Recitation 1

21256: Section C

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9/3/2020

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Table of contents

Intro

About me

- Junior at CMU, studying Math!
- Hobbies: Sports, Gaming, Music

Class Stuff

- OH: Thursday 2:30-3:30 PM, 8:30-9:30 PM
- Office hours are for theory questions/homework help. You may email me to set up an appointment if you wish
- Use Piazza for questions, please make posts public
- Recitation notes and videos will be posted at: www.lieric.com/teaching
- Feel free to email me at ericl2@andrew.cmu.edu if you have any questions or concerns!

Recitation Strategy

- Generally, we will spend 5-10 minutes covering theory, then the rest of recitation doing fun problems. We may occasionally break into groups to work on problems together.
- I will send polls occasionally to see how things are going
- Last 5-10 minutes, I will split you into breakout rooms.

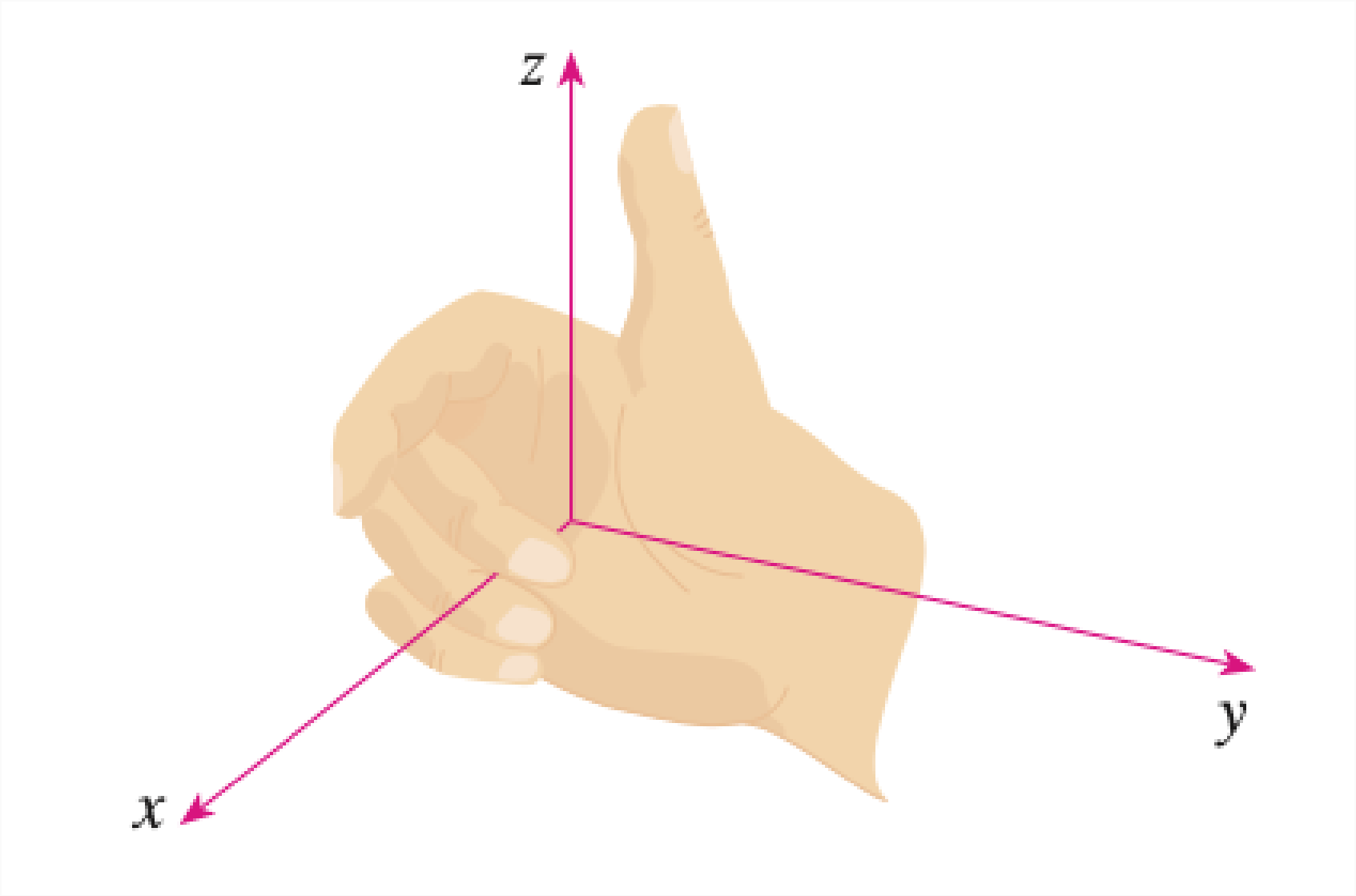
Why Multivariate Calculus?

Multivariate Calculus is everywhere, including:

- Machine Learning
- Statistics/Data Science
- Numerical Methods
- Physics
- Portfolio Optimization
- Future classes at CMU

3D Space

Right Hand Rule



Formulas

Distance Formula for Three Dimensions

The distance $|P_1P_2|$ between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

$$|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Equation of a Sphere

The equation of a sphere centered at $C = (h, k, l)$ and with radius r is:

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

Problems

Problem 1

Problem 1

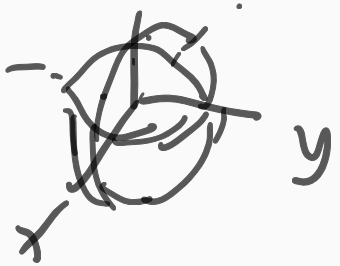
Find an equation of a sphere with center $(-3, 2, 5)$ and radius 4. What is the intersection of this sphere with the yz-plane?

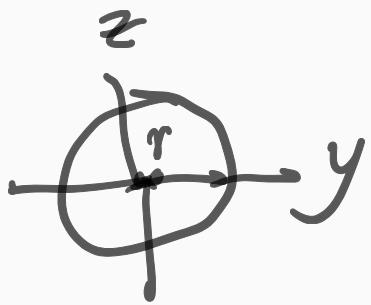
$C = (h, k, l) = (-3, 2, 5)$, $r = 4$

$(x-(-3))^2 + (y-2)^2 + (z-5)^2 = 4^2 = 16$

$(0-(-3))^2 + (y-2)^2 + (z-5)^2 = 16 \Rightarrow (y-2)^2 + (z-5)^2 = 7$

0 $\downarrow$ 9





Problem 2

Problem 2

Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has the center $(3, 8, 1)$.

$C = (3, 8, 1)$, $r = ?$

$$(x-3)^2 + (y-8)^2 + (z-1)^2 = r^2$$

$$(4-3)^2 + (3-8)^2 + (-1-1)^2 = r^2$$

$$1^2 + (-5)^2 + (-2)^2 = 1 + 25 + 4 = 30 = r^2$$

$$(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$$

Problem 3

Show that the following equation represents a sphere, and find its center and radius:

$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$$

same

+16 +16

$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

$(x^2+8x+16) + (y^2-6y+9) + (z^2+2z+1) + 17 = 16+1+9$

$(x+4)^2 + (y-3)^2 + (z+1)^2 = 16+9-17$

$(x+4)^2 + (y-3)^2 + (z+1)^2 = 9$

$C = (-4, 3, -1) \quad r = \sqrt{9} = 3$

Problem 4

Show that the following equation represents a sphere, and find its center and radius:

$$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$$

$$x^2 + y^2 + z^2 = 4x - 12z + \frac{1}{2}$$

$$x^2 - 4x + y^2 + z^2 + 12z = \frac{1}{2}$$

$$(x-2)^2 + y^2 + (z+6)^2 = \frac{1}{2} + 4 + 36$$

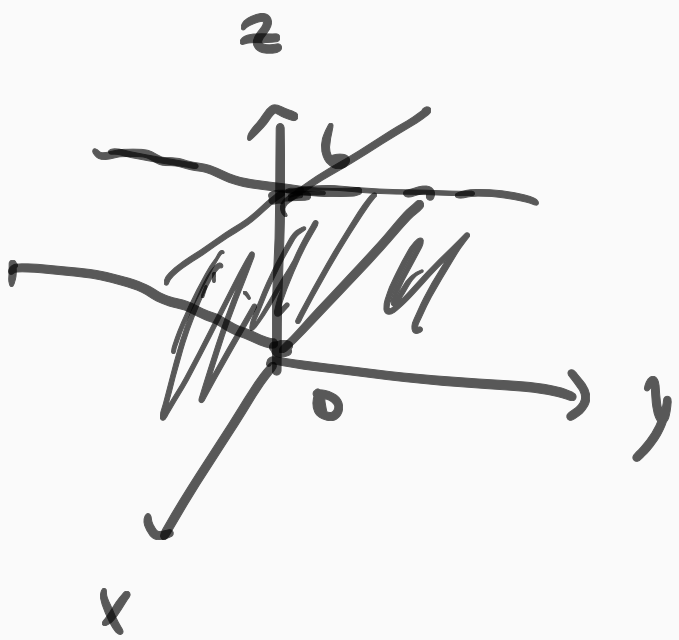
$$= 40.5$$

$C = (2, 0, -6)$ $\quad R = \sqrt{81/2}$

Problem 5

Describe in words the region of $\mathbb{R}^3$ represented by the equation(s) or inequality:

$$0 \leq z \leq 6$$



Infinite 3D object

Expands infinitely in the xy− plane

Has of height of 6 in the z plane

$$0 \leq z \leq 6$$

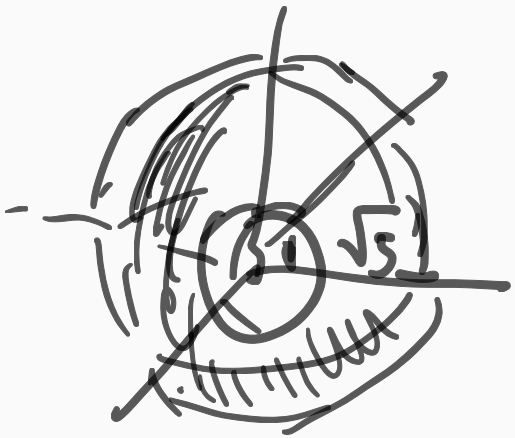
Problem 6

Describe in words the region of $\mathbb{R}^3$ represented by the equation(s) or inequality:

$$1 \leq x^2 + y^2 + z^2 \leq 5$$

$x^2+y^2+z^2 = 5$

$x^2+y^2+z^2 = 1$

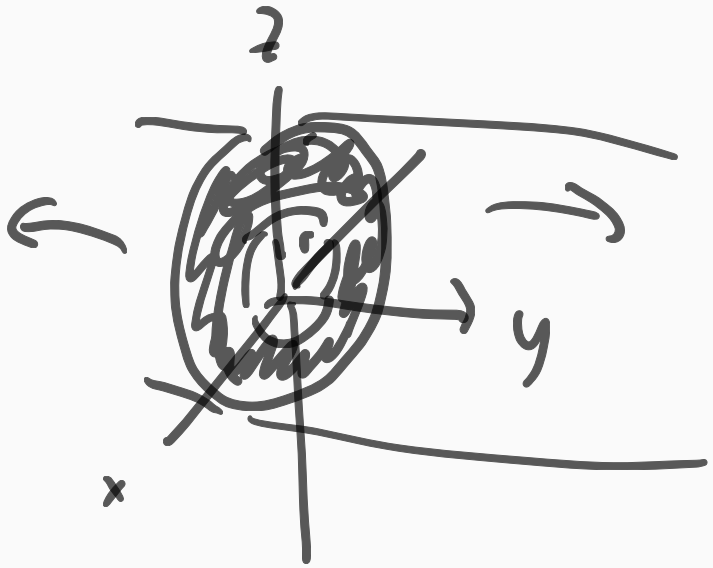


The area between a sphere radius 1 and sphere of radius $\sqrt{5}$ inclusive

Problem 7

Describe in words the region of $\mathbb{R}^3$ represented by the equation(s) or inequality:

$$1 \leq x^2 + z^2 \leq 9$$



The area in an infinite expanding cylinder in the y direction and w/ a cross section which is the area ave between 2 circles

Problem 8

Write inequalities to describe the following regions:

The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$:

$$r^2 < x^2 + y^2 + z^2 < R^2$$

Questions?