Recitation_7

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Recitation 7

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Problem 1

Find

$$\lim_{(x,y)\to(0,0)}\left|\frac{xy^4}{x^4+y^4}\right|$$

y axis: $\lim_{\substack{x=0\\y\to 0}} \frac{xy^4}{x^4+y^4} = \lim_{\substack{x=0\\y\to 0}} \frac{0}{0+y^4} = 0$

$0 \le \left|\frac{xy^4}{x^4+y^4}\right|$

x axis: $\lim_{\substack{y=0\\x\to 0}} \frac{xy^4}{x^4+y^4} = \lim_{\substack{y=0\\x\to 0}} \frac{0}{x^4} = 0$

$\frac{y^4}{x^4+y^4} \le 1 \to y^4 \le x^4+y^4$ pos $\to 0 \le x^4$

$x = y^m$: $\lim_{x=y^m\to 0} \frac{xy^4}{x^4+y^4} = \lim_{x=y^m\to 0} \frac{y^{4+m}}{y^{4m}+y^4} = 0$

$|x| \ge 0$

$|x|\left(\frac{y^4}{x^4+y^4}\right) \le |x|.$

$\left|\frac{xy^4}{x^4+y^4}\right| = |x|\left(\frac{y^4}{x^4+y^4}\right) \Rightarrow 0 \le \left|\frac{xy^4}{x^4+y^4}\right| \le |x|$

$\lim_{(x,y)\to(0,0)} |x| = 0$ (by squeeze thm.)

$\lim_{(x,y)\to(0,0)} 0 = 0$ → limit is

Problem 2

Find

$$\lim_{(x,y)\to(0,0)} \frac{x^4-4y^2}{x^2+2y^2}$$

y axis: $\lim_{\substack{x=0\\y\to 0}} \frac{x^4-4y^2}{x^2+2y^2} = \lim_{\substack{x=0\\y\to 0}} \frac{-4y^2}{2y^2} = -2$

x axis: $\lim_{y=0} \frac{x^4-4y^2}{} = \lim_{y=0} \frac{x^4}{} = \lim_{x\to 0} x^2 = 0$

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$x \to 0 \quad x^2+2y^2 \qquad x \to 0 \quad x^2$

$-2 \neq 0$, limit DNE

Problem 3

Find

$$\lim_{(x,y)\to(0,0)} \frac{xy^2 \cos y}{x^2+y^4}$$

x axis: $\lim\limits_{\substack{y=0 \\ x\to 0}} \frac{xy^2\cos y}{x^2+y^4} = \lim\limits_{x\to 0} \frac{0}{x^2} = 0$

$x = y^2$: $\lim\limits_{\substack{x=y^2 \\ y\to 0}} \frac{xy^2\cos y}{x^2+y^4} = \lim\limits_{(x,y)\to(0,0)} \frac{y^4\cos y}{2y^4} = \lim\limits_{y\to 0} \frac{\cos y}{2} = \frac{1}{2}$

$1 \neq 0$, Limit DNE

$(a+b)(a-b) = a^2-b^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\cdot\frac{\left(\sqrt{x^2+y^2+1}+1\right)}{\left(\sqrt{x^2+y^2+1}+1\right)} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1}+1\right)}{(x^2+y^2+1)-1^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1}+1\right)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2+1}+1$$

$$= \sqrt{0+0+1}+1 = 2$$

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$z^4+z^4+z^4$ ← $x=y=z^2$

z-axis: $\lim_{\substack{x=0\\y=0\\z\to 0}} \frac{xy+yz^2+xz^2}{x^2+y^2+z^4} = \lim_{z\to 0} \frac{0}{z^4} = 0$

$\lim_{\substack{x=y=z^2\\z\to 0}} \frac{xy+yz^2+xz^2}{x^2+y^2+z^4} = \lim_{z\to 0} \frac{z^4+z^4+z^4}{z^4+z^4+z^4} = 1$

$1 \neq 0$, Limit DNE

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$$w = \_\_$$

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left(x \cdot \frac{1}{y}\right) = \frac{1}{y}$$

$$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}\left(x \cdot \frac{1}{y}\right) = x \cdot \frac{-1}{y^2} = \frac{-x}{y^2}$$

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$$\rightarrow \quad (\ln(\_))' = \frac{1}{\_}$$

$$\frac{\partial w}{\partial x} = \frac{1}{x+2y+3z}$$

$$\frac{\partial w}{\partial y} = \frac{1}{x+2y+3z} \cdot 2 = \frac{2}{x+2y+3z}$$

$$\frac{\partial w}{\partial z} = \frac{1}{x+2y+3z} \cdot 3 = \frac{3}{x+2y+3z}$$

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$$y\cdot\left(\frac{\partial(xz)}{\partial x}\right) = \left(x\frac{\partial z}{\partial x} + z\frac{\partial x}{\partial x}\right)y$$

$$(e^z)' = e^z\cdot z' \qquad \frac{\partial}{\partial x}(\,)\ \frac{\partial}{\partial x}(\,)$$

$$e^z\cdot\frac{\partial z}{\partial x} = y\cdot\frac{\partial(xz)}{\partial x} = y\left(x\frac{\partial z}{\partial x} + z\frac{\partial x}{\partial x}\right) \to \text{product}$$

$$= yx\frac{\partial z}{\partial x} + yz = e^z\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x}(e^z - yx) = yz \qquad yz = e^z\frac{\partial z}{\partial x} - yx\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{yz}{e^z - yx}, \qquad \frac{\partial z}{\partial y} = \frac{xz}{e^z - xy} \qquad = \frac{\partial z}{\partial x}(e^z - yx)$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = \cos x\cos y, \qquad \frac{\partial z}{\partial y} = -\sin x\sin y$$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}}, \qquad \frac{dy}{dt} = \frac{-1}{t^2}$$

$$\frac{\partial z}{\partial t} = \frac{1}{2\sqrt{t}} \cos x \cos y + \frac{1}{t^2} \sin t \sin y$$

x^3

$$\left| \frac{x^3}{x^2+y^2} \right| \leq \frac{x^2}{x^2+y^2} |x| \qquad x^2 \leq$$

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