolute-sized dividends} \\ \dfrac{S}{(1-D_{N_D+1}) \cdot \ldots \cdot (1-D_{N^*}) \cdot e^{(r-q)(T-T^U)}} & \text{only percentage dividends} \end{cases}$$

1.1.4 Underlying instrument is a future, $T^U = T$

If the underlying future and the derivative expire on the same date, forward price is simply equal to the price of the future: $F(T) = S$.

1.2 Multiple curves for forward estimation and discounting

In classical financial mathematics theory, a single value of r and q is used for both "moving forward in time" (forward estimation) and "moving backward in time" (discounting). However, sometimes one is interested in separating those two processes. In Tbricks this can be achieved by setting up "ZCYC discount" (r_d) and "ZCYC offset discount" (r_{od}) tables in addition to the usual "ZCYC" and "Convenience yield" .

In the four curves setup, values of r and q are adjusted according to the following rules before being passed to the models:

$$\begin{aligned} r' &= r_d + r_{od} \\ q' &= r_d - r + r_{od} + q. \end{aligned}$$

Calculated values "Discounting rate" and "Synthetic yield" show r' and q' respectively. It is worth mentioning that the difference between r and q is preserved after the new variables enter the calculation:

$$r' - q' = r_d + r_{od} - (r_d - r + r_{od} + q) = r - q.$$

Note: The default values for r_d and r_{od} are r and 0, respectively. Note that

$$\begin{aligned} \lim_{r_d \to r, r_{od} \to 0} r' = r \quad \text{and} \\ \lim_{r_d \to r, r_{od} \to 0} q' = q, \end{aligned}$$

which is sufficient to provide continuity of fair market price between two- and multiple curves setups.

Note: As of Tbricks 2.13.1, the multiple curves feature is not supported for Bonds, Equity returns and FX options.

1.3 Dividend equivalent yield

In the presence of absolute sized and / or percentage dividends so called *dividend equivalent yield* approximation is used by a few of TBricks pricing models.

Let, as usual, F and S denote Forward and Spot price of the underlying contract. q_{eq} - dividend equivalent yield is then obtained by simply solving the well known forward price equation for convenience yield as if the dividends were not present:

$$F = Se^{(r-q_{eq})t} \Rightarrow q_{eq} = r - \frac{ln\left(\frac{F}{S}\right)}{t}.$$

1.4 Moneyness

Moneyness is the relative position of the forward price of an underlying asset with respect to the strike price of a derivative. Formally, moneyness is defined as:

$$M = \frac{ln(F/X)}{\sigma \cdot \sqrt{T}}.$$

This value is also known as the *standardized forward moneyness*.

In Tbricks, each option is placed to one of the following five groups depending on its moneyness value as well as "Moneyness threshold" (M_T) & "Deep moneyness threshold" (M_{DT}) settings:

1. Deep in-the-money, if $\phi \cdot M > M_{DT}$
2. In-the-money, if $M_T < \phi \cdot M \leq M_{DT}$
3. At-the-money, if $-M_T \leq \phi \cdot M \leq M_T$
4. Out-of-the-money, if $-M_{DT} \leq \phi \cdot M < -M_T$
5. Deep out-of-the-money, if $\phi \cdot M < -M_{DT}$,

where $\phi = 1$ for calls and $\phi = -1$ for puts.

References

[1] D.-H. Ahn, S. Figlewski, and B. Gao. Pricing discrete barrier options with an adaptive mesh model. *Journal of Derivatives*, 6(4):33, 1999-06-22.

[2] Giovanni Barone-Adesi and Robert E Whaley. Efficient analytic approximation of American option values. *The Journal of Finance*, 42(2):301–320, 1987.

[3] D. Benson. Derivatives. *Euromoney 1997 Guide to Germany*, pages 20–23, 1997.

[4] M. Bos and S. Vandermark. Finessing fixed dividends. *Risk*, 2002.

[5] Michael J Brennan and Eduardo S Schwartz. The valuation of american put options. *The Journal of Finance*, 32(2):449–462, 1977.

[6] M. Curran. Beyond average intelligence. *Risk*, 5(10), 1992.

[7] M. Curran. Valuing Asian and portfolio options by conditioning on the geometric mean price. *Management Science*, 40(12), 1994.

[8] Jonatan Eriksson. Explicit pricing formulas for turbo warrants. *Risk magazine*, 2006.

[9] S. Figlewski and B. Gao. The adaptive mesh model: A new approach to efficient option pricing. New York University, Leonard N. Stern School finance department working paper series, New York University, Leonard N. Stern School of Business-, 1998.

[10] Jim Gatheral. *The Volatility Surface: A Practitioner's Guide*, volume 357. John Wiley & Sons, 2011.

[11] M.B. Giles and R. Carter. Convergence analysis of Crank-Nicolson and Rannacher time-marching. *Journal of Computational Finance*, 9, 2006.

[12] E. Haug. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill Education, 1997.

[13] Marek Musiela and Marek Rutkowski. *Martingale Methods in Financial Modelling*. Springer, 2010.

[14] W. Nordström. Adaptive tree techniques in option pricing. Master's thesis, KTH, Numerical Analysis, NA, 2015.

[15] M.H. Vellekoop and J.W. Nieuwenhuis. Efficient pricing of derivatives on assets with discrete dividends. *Applied Mathematical Finance*, 13(3):265–284, 2006.

[16] R. Zvan, P.A. Forsyth, and K.R. Vetzal. Discrete Asian barrier options. *Journal of Computational Finance*, 3, 1999.