Recitation 2

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Vector Properties

For vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$:

- u + v = v + u (commutativity of addition)
- u + (v+w) = (u+v) + w (associativity of addition)
- u + 0 = u (identity for addition)
- u + (-u) = 0 (inverse for addition)
- c(u+v) = cu + cv (distributivity for vector addition)
- (c+d)u = cu+du (distributivity for scalar addition)
- (cd)u = c(du) (associativity of scalar multiplication)
- 1u = u (identity for multiplication

Dot Product

Definition:

Let $\hat{a} = \langle a_1, a_2, a_3, ..., a_n \rangle$ and $\hat{b} = \langle b_1, b_2, b_3, ...b_n \rangle$. The dot product of these vectors is:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 ... a_n b_n$$

This value is a scalar!

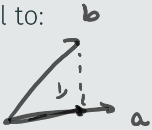
Projections

Definition:

The scalar projection of $\hat{b}$ onto $\hat{a}$, $comp_{\hat{a}}^{\hat{b}}$, is equal to:

$\hat{a} \cdot \hat{b} = |a||b| \cos\theta$

$$|\hat{b}| \cos\theta = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}|}$$



Definition:

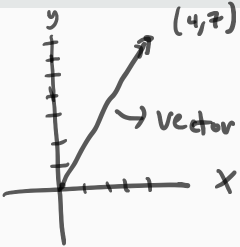
The vector projection of $\hat{b}$ onto $\hat{a}$, $proj_{\hat{a}}^{\hat{b}}$, is equal to:

$$comp_{\hat{a}}^{\hat{b}} = \left(\frac{\hat{a} \cdot \hat{b}}{|\hat{a}|}\right)\left(\frac{\hat{a}}{|\hat{a}|}\right) = \hat{a}\frac{\hat{a} \cdot \hat{b}}{|\hat{a}|^2}$$

Problems

Problem 1

What is the relationship between the point $(4,7)$ and the vector $\langle 4,7\rangle$? Illustrate with a sketch.



$\langle 4,7\rangle$ position vector of $(4,7)$

Find $\hat{a}+\hat{b}, 4\hat{a}+2\hat{b}, \|\hat{a}\|, and \|\hat{a}-\hat{b}\|$
$\hat{a}=4\hat{i}-3\hat{j}+2\hat{k}, \hat{b}=2\hat{i}-4\hat{k}$

$\hat{a}=\langle 4,-3,2\rangle$, $\hat{b}=\langle 2,0,-4\rangle$

$\hat{a}+\hat{b}=\langle 6,-3,-2\rangle$

$4\hat{a}=4\langle 4,-3,2\rangle=\langle 16,-12,8\rangle$, $2\hat{b}=2\langle 2,0,-4\rangle=\langle 4,0,-8\rangle$

$4\hat{a}+2\hat{b}=\langle 20,-12,0\rangle$

$|\hat{a}|=\sqrt{4^2+(-3)^2+(2)^2}=\sqrt{16+9+4}=\sqrt{29}$

$|\hat{a}-\hat{b}|=|\langle 2,-3,6\rangle|=\sqrt{2^2+(-3)^2+6^2}=\sqrt{4+9+36}=\sqrt{49}=7$

Find the unit vector that has the same direction as the given vector

$$\vec{a} = 8\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{u} = \frac{\vec{a}}{|\vec{a}|}$$

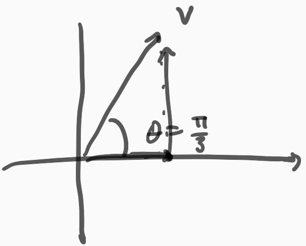
$$|\vec{a}| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{64 + 1 + 16} = \sqrt{81} = 9$$

$$\vec{u} = \frac{1}{9}(8\hat{i} - \hat{j} + 4\hat{k}) = \frac{8}{9}\hat{i} - \frac{1}{9}\hat{j} + \frac{4}{9}\hat{k}$$

$$= \left\langle \frac{8}{9}, \frac{-1}{9}, \frac{4}{9} \right\rangle$$

Problem 4

If v lies in the first quadrant and makes an angle $\frac{\pi}{3}$ with the positive $x-$axis and $|v| = 4$, find v in complement form



$|v| = 4$

x comp $= |v| \cos\theta$

y comp $= |v| \sin\theta$

$\langle |v|\cos\theta, |v|\sin\theta \rangle = \langle 4\cos\pi/3, 4\sin\pi/3 \rangle$

$= \langle \frac{4}{2}, \frac{4}{2}\sqrt{3} \rangle = \langle 2, 2\sqrt{3} \rangle$

Problem 5

Find $\hat{a} \cdot \hat{b}$

$\hat{a} = 4\hat{i} - 3\hat{j} + \hat{k}, \hat{b} = 2\hat{i} - \hat{k}$

$\hat{a} = \langle 4, -3, 1 \rangle$, $\hat{b} = \langle 2, 0, -1 \rangle$

$\hat{a} \cdot \hat{b} = 4(2) + (-3)(0) + 1(-1) = 8 - 1 = 7$

Problem 6

Find the scalar and vector projections of $\hat{b}$ onto $\hat{a}$

$\hat{a} = 3\hat{i} - 3\hat{j} + \hat{k}, \hat{b} = 2\hat{i} + 4\hat{j} - \hat{k}$

$\text{comp}_{\hat{a}} \hat{b} = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}|}$

$\hat{a} = \langle 3, -3, 1 \rangle$

$\hat{b} = \langle 2, 4, -1 \rangle$

$\hat{a} \cdot \hat{b} = 3(2) + (-3)4 + (1)(-1) = 6 - 12 - 1 = -7$

$|\hat{a}| = \sqrt{3^2 + (-3)^2 + 1^2} = \sqrt{9+9+1} = \sqrt{19}$

$\text{comp}_{\hat{a}} \hat{b} = \frac{-7}{\sqrt{19}}$, $\text{proj}_{\hat{a}} \hat{b} = \text{comp}_{\hat{a}} \hat{b} \frac{\hat{a}}{|\hat{a}|}$

$= \frac{-7}{\sqrt{19}} \left(\frac{1}{\sqrt{19}} \right) \hat{a} = \frac{-7}{19} \hat{a} = \underline{\left\langle \frac{-21}{19}, \frac{21}{19}, \frac{-7}{19} \right\rangle}$

Problem 7

Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & -4 \\ 10 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} x \end{bmatrix}$, solve, if possible, for

3×3 $\quad$ 3×1 $\quad$ 1×1

$$AB$$

$3\times 3 \cdot 3\times 1 \Rightarrow 3\times 1$

$$AB = \begin{bmatrix} 1\cdot 6 + 2\cdot 5 + 3\cdot 0 \\ 4\cdot 6 + 1\cdot 5 + (-4)0 \\ 10\cdot 6 + 1\cdot 5 + 2(0) \end{bmatrix} = \begin{bmatrix} 16 \\ 29 \\ 65 \end{bmatrix}$$

$$A: \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \quad B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \rightarrow AB - \begin{bmatrix} & \end{bmatrix}$$

Problem 8

Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & -4 \\ 10 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} x \end{bmatrix}$, solve, if possible, for

3×3 3×1

$$BA$$

$\underbrace{3\times 1 \cdot 3}\times 3$

can't solve

undefined

$AB \neq BA$

Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & -4 \\ 10 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} x \end{bmatrix}$, solve, if possible, for

3×1 1×1

$$CB$$

$CB: 1 \times \underbrace{1 \quad \cdot \quad 3}\times 1 \rightarrow$ DNE

$BC: \underline{3} \times 1 \cdot 1 \times \underline{1} \rightarrow$ solvable

$$\begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 6x \\ 5x \\ 0 \end{bmatrix}$$

Problem 10

Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & -4 \\ 10 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} x \end{bmatrix}$, solve, if possible, for

$$2A$$

$$2A = \begin{bmatrix} 2\cdot 1 & 2\cdot 2 & 2\cdot 3 \\ 4\cdot 2 & 1\cdot 2 & -4\cdot 2 \\ 10\cdot 2 & 1\cdot 2 & 2\cdot 2 \end{bmatrix}$$

Problem 10

Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & -4 \\ 10 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} x \end{bmatrix}$, solve, if possible, for

$$AB + B$$

$$AB = \begin{bmatrix} 16 \\ 29 \\ 65 \end{bmatrix}$$

$$B + C = \text{DNE}$$

$$AB + B = \begin{bmatrix} 16 \\ 29 \\ 65 \end{bmatrix} + \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 22 \\ 34 \\ 65 \end{bmatrix}$$