Recitation 3

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9/17/2020

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Problem 1

Use the scalar triple product to show that the vectors $a = \langle 1, 4, -7 \rangle$, $b = \langle 2, -1, 4 \rangle$, and $c = \langle 0, -9, 18 \rangle$ are coplanar.

$a \cdot (b \times c) \rightarrow |a \cdot (b \times c)| =$ Volume of parallelepiped

$$a \cdot (b \times c) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 1\begin{vmatrix} -1 & 4 \\ -9 & 18 \end{vmatrix} - 4\begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} - 7\begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix}$$

$$= 1(-18+36) - 4\begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} - 7\begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix} = 18 - 4(36) + 7(18)$$

$$= 0$$

Problem 2

Find two unit vectors orthogonal to both $\langle 3,2,1\rangle$ and $\langle -1,1,0\rangle$

$A \times B$ orthogonal to A and B

$B \times A$ ortogonal to both

$$A \times B = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} = i\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ -1 & 0 \end{vmatrix} + k\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix}$$

$$|A \times B| = \sqrt{5^2+1+1} = \sqrt{27} = 3\sqrt{3}$$

$$= -i - j + 5k \rightarrow \langle -1,-1,5\rangle$$

Unit vector $= \frac{1}{3\sqrt{3}}\langle -1,-1,5\rangle$

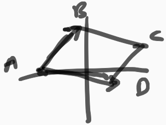
$$B \times A = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ 3 & 2 & 1 \end{vmatrix} = i\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} - j\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} + k\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = i + j - 5k = \langle 1,1,5\rangle$$

$A \times B = -B \times A$

Problem 3

Problem 3

Find the area of the parallelogram with vertices $A(-3,0), B(-1,3), C(5,2), D(3,-1)$



$|u \times v| = \text{area}$

$u = \langle -1-(-3),\ 3-0 \rangle = \langle 2, 3 \rangle$

$v = \langle 3-(-3),\ -1-0 \rangle = \langle 6, -1 \rangle$

$\begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 6 & -1 & 0 \end{vmatrix} = 0i - 0j + k \begin{vmatrix} 2 & 3 \\ 6 & -1 \end{vmatrix} = (-2-18)k = -20k$

area $= 20$

Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS.

$$P(-2,1,0), Q(2,3,2), R(1,4,-1), S(3,6,1)$$

$\vec{PQ} = \langle 2-(-2), 3-1, 2-0 \rangle = \langle 4, 2, 2 \rangle$

$\vec{PR} = \langle 1-(-2), 4-1, -1-0 \rangle = \langle 3, 3, -1 \rangle$

$\vec{PS} = \langle 3-(-2), 6-1, 1-0 \rangle = \langle 5, 5, 1 \rangle$

$$\hookrightarrow \left| \begin{matrix} 4 & 2 & 2 \\ 3 & 3 & -1 \\ 5 & 5 & 1 \end{matrix} \right| = 4\begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & 3 \\ 5 & 5 \end{vmatrix}$$

$$= 4(8) - 2(8) + 2(0) = 2(8) = 16$$