Recitation 5 Answers

Eric Li

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1 Problem 1

Determine if the following lines L_1, L_2 are parallel, skew, or intersecting.

$$L_1 = \frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3}, L_2 = \frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$$

First Step: Check normal vectors:

$$n_{L_1} = \langle 1, -2, -3 \rangle, n_{L_2} = \langle -3, 1, 3 \rangle$$

Since these two vectors are not scalar multiples of each other, it is clear that these lines are not parallel.

Next, we want to convert our lines to parametric form, as it will be easier to solve for equalities

$$L_1 : x = 2 + t, y = 3 - 2t, z = 1 - 3t, L_2 : x = 3 + s, y = -4 + 3s, z = 2 - 7s$$

We want to find t, s such that our coordinates are equal. If we cannot find these, then our lines do not intersect, meaning they are skew.

$$x = 2+t = 3+s \implies t = s+1, y = 3-2t = -4+3s \implies 2t+3s = 3+4 \implies 2(s+1)+3s = 7s \implies 5s = 5$$

Thus, we get that $s = 1, t = 2$. Thus, we can plug this in to our z equation to check for correctness. If our equations are equivalent, there is an intersecting point.

$$1 - 3t, 2 - 7s, 1 - (3)(2) = -5, 2 - 7(1) = -5$$

Since this equation is fulfilled, there is an intersecting point. Thus, our lines are intersecting.

2 Problem 2

Find the plane through the points: $P : (2, 1, 2), Q : (3, -8, 6), R : (-2, -3, 1)$
We want to find two vectors and find the cross product between them to find the normal vector of the plane.

$$\vec{PQ} = \langle 3 - 2, -8 - 1, 6 - 2 \rangle = \langle 1, -9, 4 \rangle$$

$$\vec{PR} = \langle -2 - 2, -3 - 1, 1 - 2 \rangle = \langle -4, -4, -1 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{bmatrix} i & j & k \\ 1 & -9 & 4 \\ -4 & -4 & -1 \end{bmatrix} = \langle 25, -15, 40 \rangle$$

Take the point P and this vector. Plane=

$$25(x - 2) - 15(y - 1) - 40(z - 2) = 0$$

Which simplifies to

$$5x - 3y - 8z - 10 + 3 + 16 = 5x - 3y - 8z + 9 = 0$$

3 Problem 3

Find the plane through the point $(3,1,4)$ and through the line of intersection between the planes $x+2y+3z=1, 2x-y+z=-3$.
The normal vectors of our planes are $\langle 1,2,3\rangle$ and $\langle 2,-1,1\rangle$, respectively. Next, we want to find the line of intersection. We note that the vector portion of the line will be the cross product of our two vectors.

$$\langle 1,2,3\rangle \times \langle 2,-1,1\rangle = \begin{bmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & -1 \end{bmatrix} = \langle 5,5,-5\rangle$$

Next, we want to find a point on the line of intersection. Fix $z=0$. Thus, we have that $x+2y=1, 2x-y=-3 \implies 2(1-2y)-y=-3, y=1, x=-1$ Thus, we have a point at $(-1,1,0)$. Next, we want to find a vector from this point to the point $(3,1,4)$.

$$\vec{v} = \langle 3-(-1), 1-1, 4-0\rangle = \langle 4,0,4\rangle$$

. From this, we can determine the normal vector of our plane.

$$\vec{n} = \langle 5,5,-5\rangle \times \langle 4,0,4\rangle = \begin{bmatrix} i & j & k \\ 5 & 5 & -5 \\ 4 & 0 & 4 \end{bmatrix} = \langle 20,40,-20\rangle$$

Thus, our plane will equal

$$20(x-3)+40(y-1)-20(z-4) = (x-3)+2(y-1)-(z-4) = x+2y-z-1=0$$

4 Problem 4

Find the plane through the point $(3,1,4)$ and perpendicular to the planes $2x+y-2z=2, x+3z=4$.
First, we take the normal vectors. The normal vectors of our planes are $\langle 2,1,-2\rangle$ and $\langle 1,0,3\rangle$, respectively. Next, we want to find the line of intersection. We note that this line is perpendicular to the plane we want to find, meaning the vector portion of this line is its normal vector.

$$\langle 2,1,-2\rangle \times \langle 1,0,2\rangle = \begin{bmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & 0 & 2 \end{bmatrix} = \langle 3,-8,-1\rangle$$

Next, we can find a plane through the point $(3,1,4)$ with the normal vector $\langle 3,-8,-1\rangle$. Plane:

$$3(x-3)-8(y-1)-1(z-4) = 3x-9-8y+8-z+4 = 3x-8y-z+3=0$$

5 Problem 5

Find the symmetric equations for the line of intersection of the planes $5x-2y-2z=1, 4x+y+z=6$.
First, we take the normal vectors. The normal vectors of our planes are $\langle 5,-2,-2\rangle$ and $\langle 4,1,1\rangle$, respectively. Next, we want to find the line of intersection. We note that the vector portion of this line is equal to the cross product of these two vectors.

$$\vec{n} = \langle 5,-2,-2\rangle \times \langle 4,1,1\rangle = \begin{bmatrix} i & j & k \\ 5 & -2 & -2 \\ 4 & 1 & 1 \end{bmatrix} = \langle 0,-13,13\rangle$$

Next, we want to find a point of intersection between the two planes. Fix $z = 0$. Thus, we have

$5x-2y = 1, 4x+y = 6 \implies y = 6-4x, 5x-2(6-4x) = 1, 5x-12+8x = 1, 13x = 13 \implies x = 1, y = 2$

Thus, the point $(1, 2, 0)$ is on both of the planes and is our line of intersection. Now, we can solve for the symmetric equations for this line:

$$\frac{y-2}{-13} = \frac{z}{13}, x = 1$$