Recitation 12

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Table of contents

Intro

Introduction

- Homework due tonight!
- Going over problems
- Feel free to email me if you can't make it to OH

Tips on solving some trig integrals

Solving $\cos^k(\theta)$ and $\sin^k(\theta)$

- Even Case : K is even $\rightarrow \cos^2\theta = \frac{1+\cos(2\theta)}{2}$
- Odd Case

$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \rightarrow \cos^2\theta = 1-\sin^2\theta$

Even: $\int \cos^2\theta = \int \frac{1+\cos(2\theta)}{2}$

$\int \cos^4\theta = \int \left(\frac{1+\cos(2\theta)}{2}\right)^2 = \int \frac{1}{4}\left(1+2\cos(2\theta)+\cos^2(2\theta)\right)$

$\cos^2(2\theta) \rightarrow \frac{1+\cos(4\theta)}{2}$

Odd: $\int \cos^3\theta = \int \cos^2\theta \cdot \cos\theta = \int (1-\sin^2\theta)\cdot\cos\theta$

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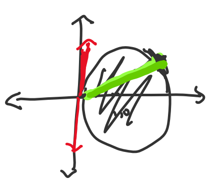


$$x^2 + y^2 - 2x = 0$$

$$(x-1)^2 + y^2 = 1$$

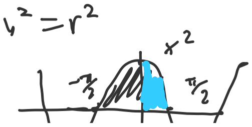
$$\iint_D \underbrace{(x^2+y^2)}_{} \; \underbrace{dA}_{\to\, r\,dr\,d\theta}$$

$x = r\cos\theta$
$y = r\sin\theta$

$$x^2 + y^2 = r^2$$

$$\int \int \underbrace{r^2} \cdot r\,dr\,d\theta$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$



$0 \le r \le 2\cos\theta \qquad x^2+y^2 = 2x \Rightarrow r^2 = 2r\cos\theta, \quad \underline{r = 2\cos\theta}$ $\qquad 8\int_0^{\pi/2} \cos \ldots$ ← here

$$\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r^3\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \left(\tfrac{1}{4}r^4\right)\Big|_0^{2\cos\theta} d\theta = \int_{-\pi/2}^{\pi/2} 4\cos^4\theta\,d\theta = 4\int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta$$

$$4\int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos(2\theta)}{2}\right)^2 d\theta = 4\cdot\frac{1}{4}\int_{-\pi/2}^{\pi/2} \left(1+2\cos(2\theta)+\frac{1+\cos(4\theta)}{2}\right)d\theta = 2\cdot\int_0^{\pi/2}\left(1+2\cos(2\theta)+\frac{1}{2}\cos(4\theta)+\frac{1}{2}\right)$$

$$2\cdot\left(\frac{3}{2}\theta + \cancel{\sin 2\theta} + \frac{1}{8}\cancel{\sin(4\theta)}\right)\Big|_0^{\pi/2} = 3\cdot\frac{\pi}{2} - 3\cdot 0 = \boxed{\frac{3\pi}{2}} \qquad \int_{-\pi/2}^{\pi/2}\ldots = 2\cdot\int_0^{\pi/2}$$

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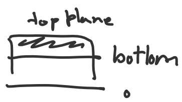
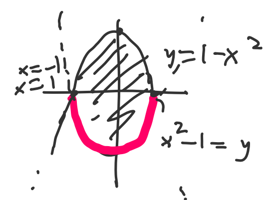


$P_1 : z = 2-x-y, \quad \iint_D (2-x-y)\,dA$

$P_2 : z = 2x+2y+10, \quad \iint_D (2x+2y+10)\,dA$



$$V = \left|\iint_D (2x+2y+10)\,dA - \iint_D (2-x-y)\,dA\right| = \iint_D (3x+3y+8)\,\underbrace{dA}_{dy\,dx}$$

$$= \int_{-1}^{1}\int_{x^2-1}^{1-x^2} (3x+3y+8)\,dy\,dx = \int_{-1}^{1} \left((3xy+8y)+\frac{3y^2}{2}\Big|_{x^2-1}^{1-x^2}\right)dx$$

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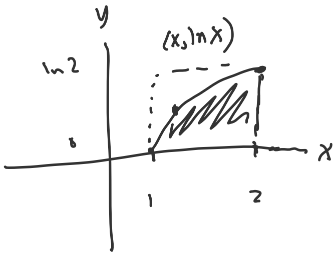

$0 \le y \le \ln 2$

$e^y \le x \le 2$

$$\int_0^{\ln 2}\int_{e^y}^{2} f(x,y)\,dx\,dy$$

left side: $y = \ln x \Rightarrow x = e^y$

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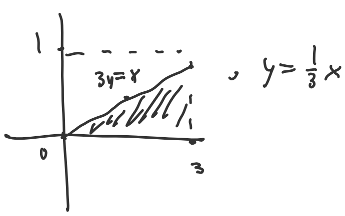

, $y = \frac{1}{3}x$

$$\int_0^3\int_0^{\frac{1}{3}x} e^{x^2}\,dy\,dx$$

$$= \int_0^3 e^{x^2}\left(y\,\Big|_0^{\frac{1}{3}x}\right) = \int_0^3 e^{x^2}\cdot\frac{1}{3}\overset{2x}{x}\,dx$$

$$= \int_0^3 \frac{1}{6}\left(e^{x^2}\cdot 2x\right)dx = \frac{1}{6}\int_0^3 e^{x^2}\cdot 2x\,dx$$

$$= \frac{1}{6}e^{x^2}\Big|_0^3 = \frac{1}{6}\left(e^9 - 1\right)$$

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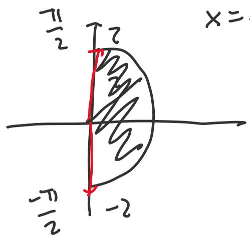

$x=\sqrt{4-y^2}$, $x^2+y^2=4$, but only pos x.

$$\iint_D e^{-x^2-y^2}\,dA$$

$$= \iint_D e^{-r^2}\,r\,dr\,d\theta,\quad \int_{-\pi/2}^{\pi/2}\int_0^2 e^{-r^2}\,r\,dr\,d\theta$$

$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$0\le r\le 2$

$u=r^2$, $du=2r\,dr$

$$\int_{-\pi/2}^{\pi/2}\int_0^4 \frac{1}{2}e^{-u}\,du\,d\theta = \int_{-\pi/2}^{\pi/2}\frac{1}{2}\,-e^{-u}\,d\theta\,\Big|_0^4 = \frac{1}{2}\int_{-\pi/2}^{\pi/2}(-e^{-4}+1)\,d\theta$$

$$= \frac{1}{2}(-e^{-4}+1)\,\theta\,\Big|_{-\pi/2}^{\pi/2} = \frac{1}{2}(-e^{-4}+1)\cdot\pi = \frac{\pi}{2}(-e^{-4}+1)$$

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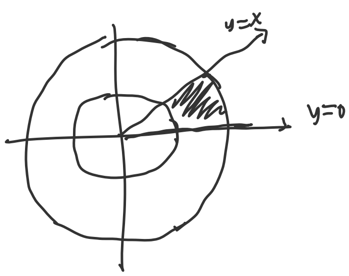
2



$$\int_0^{\pi/4}\int_1^2 \arctan\left(\frac{y}{x}\right) r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_1^2 \arctan\left(\overset{\tan\theta}{\frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}}\right) r\,dr\,d\theta$$

$0\le\theta\le\frac{\pi}{4}$

$1\le r\le 2$

$$= \int_0^{\pi/4}\int_1^2 \theta\,r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\theta\left(\frac{1}{2}r^2\Big|_1^2\right)d\theta = \int_0^{\frac{\pi}{4}}\left(2-\frac{1}{2}\right)\theta\,d\theta$$

$$= \frac{3}{2}\left(\frac{1}{2}\theta^2\right)\Big|_0^{\pi/4} = \frac{3}{2}\left(\frac{1}{2}\left(\frac{\pi}{4}\right)^2\right) = \frac{3\pi}{64}$$