Classifying Equilibria of a Dynamic Population Model

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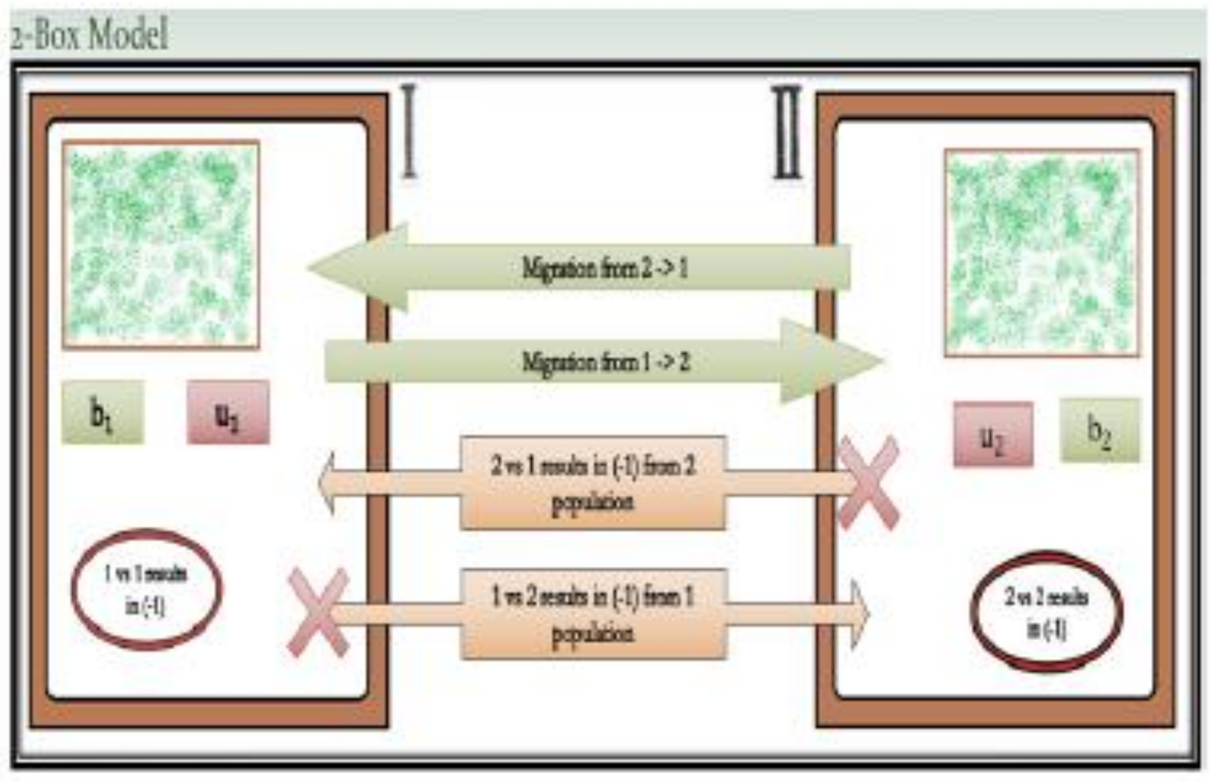
Abstract

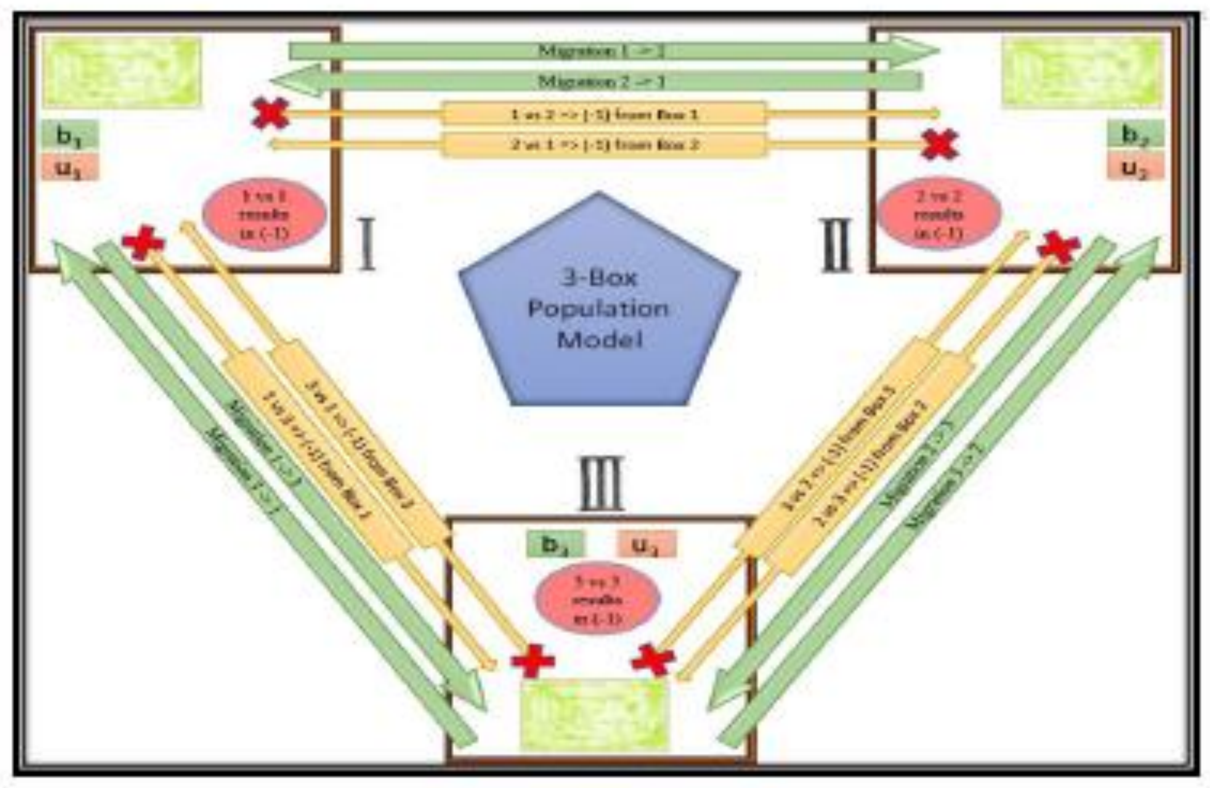
A mean field approximation is employed to analyze a model for population dynamics introduced by Bolker and Pacala. Simulations of a related random walk are run using R to search for equilibria and analyze their stability. A system of ordinary differential equations for the mean field approximation with unspecified parameters including birth, death, migration, and competition dynamics - the equilibrium points are found using *MATHEMATICA*. We then use the eigenvalue method to classify the stability of these equilibria and determine under what conditions on the given parameters, the population equilibria are stable and unstable.

Introduction

During this research we use simulated random walks to search for equilibria of the 2-Box and 3-Box Bolker-Pacala population model. In previous research we focused on rationalizing instances of stability for the equilibria of the 2-Box model. In that we found out that there exists a stable equilibrium point if certain inequalities hold true (in terms of the unspecified parameters). We use the inequalities as a starting point to make a justified guess for any set of values for our unknown parameters. Our hope is that when the values are plugged into our random walk simulation we will have stable equilibria. We can then make sure under what specific conditions, our equilibria are stable and unstable. We then repeat the process for the 3-Box Model. We also made a software application using *Shiny* to help expedite the random walk simulations.

Models





The small box of green dots in both diagrams represent the plant population that Bolker and Pacala used in their introduction of this population model. In each model, each box has the same factors that affect that population given by the unspecified parameters birth, death, migration, and competition dynamics.

Discussion

For both models we assume that the birth is greater than the death rate, otherwise the population will eventually become extinct. The parameter condition we imposed on the model is given by the caption under each plot. The sensitive nature of the 2-Box Model is seen by Figures [3] & [4], where a 1% increase in the inner competition leads the model to be unstable. In Figure [2] we can see that an inner competition of 0 in a 2-Box Model leads to a stable growth while the same for a 3-Box Model leads to an unstable equilibria. In Figure [6] we see when the outer competition is zero, the system is stable. More plots have been made that demonstrate the outcomes of varying conditions which led to several theories.

Figures

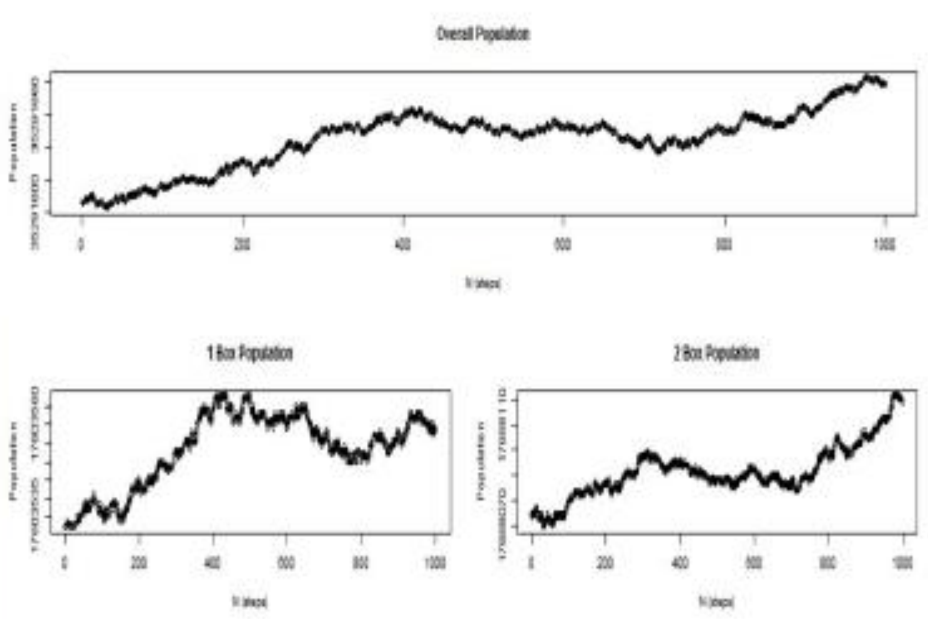


Figure 1: Equal parameters for 2-Boxes for ref..

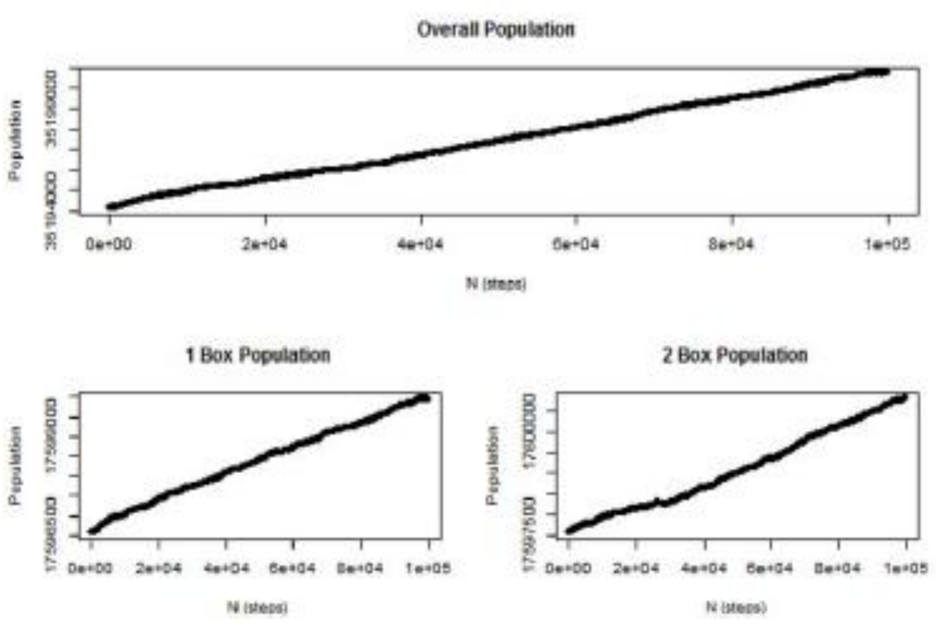


Figure 2: No inner competition.

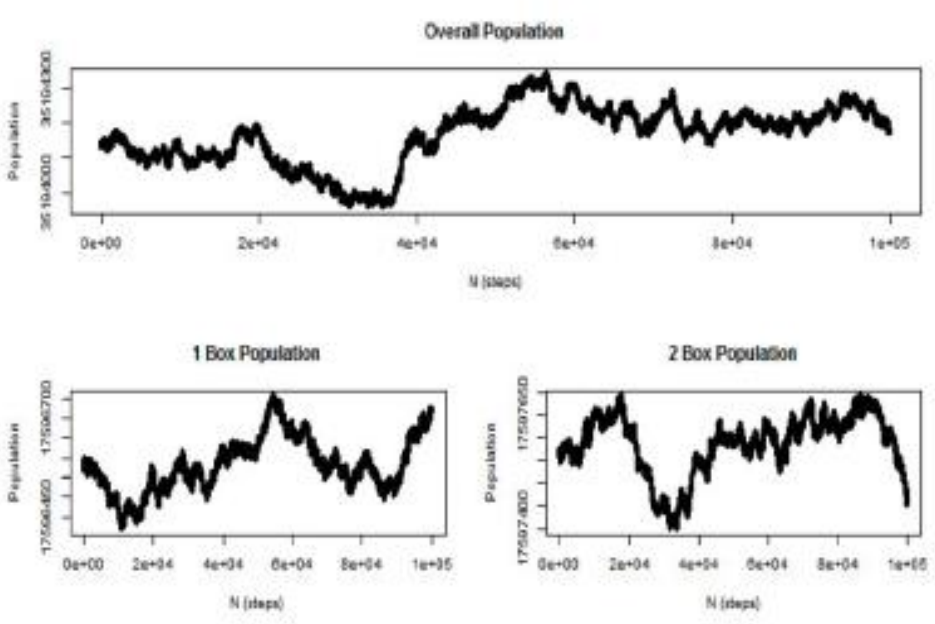


Figure 3: Inner & Outer = 0. Death = Birth

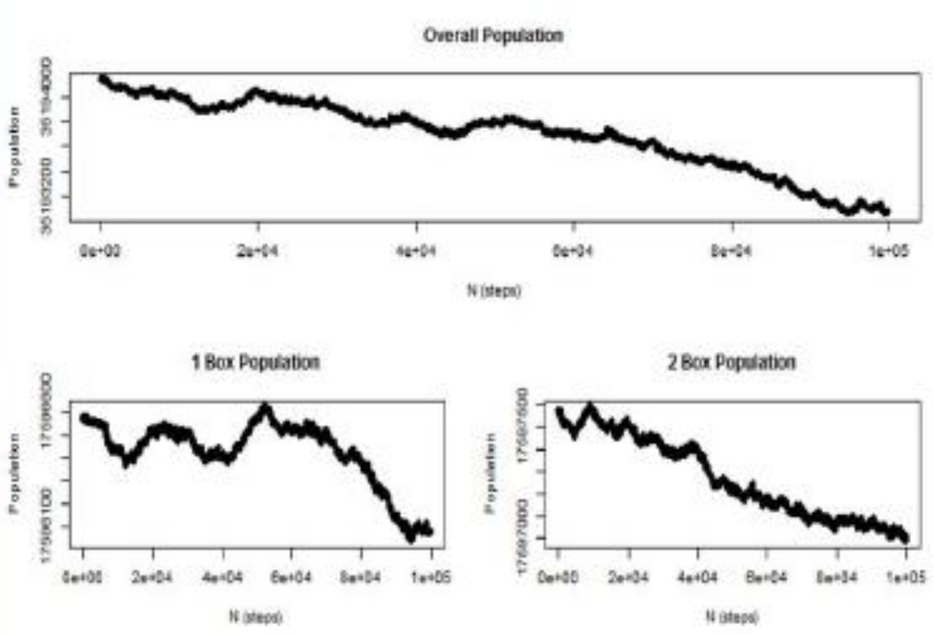


Figure 4: Figure [3] + 1% Inner Competition

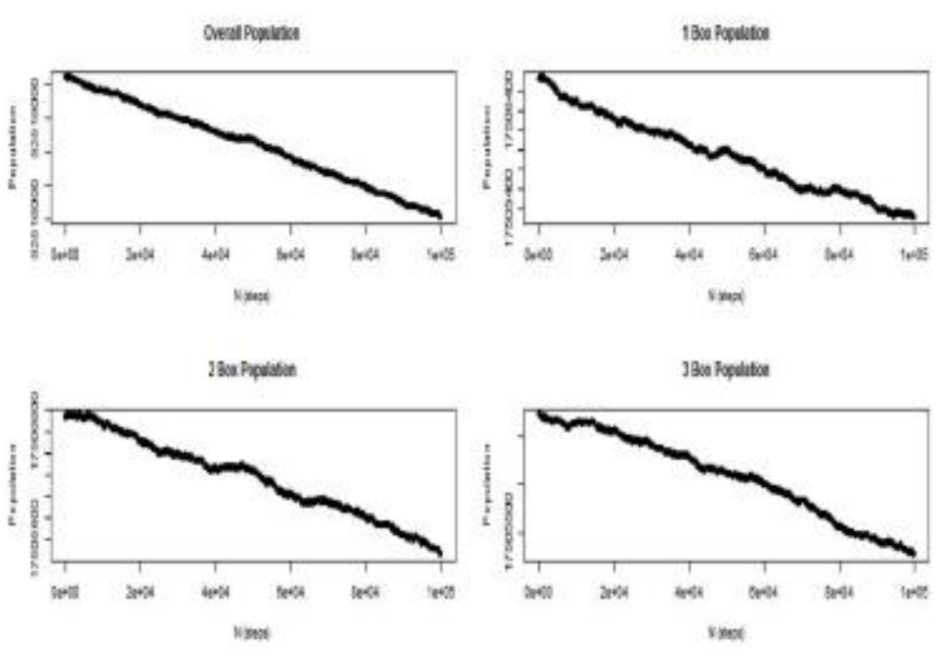


Figure 5: 3-Box No inner competition.

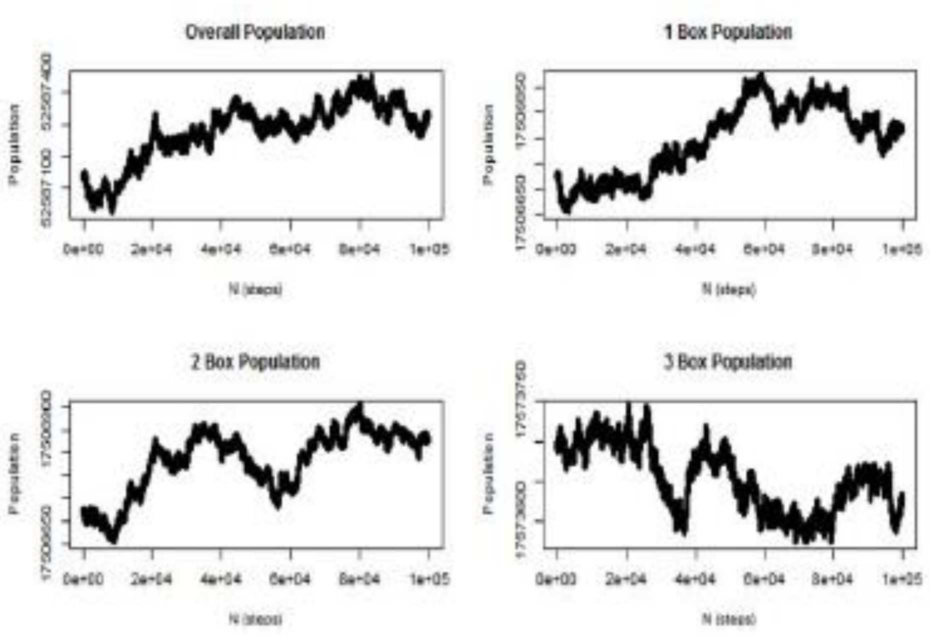


Figure 6: 3-Box Model, Outer Comp. = 0

Conclusion

For the 2-Box model we find that the conditions on the parameters that leads the population to be stable are: When there are no competition dynamics and birth is greater than deaths. The population also tends to be unstable when birth equals death with any small competition rate. For the 3-Box model the conditions for stability are the same. When the birth rates are different by more than 20% from box to box, only two boxes and the overall population are stable while one box dies out. The 3-Box Model is unstable when there is no inner competition and the migration is 0.

Acknowledgements

This research is supported by the BMCC MSEIP Grant under the supervision of Professor Mariya Bessonov.