Computer lab

General information

The information in this laboratory can be solved individually or in groups. However, every student must submit their own lab report and include the names of those with whom they worked. The report must be submitted via the Computer Computing (Tasks) tool in Cambro, in the form of a PDF. In order to facilitate the correction, the student's name should be in the filename, for example "labreport_firstname_lastname.pdf". The report must be submitted via Cambro no later than the 28th of September by 16:00. If the laboratory report is not satisfactory, a new report must be submitted by the 2nd of November by 16:00. If the report still does not pass, there is the opportunity to do another computer lab at the next time the course is given, but it will then be according to the instructions applicable at the time of the course (i.e., then it may be a new computer lab with other tasks to be solved).

You can use the campus license for MATLAB to do the computer lab. Read more about running MATLAB on a private computer at: `http://www.math.umu.se/for-vara-studenter/matlab/`.

Format for the lab report

The assignment for the lab has ten steps/tasks. The lab report should be written in such a way that the student addresses each task in order and includes all MATLAB code and graphs. The MATLAB code should be in such shape that someone could copy and paste it into a brand new MATLAB prompt and it should work. In other words, if you type `clear` into MATLAB and then paste your code it should work. Graphs should have labels on the horizontal and vertical axes (use `xlabel` and `ylabel`). For this lab report, the `plot` command should be sufficient to generate graphs, so `plot(xdata,ydata,'o-')` should work well to plot ydata versus xdata. The final task is a written account of the student's progress and logic through the previous tasks. For the written material, please write clearly and in complete sentences. There is a fine balance for the amount of written material. It should be concise but also detailed enough that a grader can follow your reasoning. One tip is to let someone else read your report and see if they understand it.

Grading

To get a passing grade of G on the lab report all of the assignment must be finished. For the written material, the graders are looking for clear writing and argumentation. There are many ways to do some aspects of this lab. We are happy to embrace creative, even inexact solutions but we need to see some evidence of critical thought underlying these approaches. The purpose of this lab is to gain experience computationally solving differential equations and wrestling with some of the details.

Lab

Background

You have a scientist friend who is studying microbial food webs. For their research, they put ten different microbes in a beaker and watch what happens. They have chosen a strange set of microbes such that each microbe feeds on two others. They come to you and the following conversation ensues...

SCIENTIST: I found something really interesting in my microbial food webs!
YOU: That's fantastic! What did you find?
SCIENTIST: After a week in a beaker, I found that all ten types of microbes were still alive! Not only that, but their populations were cycling—beautiful oscillations like you were showing me from your differential equations class. How cool is that?
YOU: That is cool! Hmmmm. But wait, what did you expect to happen?
SCIENTIST: I thought they would all go extinct or maybe there would only be one or two left. I just wish there was some way to quantify how surprising this result is.
YOU: Maybe I can help. So you have this beaker where everything is well-mixed?
SCIENTIST: Yes, that's right.
YOU: And there are ten types of microbes and each one eats another two?
SCIENTIST: Yes.
YOU: And you put some random amount of each microbe in the beaker at the start?
SCIENTIST: Yes.
YOU: I think we can model that with a set of differential equations like the ones used for predator-prey interactions.
SCIENTIST: Great!
YOU: Hmmm. Let me think about what else I need to know—oh do the organisms only get food from eating each other?
SCIENTIST: Yes.
YOU: Do they die from anything other than predation?
SCIENTIST: No. We do not culture them for long and we use media that gives them just enough to sustain life but they can only reproduce through eating other organisms.
YOU: Super! Your situation sounds strangely perfect for modeling.
SCIENTIST: We also know the ranges for predation rates based on interactions between the microbes. Is that of any use?
YOU: Oh my gosh, yes!
SCIENTIST: And we know the ranges for the initial numbers of the microbes.
YOU: We have everything we need! So, I have a plan. We will create a large set of random food webs that have the same basic properties as yours and solve the resulting differential equations. We can then see how many times they result in all ten microbes surviving.
SCIENTIST: That sounds great. I am sure glad you took that differential equations course.
YOU: Me too!

Excited by your plan you start to work. You first obtain a set of 1,000 randomized microbial food webs with similar properties to the one your scientist friend used. It is available in a MATLAB file called `randwebs.mat` , which when loaded `load randwebs` creates a variable called `randwebscell`. The variable `randwebscell` is a cell type variable in which each i^{th} entry contains a matrix that corresponds to the predator-prey interactions of the microbial food web. For example, you can obtain the third interaction matrix (i.e. random food web) by using the command `A=randwebscell{3};` An interaction matrix A

can be used to model the following predator-prey differential equation:

$$
\begin{aligned}
\frac{\partial x_1}{\partial t} &= x_1 * \sum_{i=1}^{10} A_{1,i} x_i \qquad (1)\\
\frac{\partial x_2}{\partial t} &= x_2 * \sum_{i=1}^{10} A_{2,i} x_i \\
&\vdots \\
\frac{\partial x_{10}}{\partial t} &= x_{10} * \sum_{i=1}^{10} A_{10,i} x_i
\end{aligned}
$$

In the above equations, x is a vector that tracks the abundance of the ten microbial species. The notation x_i simply means the ith element of x, that is the abundance of the ith microbial species. The A matrix is simply one of the interaction matrices.

You also can load a file `randinits.mat` that when loaded creates a variable called `randinitscell` This is a cell that contains 1,000 initial conditions (i.e. initial vectors x denoted $x(0)$) that correspond to the interaction matrices specified in `randwebscell` With this data you write down an itemized list of the steps you need to take.

Assignment

1. Write MATLAB code that solves the differential equations in Eq. 1 for a specific interaction matrix `A` and initial condition $x(0)$. Use the first ones in the loaded data, i.e. `randwebscell{1};` and `randinitscell{1};` The idea is you will use this code in step 3 for each of the 1,000 sets of interaction matrices and initial conditions. Hint: Use `ode45` and make sure to use the `NonNegative` option using `odeset` and `ode45`. The MATLAB documentation will help with this.

2. Write MATLAB code that can examine the solution to the differential equation and determine how many species are still alive.

3. Write MATLAB code that performs steps 1 and 2 for all 1,000 sets of interaction matrices and initial conditions. For each solution, compute the number of species still alive.

4. Write MATLAB code that counts how often 0,1,. . . ,10 species survive out of the 1,000 randomized food webs. Hint: `histc()` can help

5. Submit a plot that shows an example in which all ten species survive by the end. The plot should have time on the horizontal axis and the trajectories of ten different microbial species on the vertical axis.

6. Answer the question: is it more likely that all ten species survives or five species survive? Use the results of step 4 to quantify this answer, i.e. "I observe that z times out of 1,000 that ten species survive and y times out of 1,000 five species survive. I conclude that . . . "

7. Answer the question: is it more likely that all five species survives or no species survive? Use the results of step 4 to quantify this answer, i.e. "I observe that z times out of 1,000 that five species survive and y times out of 1,000 no species survive. I conclude that . . . "

8. How do you know that you solved the differential equations for a long enough time scale? Determine a way to show that your answer is valid and do it. Hint: This requires looking at some plots and thinking. There is no one right answer but there are some quick computational approaches.

9. Write MATLAB code that computes how many microbes are oscillating by the end of a differential equation solution. Hint: This requires looking at some plots and thinking. There is no one right answer but there are some quick computational approaches.

10. Write a clear description of what you did during this lab report and what your thinking was for each step. This is particularly important for steps 8 and 9. Ideally, it should be a 3-5 paragraphs that describes your approach to the problem, the challenges you found, and how you tried to solve them.