Recitation 14

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Intro

Intro

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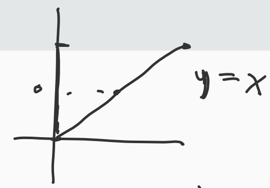
Quiz 4 Overview

Quiz Problem 1

Find the following integral, given that D is the region bounded by $x = 0, y = 0, y = x$

$y = 1, \ x = 1$

$$\int\int_D \ln(y^2)\,dA$$



$0 \le y \le 1$

$0 \le x \le y$

$\int\int \ln(y^2)\,dx\,dy$

$\downarrow$

$\int_0^1 \int_0^y \ln(y^2)\,dx\,dy \rightarrow \int_0^1 (x\ln(y^2))\Big|_0^y\,dy$

$= \int_0^1 y\ln(y^2)\,dy \rightarrow u = y^2, \ du = 2y$ $\quad \rightarrow \int \ln(x) \rightarrow$ $u = \ln x, \ du = \frac{1}{x}$, $dv = 1, \ v = x$

$\downarrow$

$= \frac{1}{2}\int_0^1 \ln(u)\,du$ $\quad uv - \int v\,du = x\ln x - \int \frac{1}{x}\cdot x = x\ln x - x$

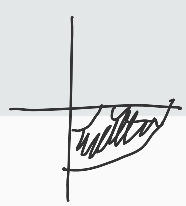
$= \frac{1}{2}(u\ln u - u)\Big|_0^1 = \frac{1}{2}(1\ln 1 - 1) - \frac{1}{2}(0\ln 0 - 0)$ $\quad \rightarrow \lim_{x\to 0} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$

$= \frac{-1}{2}$ $\quad \lim_{x\to 0} -x$

Quiz Problem 2

Find the following integral, given that D is the region bounded by $x \geq 0, y \leq 0, 1 \leq x^2 + y^2 \leq 9$

$$\int\int_D (2x + y^2)\,dA$$



$1 \le r \le 3$

$-\frac{\pi}{2} \le \theta \le 0$

$\int_{-\frac{\pi}{2}}^{0} \int_1^3 (2r\cos\theta + r^2\sin^2\theta)\,r\,dr\,d\theta$

$= \int^0 \int^3 2r^2\cos\theta + r^3\sin^2\theta\,dr\,d\theta = \int_{-\pi}^{0} \left(\frac{2r^3}{3}\cos\theta + \frac{r^4}{4}\sin^2\theta\right)\Big|_1^3 d\theta$

$=$

$= 0$

$$= \int_{-\frac{\pi}{2}}^{0} \left(\frac{54-2}{3} \cos\theta + \frac{81-1}{4} \sin^2\theta \right) d\theta = \int_{-\frac{\pi}{2}}^{0} \frac{52}{3} \cos\theta + 10(1-\cos 2\theta)\, d\theta$$

$$= \left(\frac{52}{3} \sin\theta + 10\theta - 5\sin 2\theta \right) \Big|_{-\frac{\pi}{2}}^{0} = 0 - \left(\frac{-52}{3} - 5\pi \right) = \frac{52}{3} + 5\pi$$

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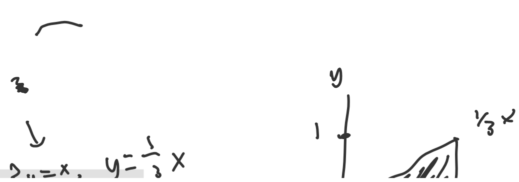
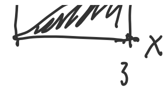


$$\int_0^3 \int_0^{\frac{1}{3}x} e^{x^2}\,dy\,dx$$



$0 \le x \le 3$

$0 \le y \le \frac{1}{3}x$

$$= \int_0^3 \left(y e^{x^2} \right)\Big|_0^{\frac{1}{3}x} dx \qquad \int_0^3 \frac{1}{3} x e^{x^2}\,dx$$

$u = x^2$, $du = 2x\,dx$ $\rightarrow$ $\int_0^9 \frac{1}{3}\cdot\frac{1}{2}\cdot e^u\,du = \frac{1}{6}\left(e^u\Big|_0^9\right) = \frac{e^9-1}{6}$

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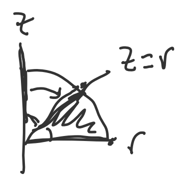
$\rightarrow z = \sqrt{\underbrace{r^2\cos^2\theta + r^2\sin^2\theta}_{r^2}} = r$

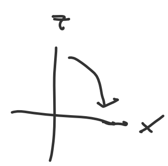
$x^2+y^2+z^2=4$, $x^2+y^2+z^2=\rho^2=4$, $\rho = 2$

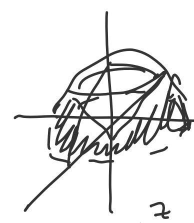
above xy plane: $\varphi = \frac{\pi}{2}$

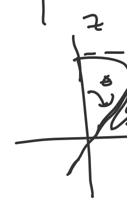
$z = \sqrt{x^2+y^2} = \sqrt{r^2} = r$, $\rightarrow$ $z = r$, $\varphi = \frac{\pi}{4}$



$\rightarrow \frac{\pi}{4}$



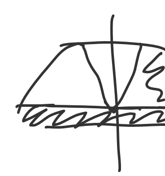




$0 \le \theta \le 2\pi$

$\longrightarrow dA = \rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta$

$$V = \iiint 1\,dA = \int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta =$$



$$= \int_0^{2\pi}\int_{\pi/4}^{\pi/2} \left(\frac{8}{3}\sin\varphi\right) d\varphi\,d\theta = \frac{8}{3}\int_0^{2\pi} -\cos\varphi\Big|_{\pi/4}^{\pi/2} d\theta = \frac{8}{3}\int_0^{2\pi} \frac{\sqrt{2}}{2}\,d\theta$$

$$= \frac{8\sqrt{2}}{3\cdot 2}\,\theta\Big|_0^{2\pi} = \frac{8\sqrt{2}}{3}\cdot\pi$$

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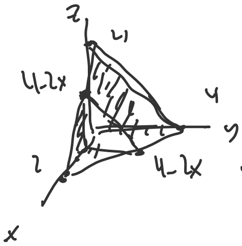
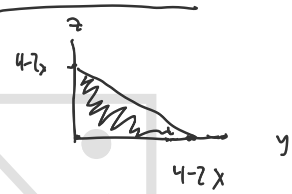
r



Fix x :



$z = (4-2x) - y$

$dz\,dy\,dx$

$0 \le y \le 4-2x$

$0 \le z \le 4-2x-y$

$0 \le x \le 2$

$$\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1\, dz\, dy\, dx = \int_0^2 \int_0^{4-2x} (4-2x-y)\, dy\, dx$$

$u = 4-2x, \quad du = -2$

$$= \int_0^2 (4-2x)y - \frac{1}{2}y^2 \Big|_0^{4-2x} dx = \frac{1}{2}\int_0^2 (4-2x)^2 dx \longrightarrow \frac{-1}{4}\int_4^0 u^2\, du$$

$$= \frac{1}{4}\int_0^4 u^2\, du = \frac{1}{12} u^3 \Big|_0^4 = \frac{16}{3}$$

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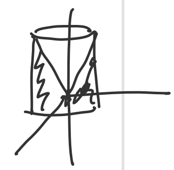
$\int$

→ within $x^2+y^2=1$, above $z=0$, below cone $z^2=4x^2+4y^2$



↓

$z^2=4r^2$

↓

$z=2r$

$r=1$

polar + z z, r, θ

→ fix θ, $0\le\theta\le 2\pi$,

$0\le z\le 2r$

$0\le r\le 1$

$$\int_0^{2\pi}\int_0^1\int_0^{2r} 1 \cdot r\,dz\,dr\,d\theta$$

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