UMEÅ UNIVERSITY

Differential equations
September 2021
Final Exam

Information about the exam:

- For each question, write only your answers in the space below the question on the exam sheet and turn in the exam sheet. Answers elsewhere may not be graded. Correct answers get full credit. Incorrect answers might earn partial credit if the work is shown clearly on attached pages.
- No calculators or books or other aids are allowed.
- When explaining your answers please write complete, logical sentences. Please write in English since it is the language of the textbook and this course offering.
- The exam is worth 60 points. To pass the exam, you need at least 31 points.
- Here are some useful Laplace transforms: $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \mathcal{L}\{1\} = \frac{1}{s}, \mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}, \mathcal{L}\{\delta(t-c)\} = e^{-cs}$

Final Exam questions

1. (6 points) Find the values of b and c in the 2nd order differential equation: $y'' + by' + cy = 0$, whose solution is $y = e^{x^2}$, and that is guaranteed to have a unique solution at x=0.

2. (6 points) Solve the differential equation: $y'' - y' - 12y = \delta(t-4), y'(0) = 0, y(0) = 1$

3. (6 points) Convert the differential equation $y'' + 4t^2y' + ty = 7t$ into a system of differential equations $\frac{dx_1}{dt}, \frac{dx_2}{dt}$ featuring the variables x_1, x_2.

4. (6 points) Find the critical points of the system $\left\{\frac{dx}{dt} = x - rxy,\ \frac{dy}{dt} = 2y - xy\right\}$ where x and y measure concentrations of organisms and r is a parameter. Also, classify each critical point based on its local dynamics (i.e. stable node, unstable spiral point, etc).

5. (6 points) What is the solution to $3y'' + 6y' - 9y = 4e^{-2t}$, if $y(0) = 14/9, y'(0) = -10/9$?

6. (6 points) Write the recurrence relationships for the coefficients a_0 and a_1 found in the power series solution to the differential equation: $y'' - xy' - y = 0$, where $x_0 = 0$.

7. (6 points) Solve the differential equation: $y' + t^2 y = 4e^{-t^3/3}, y(0) = 2$.

8. (6 points) Find the Fourier cosine series for $f(x) = \begin{cases} 0, \text{if } 0 \leq x < 1 \\ 1, \text{if } 1 \leq x < 2 \end{cases}$

9. (6 points) Determine whether the method of separation of variables can be used to replace the following partial differential equation: $t\frac{\partial^2 u}{\partial x^2} = 3x\frac{\partial u}{\partial t}$ by a pair of ordinary differential equations. If so, write the equations below. If not, explain why not.

10. (6 points) Suppose $y_1 = te^t$ and $y_2 = e^{-2t}$ are solutions to a 2nd order linear differential equation. Write the value of the Wronskian below and list all ranges of initial values of t for which a unique solution can be produced combining y_1 and y_2.