Notes about the exam:

- For each question, write your answer in the box on the exam sheet and turn in the exam sheet. Answers elsewhere will not be graded. You get full credit for correct answers. If you do not have the correct answer, you may get partial credit if you show your work clearly on other attached pages.
- There are a total of 50 points on the exam. A score between 0 and 29 points corresponds to a U, 30-37 is a 3, 38-44 is a 4, and 45-50 is a 5.
- Helpful note: $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}F(s)$, where $F(s) = \mathcal{L}\{f(t)\}$.

1. (4 pts) Write a fundamental set of solutions for: $\frac{\partial^2 y}{\partial t^2} + \mu^2 y = 0$, where $y(0) = 0$, $y(10) = 0$, and $\mu \in \mathbb{R}$.

2. (4 pts) Write the general solution of the differential equation: $\frac{\partial^3 y}{\partial t^3} - \frac{\partial^2 y}{\partial t^2} + 9\frac{\partial y}{\partial t} = 9$. If possible, write a set of initial conditions so that a unique solution exists. If it is not possible, say why it is not.

3. (4 pts) Find a polynomial series solution to $x\frac{\partial y}{\partial x} - y = 2$. Write down the first 6 terms corresponding to x^n for $n = 0, 1, 2, \ldots, 5$.

4. (4 pts) Solve the differential equation : $\frac{\partial y}{\partial t} = 1 - t + 4y$, where $y(0) = 1$.

5. (10 pts) Compute the critical points and determine their local stability. Describe each critical point using terms such as stable node, saddle, unstable spiral, etc.

$$\frac{\partial A}{\partial t} = 2A^2 - B$$
$$\frac{\partial B}{\partial t} = A^2 + 2B - 5$$

6. (8 pts) Solve the problem: $2\frac{\partial^2 y}{\partial t^2} + \frac{\partial y}{\partial t} - 1 = g(t)$, where $y(0) = 0$, $\frac{\partial y}{\partial t}(0) = 0$, and $g(t)$ is described as follows.

$$g(t) = \begin{cases} 0, & \text{if } 0 \le t < 1 \\ 1, & \text{if } 1 \le t < 2 \\ 2, & \text{if } 2 \le t < 3 \\ 1, & \text{if } 3 \le t < 4 \\ 0, & 4 \le t \end{cases}$$

7. (8 pts) Use separation of variables to solve: $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ where $0 \le x \le 10$, $t \ge 0$, $u(10,t) = 0$, $u(0,t) = 0$, and $u(x,0) = 4$.

8. (8 pts) Write a second order linear differential equation whose solution is: $y(t) = 2e^{-2t} + e^{-3t} + \cos(2t)$. Use $t = 0$ for the initial condition(s). Also note which part of the solution is the solution to the homogeneous problem and which part corresponds to the particular solution.