Recitation_10

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Recitation 10

21256

Eric Li

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Carnegie Mellon University

Table of contents

1. Introduction

2. Quiz Answers

3. Problems

Intro

Quiz Answers

Problem 1

Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2}$$

$$\lim_{\substack{y=0 \\ x\to 0}} \frac{xy}{3x^2+y^2} = \lim_{x\to 0} \frac{0}{3x^2} = 0$$

$$\lim_{\substack{y=x \\ x\to 0}} \frac{xy}{3x^2+y^2} = \lim_{x\to 0} \frac{x^2}{3x^2+x^2} = \frac{1}{4}$$

$0 \neq \frac{1}{4}$, limit DNE

Problem 2

Find an equation of the tangent plane to the surface $=\sqrt{xy}$ at the point (-1,-1, 1).

$f(x,y) = \sqrt{xy}$

$f_x(x,y) = \frac{1}{2\sqrt{xy}} \cdot y$ $\quad f_x(-1,-1) = \frac{1}{2\sqrt{-1\cdot-1}} \cdot -1 = \frac{-1}{2}$

$f_y(x,y) = \frac{1}{2\sqrt{xy}} \cdot x$ $\quad f_y(-1,-1) = \frac{1}{2\sqrt{-1\cdot-1}} \cdot -1 = \frac{-1}{2}$

$z - 1 = \frac{-1}{2}(x+1) - \frac{1}{2}(y+1)$

Problem 3

Find the local maximum and minimum values and saddle points of the following function

$$f(x, y) = -2x^2 - 2y^2 + xy + 30y$$

$f_x = -4x + y = 0 \rightarrow y = 4x$

$f_y = -4y + x + 30 = 0$ $\quad -4(4x) + x + 30 = 0, \quad -15x + 30 = 0, \quad x = 2$

$y = 8$

$f_{xx} = \underline{-4}$

$f_{yy} = -4$ $D = \det \begin{bmatrix} 1 & -4 \end{bmatrix} = 15 > 0$

$f_{xy} = f_{yx} = 1$ $f_{xx} < 0$

$\Downarrow$

$(2,8)$ is local max

4

Problems

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$f_x = y - \frac{1}{x^2} = 0$, $y = \frac{1}{x^2}$

$f_y = x - \frac{1}{y^2} = 0 \to x - \frac{1}{\frac{1}{x^2}} = 0$, $x - x^2 = 0$, $(x)(1-x) = 0$

$\cancel{x=0}$, $\underline{x=1}$

$y = 1$

$f_{xx} = -\left(\frac{-2}{x^3}\right) = \frac{2}{x^3} \to f_{xx}(1,1) = 2$

$f_{yy} = -\left(\frac{-2}{y^3}\right) = \frac{2}{y^3} \to f_{yy}(1,1) = 2 \to D = \det\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = 3$

$f_{xy} = f_{yx} = 1$

$D > 0$, $f_{xx}(1,1) > 0$, $(1,1)$ local min

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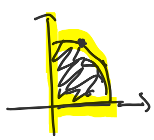

$f_x = y^2 = 0 \to \underline{y=0}$

$f_y = 2xy = 0$ x can be anything

$\to$ No points inside

$L_1 : f(0,y) = 0$

$L_2 : f(x,0) = 0$ $\to$ absolute min

$L_3 : x^2 + y^2 = 3$, $y^2 = 3 - x^2$, $y = \sqrt{3-x^2}$

$f(x, \underline{\sqrt{3-x^2}}) = x \cdot (\sqrt{3-x^2})^2 = \underline{3x - x^3}$

$\frac{df}{dx} = 3 - 3x^2$, $x = 1$ $\to (1, \sqrt{2}) \to \underline{f(1,\sqrt{2})} = 1(\sqrt{2})^2 = \underline{2}$

$y = \sqrt{3-1^2} = \sqrt{2}$

abs. max

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$\nabla f = \langle f_x, f_y \rangle = (2x, -2y)$ $\qquad \nabla f = \lambda \nabla g$ $\qquad \begin{cases} 2x = \lambda 2x \\ -2y = \lambda 2y \\ x^2+y^2 = 1 \end{cases} \rightarrow \begin{matrix} x = \lambda x \\ y = -\lambda y \end{matrix}$

$\nabla g = (g_x, g_y) = (2x, 2y)$

$x = \lambda x \rightarrow x = 0: \ 0^2 + y^2 = 1, \ y = \pm 1, \quad y = -\lambda y, \quad \lambda = -1 \rightarrow \begin{matrix}(0,1) \\ (0,-1)\end{matrix}$

$\searrow \lambda = 1: -2y = \lambda 2y, \ -2y = 2y, \ 4y = 0, \ y = 0,$

$x^2 + 0^2 = 1, \ x = \pm 1 \nearrow \begin{matrix}(1,0) \\ (-1,0)\end{matrix}$

$f(0,1) = 0 - 1^2 = -1 = f(0,-1) \rightarrow$ abs min

$f(1,0) = f(-1,0) = 1 \longrightarrow$ abs. max

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inside: $\begin{matrix} f_x = 2x + 4 = 0 \\ f_y = 2y - 4 = 0 \end{matrix} \nearrow$ critical point $(-2, 2)$

$(x,y) = \left(\frac{2}{\lambda - 1}, \frac{-2}{\lambda - 1}\right) \rightarrow \lambda = 1 + \frac{\sqrt{8}}{3}$

$\lambda = 1 - \frac{\sqrt{8}}{3}$

on boundry: $x^2 + y^2 = 9$

$\nabla f = \langle 2x+4, 2y-4 \rangle \longrightarrow \nabla f = \lambda \nabla g$ $\qquad \begin{cases} 2x + 4 = 2\lambda x \\ 2y - 4 = 2\lambda y \\ x^2 + y^2 = 9 \end{cases}$

$\nabla g = \langle g_x, g_y \rangle = (2x, 2y)$

$2x + 4 = 2\lambda x, \quad 4 = 2(\lambda - 1)x, \quad x = \frac{2}{\lambda - 1} \nearrow \left(\frac{2}{\lambda-1}\right)^2 + \left(\frac{-2}{\lambda-1}\right)^2 = 9, \quad \frac{8}{(\lambda-1)^2} = 9,$

$2y - 4 = 2\lambda y, \quad -4 = 2(\lambda - 1)y, \quad y = \frac{-2}{\lambda - 1}$ $\qquad (\lambda-1)^2 = \frac{8}{9}, \quad \lambda - 1 = \pm \sqrt{8}/3, \quad \lambda = 1 \pm \sqrt{8}/3$

$x = \frac{2}{\sqrt{8}/3}, \frac{2}{-\sqrt{8}/3} = \pm \frac{3}{\sqrt{2}}, \quad y = \pm \frac{3}{\sqrt{2}}$ $\qquad \left(\frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right), \left(\frac{-3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$