

## Homework 12 - Solution

### R-26.5

Canonical/Slack Form:

$$\begin{array}{ll} \text{Maximize:} & z = x_1 + x_2 \\ \text{Subject to:} & 3x_1 + 5x_2 + x_3 = 77 \\ & 7x_1 + 2x_2 + x_4 = 56 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array} \quad \begin{array}{l} x_1, x_2 \text{ are free;} \\ x_3, x_4 \text{ are basic} \end{array}$$

$$\begin{array}{ll} \text{Maximize:} & z = 8 - 5/7x_2 + 1/7x_4 \\ \text{Subject to:} & 29/7x_2 + x_3 - 3/7x_4 = 53 \\ & x_1 + 2/7x_2 + 1/7x_4 = 8 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{Maximize:} & z = 497/29 + 5/29x_3 + 2/29x_4 \\ \text{Subject to:} & x_2 + 7/29x_3 - 3/29x_4 = 371/29 \\ & x_1 - 2/29x_3 + 5/29x_4 = 126/29 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

The optimal solution value is  $Z = 497/29$ , with  $x_1 = 126/29$  and  $x_2 = 371/29$ .

### R-26.7

Standard Form:

$$\begin{array}{ll} \text{Maximize:} & z = -3y_1 - 2y_2 - y_3 \\ \text{Subject to:} & 3y_1 - y_2 - y_3 \leq -1 \\ & -2y_1 - y_2 + y_3 \leq -2 \\ & y_1, y_2, y_3 \geq 0 \end{array}$$

Canonical/Slack Form:

$$\begin{array}{ll} \text{Maximize:} & z = -3y_1 - 2y_2 - y_3 \\ \text{Subject to:} & -3y_1 + y_2 + y_3 - y_4 + y_6 = 1 \\ & 2y_1 + y_2 - y_3 - y_5 + y_7 = 2 \\ & y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0 \end{array} \quad \begin{array}{l} y_1, y_2, y_3 \text{ are free;} \\ y_4, y_5, y_6, y_7 \text{ are basic} \end{array}$$

$$\begin{array}{ll} \text{Maximize:} & z = -3 + y_1 - 2y_2 + y_4 + y_5 \\ \text{Subject to:} & -3y_1 + y_2 + y_3 - y_4 + y_6 = 1 \\ & 2y_1 + y_2 - y_3 - y_5 + y_7 = 2 \\ & y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{Maximize:} & z = -1 + -5y_1 + 2y_3 - y_4 + y_5 + 2y_6 \\ \text{Subject to:} & -3y_1 + y_2 + y_3 - y_4 + y_6 = 1 \\ & 5y_1 - 2y_3 + y_4 - y_5 - y_6 + y_7 = 1 \\ & y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0 \end{array}$$

Maximize:  $z = y_6 + y_7$   
 Subject to:  $y_2 - 1/5y_3 - 2/5y_4 - 3/5y_6 + 2/5y_7 = 8/5$   
 $y_1 - 2/5y_3 + 1/5y_4 - 1/5y_5 - 1/5y_6 + 1/5y_7 = 1/5$   
 $y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0$

Maximize:  $z = -19/5 + 13/5y_3 + 1/5y_4 + 9/5y_5$   
 Subject to:  $y_2 - 1/5y_3 - 2/5y_4 - 3/5y_5 = 8/5$   
 $y_1 - 2/5y_3 + 1/5y_4 - 1/5y_5 = 1/5$   
 $y_1, y_2, y_3, y_4, y_5 \geq 0$

The optimal solution value is  $z = -19/5$ , with  $x_1 = 1/5$ ,  $x_2 = 8/5$  and  $x_3 = 0$ .

### **R-26.9**

The constraints that bind the feasible region are:

- $x, y \geq 0$
- $x \leq 8$
- $y \leq 9$
- $3x + 5y - 54 \leq 0$