Recitation 11

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Table of contents

1. Introduction

2. Problems

Intro

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- Going over test
- Homework due on Saturday

Problems

Problem 1a

Show that the following limit doesn't exist

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+4y^2}$$

$$\lim_{\substack{y=0\\x\to 0}} \frac{xy}{3x^2+4y^2} = \lim_{x\to 0} \frac{0}{3x^2} = 0$$

$$\lim_{\substack{y=x\\x\to 0}} \frac{xy}{3x^2+4y^2} = \lim_{x\to 0} \frac{x^2}{3x^2+4x^2} = \frac{1}{7}$$

$0 \neq \frac{1}{7}$, limit DNE

Problem 1b

Prove that the following limit exists (Squeeze Theorem)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{3x^2+4y^2}$$

$$\lim_{\substack{y=0\\x\to 0}} \frac{x^2y}{3x^2+4y^2} = 0$$

$$\frac{x^2}{3x^2+4y^2} \leq 1$$

$0 \leq \left| \frac{x^2y}{} \right| \quad |y| \, x^2 \quad \leq \quad |y|$

$$\left| \frac{\quad}{3x^2+4y^2} \right| = \frac{\quad}{3x^2+4y^2} \le$$

$\lim_{(x,y)\to(0,0)} 0 = 0$

$\lim_{(x,y)\to(0,0)} |y| = 0$ $\rightarrow$ By squeeze thm, we have our limit

4

Problem 1b

Prove that the following limit exists (Delta Epsilon)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{3x^2+4y^2}$$

Fix $\varepsilon > 0$. Want to find δ. Take $\delta = \varepsilon$

$\forall (x,y) : 0 < \sqrt{x^2+y^2} < \delta \rightarrow |y| \le \sqrt{x^2+y^2} < \delta = \varepsilon$

$$\frac{|y|x^2}{3x^2+4y^2} = \left| \frac{yx^2}{3x^2+4y^2} \right| \le |y| < \varepsilon$$

We have our limit.

5

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5 y

$-y \qquad -2xy \qquad -2$

$$f_x = \frac{\quad}{(x^2+y^2+1)^2}\cdot 2x = \frac{\quad}{(x^2+y^2+1)^2} \rightarrow f_x(1,1) = -\frac{\quad}{9}$$

$$f_y = \frac{x^2+y^2+1-2y^2}{(x^2+y^2+1)^2} = \frac{x^2-y^2+1}{(x^2+y^2+1)^2} \rightarrow f_y(1,1) = \frac{1}{9}$$

$$f(1,1) = z_0 = \frac{1}{3}$$

$$z - z_0 = f_x(1,1)\cdot(x-x_0) + f_y(1,1)\cdot(y-y_0)$$

$$z - \frac{1}{3} = \frac{-2}{9}(x-1) + \frac{1}{9}(x-1)$$

$$z = \frac{-2}{9}x + \frac{2}{9} + \frac{y}{9} - \frac{1}{9} + \frac{1}{3} = \frac{-2x}{9} + \frac{y}{9} + \frac{4}{9}$$

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$$z = \frac{-2(1.1)}{9} + \frac{0.8}{9} + \frac{4}{9} = \frac{2.6}{9} = \frac{13}{45}$$

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$f_x = 4x + y = 0$

$f_y = 4y + x - 30 = 0$

$f_z = 2z + 6 = 0$

$z = -3$

$y = -4x$

$4(-4x) + x - 30 = 0, \quad -15x - 30 = 0$

$x = -2$

$y = 8$

Point: $(-2, 8, -3)$

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$$H = \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & \boxed{2} \end{bmatrix}$$

$$\det \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} = 16 - 1 = 15 > 0$$

$$\det \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} - 0 \cdot \_ + 0 \cdot \_ = 30 > 0$$

H positive def $\Rightarrow$ $(-2, 8, -3)$ is local min

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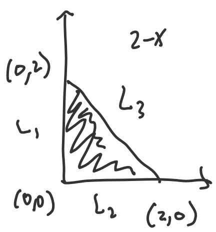

$f_x = 1 - 3y = 0$

$f_y = 2 - 3x = 0 \quad , \quad \left(\frac{2}{3}, \frac{1}{3}\right)$

$f\left(\frac{2}{3}, \frac{1}{3}\right) = \frac{2}{3} + \frac{2}{3} - \frac{6}{9} = \frac{2}{3}$ ✖

L_1: $f(0, y) = 2y$

min: 0 at $(0,0)$, max: 4 at $(0,2)$

L_2: $f(x, 0) = x$

min: 0 at $(0,0)$, max 2 at $(2,0)$

L_3: $f(x, 2-x) = x + 2(2-x) - 3x(2-x)$

$= 3x^2 - 7x + 4 = g(x)$

$g'(x) = 6x - 7, \quad x = \frac{7}{6}$

$f\left(\frac{7}{6}, \frac{5}{6}\right) = 3 \cdot \frac{49}{36} - \frac{49}{6} + 4 = \underline{\frac{-1}{12}} \to$ min

abs min: $\frac{-1}{12}$ at $\left(\frac{7}{6}, \frac{5}{6}\right)$

max: 4 at $(0, 2)$

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$\nabla f = (-2, 3)$

$\nabla g = (2x, 6y)$

$\begin{cases} -2 = 2\lambda x \\ 3 = 6\lambda y \\ x^2 + 3y^2 = 28 \end{cases} \to x = \frac{-1}{\lambda}, \quad y = \frac{1}{2\lambda}$

$\left(\frac{-1}{\lambda}\right)^2 + 3\left(\frac{1}{2\lambda}\right)^2 = 28, \quad \frac{1}{\lambda^2} + \frac{3}{4\lambda^2} = 28, \quad$ ✖ $\frac{7}{4\lambda^2} = 28, \quad \frac{1}{\lambda^2} = 16 \quad \lambda = \pm\frac{1}{4}$

$\lambda = \frac{1}{4}$: $x = -4, \quad y = 2 \quad = \quad f(-4, 2) = 14 \Rightarrow$ max

$\lambda = \frac{-1}{4}$: $x = 4$, $y = -2$ $\Rightarrow$ $f(4,-2) = -14 \Rightarrow$ min