Recitation 8

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Problems

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at the point, $f_x(x,y)$ is continuous

$f_y(x,y)$ is continuous

$f_x(x,y) = \ln(xy-5) + \frac{x}{xy-5}\cdot y \rightarrow xy > 5,\ xy \neq 5$

$f_y(x,y) = \frac{x^2}{xy-5} \rightarrow xy \neq 5$

$xy = 2\cdot 3 = 6 \pm 0.1 \Rightarrow 5.9,\ 6.1$ both have values continuous

since both f_x, f_y are continuous, $f(x,y)$ is differentiable at point

$L(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 + 2\ln(1) + \frac{2\cdot 3}{6-5}(x-2) + \frac{2^2}{6-5}(y-3) = 1 + 6(x-2) + 4(y-3)$

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$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}\cdot dy \rightarrow$ $dx = 0.05$, $dy = 2.1 - 2 = 0.1$

$\frac{\partial z}{\partial x} = 10x$, $\frac{\partial z}{\partial y} = 2y$

$dz = 10(1)(0.05) + 2(2)(.1) = .5 + .4 = .9$

$$\Delta z = 5(1.05)^2 + (2.1)^2 - \left(5(1)^2 + 2^2\right)$$

$$= \underline{0.9225}$$

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$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = -1, \quad \frac{dz}{dt} = 2$$

$$\frac{\partial w}{\partial x} = e^{y/z}, \quad \frac{\partial w}{\partial y} = x e^{y/z} \cdot \frac{1}{z}, \quad \frac{\partial w}{\partial z} = x e^{y/z} \cdot \left(\frac{-y}{z^2}\right)$$

$$\frac{dw}{dt} = e^{y/z}(2t) - \frac{x e^{y/z}}{z} + 2\left(x e^{y/z}\right)\left(\frac{-y}{z^2}\right)$$

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$$D_u f(x_0, y_0) = f_x(x_0, y_0)a + f_y(x_0, y_0)b$$, $\underline{\langle a, b\rangle \text{ unit vector}}$

$$u = \frac{v}{\|v\|} = \frac{\langle -6, 8\rangle}{\sqrt{36+64}} = \left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle$$

$$f_x(x,y) = e^x \sin y$$

$$f_y(x,y) = e^x \cos y$$

$$D_u f(0, \pi/3) = \left(e^0 \sin \frac{\pi}{3}\right)\left(\frac{-3}{5}\right) + \left(e^0 \cos \frac{\pi}{3}\right)\left(\frac{4}{5}\right) = \frac{\sqrt{3}}{2}\left(\frac{-3}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)$$

$$= \frac{4 - 3\sqrt{3}}{10}$$

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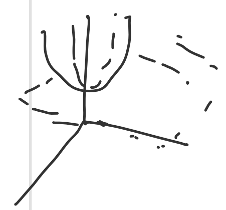

→ XX

xx

xx

any could be true

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$f_x(x,y) = 3x^2 - 3 + 3y^2$ → both to be 0

$f_y(x,y) = \underline{6xy}$

either $\underline{x=0}$, or $\underline{y=0}$ → Suppose $x=0$, $f_x(0,y) = 3y^2 - 3$

$y^2 = 1$, $y = \pm 1$ → $\underline{(0,1)}$, $\underline{(0,-1)}$ critical

Suppose $y=0$, $f_x(x,0) = 3x^2 - 3$ → $\underline{(1,0),\ (-1,0)}$ also critical

$D = f_{xx}(a,b) f_{yy}(a,b) - (f_{xy}(a,b))^2$

$f_{xx} = 6x$

$f_{yy} = 6x$

$f_{xy} = 6y$

↳ $D = 6x(6x) - (6y)^2 = \underline{36x^2 - 36y^2}$

$(0,1)$: $36(0) - 36 < 0$ ⇒ saddle point

$(0,-1)$: $36(0) - 36 < 0$ ⇒ saddle point

$(1,0)$: $36(1) - 36(0) > 0$, $f_{xx}(1,0) = 6(1) > 0$,

$(-1,0)$: $36(-1)^2 - 36(0) > 0$, $f_{xx}(-1,0) = 6(-1) < 0$

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$f_x(x,y) = \underline{e^x}\cos y$

$f_y(x,y) = -\underline{e^x}\sin y$ → want to find (x,y) such that both 0

$\cos y = 0$ → $y = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

$\sin y = 0$ → $y = k\pi$, $k \in \mathbb{Z}$

No values such that both 0, no critical points local min

, local max